Homework 7

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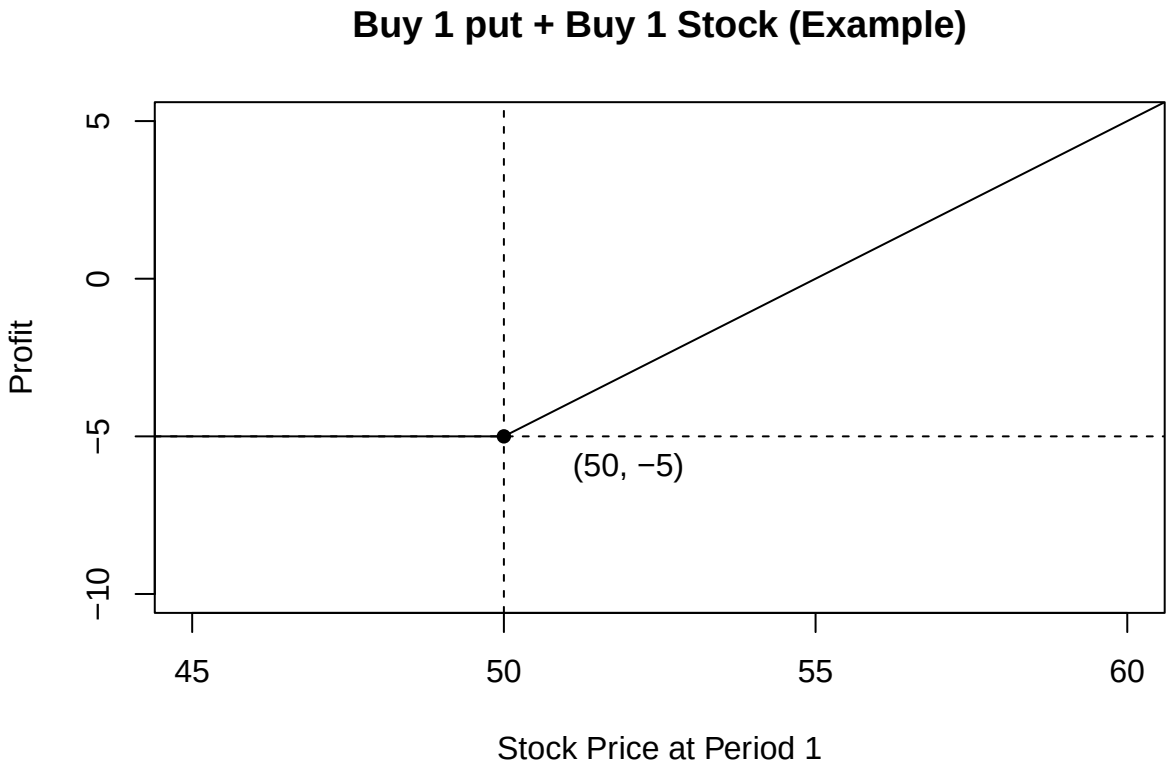
Exercise 1.

a. Buy 1 put, and buy 1 stock.
Let the price of put p, the stock price S_0 and S_1, exercise price E.

Then, (1) if $S_1 \uparrow$, then $-p + (S_1 - S_0)$,
(2) if $S_1 \downarrow$, then $-p + (E - S_1) + (S_1 - S_0) = -p + E - S_0$.

```
x7_hw7 <- seq(30, 50)
y7_hw7 <- rep(-5, length(x7_hw7))

x8_hw7 <- seq(50, 70)
y8_hw7 <- -55 + x8_hw7

plot(c(x7_hw7, x8_hw7), c(y7_hw7, y8_hw7), type='l', xlim=c(45, 60), ylim=c(-10, 5), xlab='Stock Price a
abline(v=50, lty='dashed') ; abline(h=-5, lty='dashed')
points(50,-5, pch=16) ; text(52,-6, '(50, -5)')
```



Suppose you sell 1 call option and sell 1 stock.
Then (1) if $S_1 \uparrow$, then $-c + (S_1 - S_0)$.
(2) if $S_1 \downarrow$, then $-c + (E - S_1) + (S_1 - S_0) = -c + E - S_0$.

It is equivalent to protective put, if put-call parity holds.

b. We don't know the price of call options c_1, c_2, c_3,
but we take long position of two call options at each of 50 and 60,
and take take one short position of call option at 55.

Exercise 2.

a. $S_0 = 110, c = 17, p = 5, r = 0.05, E = 105, t = 1$.
put-call parity is $p + S_0 = c + Ee^{-rt}$, so that
$5 + 110 \neq 17 + 105 * e^{-0.05} = 17 + 99.87909 = 116.87909$.
Thus, call option is overpriced.

If we take short call, long put, and long the stock, then
(1) $S < 105$: $17 + (-5) + (105 - S) + (S - 110) = 7$.
(2) $S > 105$: $17 + (-5) + (S - 110) = S - 98 > 7$.
Thus, there can be riskless profit.

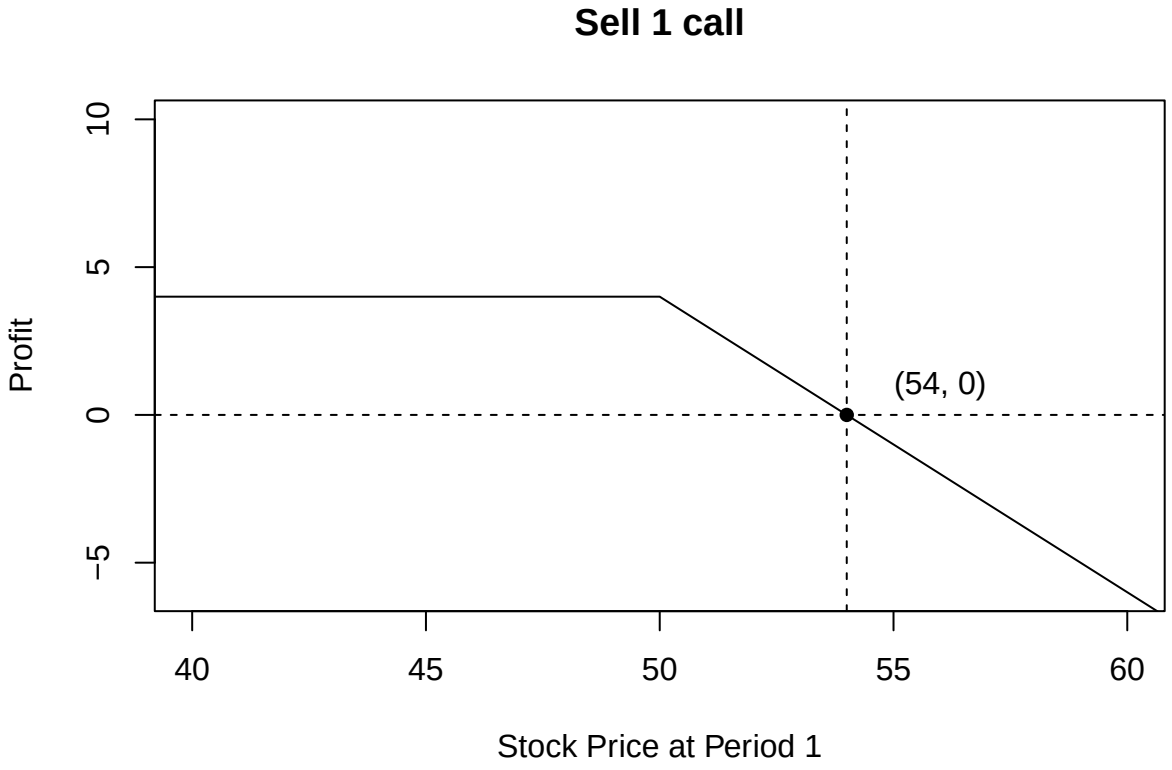
b. $p_A = 4, S_0 = 46, E = 51, t = 1, r = 0.005$.
Lower bound of put is $p \geq Ee^{-rt} - S_0$, so that
$4 \ngeq 51 * e^{-0.005} - 46 = 4.745636$. Thus, it doesn't hold.

