University of California, Los Angeles
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Statistics C183/C283 **Instructor: Nicolas Christou**

Homework 3

Exercise 1
Consider two stocks A and B with expected returns $\bar{R}_1, \bar{R}_2$, variances σ_1^2, σ_2^2, and covariance σ_{12}. Suppose short sales are allowed and risk free asset R_f exists. Show that the composition of the optimal portfolio is

$$\begin{aligned} x_1 &= \frac{\bar{R}_A \times \sigma_2^2 - \bar{R}_B \times \sigma_{12}}{\bar{R}_A \times \sigma_2^2 + \bar{R}_B \times \sigma_1^2 - (\bar{R}_A + \bar{R}_B) \times \sigma_{12}} \\ x_2 &= 1 - x_1 \end{aligned}$$

Note: $\bar{R}_A = \bar{R}_1 - R_f$ and $\bar{R}_B = \bar{R}_2 - R_f$.

Exercise 2
Given the following:

Stock	$\bar{R}$	σ
Stock A	0.12	0.20
Stock B	???	0.08

It is also given that $\rho_{AB} = 0.1$.

a. What expected return on stock B would result in an optimum portfolio of $\frac{1}{2}A$ and $\frac{1}{2}B$? Assume short sales are allowed and that $R_f = 0.04$.

b. What expected return on stock B would mean that stock B would not be held? Assume short sales are allowed and that $R_f = 0.04$.

Exercise 3
Use a numerical example of three stocks with a value of R_f of your choice to find the point of tangency G and then (1) combine G with R_f to find portfolio A on CAL and (2) verify that A can be obtained by using the formula for the weights $\mathbf{X}$ when the investor requires $\sum_{i=1}^n (\bar{R}_i - R_f)x_i + R_f = E$, where E is the expected value of portfolio A.

Exercise 4
Answer the following questions:

a. An investor has \$900000 invested in a diversified portfolio. Subsequently the investor inherits `ABC` company stock worth \$100000. His financial adviser provided him with the following forecast information:

	$\bar{R}$ (monthly)	σ (monthly)
Portfolio	0.67%	2.37%
`ABC` Compnay	1.25%	2.95%

The correlation coefficient between `ABC` company stock returns and the portfolio is 0.40.
Assume that the investor keeps the `ABC` company stock. Answer the following questions:

1. Calculate the expected return of the new portfolio which includes the `ABC` company stock.
2. Calculate the covariance between `ABC` company stock and the portfolio.
3. Calculate the standard deviation of his new portfolio which includes the `ABC` company stock.

b. Refer to question (a). If the investor sells the `ABC` company stock, he will invest the proceeds in risk-free government securities yielding 0.42% per month. Calculate the:

1. Expected return of the new portfolio which includes the government securities.
2. The standard deviation of his new portfolio which includes the government securities.

Exercise 5

Answer the following questions:

a. Consider a portfolio consisting of n risky assets. When short sales allowed, the efficient frontier of all feasible portfolios which can be constructed from these n assets is defined as the locus of feasible portfolios that have the smallest variance for a prescribed expected return E is determined by solving the problem

$$\begin{array}{cl} \min & \frac{1}{2}\mathbf{x}'\mathbf{\Sigma}\mathbf{x} \\ \text{subject to} & \bar{\mathbf{R}}'\mathbf{x} = E \\ \text{and} & \mathbf{1}'\mathbf{x} = 1 \end{array}$$

Show that the weights of the optimal portfolio $\mathbf{x}$ is given by $\mathbf{x} = \mathbf{g} + \mathbf{h}E$, where $\mathbf{g}$ and $\mathbf{h}$ are $n \times 1$ vectors, given by

$$\begin{aligned} \mathbf{g} &= \frac{1}{D}\left[B\mathbf{\Sigma}^{-1}\mathbf{1} - A\mathbf{\Sigma}^{-1}\bar{\mathbf{R}}\right] \\ \mathbf{h} &= \frac{1}{D}\left[C\mathbf{\Sigma}^{-1}\bar{\mathbf{R}} - A\mathbf{\Sigma}^{-1}\mathbf{1}\right]. \end{aligned}$$

The scalars A, B, C, D are defined as in the paper "An Analytic Derivation of the Efficient Portfolio Frontier," by Robert Merton.

b. Refer to question (a). Consider two portfolios a, b on the efficient frontier (other than the minimum risk portfolio). Show that the covariance between the two portfolios is given by

$$\text{cov}(R_a, R_b) = \frac{C}{D}\left(E_a - \frac{A}{C}\right)\left(E_b - \frac{A}{C}\right) + \frac{1}{C}.$$