Chapter 10
Sorting

Data Structures and Algorithms

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Outcomes

- **L.O.6.1** - Depict the working steps of sorting algorithms step-by-steps.
- **L.O.6.2** - Describe sorting algorithms by using pseudocode.
- **L.O.6.3** - Implement sorting algorithms using C/C++ .
- **L.O.6.4** - Analyze the complexity and develop experiment (program) to evaluate sorting algorithms.
- **L.O.6.5** - Use sorting algorithms for problems in real-life.
- **L.O.8.4** - Develop recursive implementations for methods supplied for the following structures: list, tree, heap, searching, and graphs.
- **L.O.1.2** - Analyze algorithms and use Big-O notation to characterize the computational complexity of algorithms composed by using the following control structures: sequence, branching, and iteration (not recursion).

Contents

1. **Sorting concepts**

2. **Insertion Sort**
   - Straight Insertion Sort
   - Shell Sort

3. **Selection Sort**
   - Straight Selection Sort
   - Heap Sort

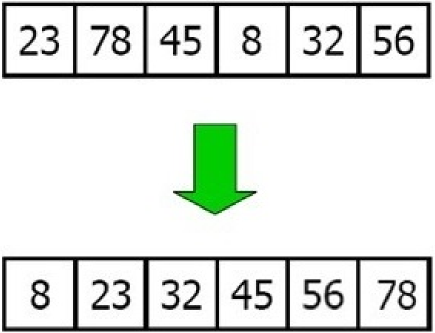
4. **Exchange Sort**
   - Bubble Sort

5. **Devide-and-Conquer**
   - Quick Sort
   - Merge Sort

Sorting concepts

Sorting

One of the most important concepts and common applications in computing.

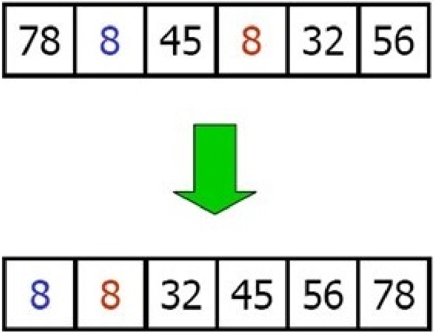


23	78	45	8	32	56

8	23	32	45	56	78

Sorting

Sort stability: data with equal keys maintain their relative input order in the output.



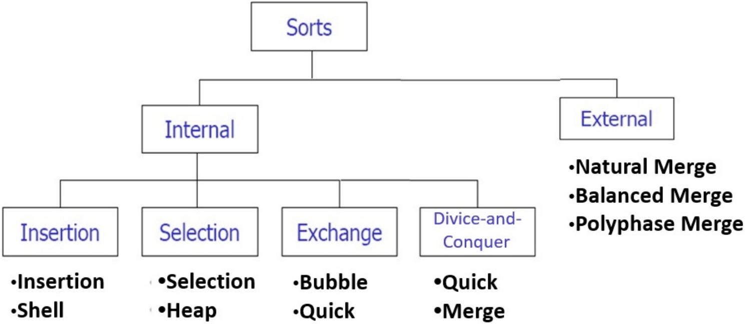
78	8	45	8	32	56

8	8	32	45	56	78

Sorting

Sort efficiency: a measure of the relative efficiency of a sort = number of comparisons + number of moves.

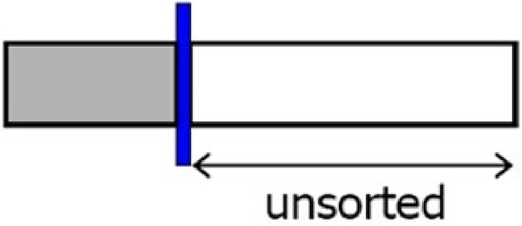
Sorting


Sorts
Internal
External
•Natural Merge
•Balanced Merge
•Polyphase Merge
Insertion
Selection
Exchange
Divice-and-Conquer
•Insertion
•Shell
•Selection
•Heap
•Bubble
•Quick
•Quick
•Merge

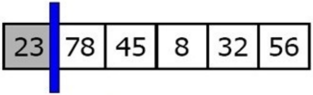
Insertion Sort

Straight Insertion Sort

- The list is divided into two parts: sorted and unsorted.
- In each pass, the first element of the unsorted sublist is inserted into the sorted sublist.