In this situation, if we lend 50$ and buy 1 put and 1 stock.
(1) $S_1 < 51$: $S_1 + (51 - S_1) - 50 * 1.005 = 0.75$.
(2) $S_1 > 51$: $S_1 - 50 * 1.005 = S_1 - 50.25 > 0.75$.
Thus, there can be riskless profit.

Exercise 3.

Sell call option at $c = 4, S_0 = 47, E = 50$.
(1) $S_1 < 50$: 4.
(2) $S_1 > 50$: $4 + (50 - S_1) = 54 - S_1$.

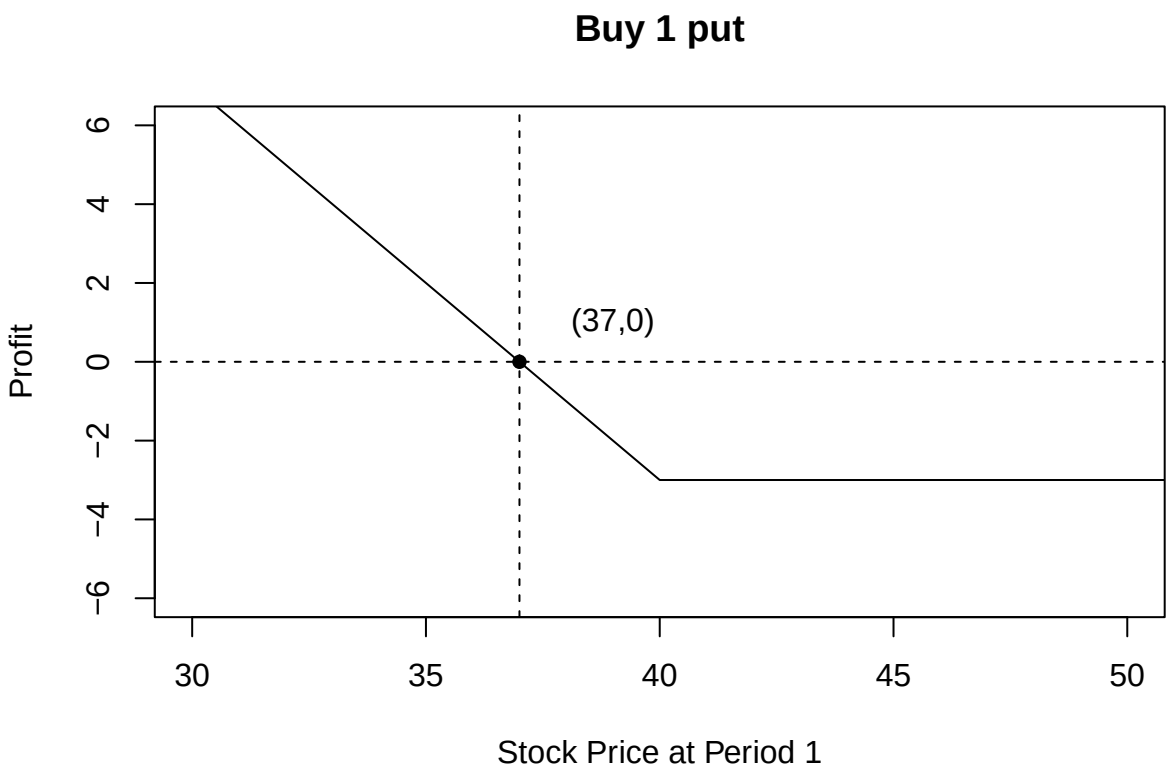
```
x_hw7 <- seq(30, 50)
y_hw7 <- rep(4,21)

x2_hw7 <- seq(50, 70)
y2_hw7 <- 54 - x2_hw7

plot(c(x_hw7,x2_hw7), c(y_hw7,y2_hw7), type='l', xlim=c(40, 60), ylim=c(-6, 10), xlab='Stock Price at P
abline(v=54, lty='dashed')
abline(h=0, lty='dashed')
points(54, 0, pch=16)
text(56, 1, '(54, 0)')
```



Exercise 4

Buy 1 put option at $p = 3, S_0 = 42, E = 40$.
(1) $S_1 < 40$: $(-3) + (40 - S_1) = 37 - S_1$.
(2) $S_1 > 40$: -3.

```
x3_hw7 <- seq(20, 40)
y3_hw7 <- 37 - x3_hw7

x4_hw7 <- seq(40, 60)
y4_hw7 <- rep(-3, length(x4_hw7))

plot(c(x3_hw7, x4_hw7), c(y3_hw7, y4_hw7), type='l', xlim=c(30, 50), ylim=c(-6,6), xlab='Stock Price at
abline(v=37, lty='dashed')
abline(h=0, lty='dashed')
points(37,0,pch=16)
text(39,1, '(37,0)')
```



