

Treasury Debt and the Pricing of Short-Term Assets

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Abstract

Since the 2008 financial crisis, the supply of short-term debt from the Treasury has been increasingly associated with changes in the yields on short-term money market assets. This puzzling pattern contrasts with the pre-crisis experience and raises questions about the Fed's ability to fulfill its mandate. In this paper, I document and rationalize these developments in an intermediary asset pricing model with heterogeneous banks subject to a liquidity management problem and regulation. The combination of large amounts of excess reserves and a more stringent capital regulation prevents traditional banks from intermediating liquidity to shadow banks. As a consequence, the pricing of reserves disconnects from the pricing of other short-term assets. The liquidity premium of these assets is then free to react to variations in the supply of Treasury bills. The quantitative model accurately predicts post-crisis variations in Treasury bill and repo yields, as well as in reverse repo volumes from the Fed.

Keywords: Repo, Treasury Bills, Money Markets, Shadow Banks.

JEL Classifications: E43, E44, E52, G12

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1 Introduction

In most advanced economies, short-term rates are tightly controlled by the central bank through variations in the supply of reserves available to banks and the interest paid on reserves. Yet since the 2008 financial crisis, fluctuations in short-term debt from the US Treasury have increasingly been associated with movements in the yields on short-term money market assets such as federal funds, repurchase agreements, and Treasury bills. For instance, in the first semester of 2018, most short-term rates puzzlingly hiked beyond what was anticipated by the Fed and prompted doubts about its ability to control short-term rates.¹² This pattern was not observed before the crisis and is not explained by the literature.

In this paper, I study how markets for short-term liquid assets have adjusted since the crisis to accommodate a new monetary policy regime and a more stringent regulatory environment. I first document two distinctive facts about post-crisis money markets: (i) the liquidity premia on T-bills and repo transactions are now higher than on reserves and (ii) the supply of short-term public assets that are available to shadow banks—which includes T-bills from the Treasury but excludes reserves from the Fed—has been strongly associated with liquidity premia since the crisis, as illustrated in Figure 1.

I then rationalize these facts in an intermediary asset pricing model with heterogeneous banks that are subject to a liquidity management problem and regulation. While traditional banks have access to both reserves and T-bills, shadow banks cannot hold reserves. Hence, when the supply of T-bills is scarce, shadow banks rely on

¹During the press conference on June 13, 2018, Chairman Powell answered a question related to the recent hike of money market rates as: “[...] [W]e’re looking carefully at that and, you know, the truth is we don’t know with any precision. Really, no one does. [...] I think there’s a lot of probability on the idea of just high bill supply leads to higher repo costs, higher money market rates generally, and the arbitrage pulls up federal funds rate towards [interest on reserves]. We don’t know that that’s the only effect.”

²Examples of this negative perception on the ability of the Fed to control short-term interest rates can be found in the financial press, as illustrated by the article by Alex Harris from May 30, 2018 titled “As Fed Loses Control of Overnight Rates, Blame Shifts to T-Bills” (Bloomberg, <https://www.bloomberg.com/opinion/articles/2018-01-16/the-fed-is-losing-control-of-the-financial-markets>, accessed on the 01/08/2019.)

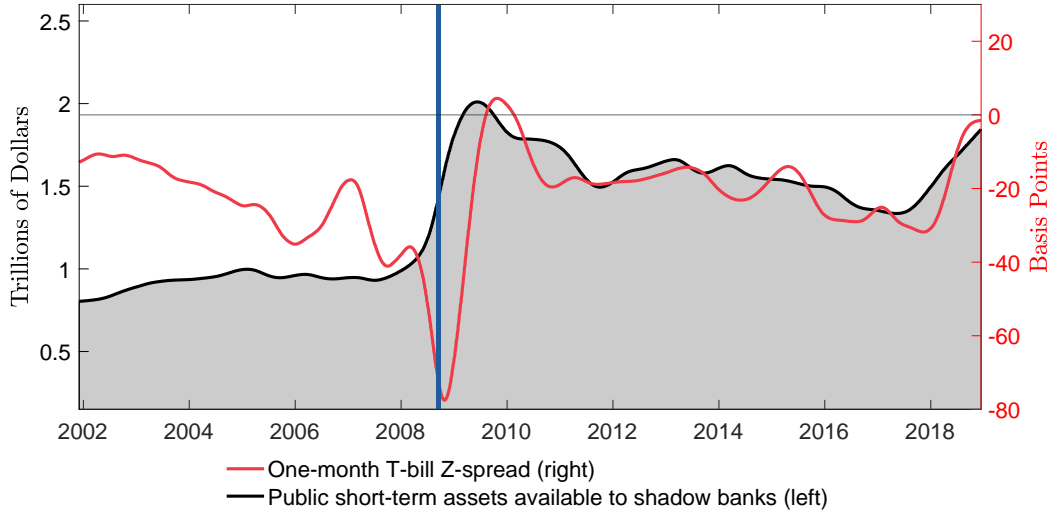


Figure 1: Public Short-Term Assets Available to Shadow Banks and Liquidity Premia. This figure plots the outstanding amount of short-term public assets available to shadow banks (left axis) along with the 1-month Z-spread: a common measure of the liquidity premium on T-bills (right axis). The outstanding amount of short-term public assets available to shadow banks is computed as the sum of outstanding T-bills and reverse repos from the Fed and adjusted by removing inflows of T-bills to money funds attributed to a change in regulation in 2016. Details about this adjustment are provided in Section 6.1. Details about the Z-spread are provided in Section 2.2. All series are smoothed using a standard HP filter with parameter $\lambda = 5 \times 10^5$ on daily data.

the intermediation of liquidity from traditional banks. In the model, the *combination* of large amounts of excess reserves with a stringent regulatory capital requirement creates an inaction region in which traditional banks cease to intermediate liquidity to shadow banks. In this region, the pricing of reserves is disconnected from the pricing of all other liquid instruments that shadow banks can hold. While reserves are in excess for traditional banks, T-bills are still scarce and valuable for shadow banks. As a consequence, changes in the supply of T-bills and other assets accessible to shadow banks impact yields by altering the liquidity premium on these assets.

As a first step, I provide evidence on the behavior of yields in post-crisis money markets, in relation to the literature. Using long historical samples, [Krishnamurthy and Vissing-Jorgensen \(2011\)](#) and [Greenwood, Hanson, and Stein \(2015\)](#) find that an increase in the supply of T-bills leads to a reduction in the liquidity premium on T-bills and other near-money assets. [Nagel \(2016\)](#) disputes that result and argues

that as reserves also provide liquidity services, the liquidity premium on reserves needs to be controlled for as it may affect the liquidity premium on T-bills. He finds that when doing so, the effect of a change in the supply of T-bills on the liquidity premium of T-bills completely disappears. These results suggest that reserves and T-bills are perfect substitutes and that the Fed is controlling the liquidity premium on T-bills as a side product of controlling the liquidity premium on reserves. My analysis concurs with these conclusions for the pre-crisis, but not for the post-crisis period: When controlling for the level of the fed funds target rate, the effect of a change in the supply of T-bills to short-term yields drops to non-statistically significant levels for the pre-crisis period but remains significant for the post-crisis period. These results hold both in level and in first difference for three different money market instruments, suggesting that reserves and other money market instruments have ceased to be perfect substitutes since the financial crisis.

I then build a dynamic intermediary asset pricing model to assess the role of structural transformations in monetary policy and regulation in driving these developments. The model features heterogeneous banks supplying liquid deposits to households and being subject to both a micro-founded liquidity management problem and regulatory capital constraints. Since holding liquid assets helps in avoiding costly fire-sales, these assets carry a liquidity premium. The banking sector is composed of traditional banks and shadow banks. While traditional banks can hold and trade both central bank reserves and T-bills, shadow banks are limited to T-bills. This assumption is in line with the institutional restriction that only financial firms carrying a banking license are allowed to have an account at the Fed. This rule excludes institutions usually associated with the liquidity management of shadow banking activities such as mutual funds and securities dealers. To account for the Dodd-Frank Act and Basel III reforms, traditional banks are assumed to be subject to a cost when increasing the size of their balance sheets. This balance sheet cost is motivated as originating from a regulatory leverage ratio that forces banks to partially finance arbitrage positions with equity.³ A Treasury and a central bank

³Financing an arbitrage with equity is therefore costly, and banks will only do so when the benefits from the arbitrage outweigh its cost. See [Myers \(1977\)](#) and [Duffie \(2018\)](#) for micro-foundations

complete the model by providing scarce public liquid assets to banks and influencing short-term rates.

To be in a position to address the research question, the model must account for both pre- and post-crisis institutional frameworks. Before the crisis, the Fed was not authorized to pay interest on reserves and monetary policy was implemented by varying the liquidity premium on reserves by changing the quantity of liquid assets available to banks. Since the crisis, the Fed has been using the interest paid on reserves as its main policy tool to lift rates while maintaining a large balance sheet.⁴

The first contribution of the paper is to propose a tractable dynamic asset pricing model with micro-foundations able to consistently match various monetary policy implementation regimes. Unlike approaches that rely on a reserve requirement constraint and assuming that agents have money-in-the-utility, the money multiplier of the model is endogenous such that excess reserves do not affect inflation once its liquidity premium has dropped to zero.⁵ In the model, the central bank controls both the liquidity premium on reserves—through its control of the supply of liquid assets available to banks—and the interest it pays on reserves. Since both of these variables affect short-term nominal rates, there is one degree of freedom in the implementation problem of monetary policy. With this feature, the model can account for both the pre-crisis period—when the Fed was constrained to pay zero nominal interest on reserves—and the post-crisis period—when the liquidity premium on reserves has been pushed down to zero as a consequence of large outstanding amounts of excess reserves.

The second contribution of the paper is to show that the combination of a large amount of excess reserves and a tight regulatory leverage ratio creates a segmentation of money markets. In this regime, traditional banks are fully satiated with liquidity as the supply of reserves is large but are unwilling to intermediate this liquidity to shadow banks. For this reason, while the liquidity premium on reserves drops to zero, the liquidity premium of other money market assets, such as T-bills, remains

for such problems that originate from a debt overhang mechanism.

⁴See [Amstad and Martin \(2011\)](#) for a detailed account of various pre- and post-crisis practices.

⁵This mechanism is put forward by [Keister and McAndrews \(2009\)](#).

positive because liquid assets are still scarce for shadow banks. This outcome has the consequence that yields on all non-reserves liquid assets have to drop. The mechanism explains the observation that most money market rates have been trading below the interest paid on reserves in various countries in which traditional banks hold large amounts of excess reserves.⁶

The third contribution of the paper is to explain why the supply of T-bills has had an effect on short-term rates since the crisis but not before. When the regulatory leverage ratio is not binding, money markets are fully integrated because traditional banks can expand their balance sheets to intermediate liquidity without any cost. In this regime, all short-term liquid assets are priced by a single liquidity factor that is controlled by the Fed. As a consequence, a new issuance of T-bills from the Treasury needs to be sterilized by the Fed through offsetting open market operations, as it would otherwise put downward pressure on the targeted federal funds rate. Since money market assets are perfect substitutes for each other, these offsetting operations fully neutralize the effect of a change in the supply of T-bills *on all liquid assets*. In contrast, when the regulatory leverage ratio is binding, it is not profitable for traditional banks to intermediate liquidity to shadow banks. In this *inaction* region, the pricing of reserves is disconnected from the pricing of other money market assets. As a consequence, variations in the supply of T-bills only affect the liquidity premium of liquid assets held by shadow banks, and the central bank does not conduct open market operations. Hence, yields on money market assets are free to react to changes in the supply of T-bills.

To bring the model to the data, I propose an extension that allows the central bank to set a lower bound on money market rates by supplying liquid assets directly to shadow banks at a given policy rate. With this addition, the model captures the introduction of the Fed’s reverse repo facility in September 2014, through which non-bank financial institutions can lend overnight to the Fed without limit. The modified model predicts that money market rates will react to exogenous changes in the supply of T-bills if and only if the repo rate is above the reverse repo facility rate.

⁶See [Duffie and Krishnamurthy \(2016\)](#) for evidence on this phenomenon in the US and [Arrata, Nguyen, Rahmouni-Rousseau, and Vari \(2017\)](#) for the Euro area.

Once this rate is reached, changes in the supply of T-bills show up as adjustments in the quantities of liquid assets created by the Fed. I estimate the equations for the liquidity premium on overnight repo transactions and T-bills as well as for the volume of reverse repo. The model accurately explains weekly movements in the repo spread, the T-bill spread, and the quantity of reverse repo operations by the Fed. In particular, the model correctly predicts the increase in money market rates observed at the beginning of 2018, as well as the subsequent drop in reverse repo facility usage. This event is interpreted by the model as follows: The increase in the supply of T-bills drives the liquidity premia on various money market instruments down. As the rate on illiquid assets is pinned down by the interest on reserves, this narrowing of the spread takes the form of an increase in money market rates, which grows closer to the interest on reserves.

Literature Review There is a large literature devoted to studying liquidity premia on short-term money market assets. In particular, the idea that there exists a convenience yield in government bonds, as in money, is found in [Patinkin \(1956\)](#), [Tobin \(1963\)](#); [Bansal and Coleman \(1996\)](#); [Duffee \(1996\)](#); [Krishnamurthy and Vissing-Jorgensen \(2012\)](#); [Greenwood, Hanson, and Stein \(2015\)](#); and [Nagel \(2016\)](#). Moreover, government bonds that can be used as an imperfect means of payment are an important feature of some neo-monetarist models with trade frictions ([Andolfatto and Williamson, 2015](#); [Venkateswaran and Wright, 2013](#)). Closely related, [Lenel, Piazzesi, and Schneider \(2019\)](#) propose to explain the convenience yields on short-term bonds as originating from intermediaries' demand for collateral to back inside money. My paper adds to this literature by addressing the increasing importance of shadow banks and T-bills in driving short-term rates in a model in which short-term liquid assets bear a liquidity premium as a hedge against liquidity risk.

This work builds on the macro-finance literature with a financial sector ([Brunnermeier and Sannikov, 2014](#); [He and Krishnamurthy, 2013](#)), and shares with these articles an incomplete market structure. As in [Brunnermeier and Sannikov \(2016b\)](#), my model features both inside and outside money while, as in [Drechsler, Savov,](#)

and Schnabl (2017), funding liquidity shocks may affect risk premia and asset prices through the balance sheet of intermediaries. In the banking literature, Holmstrom and Tirole (1998) and Diamond and Rajan (2001, 2005) characterize optimal liquidity provision when interbank markets are affected by liquidity shocks. Afonso and Lagos (2015) and Bech and Monnet (2016) develop over-the-counter models of the interbank market with random matching to understand its trading dynamics. Close to this article, Bianchi and Bigio (2014); Schneider and Piazzesi (2015); and Fiore, Hoerova, and Uhlig (2018) include interbank markets in macroeconomic models and study the effect of lender-of-last-resort monetary policy on macroeconomic variables. This paper adds to this literature by introducing a micro-founded liquidity management problem to an asset pricing model in which a subset of banks does not have direct access to central bank reserves. In this regard, it also relates to the literature on limited arbitrage following (Vayanos and Gromb, 2002). In particular, my paper broadens the fed funds market segmentation of Bech and Klee (2011) to all money market assets when taking the behavior of the Treasury and the Fed into account.

The paper is also linked to the literature on shadow banking and the shortage of safe assets. The demand for safe assets and the role of money market instruments in filling this gap and creating financial fragility is studied by Stein (2012); Caballero (2006); Lenel (2018); Sunderam (2014); and Li (2018). Plantin (2015); Huang (2018); and Ordoñez (2018) study the emergence of the shadow banking sector as a consequence of regulatory arbitrage while Gennaioli, Shleifer, and Vishny (2013) and Luck and Schempp (2014) investigate the consequences of a run on shadow banks. This paper contributes to the literature by investigating the implications of a market segmentation appearing when it is costly for traditional banks to intermediate liquidity to shadow banks. In this regard, this paper relates to recent findings in the international finance literature (Avdjiev et al., 2019; Du et al., 2018), attributing persistent and time-varying deviations to Covered Interest Parity to costly intermediary balance sheets and regulation.

2 Short-Term Liquid Assets in the Data

In this section, I document two facts on the evolution of post-crisis money markets. First, the liquidity premia on repurchase agreements, fed funds, and T-bills are larger than the liquidity premium on reserves in the post-crisis period. Second, the supply of T-bills is positively associated with yields on repo, fed funds, and T-bills in the post-crisis period but not in the pre-crisis period.

