In [ ]:
```
import numpy as np
import pandas as pd
from pandas import Series, DataFrame

import matplotlib
import matplotlib.pyplot as plt
import seaborn as sns

import scipy.stats as st
import statsmodels.formula.api as smf
import statsmodels.api as sm
```

2.

(a) Find a 95% confidence interval for the slope of the regression model, β_2. On the basis of this confidence interval decide whether there is evidence of a significant negative linear association.

In [ ]:
```
indicators = pd.read_table("/Users/user/Desktop/Yonsei/Junior/3-2/Introduction to Data Analysis and Regression/Homework/indicators2.txt", sep='\t')
```

In [ ]:
```
indicators2 = indicators[['PriceChange','LoanPaymentsOverdue']]
indicators2.head()
```

Out [ ]:

	PriceChange	LoanPaymentsOverdue
Atlanta	1.2	4.55
Boston	-3.4	3.31
Chicago	-0.9	2.99
Dallas	0.8	4.26
Denver	-0.7	3.56

In [ ]:
```
indicators2['PriceChange'].mean()
```

Out [ ]:
```
-3.4277777777777778
```

In [ ]:
```
indicators2['LoanPaymentsOverdue'].mean()
```

Out [ ]:
```
3.532222222222222
```

In [ ]:
```
len(indicators2.index)
```

Out [ ]:
```
18
```

In [ ]:
```
SXX_hw2_2 = ((indicators2['LoanPaymentsOverdue'] - indicators2['LoanPaymentsOverdue'].mean())**2).sum()
SXX_hw2_2
```

Out [ ]:
```
19.160111111111111
```

In [ ]:
```
SXY_hw2_2 = ((indicators2['PriceChange'] - indicators2['PriceChange'].mean())**2).sum()
SXY_hw2_2
```

Out [ ]:
```
347.01611111111111
```

In [ ]:
```
SXY_hw2_2 = ((indicators2['LoanPaymentsOverdue']
              - indicators2['LoanPaymentsOverdue'].mean())*(indicators2['PriceChange']
                                                            - indicators2['PriceChange'].mean())).sum()
SXY_hw2_2
```

Out [ ]:
```
-43.081888888889
```

In [ ]:
```
beta1_hw2_2 = SXY_hw2_2 / SXX_hw2_2
beta1_hw2_2
```

Out [ ]:
```
-2.2485197833578152
```

In [ ]:
```
beta0_hw2_2 = indicators2['PriceChange'].mean() - beta1_hw2_2 * indicators2['LoanPaymentsOverdue'].mean()
beta0_hw2_2
```

Out [ ]:
```
4.514493768883272
```

In [ ]:
```
s_hw2_2 = (((indicators2['PriceChange'] - beta0_hw2_2 - beta1_hw2_2 * indicators2['LoanPaymentsOverdue'])**2).sum() / (len(indicators2.index)-2))**(1/2)
s_hw2_2
```

Out [ ]:
```
3.9539902267202397
```

In [ ]:
```
st.t.ppf(0.975, len(indicators2.index)-2)
```

Out [ ]:
```
2.1199052992210112
```

In [ ]:
```
beta1_hw2_2 - st.t.ppf(0.975, len(indicators2.index)-2) * s_hw2_2 / (SXX_hw2_2)**(1/2)
```

Out [ ]:
```
-4.163454263593569
```

In [ ]:
```
beta1_hw2_2 + st.t.ppf(0.975, len(indicators2.index)-2) * s_hw2_2 / (SXX_hw2_2)**(1/2)
```

Out [ ]:
```
-0.3335853035627354
```

Thus, the 95% confidence interval is (-4.163454263593569, -0.3335853035627354).