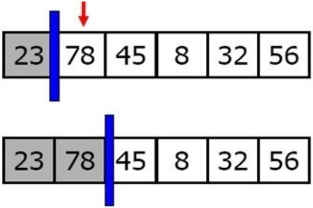


Straight Insertion Sort



23	78	45	8	32	56

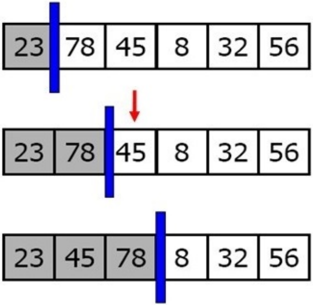
Straight Insertion Sort



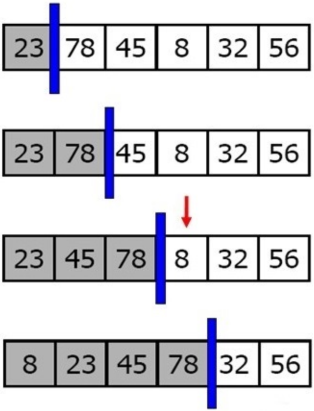
23	78	45	8	32	56

23	78	45	8	32	56

Straight Insertion Sort



Straight Insertion Sort



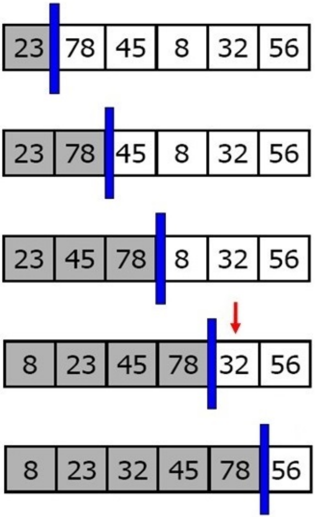
23 | 78 45 8 32 56

23 78 | 45 8 32 56

23 45 78 | 8 32 56

8 23 45 78 | 32 56

Straight Insertion Sort



23 | 78 45 8 32 56

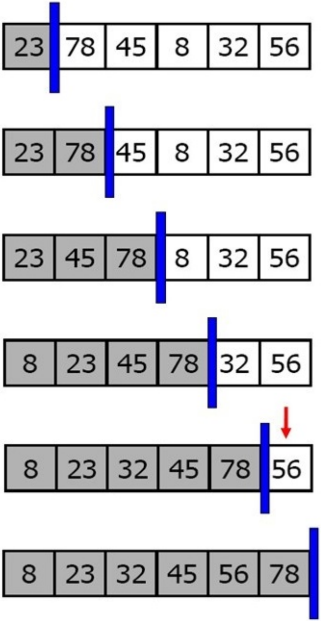
23 78 | 45 8 32 56

23 45 78 | 8 32 56

8 23 45 78 | 32 56

8 23 32 45 78 | 56

Straight Insertion Sort



23 | 78 45 8 32 56

23 78 | 45 8 32 56

23 45 78 | 8 32 56

8 23 45 78 | 32 56

8 23 32 45 78 | 56

8 23 32 45 56 78 |

Straight Insertion Sort

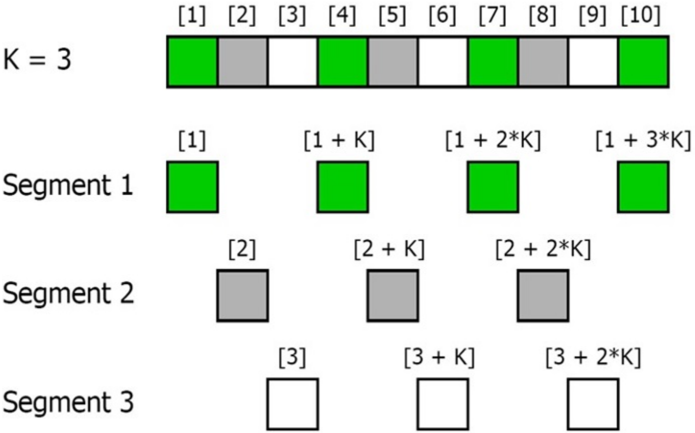
Algorithm InsertionSort()
Sorts the contiguous list using straight insertion sort.

```
if count > 1 then
    current = 1
    while current < count do
        temp = data[current]
        walker = current - 1
