Homework 4

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Exercise 1

$MSE = \frac{1}{m}\sum_{j=1}^{m}\left(A_j - P_j\right)^2 = \frac{1}{m}\sum_{j=1}^{m}\left[A_j - P_j + \bar{A} - \bar{A} + \bar{P} - \bar{P}\right]^2$
$= \frac{1}{m}\sum_{j=1}^{m}\left[\left(A_j - \bar{A}\right) - \left(P_j - \bar{P}\right) + (\bar{A} - \bar{P})\right]^2 \quad = \frac{1}{m}\sum_{j=1}^{m}\left[\left(A_j - \bar{A}\right)^2 + \left(P_j - \bar{P}\right)^2 + (\bar{A} - \bar{P})^2 - 2\left(A_j - \bar{A}\right)\left(P_j - \bar{P}\right) - 2\left(P_j - \bar{P}\right)(\bar{A} - \bar{P}) + 2\left(A_j - \bar{A}\right)(\bar{A} - \bar{P})\right]$
$= \frac{1}{m}\sum_{j=1}^{m}\left(A_j - \bar{A}\right)^2 + \frac{1}{m}\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2 + \frac{m(\bar{A} - \bar{P})^2}{m} - \frac{2}{m}\sum_{j=1}^{m}\left(A_j - \bar{A}\right)\left(P_j - \bar{P}\right)$ (*)

Let $\frac{\sum_{j=1}^{m}\left(A_j - \bar{A}\right)^2}{m} = S_A^2,\ \frac{\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2}{m} = S_P^2.$

(*) $= S_A^2 + S_P^2 + (\bar{A} - \bar{P})^2 - \frac{2}{m}\sum_{j=1}^{m}\left(A_j - \bar{A}\right)\left(P_j - \bar{P}\right)$ (**)

Because $A_i \rightarrow y$ and $P_j \rightarrow x$,
$\widehat{\beta_1} = \frac{Sxy}{Sxx} = \frac{\sum_{j=1}^{m}(A_j - \bar{A})(P_j - \bar{P})}{\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2}$, so that
$\sum_{j=1}^{m}\left(A_j - \bar{A}\right)\left(P_j - \bar{P}\right) = \widehat{\beta_1}\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2.$

(**) $= S_A^2 + S_P^2 + (\bar{A} - \bar{P})^2 - \frac{2}{m}\widehat{\beta_1}\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2$
$= S_A^2 + S_P^2 + (\bar{A} - \bar{P})^2 - 2\widehat{\beta_1}S_P^2$
$= S_A^2 + S_P^2 + (\bar{A} - \bar{P})^2 - 2\widehat{\beta_1}S_P^2$
$= S_A^2 + S_P^2 + (\bar{A} - \bar{P})^2 - 2\widehat{\beta_1}S_P^2 + \widehat{\beta_1}S_P^2 - \widehat{\beta_1}S_P^2$
$= S_A^2 + \left(1 - \widehat{\beta_1}\right)S_P^2 - \widehat{\beta_1}S_P^2 + (\bar{A} - \bar{P})^2$ (***)

And $\mathrm{R}^2 = \frac{\mathrm{SSR}}{\mathrm{SST}} = \frac{\mathrm{Sxy*Sxy}}{\mathrm{Sxx*Syy}} = \frac{\left(\widehat{\beta_1}\sum_{j=1}^{m}(P_j - \bar{P})\right)^2}{\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2\sum_{j=1}^{m}\left(A_j - \bar{A}\right)^2} = \frac{\widehat{\beta_1}^2\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2}{\sum_{j=1}^{m}\left(A_j - \bar{A}\right)^2}$
$\rightarrow \sum_{j=1}^{m}\left(A_j - \bar{A}\right)^2 * R^2 = \widehat{\beta_1}^2\sum_{j=1}^{m}\left(P_j - \bar{P}\right)^2$
$\rightarrow S_A^2 * R^2 = \widehat{\beta_1}^2 * S_P^2.$

