Homework 6

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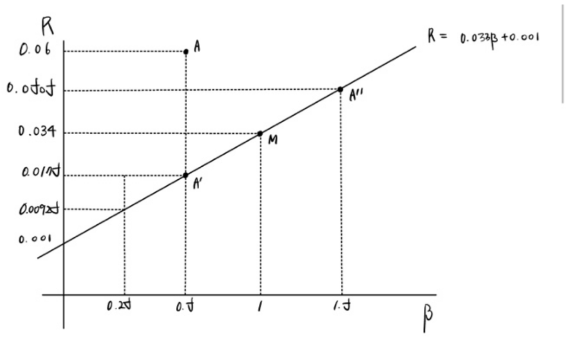
Exercise 1

a. $\overline{R_A} = 0.06,\ \beta_A = 0.5,\ \overline{R_M} = 0.034,\ \sigma_M^2 = 0.015,\ R_f = 0.001$, and total risk is 0.03375.



Thus, $\overline{R_A} - R_F = (\overline{R_A} - \overline{R_{A'}}) + (\overline{R_{A'}} - R_F) = 0.0425 + 0.0165$, so that
The Return from Selectivity is 0.0425.

Net Selectivity $= 0.06 - 0.0505 = 0.0095$,
Diversification $= 0.0505 - 0.034 = 0.0165$.

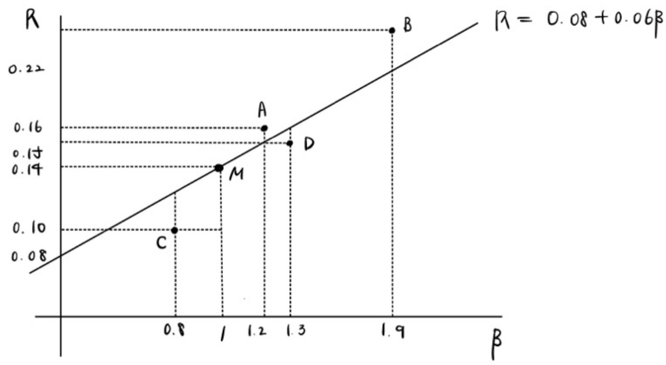
Risk from Manager $= 0.0175 - 0.00925 = 0.00825$.
Risk from Investor $= 0.00925 - 0.001 = 0.00825$.

b. Sharpe Measure: $\frac{\overline{R_p} - R_F}{\sigma_p}$, and $R_F = 0.14$.
$A = \frac{0.16-0.14}{0.19} = \frac{2}{19}$
$B = \frac{0.22-0.16}{0.16} = \frac{6}{16}$.

Let $\overline{R_B} = k$, then claim $\frac{k-0.16}{0.16} = \frac{2}{19}$.
$\rightarrow k = \frac{0.32+19*0.16}{19} = 0.1768421$.

c. Treynor Measure: $\frac{\overline{R_B} - R_F}{\beta_B}$.
$T_A = \frac{\overline{R_A} - R_F}{\beta_A} = \frac{0.16-0.14}{1.2} = \frac{1}{60}$,
$S_B = \frac{\overline{R_B} - R_F}{\sigma_B} = \frac{0.22-0.14}{0.16} = \frac{1}{2}$.

d. Jensen Differential Performance Index: $\overline{R_A} - \overline{R_{A'}}$.

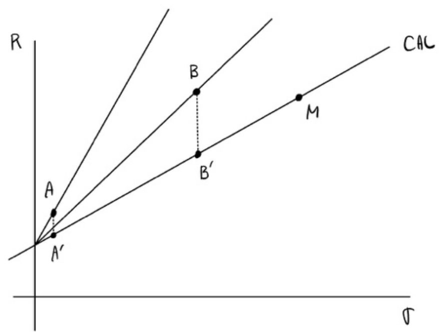


$\overline{R_A} - \overline{R_{A'}} = 0.16 - 0.152 = 0.008,$
$\overline{R_B} - \overline{R_{B'}} = 0.22 - 0.194 = 0.026,$
$\overline{R_C} - \overline{R_{C'}} = 0.1 - 0.128 = -0.028,$
$\overline{R_D} - \overline{R_{D'}} = 0.15 - 0.158 = -0.008.$

e. $\sigma_{ik} = Cov(R_i, R_k) = \beta_i \beta_k \sigma_j^2$, where stock i and k in same industry j.
And because $\sigma_j^2 . = b_j^2 \sigma_m^2 + \sigma_{c_j}^2$,
So that $\sigma_{ik} = \beta_i \beta_k \left(b_j^2 \sigma_m^2 + \sigma_{c_j}^2 \right)$.

