hw9

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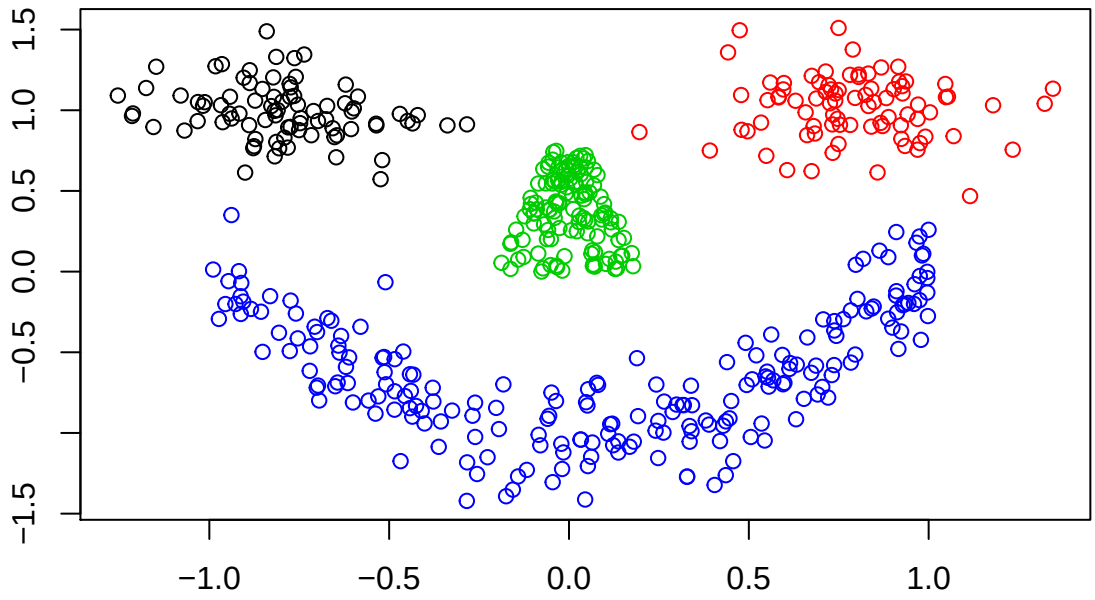
Extra 67

Make 500 smiley data points with sd1=sd2=.2

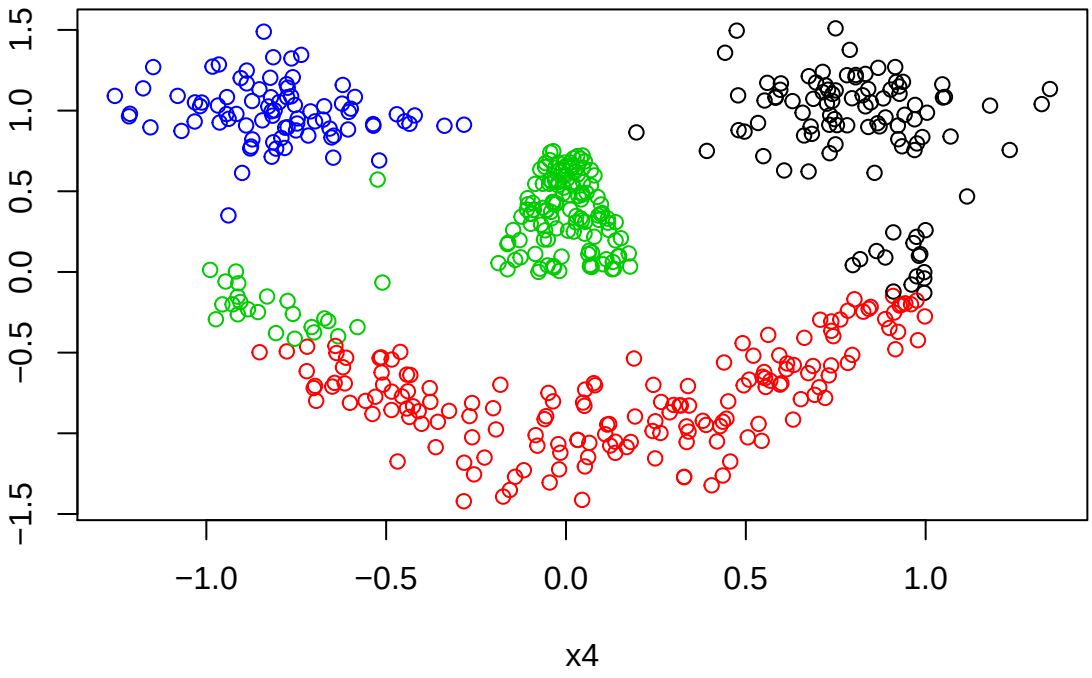
(a)

Demonstrate with a colored plot that k means with 4 clustesr is incapable of recovering the 4 original clusters exactly. Do another run of k-means and use a confusion matrix to show that the 4 original clusters are not recovered exactly.

```
set.seed(100)
test.smiley <- mlbench.smiley(n=500, sd1=.2, sd2=.2)
plot(test.smiley)
```




```
km.out <- kmeans(test.smiley$x, 4, nstart=10)
plot(x=test.smiley$x, col=(km.out$cluster))
```




```
table(actual=test.smiley$classes, pred=km.out$cluster)
```

```
##       pred
## actual   1   2   3   4
##      1   0   0   1  82
##      2  83   0   0   0
##      3   0   0 125   0
##      4  16 168  24   1
```

(b)

Try to use hierarchical clustering with a suitable choice of linkage to recover the 4 clusters. Explain your choice of linkage. Use a confusion matrix to show whether this attempt is successful.

After testing all 3 different linkage methods, the best method was found to be complete. The confusion matrix shows that this attempt was almost successful.

```
hc <- hclust(dist(test.smiley$x), method='complete')
pred.label <- cutree(hc, 4)
table(test.smiley$classes, pred.label)
```

```
##    pred.label
##       1   2   3   4
##   1  83   0   0   0
##   2   0  83   0   0
##   3   0   0 125   0
##   4   1   0 123  85
```

Book 2

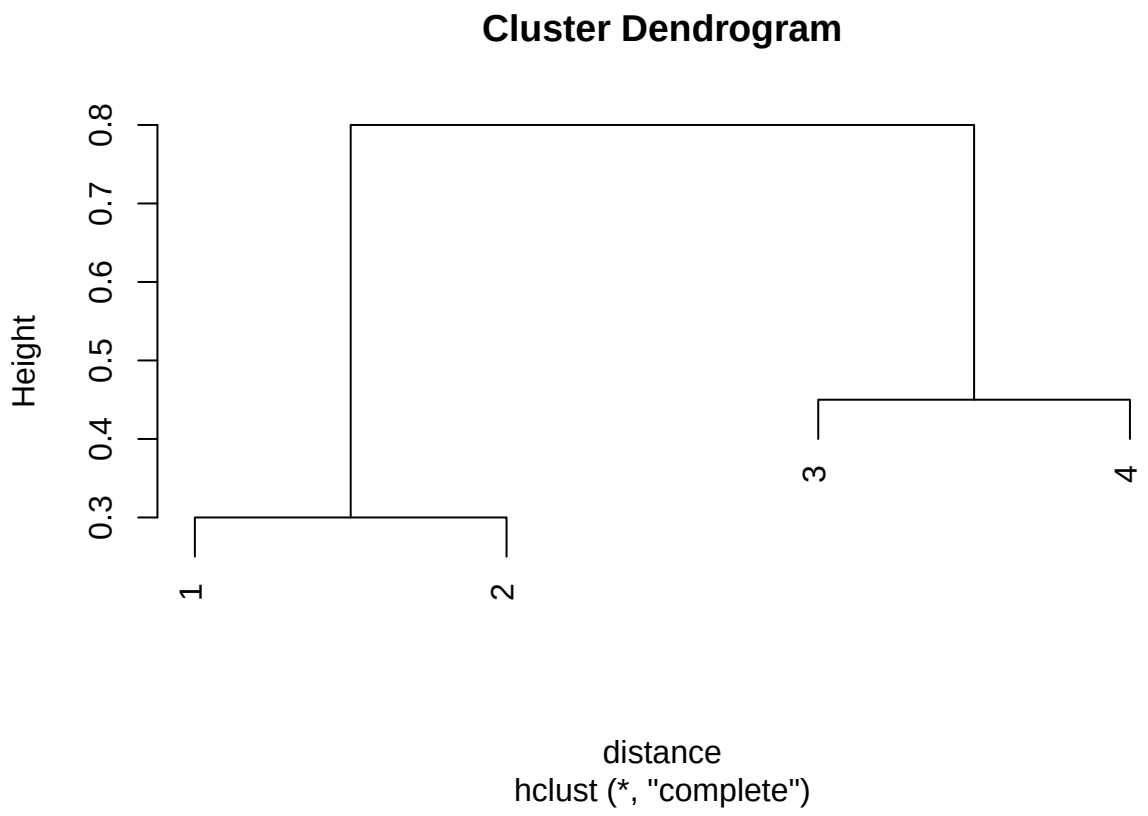
Suppose that we have 4 observations, for which we compute a dissimilarity matrix, given by picture in book. For instance, the dissimilarity between the 1st and second observations is .3, and the dissimilarity between the 2nd and 4th observations is .8.

(a)

On the basis of this dissimilarity matrix, sketch the dendrogram that results from hierarchically clustering these 4 observations using complete linkage. Be sure to indicate on the plot the height at which each fusion occurs, as well as the observations corresponding to each leaf in the dendrogram.

0.3 is the minimum dissimilarity, so fuse observations 1 and 2 to form cluster (1,2) at hight 0.3. Now, the minimum dissimilarity is 0.45, so fuse observations 3 and 4 to form cluster (3,4) at height 0.45. Lastly, fuse clusters (1,2) and (3,4) to form cluster ((1,2),(3,4)) at height 0.8.

```
distance = as.dist(matrix(c(0, 0.3, 0.4, 0.7,
                    0.3, 0, 0.5, 0.8,
                    0.4, 0.5, 0.0, 0.45,
                    0.7, 0.8, 0.45, 0.0), nrow = 4))
plot(hclust(distance, method = "complete"))
```



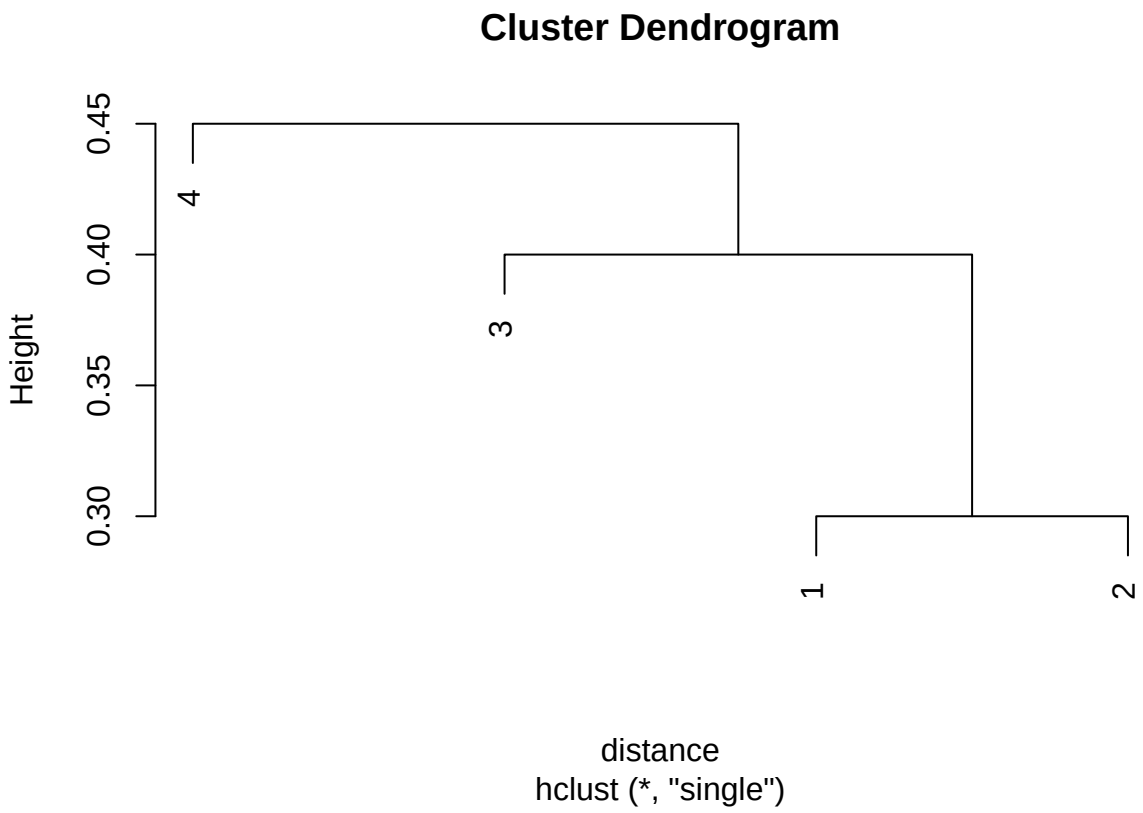
(b)

Repeat (a), this time using single linkage clustering.

0.3 is the minimum dissimilarity, so fuse observations 1 and 2 to form cluster (1,2) at height 0.3. The minimum dissimilarity is 0.4, so fuse cluster (1,2) and observation 3 to form cluster ((1,2),3) at height 0.4.

Lastly, fuse clusters ((1,2),3) and observation 4 to form cluster (((1,2),3),4) at height 0.45.

```
plot(hclust(distance, method = "single"))
```



(c)

Suppose that we cut the dendogram obtained in (a) such that two clusters result. WHich observations are in each cluster?

Points 1 and 2 are in cluster 1, and points 3 and 4 are in cluster 2.

(d)

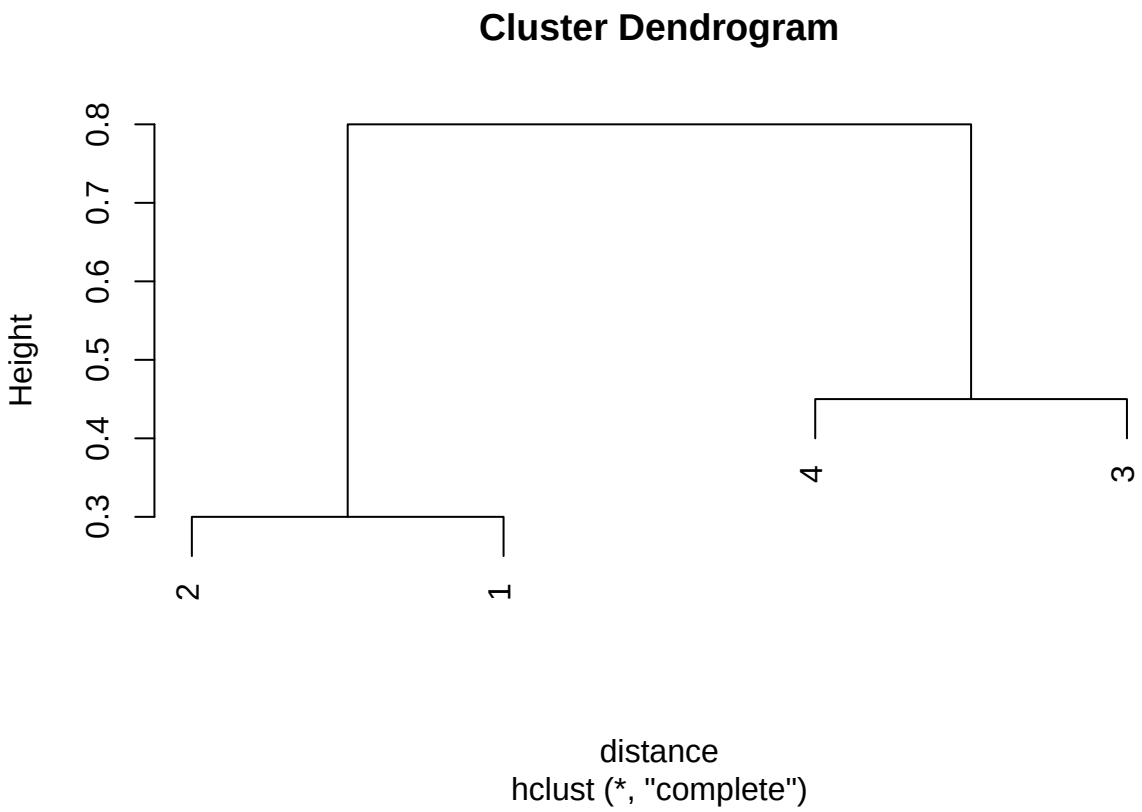
Suppose that we cut the dendrogram obtained in (b) such that two clusters result. Which observations are in each cluster?

Points 1, 2, and 3 are in cluster 1 and point 4 is in cluster 2.

(e)

It is mentioned in the chapter that at each fusion in the dendogram, the position of the two clusters being fused can be swapped without changing the meaning of the dendrogram. Draw a dendrogram that is equivalent to the dendrogram in (a), for which two or more of the leaves are repositioned, but for which the meaning of the dendrogram is the same.

```
plot(hclust(distance, method = "complete"), labels = c(2,1,4,3))
```



Extra 72

Consider the concrete strength data from problem 37. There are 8 numerical predictors and 1 numerical response. Load the data and split them into a training and test set (70%/30%). We want to predict strength.

(a)