2.1 Data

I use data from different sources. The main variable of interest is the supply of T-bills. These data are indirectly available on the auction website of the Treasury at www.treasurydirect.com. To build a dataset of the daily supply of T-bills, I rely on the daily auction reports, which are available starting from 1981. The database is formed by incrementally adding new issuance and removing maturing volumes and buybacks. I acquire the daily data on various money market rates from Bloomberg Professional and the Federal Reserve System. The overnight tri-party repo rate is provided by the Federal Reserve Bank of New York starting in August 2014. I extend the series to January 2010 by using data on overnight general collateral repo transactions from Bloomberg Professional. The overnight effective fed funds rate, the 1-month T-bill yield on secondary markets, and the rate paid by the Fed at the reverse repo facility are retrieved from FRED, the statistical service of the Federal Reserve Bank of St. Louis. The series for overnight yield on T-bills is then created by discounting the 1-month yields using the OIS curve computed and made available by Bloomberg Professional.⁷

2.2 Liquidity Premia

Measuring liquidity premia—that is, the interest that agents are willing to forgo to hold assets with superior liquidity services—is challenging. As argued by [Acharya](#)

⁷See [Duffie and Krishnamurthy \(2016\)](#) for a description of the method applied.

	2001-2007 sample			2010-2018 sample		
Liquidity Premia	# of Obs.	Mean	St. dev	# of obs.	Mean	St. dev
Reserves	1,603	2.94	1.54	2,384	0.04	0.24
3-month T-bills	1,603	0.18	0.16	2,250	0.16	0.08
3-month Repo	1,603	0.35	0.61	2,251	0.15	0.16

Table 1: Liquidity Premia on Money Market Instruments. This table describes the first and second moments of the liquidity premia on three short-term money market instruments before and after the 2008-crisis. The liquidity premia on reserves, 3-month T-bills, and 3-month repo are computed as the spread between observed yields and the shadow rate of the same maturity inferred from longer maturity assets. Details about this shadow rate are provided in Section C.

and Skeie (2011) for the short end of the yield curve, there is a liquidity premium embedded in the term structure, as short-maturity assets will automatically turn into cash in a short time-frame. Holding short-term assets is therefore a way for banks to hedge their liquidity risk, along with holding assets for which there is a liquid market. Estimating the liquidity premium on a given short-term asset requires finding a way to measure the yields on an unobservable asset of short maturity that does not provide any liquidity services—usually referred to as the *shadow rate*.

To do so, I follow Greenwood, Hanson, and Stein (2015) and Lenel, Piazzesi, and Schneider (2019) and exploit information extracted from longer-maturity safe assets. More precisely, I make use of the affine term structure model of the yield curve constructed by Gürkaynak, Sack, and Wright (2007) using data on yields on Treasuries with maturities of 1 year and more. I then use these estimates to infer the shape of the short end of a yield curve, which does not embed liquidity services.⁸ Liquidity premia for both the pre- and post-crisis samples are then measured as the spread between the realized yield on the asset and the shadow rate defined by

$$\text{liquidity premium on A} = \text{shadow rate for the maturity of A} - \text{yield on A}.$$

⁸See Appendix C for a more detailed account of the method.

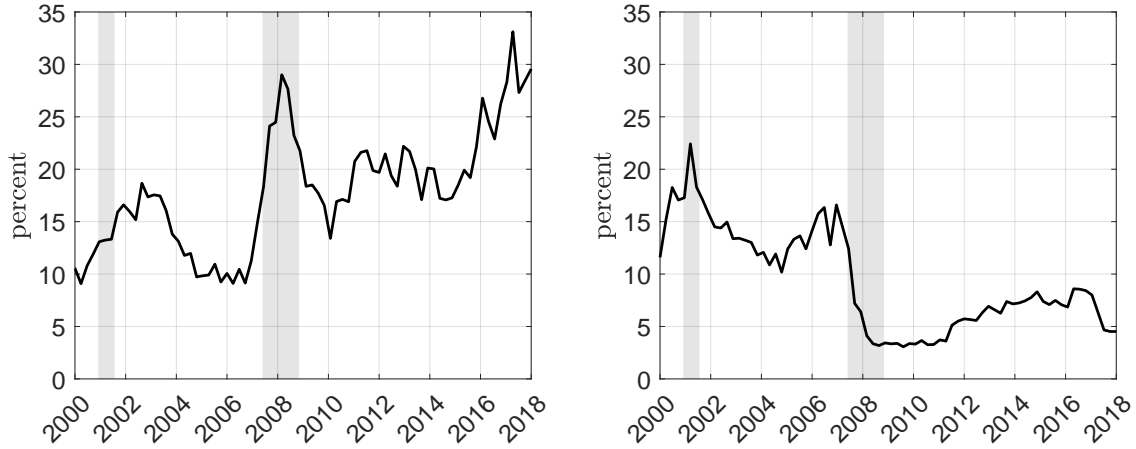
Table 1 provides the summary statistics for the liquidity premia on reserves, 3-month T-bills, and 3-month repo transactions before and after the crisis. For the pre-crisis sample from 2001 to 2008, the liquidity premium on reserves has been 2.94 pp on an average, which is significantly larger than premia on T-bills, with an average yield of 0.18 pp, and on repo transactions, with an average of 0.35 pp. For the post-crisis sample, the relationship is inverted, as the liquidity premium on reserves dropped to an average of 0.04 pp, while the average liquidity premium on T-bills has been 0.18 pp and the average liquidity premium on repo transactions has been 0.16 pp.

2.3 T-bill Holdings

T-bills are an important source of dollar liquidity because of their short maturity and highly liquid secondary markets (Adrian, Fleming, and Vogt, 2017). Unlike central bank reserves, T-bills can be purchased directly by any individuals or corporations in both primary and secondary markets. Understanding who holds these T-bills is key to assess the plausibility of the mechanism described in this paper.

Figure 2a displays the evolution of the ratio of stock of T-bills held by shadow banks over the total stock of T-bills available to the public.⁹ The share of available T-bills held by shadow banks increases after the crisis and remains at a higher level than before the crisis. Unfortunately, there is no data available on traditional banks' holdings of T-bills. Instead, I use data from call reports on bank holdings of assets of maturity of less than a year to create an upper bound measure of the ratio of traditional banks' holdings to available supply. Figure 2b displays the evolution of this series across time. The upper-bound measure falls significantly at the time of the crisis and remains at low level since. Note that this upper-bound measure is likely to be significantly above traditional banks' actual holdings as it includes any loans and securities that banks hold to maturity. Overall, this figure suggests that

⁹Shadow banks' holdings is computed as the sum of holdings of money market funds, mutual funds and insurance corporations as data on holdings of securities' dealers and asset managers are not available. The stock of T-bills available to the public is the total outstanding minus quantities held by the Fed.



(a) Shadow Banks (Observed)

(b) Traditional Banks (Upper Bound)

Figure 2: Share of Available T-bills Held by Shadow and Traditional Banks. The left panel displays the ratio of the value of T-bills held by shadow banks to the stock of T-bills available. The right panel displays the ratio of an upper bound measure of the share of T-bills held by traditional banks to the stock of T-bills available. Shadow banks' holdings are computed as the sum of holdings from money funds, mutual funds and insurance companies from flow of funds data. The stock of available T-bills is computed as the total amount outstanding minus the holdings of the Fed from flows of funds data. The upper bound measure for traditional banks is computed as the total value of assets with a maturity of less than a year in depository institutions from call report data.

traditional banks have not been holding significant portions of T-bills since the crisis. This evidence concurs with [Hanson, Shleifer, Stein, and Vishny \(2015\)](#), who find that traditional banks held less than 1% of their total assets in T-bills in 2012.

2.4 T-bill Supply and Short-Term Yields

The existence of a negative association between the supply of T-bills and the liquidity premium on the same asset has been documented by [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and [Greenwood, Hanson, and Stein \(2015\)](#). The conventional interpretation of these results is the following: T-bills are a scarce source of

	T-Bills - IOR		Fed Funds - IOR		Repo - IOR	
	level	difference	level	difference	level	difference
Panel A: August 2001 - July 2008 ($N=1,598$)						
T-Bills/GDP	0.93 (0.68)	-0.05 (1.10)	0.16 (0.12)	-0.28* (0.16)	-0.05 (0.29)	-0.20 (0.49)
Fed Funds Target	0.94*** (0.03)	1.03*** (0.07)	0.99*** (0.01)	0.96*** (0.04)	0.99*** (0.00)	0.92*** (0.03)
VIX	0.19 (0.39)	-0.15 (0.45)	-0.34 (0.16)	-0.80** (0.37)	-0.91*** (0.21)	-0.05 (0.36)
Intercept	-81.82 (57.26)	-1.99 (1.76)	-3.76 (10.94)	-0.70 (0.53)	10.83 (25.42)	-0.12 (0.98)
Panel B: January 2010 - December 2018 ($N=2,218$)						
T-Bills/GDP	0.33*** (0.04)	0.23** (0.10)	0.15*** (0.03)	0.11*** (0.03)	0.15*** (0.05)	0.27** (0.14)
Fed Funds Target	0.02 (0.02)	-0.03 (0.05)	0.04*** (0.01)	0.01 (0.02)	0.05*** (0.01)	0.11 (0.09)
VIX	0.01 (0.11)	0.04 (0.05)	-0.03 (0.07)	0.03* (0.20)	0.13 (0.14)	0.08 (0.11)
Intercept	-55.23*** (4.59)	0.40 (0.39)	-26.88*** (2.86)	0.15 (0.19)	-28.45*** (5.83)	-0.13 (0.60)

Table 2: The Supply of T-Bills and Money Market Yields The table reports daily regressions of money market spreads on the supply on T-bills scaled by GDP and controlled for the fed funds target rate the VIX index. The three money market instruments considered are the overnight T-bill, the fed funds rate, and the overnight general collateral repo rate. The overnight T-bill rate is obtained by discounting the 1-month T-bill rate with the OIS curve as described in [Duffie and Krishnamurthy \(2016\)](#). The target rate for the post-crisis sample is computed as the midpoint between the top and the bottom of the target band. I estimate equation 1 both in level and in 4-week difference by ordinary least square. I report Newey-West (1987) standard errors, allowing for serial correlation up to twelve weeks.

liquidity, and their marginal utility is higher when there is little of it. [Nagel \(2016\)](#) argues that other sources of liquidity, such as central bank reserves, should be taken into account when considering this relationship, as they may be a substitute for liquidity provision. He then shows that when controlling for the liquidity premium of reserves (as proxied by the fed funds rate), the effect of the supply of T-bills is reduced to non-statistically significant levels.

I provide evidence on the relationship between short-term yields and the supply of Treasury bills before and after the crisis. To do so, I follow [Nagel \(2016\)](#) and estimate a linear regression model of money market spreads on the supply of T-bills

while controlling for the level of the fed funds rate and the VIX. To construct the dependent variable, I take the spread between, respectively, the yields on T-bills, fed funds and repo, and the interest paid on reserves (IOR). Subtracting the interest on reserves from yields is necessary to account for the change in monetary policy regime and compare the pre- and post-crisis samples.¹⁰ I then estimate by ordinary least squares the following equation:

$$(\text{Yield} - \text{IOR})_t = \beta_0 + \beta_1(\text{T-Bills/GDP})_t + \beta_2\text{Fed Funds Target}_t + \beta_3\text{VIX}_t. \quad (1)$$

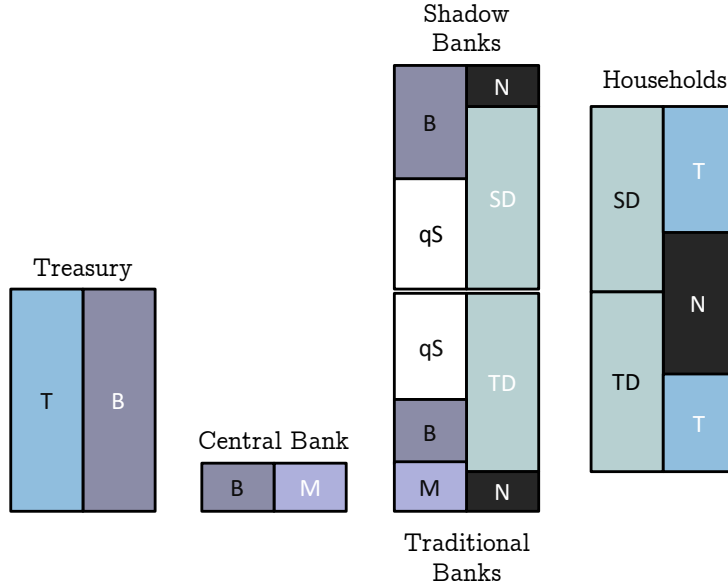
Estimated coefficients and 3-months maximum lag Newey-West standard errors are reported in Table 2. For the pre-crisis sample, the coefficient on the T-Bills/GDP variable is non-statistically significant when controlling for the fed funds target. This result, akin to Nagel (2016), is consistent across the different assets and specifications. For the post-crisis sample, even when controlling for the fed funds target, the coefficients on the T-Bills/GDP variable remain statistically significant. These results suggest that since 2010, reserves have not been a perfect substitute for other money market instruments. In Appendix D, I show that this pattern—the neutrality of T-Bills/GDP when controlling for the fed funds target—is robust to a variety of specifications, including when estimating the same equations as Nagel (2016) with an extended dataset.

3 The Model

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions and assume that all stochastic processes are adapted. The economy evolves in continuous time with $t \in [0, \infty)$ and is populated by a continuum of traditional banks, shadow banks, and households in mass one as well as a treasury and a central bank. There are

¹⁰In 2008, the Fed started paying interest on reserves as its main policy tool. Since this rate serves as the reference point for all liquidity premia, both in the pre- and post-crisis samples, not subtracting it would lead to an overestimation of the relationship between supply and yields in the post-crisis sample. In Appendix D, I show that the results are unchanged for the pre-crisis period, but stronger for the post-crisis period, when regressing on yields rather than spreads.

Figure 3: Sketch of Agents' Balance Sheets



two goods in positive supply: the final consumption good and securitized physical capital. Figure 3 introduces the balance sheet of the different sectors in the economy. The Treasury issues T-bills B against future tax liabilities T ; the central bank holds some of the outstanding T-bills by issuing reserves M to the banking sector; and households hold their wealth in both traditional bank deposits TD and shadow bank deposits SD . The two types of banks issue these two instruments to finance their holdings of securitized capital S valued at price q and of liquid assets. N denotes the net worth of a given sector. Reserves M is the unit of account asset, while output Y is the numeraire in the economy. All rates and asset prices are expressed in real terms except when explicitly specified otherwise.

3.1 Environment

Preferences All agents have logarithmic preferences over their consumption rate c_t of their net worth n_t with a time preference ρ :

$$V_t = \mathbb{E}_t \left[\int_t^\infty e^{-\rho u} \log(c_u n_u) du \right].$$

Technology There is a positive supply of real capital that produces a flow of output with constant productivity a . All units of capital are pooled in an economy-wide diversified asset-backed security vehicle in quantity S_t . The law of motion for the stock of securities is given by

$$dS_t = \Phi S_t dt + \sigma S_t dZ_t,$$

where $\sigma S_t dZ_t$ is an aggregate capital quality shock and Z_t is an adapted standard Brownian. The supply of securities grows deterministically at a rate of Φ .

Nominal Definition As in the second chapter of [Woodford \(2003\)](#), the economy does not feature any nominal frictions and the ability of the central bank to influence inflation is derived from the role of reserves as the unit of account. I define the nominal output $Y_t^\$ = Y_t/P_t$, where P_t is the price level. Inflation π_t is then defined by the drift of the deterministic law of motion of aggregate prices: $dP_t/P_t = \pi_t dt$.

Returns As there is only one aggregate stochastic process dZ_t in the model, the stochastic law of motion of the price of a unit of securities q_t can be written as

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dZ_t,$$

where μ_t^q and σ_t^q are determined endogenously by equilibrium conditions. Applying Ito's lemma, the flow of return generated by holding securities is given by

$$dr_t^s = \underbrace{\left(\frac{a}{q_t} + \Phi + \mu_t^q + \sigma \sigma_t^q \right)}_{\equiv \mu_t^s} dt + \underbrace{(\sigma + \sigma_t^q)}_{\equiv \sigma_t^s} dZ_t.$$

The drift of this process, μ_t^s , is composed of the dividend-price ratio plus capital gains. This formulation assumes, without loss of generality, that new capital formation is distributed proportionally to securities holdings. The loading factor σ_t^s consists of the sum of the exogenous (fundamental shock) and endogenous volatility (corresponding response in asset prices).