(***) $= S_A^2 + \left(1 - \widehat{\beta_1}\right)S_P^2 - S_A^2 * R^2 + (\bar{A} - \bar{P})^2$
$= (\bar{A} - \bar{P})^2 + \left(1 - \widehat{\beta_1}\right)S_P^2 + (1 - R^2)S_A^2.$ QED

```r
a1 <- read.csv("/Users/user/Desktop/Yonsei/Junior/3-2/Statistical Models in Finance/stockData_hw4_1.csv", sep=",", header=TRUE)
a2 <- read.csv("/Users/user/Desktop/Yonsei/Junior/3-2/Statistical Models in Finance/stockData_hw4_2.csv", sep=",", header=TRUE)
a3 <- read.csv("/Users/user/Desktop/Yonsei/Junior/3-2/Statistical Models in Finance/stockData_hw4_3.csv", sep=",", header=TRUE)

r1 <- (a1[-1,3:ncol(a1)]-a1[-nrow(a1),3:ncol(a1)])/a1[-nrow(a1),3:ncol(a1)]
r2 <- (a2[-1,3:ncol(a2)]-a2[-nrow(a2),3:ncol(a2)])/a2[-nrow(a2),3:ncol(a2)]
r3 <- (a3[-1,3:ncol(a3)]-a3[-nrow(a3),3:ncol(a3)])/a3[-nrow(a3),3:ncol(a3)]

covmat1 <- var(r1)
covmat2 <- var(r2)
covmat3 <- var(r3)

beta1 <- covmat1[1,-1] / covmat1[1,1]
beta2 <- covmat2[1,-1] / covmat2[1,1]
beta3 <- covmat3[1,-1] / covmat3[1,1]

PRESS1 <- sum((beta2-beta3)^2) / 30

U1 <- ( mean(beta3) - mean(beta2) )^2

q1 <- lm(beta3 ~ beta2)
Sp12 <- (29/30)*var(beta2)
U2 <- (1-q1$coef[2])^2*Sp12

Sa2 <- (29/30)*var(beta3)
rap12 <- ( cor(beta2,beta3) )^2
U3 <- (1-rap12)*Sa2

U1+U2+U3 ; PRESS1
```

```
> U1+U2+U3 ; PRESS1
    beta2
0.6624729
[1] 0.6624729
```

Exercise 2

a. $\frac{1}{m}\sum_{j=1}^{m}\left(A_j - P_j\right)^2 = (\bar{A}-\bar{P})^2 + (1-\beta_1)S_P^2 + (1-R^2)S_A^2.$

$= (\bar{A}-\bar{P})^2 + \left(1 - \frac{\sum_{j=1}^{m}(A_j-\bar{A})(P_j-\bar{P})}{\sum_{j=1}^{m}(P_j-\bar{P})^2}\right)\frac{\sum_{j=1}^{m}(P_j-\bar{P})^2}{m-1} + \left(1 - \frac{\beta_1^2\sum_{j=1}^{m}(P_j-\bar{P})^2}{\sum_{j=1}^{m}(A_j-\bar{A})^2}\right)$

$= (\bar{A}-\bar{P})^2 + (1-\beta_1)\frac{V(\beta_1)}{48} + \left(1 - \frac{\beta_1^2 V(\beta_1)}{V(\beta_2)}\right)$, (*) where

$\widehat{\beta_1} = \frac{\sum_{j=1}^{m}(A_j-\bar{A})(P_j-\bar{P})}{\sum_{j=1}^{m}(P_j-\bar{P})^2} = \frac{\sum_{j=1}^{m}(A_j-\bar{A})(\widehat{A_\iota}-\bar{A})}{\sum_{j=1}^{m}P_j^2-2\bar{P}\sum_{j=1}^{m}P_j+30\overline{P^2}} = \frac{4.299189}{51.70104-2*\frac{32.44349}{30}*32.44349+\left(\frac{32.44349}{30}\right)^2} = 0.2587529.$