Compute the principal components of the matrix of predictors for the training set. Fit a linear model to predict strength from the 1st principal component (simple regresssion).

```
trainIdx <- sample(nrow(concrete), .7*nrow(concrete), replace=FALSE)
train <- concrete[trainIdx,]
test <- concrete[-trainIdx,]
```

```
pr.out <- prcomp(train[,1:8], scale =TRUE)
pr.out$x
```

```
##              PC1         PC2          PC3          PC4          PC5
## 382   0.368601736  0.82705751 -0.658950222 -2.245086373 -0.337224872
## 89    0.336680719 -1.39308898 -1.372963397 -1.092747146 -0.118394184
## 307   2.011148126  0.42185879 -0.267418417  0.632867753 -0.235524402
## 768  -0.898442070  0.89288148 -0.912055391  0.133755275 -0.640698150
## 918  -0.029967243 -0.66668697  0.876699033 -0.825830673 -1.553411397
## 786  -0.559212810  0.87906062 -0.403600438  0.259843080 -1.337316778
## 891   0.420186866 -0.46239256  1.901721570  1.196834933  1.752611446
## 630  -0.088261986  1.83415312  0.554345665 -0.805859777 -1.574494139
```

```
## 849   0.309815690 -1.48361661  0.173247027  1.166932721 -0.262806105
## 179   0.703469466 -0.92948021 -0.874558480 -0.866514543 -0.287110200
## 804  -0.963387856  0.64603695 -0.956363663  0.161860812 -0.696592800
## 912   2.499247443 -1.00095033  1.105367184  2.056205472  0.397844524
## 960  -0.126967878 -1.38340929 -0.833959428 -0.416036512 -0.210143278
## 47   -0.649479886  1.45946526 -0.396024490 -0.352854465 -1.170091542
## 603  -1.322272458  0.87599430 -0.416069067  0.809235451 -0.562197950
## 945   0.325470253 -0.62831325  1.387961849 -1.036565860  0.959452199
## 148   1.267592343 -1.24757633 -1.968290770  1.249245654 -1.190548269
## 874  -1.801557769 -1.43378945  1.029867230  0.951417481  0.676758476
## 232   0.097467751  0.33057073  0.507335523 -0.955177594 -0.508217003
## 381  -0.006040313  2.19148175 -1.552156615 -1.732755267  1.082463557
## 1012 -0.034271592 -1.11052627  0.062683254 -0.108577210  1.052601964
## 979   0.194728921 -1.99438297  1.490754680 -0.050406638  1.187879702
## 133   0.214176743 -1.36422852 -1.370653675 -0.895558837 -0.002162188
## 614  -0.285898064  0.71608345 -0.025438426  0.437267086 -1.703792502
## 286   1.596975468  1.15514177  1.399914697 -0.303886721  0.678001290
## 265   1.737082811  1.40895411  0.806652430 -0.713101697  0.153798057
## 955   1.909174600  0.86184179  1.401989074 -0.850938216  2.204668505
## 824  -1.793610281  0.98854558 -0.191897003  1.636689073 -0.322627586
## 454   1.487828795  1.25124904  0.964964374 -0.177881025  0.520860784
## 584  -1.600451779 -1.87098093  0.770087937 -1.887562301 -0.486556236
## 346   2.172662363  1.11472867  0.901185498 -0.427435124  0.953375914
## 28   -3.166957429  0.25331692 -0.084398252  1.342803098  0.810126124
## 460   0.488072520 -0.30905615  1.375234571 -0.091427171  0.990893947
## 384   0.366291758  1.00456308 -2.049177935 -0.635061490  0.330753036
## 109   0.247464790  0.24415431 -1.469766218 -3.144240191  1.502933361
## 51   -3.726515848 -0.62755985  0.098557608  0.442180980  1.371725813
## 727  -0.555857904  1.25521915 -0.305062735 -0.126993240 -1.318760671
## 269   1.261767383  1.52093268  0.815614150  0.051988939  0.604778204
## 17   -1.172443695 -0.47202364  1.429951496 -0.720094032 -1.222422266
## 323   2.132926404  0.48311978 -0.088516681  0.793905721 -0.187674750
## 922   1.383845153 -0.46866502  1.289890167  1.728170758 -0.144349123
## 903  -0.528369721 -2.24355394 -0.898870893  0.708168992 -0.636559803
## 995  -0.603039178  0.89155253 -0.981587501 -0.338359981  0.063992501
## 70    0.156247010  2.44932599 -1.843328960 -1.168416014 -0.835646773
## 829   0.567376712  0.72482467 -2.598145268  0.941695060 -1.603893511
## 533  -0.561492454  0.40227060 -0.212484041  1.734850779 -1.614788956
## 744  -1.054140553  1.50449642 -0.847651666 -0.542207852 -0.371942154
## 569  -0.942223093  1.48615422 -0.852018488 -0.713757733 -0.472441346
## 118  -1.072225725 -1.13380706 -0.435191949 -2.627411528  0.914978025
## 324   1.917319406  0.53391419 -0.084451571  1.140957143  0.016893564
## 883   0.001909363 -1.25758221  1.536358812  0.171411984  0.278959292
## 224   0.761816793  1.38625544  1.730944155  0.427840303  0.811111020
## 303   1.412421118  0.46780514 -0.200479437  0.971454904 -0.153085133
## 754  -1.526519024  2.43334544 -1.918690280 -1.885725127  0.862070834
## 946   0.031688231 -1.27535768  1.524327215  0.179086876  0.304600393
## 666  -0.398192903 -0.55979158  0.858076470  0.054324398 -1.926531776
## 348   1.966855683  1.16321424  0.905065831 -0.096158766  1.148645668
## 581  -1.304716013 -0.60059158  0.077206189 -1.519974971 -0.509085217
## 796  -2.648793733  2.53477217 -1.593609589 -0.518710816  1.565435572
## 587  -0.665477137  1.99828364 -0.598943507 -0.908989076 -0.913955499
## 376  -1.529133898  1.07921105 -0.414467810  0.059034494 -0.676112205
## 921   1.093199277 -1.47272596  1.286093340 -0.198502960  0.508428311
```

```
## 331   2.376078384  1.32580577  0.164470116 -0.791479112  0.504677707
## 811  -0.405809913  0.78752639 -0.260427101  0.442914911 -1.581606127
## 18   -4.073486852  0.46688431 -0.067306311  2.801996580  1.670242900
## 352   2.223490878  1.04370614  0.827975103 -0.318323017  1.119375603
## 1025  0.337643924 -3.04793603  0.197808941 -0.104684075 -0.863447505
## 425   1.660640128  0.42392327  1.442719773 -0.142165461  0.473064263
## 253   0.648567455  0.50317266  0.508738800  1.180875707 -0.419024673
## 377  -1.597736125  1.09537291 -0.413174366  0.169459947 -0.611022287
## 479   0.852205716  0.35656183 -1.642712217 -0.451537316  1.136076541
## 855  -0.059893925 -1.10911274  0.079265311 -0.102878229  1.039193251
## 529   1.952567436  0.22285540 -0.690101765  0.285803864  0.693968605
## 328   2.535914336  0.45286192 -0.321091692  0.666354821 -0.106048521
## 435   0.252578086 -0.17124905  1.445823057 -0.653595227  0.277450169
## 132   0.144561450  0.26839709 -1.467826052 -2.978602011  1.600568238
## 831   1.577812646 -2.80423746  0.765015464  0.287442523  1.493464900
## 688  -1.504071862 -0.51508131  0.483374605 -0.220558127 -0.557167441
## 694  -0.020744007 -0.90422480  0.873267794  1.154850633 -2.583748609
## 794  -0.867534755  0.76924229 -0.054766804  0.411568345 -1.269069390
## 628   0.032130238  1.89686235  0.758002720 -0.845552166 -1.715178177
## 715   0.082159333 -0.92846758  0.871327628  0.989212454 -2.681383487
## 193   0.283864517  0.40460380  0.773958698  1.310019027 -0.423591362
## 807  -0.880067223  1.51667704 -0.732703733 -0.746170264 -0.535946622
## 116  -0.068300112 -1.35327876 -0.966392470 -0.958371791 -0.263017851
## 535  -0.825295725  0.66086598 -0.955449260  0.303409872 -0.929371090
## 785  -0.547533574  0.06934506 -0.612200445  0.932148414 -1.363098628
## 39   -3.219729705  0.65129832 -1.146892954  0.449195718  1.264162522
## 1029  0.377962538 -1.32984899  0.419459340 -0.970475968 -0.600879724
## 1016 -0.659069156 -0.97881354 -0.696345210 -0.776159188  0.174945351
## 641  -0.926448569  1.36426079 -0.891899630 -0.531476554 -0.557511158
## 813  -0.812523114  0.88334311 -0.252758825  1.097580095 -1.195715898
## 266   1.683181061  1.42165271  0.807668707 -0.626338842  0.204940135
## 949   0.991408646 -1.67495936  1.102938240  0.635913903 -0.191656508
## 389   2.289385708 -0.36999974 -1.383273781  0.671959970  1.678810669
## 1010  0.626961215 -0.11021799  0.011776042  1.304949241  0.037262092
## 438   1.592037901  0.44008513  1.444013217 -0.031740008  0.538154182
## 734  -0.760760378  1.56227347 -0.374811175 -0.208558408 -1.070921093
## 202   1.593458681  1.43223148  1.023269979 -0.495727875  0.207843465
## 845  -0.798133442 -1.31346118  0.339821990 -0.027833319  1.315469958
## 456   1.953111536  0.98998090  0.868809787 -0.005097692  1.285075654
## 909   1.349920753 -0.32245788 -1.225608925 -1.148497004 -0.889713445
## 248   0.722772281  0.34427987  0.498033870  1.138755834 -0.234112048
## 835   1.303317583 -1.76211216 -0.770725068 -0.689751798  2.402848915
## 252   0.785771908  0.47084895  0.506151911  0.960024801 -0.549204509
## 506  -0.612774811 -0.43245213 -1.168451209  1.065864963  1.204527577
## 782  -0.802408810  1.66844026  0.089403860 -0.827685142 -0.878116079
## 762  -1.516306658  0.92295314 -0.413650339  0.802577891 -0.486414203
## 924  -0.472745436 -2.72753088  1.771548580  0.251467335  2.002417471
## 837  -1.464543747 -1.35006971 -0.222820031  0.029834329 -0.086022687
## 869   0.496109071 -1.31926905 -0.039394976  0.442429055  0.373006465
## 770  -2.288968954  1.28657028 -0.370987168  3.044141995  0.303879015
## 238  -0.055609976  0.37495152  0.518608052 -0.734996970 -0.391775812
## 683  -1.097592876 -0.52465466  0.951770283 -0.072730452 -1.321446331
## 221   1.183230471  1.28697546  1.722998713 -0.250487478  0.411272952
## 772  -0.539612174  0.87444294 -0.403969993  0.228292951 -1.355913898
```

```
## 740  -0.735593062  1.75304759  0.028112734 -0.741079739 -0.821404372
## 49   -2.922636219 -1.74911143  0.907331376 -1.400290546  0.360152464
## 166   1.115304016 -0.89361280 -2.428593996  0.174409243  0.475068987
## 364   1.897655693  0.85693835  0.389830702 -0.375459965  0.874260589
## 752  -1.423615684  2.40910265 -1.920630446 -2.051363306  0.764435957
## 884   0.938507281 -2.95226287  0.175124594  0.358947344 -1.424096766
## 3    -4.167530162 -0.52366220  0.106872606  1.152058890  1.790161001
## 953  -0.381843809 -0.95451283  0.459637887  0.257137826  0.381469146
## 450   1.835443537  0.93209470  1.418924025  0.606873748 -0.017096414
## 1009  0.188833928 -2.56031071  1.391697485  0.460809736  1.269756748
## 434   0.840883971 -0.39217426  1.368582572 -0.659329500  0.656145796
## 333   2.170271704  1.37429134  0.168350448 -0.460202754  0.699947461
## 86    0.267065426  0.23953663 -1.470135774 -3.175790320  1.484336242
## 139  -0.171203452 -1.32903597 -0.964452304 -0.792733612 -0.165382974
## 14   -2.796099732 -1.32552026  1.226898018 -0.131760304  0.060094245
## 959   0.801999433 -0.74936154 -0.050501177 -0.118405837  0.391419114
## 427   1.334730781 -0.13626960  1.345308865 -0.563522589  0.951439592
## 36   -4.211378049 -1.44549941  0.931629648  0.674130458  1.582913070
## 22   -0.868633834 -0.54359757  1.424223386 -1.209121037 -1.510677617
## 864   0.130793284 -1.01907437 -0.594200875  0.189035769 -0.982125380
## 50   -2.856864390 -0.36635561 -0.329804188 -0.683397898  0.671024362
## 73    1.734039808 -1.54645934 -2.493639281  0.542087361 -0.778234851
## 106   1.337769146 -0.45126033 -1.973355501 -0.296663266 -0.827449940
## 927   0.022898292 -2.75302280  1.548759225  0.280906476  1.561640700
## 1005  0.942503865 -1.75634833 -0.196265453  0.843483487  0.163267430
## 878  -0.023877427 -0.97094726  0.105839196  1.902246949  0.707045538
## 262   2.178429997  0.64549304  0.356014629  0.548137015 -0.640858462
## 598  -1.311061285 -0.39371147  0.229567871 -1.352230383 -0.514791536
## 217   1.761228847  1.30856760  0.923741472 -0.494498847  0.354220476
## 495   1.800967328  0.05516271 -1.337780077  0.832235693  0.440984504
## 339   1.199088335  1.24751312  0.155921262  0.173236757  0.997116134
## 335   1.674403763  1.13553454  0.146959542 -0.591853879  0.546135986
## 599  -0.895958621  0.77555991 -0.424106898  0.123020137 -0.966685298
## 947   0.908774684 -2.93209268  0.186493369  0.357161153 -1.447182423
## 851  -0.307423255 -1.55359092  0.638987667 -1.901717080  0.114519405
## 826  -1.029255116  1.50753993 -0.859934600 -0.552977370 -0.373199444
## 169   2.655328590 -2.22958630 -3.711069589  0.766128495  1.091952430
## 424   2.041250216  0.88360913  1.415043693  0.275597389 -0.212366168
## 957   0.520771857 -0.77341268  1.239823142 -0.090819555 -0.030961665
## 140  -0.187416797 -1.40431336 -1.362425242 -0.478003163  0.045312055
## 954   0.425953883 -0.46341530  1.893685506  1.195037677  1.752876122
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## 576  -0.544325035  1.89764300  0.526759389 -0.858043828 -1.081421436
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## 841   0.403049702 -1.24440682 -0.972158447 -0.183905875 -0.565357685
## 407   1.666457715  1.12688529  1.532993818  0.489316366 -1.132997691
## 728  -0.575458540  1.25983683 -0.304693180 -0.095443110 -1.300163551
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## 739  -0.632689722  1.72880480  0.026172568 -0.906717918 -0.919039249
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## 540  -1.739234434  0.84608039 -1.619903407  0.657501447  0.229934223
## 642  -0.213735823  1.76295108  0.348597866 -0.764340883 -1.425447100
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## 607  -0.212784773  0.66081943  0.325057168  0.698061849 -1.957586204
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## 511   0.744337465 -0.64818197 -1.091888055  1.108819132  1.016131258
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## 679  -1.496370566 -1.12071197  0.032341327 -0.737997917 -0.524724126
## 600  -0.915559257  0.78017758 -0.423737343  0.154570267 -0.948088179
## 928   0.043634981 -2.72006058  1.524967273  0.358881259  1.186849641
## 534  -0.135178617  0.30183621 -0.220521872  1.048635466 -2.019276305
## 165   1.302825812 -1.44487053 -2.485509061  1.236190207 -0.369098223
## 554  -1.745121538 -2.49027153  0.518742167 -2.131202331 -0.357495984
## 876  -0.684508108 -0.80882552  0.886335625 -1.658026844  0.046016047
## 664  -0.690215699 -1.44040214  1.212794524 -0.145354444 -1.796325019
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## 338   1.414695333  1.19671871  0.151856152 -0.173814666  0.792547819
## 486   1.938171781  0.02283900 -1.340366965  0.611384788  0.310804667
## 515   0.785586696 -0.58163152 -1.228249614  1.387222408  1.257504841
## 213   0.034698019  1.07408318  1.106257164  0.269854418  0.553800066
## 414   1.388632531 -0.14896820  1.344292587 -0.650285444  0.900297514
## 430   2.158918216  0.94149533  0.864929455 -0.336374050  1.089805900
## 730  -0.982171741  1.35565354 -0.297024904  0.559222074 -0.914273322
## 990   0.418742587 -1.64342577  1.199290087  1.042871369 -0.339193878
## 465   1.177871922  0.23199813  1.412957054  0.254322214  1.082689198
## 791  -1.080295162  1.55957520 -0.387975324  0.328606017 -0.758012177
## 706  -1.002962180 -1.15643896  1.479541298 -0.570670404 -1.572323913
## 881   1.086995643  0.47908678  1.282907387  0.451299588 -0.028253872
## 890  -0.371586446 -0.95875153  0.453391585  0.253734853  0.376619350
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## 400  -0.586397167 -0.80230454  0.206174665 -0.745600000 -0.557596116
## 319   2.242028687  0.74884770 -0.188908626  0.943646051 -0.101033814
## 35   -4.143643470 -1.00805523  1.252304957  2.037311089  1.338646209
## 673  -0.378592267 -0.56440925  0.857706915  0.022774268 -1.945128896
## 680  -1.419803074 -0.52956915  0.787341110 -0.518808255 -0.866291802
## 242   0.295939752  0.95440525  1.031163769  0.047900079  0.528304508
## 513   1.045295126 -0.64281569 -1.233146224  0.969183194  1.011093008
## 929   0.150309547 -1.19799588  1.994513485 -0.572521287  2.166524547
## 16   -2.708403960  0.51815416 -0.422616067  0.824096561  0.474590108
## 610  -1.001325738  0.85549638  0.328110786  1.996622699 -1.187063036
## 525   1.849664096  0.24709818 -0.688161599  0.451442044  0.791603482
## 483   0.612097923  0.41312833 -1.638185163 -0.065048231  1.363891254
## 342   0.961366438  0.91119589  0.096019403 -0.142166588  0.858832672
## 1003 -0.803897694 -1.55622455 -0.608216637  0.120464791 -0.635430615
## 94    0.052690997 -1.46087985 -1.366952297 -0.864492247 -0.182502659
## 830  -0.713222687 -1.09360114  0.359667317  0.087641280  1.093236276
## 478   0.871806352  0.35194416 -1.643081773 -0.483087445  1.117479421
## 606  -0.158883023  0.64812083  0.324040890  0.611298993 -2.008728283
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## 697  -0.994689536 -0.54889745  0.949830117 -0.238368631 -1.419081208
## 91    1.616869041 -1.82085738 -2.284289583 -1.299875103  0.579808591
## 386   0.187076778  0.03433109 -2.608577796  1.269411596 -0.894248161
## 155   0.007356997  0.30072080 -1.465239164 -2.757751106  1.730748075
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## 75    3.583885529 -1.76366416 -2.768294341  0.848394218 -1.321643321
## 640  -1.029351909  1.38850357 -0.889959464 -0.365838374 -0.459876281
## 10   -2.915919844  0.57972439 -1.152621064 -0.039831287  0.975907170
## 653   0.072638805 -1.43649513  1.311632895  0.702388409 -2.802033464
## 528   1.952567436  0.22285540 -0.690101765  0.285803864  0.693968605
## 469   1.737504538  1.04077531  0.872874897  0.341953731  1.489643968
## 537  -0.947799701  0.68972644 -0.953139539  0.500598180 -0.813139093
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## 646  -2.208347648 -2.00699494  1.002576350 -2.028812960 -0.067138938
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## 141  -1.209430179 -1.10148335 -0.432605060 -2.406560623  1.045157861
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## 905   0.157162859 -2.12967222  0.526614575  0.454031943 -1.408804470
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## 46   -2.834940447  0.09456300 -0.742182709 -0.444433682  0.774648327
## 512   0.504229672 -0.59161548 -1.087361001  1.495308217  1.243945971
## 410   0.901432034 -0.41477169  1.345553260 -0.871357309  0.495561833
## 200   1.715962657  1.40337102  1.020960257 -0.692916184  0.091611468
## 602  -1.018462597  0.80442037 -0.421797177  0.320208446 -0.850453302
## 210   0.294406449  1.01289901  1.101360554 -0.148184795  0.307388233
## 908   0.086677172 -1.43824222 -0.113764116 -0.034357363  1.190928928
## 87    0.336680719 -1.39308898 -1.372963397 -1.092747146 -0.118394184
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## 437   1.972647990  0.89977099  1.416337137  0.386022842 -0.147276250
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## 218   1.624024394  1.34089132  0.926328360 -0.273647942  0.484400312
## 411   2.095151966  0.87091053  1.414027415  0.188834534 -0.263508247
## 662  -1.357162213 -1.25340532  1.395847367 -1.036675082 -1.105625528
## 274   0.644499450  1.31722111  0.816264427  0.306329701  0.731999892
## 686  -0.587312359 -1.46464492  1.210854357 -0.310992623 -1.893959896
## 880   0.080124775 -1.55855202 -0.878948417  0.356898305 -0.852115010
## 153   0.076972289 -1.33190481 -1.368066787 -0.674707932  0.128017649
## 910  -0.272140699 -0.85646366  0.950527797 -0.657981169 -1.384364694
## 403  -0.305444921 -0.61679072 -0.437539483 -1.926217894  0.215738618
## 244  -0.056871699  1.03752337  1.037815767  0.615802407  0.863052658
## 977   1.591765846  0.20946685 -0.410967933  0.361906179  0.718903128
## 608  -0.281387000  0.67698129  0.326350612  0.808487302 -1.892496286
## 227   2.496287854  1.07559395  1.031387022 -0.929091497  0.466937890
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## 510   0.922791149 -0.61395524 -1.230836502  1.166371502  1.127325005
## 228   2.359083400  1.10791766  1.033973910 -0.708240591  0.597117726
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## 295   1.960054409  0.80741194  1.022873319 -0.922268377  0.278916259
## 738  -0.553137033  1.89807926  0.531632120 -0.857847866 -1.078655346
## 776  -0.688748410  1.84335359  0.172007133 -0.818402012 -0.886008662
## 902   0.647072076 -2.34404927  1.386324412  0.994452080  0.113052922
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## 1013  0.027076722 -1.63611718 -1.011346652 -0.068619695 -0.504027731
## 257   2.042240113  0.76500287  0.440717517  0.519196988 -0.750223093
## 349   1.751248685  1.21400864  0.909130941  0.250892657  1.353213982
## 741  -0.813127479  1.31126230 -0.086187573 -0.735865304 -0.625859992
## 117  -0.050212343 -1.43663707 -1.365012131 -0.698854068 -0.084867782
## 806  -1.389701693  0.74647134 -0.948325832  0.848076126 -0.292105451
## 374   1.521181213  1.29807048  0.580069959  0.066470280  0.876268094
## 113  -0.576396366 -1.70543177 -1.811838367 -0.582963172  0.420616069
## 482   0.612097923  0.41312833 -1.638185163 -0.065048231  1.363891254
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## 1    -0.741338122  1.67046992 -2.346250161 -1.069472669  0.473954360
## 647  -2.102611836 -0.58378424  0.537700895 -0.465407463 -0.062260536
## 872   0.343748853 -1.31501215  0.439834731 -0.969643943 -0.621132066
## 170   1.302825812 -1.44487053 -2.485509061  1.236190207 -0.369098223
## 204   1.240647230  1.51534960  1.029921977  0.072174453  0.542591615
## 491   1.841282850  0.13202376 -1.220776600  0.416833232  0.018183218
## 911   0.664030902 -0.99294765  1.311094614 -0.136951994  0.104512964
## 562  -1.045126433  1.51039701 -0.850078321 -0.548119554 -0.374806469
## 458   0.146981958  1.17073558  0.886696761  0.123003329  0.699674481
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## 586  -1.407619353 -0.57634880  0.079146355 -1.354336792 -0.411450340
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## 634  -0.334128047  1.70024185  0.144940811 -0.724648494 -1.284763062
## 795  -1.680961157  0.96087573 -0.039430252  1.720898713 -0.497288931
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## 125   1.404796796 -1.27990004 -1.970877658  1.028394749 -1.320728105
## 948  -0.049822488 -0.39869743  0.220438781  1.151144980  1.161673903
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## 476   0.871806352  0.35194416 -1.643081773 -0.483087445  1.117479421
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## 412   1.714541877  0.41122467  1.441703495 -0.228928317  0.421922185
## 163  -0.358922363 -1.36390871 -1.359191632 -0.201939531  0.208036850
## 669   0.101759969 -0.93308526  0.870958073  0.957662325 -2.699980606
## 1008 -0.341135244 -1.54395655  0.659798202 -1.902265590  0.096893941
## 395  -0.980110739  2.18801919 -0.875078076 -1.433245821 -0.038476491
## 622  -1.464500350  0.94916410 -0.212986862  1.630846671 -0.460305254
## 675  -1.077758025 -0.61551570  0.475336774 -0.906773440 -0.961654789
## 150   1.474331379 -1.48527517 -2.488742671  0.960126575 -0.531823018
## 191   0.489671197  0.35611823  0.770078366  0.978742669 -0.618861116
## 516   0.039633864  0.35800467  1.556078035  1.014456267  0.015537412
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## 201   1.662060908  1.41606962  1.021976534 -0.606153328  0.142753546
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## 839   1.137144312 -1.37994543  0.904871206 -0.576928416  1.175672819
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## 631  -0.191165326  1.85839590  0.556285831 -0.640221597 -1.476859262
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## 594  -0.486893592  1.83737992  0.399807948 -0.994873477 -1.114896225
## 108  -0.857817902 -2.24778019 -0.689522526 -1.805326911  0.529033117
## 940  -0.594238647 -2.67354070  0.910148792  0.709164882 -1.842000144
## 963   1.539315640  0.24426708  1.276622678  0.538029377  0.369936381
## 690  -0.976246849 -1.66842819  1.082908479 -0.838590501 -1.388882319
## 92    0.336680719 -1.39308898 -1.372963397 -1.092747146 -0.118394184
## 580  -0.900043781  0.39447065  0.843833638 -1.368022640 -0.969481071
## 445   0.284186411  1.13841186  0.884109872 -0.097847576  0.569494645
## 461  -0.100233365 -0.08813093  1.452475055 -0.085692899  0.612198320
## 923  -0.083272788 -0.71858293  0.365587216  0.131610912  0.225207612
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## 668  -1.549358758 -2.07075663  0.861174090 -1.214297292 -0.719972800
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## 692  -1.297299097 -0.55842961  0.785031389 -0.715996563 -0.982523799
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## 625  -0.719676175  0.77369252 -0.227029970  0.431941756 -1.166995794
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## 423   0.847530284 -0.40207309  1.346569538 -0.784594453  0.546703911
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## 101   1.714439172 -1.54184167 -2.493269726  0.573637491 -0.759637732
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## 551  -0.844896361  0.66548366 -0.955079705  0.334960001 -0.910773970
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## 215   1.883732823  1.27970715  0.921431750 -0.691687156  0.237988479
## 57   -4.567273443  0.96876335 -1.121486016  2.618267111  2.542714486
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## 417   2.212819965  0.92879673  0.863913177 -0.423136906  1.038663821
## 743  -1.695236368  0.84762628 -1.620434904  0.705415069  0.155686660
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## 372   1.873992664  1.21495237  0.573417960 -0.501432049  0.541519944
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## 766  -0.795538730  0.86863869 -0.913995557 -0.031882904 -0.738333027
## 312   1.383837525  0.40713992 -0.195954944  0.800221940 -0.259718596
## 573  -0.918944226  0.12949433  0.677470205 -1.511968360 -0.922199306
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## 115   0.317080083 -1.38847131 -1.372593842 -1.061197016 -0.099797065
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## 731  -1.129176512  1.39028609 -0.294253238  0.795848044 -0.774794926
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## 378  -1.734940578  1.12769662 -0.410587477  0.390310852 -0.480842450
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## 468   0.286566699  0.48430081  0.559102129  1.611252945 -0.188828457
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## 917  -0.767566898 -1.01997436 -0.287452582 -0.155905912 -0.448088460
## 162  -0.308407905 -1.29671226 -0.961865416 -0.571882706 -0.035203138
## 74    1.546518012 -0.99520161 -2.436724216 -0.519693603  0.065932359
## 42   -4.589197386  0.50784475 -0.709107495  2.379302895  2.439090520
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## 64   -3.678128361 -1.11772497  1.243528014  1.287995517  0.896964622
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## 719  -1.022562816 -1.15182129  1.479910853 -0.539120275 -1.553726793
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## 67   -2.495486638 -0.16033069  1.454896490  1.409539699  0.032883299
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## 926  -0.331679160 -1.13984699  1.565589876 -0.838549731  1.508776644
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## 1001  1.238693317 -1.86603216  1.163128916  0.206806704  0.422662631
## 79    1.527300772 -1.30876050 -1.973187379  0.831206440 -1.436960102
## 24   -1.613458010 -0.36812599  1.438266494 -0.010216122 -0.803987077
##                PC6           PC7           PC8
## 382  -1.382528e+00  3.904189e-01 -0.0360841344
## 89   -1.944503e-01  4.051115e-02 -0.1285496923
## 307  -1.010061e-01 -1.832722e-01 -0.0854289565
## 768   7.628175e-01  2.860723e-01  0.3170717180
## 918  -8.576377e-01  2.863149e-01  0.1869463229
## 786   8.141336e-01 -7.094231e-02  0.0549853650
## 891   1.707044e+00 -9.160947e-01 -0.0086196592
## 630  -2.023595e-01 -4.332086e-01  0.0512597153
## 849   8.571405e-01 -2.031906e-02  0.0051128677
## 179  -2.034231e+00  3.505170e-01 -0.0440133770
## 804   1.279314e+00  2.065233e-01  0.3247715625
## 912   8.056616e-01 -4.380957e-01  0.1310111646
## 960  -8.085623e-02 -1.176787e-01  0.1165616225
## 47    6.164034e-01 -2.313955e-01 -0.2360870457
## 603   1.392150e-01 -1.331461e-02  0.1919187733
## 945  -1.155336e-01  7.028973e-02  0.2006446202
## 148  -4.645039e-01  1.877912e-01 -0.1564725587
## 874   1.939142e+00 -1.127604e+00 -0.2252210272
## 232  -5.591772e-01 -6.260965e-01 -0.1543418575
## 381  -1.876469e-03  1.332994e+00  0.4262622925
## 1012  7.018164e-01  1.102953e-01  0.0936308224
## 979   6.299640e-01  2.072593e-01  0.0281424209
## 133  -4.670752e-01  7.229960e-02 -0.1346315818
## 614   7.210545e-01 -1.544557e-01  0.1825825561
## 286   2.096847e-01  4.469495e-02 -0.0554822094
## 265  -1.263614e-01 -5.742258e-03  0.0052494960
## 955  -9.952819e-02 -7.104346e-01  0.3954811474
## 824  -8.377677e-01 -1.080872e-01  0.0038746052
## 454  -6.450833e-01 -3.771216e-01 -0.0425372600
## 584  -5.207259e-01  8.365450e-01 -0.1961706825
## 346  -9.424043e-03  2.317750e-01  0.0241521358
## 28   -5.990753e-02 -3.585666e-01  0.1608489419
## 460  -9.434932e-01  2.704640e-01 -0.1647796413
## 384  -3.186051e-02 -4.942888e-01  0.1224613672
## 109  -9.510897e-01 -7.070826e-01  0.1038179554
## 51   -2.727482e-01 -3.632626e-02  0.0788150039
## 727   6.534121e-01 -2.126068e-01 -0.1355854236
## 269  -1.184146e+00  1.175969e-01 -0.0183482354
## 17   -1.423416e+00  3.393983e-02 -0.4305871944
## 323  -6.143325e-01 -2.980371e-01 -0.0799964859
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## 533  -1.617736e-02  9.611417e-02  0.1290970827
## 744   5.083159e-01  1.097391e-01  0.0865589268
## 569   7.289669e-01  8.830699e-02  0.0927880106
## 118  -7.222591e-01  3.551383e-02 -0.1510421440
## 324  -1.094152e+00 -2.420894e-01 -0.0907006115
## 883   5.157795e-01 -2.380133e-01  0.0380623218
## 224  -5.866501e-01  2.669757e-01 -0.1128142427
## 303  -1.136471e-01  1.693560e-03 -0.1418391963
## 754   4.172203e-01  5.441298e-01 -0.0201698209
## 946   5.111550e-01 -2.892801e-01  0.0275274438
## 666   2.073462e-01 -2.664898e-04  0.1081152189
## 348  -4.674339e-01  2.851796e-01  0.0139345613
## 581   1.386272e-01  5.569492e-01 -0.0828896406
## 796  -9.992869e-01  3.206696e-01 -0.2841938542
## 587   3.131741e-01 -1.633359e-01 -0.3999758579
## 376   1.212952e+00 -5.496363e-01 -0.3279812826
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## 811   7.610622e-01 -1.063097e-01  0.0461403491
## 18   -2.077332e+00 -1.233321e-01  0.1158429594
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## 1025 -5.514700e-01 -4.904967e-01  0.0075880181
## 425   1.965681e-01  5.800891e-01 -0.0783448514
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## 855   7.138970e-01  1.251913e-01  0.0875849417
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## 694   8.363655e-02  6.725163e-02  0.0615404947
## 794   9.007050e-01 -3.713400e-01  0.0379366993
## 628  -3.452054e-01 -5.043603e-01  0.0569136366
## 715   3.126415e-01  4.054934e-02  0.0666492819
## 193   4.960844e-01 -2.647122e-01 -0.2565839348
## 807   6.500141e-01  5.186760e-02  0.1150777574
## 116  -5.878784e-02 -1.552014e-02 -0.1871939104
## 535   1.223241e+00  1.884163e-01  0.1735359212
## 785   1.304466e+00  2.166439e-01  0.5047610539
## 39    1.451202e+00 -2.560483e-01  0.2186671232
## 1029 -9.327776e-01 -7.395900e-01  0.1700259385
## 1016  8.722134e-02 -2.794873e-01  0.1101578752
## 641   8.064570e-01  1.031174e-01  0.0695217495
## 813  -1.440525e-01 -7.720387e-04  0.0259484758
## 266  -2.463163e-01  8.244658e-03  0.0025734646
## 949   2.306564e-01  3.884333e-01  0.0832387139
## 389   5.060337e-01 -3.114128e-01  0.1007026562
## 1010  1.114265e+00  2.378801e-01  0.1061042121
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## 438   4.389818e-02  5.978907e-01 -0.0817507096
## 734   2.991078e-01 -2.274563e-01 -0.2942692723
## 202  -4.153064e-01 -1.063791e-01 -0.0151185325
## 845   1.230222e+00  3.847359e-01  0.0118269955
## 456  -3.837010e-01  4.257389e-02 -0.0067253876
## 909  -1.337253e+00  9.493904e-01  0.4150410731
## 248   2.436643e-01 -2.590132e-01 -0.0751596162
## 835   9.145491e-02 -6.252061e-01  0.2202942046
## 252   5.725084e-01 -1.412296e-01 -0.2357791270
## 506   2.481635e+00  9.081918e-01 -0.2987471899
## 782   3.792853e-01 -4.254631e-01  0.0417978444
## 762   2.464616e-01 -1.487669e-01  0.0410769219
## 924   1.402726e+00  2.980192e-02 -0.1126140887
## 837   8.536640e-01 -5.878448e-01 -0.0708676100
## 869   5.732891e-01  3.161915e-01  0.0956353343
## 770  -3.035330e+00  3.779105e-01 -0.0308909152
## 238  -8.598546e-01 -5.688748e-01 -0.1612162723
## 683   2.260695e-01 -2.117038e-01  0.0633659182
## 221   3.511796e-01  1.576235e-01 -0.0918925427
## 772   8.577536e-01 -7.602846e-02  0.0559584673
## 740   1.865365e-02 -2.714796e-01  0.1015905065
## 49    1.118816e+00 -1.861132e-01  0.0422634682
## 166  -1.238743e+00 -4.011491e-01 -0.0311955298
## 364  -1.718603e+00  2.990567e-01  0.0579069950
## 752   6.462252e-01  5.174276e-01 -0.0150610337
## 884  -7.118142e-01 -4.577794e-01  0.0259676521
## 3    -1.254198e+00  7.811214e-02  0.0569202016
## 953   8.756627e-01  2.662316e-01  0.0390787072
## 450  -3.388033e-01  9.410267e-01 -0.0635265627
## 1009  9.463576e-01 -6.691596e-02 -0.0523129709
## 434  -1.583334e-01  1.789132e-01 -0.1472637995
## 333  -1.026583e+00  4.632002e-02  0.0760264961
## 86   -9.074697e-01 -7.121688e-01  0.1047910577
## 139  -2.877928e-01  1.118216e-02 -0.1923026976
## 14    1.295416e-01 -3.607025e-01  0.0569226062
## 959   6.552500e-02  5.507166e-02  0.1988837328
## 427   1.297479e-01  1.201156e-01 -0.1369435606
## 36   -1.749198e+00  1.483012e-01 -0.0217180097
## 22   -7.473064e-01 -4.489552e-02 -0.4155041083
## 864  -3.363244e-02  3.017810e-03  0.1228774382
## 50    1.861316e+00 -2.913968e-01  0.1602207852
## 73    3.210329e-02 -1.333035e-01 -0.0974137574
## 106  -3.395116e-01  2.400075e-01 -0.0245778606
## 927   1.041951e+00  3.235949e-01 -0.0633420761
## 1005  5.656059e-01  6.570236e-01  0.0910703850
## 878   1.900873e+00 -9.404609e-01 -0.0861706470
## 262  -3.861925e-01 -2.243188e-01 -0.0813427519
## 598  -1.697301e-01  4.661140e-01 -0.0731047250
## 217  -4.673352e-01 -3.286798e-01 -0.0114332222
## 495  -1.796294e-01  5.704881e-02 -0.2715322696
## 339  -9.378566e-01  9.507978e-02 -0.0324595579
## 335   1.199280e-01 -2.825938e-02 -0.0088618265
## 599   1.087950e+00 -1.239384e-01  0.2130837489
## 947  -7.016093e-01 -4.070265e-01  0.0336118490
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## 851  -1.010474e+00 -7.007800e-01  0.1618721142
## 826   4.914274e-01  1.338107e-01  0.1047409686
## 169  -1.489405e+00 -1.223829e+00  0.0327002729
## 424   1.192065e-01  8.876221e-01 -0.0533089883
## 957   2.908919e-02 -5.139876e-02  0.1383709727
## 140  -5.391655e-01  2.271986e-01 -0.1925091180
## 954   1.703936e+00 -9.076371e-01 -0.0023793270
## 769  -1.072431e+00  1.490337e-01  0.0128986894
## 576  -2.681925e-01 -4.833417e-01  0.1137748247
## 481   6.873145e-01  2.179355e-01  0.0751470762
## 841  -2.769052e-01 -5.245071e-03  0.1611701727
## 407   3.131556e-01  9.197464e-01 -0.1063296228
## 728   6.097921e-01 -2.075206e-01 -0.1365585259
## 518   1.441573e+00  6.350179e-02 -0.2621876133
## 151  -9.879067e-01  2.062022e-01 -0.1822780386
## 277   5.456960e-01 -2.231633e-01 -0.1651561242
## 249  -2.361556e-01 -2.030656e-01 -0.0858637418
## 449  -4.635477e-01  2.262049e-01 -0.1575664696
## 986   5.714769e-01  8.680117e-02  0.0927585468
## 861   6.363734e-01  1.522000e-01  0.2306286570
## 182  -1.711218e+00 -4.797815e-01 -0.0147444346
## 724   3.812139e-01 -1.919358e-01 -0.0417495169
## 756  -1.240339e+00  7.374036e-01 -0.0571477093
## 275   8.183209e-01 -2.549517e-01 -0.1590742347
## 761   6.172314e-01 -1.919992e-01  0.0493482917
## 592  -1.980384e-01 -4.287731e-01  0.1120081558
## 563   6.586184e-01  3.352783e-02  0.0949804563
## 110  -2.380703e-01  4.559731e-02 -0.1295227946
## 211   8.021088e-01 -3.256699e-01 -0.1964374781
## 83   -2.958916e-01  2.349214e-01 -0.0236047583
## 739   2.476586e-01 -2.981819e-01  0.1066992937
## 159   1.480329e-01 -1.297882e-02 -0.2559869349
## 394   1.056473e+00 -5.844078e-01  0.0135264760
## 886   2.229202e-01  4.008498e-01  0.0936621835
## 223  -1.068302e-01  2.110281e-01 -0.1021101172
## 334  -1.506403e+00  1.022677e-01  0.0653223706
## 570   8.416917e-02 -1.366336e-01 -0.4050846451
## 29    1.879812e+00 -2.997891e-01  0.1944311036
## 578  -8.817369e-01  2.360451e-02 -0.1305857227
## 497  -1.045161e-01  3.548378e-01 -0.2471262913
## 691   5.182547e-01  5.841151e-01  0.0470237785
## 636   2.390700e-01 -2.126573e-01  0.0541266722
## 695   5.874569e-02  2.632003e-01  0.0888330715
## 755  -2.588895e-01  6.229652e-01 -0.0352529070
## 383   1.336167e+00  5.429885e-01 -0.2863921228
## 893   9.479346e-01 -8.748387e-01  0.0905449248
## 605  -2.859659e+00  3.363583e-01  0.1250179884
## 439  -8.734801e-02  4.957208e-01 -0.0215232052
## 517   1.485193e+00  5.841564e-02 -0.2612145110
## 593   4.296135e-01  6.023012e-02  0.0898716691
## 321  -1.563226e-01 -3.514417e-01 -0.0697789115
## 474   4.583096e-01  2.446378e-01  0.0700382890
## 123  -8.023903e-01 -1.303936e+00  0.0480266345
## 25   -1.879332e+00 -1.514077e-01  0.1472982439
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## 1023  1.027908e+00  3.245265e-01 -0.0140842172
## 818   8.109424e-01  1.095943e-01 -0.2438101077
## 361  -7.807736e-01  1.897044e-01  0.0788286950
## 652   4.986944e-01 -2.434922e-01  0.0694478077
## 146  -1.107730e+00 -1.268333e+00  0.0412149182
## 399   5.247515e-01  5.916872e-01 -0.0621387035
## 566   2.353059e-01 -2.956979e-01  0.1050444563
## 154  -6.046085e-01  4.390754e-02 -0.2596245110
## 172  -9.095056e-01 -1.112633e+00  0.0406260537
## 556  -6.433673e-01  5.332739e-01 -0.1483490767
## 330  -4.486184e-01 -2.107149e-02  0.0889201019
## 779   3.235829e-01 -2.141341e-01  0.2243396522
## 504   1.828552e+00  5.764753e-01 -0.7463076364
## 775   2.911089e-01 -1.880366e-01 -0.4353843315
## 828   6.413458e-01  1.638971e+00  0.0678099734
## 463  -8.186231e-01  9.969744e-01 -0.0742306883
## 388   1.046829e+00  5.205806e-01  0.1044718368
## 895   3.532153e-01  1.122931e-01  0.1212299118
## 716  -7.821441e-01  2.496513e-02 -0.0049821953
## 934   7.828238e-01  9.640589e-01  0.0889302053
## 341   4.835700e-01 -2.081935e-01 -0.0883945373
## 96   -1.151669e-02 -1.282173e-01 -0.0983868597
## 540   6.054382e-01  5.386946e-01  0.1968573646
## 642  -5.306770e-02 -3.585088e-01  0.0555201544
## 812   5.320572e-01 -7.960738e-02  0.0410315618
## 484   1.529697e-01  2.802409e-01  0.0632265728
## 203  -7.206463e-01 -7.077606e-02 -0.0219302488
## 131  -2.992686e-01  8.304477e-03 -0.2528127948
## 111  -1.118211e+00  2.437078e-01 -0.0235782282
## 787   5.851287e-01 -4.424001e-02  0.0498765778
## 216  -3.146653e-01 -3.464814e-01 -0.0080273641
## 222   1.985096e-01  1.754250e-01 -0.0952984009
## 749   1.024444e+00 -2.167409e-01 -0.3356654333
## 145   3.922637e-01  3.112758e-01 -0.2516573509
## 952  -7.839745e-01 -6.183243e-01  0.1702684483
## 147  -5.458615e-01 -6.591197e-02 -0.1103073632
## 980   3.314402e-01 -2.310150e-01  0.0398975338
## 236  -4.018447e-01 -6.222794e-01 -0.1509986979
## 100  -5.733854e-01 -1.330638e+00  0.0531354217
## 862   1.133715e+00  1.499853e-02  0.1504704503
## 938   6.695468e-01 -4.831864e-03  0.2559532876
## 325  -2.683784e-01 -2.290537e-01  0.1211654023
## 722   6.538388e-01 -2.237242e-01 -0.0356676274
## 735  -3.770020e-01 -1.486210e-01 -0.3093523584
## 397   1.050406e+00  6.889416e-01  0.6587925786
## 84   -1.944503e-01  4.051115e-02 -0.1285496923
## 199  -1.036195e+00 -8.236552e-03 -0.1328369292
## 698   1.781021e-01 -2.513057e-01  0.0194890374
## 966   6.512799e-01 -1.887420e-02 -0.0572280261
## 314  -4.752883e-01  6.025701e-02 -0.1700011973
## 299  -1.475755e+00 -1.543556e-01 -0.0222741381
## 207   1.721391e-01 -1.783887e-01 -0.1159266114
## 45    1.203702e+00 -2.209537e-01  0.1793480176
## 121  -1.147158e+00 -4.182892e-01  0.0319924875
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## 981  -8.615062e-01  2.590096e-01  0.1791545636
## 370  -1.821953e-01 -3.465352e-01  0.0125856829
## 320  -3.636767e-02 -3.654286e-01 -0.0671028801
## 143  -8.570682e-01 -4.456529e-01 -0.0226808845
## 819   6.582725e-01  1.273958e-01 -0.2472159659
## 135  -4.670752e-01  7.229960e-02 -0.1346315818
## 134  -1.347216e+00  2.704101e-01 -0.0286870154
## 685  -8.738568e-01  7.680905e-01 -0.0335129843
## 897  -8.750551e-02 -1.706779e-01  0.1035817158
## 736  -7.041518e-01 -1.104749e-01 -0.3166506258
## 190   1.074049e+00 -3.321037e-01 -0.2436903290
## 932   4.795609e-01  3.464503e-01  0.2838328946
## 322  -3.089926e-01 -3.336402e-01 -0.0731847697
## 760   9.225713e-01 -2.276023e-01  0.0561600080
## 246   7.016741e-01 -3.124178e-01 -0.0649420418
## 81    3.210329e-02 -1.333035e-01 -0.0974137574
## 1022  1.503439e+00 -8.339965e-01  0.0161716574
## 919   1.434931e+00 -1.412187e-01 -0.0083266724
## 898  -4.022254e-01  4.581699e-02  0.2158991711
## 502   2.406517e+00  5.090838e-01 -0.7334140306
## 810  -2.075451e-01 -4.110246e-01  0.1311987851
## 419   8.306184e-01 -4.037904e-01 -0.2896868789
## 553  -5.121813e-01  2.642969e-01 -0.1043869571
## 300   4.643177e-01 -6.569794e-02 -0.1289455905
## 607   5.159753e-01 -3.161857e-01  0.1438798675
## 176  -1.154090e+00  1.524065e-01 -0.1499579435
## 453  -3.282619e-01  1.735202e-01 -0.1471611350
## 416   9.688390e-01 -2.308574e-01 -0.2500994914
## 53    2.137012e-01 -8.057560e-02  0.0220715950
## 20    4.697523e-01 -1.416099e-01  0.1967723209
## 802  -2.075451e-01 -4.110246e-01  0.1311987851
## 160  -1.329543e+00 -5.242853e-01 -0.0062297893
## 987   1.091556e+00 -5.300260e-01 -0.0058669547
## 973  -6.234034e-01 -9.353843e-02  0.1227461912
## 964  -1.300207e+00 -8.071742e-01  0.2339537155
## 889  -7.870269e-01 -6.152005e-01  0.1689998527
## 920  -2.335440e-03 -6.907433e-01  0.2251600737
## 5    -3.727976e+00  4.742535e-01 -0.1449915929
## 942  -3.234864e-01 -1.918805e-01  0.0176832147
## 44    1.380926e-01 -3.866422e-01  0.1923042265
## 220   4.711345e-01  1.436366e-01 -0.0892165113
## 765   1.035442e+00  2.542838e-01  0.3231536076
## 539   1.510553e+00  4.331569e-01  0.2170492379
## 707  -5.924732e-01  1.460870e-01  0.0464574087
## 196  -9.836570e-02 -1.175888e-01 -0.1119152292
## 827   1.538060e+00  4.234543e-01  0.1715762039
## 567  -2.488880e-01  3.487871e-01 -0.0586367990
## 867  -1.358538e+00 -5.677783e-01  0.2497968231
## 892  -1.038484e-01 -7.613199e-01  0.3891723407
## 260  -1.135676e-01 -2.561073e-01 -0.0752608624
## 913   6.713333e-01  1.181452e-01  0.1027876566
## 856  -2.030648e-02  1.626252e-01  0.0964270059
## 511   1.782360e+00  8.215456e-01  0.0558760788
## 524   3.575486e-01  6.806183e-01 -0.1141452557
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## 368  -6.351585e-01  3.094796e-01  0.1931082902
## 503   2.362897e+00  5.141699e-01 -0.7343871330
## 254  -2.126513e-01 -4.967890e-02 -0.2532949689
## 888   1.755293e+00 -1.315820e-01  0.0092447915
## 992   8.655074e-02 -6.309187e-01  0.2226652701
## 1021 -2.661485e-02 -5.768761e-03  0.1116643497
## 208  -1.332008e-01 -1.427856e-01 -0.1227383277
## 272   8.318593e-02 -9.152319e-02 -0.0845522007
## 759   1.075241e+00 -2.454038e-01  0.0595658661
## 800   6.898452e-01  5.123414e-01 -0.0140879314
## 441  -3.397434e-01 -4.127247e-01 -0.0357255438
## 1017  1.000566e+00 -5.209769e-02  0.1309574077
## 390   7.144946e-01 -5.190492e-01 -0.3666739355
## 623  -3.049371e+00  3.469120e-01  0.1671837079
## 294  -1.068666e+00 -5.819208e-02 -0.0921658169
## 72   -4.496342e-01  3.725386e-03 -0.1449602545
## 494   4.298288e-01  2.925324e-01 -0.2352057878
## 747   1.089874e+00 -2.243701e-01 -0.3342057798
## 542   1.038785e+00  5.612209e-02  0.1648985374
## 789   5.745236e-01 -2.257767e-01 -0.2322172472
## 699  -2.280727e-02  4.374833e-01 -0.0534842313
## 1024 -1.359476e+00 -6.008918e-01  0.2391315018
## 648  -9.232074e-01  4.913811e-01  0.0042669467
## 1014  1.754834e+00  9.533840e-02  0.0356775456
## 1027  1.514291e+00 -1.875245e-01  0.0259478487
## 834  -3.367228e-01 -3.772948e-01  0.2130185757
## 1030  9.481410e-01 -2.559660e-01 -0.0031567669
## 532   2.061897e+00  2.602538e-02  0.3295542138
## 950  -4.080752e-01 -7.080761e-01  0.3572024248
## 651  -7.651268e-01 -8.468088e-02  0.0015357510
## 500   2.133892e+00  5.408722e-01 -0.7394959202
## 595   2.574689e-01  7.319698e-01 -0.1038148693
## 672   2.877506e-01  2.364980e-01  0.0939418587
## 112  -2.380703e-01  4.559731e-02 -0.1295227946
## 428  -1.870735e-01 -4.305262e-01 -0.0323196856
## 545   8.889373e-01 -9.423464e-03  0.1492889559
## 684  -9.283351e-01  3.277583e-01  0.0137016019
## 40   -7.677485e-01  3.386281e-02  0.0001767926
## 119  -2.405216e-01 -1.015150e-01 -0.1034956469
## 616  -1.209130e+00  7.060645e-02  0.1395227782
## 189  -2.663657e-01 -7.326136e-02 -0.2624017604
## 635  -1.392266e-01 -2.606548e-01  0.0447574459
## 7    -2.042672e+00  1.638137e-01  0.0731281270
## 332  -7.212433e-01  1.071696e-02  0.0828382124
## 464  -7.412615e-01  6.894414e-01 -0.0992665514
## 183  -1.154090e+00  1.524065e-01 -0.1499579435
## 54    7.235420e-01 -4.449293e-01  0.1512884597
## 366  -1.771487e-01  2.560750e-01  0.2033258646
## 663  -2.470976e-01  4.125457e-01  0.0193500327
## 956   9.500134e-01 -8.275569e-01  0.1012450131
## 969   4.481668e-01 -2.992597e-01  0.2162358878
## 114  -7.951984e-01 -5.865906e-01  0.0056907142
## 679   2.892498e-01  6.108174e-01  0.0419149913
## 600   1.044330e+00 -1.188522e-01  0.2121106465
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## 928   9.404474e-01  3.939840e-01 -0.0503742828
## 534   9.325573e-01 -1.450962e-02  0.1502620582
## 165  -9.275364e-01 -2.140815e-02 -0.1188220085
## 554  -1.688852e-01  1.113324e+00 -0.2393092780
## 876  -6.521275e-01 -1.039230e+00  0.1450211859
## 664  -2.522254e-01  2.489230e-01  0.0287846879
## 66   -1.041357e+00 -2.441282e-01  0.1389541396
## 338  -4.580368e-01  3.913212e-02 -0.0217554323
## 486   1.257105e-01  2.144576e-02 -0.2647205533
## 515   1.068002e+00  1.080450e+00  0.2221602920
## 213   3.440989e-01 -2.722653e-01 -0.2066550525
## 414   2.497029e-01  1.061286e-01 -0.1342675292
## 430   7.430888e-02 -1.083070e-02  0.0034921869
## 730  -2.953225e-01 -1.019830e-01 -0.1567503992
## 990   6.251429e-01 -4.763079e-01 -0.0237057261
## 465  -8.725077e-01  5.872715e-01 -0.0390390470
## 791  -3.305911e-01 -1.202391e-01 -0.2524091204
## 706   1.836078e-01 -1.953047e-01  0.0227007266
## 881   2.436852e-01 -5.284905e-01  0.2085160945
## 890   8.672315e-01  2.772954e-01  0.0427279681
## 531   5.220872e-02  7.162214e-01 -0.1209569720
## 400  -2.156575e-01 -3.536484e-01  0.0069155600
## 319  -1.379416e+00  4.245197e-02 -0.0275162457
## 35   -2.869332e+00 -1.102956e-02 -0.0099781787
## 673   2.509661e-01 -5.352641e-03  0.1090883213
## 680   2.522302e-01 -1.117234e-01  0.0532993989
## 242   6.094927e-01 -4.628342e-01 -0.1948051828
## 513   1.645967e+00  1.013059e+00  0.2350538978
## 929   8.037997e-01  4.434282e-01  0.1102907109
## 16    1.119542e+00 -5.010806e-01  0.2141990288
## 610  -1.311143e+00 -8.103883e-02  0.1216781628
## 525   3.575486e-01  6.806183e-01 -0.1141452557
## 483   1.529697e-01  2.802409e-01  0.0632265728
## 342   3.309000e-01 -1.903920e-01 -0.0918003955
## 1003  4.625863e-01  2.806550e-01  0.0058578317
## 94   -4.820696e-03  1.648932e-01 -0.1805886145
## 830   1.097490e+00 -5.361838e-01 -0.0144603052
## 478   7.309345e-01  2.128493e-01  0.0761201786
## 606   6.359303e-01 -3.301726e-01  0.1465558989
## 356  -7.322406e-01 -4.659445e-01 -0.0426328443
## 697   4.550744e-01 -2.384060e-01  0.0684747054
## 91   -7.515784e-01 -5.916768e-01  0.0066638165
## 386   9.333927e-01  6.556939e-01 -0.0056676091
## 155  -1.485435e+00 -6.447773e-01  0.0918974519
## 304  -5.934669e-01  5.764122e-02 -0.1525433219
## 97   -3.227234e-01 -5.079583e-01 -0.0107603810
## 466  -8.080818e-01  2.294678e-01 -0.1578652606
## 611  -3.328567e+00  1.541957e-01  0.0766721802
## 178  -1.154090e+00  1.524065e-01 -0.1499579435
## 75   -8.745328e-01 -4.500777e-01  0.0380743770
## 640   5.774521e-01  1.298197e-01  0.0644129623
## 10    2.127312e+00 -3.348836e-01  0.2337502092
## 653   9.716359e-02  7.675089e-02  0.0358585418
## 528   5.865535e-01  6.539160e-01 -0.1090364685
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## 469  -8.635208e-01  9.852155e-02 -0.0174295131
## 537   9.506161e-01  2.202047e-01  0.1674540317
## 137  -1.024203e+00 -5.598883e-01  0.0005819270
## 646   2.061976e-01  4.107811e-01 -0.0483754441
## 420   1.932006e-01  9.337333e-01 -0.1090056542
## 520   1.248455e+00  8.007158e-01  0.2731263720
## 141  -1.027599e+00  7.111689e-02 -0.1578538603
## 526   6.301735e-01  6.488299e-01 -0.1080633662
## 809   3.834832e-02 -4.617985e-01  0.1181255305
## 467  -1.124903e+00 -3.211740e-01 -0.0532413856
## 905  -2.156830e-01 -6.378559e-01 -0.0044588646
## 790   3.455187e-01 -1.990744e-01 -0.2373260344
## 46    2.108816e+00 -3.264914e-01  0.1995398908
## 512   1.248016e+00  8.838510e-01  0.0439555753
## 410   1.144171e-01  1.588134e-01 -0.1446728637
## 200  -1.426815e-01 -1.381676e-01 -0.0090366430
## 602   8.153247e-01 -9.214995e-02  0.2070018593
## 210   9.220637e-01 -3.396568e-01 -0.1937614467
## 908   8.029147e-01  7.892081e-01  0.1019668719
## 87   -1.944503e-01  4.051115e-02 -0.1285496923
## 437  -3.346341e-02  9.054237e-01 -0.0567148464
## 59   -2.579077e-01 -3.304909e-01  0.1293936574
## 218  -7.726751e-01 -2.930768e-01 -0.0182449385
## 411   2.391615e-01  8.736352e-01 -0.0506329569
## 662   1.229706e-01 -8.057251e-02  0.0152096780
## 274  -7.019738e-01  2.753712e-05 -0.1020680425
## 686  -2.322046e-02  2.222207e-01  0.0338934752
## 880   1.203249e-01  1.137398e-01  0.0799317520
## 153  -7.724151e-01  1.079027e-01 -0.1414432981
## 910  -6.125990e-01 -5.076869e-02  0.1192492695
## 403  -7.945408e-01  1.854051e-01  0.2511345178
## 244  -1.756670e-01 -3.712835e-01 -0.2123210247
## 977   2.701627e-01 -1.503760e-01  0.2886501420
## 608   3.633054e-01 -2.983842e-01  0.1404740094
## 227  -8.911122e-01  1.486021e+00  0.6628143049
## 219  -1.252495e+00 -2.371291e-01 -0.0289490641
## 510   1.373342e+00  1.044847e+00  0.2289720083
## 228  -1.196452e+00  1.521624e+00  0.6560025886
## 38    7.087015e-01 -1.507647e-01  0.1007098063
## 295  -4.179702e-01 -2.776948e-01  0.0013235933
## 738  -2.636427e-01 -4.911875e-01  0.1094536174
## 776  -8.335027e-02 -3.474144e-01  0.0750849919
## 902   7.093109e-01  3.943895e-01 -0.0253416893
## 404  -5.954344e-01 -2.136485e-02  0.3664646492
## 1013 -1.780382e-02  1.751325e-01  0.0988131666
## 257  -3.584445e-01 -4.633228e-02 -0.0807892815
## 349  -9.472537e-01  3.411272e-01  0.0032304358
## 741   5.428896e-01 -1.275493e-01  0.5174445310
## 117  -2.338256e-01  1.915955e-01 -0.1856974017
## 806   3.305797e-01  3.171471e-01  0.3036065870
## 374  -1.239980e+00 -2.231960e-01 -0.0110120485
## 113   6.823777e-01 -7.528418e-02 -0.2440664314
## 482   1.529697e-01  2.802409e-01  0.0632265728
## 243   3.041528e-01 -4.272312e-01 -0.2016168991
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## 1     4.236241e-01  7.471893e-01  0.2035670055
## 647  -3.437612e-01  1.407598e-01  0.0276461711
## 872  -9.208488e-01 -7.167299e-01  0.1620135685
## 170  -9.275364e-01 -2.140815e-02 -0.1188220085
## 204  -1.200466e+00 -1.482840e-02 -0.0326343744
## 491   4.734487e-01  2.874463e-01 -0.2342326855
## 911  -2.181212e-04  2.387531e-01  0.1617229743
## 562   4.999620e-01  1.150093e-01  0.0876792234
## 458   2.526537e-01 -3.363989e-01 -0.3025804847
## 313   4.531502e-03  4.309345e-03 -0.1592970717
## 720  -7.663946e-01  2.091787e-01 -0.0242516804
## 105  -4.535619e-01  1.438969e-01 -0.1703575351
## 4    -2.290172e+00  1.989082e-01  0.0338090214
## 887  -4.064213e-01 -6.931734e-01  0.3576188831
## 34   -5.116973e-01 -2.717147e-02  0.1748775186
## 586  -9.037776e-02  5.836515e-01 -0.0879984278
## 1028  4.879380e-01  1.577949e-01  0.0388080685
## 634   8.977827e-02 -2.873571e-01  0.0498662331
## 795  -9.095243e-01 -1.602648e-01 -0.0024470472
## 815  -3.088401e+00  3.425432e-01 -0.0397359311
## 125  -1.591640e-01  1.521881e-01 -0.1496608424
## 948   1.655607e+00 -6.569544e-01  0.0075379407
## 1026  5.701603e-01  3.388753e-01  0.0982691756
## 476   7.309345e-01  2.128493e-01  0.0761201786
## 808   4.210092e-01  7.856990e-02  0.1099689702
## 412   3.165231e-01  5.661022e-01 -0.0756688200
## 163  -9.208404e-01  2.717024e-01 -0.2010237633
## 669   3.562615e-01  3.546319e-02  0.0676223842
## 1008 -9.988925e-01 -6.741563e-01  0.1599039152
## 395   5.252512e-02  1.779505e-01  0.0996895197
## 622  -1.031947e+00  1.116775e-01  0.2121896905
## 675   3.313706e-01  2.314119e-01  0.0949149611
## 150  -5.458615e-01 -6.591197e-02 -0.1103073632
## 191   9.540942e-01 -3.181167e-01 -0.2463663604
## 516   1.212568e+00  9.020409e-02 -0.2672964005
## 347  -1.620940e-01  2.495765e-01  0.0207462776
## 557  -5.942360e-01  6.550966e-01 -0.1680438492
## 58    1.651078e-01  2.031453e-02 -0.0581422103
## 201  -2.626364e-01 -1.241807e-01 -0.0117126744
## 251   7.251783e-01 -1.590311e-01 -0.2323732689
## 318  -8.995958e-01 -1.349569e-02 -0.0168121202
## 839  -1.998637e-01 -4.510944e-01  0.1910783986
## 188   2.134541e-01 -1.292090e-01 -0.2516976349
## 631  -4.313644e-01 -4.065063e-01  0.0461509281
## 999   1.009305e+00 -6.006355e-01  0.1407255432
## 594   3.096650e-02 -4.554754e-01  0.1171169430
## 108  -7.026369e-02 -1.839782e-02 -0.2477040075
## 940   1.459751e-01  1.262513e-02 -0.1231656879
## 963   2.335818e-01 -6.496257e-01  0.2226190723
## 690  -1.809268e-02  3.858434e-01  0.0244588199
## 92   -1.944503e-01  4.051115e-02 -0.1285496923
## 580  -5.284630e-01 -2.648832e-02 -0.0744628730
## 445   5.579935e-01 -3.720019e-01 -0.2957687684
## 461  -7.727869e-01  6.032022e-01 -0.2014681714
```