Liquidity Management The two types of banks are subject to idiosyncratic funding shocks. After a negative realization of the funding shock, some deposits in a given bank are transferred to another bank. This process can be interpreted both as a feature of normal payment flows from depositors or as an abnormal run on a given bank. This reshuffling creates a funding gap for one (the deficit bank) and a funding surplus for the other (the surplus bank). The sequence of action takes place in a short period of time in which loans can only be traded at a discount with respect to their fundamental value.

To avoid having to bear the cost of these fire-sales, banks can hold liquid money market instruments as a buffer against funding shocks. Two assets with this property are issued by the public sector: T-bills and central bank reserves.¹¹ There are two differences between these assets. First, they differ in terms of their liquidity services, such that reserves are more liquid than T-bills.¹² Second, reserves can only be held

¹¹In this section, I propose a minimalist model and assume that banks cannot create liquid assets for other banks. I relax this assumption in Section 5 and show that the results still hold when we also add a regulatory constraint.

¹²This element of the model follows from the fact that reserves are always accepted by banks as a means of settlement. Hence, reserves can be transferred without delay or cost to meet a funding shock whereas T-bills have to be first sold. The higher liquidity value of reserves when compared to T-bills concurs with the empirical evidence in the pre-crisis period that the liquidity premium on reserves is higher.

by traditional banks (and not shadow banks), while all types of banks can hold T-bills. Note that reserves and T-bills are always an asset for banks as they are, by definition, a liability of the government.

In Appendix B, I show that such a problem converges in continuous time to the following idiosyncratic, but not diversifiable, transfers of wealth:

$$\begin{aligned} \text{traditional banks:} \quad \psi_t &= \lambda \max \{ \sigma^d w_t^d - \theta^m w_t^m - \theta^b w_t^b, 0 \}, \\ \text{shadow banks:} \quad \bar{\psi}_t &= \lambda \max \{ \sigma^d \bar{w}_t^d - \theta^b \bar{w}_t^b, 0 \}. \end{aligned}$$

These equations have the following interpretation. When a negative funding shock of size $\sigma^d w_t^d$ hits a bank, it has to pay a fire-sale cost λ on the amount remaining after having disbursed liquid reserves w_t^m and T-bills w_t^b . When a bank receives a positive shock, it has the extra resources to purchase the asset sold by the deficit bank at a discount and make a profit on the operation. The liquidity parameters θ^m and θ^b reflect that reserves provide more liquidity services per unit than T-bills $\theta^m > \theta^b$. The maximum operator guarantees that holding liquidity assets never increases the exposure to liquidity risks.

Treasury The Treasury issues T-bills against future tax liabilities of other agents and is responsible for administrating redistributive lump-sum tax policies. The ratio of Treasury T-bills to the total wealth of the economy $b_t = B_t/(q_t S_t)$ follows an exogenous stochastic process:

$$db_t = \kappa(b^{ss} - b_t)dt + \sigma^b \sqrt{b_t} dZ_t, \quad (2)$$

where b^{ss} is the long-run mean and κ is the mean reversion parameter. Lump-sum tax policies have two purposes in this economy. First, they allow the Treasury to issue T-bills. Correspondingly, the net present value of future tax liabilities must equal the outstanding amount of T-bills: $T_t + \bar{T}_t = B_t$, where T_t and \bar{T}_t are the tax liabilities of the traditional and shadow banking sector, respectively. Second, taxes redistribute wealth from the banking sector to the household sector to prevent

the economy from converging to the trivial state in which banks hold all of the wealth in the economy. To do so, I set up the transfer rules such that the Treasury (i) runs a balanced budget with zero net worth, and (ii) performs further transfers (independent from the level of T-bills supply) between different sectors in order for the distribution of wealth to remain constant.

Central Bank The central bank controls the supply of liquidity to the banking sector by swapping reserves for T-bills (and conversely) through open market operations. As reserves are more liquid than T-bills, the purchase of T-bills financed by issuing new reserves increases the effective supply of liquidity to the banking sector. In other words, the central bank decides on the stock of reserves available to banks m_t and the amount of T-bills removed from the market and held by the central bank \underline{b}_t subject to the balance sheet constraint:

$$\underline{b}_t = m_t.$$

The monetary policy variables \underline{b}_t and m_t are also expressed as a fraction of the total wealth in the economy: $\underline{b}_t = \underline{B}_t/(q_t S_t)$ and $m_t = M_t/(q_t S_t)$. The underline differentiates the central bank's holdings of T-bills \underline{B}_t from the T-bills issued by the Treasury B_t . For simplicity, I assume that the central bank always operates with zero net worth and instantaneously transfers all seigniorage revenues to the Treasury. Moreover, the central bank also decides on the nominal interest rate it pays on its reserves i_t^m . Hence, the set of monetary policy decisions is $\{m_t, i_t^m\}$.

3.2 Agents' Problems

Traditional Banks Traditional banks face a [Merton's \(1969\)](#) portfolio choice problem augmented by the liquidity management component. Bankers maximize their

lifetime expected logarithmic utility:

$$\max_{\{w_\tau^s \geq 0, w_\tau^i, w_\tau^b \geq 0, w_\tau^m \geq 0, w_\tau^d, c_\tau\}_{\tau=t}^\infty} \mathbb{E}_t \left[\int_t^\infty e^{-\rho\tau} \log(c_\tau n_\tau) d\tau \right], \quad (3)$$

subject to the law of motion of wealth:

$$\begin{aligned} dn_t = & (w_t^s \mu_t^s + w_t^i r_t^i + w_t^b r_t^b + w_t^m r_t^m - w_t^d r_t^d - c_t + \mu_t^\tau) n_t dt + (w_t^s \sigma_t^s + \sigma_t^\tau) n_t dZ_t \\ & + \lambda \max \{ \sigma_t^d w_t^d - \theta^m w_t^m - \theta^b w_t^b, 0 \} n_t d\tilde{Z}_t, \end{aligned} \quad (4)$$

and the balance sheet constraint:

$$w_t^s + w_t^i + w_t^b + w_t^m = 1 + w_t^d + w_t^\tau, \quad (5)$$

Traditional banks choose their portfolio weights for risky securities w_t^s , illiquid interbank loans w_t^i , T-bills w_t^b , reserves w_t^m , and deposits w_t^d given their respective interest rates μ_t^s , r_t^b , r_t^m , and, r_t^d . Traditional bankers also choose their consumption rate c_t . A negative portfolio weight for an asset corresponds to a short (liability) position for a bank, while a positive weight is a long (asset) position. The portfolio weights on productive securities, reserves, and T-bills are subject to a non-negativity constraint because, by definition, these assets cannot be issued by banks. Since an interbank loan is always the liability of some banks, this constraint does not apply to this asset category. When holding illiquid securities, banks increase their exposure to funding risk defined by the standard adapted Brownian \tilde{Z}_t , which is idiosyncratic to the individual bank. Moreover, traditional banks receives a flow of transfers per unit of wealth $d\tau_t = \mu_t^\tau dt + \sigma_t^\tau dZ_t$ from the Treasury (this flow can be negative). The variable $w_t^\tau = T_t/n_t$ is the ratio of implicit tax liabilities per unit of wealth, as determined by the tax policy of the government.

Shadow Banks Shadow banks face the same problem as traditional banks except that they cannot hold reserves:

$$\max_{\{\bar{w}_\tau^s \geq 0, \bar{w}_\tau^i, \bar{w}_\tau^b \geq 0, \bar{w}_\tau^d, \bar{c}_\tau\}_{\tau=t}^\infty} \mathbb{E}_t \left[\int_t^\infty e^{-\rho\tau} \log(\bar{c}_\tau \bar{n}_\tau) d\tau \right], \quad (6)$$

subject to the law of motion of wealth:

$$\begin{aligned} d\bar{n}_t = & (\bar{w}_t^s \mu_t^s + \bar{w}_t^i r_t^i + \bar{w}_t^b r_t^b - \bar{w}_t^d r_t^d - \bar{c}_t + \bar{\mu}_t^\tau) \bar{n}_t dt + (\bar{w}_t^s \sigma_t^s + \bar{\sigma}_t^\tau) \bar{n}_t dZ_t \\ & + \lambda \max\{\sigma_t^d \bar{w}_t^d - \theta^b \bar{w}_t^b, 0\} \bar{n}_t d\tilde{Z}_t, \end{aligned} \quad (7)$$

and the balance sheet constraint:

$$\bar{w}_t^s + \bar{w}_t^i + \bar{w}_t^b = 1 + \bar{w}_t^d + \bar{w}_t^\tau. \quad (8)$$

The interpretation of the variables, overlined to denote shadow bankers, is the same as for traditional banks. Shadow bank and traditional bank deposits are assumed to be perfectly non-substitutable. Thus, the interest rate on traditional bank deposits r_t^d might deviate from the interest rate on shadow bank deposits \bar{r}_t^d .

Households Households maximize their lifetime utility function subject to the additional assumption that they can only invest in shadow and traditional bank deposits:

$$\max_{\{c_\tau^h\}_{\tau=t}^\infty} E_t \left[\int_t^\infty e^{-\rho\tau} \log(c_\tau^h n_\tau^h) d\tau \right], \quad (9)$$

subject to the law of motion of wealth:

$$dn_t^h = \left((\gamma r_t^d + (1 - \gamma) \bar{r}_t^d) - c_t^h + \mu_t^{\tau, h} \right) n_t^h dt + \sigma_t^{h, \tau} n_t^h dZ_t.$$

Shadow bank and traditional bank deposits are assumed are held in fixed proportion γ for traditional and $1 - \gamma$ for shadow deposits. Households decide on their consumption

rate c^h and overall deposit investment w_t^h subject to the balance sheet constraint:

$$w_t^h = 1,$$

where the variables indexed by h refer to households.

Treasury Balanced Budget Constraint The balanced budget constraint for the Treasury is given by:

$$r_t^b B_t dt = \mu^\tau n_t dt + \bar{\mu}^\tau \bar{n}_t dt + \mu_t^{\tau,h} n_t^h dt + (r_t^b - r_t^m) M_t dt.$$

To pay the interest on T-bills, the Treasury collects taxes from traditional banks, shadow banks, and households and seigniorage revenues from the central bank. I assume that tax liabilities are shared amongst traditional and shadow banks in proportion of their share of wealth. That is, $T_t/\bar{T}_t = n_t/\bar{n}_t$.

Definition 1. *Given an initial allocation of all asset variables at $t = 0$, monetary policy decisions $\{m_t, i_t^m : t \geq 0\}$, and transfer rules $\{\mu_t^\tau, \bar{\mu}_t^\tau, \mu_t^{\tau,h}, \sigma_t^\tau, \bar{\sigma}_t^\tau, \sigma_t^{\tau,h} : t \geq 0\}$; a **sequential equilibrium** is a set of adapted stochastic processes for (i) prices $\{q_t, r_t^b, r_t^m, r_t^d, \bar{r}_t^d, r_t^i : t \geq 0\}$; (ii) individual controls for regular bankers $\{c_t, w_t^s, w_t^m, w_t^b, w_t^d : t \geq 0\}$, (iii) shadow bankers $\{\bar{c}_t, \bar{w}_t^s, \bar{w}_t^b, \bar{w}_t^d : t \geq 0\}$; (iv) households $\{c_t^h : t \geq 0\}$; (v) aggregate security stock $\{S_t : t \geq 0\}$; and (vi) agents' net worth $\{n_t, \bar{n}_t, n_t^h : t \geq 0\}$ such that:*

1. *Agents solve their respective problems defined in equations (3), (6), and (9).*

2. *Markets clear:*

$$\begin{aligned}
(a) \text{ securities:} & \quad \int_0^1 w_t^s n_t^s di + \int_0^1 \bar{w}_t^s \bar{n}_t dj = q_t S_t, \\
(b) \text{ T-bills:} & \quad \int_0^1 w_t^b n_t di + \int_0^1 \bar{w}_t^b \bar{n}_t dj + \underline{B}_t = B_t, \\
(c) \text{ reserves:} & \quad \int_0^1 w_t^m n_t di = M_t, \\
(d) \text{ traditional deposits:} & \quad \int_0^1 w_t^d n_t di = \int_0^1 \gamma n_t^h dh, \\
(e) \text{ shadow deposits:} & \quad \int_0^1 \bar{w}_t^d \bar{n}_t dj = \int_0^1 (1 - \gamma) n_t^h dh, \\
(f) \text{ output:} & \quad \int_0^1 c_t n_t di + \int_0^1 \bar{c}_t \bar{n}_t dj + \int_0^1 c_t^h n_t^h dh = a S_t.
\end{aligned}$$

3.3 Discussion of Assumptions

Access to Reserves The model assumes that shadow banks don't have access to reserves. In the US, only financial institutions classified as depository institutions can hold reserves at the Fed. Other financial institutions providing liquidity transformation services in dollars such as money market mutual funds, securities dealers, or foreign banks without a depository subsidiary in the US, rely on other money market instruments to solve their liquidity management problems. This assumption is critical for the results.

Substitutability of Deposits The model assumes that traditional and shadow deposits are not substitutes. This hypothesis follows [Begenau and Landvoigt \(2018\)](#), who find an imperfect elasticity of substitution in the demand function for shadow and traditional deposits. [Adrian and Ashcraft \(2012\)](#) and [Pozsar, Adrian, Ashcraft, and Boesky \(2012\)](#) argue that this non-substitutability is partly driven by a demand for liquid assets from institutions, usually referred to as "cash pools", dealing with amounts much too large to benefit from the deposit insurance of the Federal Deposit Insurance Corporation on traditional bank deposits. This assumption could be relaxed without affecting the results, as long as some imperfection prevents all of the deposits from flowing into the sector with the less liquidity risk for a small change in spreads.

Risk Sharing The assumption that households cannot hold risky securities has the consequence that the stochastic discount factor of financial intermediaries is pricing the assets in the economy. This hypothesis is a parsimonious mean to generate this feature for which there is strong empirical evidence (see, for instance, [Adrian, Etula, and Muir, 2014](#), and [He, Kelly, and Manela, 2017](#)). I refer to [Brunnermeier and Sannikov \(2016a\)](#) and [He and Krishnamurthy \(2013\)](#) for a micro-foundation originating from agency frictions that compel bankers to keep sufficient skin in the game in their banks. The model could allow banks to issue some limited outside equity to households without affecting the main results of the paper.

3.4 Solving

The homotheticity of logarithmic preferences generates optimal strategies that are linear in the net worth of each agent. Hence, the distribution of net worth within each sector does not affect the equilibrium. I follow [Brunnermeier and Sannikov \(2014\)](#) and [Di Tella \(2017\)](#) in using a recursive formulation of the problem taking advantage of the scale invariance of the model to abstract from the level of aggregate capital. I guess and verify that the value function of each agent has the following additive form:

$$V(\xi_t, n_t) = \xi_t + \frac{\log(n_t)}{\rho}, \quad \bar{V}(\bar{\xi}_t, \bar{n}_t) = \bar{\xi}_t + \frac{\log(\bar{n}_t)}{\rho}, \quad V^h(\xi_t^h, n_t^h) = \xi_t^h + \frac{\log(n_t^h)}{\rho},$$

for some stochastic processes $\{\xi_t, \bar{\xi}_t, \xi_t^h\}$ that capture time variations in the set of investment opportunities for a given type of agent. A unit of net worth has a higher value for a regular bank, a shadow bank, or a household in states in which ξ_t , $\bar{\xi}_t$ or ξ_t^h are respectively high. I postulate that these wealth multipliers follow geometric Brownian motions.

Recursive Formulation Thanks to the homotheticity of preferences and the linearity of technology, all agents of the same type choose the same set of control variables when stated as a proportion of their net worth. Hence, we only have to

track the distribution of wealth between types and not within types. Two of the three state variables of the economy are the share of wealth in the hands of the regular and shadow banking sectors:

$$\eta_t \equiv \frac{n_t}{n_t + \bar{n}_t + n_t^h}, \quad \bar{\eta}_t \equiv \frac{\bar{n}_t}{n_t + \bar{n}_t + n_t^h},$$

where the total net worth in the economy is given by $n_t + \bar{n}_t + n_t^h = q_t S_t$. The last state variable is the supply of T-bills from the Treasury b_t . From here on, I characterize the economy as a recursive Markov equilibrium.

