Homework 5

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Exercise 1

a. Actually, there is no CCM condition, so that I can't use constant ρ.
However, if CCM holds,

$C_i = \frac{\rho}{1-\rho+i\rho}\sum_{j=1}^{i}\frac{\overline{R_j}-R_f}{\sigma_j}$.

Thus, $C_1 = 0.5 * 1 = 0.5$, $C_6 = 0.2 * 3.9 = 0.78$.

$\therefore\ C_4 = 0.7668$, $C_6 = 0.78$.

b. Stock 1, 2, 5, and 6 can be included.
$z_i = \frac{1}{(1-\rho)\sigma_i}\left[\frac{\overline{R_i}-R_f}{\sigma_i} - C_i\right]$.

$z_1 = \frac{1}{(1-0.5)0.1}[1-0.5] = \frac{1}{0.05}0.5 = 10.$
$z_2 = \frac{1}{(1-0.5)0.15}[1-0.6667] = 4.444.$
$z_5 = \frac{1}{(1-0.5)0.05}[1-0.75] = \frac{1}{0.5*0.05}0.25 = 10.$
$z_6 = \frac{1}{(1-0.5)0.1}[1-0.78] = \frac{1}{0.5*0.1}0.22 = 4.4.$

Thus, $\sum_{1,2,5,6} z_i = 28.844$.
$x_1 = \frac{10}{28.844} = 0.3466926$, $x_2 = 0.1540702$, $x_5 = 0.3466926$, $x_6 = 0.1525447$.

c. $E = \sum_{1,2,5,6} x_i R_i = 0.1388434$,

$$\sigma^2 = \boldsymbol{x}'\Sigma\boldsymbol{x} = (x_1 \quad x_2 \quad x_3 \quad x_4)\begin{pmatrix}\sigma_1^2 & \rho & \rho & \rho \\ \rho & \sigma_2^2 & \rho & \rho \\ \rho & \rho & \sigma_3^2 & \rho \\ \rho & \rho & \rho & \sigma_4^2\end{pmatrix}\begin{pmatrix}x_1 \\ x_2 \\ x_3 \\ x_4\end{pmatrix} = 0.3585698.$$

Thus, Combination $E' = 0.8E + 0.2R_f = 0.1210747$,
$\sigma^{2\prime} = 0.8^2 * \sigma^2 + 0.2^2 * 0 = 0.2294846$.

Exercise 2

a. (a) 0.2 (b) 0.72625 (c) $C_i = \frac{\sigma_m^2 \sum_{j\in\Omega}\frac{(\overline{R_j}-R_f)\beta_j}{\sigma_{\varepsilon_j}^2}}{1+\sigma_m^2\sum_{j\in\Omega}\frac{\beta_j^2}{\sigma_{\varepsilon_j}^2}}$, so that $C_5 = \frac{10*3.55}{1+10*0.55125} = 5.451026$.

b. Stock 1, 2, 3, 4, and 5.
$z_1 = \frac{\beta_1}{\sigma_{\varepsilon_1}^2}\left(\frac{\overline{R_1}-R_f}{\beta_1} - C_5\right) = \frac{1}{\beta_1}\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}\left(\frac{\overline{R_1}-R_f}{\beta_1} - 5.451056\right) = \frac{1}{1} * 0.02 * (10 - 5.451056) = 0.09097888.$

c. $C^* = 4.52$.
$z_1 = \frac{1}{1} * 0.02 * (10 - 4.52) = 0.1096.$

d. $\sigma_{1m} = \beta_1\beta_m\sigma_m^2$, where $\beta_m = 1$.
$= \beta_1\sigma_m^2 = 1 * 10 = 10$. QED

Exercise 3

$R_f = 0.05,\ \rho = 0.45.$

a. $a = \frac{0.45}{1-0.45+5*0.45} = 0.1607143.$
   $b = 22.832 + 4.746 = 27.578.$
   $c = a * b = 4.432179.$

b. If short sales are not allowed, then $C_5 = C^* = 4.432179.$

c. If short sales are allowed, then $C^* = 3.271.$

d. $\sigma^2 = x'\Sigma x = (x_1 \quad \cdots \quad x_{12}) \begin{pmatrix} \sigma_1^2 & \rho & \cdots & \rho \\ \rho & \sigma_2^2 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & \sigma_{12}^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{12} \end{pmatrix}.$

e. $\bar{R} = 0.055, \sigma = 0.025.$

$\bar{R}_i$	σ_i	$\frac{\bar{R}_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum_{j=1}^{i} \frac{R_j - R_f}{\sigma_j}$	C_i
0.055	0.025	0.2	0.0703125	43.452	3.055219

