

Homework 2

Exercise 1

Refer to the lecture material on Friday, 04/02. In order to find the Lagrange multipliers λ_1 and λ_2 we must invert the matrix $\begin{pmatrix} B & A \\ A & C \end{pmatrix}$, where $A = \mathbf{1}'\Sigma^{-1}\bar{\mathbf{R}}$, $B = \bar{\mathbf{R}}'\Sigma^{-1}\bar{\mathbf{R}}$, and $C = \mathbf{1}'\Sigma^{-1}\mathbf{1}$. Show that $BC - A^2 > 0$. Note: Begin with $(A\bar{\mathbf{R}} - B\mathbf{1})'\Sigma^{-1}(A\bar{\mathbf{R}} - B\mathbf{1}) > 0$ because Σ is positive definite matrix.

Exercise 2

In the paper “An Analytic Derivation of the Efficient Portfolio Frontier,” *The Journal of Financial and Quantitative Analysis*, Vol. 7, No. 4, Robert Merton gives on page 1854 the proportion of the k_{th} risky asset held in the frontier portfolio with expected return E by

$$x_k = \frac{E \sum_{j=1}^m v_{kj}(CE_j - A) + \sum_{j=1}^m v_{kj}(B - AE_j)}{D}, \quad k = 1, \dots, m. \quad (1)$$

Prove equation (1).

On the same page, it is shown that the expected return of the minimum risk portfolio is $\bar{E} = \frac{A}{C}$. Using equation (1) above show that the proportion of the k_{th} risky asset of the minimum risk portfolio is $x_k = \frac{\sum_{j=1}^m v_{kj}}{C}$, $k = 1, \dots, m$.

Exercise 3

Find an expression of the correlation coefficient of two portfolios on the efficient frontier. See homework 1 for the covariance between two portfolios.

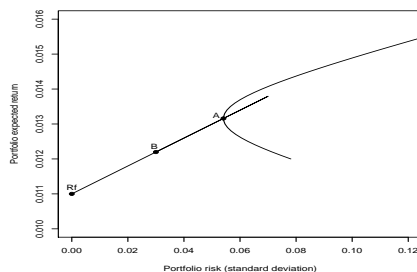
Exercise 4

The covariance matrix \mathbf{Q} of the returns of two stocks has the following inverse:

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> solve(Q)
      [,1]      [,2]
[1,] 166.21139 -22.40241
[2,] -22.40241 220.41076
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Answer the following questions:

- Find the composition of the minimum risk portfolio.
- It is given that the minimum risk portfolio (point A on the graph below) has standard deviation equal to 0.05408825 and expected return equal to 0.01315856. Portfolio B (see graph below) has expected return equal to 0.01219724. What is the composition of portfolio B in terms of portfolio A and the risk free asset? Assume $R_f = 0.011$.



- The standard deviation of portfolio B is equal to 0.03. Given this level of risk, can you do better than the expected return of portfolio B? Please explain.

Exercise 5

Show that two portfolios on the capital allocation line are perfectly correlated.