Definition 2. A *Markov equilibrium* \mathcal{M} in $x_t = (\eta_t, \bar{\eta}_t, b_t)$ is a set of functions $g_t = g(\eta_t, \bar{\eta}_t, b_t)$ for (i) prices $\{q_t, r_t^b, r_t^m, r_t^d, \bar{r}_t^d, r_t^i\}$; (ii) individual controls for traditional banks $\{c_t, w_t^s, w_t^m, w_t^b, w_t^i, w_t^d\}$, shadow banks $\{\bar{c}_t, \bar{w}_t^s, \bar{w}_t^b, \bar{w}_t^i, \bar{w}_t^d\}$, and households $\{c_t^h\}$; (iii) monetary policy functions $\{m_t, i_t^m\}$; and (iv) transfer rules $\{\mu_t^\tau, \bar{\mu}_t^\tau, \mu_t^{\tau,h}, \sigma_t^\tau, \bar{\sigma}_t^\tau, \sigma_t^{\tau,h}\}$; such that:

1. Wealth multipliers $\{\xi_t, \bar{\xi}_t, \xi_t^h\}$ solve their respective Hamilton-Jacobi-Bellman equations with optimal controls (ii), given prices (i), monetary policy (iii) and transfer rules (iv).

2. Markets clear:

$$\begin{aligned} (a) \text{ securities:} & \quad w_t^s \eta_t + \bar{w}_t^s \bar{\eta}_t = 1, \\ (b) \text{ T-bills:} & \quad w_t^b \eta_t + \bar{w}_t^b \bar{\eta}_t + \underline{b}_t = b_t, \\ (c) \text{ reserves:} & \quad w_t^m \eta_t = m_t, \\ (d) \text{ traditional deposits:} & \quad w_t^d \eta_t = \gamma(1 - \eta_t - \bar{\eta}_t), \\ (e) \text{ shadow deposits:} & \quad \bar{w}_t^d \bar{\eta}_t = (1 - \gamma)(1 - \eta_t - \bar{\eta}_t), \\ (f) \text{ output:} & \quad c_t \eta_t + \bar{c}_t \bar{\eta}_t + c_t^h(1 - \eta_t - \bar{\eta}_t) = a/q_t. \end{aligned}$$

3. Monetary policy variables m_t, i_t^m are set as functions of the state variables only.

4. Transfer rules $\{\mu_t^\tau, \bar{\mu}_t^\tau, \mu_t^{\tau,h}, \sigma_t^\tau, \bar{\sigma}_t^\tau, \sigma_t^{\tau,h}\}$ are set as functions of the state variables only.

5. The laws of motion for the state variables in $x_t = \{\eta_t, \bar{\eta}_t, b_t\}$ are consistent with transfer and monetary policy rules.

Hamilton-Jacobi-Bellman Equation Using the guess and substituting for the balance sheet identity, the HJB equation for traditional banks can be written as:

$$\rho \xi_t + \log(n_t) = \max_{\{w_t^s, w_t^b, w_t^m, w_t^d, c_t\}} \left\{ \log(c_t n_t) + \frac{\mu_t^n}{\rho} - \frac{1}{2} \frac{(\sigma_t^n)^2}{\rho} - \frac{1}{2} \frac{\psi_t^2}{\rho} + \mu_t^\xi \xi_t \right\}, \quad (10)$$

where

$$\begin{aligned} \mu_t^n &= w_t^s(\mu^s - r_t^i) + w_t^b(r_t^b - r_t^i) + w_t^m(r_t^m - r_t^i) - w_t^d(r_t^d - r_t^i) - c_t + \mu_t^\tau, \\ \sigma_t^n &= w_t^s \sigma_t^s + \sigma_t^\tau, \\ \psi_t &= \lambda \max\{\sigma^d w_t^d - \theta^m w_t^m - \theta^b w_t^b, 0\}. \end{aligned}$$

Shadow banks' and households' problems are nested by the one of traditional banks such that their respective HJB equations can be inferred from equation (10).

First-Order Conditions Applying the maximum principle, I derive the first-order conditions for the three types of agents.

Traditional banks:

$$\begin{aligned} c_t &= \rho \\ \mu_t^s - r_t^i &\leq w_t^s (\sigma_t^s)^2 + \sigma_t^\tau \sigma_t^s && \text{with equality if } w_t^s > 0 \end{aligned} \quad (11)$$

$$r_t^i - r_t^d \leq \lambda \sigma^d \psi_t \quad \text{with equality if } w_t^d > 0 \quad (12)$$

$$r_t^i - r_t^b \geq \lambda \theta^b \psi_t \quad \text{with equality if } w_t^b > 0 \quad (13)$$

$$r_t^i - r_t^m \geq \lambda \theta^m \psi_t \quad \text{with equality if } w_t^m > 0 \quad (14)$$

Shadow banks:

$$\begin{aligned} \bar{c}_t &= \rho \\ \mu_t^s - r_t^i &\leq \bar{w}_t^s (\sigma_t^s)^2 + \bar{\sigma}_t^s \sigma_t^s && \text{with equality if } \bar{w}_t^s > 0 \end{aligned} \quad (15)$$

$$r_t^i - \bar{r}_t^d \leq \lambda \sigma^d \bar{\psi}_t \quad \text{with equality if } \bar{w}_t^d > 0 \quad (16)$$

$$r_t^i - r_t^b \geq \lambda \theta^b \bar{\psi}_t \quad \text{with equality if } \bar{w}_t^b > 0 \quad (17)$$

Households:

$$c_t^h = \rho$$

With logarithmic preferences, every agent always consumes a fixed proportion ρ . When a given type of agent holds a positive amount of risky securities, the excess return on this asset is equal to the volatility of its stochastic discount factor in equations (11) and (15). In equations (12) and (16), banks equalize the marginal benefits of issuing deposits (its liquidity risk premium) to its marginal cost (the marginal increase in liquidity risk). The first-order conditions for reserves and T-bills, given in equations (13), (14), and (17) have a similar structure but an inverse interpretation. The marginal cost is the forgone interest of holding a unit of liquid asset, the liquidity premium, on the left-hand side. The marginal benefits are the marginal reduction in liquidity risk on the right-hand side.

4 Theoretical Analysis

This section exposes the main theoretical results of the paper. When money markets are integrated, exposure to liquidity risk is equalized between the two banking sectors. When money markets are segmented, the liquidity risk of the two banking sectors disconnect. This case arises when traditional banks run out of T-bills to sell to shadow banks. As a consequence, the liquidity premium on T-bills responds to a change in the supply of T-bills only in segmented market equilibria.

To clarify the exposition, I define the two sets of equilibria corresponding to changes in pricing dynamics. First, let's define the set of equilibria in which the nonnegativity constraint on traditional banks' T-bills holdings is binding.

Definition 3. *Let \mathcal{S} be the set of a segmented money markets equilibria defined as $\{\mathcal{M}(x) \in \mathcal{S} \mid r^i(x) - r^b(x) > \lambda\theta^b\psi(x)\}$.*

As the marginal benefits of holding T-bills are higher than their marginal cost, this definition corresponds to cases in which the nonnegativity constraint is binding. Second, let's define the set of equilibria for which traditional banks have liquidity in excess of their needs.

Definition 4. *Let \mathcal{E} be the set of traditional bank satiation equilibria defined as $\{\mathcal{M}(x) \in \mathcal{E} \mid \psi(x) = 0\}$.*

I also restrict the analysis in the rest of the paper to the equilibria that satisfy the following condition:

$$\gamma \leq \frac{\eta}{\eta + \bar{\eta}}. \tag{C1}$$

That is, I discard equilibria of which the fraction of deposits in the shadow banking sector is too small compared to the relative wealth of the traditional banks. In such an equilibrium, the quantity of liquidity risk in the shadow banking sector might be so low that it is not optimal for shadow banks to hold any T-bills. I also assume that both T-bills and central bank reserves are in strictly positive supply.

All proofs of lemmas and propositions are relegated to Appendix [A](#).

4.1 Integrated Money Markets

This section describes an economy in which the nonnegativity constraint on traditional banks' holdings of T-bills is never binding, as a reference point for the analysis. Under this assumption, equilibrium conditions imply that banks of all types have the same exposure to liquidity risk.

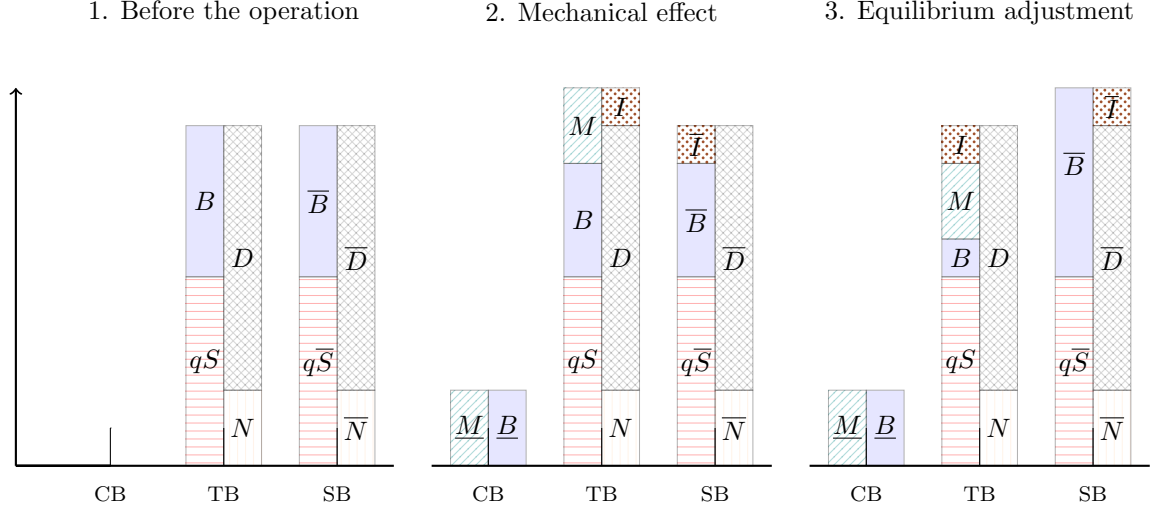


Figure 4: Balance Sheet Adjustments After an Open Market Operation

Lemma 1. *In an equilibrium in which money markets are not segmented, $\mathcal{M}(x) \notin \mathcal{S}$, traditional and shadow banks have the same exposure to liquidity risk per unit of wealth:*

$$\psi(x) = \bar{\psi}(x).$$

Lemma 1 has an intuitive interpretation: Risk-averse agents exploit the benefits of risk-sharing by trading liquid assets in order to equalize their marginal utility of holding these liquid assets. This result unfolds from the first-order conditions (13) and (17) holding with equality.

In an expansionary open market operation, the central bank increases the supply of reserves by purchasing T-bills. As reserves are more liquid than T-bills, the net impact of this operation is to increase the effective supply of liquidity in the economy. Figure 4 illustrates the sequence of balance-sheet adjustments that follow such an open market operation. In the first stage, the central bank does not hold any T-bill, such that shadow and traditional banks are similar. In the second stage, the central bank purchases T-bills from both sectors. As only traditional banks can hold reserves, these purchases result in a mechanical outcome in which traditional banks

have more liquid assets. This second stage is illustrative only as it cannot be an equilibrium outcome. For the asset pricing condition in Lemma 1 to hold, shadow banks must purchase T-bills from traditional banks, as illustrated in the third stage.

Proposition 1. *Consider a set of monetary policy rules $m(x)$ for a subset of equilibria in which money markets are not segmented and traditional banks are not liquidity satiated, $\mathcal{M}(x) \in \mathcal{S}^c \cap \mathcal{E}^c$. For any given x , an equilibrium with more reserves $m^*(x) > m^{**}(x)$ implies:*

- *less liquidity risk: $\bar{\psi}(x; m^*) = \psi(x; m^*) < \psi(x; m^{**}) = \bar{\psi}(x; m^{**})$,*
- *a lower premium on reserves: $r^i(x; m^{**}) - r^m(x; m^{**}) > r^i(x; m^*) - r^m(x; m^*)$,*
- *a lower premium on T-bills: $r^i(x; m^{**}) - r^b(x; m^{**}) > r^i(x; m^*) - r^b(x; m^*)$.*

Proposition 1 entails that an increase the supply of reserves reduces liquidity risk and liquidity premia. This effect appears when taking the partial derivative of the function for liquidity risk with respect to the policy rule for reserves:

$$\frac{\partial \psi(x; m)}{\partial m} = \frac{\partial \bar{\psi}(x; m)}{\partial m} = - \underbrace{\frac{\lambda \theta^m}{\eta + \bar{\eta}}}_{\text{more reserves}} + \underbrace{\frac{\lambda \theta^b}{\eta + \bar{\eta}}}_{\text{fewer T-bills}} < 0.$$

The effect of a change in the supply of reserve m to the liquidity of traditional banks has two parts. The first term is the higher holdings of reserves by traditional banks. The second term is the decrease in the supply of T-bills available to banks. The net effect is to decrease liquidity risk in both sectors as the liquidity provided by reserves is superior to the liquidity provided by T-bills. As a consequence of Lemma 1, increasing reserves also implies a rebalancing of liquidity across the two banking sectors: Since traditional banks hold more reserves, they sell T-bills to shadow banks such that liquidity risk is perfectly shared.

Figures 5a and 5b illustrate that result. Liquidity risk in the two banking sectors is monotonically decreasing in the quantity of reserves up to the threshold m^S , defined

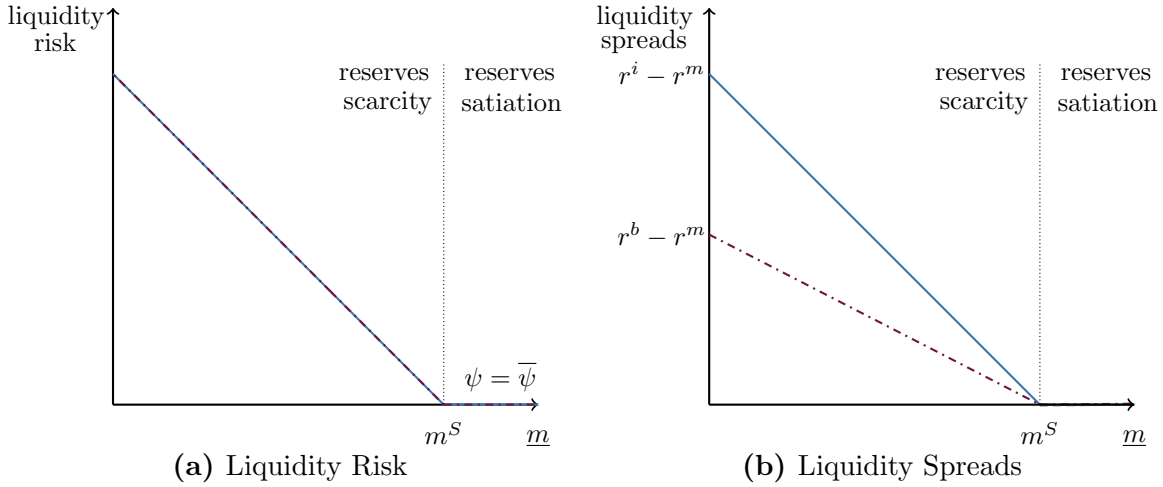


Figure 5: Reserves Supply in Integrated Markets

as the point at which the liquidity risk of traditional banks reaches zero. From this point onward, liquidity risk does not depend on the supply of reserves. Figure 5b displays liquidity spreads between the illiquid rate and interest on reserves, and the T-bill rate and interest on reserves as decreasing functions of the supply of reserves. Since the marginal value of holding liquid assets is proportional to exposure to liquidity risk, liquidity premia on reserves and T-bills must drop as a reaction to a larger supply of reserves.

4.2 Segmented Money Markets

In this section, I analyze how monetary policy affects the economy in equilibrium with money market segmentation. In this region, traditional banks do not have any T-bills to sell to shadow banks. Hence, they cannot intermediate the liquidity received from additional reserves. In this case, the two banking sectors face a different exposure to liquidity risk. This divergence leads to unrealized gains from risk-sharing, as shadow banks have larger marginal benefits of holding liquid assets.

Lemma 2. *In an equilibrium in which money markets are segmented, $\mathcal{M}(x) \in \mathcal{S}$, shadow banks have a larger exposure to liquidity risk per unit of wealth than traditional*

banks:

$$\psi(x) < \bar{\psi}(x).$$

Markets for liquid assets are segmented, as shadow banks hold all of the supply of T-bills, and traditional banks hold all of the supply of reserves. In this case, the marginal value of T-bills for shadow banks determines the liquidity premium on T-bills, while the marginal value of reserves for traditional banks determines the liquidity premium on reserves. The pricing factor of the two assets is therefore disconnected, and the supply of reserves only matters for the liquidity premium on reserves, while the supply of T-bills only matters for the liquidity premium on T-bills. The following proposition characterizes how open market operations affect liquidity risk and liquidity premia when money markets are segmented.