        while walker >= 0 AND temp.key <
        data[walker].key do
            data[walker+1] = data[walker]
            walker = walker - 1
        end
        data[walker+1] = temp
        current = current + 1
    end
end
End InsertionSort
```

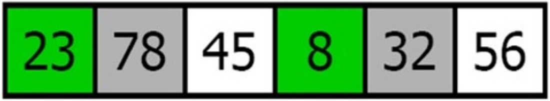
Shell Sort

- Named after its creator Donald L. Shell (1959).
- Given a list of N elements, the list is divided into K segments (K is called the increment).
- Each segment contains N/K or more elements.
- Segments are dispersed throughout the list.
- Also is called diminishing-increment sort.

Shell Sort


K = 3
[1]
[2]
[3]
[4]
[5]
[6]
[7]
[8]
[9]
[10]
Segment 1
[1]
[1 + K]
[1 + 2*K]
[1 + 3*K]
Segment 2
[2]
[2 + K]
[2 + 2*K]
Segment 3
[3]
[3 + K]
[3 + 2*K]

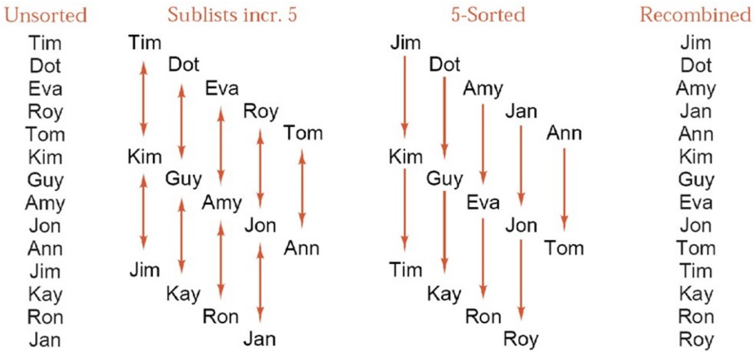
Shell Sort



23	78	45	8	32	56

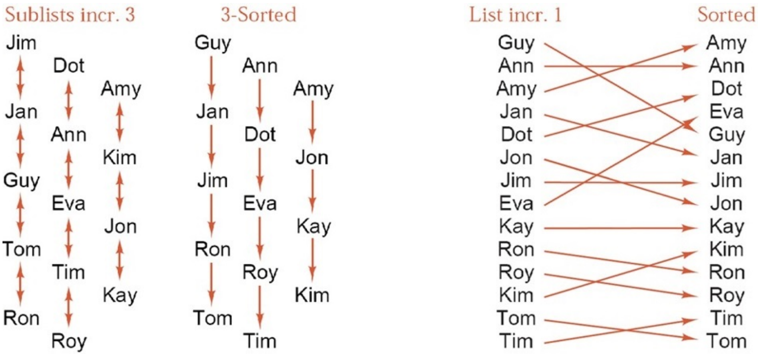
- For the value of K in each iteration, sort the K segments.
- After each iteration, K is reduced until it is 1 in the final iteration.