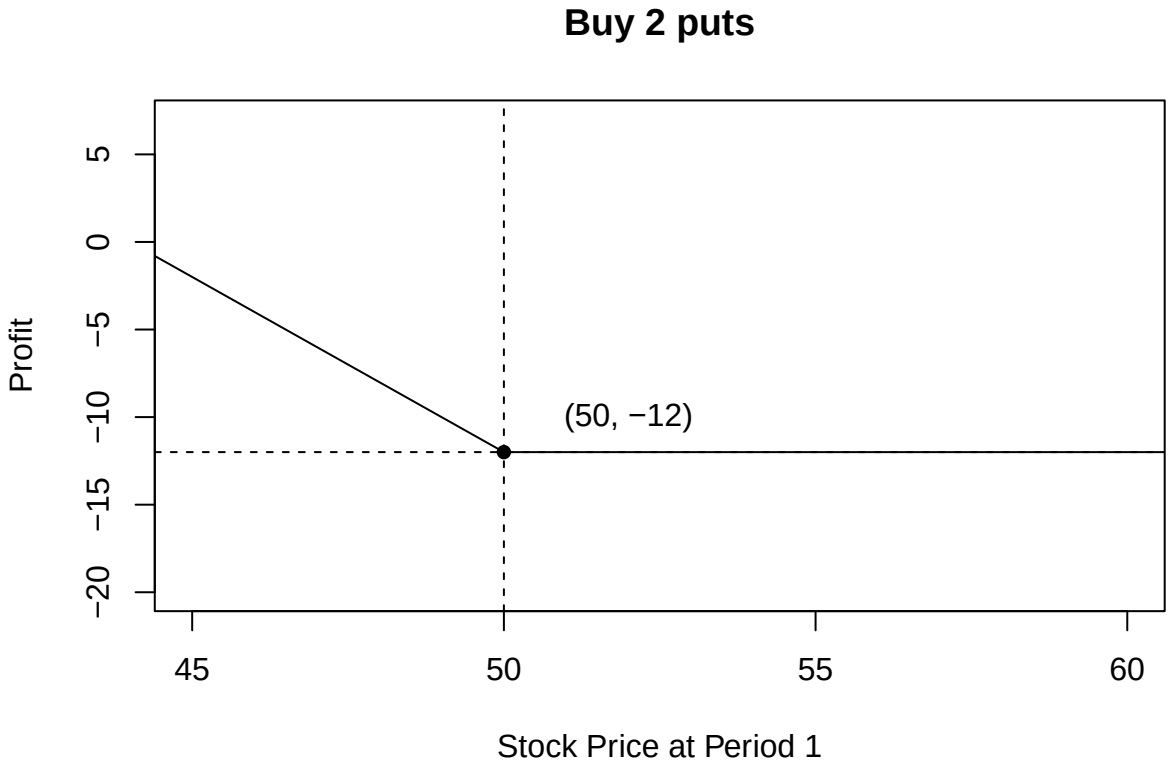
Exercise 5

Buy 2 put at $6 * 2 = 12$, 1 call at 5. Let $E = 50$.

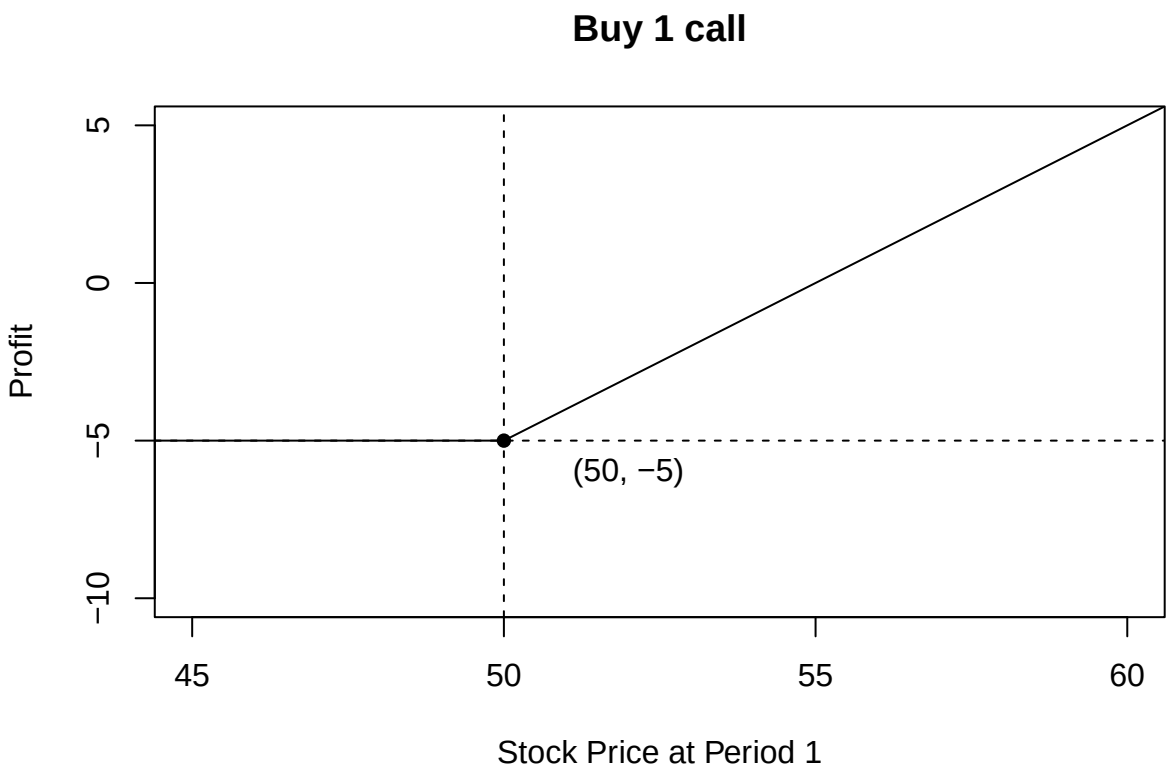
a. (1) $S < 50$: $-12 + 2 * (50 - S) = 88 - 2S$.
(2) $S > 50$: -12.

b. (1) $S < 50$: -5.
(2) $S > 50$: $-5 + (S - 50) = -55 + S$.