Proposition 2. *Consider a set of monetary policy rules $m(x)$ for a subset of equilibria in which money markets are segmented and traditional banks are not liquidity satiated, $\mathcal{M}(x) \in \mathcal{S} \cap \mathcal{E}^c$. For any given x , an equilibrium with more reserves $m^* > m^{**}$ implies:*

- *less liquidity risk for traditional banks: $\psi(x; m^*) < \psi(x; m^{**})$,*
- *more liquidity risk for shadow banks: $\bar{\psi}(x; m^*) > \bar{\psi}(x; m^{**})$,*
- *a lower premium on reserves: $r^i(x; m^{**}) - r^m(x; m^{**}) > r^i(x; m^*) - r^m(x; m^*)$,*
- *a larger premium for T-bills: $r^i(x; m^{**}) - r^b(x; m^{**}) < r^i(x; m^*) - r^b(x; m^*)$.*

The disconnection in the pricing kernel of the two liquid assets also appears when taking the partial derivative of liquidity risk exposure with respect to the supply of reserves.

$$\frac{\partial \psi(x; m)}{\partial m} = -\frac{\lambda \theta^m}{\eta} < 0, \quad \frac{\partial \bar{\psi}(x; m)}{\partial m} = \frac{\lambda \theta^b}{\bar{\eta}} > 0.$$

In an expansionary open market operation, liquidity risk decreases for traditional banks but increases for shadow banks. From the previous proposition, we can infer

that the quantity of reserves may shift an equilibrium from a nonsegmented to a segmented region. As a reaction to an increase in reserves, traditional banks sell T-bills to shadow banks up to fully depleting their portfolios and hitting their nonnegativity constraint. From this point on, the constraint becomes binding, and the economy enters the segmented markets regime. I define this threshold as m^T and characterize a set of equilibria for which the condition $0 < m^T < m^S$ holds.

Proposition 3. *Consider a set of monetary policy rules $m(x)$ for a subset of equilibria for which $m^T < m^S$. That is, traditional banks are satiated only when markets are segmented. For any given x , liquidity risk can be expressed as a function of the monetary policy rules as:*

$$\psi(x; m) = \begin{cases} \lambda \left(\sigma^d \frac{1-\eta-\bar{\eta}}{\eta+\bar{\eta}} - m \frac{\theta^m - \theta^b}{\eta+\bar{\eta}} - b \frac{\theta^b}{\eta+\bar{\eta}} \right), & \text{if } m < m^T \\ \lambda \left(\sigma^d \frac{\gamma}{\eta} (1 - \eta - \bar{\eta}) - m \frac{\theta^m}{\eta} \right), & \text{if } m^T < m < m^S \\ 0, & \text{if } m^S < m \end{cases}$$

$$\bar{\psi}(x; m) = \begin{cases} \lambda \left(\sigma^d \frac{1-\eta-\bar{\eta}}{\eta+\bar{\eta}} - m \frac{\theta^m - \theta^b}{\eta+\bar{\eta}} - b \frac{\theta^b}{\eta+\bar{\eta}} \right), & \text{if } m < m^T \\ \lambda \left(\sigma^d \frac{1-\gamma}{\eta} (1 - \eta - \bar{\eta}) - b \frac{\theta^b}{\eta} + m \frac{\theta^b}{\eta} \right), & \text{if } m^T < m < m^S \\ \lambda \left(\sigma^d \frac{1-\gamma}{\eta} (1 - \eta - \bar{\eta}) - b \frac{\theta^b}{\eta} + m \frac{\theta^b}{\eta} \right), & \text{if } m^S < m. \end{cases}$$

Liquidity risk exposures are continuous functions of the supply of liquid assets. Figure 6a provides a graphical representation of these relations between the supply of reserves, liquidity risk, and liquidity spreads. When the supply of reserves is below the threshold m^T , the equilibrium is similar to the previous section without market segmentation: An increase in the supply yields a decrease in liquidity risk for both sectors. Starting from m^T , further injections of reserves improve the liquidity risk of the traditional banks at the expense of shadow banks. As T-bills are the only asset that shadow banks can own to mitigate liquidity risk, an open market operation that removes T-bills deteriorates their liquidity position. From the satiation threshold m^S onward, the liquidity risk of traditional banks reaches the constant zero. Proceeding

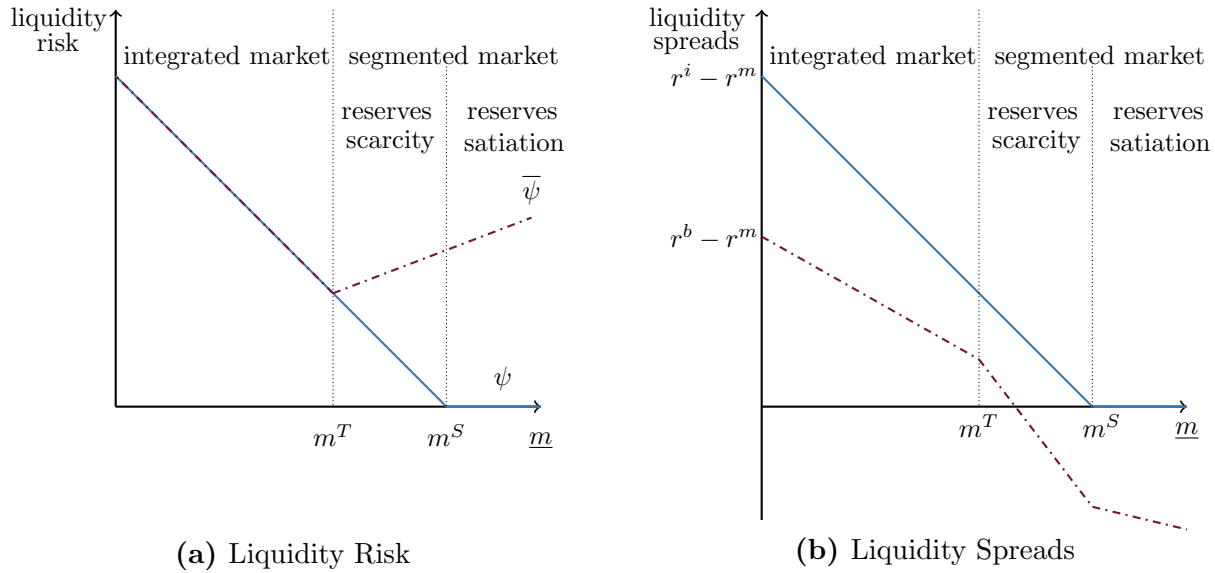


Figure 6: Reserves Supply in Segmented Markets

to further open market operations after this point increases the liquidity risk of shadow banks without any benefits for traditional banks.

Figure 6b describes the effect of an increase in reserves supply by an open market operation on liquidity spreads. Up to the threshold m^T , markets are integrated, and an increase in reserves supply lowers liquidity spreads. After the threshold m^T , the liquidity premium on reserves still decreases to reflect the reduction in the liquidity risk of traditional banks. In contrast, since the liquidity risk of shadow banks is rising, the liquidity premium on T-bills has to increase. This effect translates into the blue line moving away from the red line. For a large enough supply of reserves, the T-bill rate moves below the interest on reserves to reflect a larger liquidity premium on T-bills than on reserves.

4.3 Monetary Policy and T-Bill Supply

In this section, I investigate the implementation of targeting inflation by the central bank. When the central bank decides on the interest on reserves, there is a degree

of freedom in its target function. This feature means that several monetary policy frameworks are feasible. When money markets are segmented, the central bank does not have to offset changes in the supply of T-bills to stabilize inflation. This result implies that the liquidity premium on T-bills reacts to the supply of T-bills if and only if money markets are segmented.

Monetary Policy Implementation As demonstrated in previous sections, the central bank is able to manage the liquidity premium on reserves by controlling the supply of liquid assets. However, the central bank has a second tool in the model as it also decides on the nominal interest to pay on reserves i^m . This excess in the number of policy tools gives rise to a degree of freedom in the objective function of the central bank.

Lemma 3. *For any monetary policy rule couple $\{m(x), i^m(x)\}$ able to implement a given inflation target π^* : $\pi(x; i^m(x), m(x)) = \pi^*$, $\forall x \in [0, 1] \times [0, 1] \times \mathbb{R}^+$, there exists a linear combination of $m(x)$ and $i^m(x)$ able to implement the same target π^* .*

This result appears when substituting for the liquidity premium on reserves in the definition of the nominal rate on reserves $i^m(x) = r^m(x) + \pi(x)$. Doing so yields the following Fischer equation:

$$\pi(x) = \underbrace{i^m(x) + \theta^m \psi(x)}_{\text{nominal illiquid rate}} - r^i(x). \quad (18)$$

As both the nominal interest on reserves $i^m(x)$ and the liquidity premium $\psi(x)$ are in the control of the central bank, there is one degree of freedom in implementing a given inflation target. For instance, in the post-crisis period,¹³ monetary policy is implemented by varying the nominal interest on reserves. In the model, this case corresponds to an equilibrium in which banks are fully liquidity satiated, such that

¹³In the post-crisis period, monetary policy has been operated in a floor regime. This implementation regime corresponds to the traditional assumption of Neo-Keynesian models that monetary policy is implemented by varying the nominal rate without any role for the supply of money (see [Woodford, 2003](#)).

liquidity risk is zero for traditional banks ($\psi(x) = 0$). In contrast, in the pre-crisis period, interest on reserves was fixed to a nominal zero rate, as the Fed did not have the authority to pay any interest on reserves. In this case, the central bank has to vary the liquidity premium on reserves by restricting the supply of liquidity available to banks through open market operations ($\psi(x) > 0$).

Central Bank Reaction Function An increase in T-bills boosts the net supply of liquid assets and, in turn, decreases liquidity risk. If the central bank does not react, the nominal rate on illiquid assets must decrease to reflect the lower marginal value of liquid assets. Hence, to keep inflation on target, the central bank has to sterilize the surge in liquidity created by the Treasury by offsetting open market operations.

Proposition 4. *Consider an equilibrium in which money markets are not segmented and traditional banks are not liquidity satiated. Any policy rule $\{m(x), i^m(x)\}$ such that interest paid on reserves is constant $i^m(x) = i^m$ and implementing a constant inflation target π^* fully neutralizes the effect of a change in T-bills: $\mathcal{M}(\eta, \bar{\eta}, b^*) = \mathcal{M}(\eta, \bar{\eta}, b^{**})$ for any b^* and b^{**} such that $\mathcal{M}(x) \notin \mathcal{S} \cup \mathcal{E}$.*

The intuition about this proposition is that when money markets are integrated, equation (18) can be written as

$$\pi(x) = i^m + \underbrace{\theta^m \lambda^2 \left(\sigma^d \frac{1 - \eta - \bar{\eta}}{\eta + \bar{\eta}} - m \frac{\theta^m - \theta^b}{\eta + \bar{\eta}} - b \frac{\theta^b}{\eta + \bar{\eta}} \right)}_{\lambda \psi(x)} - r^i(x). \quad (19)$$

When the nominal interest on reserves i^m is held constant, the liquidity risk of traditional banks $\psi(x)$ will drop when b increases. If the central bank wants to keep inflation to a target, it needs to adjust the supply of reserves m to prevent $\psi(x)$ from falling. More precisely, to keep liquidity risk $\psi(x)$ constant, the central bank follows the reaction function:

$$dm_t = -\frac{\theta^b}{\theta^m - \theta^b} db_t.$$

This reaction function implies that the central bank will decrease the amount of reserves available to banks to offset any exogenous increase in T-bills. As a side product of the withdrawal of reserves, the central bank increases the supply of T-bills available to banks. As T-bills are less liquid than reserves, the net effect of these operations is to decrease the amount of aggregate liquidity and bring liquidity risk back to its initial position. A similar analysis is applied to the case in which banks are fully satiated with reserves.

Proposition 5. *Consider an equilibrium in which traditional banks are liquidity satiated and money markets are not segmented. The supply of T-bills does not affect the equilibrium: $\mathcal{M}(\eta, \bar{\eta}, b^*) = \mathcal{M}(\eta, \bar{\eta}, b^{**})$ for any b^* and b^{**} such that $\mathcal{M}(x) \in \mathcal{S}^c \cap \mathcal{E}$.*

When monetary policy has reached a floor, and liquidity risk is zero in both banking sectors ($\psi_t = \bar{\psi}_t = 0$), changes in the supply of T-bills are completely inconsequential such that the central bank does not have to proceed to offsetting open market operations.

With segmented markets, the liquidity premium on T-bills is disconnected from the liquidity premium on reserves. As the central bank implements its policy target through this liquidity premium on reserves, a change in the supply of T-bills does not have to be sterilized. This result is formalized in Proposition 6 and Corollary 1.

Proposition 6. *When money markets are segmented, the supply of T-bills does not affect the liquidity premium on reserves: $\lambda\theta^m\psi(\eta, \bar{\eta}, b^*) = \lambda\theta^m\psi(\eta, \bar{\eta}, b^{**})$ for any b^* and b^{**} such that $\mathcal{M}(x) \in \mathcal{S}$.*

Corollary 1. *When money markets are segmented, the central bank keeps inflation on target with a policy rule $\{m(x), i^m(x)\}$ that does not react to the supply of T-bills.*

5 Limited Arbitrage and Reverse Repo

This section extends the model to account for further essential determinants of the pricing of money market instruments in the data. The first addition is to allow

traditional banks to issue a liquid asset that may be held by shadow banks, such as an overnight repurchase agreement. The second addition is a cost for traditional banks to extend their balance sheets to produce this liquid asset. This cost creates a locus in which it is not profitable for traditional banks to intermediate liquidity. In this region, money markets are segmented, and the conclusions of the previous section hold. Third, the central bank is also able to directly provide liquid assets to shadow banks by setting up a reverse repo facility at the policy rate of its choice. This last addition allows the central bank to set a lower bound on money market yields.

5.1 Liquidity Intermediation with a Costly Balance Sheet

I relax the assumption that banks do not issue liquid assets to shadow banks by defining f_t , an additional asset issued by banks with liquidity services $\theta^f < \theta^b < \theta^m$. Holding this asset, with a positive portfolio weight $w_t^f > 0$, decreases liquidity risk; while issuing this asset, with a negative portfolio weight $w_t^f < 0$, increases liquidity risk. Moreover, to create the liquid asset, banks have to expand their balance sheets. As balance sheet space is costly due to regulation, so is creation of liquid assets.

Traditional Banks Traditional bankers maximize their lifetime expected logarithmic utility:

$$\max_{\{w_\tau^s \geq 0, w_\tau^b \geq 0, w_\tau^m \geq 0, w_\tau^d \geq 0, w_\tau^i, w_\tau^f, c_\tau\}_{\tau=t}^\infty} \mathbb{E}_t \left[\int_t^\infty e^{-\rho\tau} \log(c_\tau) d\tau \right], \quad (20)$$

subject to the law of motion of wealth:

$$\begin{aligned} dn_t = & \left(w_t^s \mu_t^s + w_t^i r_t^i + w_t^b r_t^b + w_t^m r_t^m + w_t^f r_t^f - w_t^d r_t^d - c_t + \chi_t^- w_t^f + \mu_t^\tau \right) n_t dt \\ & + w_t^s \sigma_t^s n_t dZ_t + \lambda \max \left\{ \sigma_t^d w_t^d - \theta^m w_t^m - \theta^f w_t^f - \theta^b w_t^b, 0 \right\} n_t d\tilde{Z}_t, \end{aligned} \quad (21)$$

where $\chi_t^- = \chi$ if $w_t^f < 0$ and $\chi_t^- = 0$ otherwise, and the balance sheet constraint:

$$w_t^s + w_t^i + w_t^b + w_t^m + w_t^f = 1 + w_t^d + \tau_t. \quad (22)$$

In this updated problem, banks have to pay a cost $\chi_t w_t^f$ to issue this liquid interbank asset. I refer to [Duffie and Krishnamurthy \(2017\)](#) for a micro-foundation for this balance sheet space cost as originating from a regulatory leverage ratio that compels banks to issue outside equity to households when expanding their balance sheet. Issuing outside equity to households is costly for bankers, because it generates a debt-overhang effect as in [Myers \(1977\)](#). I discuss the interpretation of this cost in the next section.