Example of Shell Sort



Unsorted	Sublists incr. 5	5-Sorted	Recombined
Tim	Tim	Jim	Jim
Dot	Dot	Dot	Dot
Eva	Eva	Amy	Amy
Roy	Roy	Jan	Jan
Tom	Tom	Ann	Ann
Kim	Kim	Kim	Kim
Guy	Guy	Guy	Guy
Amy	Amy	Eva	Eva
Jon	Jon	Jon	Jon
Ann	Ann	Tom	Tom
Jim	Jim	Tim	Tim
Kay	Kay	Kay	Kay
Ron	Ron	Ron	Ron
Jan	Jan	Roy	Roy

Example of Shell Sort



Sublists incr. 3

Jim
Dot
Amy
Jan
Ann
Kim
Guy
Eva
Jon
Tom
Tim
Kay
Ron
Roy

3-Sorted

Guy
Ann
Amy
Jan
Dot
Jon
Jim
Eva
Kay
Ron
Roy
Kim
Tom
Tim

List incr. 1

Guy
Ann
Amy
Jan
Dot
Jon
Jim
Eva
Kay
Ron
Roy
Kim
Tom
Tim

Sorted

Amy
Ann
Dot
Eva
Guy
Jan
Jim
Jon
Kay
Kim
Ron
Roy
Tim
Tom

Choosing incremental values

- From more of the comparisons, it is better when we can receive more new information.
- Incremental values should not be multiples of each other, other wise, the same keys compared on one pass would be compared again at the next.
- The final incremental value must be 1.

Choosing incremental values

- Incremental values may be:
  $1, 4, 13, 40, 121, \ldots$
  $k_t = 1$
  $k_{i-1} = 3 * k_i + 1$
  $t = |\log_3 n| - 1$

- or:
  $1, 3, 7, 15, 31, \ldots$
  $k_t = 1$
  $k_{i-1} = 2 * k_i + 1$
  $t = |\log_2 n| - 1$

Shell Sort

Algorithm ShellSort()

Sorts the contiguous list using Shell sort.

```
k = first_incremental_value
while $k >= 1$ do
    segment = 1
    while $segment <= k$ do
        SortSegment(segment)
        segment = segment + 1
    end
    k = next_incremental_value
end
End ShellSort
```

Shell Sort

Algorithm SortSegment(val segment <int>, val k <int>)
Sorts the segment beginning at segment using insertion sort, step between elements in the segment is k.

```
current = segment + k
while current < count do
    temp = data[current]
    walker = current - k
    while walker >=0 AND temp.key < data[walker].key do
        data[walker + k] = data[walker]
        walker = walker - k
    end
    data[walker + k] = temp
    current = current + k
end
End SortSegment
```

Insertion Sort Efficiency

- Straight insertion sort:
  $f(n) = n(n+1)/2 = O(n^2)$
- Shell sort:
  $O(n^{1.25})$ (Empirical study)

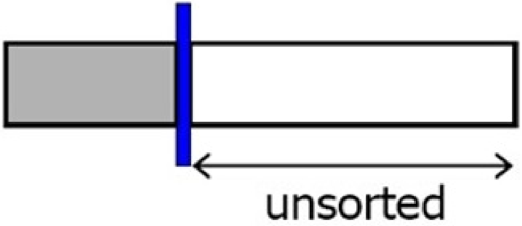
Selection Sort

Selection Sort

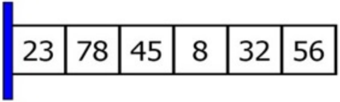
In each pass, the smallest/largest item is selected and placed in a sorted list.

Straight Selection Sort

- The list is divided into two parts: sorted and unsorted.
- In each pass, in the unsorted sublist, the smallest element is selected and exchanged with the first element.