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## 923   5.914857e-01  1.324333e-01  0.0918798787
## 984  -1.272100e-01  2.440608e-01  0.1676328915
## 860   9.866594e-01 -9.148679e-02  0.1337803961
## 840  -4.090579e-01 -5.810373e-01  0.0425441980
## 353  -5.076056e-01  1.254533e-01  0.0181001737
## 310   5.824963e-01 -6.308216e-02 -0.1464034659
## 677   1.387201e-01  1.036410e-01 -0.0040598071
## 379   2.751224e-01 -4.402841e-01 -0.3489029826
## 844   4.411097e-02  2.753407e-01  0.1111292466
## 240   8.821176e-01 -4.946227e-01 -0.1887232933
## 69    8.056514e-01 -4.395378e-01  0.0720056922
## 279  -2.394637e-01 -1.316125e-01 -0.1826719661
## 629  -5.742104e-01 -4.776580e-01  0.0518048494
## 968  -2.154161e-01 -6.558036e-01 -0.0112924357
## 214  -1.357209e-01 -2.163176e-01 -0.2173591781
## 285   3.296397e-01  3.070803e-02 -0.0528061780
## 771   6.181436e-01 -2.308628e-01 -0.2312441449
## 817   8.872774e-01  1.006936e-01 -0.2421071787
## 962   3.113205e-01 -1.827342e-01  0.1881280239
## 234  -1.344337e+00 -5.345458e-01 -0.1718576994
## 668  -1.977471e-01  6.892552e-01 -0.0184298982
## 704  -1.077804e-02  1.066111e-01  0.0895506923
## 284  -2.561556e-01 -1.977879e-01 -0.1880955168
## 848   5.716161e-01  7.205314e-01  0.1037808317
## 780   9.457799e-02 -1.874318e-01  0.2192308650
## 619  -1.677144e-01 -2.649107e-01  0.0944049499
## 426   6.532193e-02  4.779193e-01 -0.0181173470
## 612   6.010995e-01 -1.404688e-01  0.1799065247
## 692   5.248551e-01 -1.435118e-01  0.0593812884
## 82   -4.099419e-01  1.388107e-01 -0.1693844328
## 48   -2.524801e-02 -7.142080e-02  0.1181341096
## 433   4.053068e-02  9.515348e-01 -0.1124115124
## 1018  6.288890e-01  1.401200e-01  0.2339354579
## 181  -2.336420e-01  3.152500e-02 -0.2645015802
## 129  -5.685165e-01  2.667098e-01 -0.0296866478
## 915  -2.277989e-01 -2.054681e-01  0.1943633501
## 975   8.128487e-01 -4.468434e-01  0.1215608877
## 649  -1.060344e-01 -5.387022e-02  0.0101008908
## 152  -8.738564e-01  3.023129e-01 -0.0364983641
## 670   5.618747e-01  5.790290e-01  0.0479968809
## 645   3.022029e-01 -4.010343e-02  0.0476244028
## 705   6.049734e-01  3.013601e-02  0.0488111467
## 604  -8.422347e-01  1.011238e-01  0.1700239710
## 625   6.256124e-01 -8.159626e-02  0.2491675788
## 351  -4.959580e-02  7.204869e-02  0.0283177481
## 936   1.610610e+00  1.476289e-01 -0.1079106402
## 124  -2.405216e-01 -1.015150e-01 -0.1034956469
## 452  -3.926879e-01  5.313239e-01 -0.0283349214
## 546   6.033148e-01 -1.455497e-02  0.1275891517
## 423  -5.537886e-03  1.728003e-01 -0.1473488951
## 271   2.358559e-01 -1.093247e-01 -0.0811463425
## 101  -1.151669e-02 -1.282173e-01 -0.0983868597
## 32   -2.473332e+00 -6.718081e-02  0.0529323904
## 551   1.179621e+00  1.935025e-01  0.1725628189
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## 472   4.583096e-01  2.446378e-01  0.0700382890
## 983  -2.629144e-04 -6.930137e-01  0.2249030077
## 972  -1.330823e+00  9.414523e-01  0.4126732603
## 575  -6.527320e-01 -3.097779e-03 -0.1254769355
## 885   1.662520e+00 -6.781392e-01 -0.0057357254
## 31   -2.275332e+00 -9.525644e-02  0.0843876749
## 258  -6.637844e-01 -1.072922e-02 -0.0876009978
## 783   1.502804e-01 -3.987608e-01  0.0366890572
## 359  -1.670070e+00 -3.565923e-01 -0.0635545442
## 326  -3.883334e-01 -2.150668e-01  0.1184893709
## 371  -3.021502e-01 -3.325483e-01  0.0099096515
## 289  -7.281450e-01  1.540472e-01 -0.0764039094
## 396  -7.911059e-01 -9.908150e-02 -0.2423053482
## 350   7.035915e-02  5.806177e-02  0.0309937795
## 588  -3.274702e-01  2.323359e-01 -0.0437707551
## 597  -1.371985e-01  1.005640e-01  0.3844998107
## 703   2.182269e-01  7.990885e-02  0.0946594795
## 215  -1.947103e-01 -3.604683e-01 -0.0053513327
## 57   -1.547672e+00  9.362463e-02  0.1517663383
## 41    9.215421e-01 -4.730050e-01  0.1827437443
## 967   4.690422e-01 -7.908658e-01  0.2264220224
## 654  -8.515711e-01  1.873747e-01  0.0146935662
## 803   2.497413e-01 -5.929060e-02  0.1534043146
## 712  -1.794432e-01  4.346012e-01  0.0012520145
## 417   1.942638e-01 -2.481762e-02  0.0061682183
## 743   5.893254e-01  5.340781e-01  0.1504112284
## 821  -1.980737e+00  4.351080e-01 -0.3060886565
## 315  -3.216310e-01 -8.088720e-02 -0.0039185144
## 372  -4.548202e-01 -3.147467e-01  0.0065037934
## 21    2.222522e-01 -1.065153e-01  0.1574532152
## 678   5.354361e-02  8.183704e-02  0.0348854395
## 871   5.013344e-01  1.649092e-01  0.0336131803
## 965   6.849134e-01  3.461271e-01 -0.0220330358
## 693  -4.238796e-01 -3.288805e-02  0.0382163128
## 766   9.918224e-01  2.593700e-01  0.3221805052
## 312   3.098714e-01 -3.129371e-02 -0.1524853554
## 573  -1.787200e-01  8.750478e-02 -0.0246749370
## 609  -3.128044e-01 -2.195489e-01  0.1253909234
## 894   3.573678e-02 -3.862201e-02  0.1370779651
## 906   4.462773e-01 -2.922161e-01  0.2162847086
## 1015  4.127023e-01  1.102781e+00  0.0697283831
## 316  -4.415860e-01 -6.690028e-02 -0.0065945458
## 764   1.057252e+00  2.517407e-01  0.3236401587
## 816   9.308974e-01  9.560740e-02 -0.2411340763
## 836  -9.156974e-02 -5.461886e-01  0.3242324266
## 99    9.266085e-01  2.489705e-01 -0.2397368474
## 63    7.990098e-01 -1.147072e-01  0.0393514914
## 13   -7.591975e-01  7.923069e-03  0.1355584129
## 659   7.256715e-01  3.290636e-01  0.0214438877
## 499  -2.504092e-01  3.761064e-01 -0.1762184487
## 337  -1.526969e-01  3.529061e-03 -0.0149437160
## 527   6.301735e-01  6.488299e-01 -0.1080633662
## 572  -4.473480e-01  1.018488e+00 -0.2252318466
## 250   8.451333e-01 -1.730181e-01 -0.2296972375
```