$\sigma_{ik} = \beta_i \beta_k b_j b_l \sigma_m^2$, where stock i and k in different industry j and l.

f. In below case, there is $\frac{\overline{R_A} - R_F}{\sigma_A} > \frac{\overline{R_B} - R_F}{\sigma_B}$, but the differential excess return is $D_A < D_B$.



g.

$$
\begin{aligned}
\overline{R_1} - R_f &= z_1\sigma_1^2 + z_2\sigma_{12} + z_3\sigma_{13} + z_4\sigma_{14} + \cdots + z_9\sigma_{19} \\
&= z_1\left(\beta_1^2\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + \sigma_{\varepsilon_1}^2\right) + z_2\beta_1\beta_2\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + z_4\beta_1\beta_4 b_1 b_4\sigma_m^2 + \cdots + z_9\beta_1\beta_9 b_1 b_9\sigma_m^2 \\
&= z_1\sigma_{\varepsilon_1}^2 + \beta_1\left[z_1\beta_1\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + z_2\beta_2\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + z_3\beta_3\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\right] \\
&\quad + \beta_1\textstyle\sum_{i=4}^6 z_i\beta_i b_1 b_i\sigma_m^2 + \beta_1\sum_{i=7}^9 z_i\beta_i b_1 b_i\sigma_m^2 \\
&= z_1\sigma_{\varepsilon_1}^2 + \beta_1\left[b_1^2\sigma_m^2(z_1\beta_1 + z_2\beta_2 + z_3\beta_3) + \sigma_{c_1}^2(z_1\beta_1 + z_2\beta_2 + z_3\beta_3)\right] \\
&\quad + \beta_1[b_1 b_4\sigma_m^2(z_4\beta_4 + z_5\beta_5 + z_6\beta_6)] + \beta_1[b_1 b_7\sigma_m^2(z_7\beta_7 + z_8\beta_8 + z_9\beta_9)] \qquad (*) \\
&\quad (\because b_4 = b_5 = b_6,\ b_7 = b_8 = b_9)
\end{aligned}
$$

Let $\phi_1 = z_1\beta_1 + z_2\beta_2 + z_3\beta_3$, $\phi_2 = z_4\beta_4 + z_5\beta_5 + z_6\beta_6$, $\phi_3 = z_7\beta_7 + z_8\beta_8 + z_9\beta_9$.

(*) $\quad = z_1\sigma_{\varepsilon_1}^2 + \beta_1\left[b_1^2\sigma_m^2\phi_1 + \sigma_{c_1}^2\phi_1\right] + \beta_1 b_1 b_4\sigma_m^2\phi_2 + \beta_1 b_1 b_7\sigma_m^2\phi_3.$

$z_1\sigma_{\varepsilon_1}^2 = \left(\overline{R_1} - R_f\right) - \beta_1\left[b_1^2\sigma_m^2\phi_1 + \sigma_{c_1}^2\phi_1\right] - \beta_1 b_1 b_4\sigma_m^2\phi_2 - \beta_1 b_1 b_7\sigma_m^2\phi_3.$

Thus, $\quad z_1 = \frac{\beta_1}{\sigma_{\varepsilon_1}^2}\left[\frac{\overline{R_1}-R_f}{\beta_1} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right]$

$z_2 = \frac{\beta_2}{\sigma_{\varepsilon_2}^2}\left[\frac{\overline{R_2}-R_f}{\beta_2} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right]$

$z_1 = \frac{\beta_3}{\sigma_{\varepsilon_3}^2}\left[\frac{\overline{R_3}-R_f}{\beta_3} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right]$

$$
\begin{aligned}
\beta_1 z_1 &= \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}\left[\frac{\overline{R_1}-R_f}{\beta_1} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right] \\
\beta_2 z_2 &= \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}\left[\frac{\overline{R_2}-R_f}{\beta_2} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right] \\
\beta_3 z_3 &= \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\left[\frac{\overline{R_3}-R_f}{\beta_3} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right]
\end{aligned}
$$