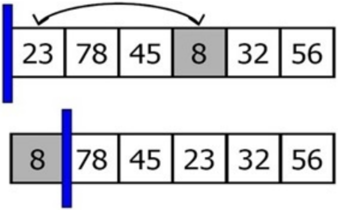


Straight Selection Sort



23	78	45	8	32	56

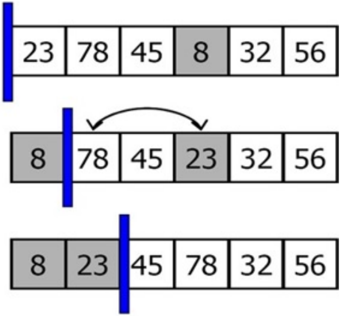
Straight Selection Sort



23 78 45 8 32 56

8 78 45 23 32 56

Straight Selection Sort

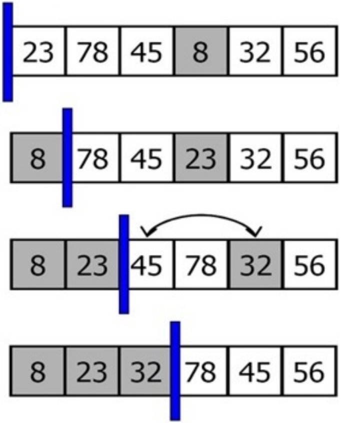


23 78 45 8 32 56

8 78 45 23 32 56

8 23 45 78 32 56

Straight Selection Sort



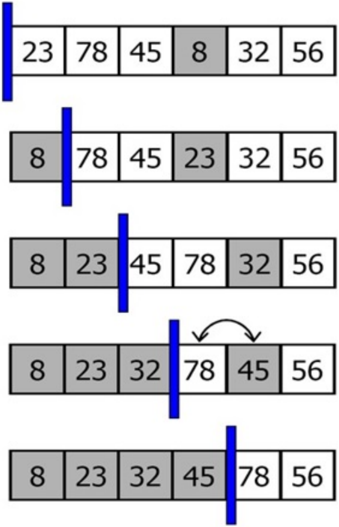
23 78 45 8 32 56

8 78 45 23 32 56

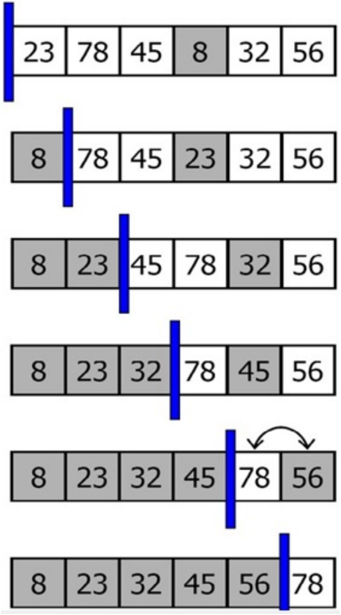
8 23 45 78 32 56

8 23 32 78 45 56

Straight Selection Sort



Straight Selection Sort



23 | 78 | 45 | 8 | 32 | 56

8 | 78 | 45 | 23 | 32 | 56

8 | 23 | 45 | 78 | 32 | 56

8 | 23 | 32 | 78 | 45 | 56

8 | 23 | 32 | 45 | 78 | 56

8 | 23 | 32 | 45 | 56 | 78

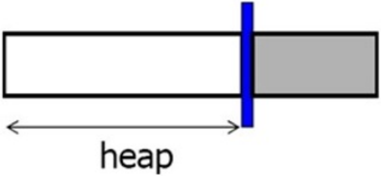
Straight Selection Sort

Algorithm SelectionSort()
Sorts the contiguous list using straight selection sort.