```
## 814  -1.125502e+00  1.136664e-01  0.0040536735
## 65   -4.559078e-01 -3.024153e-01  0.0979383729
## 245   8.216291e-01 -3.264047e-01 -0.0622660104
## 899   3.218759e-01 -1.427870e-01  0.1900715322
## 276   6.983659e-01 -2.409648e-01 -0.1617502661
## 708  -6.868878e-01  1.854465e-01  0.0744676063
## 733   5.281127e-01 -2.541586e-01 -0.2891604851
## 882  -1.252036e-01  8.164180e-02  0.2080937817
## 832   9.464335e-01 -9.043205e-01 -0.0419490860
## 239  -1.339674e+00 -5.129272e-01 -0.1719203979
## 665   5.613535e-01  3.522216e-02  0.0478380444
## 774   4.378978e-01  5.449827e-02  0.0917869285
## 373  -7.601600e-01 -2.791437e-01 -0.0003079229
## 657   2.618469e-01  7.482270e-02  0.0956325818
## 477   7.309345e-01  2.128493e-01  0.0761201786
## 564  -2.917210e-01  8.098427e-01 -0.1910618953
## 6     5.255419e-01 -4.168537e-01  0.1198331752
## 413   1.852769e-01  4.639324e-01 -0.0154413156
## 235  -2.818898e-01 -6.362663e-01 -0.1483226665
## 263  -6.915324e-01 -1.887158e-01 -0.0881544682
## 226  -7.384422e-01  1.468219e+00  0.6662201631
## 78    3.210329e-02 -1.333035e-01 -0.0974137574
## 115  -2.380703e-01  4.559731e-02 -0.1295227946
## 519   9.072278e-01  1.258071e-01 -0.2741081168
## 136   4.533728e-01 -4.858188e-02 -0.2491752186
## 26   -8.433570e-01 -2.722038e-01  0.1704094241
## 763  -7.349881e-01 -3.432853e-02  0.0191821196
## 561  -4.077250e-01  1.142071e-01 -0.0297837242
## 444  -1.275179e-01 -1.832577e-01 -0.1388161560
## 340   6.035249e-01 -2.221804e-01 -0.0857185059
## 336  -2.693423e-05 -1.427247e-02 -0.0115378579
## 1020  1.560097e+00 -5.676477e-01 -0.1811541920
## 309  -8.861659e-01 -9.172148e-02 -0.1029447983
## 406   5.143952e-02  2.439649e-01  0.6598595940
## 184  -9.748075e-01  9.128904e-02 -0.2076290592
## 859   1.031971e-01 -2.287747e-01  0.1138262615
## 380  -7.575879e-02  1.141112e+00  0.1433552090
## 731  -6.224724e-01 -6.383687e-02 -0.1640486666
## 655  -8.901707e-02 -1.635162e-01  0.0166188370
## 850   2.913601e-01 -7.244429e-01  0.2565119024
## 549   6.599324e-01  1.727883e-02  0.1441801687
## 916   6.346863e-01  2.217838e-01  0.0233344563
## 914   2.702318e-01 -1.844437e-01  0.2838555188
## 470  -9.126776e-01 -9.170693e-02 -0.1563319978
## 933   1.775161e+00 -1.971793e-02 -0.1429506235
## 62   -1.437357e+00 -1.879769e-01  0.0760435706
## 638   6.571652e-01  2.841764e-02  0.0652613104
## 52   -8.519080e-01 -2.462641e-01  0.0350278039
## 797   1.193290e-01 -1.112033e-01 -0.3558573065
## 626  -5.049737e-02 -2.760920e-03  0.2340844928
## 571   6.301009e-03 -2.689956e-01  0.0999356691
## 198  -5.563755e-01 -6.418422e-02 -0.1221328036
## 378   7.549422e-01 -4.962317e-01 -0.3381988570
## 9     1.201652e+00 -4.956891e-01  0.1349162613
```

