Computer Architecture and Programming Languages
Jacobs University Bremen
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Course: CO20-320241
September 16th, 2019

Homework 2

Problem 2.1
Solution:

First, we add 2 digits. If greater than 8/16, we convert the decimal number into it's octal/hexadecimal value. The remainder is written below while the quotient is carried forward. For example, in the first part, 777 + 1 is 778. We replace 8 by 0 and take 1 as a carry. This is repeated for all digits until we get 1000 as the answer.

(a) $777_8 + 1_8 = 1000_8$

(b) $888_{16} + 1_{16} = 889_{16}$

(c) $32007_8 + 1_8 = 32010_8$

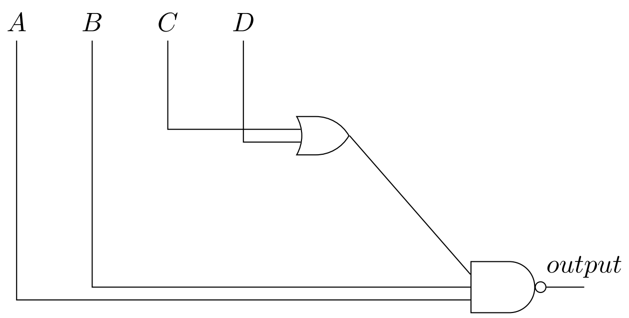
(d) $32108_{16} + 1_{16} = 32109_{16}$

(e) $8BFF_{16} + 1_{16} = 8C00_{16}$

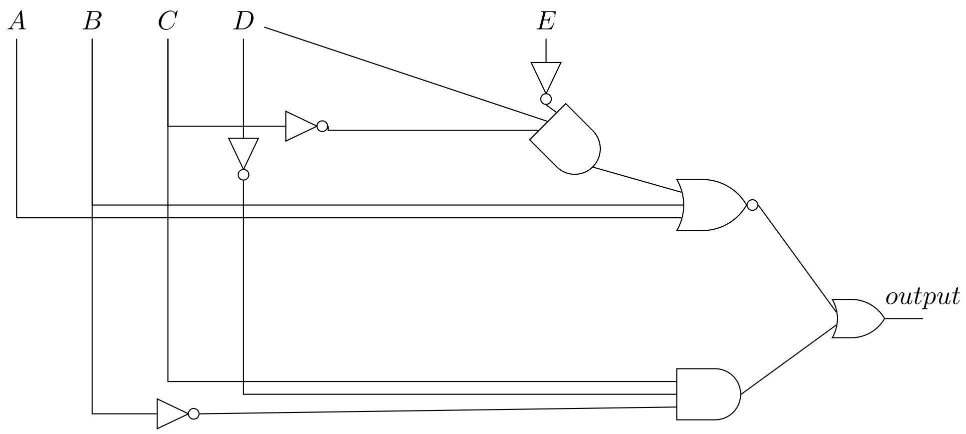
(f) $1219_{16} + 1_{16} = 121A_{16}$

Problem 2.2
Solution:

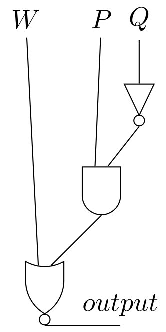
(a) $\overline{A \cdot B \cdot (C + D)}$



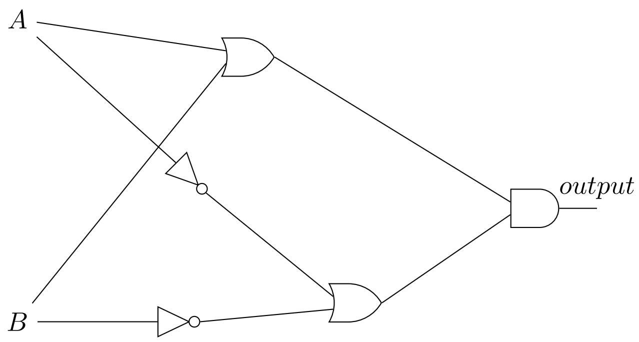
(b) $\overline{A + B + \overline{\overline{C} \cdot D \cdot \overline{E}}} + \overline{B} \cdot C \cdot \overline{D}$



(c) $\overline{W + P \cdot \overline{Q}}$



(d) (A+B)$\cdot(\overline{A}+\overline{B})$



Problem 2.3

Solution:

The given diagram represents a logic circuit with the following algebraic expression:

$(\overline{\overline{(M \cdot N \cdot Q)} \cdot \overline{(M \cdot \overline{N} \cdot Q)} \cdot \overline{(\overline{M} \cdot N \cdot Q)}})$

Let $(\overline{\overline{(M \cdot N \cdot Q)} \cdot \overline{(M \cdot \overline{N} \cdot Q)} \cdot \overline{(\overline{M} \cdot N \cdot Q)}})$ = X where X is the final answer of the expression.

