Homework 9

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Exercise 1

a. Let the period 1, so that S_1.

$S_1 = S_0 + \Delta S_1 = S_0 + \varepsilon_1 \sqrt{\Delta t}$.

$E(S_1) = S_0 = 50$,

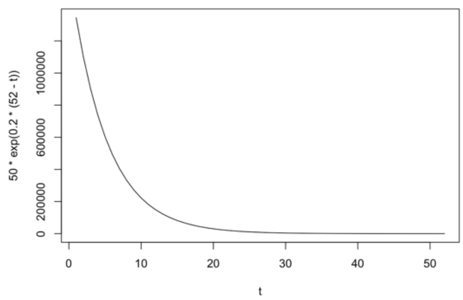
$V(S_1) = V\left(S_0 + \varepsilon_1 \sqrt{\Delta t}\right) = V(\varepsilon_1) = 1$.

Thus, $S_1 \sim N(50,1)$.

It holds for any period having the amount of Δt of time.

b. Stock price $= Se^{\mu(T-t)} = 50e^{0.2(52-t)}$ for $t = 1, 2, \ldots, 52$.

```
t <- seq(1, 52)
plot(t, 50 * exp(0.2 * (52 - t)), type='l')
```



Exercise 2

$\mu = 0.16, \sigma = 0.3, S_0 = 50$.

a. $\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t} = 0.16 * 50 * \Delta t + 0.3 * 50 * \varepsilon \sqrt{\Delta t}$,

so that the expected price of tomorrow is as follows:

$\Delta S|_{t=1/365} = 0.16 * 50 * \sqrt{1/365} + 0.3 * 50 * \varepsilon * \sqrt{1/365}$,

$E\left(\Delta S|_{t=1/365}\right) = 0.16 * 50 * \sqrt{1/365} = 0.4187391$,

$\therefore\ S_1 = 50.4187391$.

b. $V\left(\Delta S|_{t=1/365}\right) = 0.3 * 50 * 1 * \sqrt{1/365} = 0.7851359$, so that

$sd\left(\Delta S|_{t=1/365}\right) = 0.8860789$.

Exercise 3

$S_0 = 38,\ \mu = 0.16,\ \sigma = 0.35.$

a. $E = 40$, 6-months. Because it follows log-normal distribution,
$E(S_T) = 38e^{0.16(1-0.5)} = 38e^{0.08} = 41.16491,$
$V(S_T) = 38^2 e^{2*0.16*(1-0.5)}\left(e^{0.35^2(1-0.5)} - 1\right) = 107.0357.$

The probability that a European call will be exercised: $P(S_T > 40)$
$= P\left(\frac{S_T - 41.16491}{\sqrt{107.0357}} > \frac{40 - 41.16491}{\sqrt{107.0357}}\right) = P(Z > -0.1125973) = 0.5448251.$

b. The probability that a European put will be exercised:
$P(S_T < 40) = 1 - P(S_T > 40)$ if continuous.
$= 1 - 0.5448251 = 0.4551749.$

Exercise 4

$\boldsymbol{S_0 = 40, \mu = 0.1, \sigma = 0.15.}$

$$P\left(Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)-1.96\sigma\sqrt{T-t}} \le S_T \le Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)+1.96\sigma\sqrt{T-t}}\right)$$
$$= P\left(Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)-1.96\sigma\sqrt{T-t}} - Se^{\mu(T-t)} \le S_T - Se^{\mu(T-t)} \le Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)+1.96\sigma\sqrt{T-t}} - Se^{\mu(T-t)}\right)$$
$$= P\left(\frac{Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)-1.96\sigma\sqrt{T-t}} - Se^{\mu(T-t)}}{Se^{\mu(T-t)}\left[e^{\sigma^2(T-t)}-1\right]^{1/2}} \le Z \le \frac{Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)-1.96\sigma\sqrt{T-t}} + Se^{\mu(T-t)}}{Se^{\mu(T-t)}\left[e^{\sigma^2(T-t)}-1\right]^{1/2}}\right)$$
$$= P\left(\frac{e^{-\frac{\sigma^2}{2}(T-t)-1.96\sigma\sqrt{T-t}}-1}{\left[e^{\sigma^2(T-t)}-1\right]^{1/2}} \le Z \le \frac{e^{-\frac{\sigma^2}{2}(T-t)+1.96\sigma\sqrt{T-t}}-1}{\left[e^{\sigma^2(T-t)}-1\right]^{1/2}}\right).$$
$$= P(-1.96 \le Z \le 1.96) = 0.95.$$

a. $\left[Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)-1.96\sigma\sqrt{T-t}}, Se^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)+1.96\sigma\sqrt{T-t}}\right]$
$= \left[40e^{\left(0.1-\frac{0.15^2}{2}\right)(12-10)-1.96*0.15\sqrt{12-10}}, 40e^{\left(0.1-\frac{0.15^2}{2}\right)(12-10)+1.96*0.15\sqrt{12-10}}\right]$
$= [31.51932, 72.39652].$

b. $E(S_T) = Se^{\mu(T-t)} = 40e^{0.1(12-10)} = 48.85611.$

c. $V(S_T) = S^2e^{2\mu(T-t)}\left[e^{\sigma^2(T-t)} - 1\right] = 40^2e^{2*0.1(12-10)}\left[e^{0.15^2(12-10)} - 1\right] = 109.8648.$
$sd(S_T) = 10.48164.$

Exercise 5

```r
s1 <- read.csv("/Users/user/Desktop/Yonsei/Junior/3-2/Statistical Models in Finance/AAPL.csv",
               sep=',', header=T)

head(s1) ; tail(s1) ; nrow(s1)

stock <- rep(0, nrow(s1)-1)

for (i in 1:nrow(s1)-1) {
  stock[i] <- s1$AAPL[i+1] / s1$AAPL[i]
}

stock

log_stock <- log(stock)

length(stock)

s <- (1/(length(stock)-1) * (sum(log_stock^2) - ((sum(log_stock))^2)/length(stock)))^(1/2)
s

annual_hat_sigma <- (252)^(1/2) * s
annual_hat_sigma

```

54:17 (Top Level) R Scrip

Console Terminal Background Jobs

R 4.2.3 · ~/

```
> s <- (1/(length(stock)-1) * (sum(log_stock^2) - ((sum(log_stock))^2)/length(stock)))
> s
[1] 0.000433692
> s <- (1/(length(stock)-1) * (sum(log_stock^2) - ((sum(log_stock))^2)/length(stock)))^(1/2)
> s
[1] 0.02082527
> annual_hat_sigma <- (252)^(1/2) * s
> annual_hat_sigma
[1] 0.330591
>
```

Thus, $\hat{\sigma} = \sqrt{252} * 0.02082527 = 0.330591$. QED