```
## 306   5.166380e-02 -2.010737e-01 -0.0820230983
## 671   4.966666e-01  3.557659e-01  0.0163351005
## 496  -1.685015e-01  1.011657e-01 -0.2679165691
## 402   1.115263e-01 -1.652773e-01  0.4742477531
## 142  -5.458615e-01 -6.591197e-02 -0.1103073632
## 751   7.954387e-01 -1.900386e-01 -0.3407742205
## 543  -7.279498e-02  6.428038e-02  0.1125060657
## 674   4.812351e-01 -1.384257e-01  0.0584081861
## 225  -6.184873e-01  1.454232e+00  0.6688961945
## 187   5.187940e-01 -1.648121e-01 -0.2448859186
## 436  -1.582078e-01  1.906018e-01 -0.1507547533
## 468  -8.894557e-02 -1.075182e-01 -0.2736972228
## 273  -2.221540e-01 -5.592013e-02 -0.0913639169
## 37    8.898109e-01 -1.594109e-01  0.0371546810
## 931   1.002963e+00  5.651681e-01 -0.0298627667
## 773   6.669027e-01  2.779597e-02  0.0968957157
## 917   3.188839e-01 -2.873600e-01  0.0330254784
## 162  -5.931326e-01  4.678522e-02 -0.1991144139
## 74   -2.791034e-01 -5.130444e-01 -0.0097872787
## 42   -1.795172e+00  1.287192e-01  0.1124472327
## 64   -1.833358e+00 -1.318257e-01  0.0131330015
## 127  -2.405216e-01 -1.015150e-01 -0.1034956469
## 719   1.399878e-01 -1.902185e-01  0.0217276242
## 446  -2.648092e-01  9.871379e-01 -0.1192232286
## 67   -4.367765e+00  3.772550e-01 -0.4962716013
## 175  -1.255531e+00  3.468167e-01 -0.0450130095
## 926   4.910011e-01 -6.889869e-02  0.0798077722
## 451  -2.614417e-01  6.334937e-01 -0.0885624258
## 1001  3.968728e-02  2.964055e-01  0.1255553855
## 79    1.134609e-01  1.203997e-01 -0.1435789529
## 24   -2.404866e+00  1.483782e-01 -0.4524819967
```

```
train.prc <- data.frame(pc1 = pr.out$x[,1], y = train$y)

lm.fit <- lm(y ~ pc1, data=train.prc)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = y ~ pc1, data = train.prc)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -33.01 -12.32  -1.14  10.53  46.48
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 36.23307    0.62099  58.347   <2e-16 ***
## pc1         -0.09753    0.40796  -0.239    0.811
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 16.67 on 719 degrees of freedom
## Multiple R-squared:  7.949e-05,  Adjusted R-squared:  -0.001311
## F-statistic: 0.05716 on 1 and 719 DF,  p-value: 0.8111
```

(b)

Make predictions for the test set, using the same model. You have to use the loading vectors which were found from the principal component analysis of the training data.

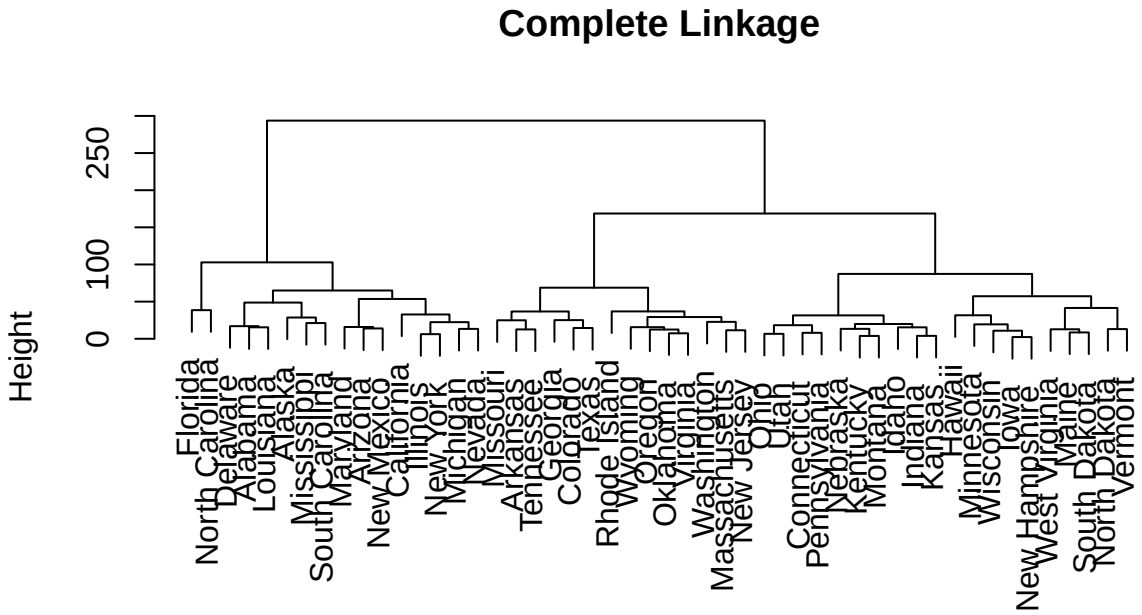
Book 9

Consider the USArrests data. We will now perform hierarchical clustering on the states.

(a)

Using hierarchical clustering with complete linkage and Euclidean distance, cluster the states.

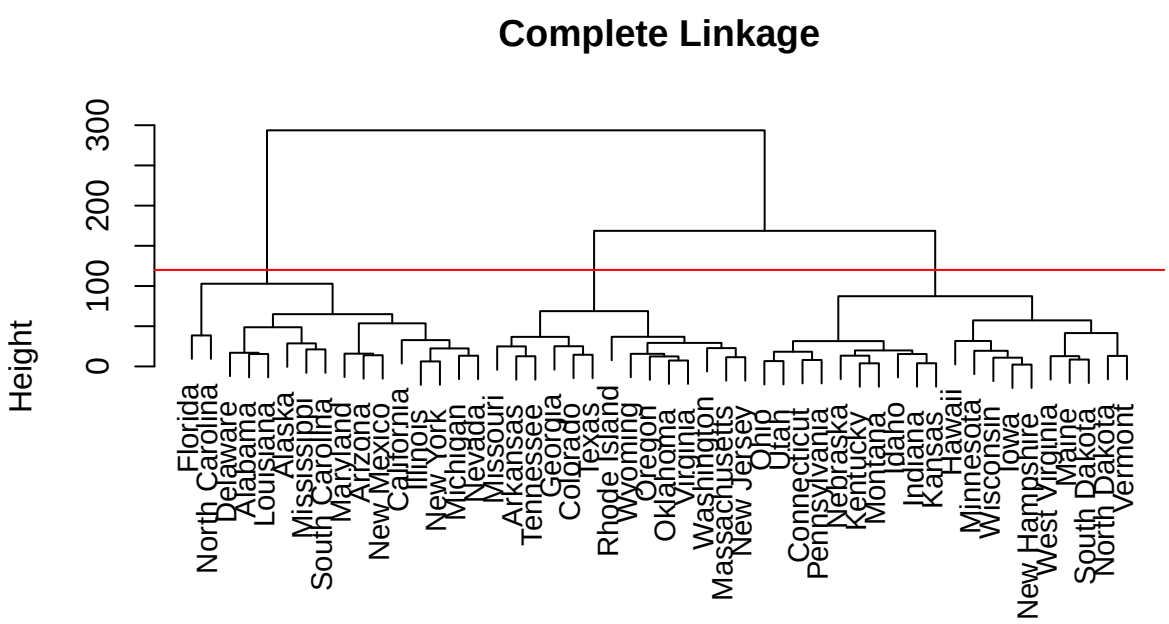
```
library(ISLR)
set.seed(100)
data(USArrests)
hc.complete = hclust(dist(USArrests, method = "euclidean"), method = "complete")
plot(hc.complete, main = "Complete Linkage", xlab = "", sub = "", cex = 1)
```



(b)

Cut the dendrogram at a height that results in 3 distinct clusters. Which states belong to which clusters?