If short sales are not allowed, then it doesn't affect anything.
If short sales are allowed,
$z_1 \sim z_{12} = \frac{1}{(1-\rho)\sigma_i}\left(\frac{\bar{R}_i - R_f}{\sigma_i} - C^*\right) \uparrow,$
$z_{13} = \frac{1}{(1-\rho)\sigma_{13}}\left(\frac{\bar{R}_{13} - R_f}{\sigma_{13}} - C^*\right) < 0$, so that $x_1 \sim x_{12} \uparrow,\ x_{13} < 0.$

Exercise 4

1. Short sales are allowed: It doesn't choose stock 1.
2. Short sales are not allowed: $\frac{\rho}{1-\rho+\rho} = \rho = 0.21005601$, so that
   $z_1 = \frac{1}{(1-\rho)\sigma_1}\left(\frac{\bar{R}_1 - R_f}{\sigma_1} - C_1\right) = -0.0004177928.$

Exercise 5

a. $A_3 = \begin{pmatrix} \frac{N_1\rho_{13}}{1-\rho_{11}} \\ \frac{N_2\rho_{23}}{1-\rho_{22}} \\ 1 + \frac{N_3\rho_{33}}{1-\rho_{33}} \\ \vdots \\ \frac{N_{13}\rho_{13,3}}{1-\rho_{13,13}} \end{pmatrix}.$

Industry 3 has only 5 stocks. Thus, there is no z_{13}.
If it is z_3, then $z_3 = \frac{1}{\sigma_3(1-\rho_{33})}\left[\frac{\bar{R}_3 - R_f}{\sigma_3} - \sum_{g=1}^{5} \rho_{kg}\Phi_g\right].$

b. $M_2 = \left(\frac{N_2\rho_{21}}{1-\rho_{22}} \quad 1 + \frac{N_2\rho_{22}}{1-\rho_{22}} \quad \frac{N_2\rho_{23}}{1-\rho_{22}} \quad \cdots \quad \frac{N_2\rho_{2,13}}{1-\rho_{22}}\right).$
   $z_4 = \frac{1}{\sigma_4(1-\rho_{44})}\left[\frac{\bar{R}_4 - R_f}{\sigma_4} - \sum_{g=1}^{5} \rho_{kg}\Phi_g\right].$