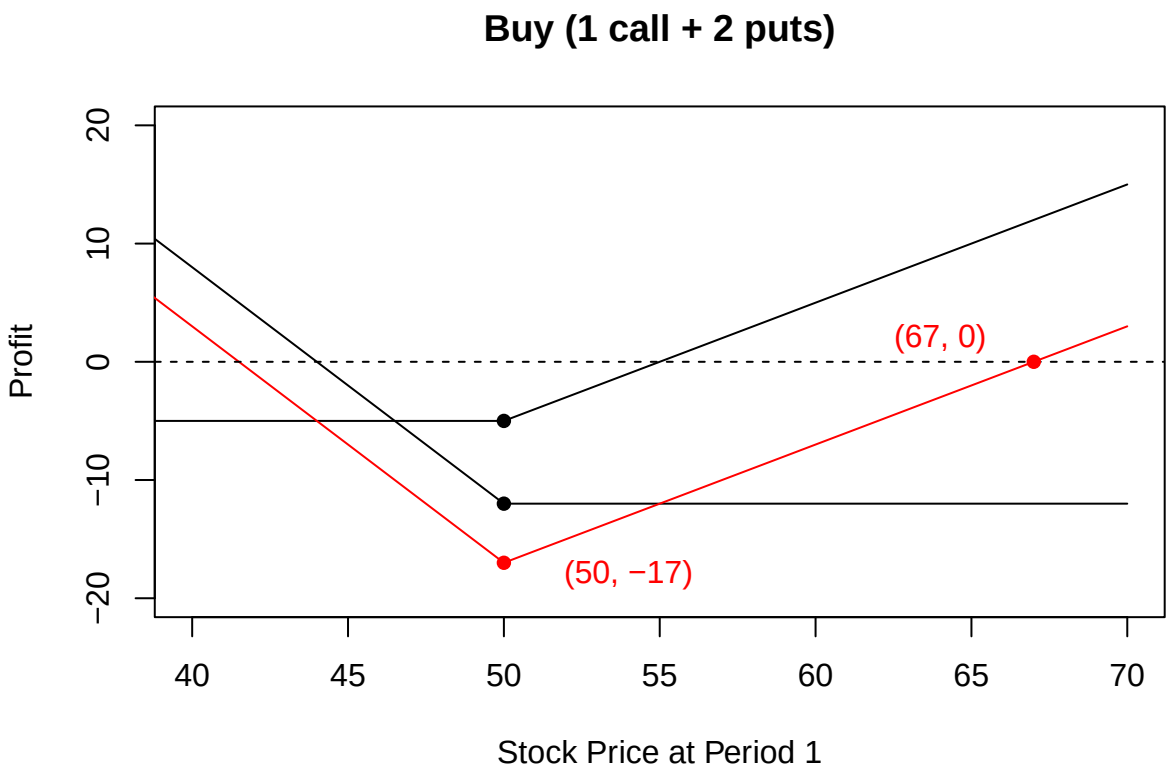
```
x6_hw7 <- seq(50, 70)
y6_hw7 <- rep(-12, length(x6_hw7))

x5_hw7 <- seq(30, 50)
y5_hw7 <- 88 - 2 * x5_hw7

plot(c(x5_hw7, x6_hw7), c(y5_hw7, y6_hw7), type='l', xlim=c(45, 60), ylim=c(-20, 7), xlab='Stock Price a
abline(v=50, lty='dashed')
abline(h=-12, lty='dashed')
points(50,-12,pch=16)
text(52,-10, '(50, -12)')
```




```
x7_hw7 <- seq(30, 50)
y7_hw7 <- rep(-5, length(x7_hw7))

x8_hw7 <- seq(50, 70)
y8_hw7 <- -55 + x8_hw7

plot(c(x7_hw7, x8_hw7), c(y7_hw7, y8_hw7), type='l', xlim=c(45, 60), ylim=c(-10, 5), xlab='Stock Price a
abline(v=50, lty='dashed')
abline(h=-5, lty='dashed')
points(50,-5, pch=16)
text(52,-6, '(50, -5)')
```




```
plot(c(x7_hw7, x8_hw7), c(y7_hw7, y8_hw7), type='l', xlim=c(40, 70), ylim=c(-20, 20), xlab='Stock Price
lines(c(x5_hw7, x6_hw7), c(y5_hw7, y6_hw7))
lines(c(x5_hw7, x6_hw7), c(y5_hw7 + y7_hw7, y6_hw7 + y8_hw7), col='red')
points(50, -5, pch=16)
points(50, -12, pch=16)
points(50, -17, pch=16, col='red')
abline(h=0, lty='dashed')
points(67, 0, pch=16, col='red')
text(54, -18, '(50, -17)', col='red')
text(64, 2, '(67, 0)', col='red')
```



Exercise 6.

Sell two calls at $E_a = 45, C_a = 5$, buy one call at $E_b = 40, C_b = 8$.

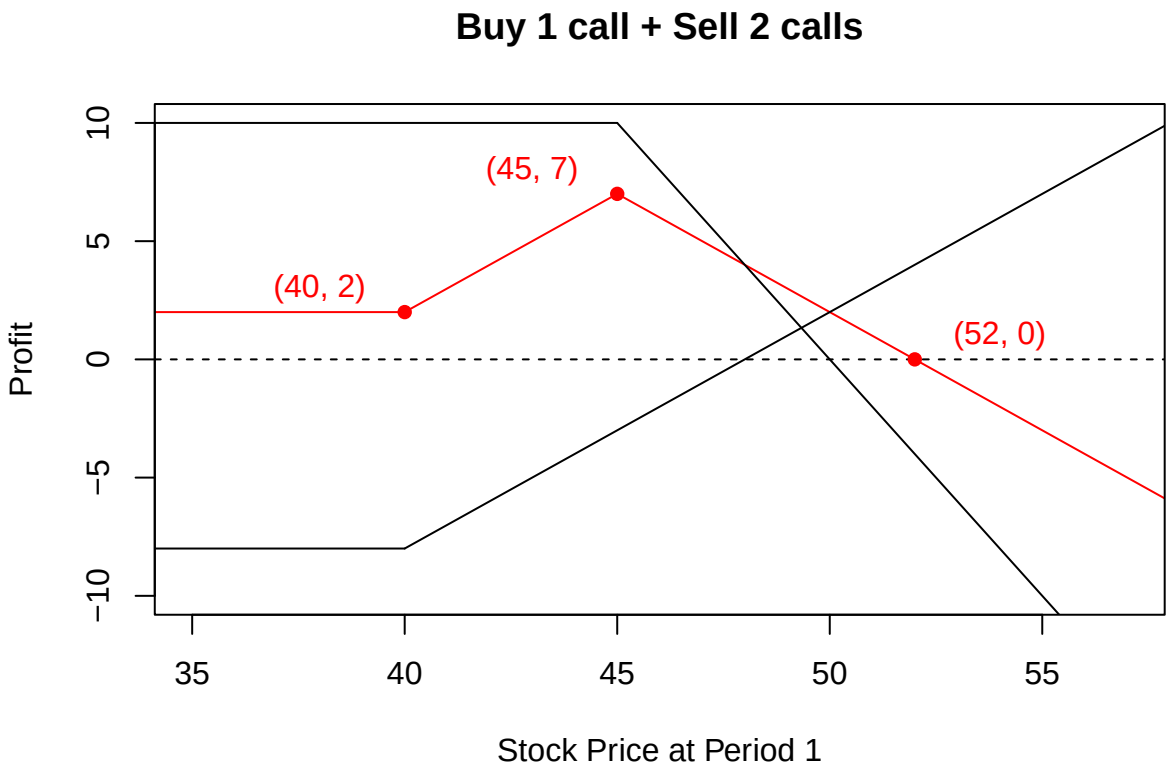
a.
(1) $S < 45$: 10.
(2) $S > 45$: $10 + 2 * (45 - S) = 100 - 2S$.

b.
(1) $S < 40$: -8.
(2) $S > 40$: $-8 + (S - 40) = S - 48$.