```
plot(hc.complete, main = "Complete Linkage", xlab = "", sub = "", cex = .9)
abline(h = 120, col = "red")
```




```
cutree(hc.complete, 3)
```

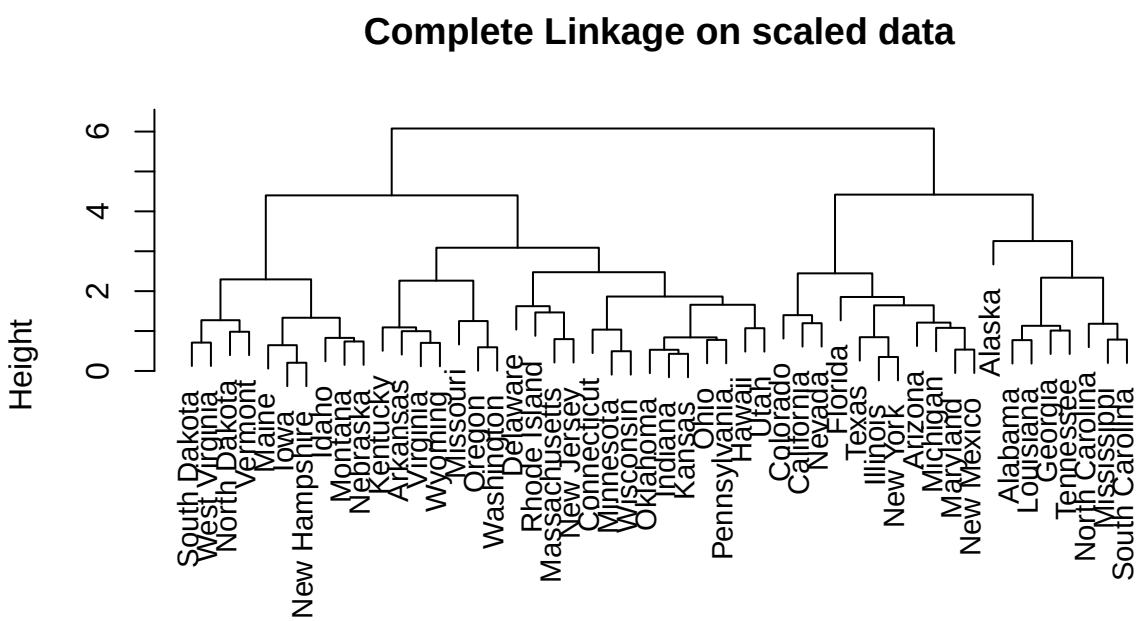
```
##        Alabama         Alaska        Arizona       Arkansas     California
##              1              1              1              2              1
##       Colorado    Connecticut       Delaware        Florida        Georgia
##              2              3              1              1              2
##         Hawaii          Idaho       Illinois        Indiana           Iowa
##              3              3              1              3              3
##         Kansas       Kentucky      Louisiana          Maine       Maryland
##              3              3              1              3              1
##  Massachusetts       Michigan      Minnesota    Mississippi       Missouri
##              2              1              3              1              2
##        Montana       Nebraska         Nevada  New Hampshire     New Jersey
##              3              3              1              3              2
##     New Mexico       New York North Carolina   North Dakota           Ohio
##              1              1              1              3              3
##       Oklahoma         Oregon   Pennsylvania   Rhode Island South Carolina
##              2              2              3              2              1
##   South Dakota      Tennessee          Texas           Utah        Vermont
##              3              2              2              3              3
##       Virginia     Washington  West Virginia      Wisconsin        Wyoming
##              2              2              3              3              2
```

(c)

Hiearchically cluster the states using complete linkage and Euclidean distance, after scaling the variables to have standard deviation 1.

```r
UsArrests.scale = scale(USArrests)

hc.complete = hclust(dist(UsArrests.scale, method = "euclidean"), method = "complete")
plot(hc.complete, main = "Complete Linkage on scaled data", xlab = "", sub = "", cex = .9)
```



(d)

What effect does scaling the variables have on the hierarchical clustering obtained? In your opinion, should the variables be scaled before the inter-observation dissimilarities are computed? Provide a justification for your answer.

The variables should be scaled before the interobservation dissimilarities are computed. The reason is that the variables are in different units and the range of some variables are larger than others. Therefore, it makes the most sense to scale the variables before any similarity measure is computed.

Extra 69

In this problem, you will use k-means clustering for the smiley data, for different values of sd=sd1=sd2. Use 500 points and 4 clusters throughout.

(a)

Demonstrate that for small values of sd, k-means clustering recovers the 4 clusters in the data reasonably well. Use confusion matrices to show this.

When sd1=sd2=.0001, the algorithm was able to perfectly recover the 4 clusters. However, when sd=.1, the recovery is not as well.

```
set.seed(100)
smiley <- mlbench.smiley(n=500, sd1 = 0.0001, sd2 = 0.0001)
km.out <- kmeans(smiley$x, 4, nstart=100)
table(actual=smiley$classes,  predicted=km.out$cluster)
```

```
##       predicted
## actual   1   2   3   4
##      1  83   0   0   0
##      2   0   0  83   0
##      3   0   0   0 125
##      4   0 168  26  15
```

```
set.seed(100)
smiley <- mlbench.smiley(n=500, sd1 = 1, sd2 = 1)
km.out <- kmeans(smiley$x, 4, nstart=100)
table(actual=smiley$classes,  predicted=km.out$cluster)
```