Exercise 6

a.

```r
######

hw <- read.csv("/Users/user/Desktop/Yonsei/Junior/3-2/Statistical Models in Finance/stockData.csv",sep=',', header=T)

r_hw5 <- (hw[-1, 3:ncol(hw)]-hw[-nrow(hw),3:ncol(hw)])/hw[-nrow(hw),3:ncol(hw)]

covmat_hw5 <- var(r_hw5)
beta_hw5 <- covmat_hw5[1,-1] / covmat_hw5[1,1]

rrr_hw5 <- r_hw5[,-c(1,which(beta_hw5<0)+1)]

beta_new_hw5 <- rep(0,ncol(rrr_hw5))
alpha_hw5 <- rep(0,ncol(rrr_hw5))
mse_hw5 <- rep(0,ncol(rrr_hw5))
Ribar_hw5 <- rep(0,ncol(rrr_hw5))
Ratio_hw5 <- rep(0,ncol(rrr_hw5))
stock_hw5 <- rep(0,ncol(rrr_hw5))

rf_hw5 <- 0.001

for(i in 1:ncol(rrr_hw5)) {
  q_hw5 <- lm(data=rrr_hw5, formula=rrr_hw5[,i]~r_hw5[,1])
  beta_new_hw5[i] <- q_hw5$coefficients[2]
  alpha_hw5[i] <- q_hw5$coefficients[1]
  mse_hw5[i] <- summary(q_hw5)$sigma^2
  Ribar_hw5[i] <- q_hw5$coefficients[1] + q_hw5$coefficients[2] * mean(r_hw5[,1])
  Ratio_hw5[i] <- (Ribar_hw5[i] - rf_hw5) / beta_new_hw5[i]
  stock_hw5[i] <- i
}

xx_hw5 <- (cbind(stock_hw5, alpha_hw5, beta_new_hw5, Ribar_hw5, mse_hw5, Ratio_hw5))

head(xx_hw5)

A_hw5 <- xx_hw5[order(-xx_hw5[,6]),]

col1_hw5 <- rep(0,nrow(A_hw5))
col2_hw5 <- rep(0,nrow(A_hw5))
col3_hw5 <- rep(0,nrow(A_hw5))
col4_hw5 <- rep(0,nrow(A_hw5))
col5_hw5 <- rep(0,nrow(A_hw5))

col1_hw5 <- (A_hw5[,4]-rf_hw5)*A_hw5[,3]/A_hw5[,5]
col3_hw5 <- A_hw5[,3]^2 / A_hw5[,5]
for(i in 1:nrow(A_hw5)) {
  col2_hw5[i] <- sum(col1_hw5[1:i])
  col4_hw5[i] <- sum(col3_hw5[1:i])
}

head(cbind(A_hw5, col1_hw5, col2_hw5, col3_hw5, col4_hw5))

for(i in 1:nrow(A_hw5)) {
  col5_hw5[i] <- var(r_hw5[,1])*col2_hw5[i]/(1+var(r_hw5[,1])*col4_hw5[i])
}

z_short_hw5 <- (A_hw5[,3]/A_hw5[,5])*(A_hw5[,6]-col5_hw5[nrow(A_hw5)])
x_short_hw5 <- z_short_hw5/sum(z_short_hw5)


A_hw5
col1_hw5

covmat_hw5

x_short_hw5

length(colnames(r_hw5))
length(col1_hw5)

covmat_market_hw5 <- var(r_hw5[-1])

covmat_market_hw5

var_market_hw5 <- t(as.matrix(x_short_hw5)) %*% covmat_market_hw5 %*% as.matrix(x_short_hw5)

C_hw5 <- (A_hw5[,4]-rf_hw5)*A_hw5[,3]*as.numeric(var_market_hw5)/A_hw5[,5]

C_hw5
```

Then,

```
> C_hw5 <- (A_hw5[,4]-rf_hw5)*A_hw5[,3]*as.numeric(var_market_hw5)/A_hw5[,5]
> C_hw5
 [1]  0.0242051978  0.0041774703  0.0225339884  0.1105829536  0.0064280686  0.0988315572
 [7]  0.0726046809  0.0271796902  0.0175490983  0.0155415125  0.0282530048  0.0673594072
[13]  0.0200742461  0.0406608906  0.0349287150  0.0291048331  0.0288407291  0.0137018356
[19]  0.0444328832  0.0453015674  0.0391336984  0.0257792440  0.0174651289  0.0090034408
[25]  0.0042025251  0.0073061709  0.0001557249 -0.0006209942 -0.0065418831 -0.0074156409
```

It can be computed as follows.

b.

1. The table is

Stock	$\overline{R_\iota}$	σ_i	$\frac{\overline{R_\iota}-R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum_{j=1}^{i}\frac{R_j-R_f}{\sigma_j}$	C_i
1	0.29	0.03	8	0.5	8	4
2	0.19	0.02	7	0.333	15	5
3	0.08	0.15	0.2	0.25	15.2	3.8

Thus, cut-off rate C^* when short sales are allowed: $C^* = 3.8$
If not allowed, then $C^* = C_2 = 5$.

2. $z_1 = \frac{1}{(1-\rho)\sigma_1}\left(\frac{\overline{R_1}-R_f}{\sigma_1} - C^*\right) = \frac{1}{(1-0.5)0.03}(8-3.8) = 280$
$z_2 = \frac{1}{(1-0.5)0.02}(7-3.8) = 320,$
$z_3 = \frac{1}{(1-0.5)0.15}(0.2-3.8) = -48.$ QED

Exercise 7

1.

Stock	$\widehat{\beta_\iota}$	R_i	$\widehat{\sigma_{\varepsilon_\iota}}^2$	$\frac{R_i-R_f}{\widehat{\beta_\iota}}$	$\frac{(R_i-R_f)\widehat{\beta_\iota}}{\widehat{\sigma^2_{\varepsilon_\iota}}}$	$\sum_{j=1}^{i}\frac{(R_i-R_f)\widehat{\beta_\iota}}{\widehat{\sigma^2_{\varepsilon_\iota}}}$
A	0.94	0.006	0.0033	0.00106383	0.2848485	0.2848485
B	0.61	0.011	0.0038	0.009836066	0.9631579	1.248006
C	1.12	0.015	0.0046	0.008928571	2.434783	3.682789