Therefore,
$$
\begin{aligned}
\phi_1 &= \sum_{i=1}^3 \frac{\beta_i\left(\overline{R_i}-R_f\right)}{\sigma_{\varepsilon_i}^2} - \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} b_1 b_4\sigma_m^2\phi_2 - \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} b_1 b_7\sigma_m^2\phi_3 \\
&\quad - \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} b_1 b_4\sigma_m^2\phi_2 - \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} b_1 b_7\sigma_m^2\phi_3 \\
&\quad - \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2} b_1 b_4\sigma_m^2\phi_2 - \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2} b_1 b_7\sigma_m^2\phi_3.
\end{aligned}
$$

$\therefore$
$$
\begin{aligned}
\sum_{i=1}^3 \frac{\beta_i\left(\overline{R_i}-R_f\right)}{\sigma_{\varepsilon_i}^2} &= \left[1 + \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_1 \\
&\quad + \left[b_1 b_4\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_2 + \left[b_1 b_7\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_3.
\end{aligned}
$$

And,
$$
\begin{aligned}
\sum_{i=4}^6 \frac{\beta_i\left(\overline{R_i}-R_f\right)}{\sigma_{\varepsilon_i}^2} &= \left[b_1 b_2\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_1 \\
&\quad + \left[1 + \left(b_2^2\sigma_m^2 + \sigma_{c_2}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_2 + \left[b_1 b_7\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_3.
\end{aligned}
$$

Also,
$$
\begin{aligned}
\sum_{i=7}^9 \frac{\beta_i\left(\overline{R_i}-R_f\right)}{\sigma_{\varepsilon_i}^2} &= \left[b_1 b_2\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_1 \\
&\quad + \left[b_1 b_4\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_2 + \left[1 + \left(b_3^2\sigma_m^2 + \sigma_{c_3}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_3.
\end{aligned}
$$

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Finally, we can conclude that $R = M\phi$ $(\leftrightarrow C = A\phi)$

$$\begin{pmatrix} \sum_{i=1}^{3}\frac{\beta_i(\overline{R_i}-R_f)}{\sigma_{\varepsilon_i}^2} \\ \sum_{i=4}^{6}\frac{\beta_i(\overline{R_i}-R_f)}{\sigma_{\varepsilon_i}^2} \\ \sum_{i=7}^{9}\frac{\beta_i(\overline{R_i}-R_f)}{\sigma_{\varepsilon_i}^2} \end{pmatrix} =$$

$$\begin{pmatrix} 1+\left(b_1^2\sigma_m^2+\sigma_{c_1}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1b_4\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1b_7\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) \\ b_1b_2\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & 1+\left(b_2^2\sigma_m^2+\sigma_{c_2}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1b_7\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) \\ b_1b_2\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1b_4\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & 1+\left(b_3^2\sigma_m^2+\sigma_{c_3}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}+\frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}+\frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) \end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}$$

QED

h. 1. Short sales are not allowed, then $C^* = C_2 = 0.224$.

2. $z_1 = \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}\left[\frac{R_1-R_F}{\beta_1} - C^*\right] = \frac{0.8^2}{0.02}[0.28-0.188] = 2.944$,

$z_2 = \frac{0.82^2}{0.01}[0.25-0.224] = 1.74824$.

Thus,

$x_1 = \frac{2.944}{2.944+1.74824} = 0.6274189$, $x_2 = 0.3683188$.

3. $\beta_p = x_1\beta_1 + x_2\beta_2 = 2.491031$.

4. $R_i = \alpha_i + \beta_i R_M + \varepsilon_i$, so that $R_i = -0.025 + 0.8 * 0.1 + \varepsilon_1$,

$\overline{R_i} = 0.055$.

Thus, $\frac{\overline{R_i}-R_F}{\beta_i} = \frac{0.055-0.002}{0.8} = 0.06625$.

This is between stock 10 and 11, so that we cannot include this stock. QED