

University of California, Los Angeles
Department of Statistics

Statistics C183/C283

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Homework 1

Exercise 1

Answer the following questions.

- a. Assume that the variance of the returns of security A is 0.16 and the variance of security B is 0.25. The variance of a portfolio consisting of 50% A and 50% B is 0.0525. What is the covariance between securities A and B ?
- b. Suppose you are constructing two portfolios using the same n stocks. What is the expression of the covariance between these two portfolios in summations form and in matrix/vector form.

Exercise 2

Answer the following questions.

1. Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz? Please explain your answer.

Portfolio	Expected return (%)	Standard deviation (%)
W	15	36
X	12	15
Y	5	7
Z	9	21

2. Suppose all stocks have $E(R) = 15\%$, $\sigma = 60\%$, and common correlation coefficient $\rho = 0.5$. What are the expected return and standard deviation of an equally weighted portfolio of $n = 25$ stocks?
3. Refer to question (2). What is the smallest number of stocks necessary to generate a portfolio with standard deviation of at most 43%?
4. Refer to question (3). As n gets larger, is it true that $\sigma_p = \sigma\sqrt{\rho}$? Please explain your answer.

Exercise 3

The mean returns and variance covariance matrix of the returns of three stocks (C, XOM, AAPL and the market SP500) are given below:

Mean returns:

C	XOM	AAPL	^GSPC
0.005174	0.010617	0.016947	0.010846

Variance-covariance matrix:

	C	XOM	AAPL	^GSPC
C	0.010025	0.000000	0.000000	0.000000
XOM	0.000000	0.002123	0.000000	0.000000
AAPL	0.000000	0.000000	0.005775	0.000000
^GSPC	0.000000	0.000000	0.000000	0.001217

Assume short sales are allowed. Compute the composition of the minimum risk portfolio using only the three stocks (do not use the SP500).

Exercise 4

Assume that the average variance of the return for an individual security is 50 and that the average covariance is 10. What is the variance of an equally weighted portfolio of 5, 10, 20, 50, and 100 securities?

Exercise 5

What is the composition of the minimum risk portfolio using n risky stocks? Show the entire derivation using matrix/vector notation. Using this composition give the expression for the expected return and variance of the minimum risk portfolio?