	$\frac{\widehat{\beta_\iota}^2}{\widehat{\sigma^2_{\varepsilon_\iota}}}$	$\sum_{j=1}^{i}\frac{\widehat{\beta_\iota}^2}{\widehat{\sigma^2_{\varepsilon_\iota}}}$	$C_i = \sigma_m^2\frac{\sum_{j\in k}\frac{(\overline{R_j}-R_f)\beta_j}{\sigma^2_{\varepsilon_j}}}{1+\sigma_m^2\sum_{j\in k}\frac{\beta_j^2}{\sigma^2_{\varepsilon_j}}}.$
A	267.7576	267.7576	0.0003457783
B	97.92105	365.6787	0.001354711
C	272.6957	638.3744	0.003084594

2. $C^* = C_A = 0.0003457783.$
$z_A = \frac{\beta_1}{\sigma^2_{\varepsilon_1}}\left(\frac{\overline{R_1}-R_f}{\beta_1} - C^*\right) = \frac{1}{0.94}267.7576(0.00106383 - 0.0003457783) = 0.204536.$
$z_B = \frac{1}{0.61}97.92105(0.009836066 - 0.0003457783) = 1.523441.$
$z_C = \frac{1}{1.12}272.6957(0.008928571 - 0.0003457783) = 2.089724.$

3. Because $C^* = C_A$ also holds in not-allowed situations, so that $\mathbf{z} = (z_A \quad z_B \quad z_C)$ is same with above.

Exercise 8

$$
\begin{aligned}
\overline{R_1} - R_f &= z_1\sigma_1^2 + z_2\sigma_{12} + z_3\sigma_{13} + z_4\sigma_{14} + \cdots + z_9\sigma_{19} \\
&= z_1\left(\beta_1^2\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + \sigma_{\varepsilon_1}^2\right) + z_2\beta_1\beta_2\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + z_4\beta_1\beta_4 b_1 b_4\sigma_m^2 + \cdots + z_9\beta_1\beta_9 b_1 b_9\sigma_m^2 \\
&= z_1\sigma_{\varepsilon_1}^2 + \beta_1\left[z_1\beta_1\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + z_2\beta_2\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right) + z_3\beta_3\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\right] \\
&\quad + \beta_1\textstyle\sum_{i=4}^{6} z_i\beta_i b_1 b_i\sigma_m^2 + \beta_1\sum_{i=7}^{9} z_i\beta_i b_1 b_i\sigma_m^2 \\
&= z_1\sigma_{\varepsilon_1}^2 + \beta_1\left[b_1^2\sigma_m^2(z_1\beta_1 + z_2\beta_2 + z_3\beta_3) + \sigma_{c_1}^2(z_1\beta_1 + z_2\beta_2 + z_3\beta_3)\right] \\
&\quad + \beta_1\left[b_1 b_4\sigma_m^2(z_4\beta_4 + z_5\beta_5 + z_6\beta_6)\right] + \beta_1\left[b_1 b_7\sigma_m^2(z_7\beta_7 + z_8\beta_8 + z_9\beta_9)\right] \qquad (*) \\
&\quad (\because b_4 = b_5 = b_6,\ b_7 = b_8 = b_9)
\end{aligned}
$$

Let $\phi_1 = z_1\beta_1 + z_2\beta_2 + z_3\beta_3$, $\phi_2 = z_4\beta_4 + z_5\beta_5 + z_6\beta_6$, $\phi_3 = z_7\beta_7 + z_8\beta_8 + z_9\beta_9$.

(*) $\quad = z_1\sigma_{\varepsilon_1}^2 + \beta_1\left[b_1^2\sigma_m^2\phi_1 + \sigma_{c_1}^2\phi_1\right] + \beta_1 b_1 b_4\sigma_m^2\phi_2 + \beta_1 b_1 b_7\sigma_m^2\phi_3.$

$$z_1\sigma_{\varepsilon_1}^2 = \left(\overline{R_1} - R_f\right) - \beta_1\left[b_1^2\sigma_m^2\phi_1 + \sigma_{c_1}^2\phi_1\right] - \beta_1 b_1 b_4\sigma_m^2\phi_2 - \beta_1 b_1 b_7\sigma_m^2\phi_3.$$

