Computer Vision

Jacobs University Bremen

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Homework 5

This notebook includes both coding and written questions. Please hand in this notebook file with all the outputs and your answers to the written questions.

This assignment covers K-Means and HAC methods for clustering and image segmentation.

In [1]:
```
# Setup
from __future__ import print_function
from time import time
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import rc
from skimage import io


%matplotlib inline
plt.rcParams['figure.figsize'] = (15.0, 12.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading extenrnal modules
%load_ext autoreload
%autoreload 2
```

Introduction

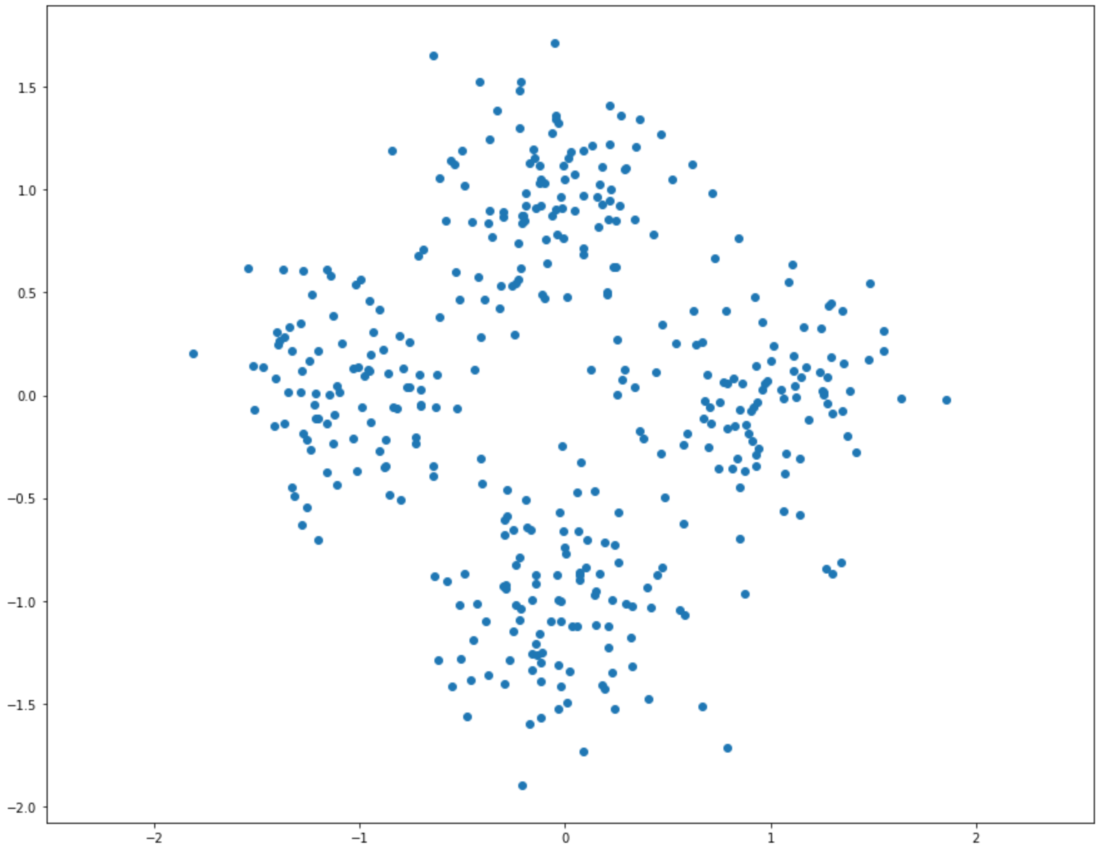
In this assignment, you will use clustering algorithms to segment images. You will then use these segmentations to identify foreground and background objects.

Your assignment will involve the following subtasks:

- **Clustering algorithms**: Implement K-Means clustering and Hierarchical Agglomerative Clustering.
- **Pixel-level features**: Implement a feature vector that combines color and position information and implement feature normalization.
- **Quantitative Evaluation**: Evaluate segmentation algorithms with a variety of parameter settings by comparing your computed segmentations against a dataset of ground-truth segmentations.

1 Clustering Algorithms (40 points)

In [2]:
```
# Generate random data points for clustering

# Set seed for consistency
np.random.seed(0)

# Cluster 1
mean1 = [-1, 0]
cov1 = [[0.1, 0], [0, 0.1]]
X1 = np.random.multivariate_normal(mean1, cov1, 100)

# Cluster 2
mean2 = [0, 1]
cov2 = [[0.1, 0], [0, 0.1]]
X2 = np.random.multivariate_normal(mean2, cov2, 100)

# Cluster 3
mean3 = [1, 0]
cov3 = [[0.1, 0], [0, 0.1]]
X3 = np.random.multivariate_normal(mean3, cov3, 100)

# Cluster 4
mean4 = [0, -1]
cov4 = [[0.1, 0], [0, 0.1]]
X4 = np.random.multivariate_normal(mean4, cov4, 100)

# Merge two sets of data points
X = np.concatenate((X1, X2, X3, X4))

# Plot data points
plt.scatter(X[:, 0], X[:, 1])
plt.axis('equal')
plt.show()
```