(*) $= \left(\frac{32.26206}{30} - \frac{32.44349}{30}\right)^2 + (1-0.2587529)\frac{0.4558062}{48} + \left(1 - \frac{0.2587529^2*0.4558062}{0.3235921}\right) = 0.9127665.$

b. Vasicek $\rightarrow \widehat{\beta_2} = \frac{S_{\beta_1}^2}{S_{\overline{\beta_1}}^2+S_{\beta_1}^2}\overline{\beta_1} + \frac{S_{\overline{\beta_1}}^2}{S_{\overline{\beta_1}}^2+S_{\beta_1}^2}\beta_2.$

$\rightarrow \sum_{j=1}^{m}\left(A_j-\bar{A}\right)\left(P_j-\bar{P}\right)\downarrow\rightarrow \beta_1\downarrow\rightarrow R^2$

$\rightarrow (1-\beta_1)\uparrow, (1-R^2)\uparrow$

$\rightarrow (\bar{A}-\bar{P})^2\downarrow, (1-\beta_1)S_P^2\uparrow, (1-R^2)S_A^2.$

Exercise 3

Stock	α_i	β_i	$\overline{R_i}$	σ_ε^2
3	0.08	1.22	0.08 + 0.122 = 0.202	0.001
1	0.01	1.08	0.01 + 0.108 = 0.118	0.003
2	0.04	0.80	0.04 + 0.080 = 0.120	0.006

Stock	$\frac{\overline{R_i}-R_f}{\beta_i}$	$\frac{(\overline{R_i}-R_f)\widehat{\beta_i}}{\widehat{\sigma_{\varepsilon_i}}^2}$	$\frac{\sum_{j=1}^{i}(\overline{R_j}-R_f)\widehat{\beta_j}}{\widehat{\sigma_{\varepsilon_i}^2}}$	$\frac{\widehat{\beta_i}^2}{\widehat{\sigma_{\varepsilon_i}}^2}$	$\sum_{j=1}^{i}\frac{\widehat{\beta_i}^2}{\widehat{\sigma_{\varepsilon_i}}^2}$
3	0.0836	124.44	124.44	1488.4	1488.4
1	0.0167	6.48	130.92	388.8	1877.2
2	0.0250	2.667	133.587	106.667	1983.867

Stock	$C^* = \frac{\sigma_m^2\sum_{j=1}^{i}\frac{(\overline{R_i}-R_f)\beta_j}{\sigma_{\varepsilon_i}^2}}{1+\sigma_m^2\sum_{j=1}^{i}\frac{\beta_j^2}{\sigma_{\varepsilon_j}^2}}$
3	0.06258298
1	0.0550732
2	0.05378187

a. $R_{it} = 0.3R_{1t} + 0.5R_{2t} + 0.2R_{3t}$.

$\beta_i = \frac{Cov(0.3R_{1t}+0.5R_{2t}+0.2R_{3t}, 0.3R_{1t}+0.5R_{2t}+0.2R_{3t})}{\sigma_m^2}$

$= \frac{0.3^2V(R_{1t})+0.5^2V(R_{2t})+0.2^2V(R_{3t})}{\sigma_m^2}$, and $\beta_{1t} = \frac{V(R_{1t})}{\sigma_m^2}$, so that

$= 0.3^2\beta_{1t} + 0.5^2\beta_{2t} + 0.2^2\beta_{3t} = 0.3^2 * 1.08 + 0.5^2 * 0.8 + 0.2^2 * 1.22 = 0.346.$

b. $z_3 = \frac{1}{1.22} * 1488.4 * (0.0836 - 0.06258298) = 25.64076$

$z_1 = \frac{1}{1.08} * 1877.2 * (0.0167 - 0.0550732) = -66.69831$

$z_2 = \frac{1}{0.8} * 1983.867 * (0.025 - 0.05378187) = -71.37425$, so that

$x_3 = -0.2280562,\ x_1 = 0.5932335,\ x_2 = 0.6348227.$

Thus,

$E = (0.08 + 1.22 * x_3 * 300000) + (0.01 + 1.08 * x_3 * 300000) + (0.04 + 0.8 * x_2 * 300000) + (0.08 + 1.22 * 0.2 * 500000) + (0.01 + 1.08 * 0.3 * 500000) + (0.04 + 0.8 * 0.5 * 500000) = 400532.9.$