In [ ]:
```
Estimate4_hw2_2 = beta0_hw2_2 + beta1_hw2_2 * 4
Estimate4_hw2_2
```

Out [ ]:
```
-4.479585354429337
```

In [ ]:
```
lm_model_hw2_2 = smf.ols(formula='PriceChange ~ LoanPaymentsOverdue', data=indicators2).fit()
```

In [ ]:
```
lm_model_hw2_2.summary()
```

Out [ ]:
```
/Users/user/opt/anaconda3/lib/python3.9/site-packages/scipy/stats/stats.py:1541: UserWarning: kurtosistest only valid for n>=20 ... continuing anyway, n=18
  warnings.warn("kurtosistest only valid for n>=20 ... continuing "
```

OLS Regression Results

Dep. Variable:	PriceChange	**R-squared:**	0.279
Model:	OLS	**Adj. R-squared:**	0.234
Method:	Least Squares	**F-statistic:**	6.196
Date:	Sat, 21 Jan 2023	**Prob (F-statistic):**	0.0242
Time:	23:16:51	**Log-Likelihood:**	-49.226
No. Observations:	18	**AIC:**	102.5
Df Residuals:	16	**BIC:**	104.2
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	4.5145	3.324	1.358	0.193	-2.532	11.561
LoanPaymentsOverdue	-2.2485	0.903	-2.489	0.024	-4.163	-0.334

Omnibus:	2.121	**Durbin-Watson:**	2.009
Prob(Omnibus):	0.346	**Jarque-Bera (JB):**	1.403
Skew:	0.448	**Prob(JB):**	0.496
Kurtosis:	1.966	**Cond. No.**	14.0

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

(b) Use the fitted regression model to estimate $E(Y|X = 4)$. Find a 95% confidence interval for $E(Y|X = 4)$. Is 0% a feasible value for $E(Y|X = 4)$? Give a reason to support your answer.

In [ ]:
```
lm_model_hw2_2.params
```

Out [ ]:
```
Intercept            4.514494
LoanPaymentsOverdue   -2.248520
dtype: float64
```

In [ ]:
```
lm_model_hw2_2.predict(DataFrame({"LoanPaymentsOverdue": [4]}))
```

Out [ ]:
```
0   -4.479585
dtype: float64
```

In [ ]:
```
lm_model_hw2_2.bse.LoanPaymentsOverdue
```

Out [ ]:
```
0.9033113322218041
```

In [ ]:
```
lm_model_hw2_2.predict(DataFrame({"LoanPaymentsOverdue": [4]})) - st.t.ppf(0.975, len(indicators2.index)-2) * s_hw2_2 * ((1/len(indicators2.index) + (4 - indicators2[
```

Out [ ]:
```
0   -6.648849
dtype: float64
```

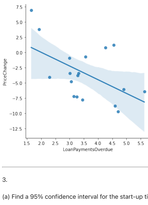
In [ ]:
```
lm_model_hw2_2.predict(DataFrame({"LoanPaymentsOverdue": [4]})) + st.t.ppf(0.975, len(indicators2.index)-2) * s_hw2_2 * ((1/len(indicators2.index) + (4 - indicators2[
```

Out [ ]:
```
0   -2.310322
dtype: float64
```

Thus, the 95% confidence interval for beta1 where x=4 is (-6.39386, -2.565311).
Therefore, 0% = 0.00 is not a feasible value for E(Y|X=4).

In [ ]:
```
sns.lmplot(x='LoanPaymentsOverdue', y='PriceChange', data=indicators)
```

Out [ ]:
```
<seaborn.axisgrid.FacetGrid at 0x7fdd1c03f160>
```



3.

(a) Find a 95% confidence interval for the start-up time, i.e. β_0.

