Note

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$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$, $\hat{\beta}_1 = \frac{Sxy}{Sxx}$.

Residual Standard Error $S = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}\hat{e_i}^2} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}[y_i - \hat{y_i}]} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}[y_i - (\hat{\beta}_0 + \hat{\beta}_1 * x_i)]}$.

$se(\hat{\beta}_1) = \frac{S}{\sqrt{Sxx}}$, $se(\hat{\beta}_0) = S\sqrt{(\frac{1}{n} + \frac{\bar{x}^2}{Sxx})}$.

1. Confidence Interval for β_1: $\hat{\beta}_1 \pm t_{\alpha/2,\ n-2} * se(\hat{\beta}_1)$.
2. Confidence Interval for β_0: $\hat{\beta}_0 \pm t_{\alpha/2,\ n-2} * se(\hat{\beta}_0)$.

1. Confidence Interval for mean Y (regression line) at $X = x^*$: $\hat{y}^* \pm t_{\alpha/2,\ n-2} * S\sqrt{(\frac{1}{n} + \frac{(x^*-\bar{x})^2}{Sxx})}$
2. Prediction Interval for Single Y at $X = x^*$: $\hat{y}^* \pm t_{\alpha/2,\ n-2} * S\sqrt{(1 + \frac{1}{n} + \frac{(x^*-\bar{x})^2}{Sxx})}$

Note that One sample t-test for mean has statistic $T = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$, where $s = \sqrt{\frac{Sxx}{n-1}}$,
and the confidence interval is $\bar{x} \pm t_{\alpha/2,\ n-1} * \frac{s}{\sqrt{n}}$.

Variation: $\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 + \sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$.
$\rightarrow\ SST = SSE(RSS) + SSR(SSreg)$.

Simple notations:

$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2 = Syy$.

$SSR = \hat{\beta}_1 Sxy$.

$SSE = \sum_{i=1}^{n}\hat{e_i}^2 = Syy - \hat{\beta}_1 Sxy$.

ANOVA table:

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F	p-value
Regression	1	SSR	MSR	$\frac{MSR}{MSE} = \frac{SSR/1}{SSE/(n-2)}$	$P(F < F_{df_1,\ df_2})$
Residual	$n-2$	SSE	MSE		
Total	$n-1$	SST			