M	N	Q	X
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	0
0	0	1	0
1	0	1	1
0	1	1	1
1	1	1	1

Sum of Products:

This is also known as Disjunctive Normal Form (DNF). To find this expression, we see the rows in our truth table where the final answer is 1 . Then, we derive corresponding expressions called conjuctions (using multiplication) for those rows and sum them all up. In this case, the sum of expressions is the following:

$(M \cdot \overline{N} \cdot Q) + (\overline{M} \cdot N \cdot Q) + (M \cdot N \cdot Q)$ (1)

Applying Distributive Law to (1):

$Q(M \cdot \overline{N} + M \cdot N) + (\overline{M} \cdot N \cdot Q)$

$Q(M(\overline{N} + N)) + (\overline{M} \cdot N \cdot Q)$ (2)

Applying Complement Rule to (2):

$Q \cdot M + (\overline{M} \cdot N \cdot Q)$

$Q(M + \overline{M} \cdot N)$ (3)

Applying Double Negation on N and M to (3):

$Q(\overline{\overline{M}} + \overline{M} \cdot \overline{\overline{N}})$ (4)

Applying De Morgan's First Law in reverse to (4):

$Q(\overline{\overline{M}} + \overline{M + \overline{N}})$ (5)

Applying De Morgan's Second Law in reverse to (5):

Q$(\overline{\overline{M} \cdot (M + \overline{N})})$ (6)

Applying Distributive Law to (6):

Q$(\overline{\overline{M} \cdot M + \overline{M} \cdot \overline{N}})$ (7)

Applying Complement Rule to (7):

Q$(\overline{\overline{M} \cdot \overline{N}})$ (8)

Applying De Morgan's 2nd Rule to (8):

Q$(\overline{\overline{M}} + \overline{\overline{N}})$

$Q(M + N)$ (9)

Applying Distributive Law to (9):

Q$\cdot M + Q \cdot N$

Problem 2.4

Solution:

(a)

X	Y	$\overline{X}$	$\overline{X} \cdot Y$	$X + \overline{X} \cdot Y$	$X + Y$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

(b)

X	Y	$\overline{X}$	$X \cdot Y$	$\overline{X} + X \cdot Y$	$\overline{X} + Y$
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	1	1

Problem 2.5

Solution:

(a) $A + 1 = 1$

(b) $A \cdot A = A$

(c) $B \cdot \overline{B} = 0$

(d) $C + C = C$

(e) $x \cdot 0 = 0$

(f) $D \cdot 1 = D$

(g) $D + 0 = D$

(h) $C + \overline{C} = 1$

(i) $G + G \cdot F = G$

(j) $y + \overline{w} \cdot y = y$

Problem 2.6

Solution:
There are 2 rules that fall under de Morgan's Theorems.

Rule 1: $(\overline{A + B}) = \overline{A} \cdot \overline{B}$

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Rule 2: $(\overline{A \cdot B}) = \overline{A} + \overline{B}$

A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Problem 2.7

Solution:
To find the sum of expressions (DNF), we see the rows where the final output is 1. For these rows, we write down the product of the input values (conjuctions) and obtain an expression . Then, we add all these expressions. For the given truth table, we get the following expression:

$$(\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D)+(\overline{A}\cdot\overline{B}\cdot C\cdot D)+(\overline{A}\cdot B\cdot\overline{C}\cdot D)+(\overline{A}\cdot B\cdot C\cdot D)+(A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D})+(A\cdot B\cdot\overline{C}\cdot D)+(A\cdot B\cdot C\cdot D)$$

We use the distributive law (R1) and take out commons:

$$= \overline{A} \cdot \overline{B} \cdot D \cdot (\overline{C} + C) + \overline{A} \cdot B \cdot D \cdot (\overline{C} + C) + A \cdot B \cdot D \cdot (\overline{C} + C) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} \qquad (1)$$

Now we use the Identity Law according to which C$\cdot\overline{C} = 1$.

$$= \overline{A} \cdot \overline{B} \cdot D + B \cdot D \cdot (\overline{A} + A) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$$

We apply Identity Rule again.

=B$\cdot D + \overline{A} \cdot \overline{B} \cdot D + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

=D$(\overline{A} \cdot \overline{B} + B) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

Applying De Morgan's First Rule (in reverse):

$= D(\overline{\overline{A+B}} + B) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

Applying double negation to B as value remains same:

$= D(\overline{\overline{A+B}} + \overline{\overline{B}}) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

Applying De Morgan's 2nd rule (in reverse):

$= D(\overline{(A+B) \cdot \overline{B}}) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

Applying Distributive Law:

$= D(\overline{A \cdot \overline{B} + B \cdot \overline{B}}) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

Applying Complement Law:

$= D(\overline{A \cdot \overline{B}}) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

Applying De Morgan's Second Rule:

$= D(\overline{A} + B) + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

$= B \cdot D + \overline{A} \cdot D + A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

Problem 2.8

Solution:

To create a K-map, we first look at the original sum of products expression. We check the number of variables. In this case, we had 4 different variables and $2^4 = 16$. This gives us the number of boxes we need inside our table, apart from our label boxes.Then, we evaluate each product within our initial expression, and fill it in our K-map. We write a 1 in our table where the expression is found and 0 when there's nothing in a box.
Instead of the initial expression, we can also evaluate the simplified expression if we're sure it is correct.

	$\overline{C} \cdot \overline{D}$	$\overline{C} \cdot D$	$C \cdot D$	$C \cdot \overline{D}$
$\overline{A} \cdot \overline{B}$	0	1	1	0
$\overline{A} \cdot B$	0	1	1	0
$A \cdot B$	0	1	1	0
$A \cdot \overline{B}$	1	0	0	0