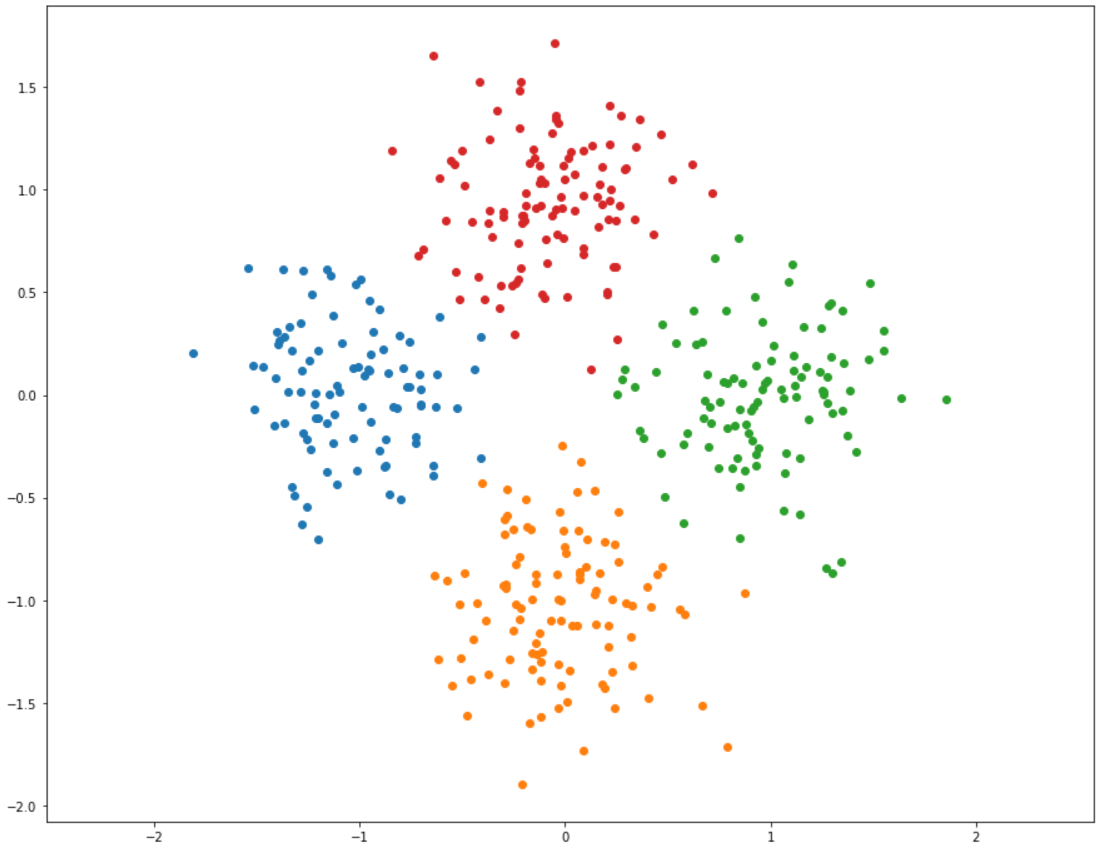


1.1 K-Means Clustering (20 points)

As discussed in class, K-Means is one of the most popular clustering algorithms. We have provided skeleton code for K-Means clustering in the file `segmentation.py`. Your first task is to finish implementing **`kmeans`** in `segmentation.py`. This version uses nested for loops to assign points to the closest centroid and compute a new mean for each cluster.

In [3]:
```
from segmentation import kmeans

np.random.seed(0)
start = time()
assignments = kmeans(X, 4)
end = time()

kmeans_runtime = end - start

print("kmeans running time: %f seconds." % kmeans_runtime)

for i in range(4):
    cluster_i = X[assignments==i]
    plt.scatter(cluster_i[:, 0], cluster_i[:, 1])

plt.axis('equal')
plt.show()
```

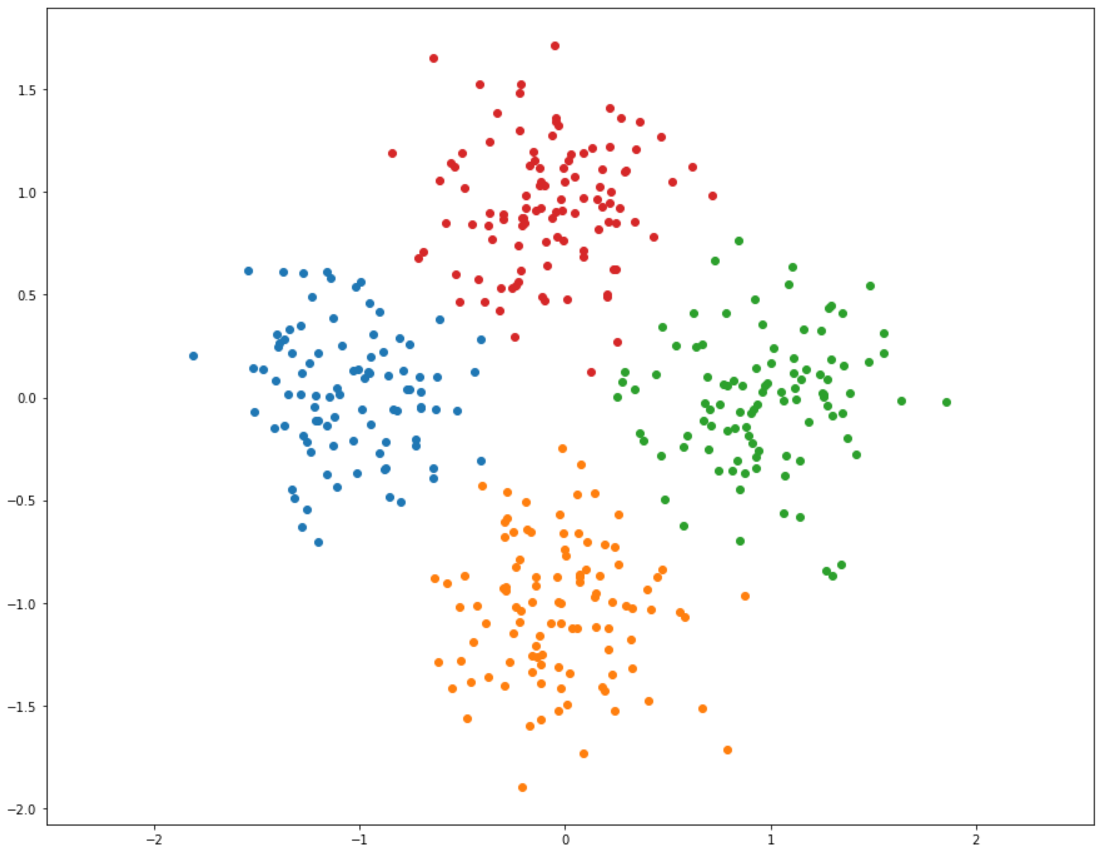
```
kmeans running time: 0.392223 seconds.
```



We can use numpy functions and broadcasting to make K-Means faster. Implement **`kmeans_fast`** in `segmentation.py`. This should run at least 10 times faster than the previous implementation.

In [4]:
```
from segmentation import kmeans_fast

np.random.seed(0)
start = time()
assignments = kmeans_fast(X, 4)
end = time()

kmeans_fast_runtime = end - start
print("kmeans running time: %f seconds." % kmeans_fast_runtime)
print("%f times faster!" % (kmeans_runtime / kmeans_fast_runtime))

for i in range(4):
    cluster_i = X[assignments==i]
    plt.scatter(cluster_i[:, 0], cluster_i[:, 1])

plt.axis('equal')
plt.show()
```

```
kmeans running time: 0.017731 seconds.
22.121087 times faster!
```



1.2 K-Means Convergence (10 points)

Implementations of the K-Means algorithm will often have the parameter `num_iters` to define the maximum number of iterations the algorithm should run for. Consider that we opt to not include this upper bound on the number of iterations, and that we define the termination criterion of the algorithm to be when the cost L stops changing.

Recall that L is defined as the sum of squared distance between all points x and their nearest cluster center c:

$$L = \sum_{i \in clusters} \sum_{x \in cluster_i} (x - c_i)^2$$

Show that for any set of points D and any number of clusters k, the K-Means algorithm will terminate in a finite number of iterations.

Your answer here:

There are only a finiite number of possible cluster assignments. For example, if we have D data points and we need to assign them in K clusters, the max possible number of assignments is K^D. For each iteration of the algorithm, we produce a new clustering based only on the old clustering. There are two things to consider:

a) If the old clustering is the same as the new, then the next clustering will again be the same. b) If the new clustering is different from the old then the newer one has a lower cost.

The domain for k-means is a finite set. So, the iteration will eventually enter a cycle. This cycle can not have a length greater than 1 because otherwise by (b), we will end up with a clustering that has a lower cost than itself which is impossible. This is impossible because any reasonable K-means algorithm will strictly reduce the error on each step, so you could not possibly come back to the same assignment.

Thus, the cycle must have length exactly 1. Hence k-means converges in a finite number of iterations.

References:

https://stats.stackexchange.com/questions/181319/why-k-means-algorithm-will-terminate-in-a-finite-number-of-iterations (https://stats.stackexchange.com/questions/181319/why-k-means-algorithm-will-terminate-in-a-finite-number-of-iterations)

https://stats.stackexchange.com/questions/188087/proof-of-convergence-of-k-means (https://stats.stackexchange.com/questions/188087/proof-of-convergence-of-k-means)

1.2 Hierarchical Agglomerative Clustering (10 points)

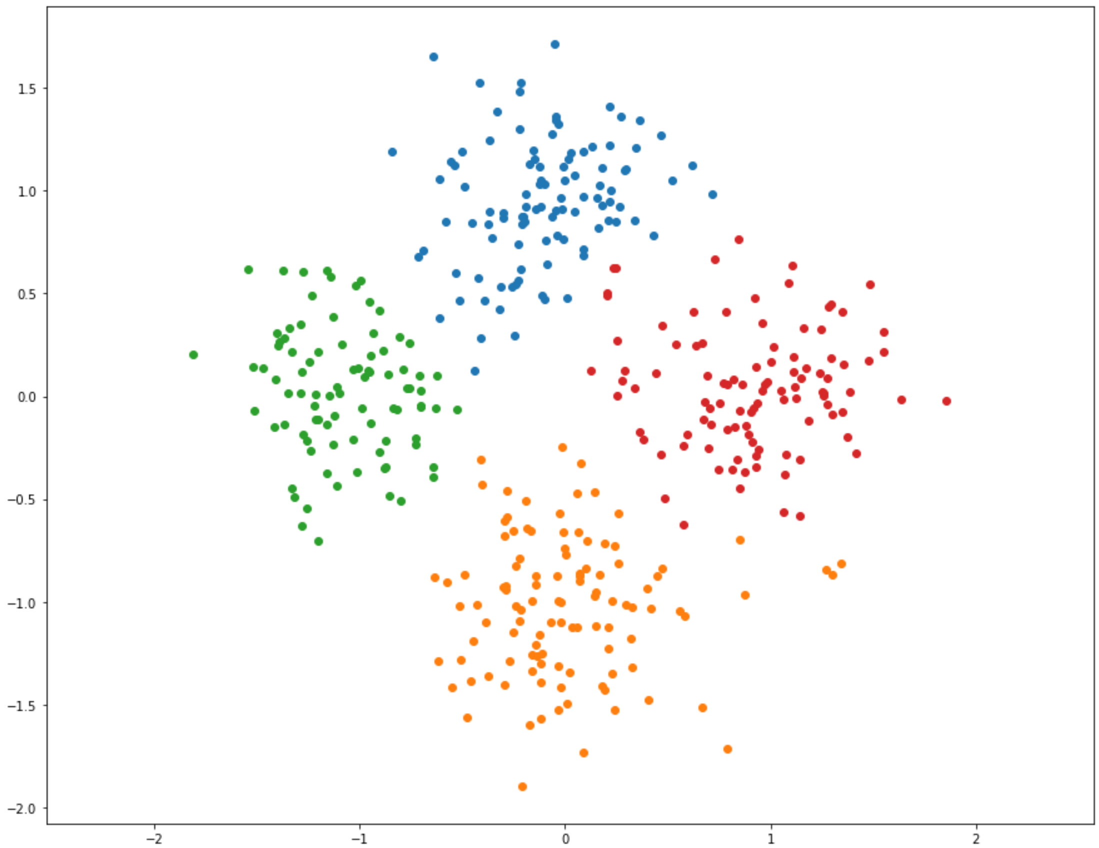
Another simple clustering algorithm is Hieararchical Agglomerative Clustering, which is somtimes abbreviated as HAC. In this algorithm, each point is initially assigned to its own cluster. Then cluster pairs are merged until we are left with the desired number of predetermined clusters (see Algorithm 1).

Implement **`hiererachical_clustering`** in `segmentation.py`.

algo1.png

In [5]:
```
from segmentation import hierarchical_clustering

start = time()
assignments = hierarchical_clustering(X, 4)
end = time()

print("hierarchical_clustering running time: %f seconds." % (end - start))

for i in range(4):
    cluster_i = X[assignments==i]
    plt.scatter(cluster_i[:, 0], cluster_i[:, 1])

plt.axis('equal')
plt.show()
```

```
hierarchical_clustering running time: 0.197994 seconds.
```



2 Pixel-Level Features (30 points)

Before we can use a clustering algorithm to segment an image, we must compute some *feature vector* for each pixel. The feature vector for each pixel should encode the qualities that we care about in a good segmentation. More concretely, for a pair of pixels p_i and p_j with corresponding feature vectors f_i and f_j, the distance between f_i and f_j should be small if we believe that p_i and p_j should be placed in the same segment and large otherwise.

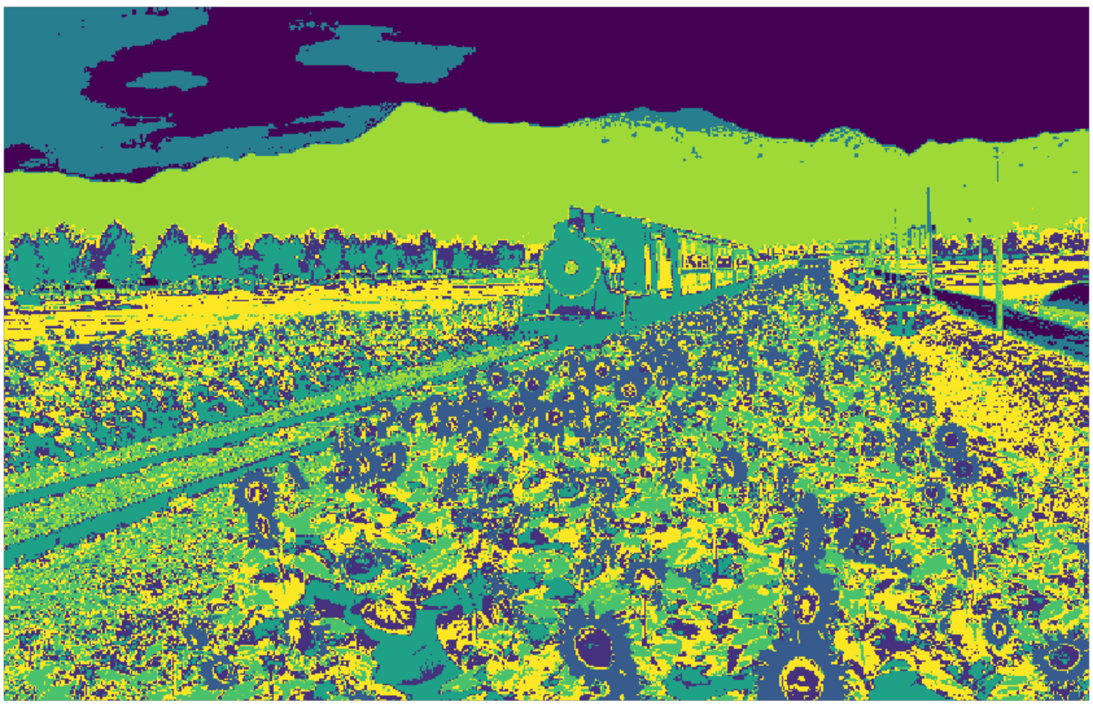
In [6]:
```
# Load and display image
img = io.imread('train.jpg')
H, W, C = img.shape

plt.imshow(img)
plt.axis('off')
plt.show()
```

2.1 Color Features (15 points)

One of the simplest possible feature vectors for a pixel is simply the vector of colors for that pixel. Implement **`color_features`** in `segmentation.py`. Output should look like the following:

color_features.png

In [7]:
```
from segmentation import color_features
np.random.seed(0)

features = color_features(img)

# Sanity checks
assert features.shape == (H * W, C),\
    "Incorrect shape! Check your implementation."

assert features.dtype == np.float,\
    "dtype of color_features should be float."

assignments = kmeans_fast(features, 8)
segments = assignments.reshape((H, W))

# Display segmentation
plt.imshow(segments, cmap='viridis')
plt.axis('off')
plt.show()
```



In the cell below, we visualize each segment as the mean color of pixels in the segment.

In [8]:
```
from utils import visualize_mean_color_image
visualize_mean_color_image(img, segments)
```

2.2 Color and Position Features (15 points)

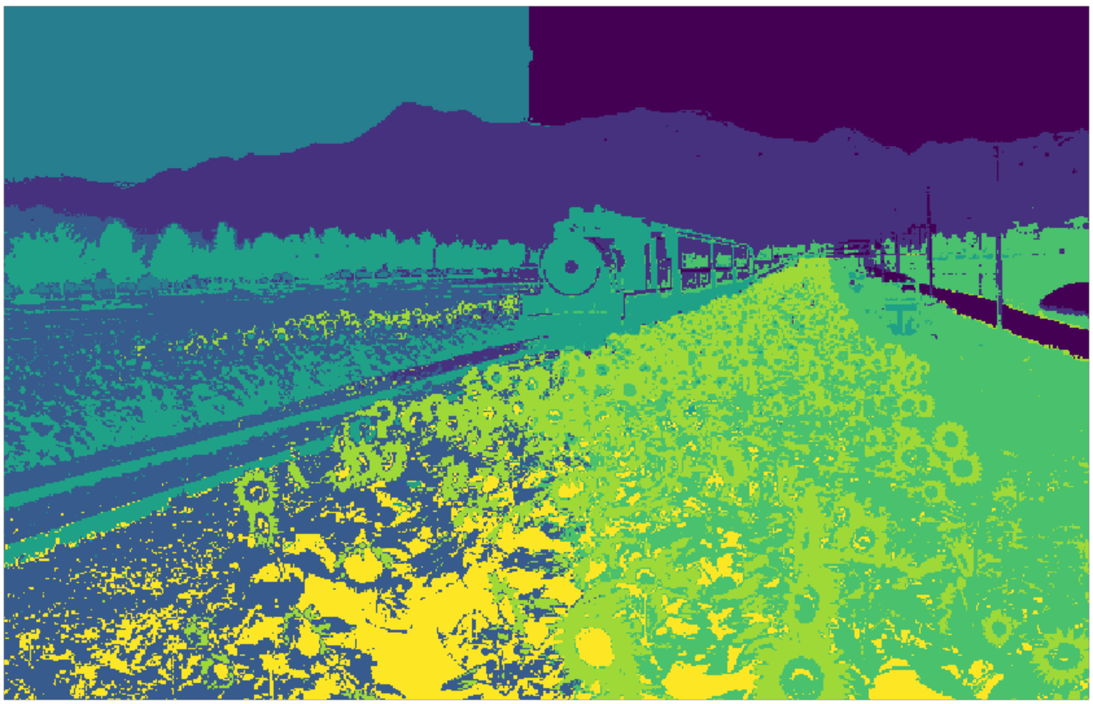
Another simple feature vector for a pixel is to concatenate its color and position within the image. In other words, for a pixel of color (r, g, b) located at position (x, y) in the image, its feature vector would be (r, g, b, x, y). However, the color and position features may have drastically different ranges; for example each color channel of an image may be in the range $[0, 1)$, while the position of each pixel may have a much wider range. Uneven scaling between different features in the feature vector may cause clustering algorithms to behave poorly.

One way to correct for uneven scaling between different features is to apply some sort of normalization to the feature vector. One of the simplest types of normalization is to force each feature to have zero mean and unit variance.

Implement **`color_position_features`** in `segmentation.py`.

Output segmentation should look like the following:
color_position_features.png

In [9]:
```
from segmentation import color_position_features
np.random.seed(0)

features = color_position_features(img)

# Sanity checks
assert features.shape == (H * W, C + 2),\
    "Incorrect shape! Check your implementation."

assert features.dtype == np.float,\
    "dtype of color_features should be float."

assignments = kmeans_fast(features, 8)

segments = assignments.reshape((H, W))

# Display segmentation
plt.imshow(segments, cmap='viridis')
plt.axis('off')
plt.show()
```



In [10]:
```
visualize_mean_color_image(img, segments)
```

Extra Credit: Implement Your Own Feature

For this programming assignment we have asked you to implement a very simple feature transform for each pixel. While it is not required, you should feel free to experiment with other feature transforms. Could your final segmentations be improved by adding gradients, edges, SIFT descriptors, or other information to your feature vectors? Could a different type of normalization give better results?

Implement your feature extractor **`my_features`** in `segmentation.py`

Depending on the creativity of your approach and the quality of your writeup, implementing extra feature vectors can be worth extra credit (up to 1 point).

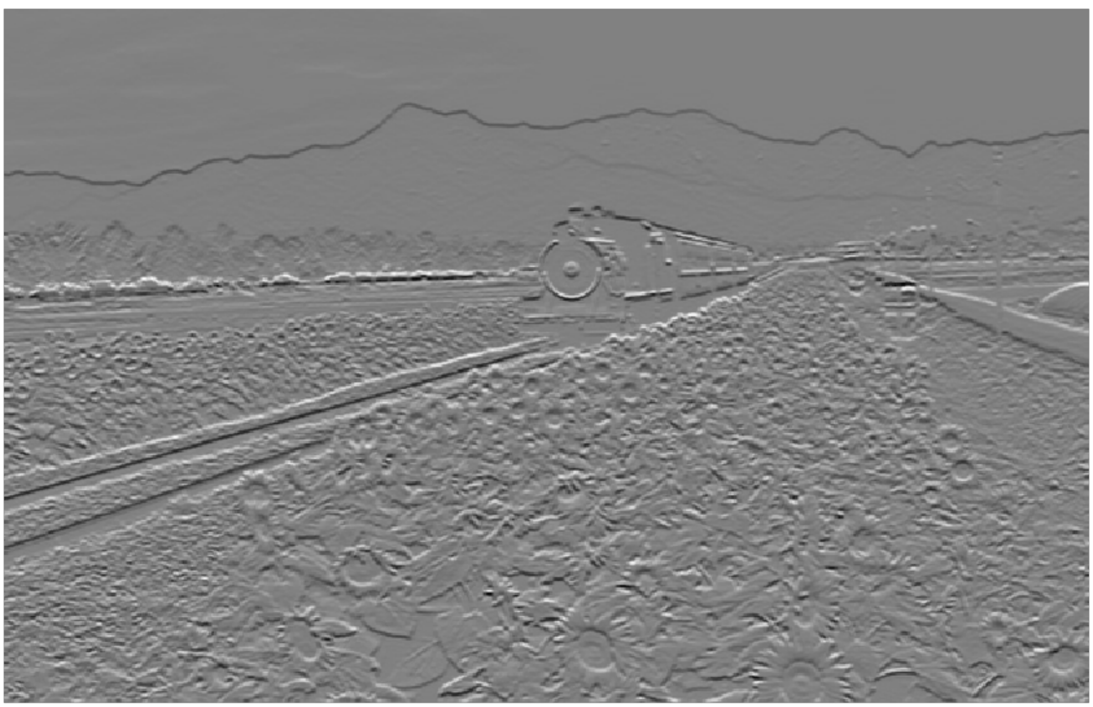
Describe your approach: I have extracted horizontal edge features here using the x-direction Prewitt kernel. An edge is basically where there is a sharp change in color. This can be detected by observing changes in pixel values in image matrices.

If we have an image matrix, we consider one pixel value at a time. To identify if a pixel is an edge or not, we will simply subtract the values on either side of the pixel. For example, if our selected pixel has a value of 85, and the values on its sides are 89 and 78, we subtract values and see the differences. Since this differences aren't very large, we can say that there is no edge around this pixel.

To extract edges, other the Sobel kernel could also be used.

Reference: [https://www.analyticsvidhya.com/blog/2019/08/3-techniques-extract-features-from-image-data-machine-learning-python/ (https://www.analyticsvidhya.com/blog/2019/08/3-techniques-extract-features-from-image-data-machine-learning-python/)](https://www.analyticsvidhya.com/blog/2019/08/3-techniques-extract-features-from-image-data-machine-learning-python/)

In [11]:
```
# PENDING
from segmentation import my_features
from segmentation import kmeans_fast

# Feel free to experiment with different images
# and varying number of segments
img = io.imread('train.jpg')
num_segments = 8

H, W, C = img.shape

# Extract pixel-level features
features = my_features(img)

# Run clustering algorithm

assignments = kmeans_fast(features, num_segments)

#segments = assignments.reshape((H, W))

# Display segmentation
plt.imshow(features, cmap='gray')
plt.axis('off')
plt.show()
```



3 Quantitative Evaluation (30 points)

Looking at images is a good way to get an idea for how well an algorithm is working, but the best way to evaluate an algorithm is to have some quantitative measure of its performance.

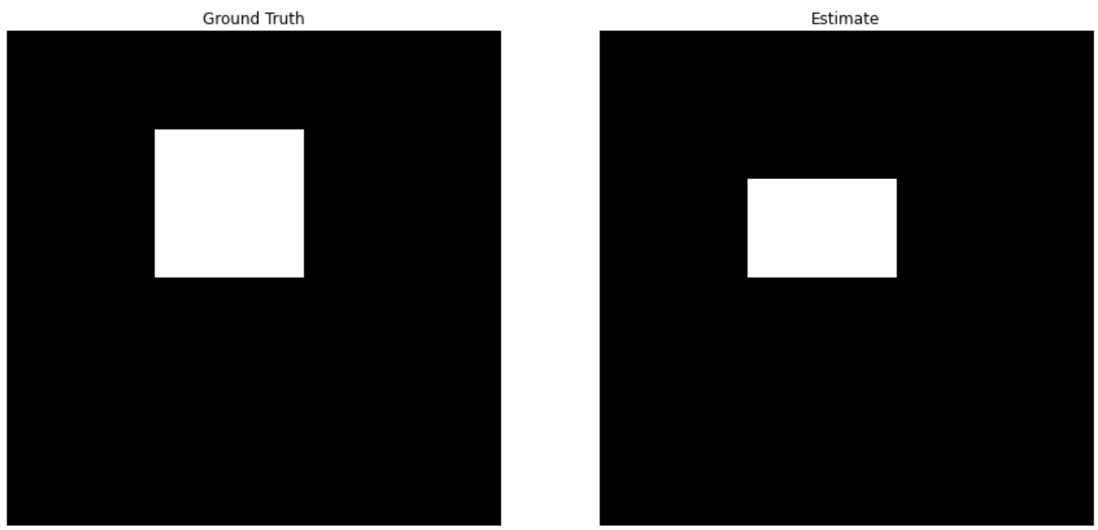
For this project we have supplied a small dataset of cat images and ground truth segmentations of these images into foreground (cats) and background (everything else). We will quantitatively evaluate different segmentation methods (features and clustering methods) on this dataset.

We can cast the segmentation task into a binary classification problem, where we need to classify each pixel in an image into either foreground (positive) or background (negative). Given the ground-truth labels, the accuracy of a segmentation is $(TP + TN)/(P + N)$.

Implement **`compute_accuracy`** in `segmentation.py`.

In [12]:
```
from segmentation import compute_accuracy

mask_gt = np.zeros((100, 100))
mask = np.zeros((100, 100))

# Test compute_accracy function
mask_gt[20:50, 30:60] = 1
mask[30:50, 30:60] = 1

accuracy = compute_accuracy(mask_gt, mask)

print('Accuracy: %0.2f' % (accuracy))
if accuracy != 0.97:
    print('Check your implementation!')

plt.subplot(121)
plt.imshow(mask_gt)
plt.title('Ground Truth')
plt.axis('off')

plt.subplot(122)
plt.imshow(mask)
plt.title('Estimate')
plt.axis('off')

plt.show()
```

```
Accuracy: 0.97
```



You can use the script below to evaluate a segmentation method's ability to separate foreground from background on the entire provided dataset. Use this script as a starting point to evaluate a variety of segmentation parameters.

In [13]:
```
from utils import load_dataset, compute_segmentation
from segmentation import evaluate_segmentation

# Load a small segmentation dataset
imgs, gt_masks = load_dataset('./data')

# Set the parameters for segmentation.
num_segments = 3
clustering_fn = kmeans_fast
feature_fn = color_features
scale = 0.5

mean_accuracy = 0.0

segmentations = []

for i, (img, gt_mask) in enumerate(zip(imgs, gt_masks)):
    # Compute a segmentation for this image
    segments = compute_segmentation(img, num_segments,
                                    clustering_fn=clustering_fn,
                                    feature_fn=feature_fn,
                                    scale=scale)

    segmentations.append(segments)

    # Evaluate segmentation
    accuracy = evaluate_segmentation(gt_mask, segments)

    print('Accuracy for image %d: %0.4f' %(i, accuracy))
    mean_accuracy += accuracy

mean_accuracy = mean_accuracy / len(imgs)
print('Mean accuracy: %0.4f' % mean_accuracy)
```

```
Accuracy for image 0: 0.8063
Accuracy for image 1: 0.9583
Accuracy for image 2: 0.9860
Accuracy for image 3: 0.9059
Accuracy for image 4: 0.9575
Accuracy for image 5: 0.6814
Accuracy for image 6: 0.6726
Accuracy for image 7: 0.6726
Accuracy for image 8: 0.8411
Accuracy for image 9: 0.9004
Accuracy for image 10: 0.8552
Accuracy for image 11: 0.8076
Accuracy for image 12: 0.7349
Accuracy for image 13: 0.6511
Accuracy for image 14: 0.7487
Accuracy for image 15: 0.4925
Mean accuracy: 0.7920
```

In [14]:
```
# Visualize segmentation results

N = len(imgs)
plt.figure(figsize=(15,60))
for i in range(N):

    plt.subplot(N, 3, (i * 3) + 1)
    plt.imshow(imgs[i])
    plt.axis('off')

    plt.subplot(N, 3, (i * 3) + 2)
    plt.imshow(gt_masks[i])
    plt.axis('off')

    plt.subplot(N, 3, (i * 3) + 3)
    plt.imshow(segmentations[i], cmap='viridis')
    plt.axis('off')

plt.show()
```

In [15]:
```
from utils import load_dataset, compute_segmentation
from segmentation import evaluate_segmentation
from segmentation import color_position_features

# Load a small segmentation dataset
imgs, gt_masks = load_dataset('./data')

# Set the parameters for segmentation.
num_segments = 2
clustering_fn = kmeans_fast
feature_fn = color_position_features
scale = 0.5

mean_accuracy = 0.0

segmentations = []

for i, (img, gt_mask) in enumerate(zip(imgs, gt_masks)):
    # Compute a segmentation for this image
    segments = compute_segmentation(img, num_segments,
                                    clustering_fn=clustering_fn,
                                    feature_fn=feature_fn,
                                    scale=scale)

    segmentations.append(segments)

    # Evaluate segmentation
    accuracy = evaluate_segmentation(gt_mask, segments)

    print('Accuracy for image %d: %0.4f' %(i, accuracy))
    mean_accuracy += accuracy

mean_accuracy = mean_accuracy / len(imgs)
print('Mean accuracy: %0.4f' % mean_accuracy)
```

```
Accuracy for image 0: 0.8212
Accuracy for image 1: 0.9045
Accuracy for image 2: 0.9832
Accuracy for image 3: 0.6043
Accuracy for image 4: 0.6986
Accuracy for image 5: 0.8733
Accuracy for image 6: 0.5759
Accuracy for image 7: 0.8672
Accuracy for image 8: 0.8996
Accuracy for image 9: 0.9382
Accuracy for image 10: 0.7032
Accuracy for image 11: 0.6050
Accuracy for image 12: 0.5623
Accuracy for image 13: 0.6537
Accuracy for image 14: 0.7745
Accuracy for image 15: 0.5076
Mean accuracy: 0.7483
```

Include a detailed evaluation of the effect of varying segmentation parameters (feature transform, clustering method, number of clusters, resize) on the mean accuracy of foreground-background segmentations on the provided dataset. You should test a minimum of 10 combinations of parameters. To present your results, add rows to the table below (you may delete the first row).

Note: I changed some values in the existing cells instead of redoing everything in each step.

Feature Transform	**Clustering Method**	**Number of segments**	**Scale**	**Mean Accuracy**
Color	K-Means (Fast)	2	0.5	0.7632
Color	K-Means (Fast)	3	0.5	0.7920
Color	K-Means (Fast)	5	0.5	0.7667
Color-Position	K-Means (Fast)	2	0.5	0.7224
Color-Position	K-Means (Fast)	3	0.5	0.7836
Color-Position	K-Means (Fast)	5	0.5	0.8004
...	...	...	...	...

Observe your results carefully and try to answer the following question:

1. Based on your quantitative experiments, how do each of the segmentation parameters affect the quality of the final foreground-background segmentation?
2. Are some images simply more difficult to segment correctly than others? If so, what are the qualities of these images that cause the segmentation algorithms to perform poorly?
3. Also feel free to point out or discuss any other interesting observations that you made.

Write your analysis in the cell below.

Your answer here:

Generally, an increase in the number of segments leads to a higher mean accuracy. However, this might not always be the case if we put a very large number, depending on our dataset.

We also notice that images with multiple colors (for instance cats with colored patches or stripes) are harder to segment. For instance, the accuracy for images 9 and 15 are lower.

The difference in accuracies from color-position & color features isn't very different so we conclude that including the position does not necessarily help a lot.