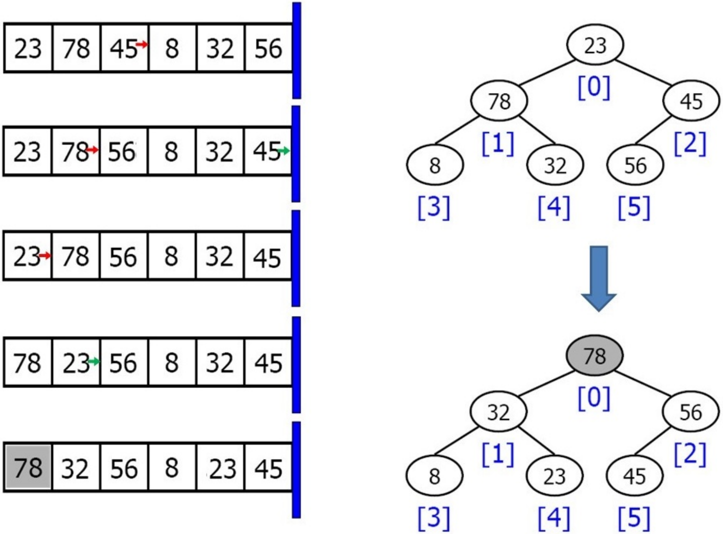
```
current = 0
while current < count - 1 do
    smallest = current
    walker = current + 1
    while walker < count do
        if data [walker].key < data [smallest].key then
            smallest = walker
        end
        walker = walker + 1
    end
    swap(current, smallest)
    current = current + 1
end
End SelectionSort
```

Heap Sort

- The unsorted sublist is organized into a heap.
- In each pass, in the unsorted sublist, the largest element is selected and exchanged with the last element.
- The the heap is reheaped.

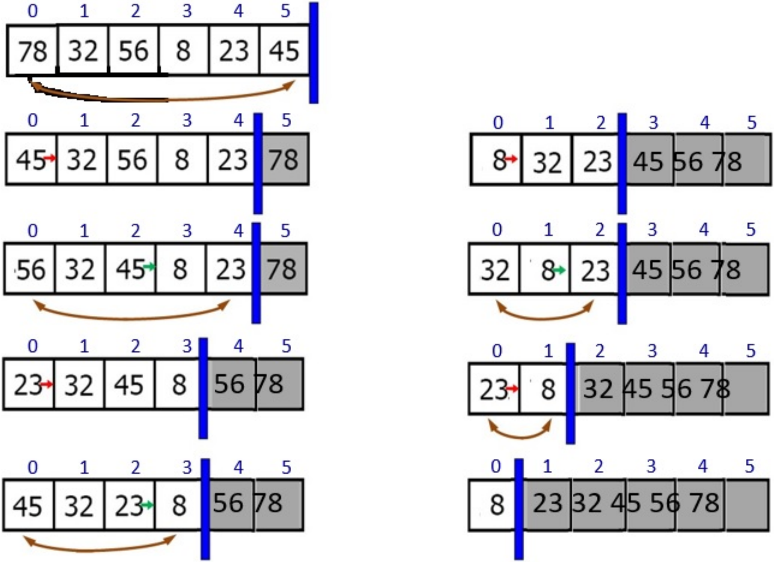


Heap Sort



23	78	45	8	32	56
23	78	56	8	32	45
23	78	56	8	32	45
78	23	56	8	32	45
78	32	56	8	23	45

Heap Sort



0	1	2	3	4	5
78	32	56	8	23	45

0	1	2	3	4	5
45	32	56	8	23	78

0	1	2	3	4	5
56	32	45	8	23	78

0	1	2	3	4	5
23	32	45	8	56	78

0	1	2	3	4	5
45	32	23	8	56	78

0	1	2	3	4	5
8	32	23	45	56	78

0	1	2	3	4	5
32	8	23	45	56	78

0	1	2	3	4	5
23	8	32	45	56	78

0	1	2	3	4	5
8	23	32	45	56	78

Heap Sort

```
Algorithm HeapSort()
Sorts the contiguous list using heap sort.

position = count/2 - 1
while position >= 0 do
    ReheapDown(position, count - 1)
    position = position - 1
end
last = count - 1
while last > 0 do
    swap(0, last)
    last = last - 1
    ReheapDown(0, last - 1)
end
End HeapSort
```