Shadow Banks Shadow banks also trade this liquid interbank asset and maximize their lifetime logarithmic utility function:

$$\max_{\{\bar{w}_\tau^s \geq 0, \bar{w}_\tau^b \geq 0, \bar{w}_\tau^d \geq 0, \bar{w}_\tau^f, \bar{w}_\tau^i, \bar{c}_\tau\}_{\tau=t}^\infty} \mathbb{E}_t \left[\int_t^\infty e^{-\rho\tau} \log(\bar{c}_\tau) d\tau \right], \quad (23)$$

subject to the law of motion of wealth:

$$\begin{aligned} d\bar{n}_t = & \left(\bar{w}_t^s \mu_t^s + \bar{w}_t^i r_t^i + \bar{w}_t^b r_t^b + \bar{w}_t^f r_t^f - \bar{w}_t^d r_t^d - \bar{c}_t + \bar{\mu}_t^r \right) \bar{n}_t dt \\ & + \bar{w}_t^s \sigma_t^s \bar{n}_t dZ_t + \lambda \max\{\sigma^d \bar{w}_t^d - \theta^f \bar{w}_t^f - \theta^b \bar{w}_t^b, 0\} \bar{n}_t d\tilde{Z}_t, \end{aligned} \quad (24)$$

and the balance sheet constraint:

$$\bar{w}_t^s + \bar{w}_t^i + \bar{w}_t^b + \bar{w}_t^f = 1 + \bar{w}_t^d + \bar{\tau}_t. \quad (25)$$

I assume that shadow banks do not face any cost when issuing liquid assets. The set of first-order conditions for traditional banks is amended by adding:

$$r_t^i - r_t^f = \begin{cases} \lambda \theta^f \psi_t & \text{when } w_t^f > 0, \\ \lambda \theta^f \psi_t + \chi_t & \text{when } w_t^f < 0. \end{cases} \quad (26)$$

When traditional banks have a long position in the liquid interbank asset ($w_t^f > 0$), the marginal cost is the liquidity premium on this asset on the left-hand side, while the marginal benefit is the marginal reduction in liquidity risk on the right-hand side. As the cost of balance-sheet space is proportional to the value of w_t^f and arises only when issuing the liquid interbank asset, it has a positive impact on the marginal cost only for a short position. The set of first-order conditions for shadow banks is modified by adding:

$$r_t^i - r_t^f = \lambda \theta^f \bar{\psi}_t. \quad (27)$$

As shadow banks do not face any balance-sheet cost when holding or issuing the interbank liquid asset, they are always marginal in the market for this asset such that equation (27) holds with equality.

5.2 Reverse Repo Facility

So far, we have assumed that the central bank does not care about liquidity premia on non-reserve money market assets. The evolution of the post-crisis monetary policy framework of the Fed suggests that this assumption is not fully accurate. In September 2013, the Fed opened a Reverse Repo Facility (RRP) with a policy rate determined by the FOMC through which shadow banking institutions can lend to the Fed against eligible collateral. While the instrument was originally limited in quantities in an experimental phase, the limits were eventually removed to constitute a fixed-rate full-allotment facility.¹⁴

This policy translates, in the model, into the central bank standing ready to supply

¹⁴According to the Fed’s website, the overnight reverse repo facility “operate[s] similarly to the way the Federal Reserve’s payment of interest on excess reserves works for depository institutions. Absent other constraints, any counterparty that is eligible to participate in the ON RRP facility should generally be unwilling to invest funds overnight with another counterparty at a rate below the facility rate. The effectiveness of the facility will depend on a range of factors, including whether a sufficiently broad set of counterparties has access to the facility, the costs associated with regulatory and balance sheet constraints, and the level of competition in the money markets.” https://www.newyorkfed.org/markets/rrp_faqs_140113.html, retrieved on July 30, 2019

a non-reserve liquid asset elastically to shadow banks at a fixed policy rate. These operations put an effective lower bound on the liquid asset rate, as quantities of liquid assets would sufficiently adjust. We capture this feature by updating the second item in Definition 2 by adding the following market-clearing condition for the liquid interbank asset:

$$(g) \text{ liquid interbank loans: } w_t^f \eta_t + \bar{w}_t^f \bar{\eta}_t = f_t.$$

The net position of the whole banking sector in the liquid interbank asset is equal to the quantity of liquid interbank assets supplied by the central bank given at the facility f_t . The balance sheet identity of the central bank is therefore updated to

$$\underline{b}_t = m_t + f_t.$$

In a fixed-rate full-allotment facility, supply must elastically adjust to demand. The expression for the movement in quantities is given by

$$f_t = \max \left\{ \frac{\underline{r}_t^f - r_t^i}{\theta^f \lambda} \bar{\eta}_t + \sigma^d \bar{w}_t^d \bar{\eta}_t + \theta^f w_t^f \eta_t - \theta^b \bar{w}_t^b \bar{\eta}_t, 0 \right\}. \quad (28)$$

If the central bank borrows whatever quantity is necessary to prevent the liquid interbank rate from dropping below the policy rate, \underline{r}^f , it does not lend at this rate. This asymmetry is captured by the maximum in equation (28). Since the reverse repo facility is a floor, quantity adjustments are asymmetric and stop whenever rates rise above the targeted floor. When the market rate is strictly above the policy floor $r_t^f > \underline{r}_t^f$, there is no demand, and volumes at the facility drop to zero.

5.3 T-bill Supply and Yields in the Extended Model

This section examines how the supply of T-bills affects yields and liquidity premia in four polar cases when traditional banks are liquidity satiated. In the first, balance sheet space is costless, and money markets are always integrated. In the second,

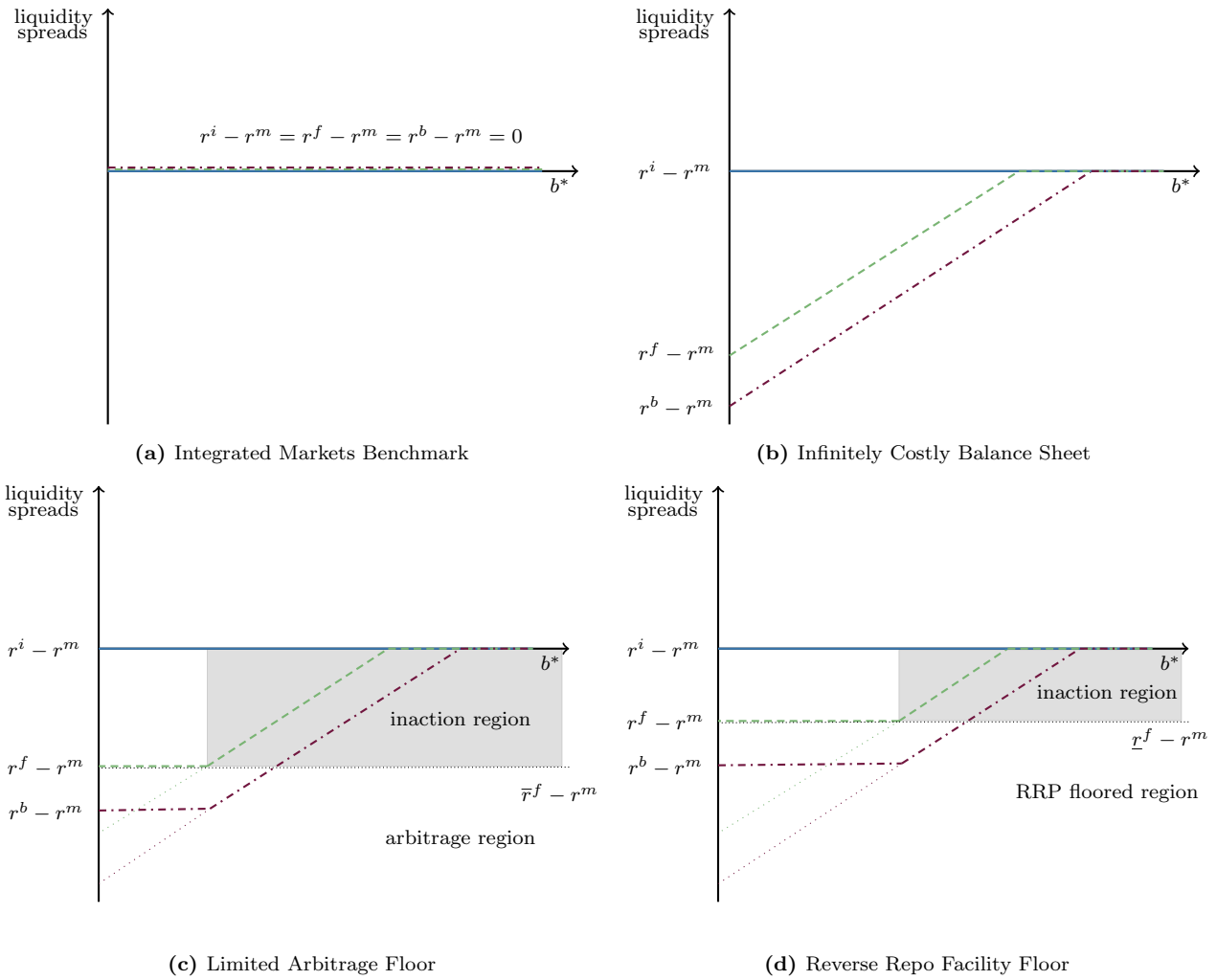


Figure 7: Extended Model Dynamics under Reserves Satiation

balance sheet space is infinitely costly, and banks always refrain from intermediating liquidity. In the third, the balance sheet cost is finite, and regulation creates a region in which traditional banks do not find it profitable to increase their balance sheets. In the last, the central bank tops up the floor created by arbitrage opportunities from traditional banks by operating a reverse repo facility.

Costless Balance Sheet When arbitrage is costless, traditional banks can always profitably intermediate liquidity to shadow banks, and thus there is no market segmentation. In this case, the repartition between T-bills and the interbank liquid asset holdings from banks is indeterminate.

Proposition 7. *In an economy in which balance sheet space is costless ($\chi = 0$), the portfolio weights $w_t^b, w_t^f, \bar{w}_t^b$, and \bar{w}_t^f are jointly indeterminate.*

According to Proposition 7, it is the overall exposure to liquidity risk that matters for banks. As T-bills and liquid loans are perfect substitutes, the same distribution of liquidity risk between the two sectors can be achieved through various combinations of T-bills and interbank loans. Shadow banks could, for example, sell half of their T-bills to traditional banks and receive a similar amount of effective liquidity in the form of liquid loans from traditional banks without impacting any other variables in the model. In contrast, traditional banks' portfolio weight on reserves is still determined by the market-clearing condition for reserves as traditional banks are the only agent that can hold reserves. Figure 7a illustrates liquidity spreads as a function of the supply of T-bills when traditional banks are liquidity satiated. The figure shows that all liquidity premia are zero, as traditional banks intermediate liquidity to shadow banks without any cost. Hence, a change in the supply of T-bills is neutral.

Infinitely Costly Balance Sheet When the cost of balance sheet space is infinitely high ($\chi \rightarrow \infty$), issuing an interbank liquid asset is always too costly for banks. Hence, the only way for traditional banks to trade liquidity is to sell T-bills to shadow banks. In this case, the model reverts to the one described in Section 4. Figure 7b illustrates that in this case, traditional banks are liquidity satiated, but shadow banks are not. The liquidity premium on reserves is, therefore, zero and independent of the supply of T-bills, while the liquidity premia on the liquid interbank rate and T-bills are decreasing functions of the supply of T-bills. Because the illiquid rate r_t^i is fixed to the interest on reserves r_t^m , the rates on T-bills r_t^b and liquid loans r_t^f must adjust upward for the liquidity premium to go down.

Limited Arbitrage Floor In the intermediate case in which the cost balance sheet space is finite ($0 < \chi < \infty$), there is an inaction region in which it is not profitable for traditional banks to issue liquid assets to shadow banks. Figure 7c illustrates this case. Inside the inaction region, the liquidity premium on the liquid interbank rate r_t^f is not large enough to compensate for the cost of increasing the size of one's balance sheet. Once the rate on liquid loans has reached the threshold \bar{r}_t^f , it becomes profitable for traditional banks to lend in the interbank market and benefit from an arbitrage trade between r_t^m and r_t^f . The existence of profitable arbitrage creates a floor, not only for the liquid interbank rate at the threshold \bar{r}_t^f , but also for the T-bill rate. The supply of liquid assets produced by traditional banks therefore matters for the pricing of all liquid assets that can be held by shadow banks.

Reverse Repo Facility Floor In the extended model, the central bank has the option to introduce its own floor by standing ready to borrow for a short maturity at a rate \underline{r}_t^f . When doing so, it creates any amount of liquid interbank loan necessary to prevent the rate from falling below this rate. Figure 7d illustrates this case when the floor set by the central bank is above the one created by the region of profitable arbitrage for traditional banks $\underline{r}^f > \bar{r}^f$. The region in which the spreads are upward sloping is therefore reduced, compared with Figure 7c.

6 Quantitative Analysis

This section proceeds to the quantitative analysis. I calibrate the effective lower bound on money market yields based on observed changes in regulation and in the reverse repo rate. I then estimate model-implied equations for liquidity premia. The model accurately predicts both movements in money market spreads and volumes of the reverse repo at the Fed. Eventually, I use the model to investigate counterfactual scenarios for alternative regulations and policies.

6.1 Identification Strategy

There are three challenges in estimating the liquidity premia equations. First, because the illiquid rate is not observed, these equations are not identified when traditional banks are not liquidity satiated. Using data on intraday overdrafts, I find evidence that traditional banks are liquidity satiated for the entire post-crisis sample. I use this evidence to identify liquidity premia by restricting the liquidity premium on reserves to zero, as predicted by the model in a satiation regime. Second, I calibrate the effective floor with observed regulatory constraints and the reverse repo rate. Third, a change in regulation in 2016 led to a sharp increase in T-bill holdings of money funds for reasons unrelated to liquidity management. I construct a measure for the supply of T-bills that takes this change in regulation into account.

Prevailing Regime As the illiquid rate is not observed, I rely on data on average daily overdrafts at the Fed to evaluate what is the prevailing liquidity regime. In the US, every traditional bank with an account at the Fed is part of a real-time gross settlement system called Fedwire. When a bank receives a negative funding shock, the transfer of deposits to another bank is made possible thanks to an intraday overdraft from the Fed. This *daylight* overdraft allows the bank to look for a new source of funding by the end of the day. In the micro-foundation of the liquidity risk problem, the temporary gap between assets and liabilities in the Section B is only possible thanks to this overdraft. The model predicts that in a regime in which banks are liquidity satiated, the quantity of liquid assets is large enough for traditional banks not to need any daily overdraft from the Fed.

Figure 8 displays the evolution of average volumes of daylight overdrafts provided by the Fed to traditional banks. The series shows a substantial drop in overdraft volumes during the crisis, lasting until 2019. From around \$60 billion in 2006, overdrafts have dropped to less than \$1 billion in the third quarter of 2011 and never recovered to more than \$4 billion since then. These elements suggest that traditional banks have been fully liquidity satiated for the whole 2010 to 2018 period. According to this evidence, I impose on my estimations the restriction that the liquidity premium

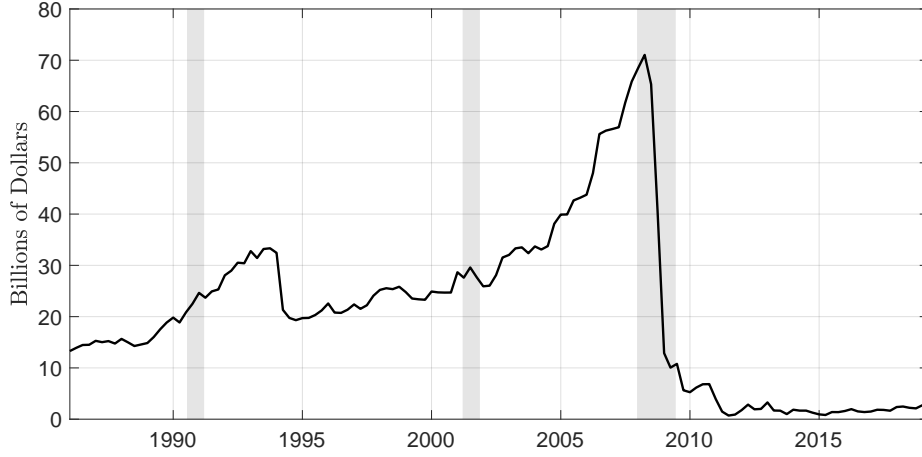


Figure 8: Average Daylight Overdraft Volume at the Fed. The figure displays the evolution of average volumes of intraday overdrafts from the Fed to depository institutions between 1986 and 2019. The series is retrieved from the Payment Systems section of the website of the Federal Reserve Board.

on reserves is zero for that period.

Effective Floor Calibration According to the model, the effective floor on the repo rate is the maximum between the floor set by banks' profitable arbitrage and the floor set by the Fed through the reverse repo facility:

$$\tilde{r}_t^f = \max\{\bar{r}_t^f, \underline{r}_t^f\}.$$

The reverse repo rate is a policy rate that is publicly announced by the Fed after each meeting of the FOMC. The limited-arbitrage floor is calibrated following [Duffie \(2018\)](#) to a 25 bps spread to the interest on reserves in the post-crisis period. This number reflects the combination of FDIC fees and the regulatory leverage ratio on both banks' and dealers' balance sheet.