```
##       predicted
## actual   1   2   3   4
##      1  44   3   8  28
##      2  10   4  54  15
##      3   0   0   0 125
##      4   2 110  14  83
```

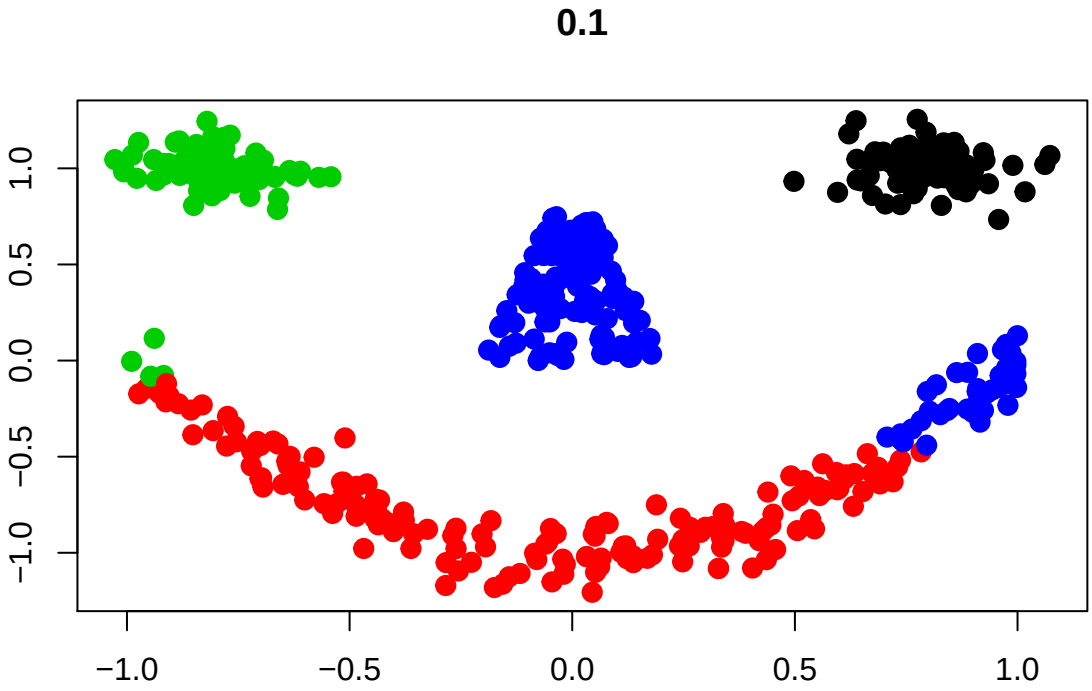
(b)

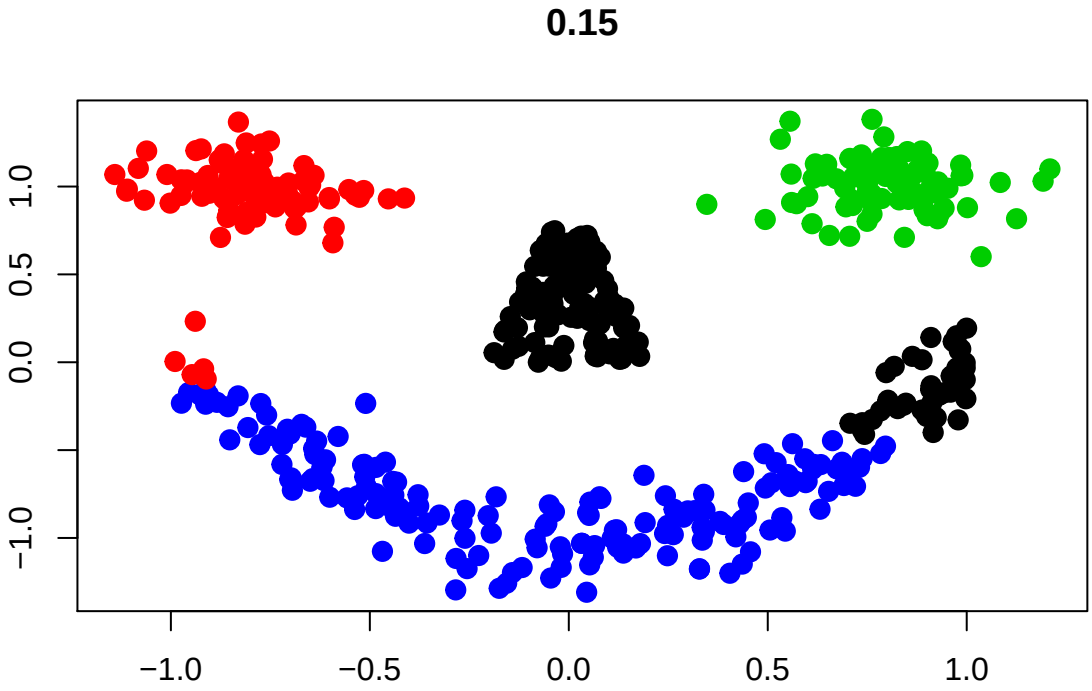
Show that if sd becomes larger, the 4 clusters are no longer recovered well. Find an approximate value of sd for which this change occurs (two decimal digits is enough), and explain how k-means clustering behaves for larger values of sd, using colored plots and two different examples.

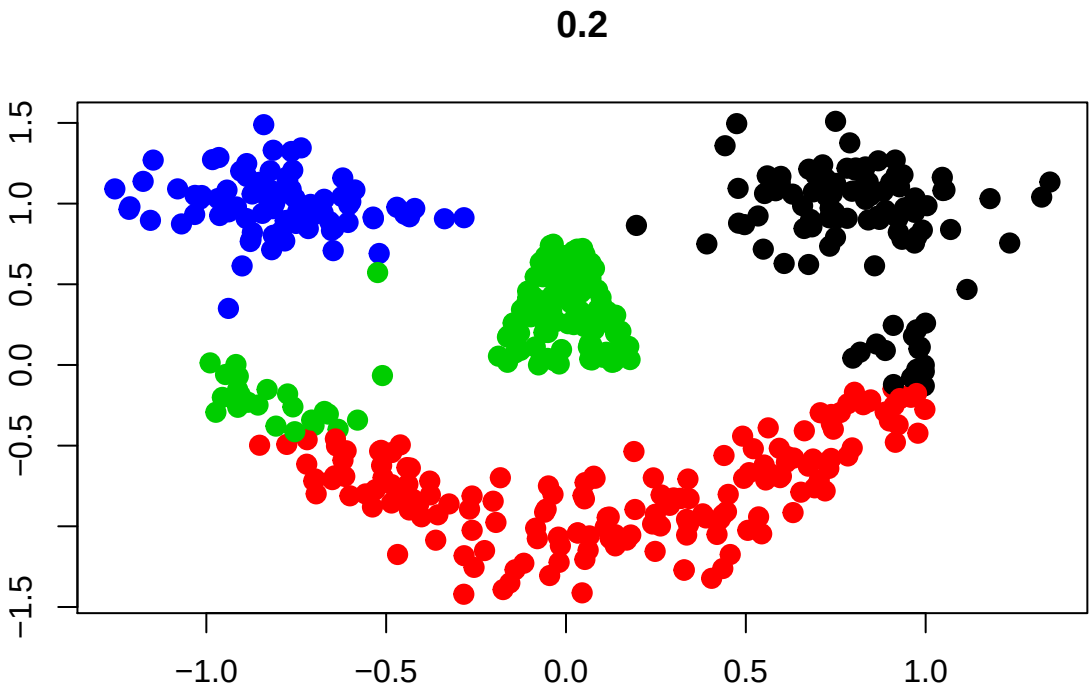
It seems that around sd=.55 is when the clusters start to really mix and become hard to distinguish. For larger values of sd, the k-means clustering is unable to correctly recover the clusters because the clusters have big overlap.

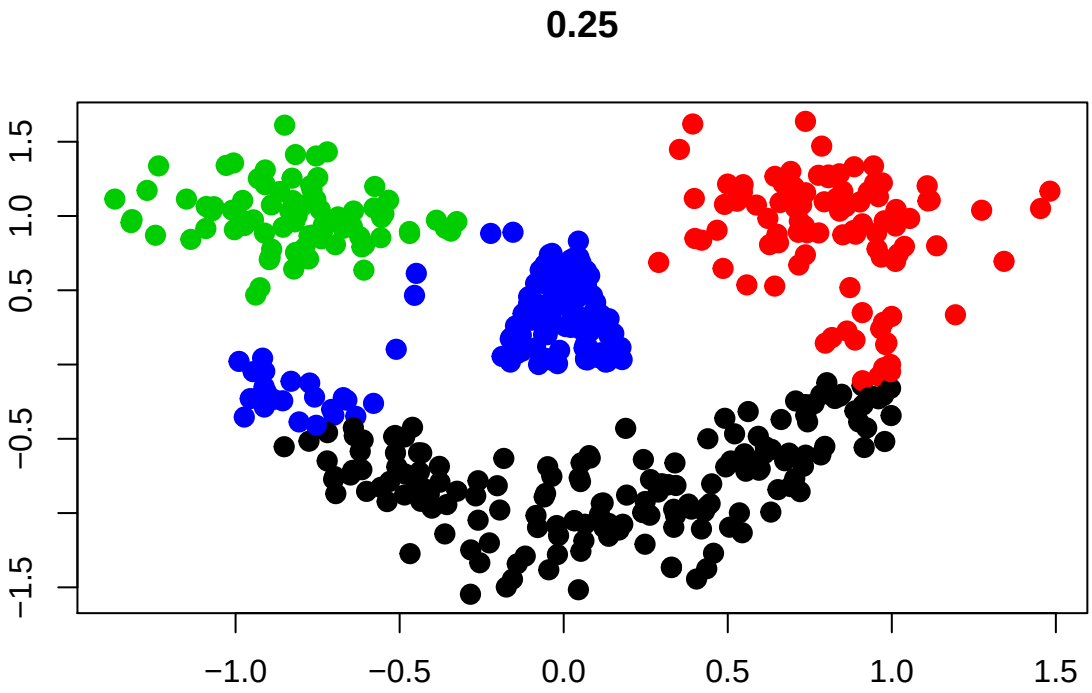
```
smiley.graph <- function(sd){
  set.seed(100)
  smiley <- mlbench.smiley(n=500, sd1=sd, sd2=sd)
  km.out <- kmeans(smiley$x, 4, nstart=100)
  # table(actual=smiley$classes, predicted=km.out$cluster)
  plot(smiley$x, col=(km.out$cluster), main=sd, xlab='', ylab='', pch=20, cex=2)
}
```

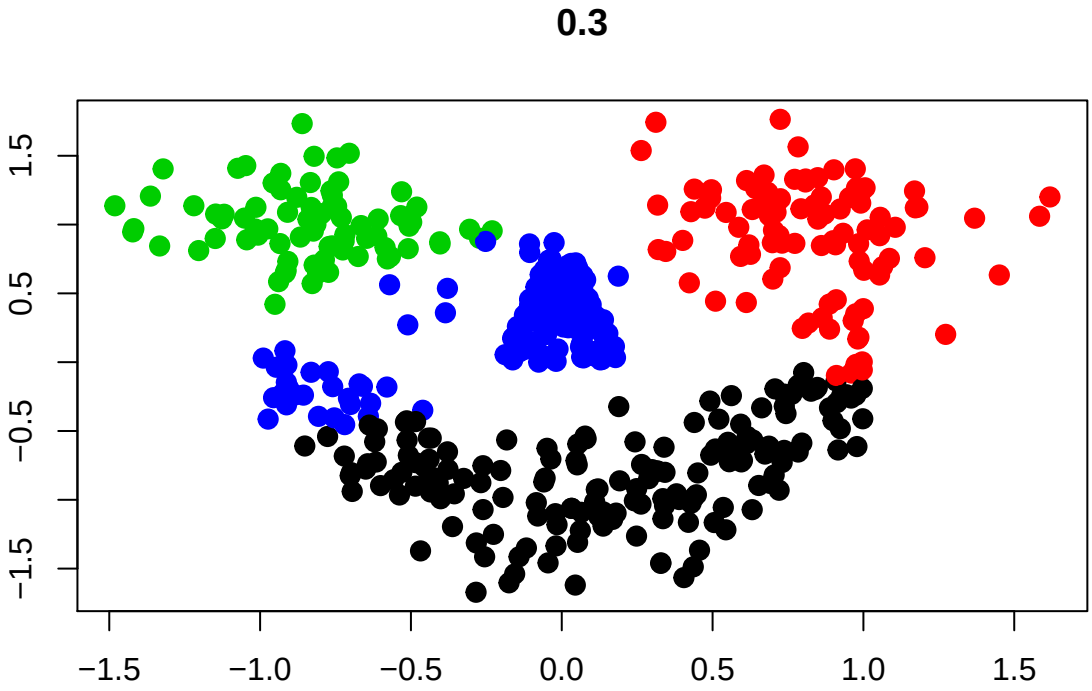
```
for (x in seq(.1, 1, .05)){
  smiley.graph(x)
}
```

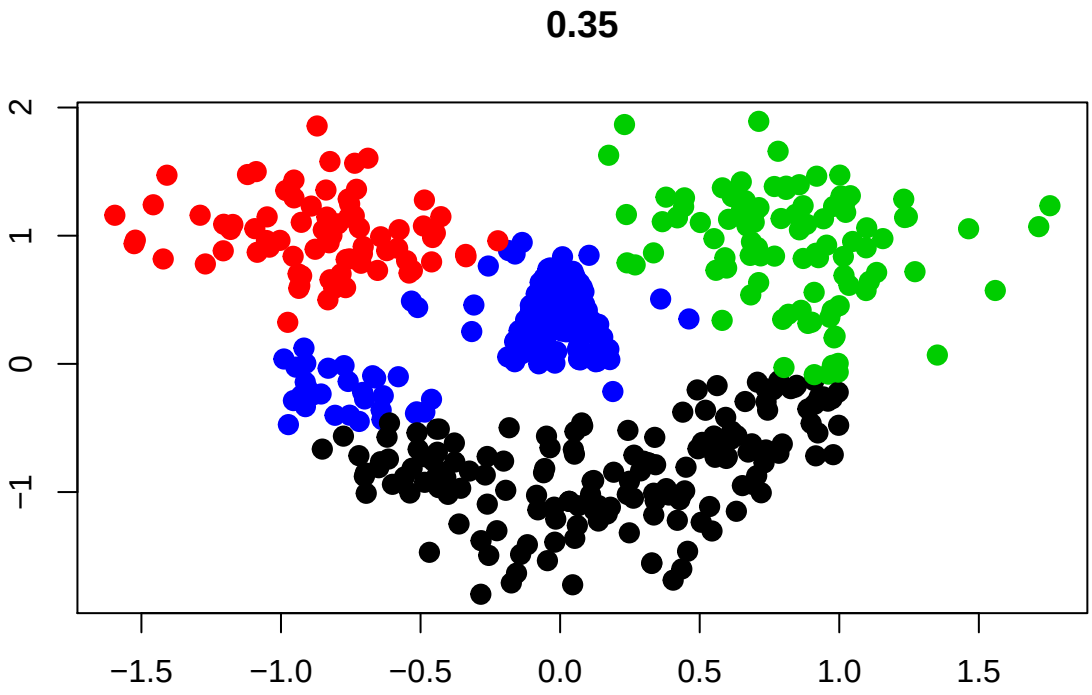

0.1
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0

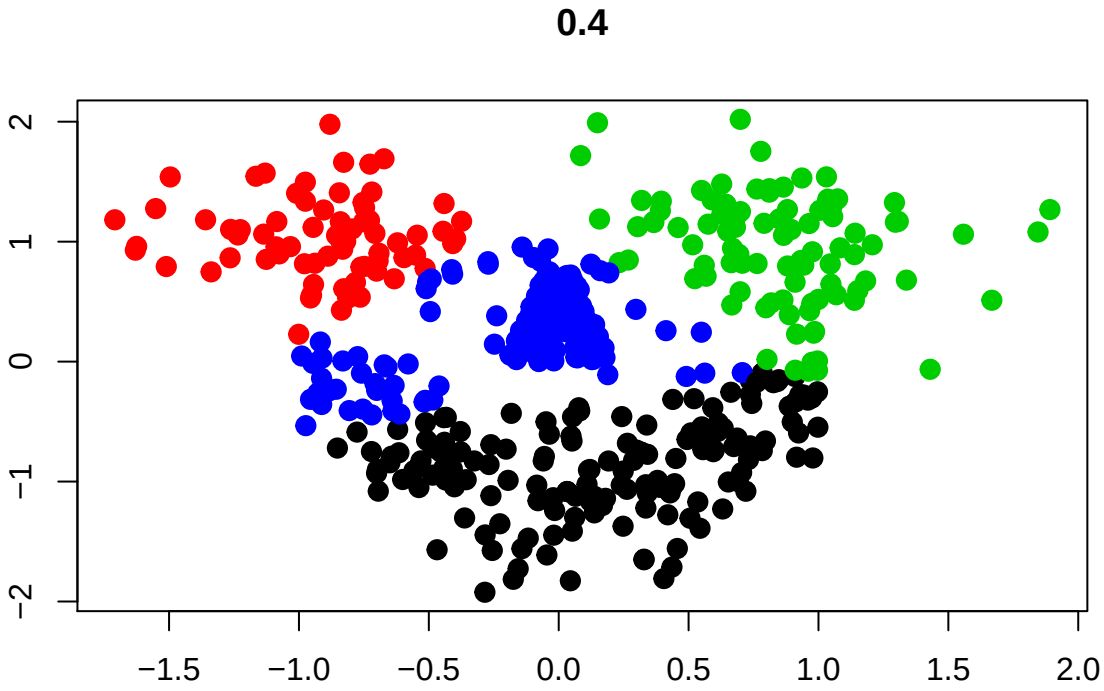

0.15
1.0
0.5
0.0
−1.0
−1.0
−0.5
0.0
0.5
1.0

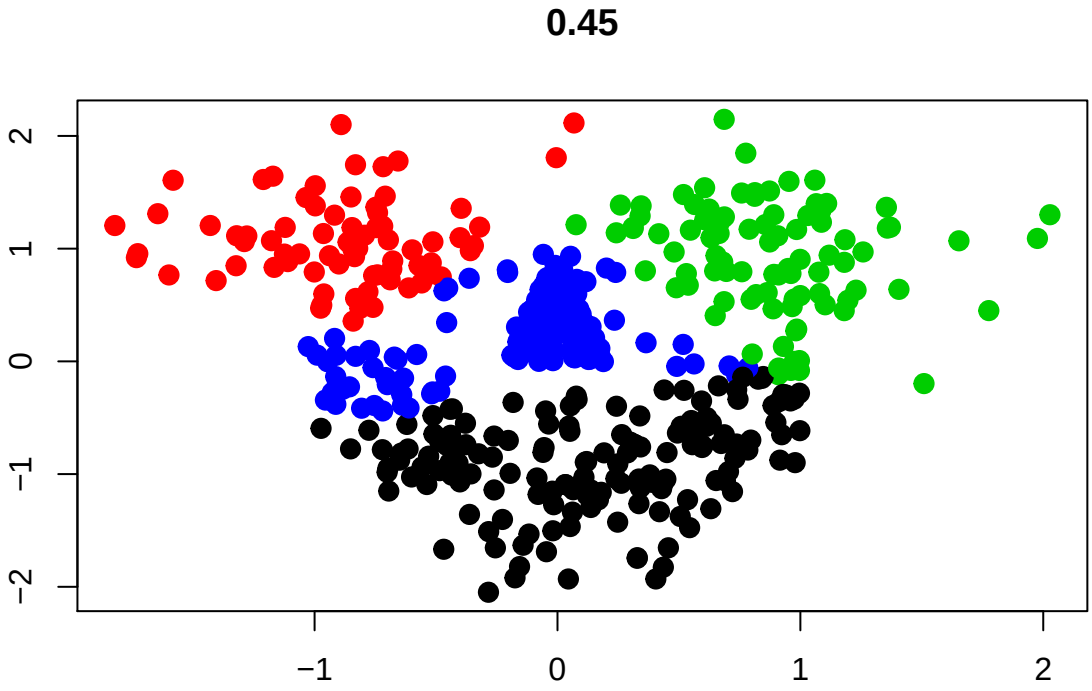

0.2
1.5
1.0
0.5
0.0
−0.5
−1.5
−1.0
−0.5
0.0
0.5
1.0

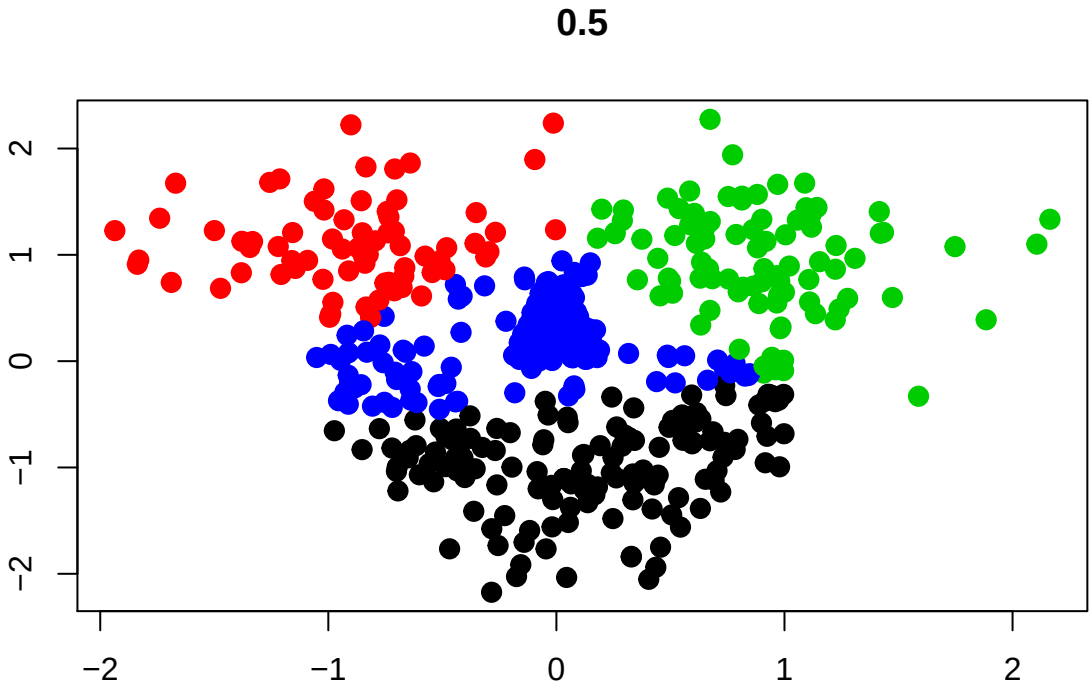

0.25
1.5
1.0
0.5
−0.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

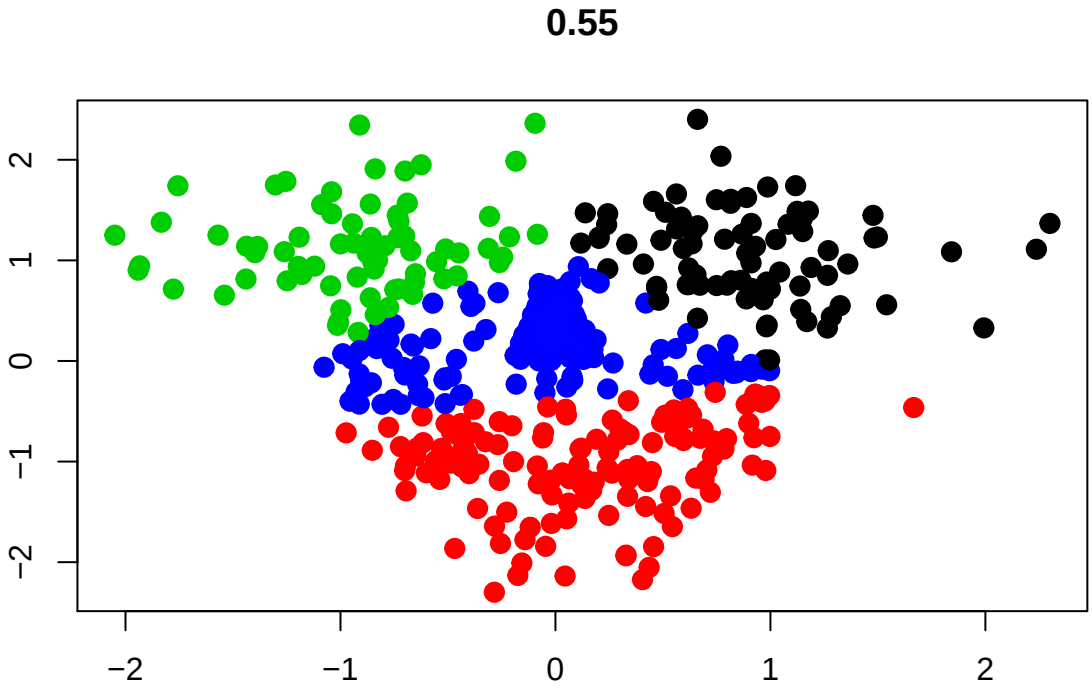

0.3
1.5
0.5
−0.5
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

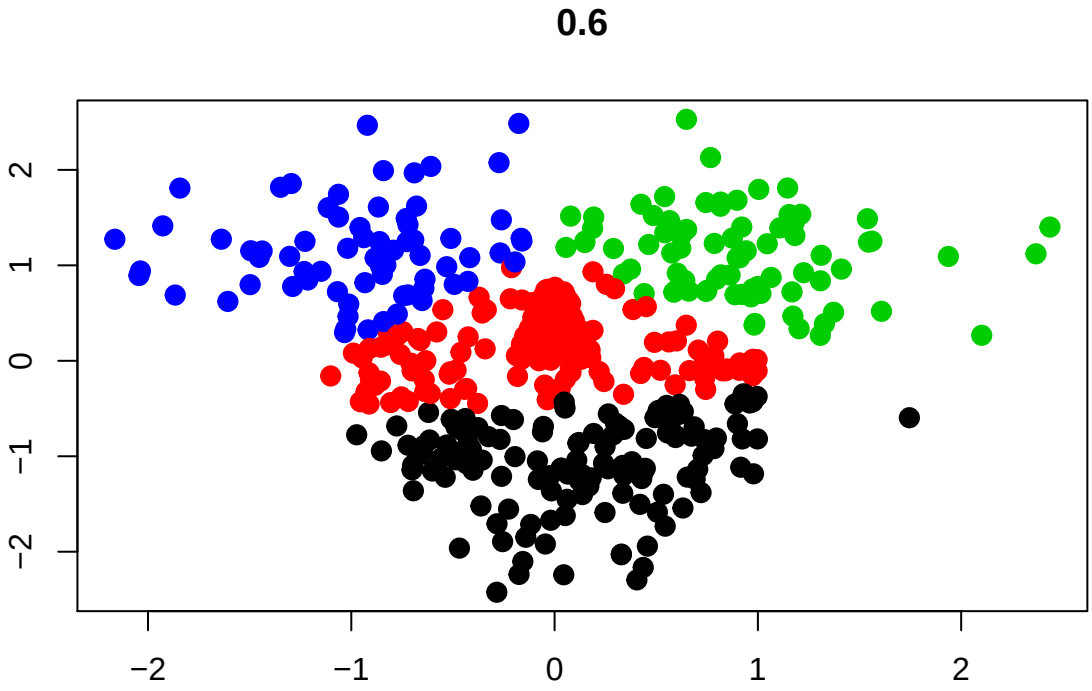

0.35
2
1
0
−1
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

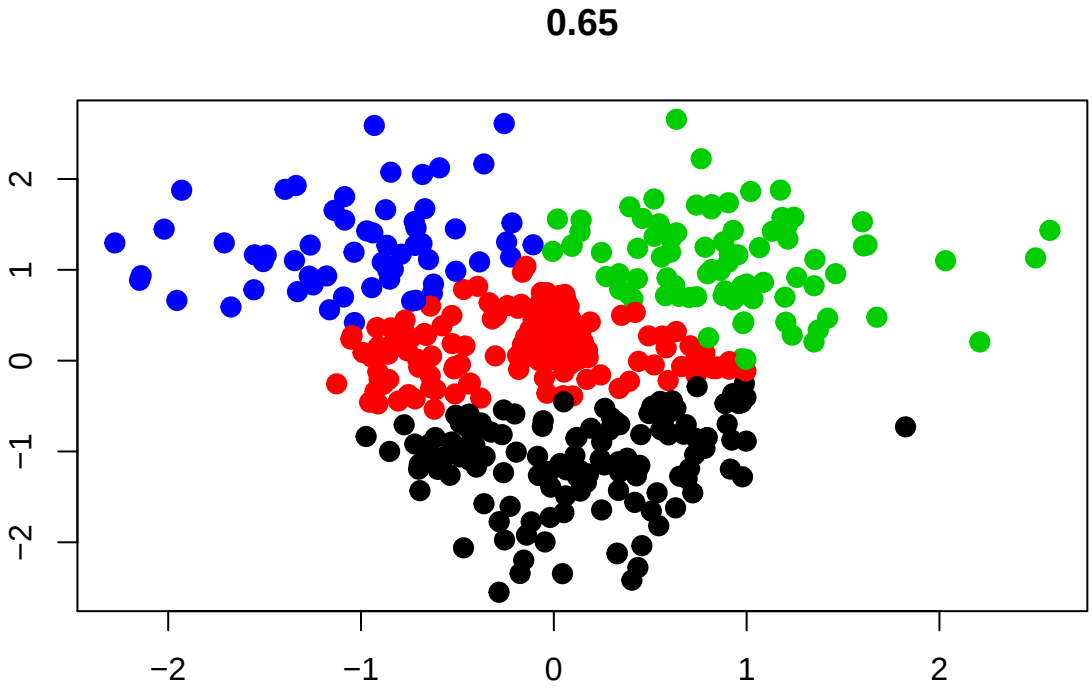

0.4
2
1
0
−1
−2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0


0.45
2
1
0
−1
−2
−1
0
1
2


0.5
2
1
0
−1
−2
−2
−1
0
1
2


0.55
2
1
0
−1
−2
−2
−1
0
1
2


0.6
2
1
0
−1
−2
−2
−1
0
1
2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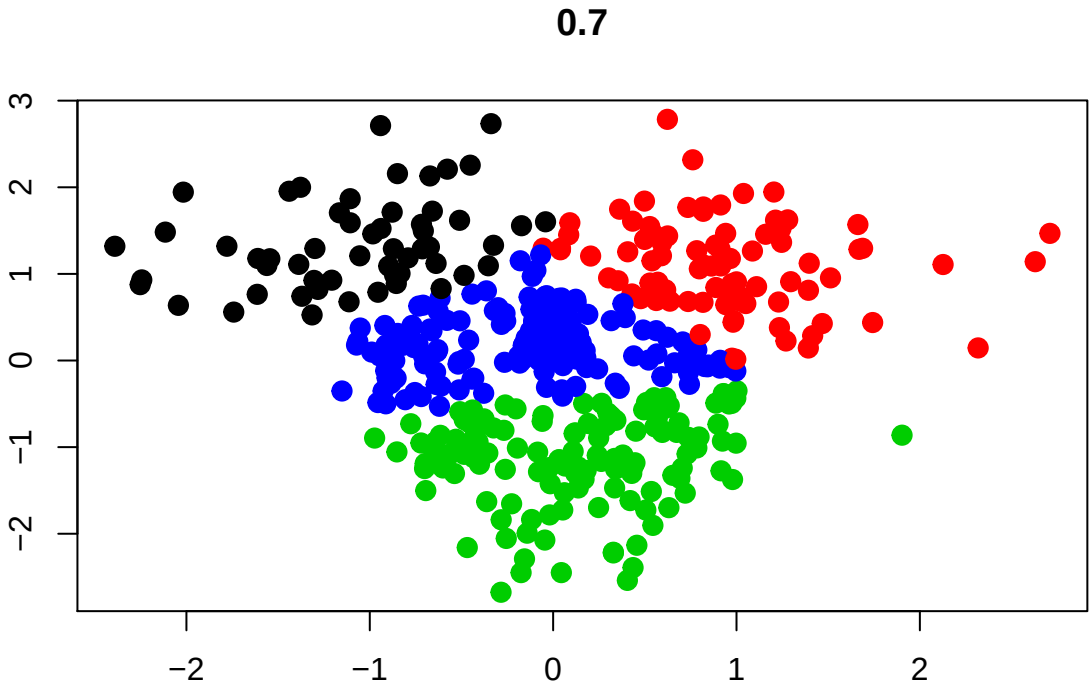
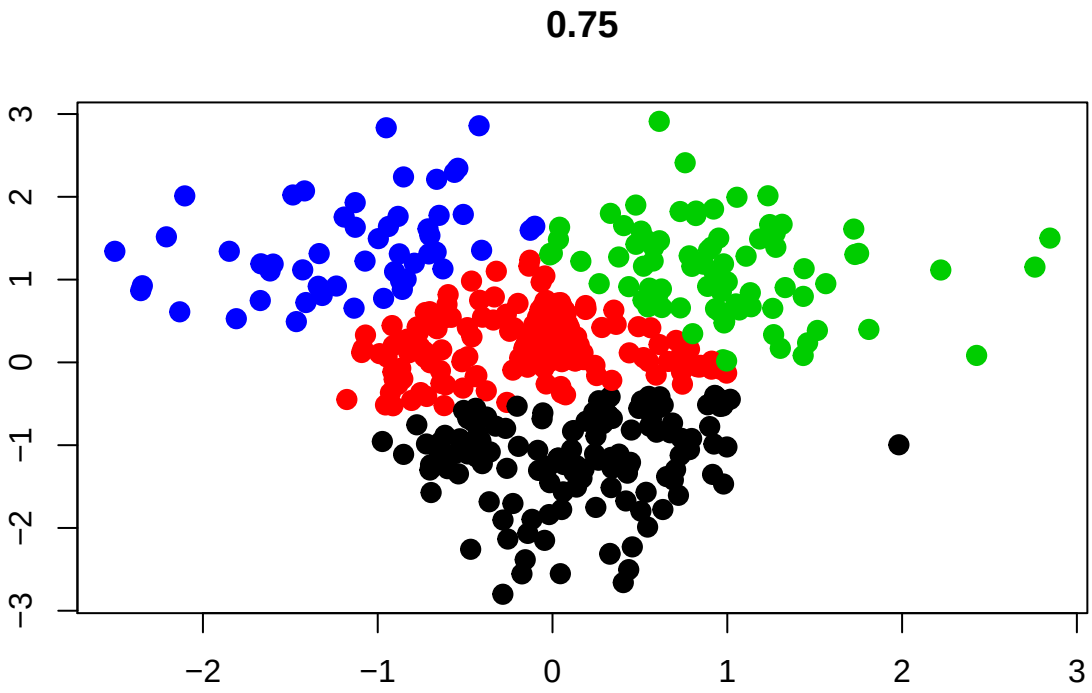
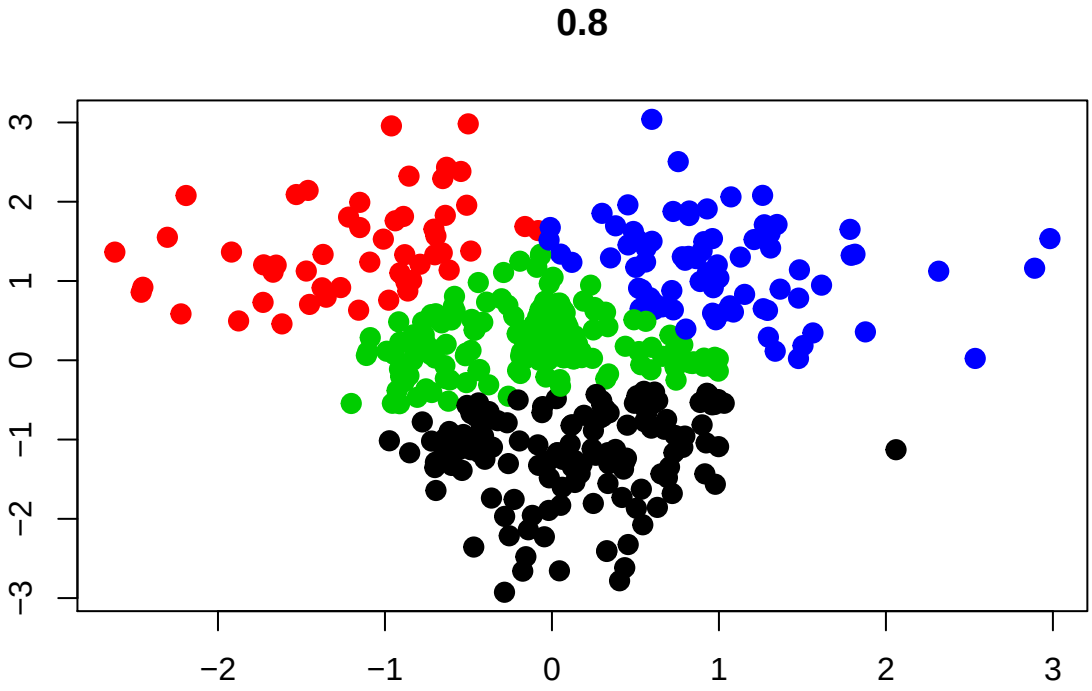
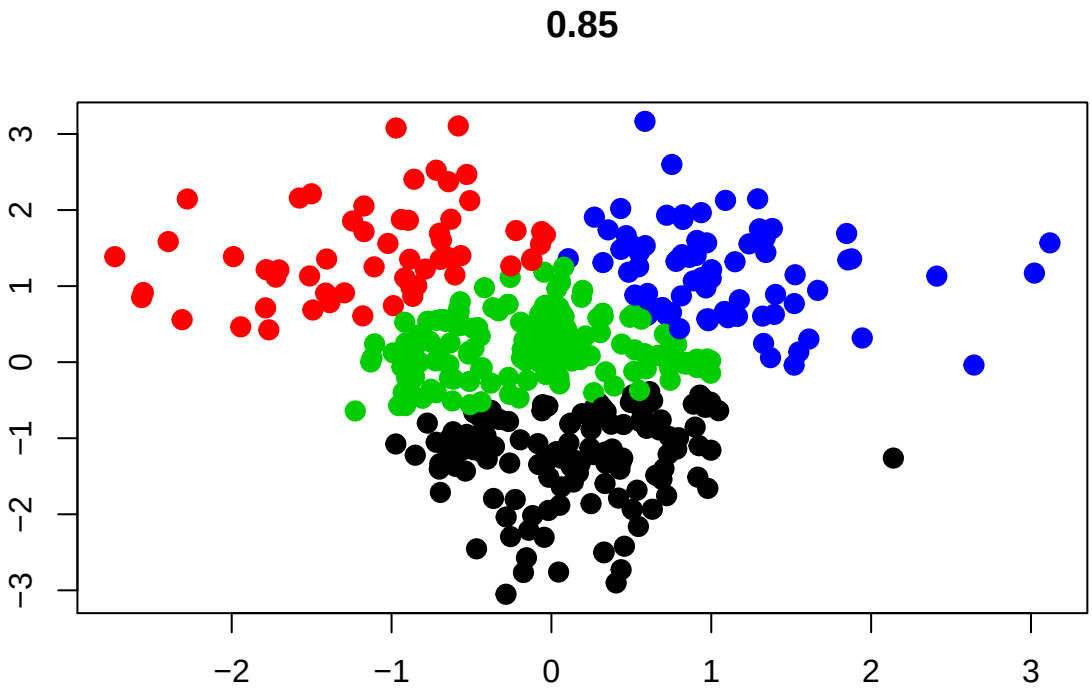
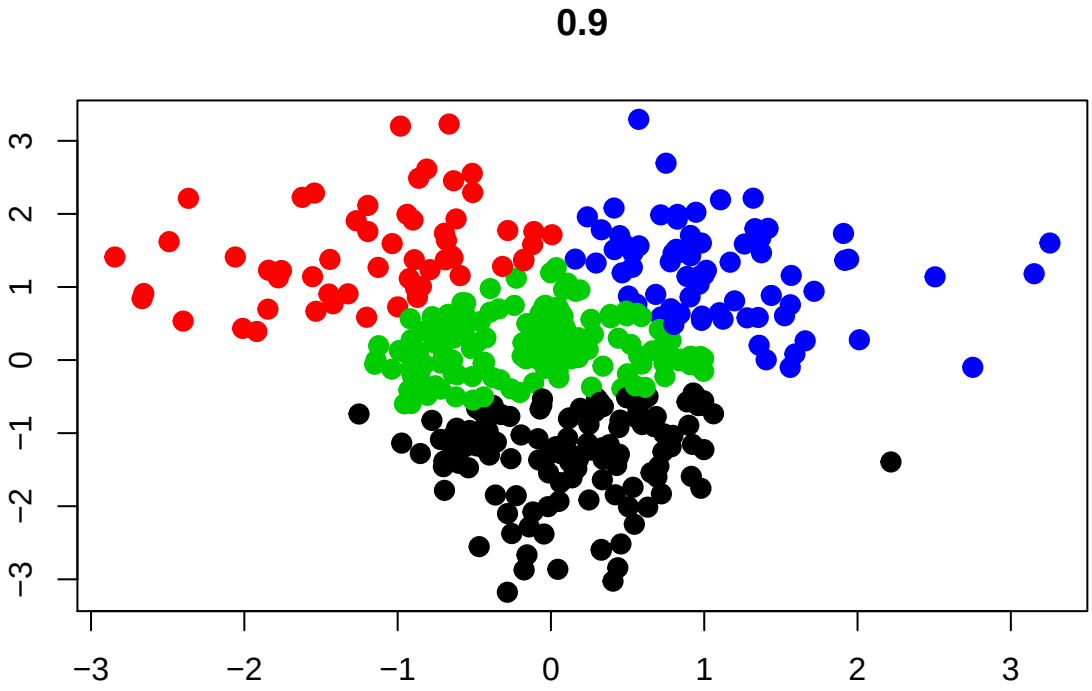
2
1
0
−1
−2
−2
−1
0
1
2

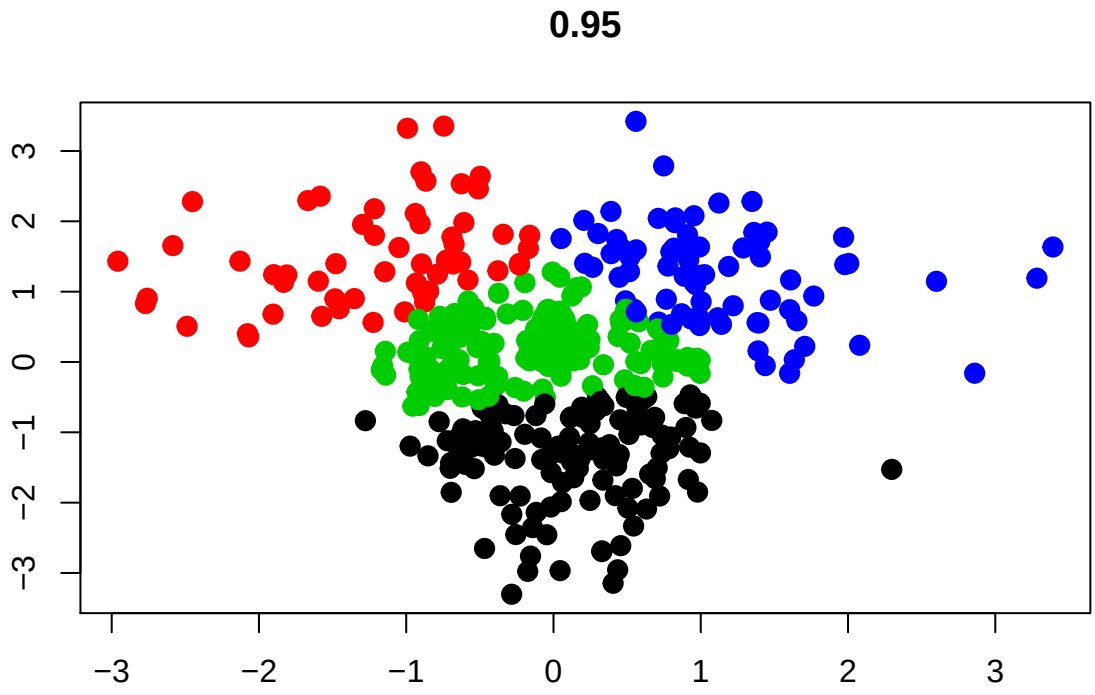

0.7
3
2
1
0
−1
−2
−2
−1
0
1
2

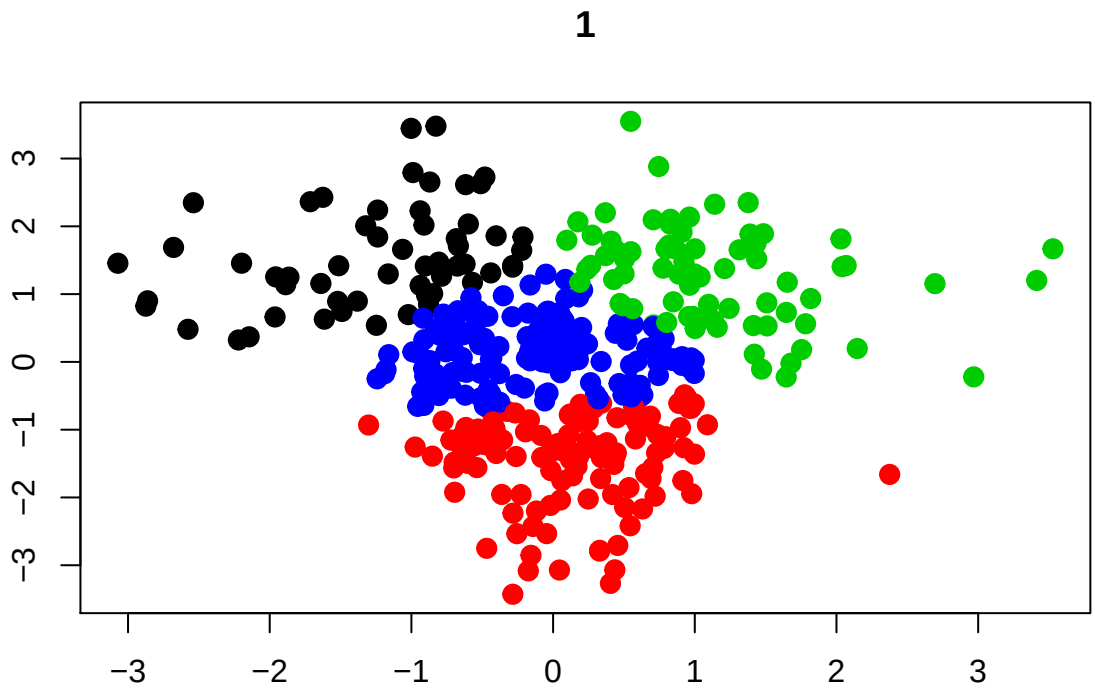

0.75
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
3


0.8
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

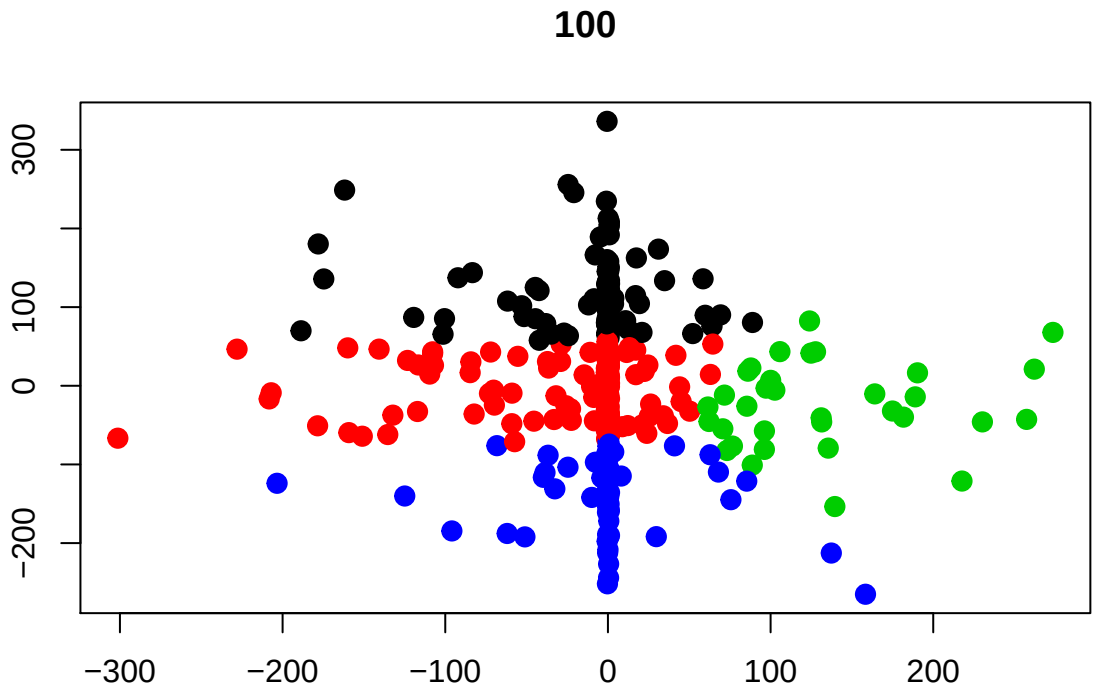

0.85
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
3


0.9
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

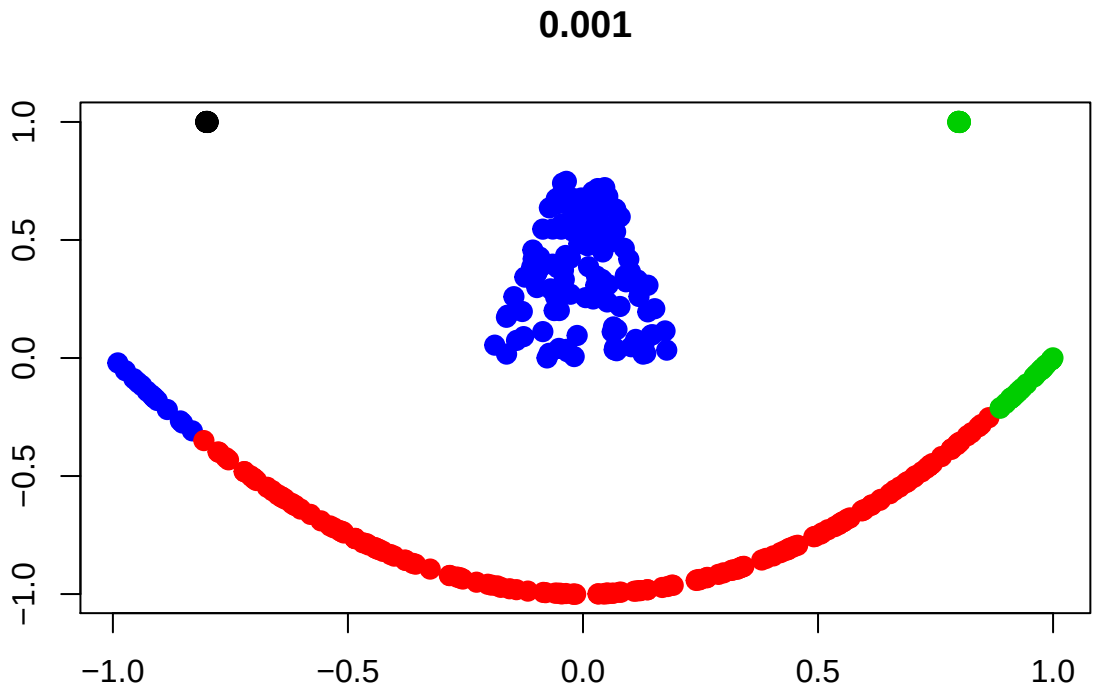

0.95
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3