a + b.
(1) $S < 40$: 2.
(2) $40 < S < 45$: $S - 48 + 10 = S - 38$.
(3) $S > 45$: $52 - S$.

```
x9_hw7 <- seq(20, 40)
y9_hw7 <- rep(2, length(x9_hw7))

x10_hw7 <- seq(40, 45)
y10_hw7 <- x10_hw7 - 38

x11_hw7 <- seq(45, 60)
y11_hw7 <- 52 - x11_hw7

x12_hw7 <- seq(20, 45)
y12_hw7 <- rep(10, length(x12_hw7))

x13_hw7 <- seq(45, 60)
y13_hw7 <- 100 - 2 * x13_hw7
```

```r
x14_hw7 <- seq(20, 40)
y14_hw7 <- rep(-8, length(x14_hw7))

x15_hw7 <- seq(40, 60)
y15_hw7 <- x15_hw7 - 48

plot(c(x9_hw7, x10_hw7, x11_hw7), c(y9_hw7, y10_hw7, y11_hw7), type='l', col='red', xlim=c(35, 57), yli
     xlab='Stock Price at Period 1', ylab='Profit', main='Buy 1 call + Sell 2 calls')
lines(x12_hw7, y12_hw7)
lines(x13_hw7, y13_hw7)
lines(x14_hw7, y14_hw7)
lines(x15_hw7, y15_hw7)
abline(h=0, lty='dashed')
points(40, 2, pch=16, col='red')
points(45, 7, pch=16, col='red')
points(52, 0, pch=16, col='red')
text(38, 3, '(40, 2)', col='red')
text(43, 8, '(45, 7)', col='red')
text(54, 1, '(52, 0)', col='red')
```

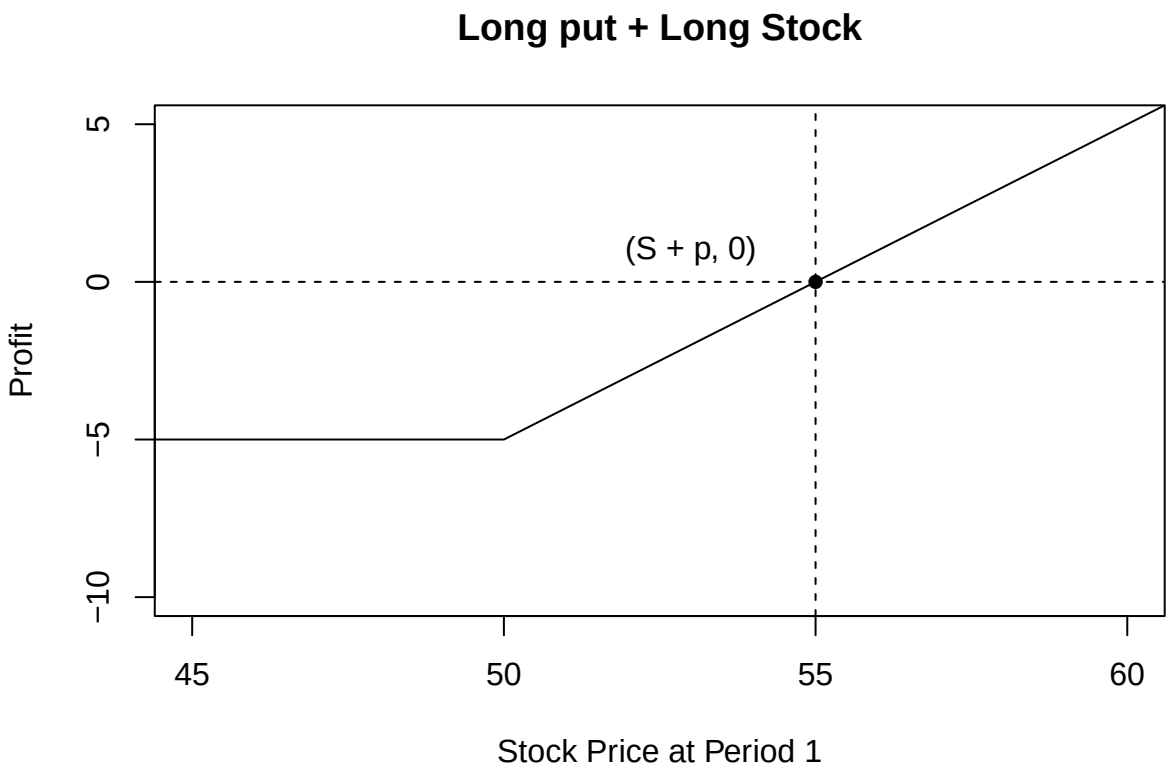


Exercise 7.

$p + S_0 = c + Ee^{-rt}$.

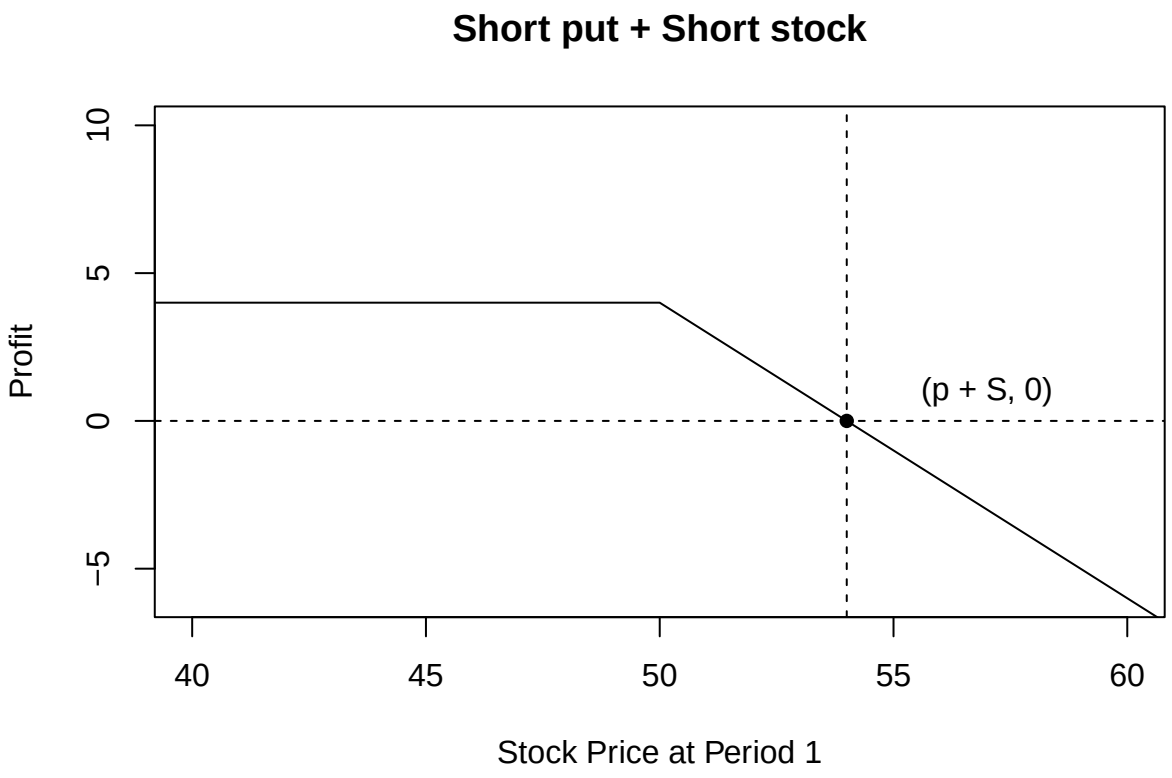
a. Long put, long stock. (1) $S_1 < E$: $-p + (E - S_1) + (S_1 - S_0) = -p + E - S_0$.
(2) $S_1 > E$: $-p + (S_1 - S_0)$.

```
plot(c(x7_hw7, x8_hw7), c(y7_hw7, y8_hw7), type='l', xlim=c(45, 60), ylim=c(-10, 5), xlab='Stock Price a
abline(v=55, lty='dashed')
abline(h=0, lty='dashed')
points(55,0, pch=16)
text(53,1, '(S + p, 0)')
```



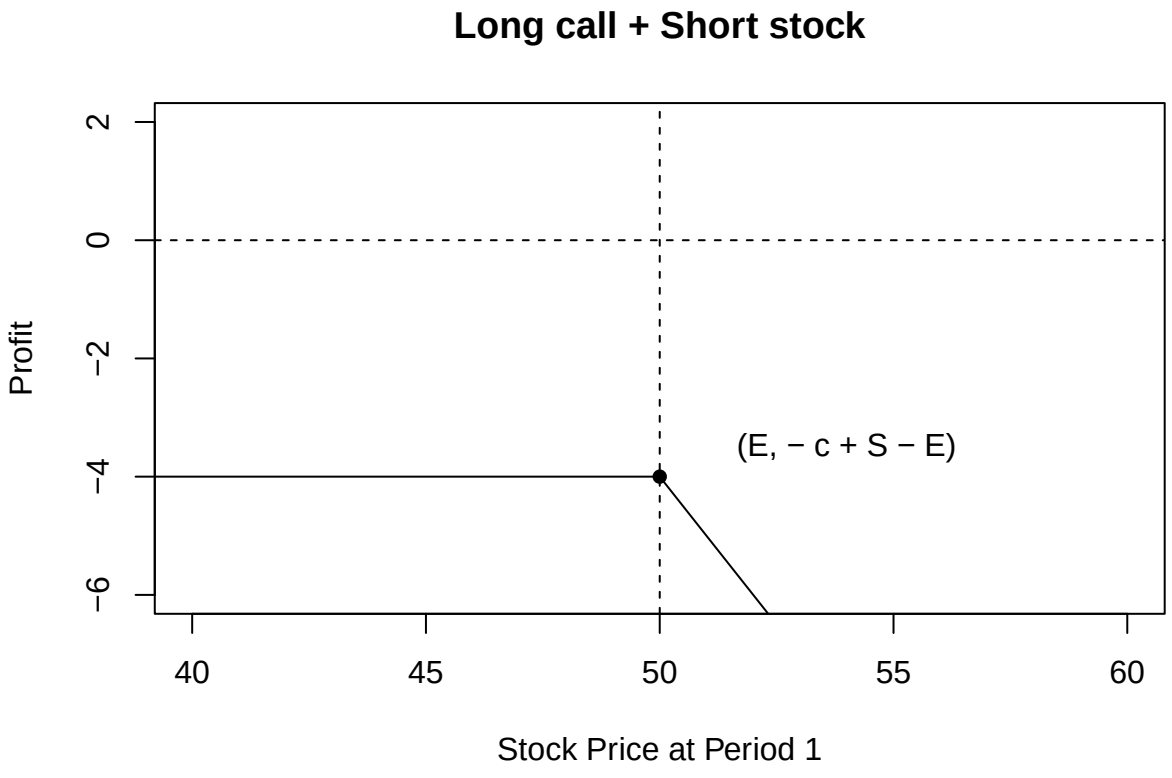
b. Short put, short stock. (1) $S_1 < E$: $p + (S_1 - E) + (S_0 - S_1) = p + S_0 - E$.
(2) $S_1 > E$: $p + (S_0 - S_1)$.

```
plot(c(x_hw7,x2_hw7), c(y_hw7,y2_hw7), type='l', xlim=c(40, 60), ylim=c(-6, 10), xlab='Stock Price at P
abline(v=54, lty='dashed')
abline(h=0, lty='dashed')
points(54, 0, pch=16)
text(57, 1, '(p + S, 0)')
```



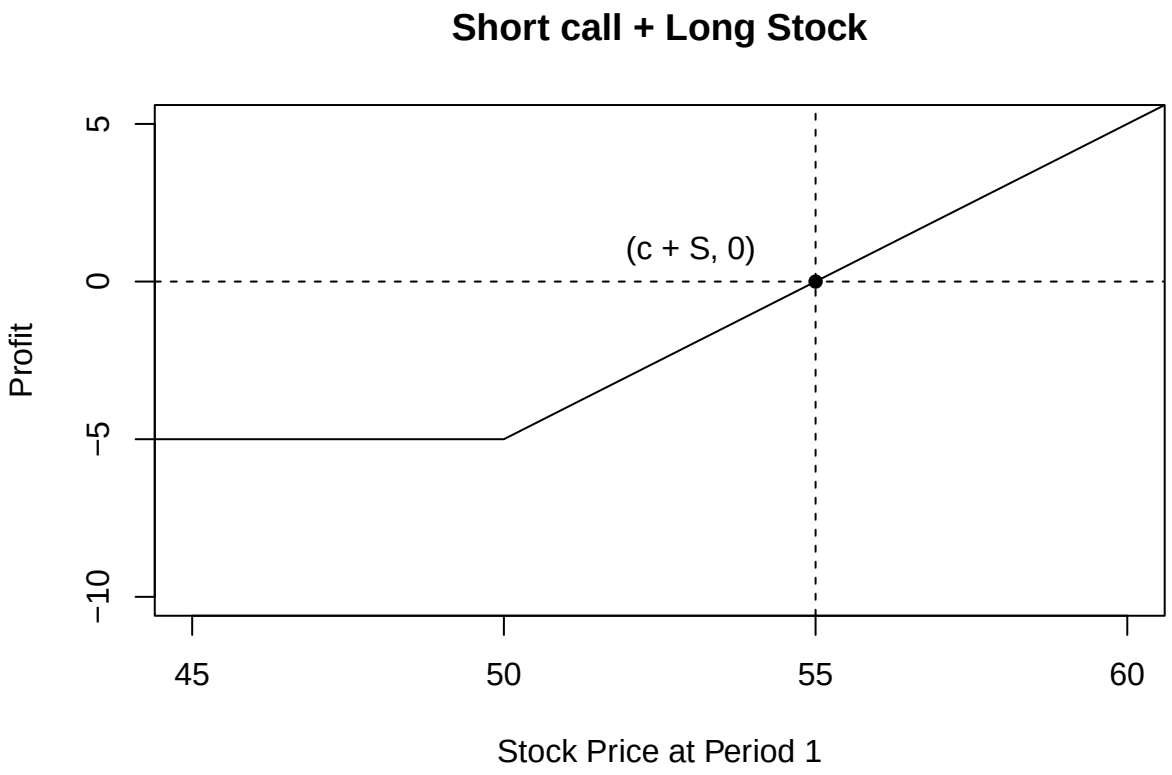
c. Long call, short stock. (1) $S_1 < E$: $-c + (S_0 - S_1)$.
(2) $S_1 > E$: $-c + (S_1 - E) + (S_0 - S_1) = -c + S_0 - E$.

```
plot(c(x_hw7,x2_hw7), c(y_hw7 - 8,y2_hw7 - 8), type='l', xlim=c(40, 60), ylim=c(-6, 2), xlab='Stock Pri
abline(v=50, lty='dashed') ; abline(h=0, lty='dashed') ; points(50, -4, pch=16) ; text(54, -3.5, '(E, -
```



d. Short call, long stock. (1) $S_1 < E$: $-c + (S_1 - S_0)$.
(2) $S_1 > E$: $-c + (E - S_1) + (S_1 - S_0) = -c + E - S_0$.

```
plot(c(x7_hw7, x8_hw7), c(y7_hw7, y8_hw7), type='l', xlim=c(45, 60), ylim=c(-10, 5), xlab='Stock Price a
abline(v=55, lty='dashed')
abline(h=0, lty='dashed')
points(55,0, pch=16)
text(53,1, '(c + S, 0)')
```



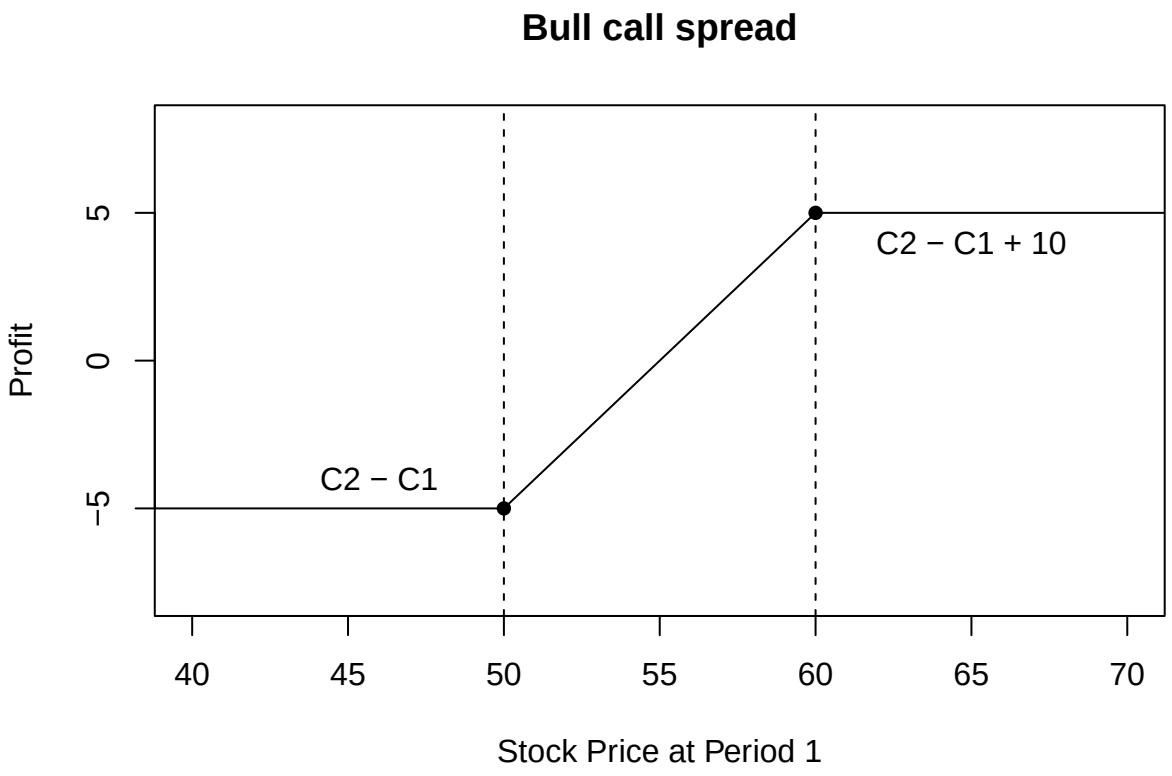
Exercise 8.

a. Bull call spread Buy one call at $c_1, E = 50$, sell one call at $c_2, E = 60$.
Larger the exercise price, then cheaper option price, because it is the right to buy more expensive option. Thus, suppose $c_1 > c_2$.

(1) $S_1 < 50$: $-c_1 + c_2$.
(2) $50 < S_1 < 60$: $-c_1 + (S_1 - 50) + c_2$.
(3) $S_1 > 60$: $-c_1 + (S_2 - 50) + c_2 + (60 - S_2) = (c_2 - c_1) + 10$.

```
x16_hw7 <- seq(-10, 50)
y16_hw7 <- rep(-5, length(x16_hw7))

x17_hw7 <- seq(50, 60)
y17_hw7 <-  -5 - 50 + x17_hw7

x18_hw7 <- seq(60, 90)
y18_hw7 <- rep(5, length(x18_hw7))

plot(c(x16_hw7, x17_hw7, x18_hw7), c(y16_hw7, y17_hw7, y18_hw7), type='l', xlim=c(40, 70), ylim=c(-8, 8
     xlab='Stock Price at Period 1', ylab= 'Profit', main='Bull call spread')
abline(v=50, lty='dashed')
```

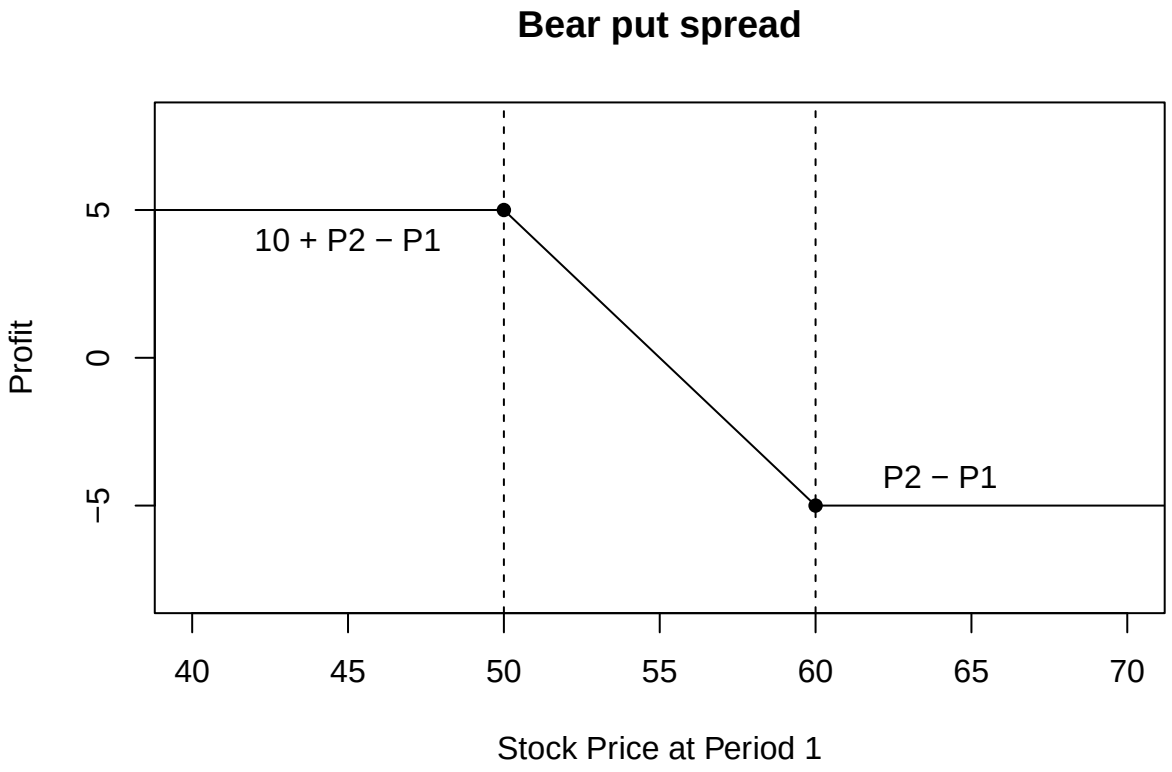
```
abline(v=60, lty='dashed')
points(50,-5, pch=16)
points(60, 5, pch=16)
text(46, -4, 'C2 - C1')
text(65, 4, 'C2 - C1 + 10')
```



b. Bear put spread Buy one put at $p_1, E = 60$, sell one put at $p_2, E = 50$.
Larger the exercise price, more expensive the option price, because it is the right to sell at that level of price. Thus, suppose $p_1 > p_2$.

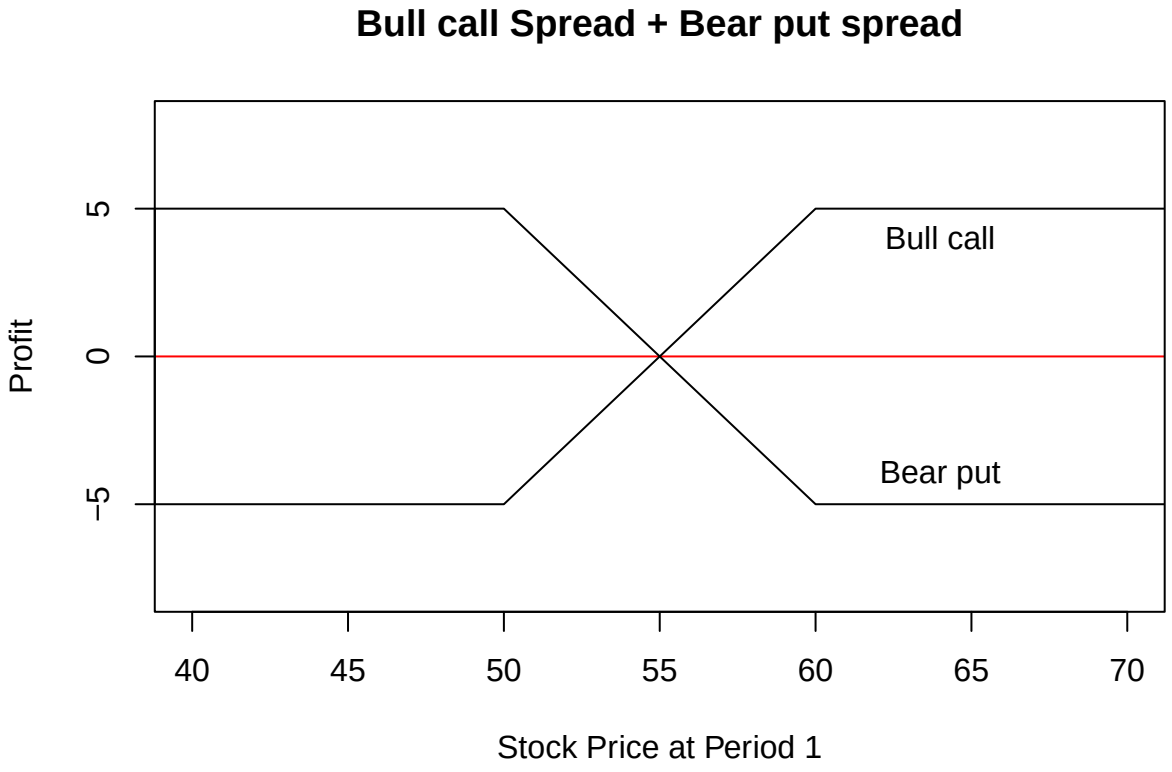
(1) $S_1 < 50$: $-p_1 + (60 - S_1) + p_2 + (S_1 - 50) = 10 + (p_2 - p_1)$.
(2) $50 < S_1 < 60$: $-p_1 + (60 - S_1) + p_2$.
(3) $S_1 > 60$: $-p_1 + p_2$.

```
plot(c(x16_hw7, x17_hw7, x18_hw7), c(y16_hw7 + 10, (-1) * y17_hw7, y18_hw7 - 10), type='l', xlim=c(40, 
     xlab='Stock Price at Period 1', ylab= 'Profit', main='Bear put spread')
abline(v=50, lty='dashed')
abline(v=60, lty='dashed')
points(50, 5, pch=16)
points(60, -5, pch=16)
text(45, 4, '10 + P2 - P1')
text(64, -4, 'P2 - P1')
```



c. Bull call spread + Bear put spread

```
plot(c(x16_hw7, x17_hw7, x18_hw7), c(y16_hw7 + y16_hw7 + 10, y17_hw7 + (-1) * y17_hw7, y18_hw7 + y18_hw
     xlab='Stock Price at Period 1', ylab= 'Profit', main='Bull call Spread + Bear put spread', col='re
lines(c(x16_hw7, x17_hw7, x18_hw7), c(y16_hw7, y17_hw7, y18_hw7), type='l') ; lines(c(x16_hw7, x17_hw7,
```



Exercise 9.

Sell put option at $p_2, E_1 = 50$, buy one put option at $p_1, E_2 = 60$, then $E_2 > E_1$.
This is same with the case of Exercise 8. a. Bear put spread,
if we take $p_2 - p_1 = 5$. QED

Exercise 10.

Sell one call at $c_1, E_1 = 50$, buy one call at $c_2, E_2 = 60$.
Suppose $c_1 > c_2$.

(1) $S_1 < 50$: $c_1 - c_2$.
(2) $50 < S_1 < 60$: $c_1 + (50 - S_1) - c_2$.
(3) $S_1 > 60$: $c_1 + (50 - S_1) - c_2 + (S_1 - 60) = c_1 - c_2 - 10$.

```
plot(c(x16_hw7, x17_hw7, x18_hw7), c(y16_hw7 + 10, (-1) * y17_hw7, y18_hw7 - 10), type='l', xlim=c(40, 
     xlab='Stock Price at Period 1', ylab= 'Profit', main='Sell one call + Buy one call')
abline(v=50, lty='dashed')
abline(v=60, lty='dashed')
points(50, 5, pch=16)
points(60, -5, pch=16)
text(46, 4, 'C1 - C2')
text(65, -4, 'C1 - C2 - 10')
```