Money Market Reform of 2016 One of the lessons of the 2008 financial crisis is the fragility of money funds, as epitomized by the run on the Reserve Primary Fund. In an attempt to lower these financial risks, the Securities and Exchange Commission (SEC) issued a new set of rules for money market mutual funds, which are usually referred to as the “Money Market Reform.” Announced in 2014 and due for implementation in October 2016, these new rules impose tighter restrictions on portfolio holdings, with an emphasis on liquidity and quality requirements. In particular, an additional requirement of the reform is to force prime funds to move from quoting a fixed parity of \$1 per share price to a floating net-asset-value system.

An important feature of the reform is that this additional rule does not affect a second category of money funds called *government-only*. These Government-only funds are still authorized to quote the \$1 fixed parity for their shares. As a consequence, the reform drastically reduced the appeal of prime money funds¹⁵ in favor of government-only funds and triggered an exodus of funds under management from the former to the latter (see Figure 9). Overall, prime funds lost over \$1 trillion during this period, as assets under management plunged from \$1.58 trillion in February 2016 to \$550 billion in December 2016. In the meantime, government-only funds gained nearly the same amount of funds in management. The shift took place as a combination of withdrawals from investors and prime funds being converted into government-only funds ahead of the reform.

In order to qualify as a government-only fund, a money market mutual fund must abide by a stricter set of rules. In particular, they are required to hold 80% of their assets as Treasuries, compared with 30% for prime funds. As a consequence, the shift from prime to government-only effectively drained hundreds of billions of T-bills for the sole purpose of being authorized to quote a \$1 fixed parity. Section 6.1 shows that from July 2015 to October 2016, the quantity of T-bills held by money market mutual funds doubled from around \$400 billion to around \$800 billion.

As the increase in T-bill holdings brought about by the reform is unrelated to the

¹⁵As argued by [Pozsar and Sweeney \(2015\)](#), by being required to float their net asset value, prime funds lost the “moneyness” of their liabilities that made them attractive to corporate treasuries and the cash desks of asset managers.

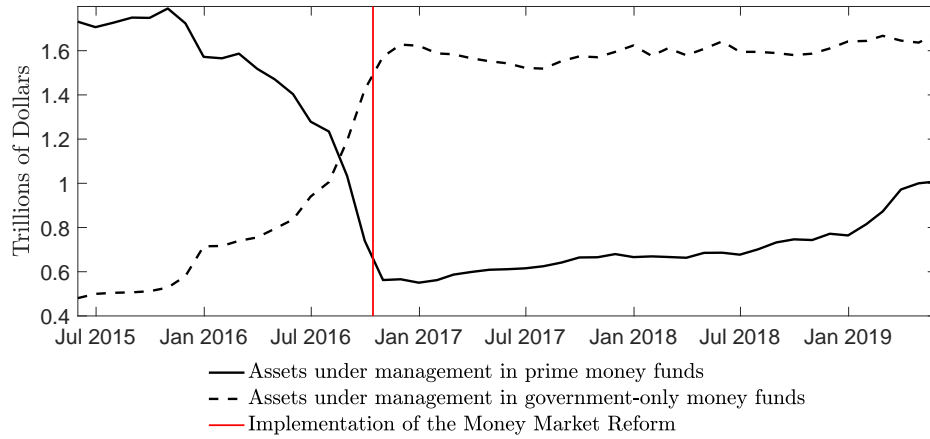


Figure 9: Assets Under Management in Money Funds The figure displays the total value of assets under management in prime (continuous line) and government-only money funds (dotted line) around the implementation of the money market reform in 2016 (vertical line). The two series are retrieved from the May 2019 Money Market Fund Statistics as published by the Securities and Exchange Commission.

liquidity management of shadow banks, I construct a measure of the effective T-bill supply available to shadow banks taking the drain into account. To do so, I track monthly flows from prime money funds to government-only money funds using data from the report of the SEC’s Division of Investment Management’s Analytics Office. I subtract 50 pp of these flows from the supply of T-bill to account for the differential in mandatory holdings (80% for government-only funds minus 30% for prime funds). This method yields an adjustment that stabilizes at around \$400 billion by the end of 2016—a magnitude that is consistent with the observed yearly increase in money fund’s T-bill holdings from the flow of funds data. I use this adjusted measure of T-bill supply in the remaining of the paper and provide more details about the effect of this adjustment in the counterfactual exercise in Section. In Section D, I also show that adding a dummy for the money market reform, as an alternative method, does not alter the results.

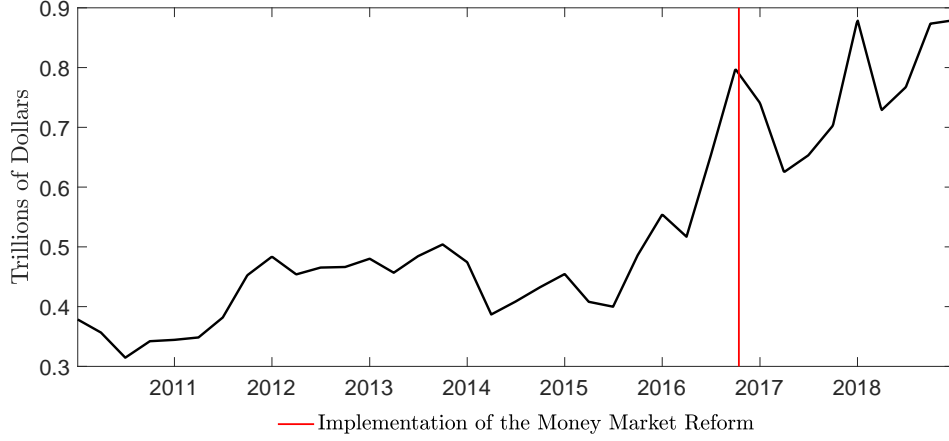


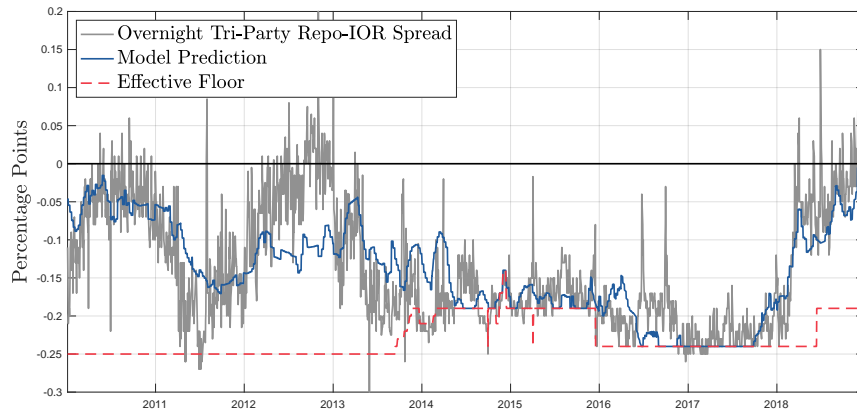
Figure 10: T-bills Held by Money Funds. The figure displays the total amount of T-bills held by money funds around the implementation of the money market reform in 2016. The series is retrieved from 2019:Q2 release of the flow of funds from the Federal Reserve Board.

6.2 Estimation

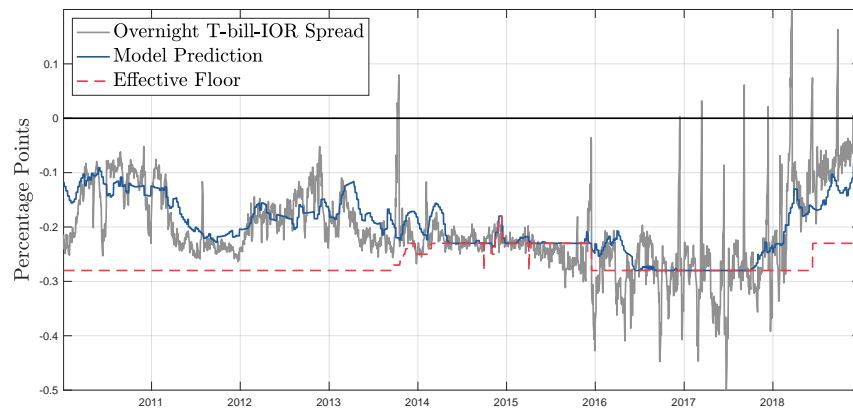
Repo Spread In the regime in which traditional banks are satiated with reserves, the interest on reserves r_t^m is equal to the illiquid rate r_t^i . I rewrite the liquidity premium on the liquid interbank asset from equation (27), interpreted as an overnight repo transaction, with the following standard (type 1) Tobit model:

$$r_t^m - r_t^f = \begin{cases} \alpha^f - \beta^f b_t & \text{when } r_t^f \geq \tilde{r}_t^f, \\ r_t^m - \tilde{r}_t^f & \text{when } r_t^f < \tilde{r}_t^f, \end{cases} \quad (29)$$

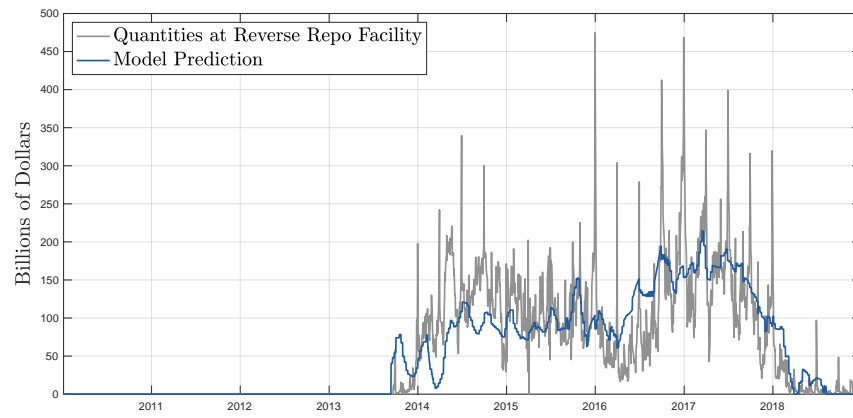
where $\alpha^f = \theta^f \lambda^2 \sigma^d (1-\gamma)(1-\eta-\bar{\eta})/\bar{\eta}$ and $\beta^f = \theta^f \lambda^2 \theta^b / \bar{\eta}$. The model predicts a linear censored relation between the supply of T-bills and the repo spread. I estimate this equation with $r_t^m - \tilde{r}_t^f$, the spread at the effective floor, as a lower censoring threshold by Maximum Likelihood. Table 5, column (1) presents the results of the regression, and Figure 11a plots the predictions of the model along with daily observed data.



(a) Model Predictions for the Spread between the Repo Rate and IOR



(b) Model Predictions for the Spread between the T-bill yield and IOR



(c) Model Predictions for Volumes at the Reverse Repo Facility

Figure 11: Model Predictions and Daily Observations

T-bill Spread The equation for the spread between the T-bill rate and the interest on reserves can be written in a similar way:

$$r_t^m - r_t^b = \begin{cases} \alpha^b - \beta^b b_t & \text{when } r_t^f \geq \tilde{r}_t^f, \\ r_t^m - \tilde{r}_t^f - \lambda(\theta^b - \theta^f)\bar{\psi}_t^* & \text{when } r_t^f < \tilde{r}_t^f, \end{cases} \quad (30)$$

where $\alpha^b = \theta^b \lambda^2 \sigma^d (1 - \gamma)(1 - \eta - \bar{\eta})/\bar{\eta}$, $\beta^b = \theta^b \lambda^2 \theta^b / \bar{\eta}$, and $\bar{\psi}_t^* = r_t^i - \tilde{r}_t^f / (\lambda \theta^f)$ is the liquidity risk of shadow banks at the floor. The effective floor for the T-bill-IOR spread is the floor on the repo spread adjusted by the difference in marginal liquidity benefits in the two instruments $-\lambda(\theta^b - \theta^f)\bar{\psi}_t^*$. I calibrate this adjustment to 3 bps by taking the mean of the spread between the overnight repo rate and the t-bill yield conditionally on the repo rate being equal to its effective floor. The censored model is estimated as for the repo spread equation by Maximum Likelihood with $r_t^m - \tilde{r}_t^f - 3$ bps as a lower censoring threshold. Table 5, column (2), reports the results of the regression and Figure 11b shows the fit of the model to observed data.

Reverse Repo Facility Volumes The model predicts that when the repo rates fall to the reverse repo facility rate, changes in the supply of T-bills show up as adjustments in quantities in the balance sheet of the Fed rather than in prices. I test this prediction by estimating the following censored equation adapted from equation (28) :

$$f_t = \begin{cases} \alpha^r - \beta^r b_t & \text{when } r_t^f \geq \tilde{r}_t^b, \\ 0 & \text{when } r_t^f < \tilde{r}_t^b \end{cases} \quad (31)$$

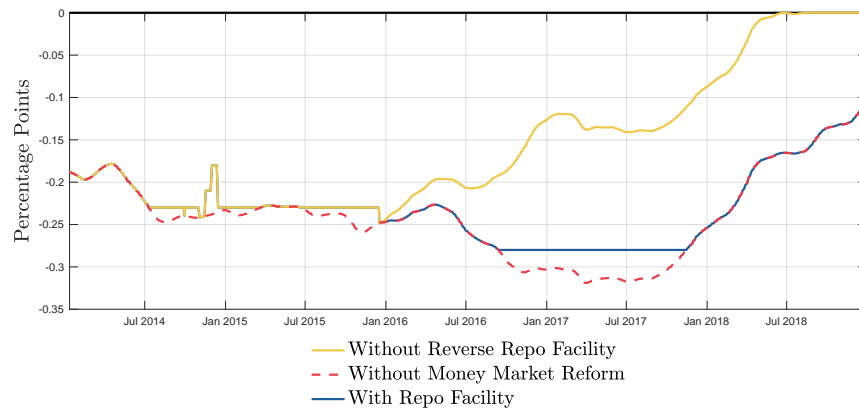
where $\alpha^r = \bar{\eta} ((\underline{r}^f - r^m) / (\theta^f \lambda^2) + \sigma^d \bar{w}^d + \theta^f w^f \eta / \bar{\eta})$ and $\beta^r = \theta^b$. The previous equation is estimated using the same Maximum Likelihood procedure as previously; results are reported in Table 5, column (3), while the fitted values of the model compared to observations is depicted in Figure 11c.

Analysis As depicted in Figure 11, the model accurately predicts most of the trend component for both short-term spreads and for reverse repo volumes. Despite the use of daily data for the stock of T-bills outstanding, the model is not able to capture high-frequency movements in these three series. In particular, the model does not predict the large spikes observed on quarter-end dates, which have been linked to a difference in reporting standards between European and American banks (Egelhof, Martin, and Zinsmeister, 2017). As the stock of T-bills is the only time-varying variable in the simple setting considered, the analysis points out to this variable as a significant driver of short-term yields. Notably, the steady increase in the stock of T-bills observed in 2018 corresponds to a significant increase in repo and T-bill yields and to a decline in reverse repo volumes up to zero. Through the lenses of the model, the increase in T-bill supply led to a decrease in the scarcity of liquid assets. Consequently, the liquidity premia on money market assets decreased, leading to an increase in yields.

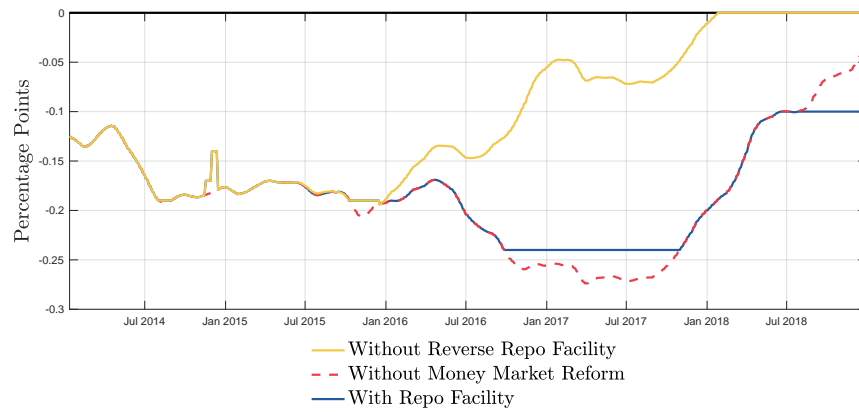
6.3 Counterfactual Exercise

In this section, I use the quantitative model to derive counterfactual estimates for three scenarios: (i) the absence of the money market reform, (ii) the absence of a reverse repo facility, and (iii) the creation of a repo facility at 10 bps below IOR by the Fed. This exercise allows us to infer the effects of these policies on short-term rates.