$\sigma^2 = 0.3^2\beta_1\sigma_m^2 + 0.5^2\beta_2\sigma_m^2 + 0.2^2\beta_3\sigma_3^2 + x_1^2 * 1.08 * \sigma_m^2 + x_2^2 * 0.8 * \sigma_m^2 + x_3^2 * 1.22 * \sigma_m^2 = 0.002223863.$

c. $\frac{Cov(Portfolio, Market)}{V(Market)} = \beta_i$, so that

$Cov(Portfolio, Market) = \beta_i V(Market) = 0.346 * 0.002 = 0.000692.$

d. $E = 0.6[(0.01 + 1.08 * 500000 * 0.3) + (0.04 + 0.8 * 500000 * 0.5) + (0.08 + 1.22 * 500000 * 0.2)] + 0.4 * 300000 * 1.002 = 410640.1$

$\sigma^2 = 0.6^2[0.3^2\beta_1\sigma_m^2 + 0.5^2\beta_2\sigma_m^2 + 0.2^2\beta_3\sigma_m^2] + 0.4^2 * 0 + 0 = 0.00024912.$

e. $\frac{Cov(Stock1,\ Market)}{V(Market)} = 1.08$, so that $Cov(Stock1,\ Market) = 1.08 * \sigma_m^2 = 0.00216.$

Exercise 4

$\overline{R_m} = 0.1,\ \sigma_m^2 = 0.2,\ \beta_p = 0.9.$

a. $V(R_p) = V\left(\frac{1}{20}R_1 + \cdots + \frac{1}{20}R_{20}\right) = \left(\frac{1}{20}\right)^2 \sum_{i=1}^{20} V(R_i) + \left(\frac{1}{20}\right)^2 \sum_{i=1}^{20}\sum_{j\neq i}^{20} Cov(R_i, R_j)$
$= \left(\frac{1}{20}\right)^2 \sum_{i=1}^{20}(\beta_i^2\sigma_m^2 + \sigma_{\varepsilon_i}^2) + \left(\frac{1}{20}\right)^2 \sum_{i=1}^{20}\sum_{j\neq i}^{20}(\beta_i\beta_j\sigma_m^2)$
$= \left(\frac{1}{20}\right)^2 * 20 * (0.9^2 * 0.2 + 0.5) + \left(\frac{1}{20}\right)^2 * 20 * 19 * (0.9^2 * 0.2) = 0.187.$

b. Blume's Technique: $\widehat{\beta_2} = \widehat{\gamma_0} + \widehat{\gamma_1}\widehat{\beta_1}.$
$\gamma_0 = \bar{y} - \beta_1\bar{x},\ \gamma_1 = \frac{Sxy}{Sxx} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{Cov(X,Y)}{V(X)} = \frac{0.2744995*0.5225564*1.154281}{1.154281^2} = 0.1242691.$
$(\because Corr(X,Y) = \frac{Cov(X,Y)}{sd(X)sd(Y)})$

$\gamma_0 = 1.042003 - 0.1242691 * 1.068617 = 0.9092069.$

Thus, $\widehat{\beta_2} = 0.9092069 + 0.1242691\widehat{\beta_1}.$

Thus, $\widehat{\beta_{19-23}} = 0.9092069 + 0.1242691 * \overline{\beta_{14-18}}$
$= 0.9092069 + 0.1242691 * 1.042003 = 1.038696.$

c. $R_m = \alpha_A + \beta_A R_A + \varepsilon_A.$
$R^2 = \frac{SSR}{SST} = \left(Corr(R_A, R_m)\right)^2 = \left(\frac{Cov(R_A,R_m)}{V(R_m)}\right)^2 = \left(\frac{\beta_A V(R_m)}{V(R_m)}\right)^2 = \beta_A^2 = 0.79^2 = 0.6241.$