```
smiley.graph(100)
```




```
smiley.graph(.001)
```



Extra 71

This problem uses the MNIST image classification data, available as mnist_all.RData that were used earlier. We use the training data only for all digits. Extract the training data and place them in suitable data frames.

(a)

Apply k-means clustering with two clusters. Can you tell which digits tend to be clustered together?

If we consider the majority, then digits 0, 2, 3, 6 tend to be clustered together, and digits 1, 4, 5, 7, 8 tend to be clustered together.

```
set.seed(100)
km.out <- kmeans(train$x, 2, nstart=100)
table(train$y, km.out$cluster)
```

```
##
##        1    2
##   0  226 5697
##   1 6729   13
##   2 2279 3679
##   3 2294 3837
##   4 5035  807
##   5 2851 2570
##   6 2394 3524
##   7 6091  174
##   8 3807 2044
```

```
##   9 5421  528
```

(b)

Apply k-means clustering with 10 clusters. How well do the cluster labels agree with the actual digit labels? Use a confusion matrix to answer this question.

The results show that correct labels are not well predicted.

```
set.seed(100)
km.out <- kmeans(train$x, 10, nstart=30)
```

```
## Warning: Quick-TRANSfer stage steps exceeded maximum (= 3000000)

## Warning: Quick-TRANSfer stage steps exceeded maximum (= 3000000)

## Warning: did not converge in 10 iterations

## Warning: Quick-TRANSfer stage steps exceeded maximum (= 3000000)

## Warning: did not converge in 10 iterations
```

```
table(train$y, km.out$cluster)
```

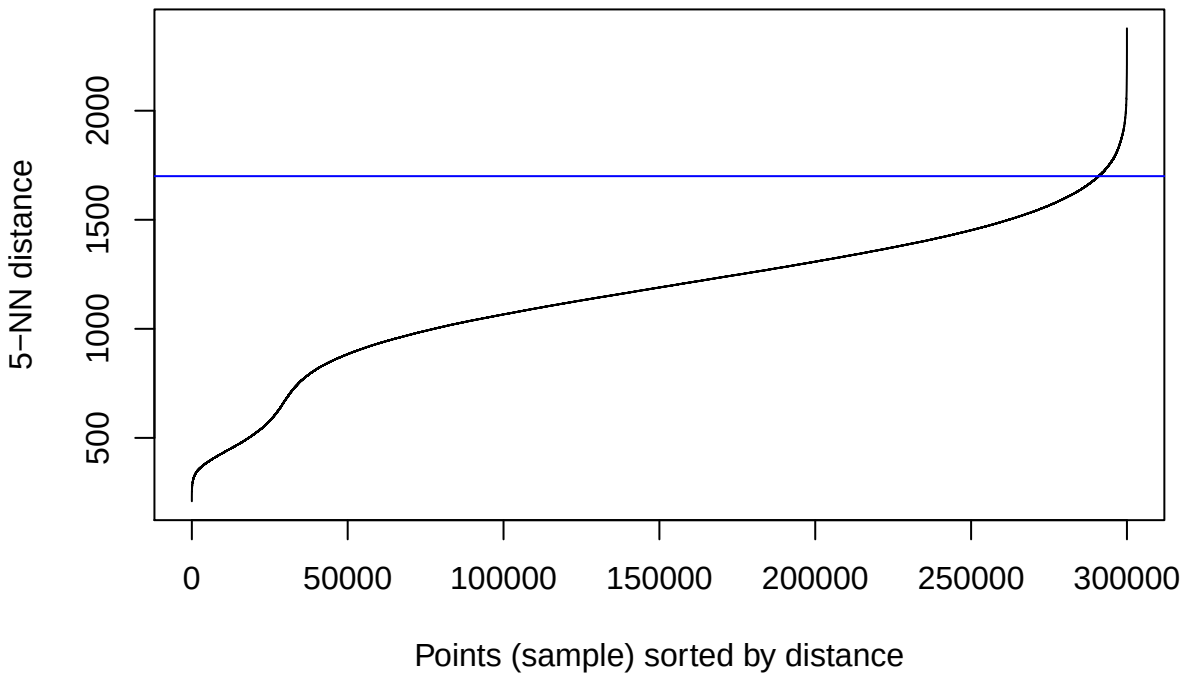
```
##
##        1    2    3    4    5    6    7    8    9   10
##   0 2497   17   38  179 2811  181  162   21   14    3
##   1    0    9    6   10    0    8    5 2978    9 3717
##   2   97 4194  173  158   11  212  327  352   69  365
##   3  127  216  175 1039   15   57 3920  106   49  427
##   4   11   38 3189   20   10  165    1  309 1940  159
##   5  259   14  376 1501   62  123 1767  801  351  167
##   6  121   85   82   91   99 4915   28  225    1  271
##   7   14   39 1795   10   15    4    5  283 3773  327
##   8   32   54  193 3456   38   47 1131  422  180  298
##   9   19   13 2901   69   37    8   85  117 2460  240
```

(c)

Apply dbscan clustering, with suitable choices of eps and minPts obtained from a k-nearest neighbor plot. Justify your choices. Then determine how well the cluster labels agree with the actual digit labels, using a confusion matrix.

The cluster labels from dbscan only created 2 diffrent clusters. Therefore, it does not agree well with the actual digit labels.

```
set.seed(100)
kNNdistplot(train$x, 5)
abline(h=1700, col=4)
```




```
set.seed(100)
db.out <- dbscan(train$x, eps=1700, minPts=20)
table(actual=train$y, pred=db.out$cluster)
```

```
##       pred
## actual    0    1
##      0   69 5854
##      1    5 6737
##      2  347 5611
##      3  221 5910
##      4   86 5756
##      5  239 5182
##      6   93 5825
##      7   29 6236
##      8  452 5399
##      9   97 5852
```