Selection Sort Efficiency

- Straight selection sort: $O(n^2)$
- Heap sort: $O(nlog_2n)$

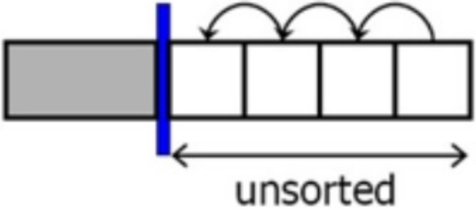
Exchange Sort

Exchange Sort

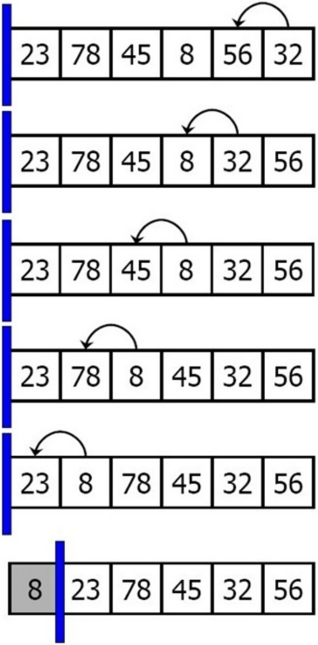
- In each pass, elements that are out of order are exchanged, until the entire list is sorted.
- Exchange is extensively used.

Bubble Sort

- The list is divided into two parts: sorted and unsorted.
- In each pass, the smallest element is bubbled from the unsorted sublist and moved to the sorted sublist.



Bubble Sort



23 78 45 8 56 32

23 78 45 8 32 56

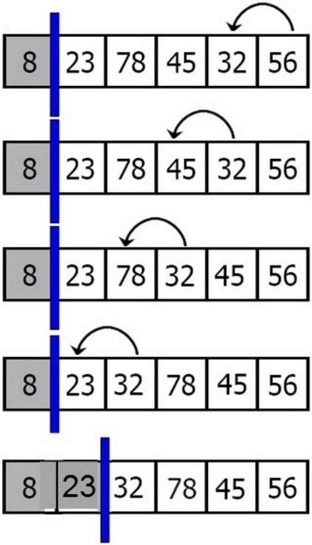
23 78 45 8 32 56

23 78 8 45 32 56

23 8 78 45 32 56

8 23 78 45 32 56

Bubble Sort



8 | 23 78 45 32 56

8 | 23 78 45 32 56

8 | 23 78 32 45 56

8 | 23 32 78 45 56

8 23 | 32 78 45 56

Bubble Sort

```
Algorithm BubbleSort()
Sorts the contiguous list using bubble sort.

current = 0
flag = False
while current < count AND flag = False do
    walker = count - 1
    flag = True
    while walker > current do
        if data [walker].key < data [walker-1].key then
            flag = False
            swap(walker, walker - 1)
        end
        walker = walker - 1
    end
    current = current + 1
end
End BubbleSort
```

Exchange Sort Efficiency

- Bubble sort:
  $f(n) = n(n+1)/2 = O(n^2)$

Devide-and-Conquer

Devide-and-Conquer Sort

```
Algorithm DevideAndConquer()
if the list has length > 1 then
    partition the list into lowlist and highlist
    lowlist.DevideAndConquer()
    highlist.DevideAndConquer()
    combine(lowlist, highlist)
end
End DevideAndConquer
```

Devide-and-Conquer Sort

	Partition	Combine
Merge Sort	easy	hard
Quick Sort	hard	easy

Quick Sort

```
Algorithm QuickSort()
Sorts the contiguous list using quick sort.

recursiveQuickSort(0, count - 1)
End QuickSort
```

Quick Sort

Algorithm recursiveQuickSort(val left <int>, val right <int>)

Sorts the contiguous list using quick sort.