In [ ]:
```
invoices = pd.read_table("/Users/user/Desktop/Yonsei/Junior/3-2/Introduction to Data Analysis and Regression/Homework/invoices.txt", sep='\t')
```

In [ ]:
```
invoices.head()
```

Out [ ]:

	Day	Invoices	Time
0	1	149	2.1
1	2	60	1.8
2	3	188	2.3
3	4	23	0.8
4	5	201	2.7

In [ ]:
```
lm_model_hw2_3 = smf.ols('Time~Invoices', data=invoices).fit()
```

In [ ]:
```
lm_model_hw2_3.summary()
```

Out [ ]:

OLS Regression Results

Dep. Variable:	Time	**R-squared:**	0.872
Model:	OLS	**Adj. R-squared:**	0.867
Method:	Least Squares	**F-statistic:**	190.4
Date:	Sat, 21 Jan 2023	**Prob (F-statistic):**	5.17e-14
Time:	23:19:01	**Log-Likelihood:**	-8.2528
No. Observations:	30	**AIC:**	20.51
Df Residuals:	28	**BIC:**	23.31
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.6417	0.122	5.248	0.000	0.391	0.892
Invoices	0.0113	0.001	13.797	0.000	0.010	0.013

Omnibus:	2.815	**Durbin-Watson:**	1.760
Prob(Omnibus):	0.245	**Jarque-Bera (JB):**	1.341
Skew:	-0.042	**Prob(JB):**	0.511
Kurtosis:	1.968	**Cond. No.**	303.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Thus, the 95% confidence interval for beta0 = (0.391, 0.892).

(b) Suppose that $\beta_1 = 0.01$. Test the null hypothesis $H_0 : \beta_1 = 0.01$ against a two-sided alternative.

In [ ]:
```
lm_model_hw2_3.HC3_se.Invoices
```

Out [ ]:
```
0.0007872634462536831
```

In [ ]:
```
(lm_model_hw2_3.params.Invoices - 0.01) / lm_model_hw2_3.HC3_se.Invoices
```

Out [ ]:
```
1.4406758109364413
```

In [ ]:
```
st.t.ppf(0.975, len(invoices.index)-2)
```

Out [ ]:
```
2.048407141795244
```

Thus, 1.64 < 2.04, so that we cannot reject the null.
Therefore, we cannot say that $\beta_1 = 0.01$.

(c) Find a point estimate and a 95% prediction interval for the time taken to process 130 invoices.

In [ ]:
```
lm_model_hw2_3.predict(DataFrame({"Invoices":[130]}))
```

Out [ ]:
```
0    2.109624
dtype: float64
```

In [ ]:
```
SXX_hw2_3 =((invoices['Invoices'] - invoices['Invoices'].mean())**2).sum()
SXX_hw2_3
```

Out [ ]:
```
162366.96666666662
```

In [ ]:
```
s_hw2_3 = (sum((lm_model_hw2_3.resid)**2) / (len(invoices.index)-2))**(1/2)
s_hw2_3
```

Out [ ]:
```
0.32977333324855559
```

In [ ]:
```
lm_model_hw2_3.resid.std() # a little bit different from residual standard error.
```

Out [ ]:
```
0.32403770733104922
```

In [ ]:
```
se130_hw2_3 = s_hw2_3 * (1 + (1/len(invoices.index)) + (((130 - invoices['Invoices'].mean())**2) / SXX_hw2_3))**(1/2)
se130_hw2_3
```

Out [ ]:
```
0.3352245025503446
```

In [ ]:
```
print(lm_model_hw2_3.predict(DataFrame({"Invoices":[130]})) - st.t.ppf(0.975, len(invoices.index)-2) * se130_hw2_3)
print(lm_model_hw2_3.predict(DataFrame({"Invoices":[130]})) + st.t.ppf(0.975, len(invoices.index)-2) * se130_hw2_3)
```

Out [ ]:
```
0    1.422947
dtype: float64
0    2.79631
dtype: float64
```

Thus, the point estimate is 2.109624, and 95% prediction interval is (1.422947, 2.7963).

In [ ]:
```
sns.lmplot(x='Invoices', y='Time', data=invoices)
```

Out [ ]:
```
<seaborn.axisgrid.FacetGrid at 0x7fdd381ec850>
```

In [ ]: