HW1 Report

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Part1

fft

The Fast Fourier Transform (FFT) is an algorithm used to efficiently compute the Discrete Fourier Transform (DFT) of a sequence or signal.
By analyzing the frequency components of a signal, the DFT and FFT enable various applications such as signal processing, image processing, audio analysis, and data compression.The FFT algorithm divides the input sequence into smaller sub-sequences and recursively applies the DFT to each sub-sequence.

Gaussian Noise

Gaussian noise, also known as white noise or Gaussian white noise, is a type of random signal that follows a Gaussian distribution. It is characterized by its statistical properties, including a mean of zero and a constant variance.
In Gaussian noise, the values of the noise samples are independent and identically distributed (i.i.d.), meaning that each sample is unrelated to the others and follows the same Gaussian distribution. It is characterized by two parameters: the mean, which represents the central tendency of the distribution, and the standard deviation, which determines the spread or variability of the distribution. The standard deviation of the Gaussian noise determines the amplitude or intensity of the noise.

Part2

LIF model

Membrane Time Constant (τ_m): The membrane time constant represents the time it takes for the neuron's membrane potential to reach approximately 63.2% of the difference between the resting potential and the threshold potential. It determines the rate at which the membrane potential changes.

Membrane Potential (V): The membrane potential represents the electrical potential difference across the neuron's cell membrane. It changes over time based on the input current and the neuron's properties.

Resting Potential (E_L): The resting potential is the membrane potential when the neuron is at rest and not receiving any input. It is the equilibrium potential of the neuron in the absence of any stimulation.

Leak Conductance (g_L): The leak conductance represents the conductance of the ion channels responsible for the passive leakage of ions across the neuron's membrane. It determines the rate at which the membrane potential leaks or decays towards the resting potential.

Input Current (I): The input current represents the sum of all incoming currents into the neuron. It can be constant or time-varying and determines the behavior of the neuron.

By altering these parameters numerically, you can observe different effects on the behavior of the LIF neuron. Here are some examples:

Changing the membrane time constant (τ_m) will affect the rate at which the membrane potential changes. A larger time constant will result in slower changes, while a smaller time constant will lead to faster changes.

Modifying the resting potential (E_L) will shift the baseline of the membrane potential. A more negative resting potential will make it harder for the neuron to reach the threshold and fire.

Adjusting the leak conductance (g_L) will change the rate at which the membrane potential leaks or decays towards the resting potential. A higher leak conductance will result in faster leakage, while a lower conductance will slow down the leakage.

Altering the input current (I) will directly influence the membrane potential. Stronger input currents will increase the likelihood of firing, while weaker currents may not reach the threshold.

By systematically varying these parameters, you can explore how different aspects of the LIF model affect the neuron's firing behavior, such as firing rate, spike timing, and responsiveness to input stimuli. This analysis helps in understanding the dynamics of individual neurons and their contribution to neural network behavior.

Euler's Method

Euler's method is a numerical approximation technique used to solve ordinary differential equations (ODEs). It is named after the Swiss mathematician Leonhard Euler, who developed the method in the 18th century. The abstract of Euler's method can be summarized as follows: Euler's method is a simple and straightforward numerical algorithm for approximating the solution of an ordinary differential equation. It involves dividing the interval of interest into small subintervals and approximating the derivative of the function at each subinterval using the slope of a tangent line. By iteratively applying this process, the method allows us to estimate the function's values at different points along the interval.

The key steps of Euler's method are as follows: 1. Choose an initial value for the function at a given point. 2. Divide the interval into smaller subintervals. 3. Approximate the derivative of the function at each subinterval using the slope of a tangent line. 4. Use the derivative approximation to update the function's value at each subinterval. 5. Repeat the process for all subintervals until the desired accuracy is achieved or the endpoint of the interval is reached.

Euler's method provides a simple and intuitive approach to numerically solve differential equations, making it a valuable tool in various scientific and engineering fields. However, it is important to note that Euler's method has limitations, such as its tendency to accumulate errors over long intervals or when dealing with stiff equations. Therefore, it is often used as a starting point for more sophisticated numerical methods that offer higher accuracy
and stability.

Hodgkin-Huxley Model

The Hodgkin-Huxley model is a mathematical model that describes the behavior of action potentials in neurons. It consists of four differential equations that govern the dynamics of the membrane potential (V) and the activation/inactivation variables (h, m, and n) for the sodium (Na) and potassium (K) ion channels. Let's describe the parameters in these equations:

1. C_M: Membrane Capacitance C_M represents the capacitance of the neuronal membrane. It determines the ability of the membrane to store electrical charge and influences the rate of change of the membrane potential.
2. g_{Na}: Sodium Conductance gNa represents the conductance of the sodium ion channels. It determines the flow of sodium ions across the membrane and influences the depolarization phase of the action potential.
3. V_{Na}: Sodium Equilibrium Potential V_{Na} represents the equilibrium potential for sodium ions. It is the membrane potential at which there is no net flow of sodium ions across the membrane.
4. g_K: Potassium Conductance g_K represents the conductance of the potassium ion channels. It determines the flow of potassium ions across the membrane and influences the repolarization phase of the action potential.
5. V_K: Potassium Equilibrium Potential V_K represents the equilibrium potential for potassium ions. It is the membrane potential at which there is no net flow of potassium ions across the membrane.
6. g_l: Leak Conductance - gl represents the conductance of the leak channels. It accounts for the passive leakage of ions across the membrane and influences the resting membrane potential.
7. V_l: Leak Equilibrium Potential V_l represents the equilibrium potential for the leak channels. It is the membrane potential at which there is no net flow of ions through the leak channels.
8. I: Applied Current I represents the applied current or external stimulus. It can be constant or time-varying and determines the input to the neuron.
9. α_h(V), β_h(V), α_m(V), β_m(V), α_n(V), β_n(V): Rate Constants α_h, β_h, α_m, β_m, α_n, and β_n are voltage-dependent rate constants that determine the opening and closing rates of the ion channels. They are functions of the membrane potential (V) and control the activation and inactivation dynamics of the sodium and potassium channels.

By adjusting these parameters, the Hodgkin-Huxley model can simulate the behavior of action potentials and provide insights into the electrical activity of neurons. The model has been instrumental in understanding the mechanisms underlying neuronal excitability and has paved the way for further research in neuroscience.

* $C_M\, dV/\, dt = -g_{Na}(V - V_{Na}) - g_k(V - V_K) - g_l(V - V_l) + I$
(V : MembranePotential) :This equation describes the dynamics of the membrane potential(V)
* $dh/\, dt = \alpha_h(V)(1 - h) - \beta_h(V)h$ (h: Na Inactivation) : This equation describes the dynamics of the inactivation variable (h) for the sodium channels.
* $dm/\, dt = \alpha_m(V)(1 - m) - \beta_m(V)m$ (m: Na Activation) : This equation describes the dynamics of the activation variable (m) for the sodium channels.

* $dn/dt = \alpha_n(V)(1-n)-\beta_n(V)n$ (n: K Activation) : This equation describes the dynamics of the activation variable (n) for the potassium channels