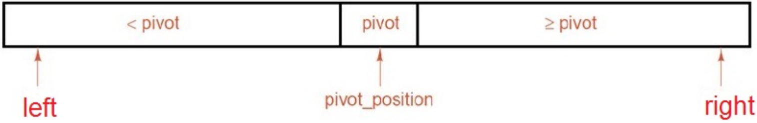
Pre: `left` and `right` are valid positions in the list

Post: list sorted

```
if left < right then
    pivot_position = Partition(left, right)
    recursiveQuickSort(left, pivot_position - 1)
    recursiveQuickSort(pivot_position + 1, right)
end
End recursiveQuickSort
```

Quick Sort

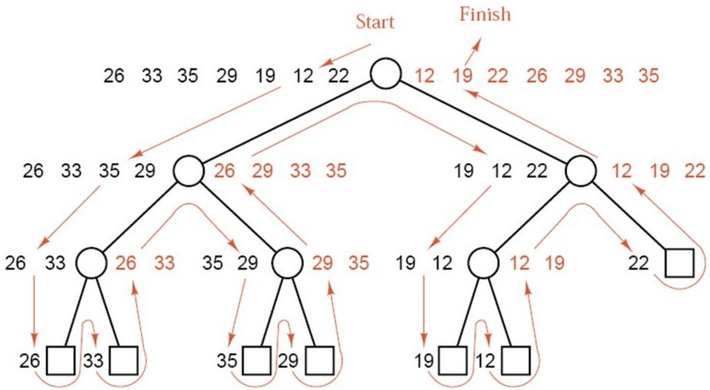
Given a pivot value, the partition rearranges the entries in the list as the following figure:



Quick Sort Efficiency

- Quick sort: $O(nlog_2n)$

Merge Sort


Start
Finish
26 33 35 29 19 12 22
12 19 22 26 29 33 35
26 33 35 29
26 29 33 35
19 12 22
12 19 22
26 33
26 33
35 29
29 35
19 12
12 19
22
26
33
35
29
19
12

Merge Sort

```
Algorithm MergeSort()
Sorts the linked list using merge sort.

recursiveMergeSort(head)
End MergeSort
```

Merge Sort

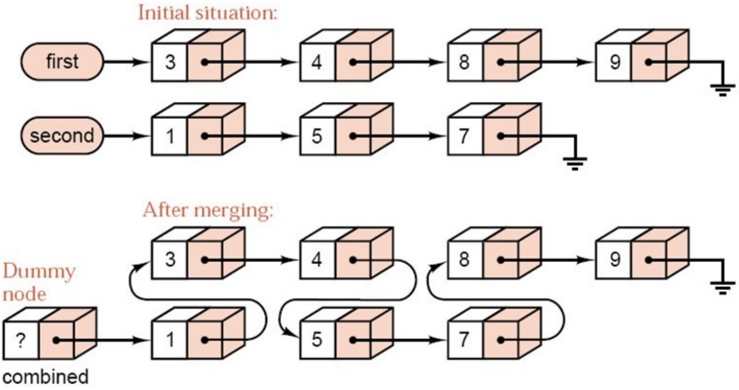
```
Algorithm recursiveMergeSort(ref sublist <pointer>)
Sorts the linked list using recursive merge sort.

if sublist is not NULL AND sublist->link is not NULL then
    Divide(sublist, second_list)
    recursiveMergeSort(sublist)
    recursiveMergeSort(second_list)
    Merge(sublist, second_list)
end
End recursiveMergeSort
```

Merge Sort

Algorithm Divide(val sublist <pointer>, ref second_list <pointer>)
Divides the list into two halves.

```
midpoint = sublist
position = sublist->link
while position is not NULL do
    position = position->link
    if position is not NULL then
        midpoint = midpoint->link
        position = position->link
    end
end
second_list = midpoint->link
midpoint->link = NULL
End Divide
```

Merge two sublists



Merge two sublists

```
Algorithm Merge(ref first <pointer>, ref second <pointer>)
Merges two sorted lists into a sorted list.

lastSorted = address of combined
while first is not NULL AND second is not NULL do
    if first->data.key <= second->data.key then
        lastSorted->link = first
        lastSorted = first
        first = first->link
    else
        lastSorted->link = second
        lastSorted = second
        second = second->link
    end
end

// ...
```

Merge two sublists

```
// ...
if first is NULL then
    lastSorted->link = second
    second = NULL
else
    lastSorted->link = first
end
first = combined.link
End Merge
```