Thus,

$$
\begin{aligned}
z_1 &= \frac{\beta_1}{\sigma_{\varepsilon_1}^2}\left[\frac{\overline{R_1}-R_f}{\beta_1} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right] \\
z_2 &= \frac{\beta_2}{\sigma_{\varepsilon_2}^2}\left[\frac{\overline{R_2}-R_f}{\beta_2} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right] \\
z_1 &= \frac{\beta_3}{\sigma_{\varepsilon_3}^2}\left[\frac{\overline{R_3}-R_f}{\beta_3} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right]
\end{aligned}
$$

$$
\begin{aligned}
\beta_1 z_1 &= \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}\left[\frac{\overline{R_1}-R_f}{\beta_1} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right] \\
\beta_2 z_2 &= \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}\left[\frac{\overline{R_2}-R_f}{\beta_2} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right] \\
\beta_3 z_3 &= \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\left[\frac{\overline{R_3}-R_f}{\beta_3} - \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - b_1 b_4\sigma_m^2\phi_2 - b_1 b_7\sigma_m^2\phi_3\right]
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\phi_1 = \textstyle\sum_{i=1}^{3}\frac{\beta_i(\overline{R_i}-R_f)}{\sigma_{\varepsilon_i}^2} &- \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2}\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} b_1 b_4\sigma_m^2\phi_2 - \frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} b_1 b_7\sigma_m^2\phi_3 \\
&- \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2}\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} b_1 b_4\sigma_m^2\phi_2 - \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} b_1 b_7\sigma_m^2\phi_3 \\
&- \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\phi_1 - \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2} b_1 b_4\sigma_m^2\phi_2 - \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2} b_1 b_7\sigma_m^2\phi_3.
\end{aligned}
$$

$\therefore$

$$
\begin{aligned}
\textstyle\sum_{i=1}^{3}\frac{\beta_i(\overline{R_i}-R_f)}{\sigma_{\varepsilon_i}^2} &= \left[1 + \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_1 \\
&\quad + \left[b_1 b_4\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_2 + \left[b_1 b_7\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_3.
\end{aligned}
$$

And,

$$
\begin{aligned}
\textstyle\sum_{i=4}^{6}\frac{\beta_i(\overline{R_i}-R_f)}{\sigma_{\varepsilon_i}^2} &= \left[b_1 b_2\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_1 \\
&\quad + \left[1 + \left(b_2^2\sigma_m^2 + \sigma_{c_2}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_2 + \left[b_1 b_7\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_3.
\end{aligned}
$$

Also,

$$
\begin{aligned}
\textstyle\sum_{i=7}^{9}\frac{\beta_i(\overline{R_i}-R_f)}{\sigma_{\varepsilon_i}^2} &= \left[b_1 b_2\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_1 \\
&\quad + \left[b_1 b_4\sigma_m^2\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_2 + \left[1 + \left(b_3^2\sigma_m^2 + \sigma_{c_3}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right)\right]\phi_3.
\end{aligned}
$$

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Finally, we can conclude that $\mathbf{R} = \mathbf{M}\phi$ $(\leftrightarrow \mathbf{C} = \mathbf{A}\phi)$

$$\begin{pmatrix} \sum_{i=1}^{3} \frac{\beta_i(\overline{R_\iota} - R_f)}{\sigma_{\varepsilon_i}^2} \\ \sum_{i=4}^{6} \frac{\beta_i(\overline{R_\iota} - R_f)}{\sigma_{\varepsilon_i}^2} \\ \sum_{i=7}^{9} \frac{\beta_i(\overline{R_\iota} - R_f)}{\sigma_{\varepsilon_i}^2} \end{pmatrix} =$$

$$\begin{pmatrix} 1 + \left(b_1^2\sigma_m^2 + \sigma_{c_1}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1 b_4 \sigma_m^2 \left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1 b_7 \sigma_m^2 \left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) \\ b_1 b_2 \sigma_m^2 \left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & 1 + \left(b_2^2\sigma_m^2 + \sigma_{c_2}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1 b_7 \sigma_m^2 \left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) \\ b_1 b_2 \sigma_m^2 \left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & b_1 b_4 \sigma_m^2 \left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) & 1 + \left(b_3^2\sigma_m^2 + \sigma_{c_3}^2\right)\left(\frac{\beta_1^2}{\sigma_{\varepsilon_1}^2} + \frac{\beta_2^2}{\sigma_{\varepsilon_2}^2} + \frac{\beta_3^2}{\sigma_{\varepsilon_3}^2}\right) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

QED