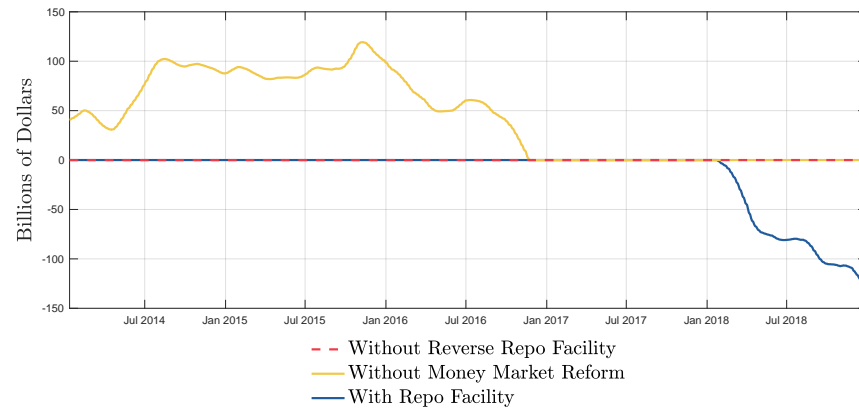
No Money Market Reform To evaluate the effect of the money market reform on the pricing of money market instruments, I derive a model-implied counterfactual using the supply of T-bills without the adjustment for the money market. The results are depicted by the yellow lines in Figure 12a. Without the money market reform draining around \$400 billion of T-bills, the supply of T-bills would have been much higher. As a consequence, yields on T-bills and repo transactions would have already reached the interest on reserves at the beginning of 2018. Consequently, the volumes of reverse repo from the Fed would have dropped to zero as early as October 2016.



(a) Spread between the Repo Rate and IOR



(b) Spread between the T-bill yield and IOR



(c) Volumes at the Reverse Repo and Repo Facility

Figure 12: Counter Factual Analysis

No Reverse Repo Facility Floor I proceed in a similar way to build a counterfactual for an economy in which the Fed does not set a floor on repo rates. The results are depicted by the red lines in Figure 12b. As a consequence of the drop in the supply of T-bills in 2016, yields on repo transactions and T-bills drift further below the IOR. At the bottom, yields on T-bills are 38 bps lower than the IOR.

Introducing a Repo Facility Ceiling Last, I also consider a scenario in which the Fed decides to impose a ceiling on the repo rate by standing ready to lend at a rate of 10 bps below the IOR. When doing so, the Fed acts in an opposite manner to the reverse repo facility: It removes valuable liquid assets for shadow banks (repo transactions) and adds worthless reserves. By adding to the scarcity, the Fed creates a floor on the liquidity premia on T-bills and repo transactions, which translates into a ceiling for repo rates. The results are depicted by the blue lines in Figure 12c. Repo volumes, depicted in negative numbers, start to grow when the yield on repo transactions reaches the 10 bps threshold as a consequence of the increase in the supply of T-bills.

7 Conclusion

The 2008 financial crisis revealed that the creation of liquidity assets by unregulated shadow banks generates financial risks. Since then, more stringent regulations have impaired the ability of banks to create liquidity and increased the burden of supplying outside money on public institutions. As a consequence, the interaction between regulatory constraints and monetary policy decisions has become critical for the pricing of short-term assets. This paper points out to the increasing role played by Treasury-created outside money when compared with Fed-created outside money in a world in which liquidity intermediation is costly. In addition, the analysis also raises questions about the ability of central banks to accommodate the liquidity needs of a broader set of financial institutions, with stark implications for financial stability and asset prices.

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Appendices

A Proofs

Lemma 1

If $\mathcal{M}(x) \notin \mathcal{S}$, then by definition, $r^i(x) - r^b(x) = \theta^b \psi(x)$. In that case, following the complementary slackness condition in (13), either $w^b(x) = 0$ or $w_t^b > 0$.

If $w^b(x) = 0$, since the supply of T-bills b is assumed to be strictly positive, we need that $\bar{w}_t^b > 0$ to satisfy the market clearing condition of T-bills. Thus, following the complementary slackness condition in (17), we have that $r^i(x) - r^b(x) = \theta^b \bar{\psi}(x)$. Thus, $\psi(x) = \bar{\psi}(x)$.

When $w_t^b > 0$, we also have two possible cases that satisfy the market clearing condition for T-bills:

- (1) $\bar{w}^b(x) > 0$: Following the complementary slackness condition in (17), we have that $r^i(x) - r^b(x) = \theta^b \bar{\psi}(x)$. Thus, $\psi(x) = \bar{\psi}(x)$.
- (2) $\bar{w}^b(x) = 0$: In that case, either $r^i(x) - r^b(x) = \theta^b \bar{\psi}(x)$ or $r^i(x) - r^b(x) > \theta^b \bar{\psi}(x)$. If $r^i(x) - r^b(x) = \theta^b \bar{\psi}(x)$, then $\psi(x) = \bar{\psi}(x)$. If $r^i(x) - r^b(x) > \theta^b \bar{\psi}(x)$, then it implies that $\bar{\psi}(x) < \psi(x)$. This condition can be reformulated as $\bar{w}^d(x)\sigma^d < w^d(x)\sigma^d - \theta^m w^m(x) - \theta^b w^b(x)$. Using the clearing market conditions, it is equivalent to:

$$(1 - \gamma)(1 - \eta - \bar{\eta})\sigma^d/\bar{\eta} < \gamma(1 - \eta - \bar{\eta})\sigma^d/\eta - \theta^m m(x)/\eta - \theta^b(b - m(x))/\eta,$$

which contradicts condition (C1).

Proposition 1

If $\mathcal{M}(x) \notin \mathcal{S}$, we have that $\psi(x) = \bar{\psi}(x)$ from Lemma 1. Since $\mathcal{M}(x) \notin \mathcal{E}$, $\psi(x) > 0$. From the definition of $\psi(x)$ and the market clearing condition for reserves, it directly follows that $\bar{\psi}(x; m^*) = \psi(x; m^*) < \psi(x; m^{**}) = \bar{\psi}(x; m^{**})$.

Since $\psi(x; m^*) < \psi(x; m^{**})$ and $m^* > 0$, using the first-order condition (14) yields that $r^i(x; m^{**}) - r^m(x; m^{**}) > r^i(x; m^*) - r^m(x; m^*)$.

Finally, since $b > 0$ and both $\psi(x; m^*) < \psi(x; m^{**})$ and $\bar{\psi}(x; m^*) < \bar{\psi}(x; m^{**})$, using the first-order conditions (13) and (17) and the market clearing condition for reserves yields that $r^i(x; m^{**}) - r^b(x; m^{**}) > r^i(x; m^*) - r^b(x; m^*)$.

Lemma 2

If $\mathcal{M}(x) \in \mathcal{S}$, then by definition, $r^i(x) - r^b(x) > \theta^b \psi(x)$. In that case, following the complementary slackness condition in (13), $w^b(x) = 0$.

If $w^b(x) = 0$, since $b > 0$, we need that $\bar{w}_t^b > 0$ to satisfy the market clearing condition of T-bills. Thus, following the complementary slackness condition in (17), we have that $r^i(x) - r^b(x) = \theta^b \bar{\psi}(x)$. Thus, $\psi(x) < \bar{\psi}(x)$.

Proposition 2

From Lemma 2, since $\mathcal{M}(x) \in \mathcal{S}$, we have that $\psi(x) < \bar{\psi}(x)$. Since $\mathcal{M}(x) \notin \mathcal{E}$, $\psi(x) > 0$. From the definition of $\psi(x)$ and the market clearing condition for reserves, it directly follows that $\psi(x; m^*) < \psi(x; m^{**})$.

By definition and since $\bar{\psi}(x; m) > 0$, we have that $\bar{\psi}(x; m) = \lambda(\bar{w}^d(x)\sigma^d - \theta^b \bar{w}^d(x))$. If $\mathcal{M}(x) \in \mathcal{S}$, then by definition, $r^i(x) - r^b(x) > \theta^b \psi(x)$. In that case, following the complementary slackness condition in (13), $w^b(x) = 0$. Using the market clearing condition for T-bills and shadow bank deposits and the budget constraint of the central bank, we can write $\bar{\psi}(x; m) = \lambda((1 - \gamma)(1 - \eta - \bar{\eta})\sigma^d - \frac{\theta^b(b-m)}{\bar{\eta}})$. Thus, $\bar{\psi}(x; m^*) > \bar{\psi}(x; m^{**})$.

Since $\psi(x; m^*) < \psi(x; m^{**})$ and $m^* > 0$, using the first-order condition (14) yields that $r^i(x; m^{**}) - r^m(x; m^{**}) > r^i(x; m^*) - r^m(x; m^*)$.

Finally, since $\psi(x; m^*) < \psi(x; m^{**})$ and $b > 0$, using the first-order condition (17) yields that $r^i(x; m^{**}) - r^b(x; m^{**}) < r^i(x; m^*) - r^b(x; m^*)$.

Proposition 3

Using the budget constraint for the central bank and the market clearing conditions for deposits, T-bills, and reserves, we get

$$\begin{aligned}\psi(x; m) &= \lambda \left(\sigma^d \frac{\gamma}{\eta} (1 - \eta - \bar{\eta}) - \frac{\theta^m m}{\eta} - \theta^b \frac{b - \bar{w}^b(x) \bar{\eta} - m}{\eta} \right), \\ \bar{\psi}(x; m) &= \lambda \left(\sigma^d \frac{(1 - \gamma)}{\bar{\eta}} (1 - \eta - \bar{\eta}) - \theta^b \bar{w}^b(x) \right).\end{aligned}$$

If $m < m^T$, then $\mathcal{M}(x) \notin \mathcal{S}$ and $\psi(x; m) = \bar{\psi}(x; m)$ from Lemma 1. Thus, we can solve for $\bar{w}^b(x)$ and get

$$\psi(x; m) = \bar{\psi}(x; m) = \sigma^d \frac{1 - \eta - \bar{\eta}}{\eta + \bar{\eta}} - m \frac{\theta^m - \theta^b}{\eta + \bar{\eta}} - \theta^b \frac{b}{\eta + \bar{\eta}}.$$

If $m^T < m < m^S$, then $\mathcal{M}(x) \in \mathcal{S} \cap \mathcal{E}^c$. If $\mathcal{M}(x) \in \mathcal{S}$, then by definition, $r^i(x) - r^b(x) > \theta^b \psi(x)$. In that case, following the complementary slackness condition in (13), $w^b(x) = 0$. Since $\mathcal{M}(x) \notin \mathcal{E}$, $\psi(x) > 0$. Thus, using the market clearing condition for T-bills and reserves, we get

$$\begin{aligned}\psi(x; m) &= \lambda \left(\sigma^d \frac{\gamma}{\eta} (1 - \eta - \bar{\eta}) - m \frac{\theta^m}{\eta} \right), \\ \bar{\psi}(x; m) &= \lambda \left(\sigma^d \frac{1 - \gamma}{\bar{\eta}} (1 - \eta - \bar{\eta}) - \theta^b \frac{b - m}{\bar{\eta}} \right).\end{aligned}$$

If $m^S < m$, then $\mathcal{M}(x) \in \mathcal{S} \cap \mathcal{E}$. If $\mathcal{M}(x) \in \mathcal{S}$, then by definition, $r^i(x) - r^b(x) > \theta^b \psi(x)$. In that case, following the complementary slackness condition in

(13), $w^b(x) = 0$. Since $\mathcal{M}(x) \in \mathcal{E}$, then by definition $\psi(x; m) = 0$. Thus, using the market clearing condition for T-bills, we get

$$\bar{\psi}(x; m) = \lambda \left(\sigma^d \frac{1 - \gamma}{\bar{\eta}} (1 - \eta - \bar{\eta}) - \theta^b \frac{b - m}{\bar{\eta}} \right).$$

Lemma 3

If $\{m(x), i^m(x)\}$ is able to implement a given inflation target π^* , that means that:

$$i^m(x) = r^m(x; m) + \pi^*.$$

For a linear combination of $\{m^*(x), i^{m,*}(x)\} \neq \{m(x), i^m(x)\}$ to be able to also implement π^* , we need to show that there exists $m^* \neq m$ such that

$$\left. \frac{\partial r^m(x; m)}{\partial m} \right|_{m=m^*} \neq 0.$$

Given condition ??, it is always possible to pick a m^* sufficiently small such that $\mathcal{M}(x) \notin \mathcal{S}$, that is $m^* < m^T$. Solving for the equilibrium prices, we find that:

$$r^m = \frac{a}{q} + \Phi - \frac{\sigma^2}{\eta + \bar{\eta}} - \sigma^\tau \sigma - \theta^m \lambda \left(\sigma^d \frac{1 - \eta - \bar{\eta}}{\eta + \bar{\eta}} - m \frac{\theta^m - \theta^b}{\eta + \bar{\eta}} - b \frac{\theta^b}{\eta + \bar{\eta}} \right),$$

and

$$\frac{\partial r^m(x; m)}{\partial m} = m \theta^m \frac{\theta^m - \theta^b}{\eta + \bar{\eta}}.$$

Proposition 4

Since $\mathcal{M} \notin \mathcal{S} \cup \mathcal{E}$, Proposition 3 implies that:

$$\psi(x; m) = \bar{\psi}(x; m) = \lambda \left(\sigma^d \frac{1 - \eta - \bar{\eta}}{\eta + \bar{\eta}} - m \frac{\theta^m - \theta^b}{\eta + \bar{\eta}} - \theta^b \frac{b}{\eta + \bar{\eta}} \right).$$

and

$$r^m(x; m) = \frac{a}{q} + \Phi - \frac{\sigma^2}{\eta + \bar{\eta}} - \sigma^\tau \sigma - \theta^m \psi(x; m)$$

The Fisher equation is given by:

$$i^m = r^m(x; m) + \pi^*. \quad (32)$$

Given that i^m , π^* , and $\frac{\partial r^m(x; m)}{\partial m}$ are constant, there is only one m such that (32) is satisfied and $\psi(x; m)$ is constant as well. Thus equilibrium prices and allocations are constant when b changes.

Proposition 5

Since $\mathcal{M}(x) \in \mathcal{S}^c \cap \mathcal{E}$, that means that traditional banks are satiated before markets become segmented and that $m \geq m^S$. Thus, we have that $\psi(x; m) = 0$. Hence, for any b^* and b^{**} , $\psi(m, b^*) = \psi(m, b^{**}) = 0 = \bar{\psi}(m, b^{**})$. Hence, equilibrium allocations and prices are not affected.

Proposition 6

When money markets are segmented, $\psi(x; m) = \gamma(1 - \eta - \bar{\eta})/\eta - \theta^m m/\eta$ and is not a function of b .

Corollary 1

Notice that by Proposition 6, $\psi(x; m)$ does not change as a consequence of a change in the supply of T-bills b . The treasury transfers are set such that the distribution of wealth is set to a constant: $\eta_t = \eta$ and $\bar{\eta}_t = \bar{\eta}$. Thus, $w^s \sigma + \sigma^\tau = \sigma$ and $r^i(x; m) = \frac{a}{q} + \Phi - \sigma - \theta^m \psi(x; m)$. Hence, by equation (19), if the central bank does not change i^m or m , π does not change either.

B Micro-Foundations for Liquidity Management

I describe the liquidity management problem of banks as a discrete-time problem with an interim period in which assets can only be traded at some cost. Then, I show that this problem converges to the continuous-time equations (4) and (7). This micro-foundation draws inspiration from [Bianchi and Bigio \(2014\)](#) (adding fire sales and liquid money market asset holdings) and [He and Xiong \(2012\)](#) (adding reserves and liquid money market asset holdings). This micro-foundation is also similar to [d’Avernas, Vandeweyer, and Darracq-Pariès \(2019\)](#), when adding T-bills and repo transactions and not allowing for interbank trade during the illiquid stage.

Timing Time is discrete with an infinite horizon. Each period is divided into two stages: the liquid stage ℓ and the illiquid stage i . Both stages last a period of time Δt . In the liquid stage, there is no liquidity friction and portfolios can be adjusted at market prices without any cost. Then, the macroeconomic shock on risky securities realizes and interest rates are paid. At the beginning of the illiquid stage, deposits are randomly reshuffled from some banks—the *deficit banks*—to others—the *surplus banks*. Deficit banks cannot contract new loans and have to rely on disbursing existing assets in order to settle their debts with the surplus banks. There are two types of liquidity frictions in the illiquid stage. First, only a fraction of assets can be mobilized to settle debts. Second, it is costly to use assets during the illiquid stage for settlement purposes. This cost depends on the liquidity of the assets, with risky securities being the most illiquid. After the end of the illiquid stage, the economy enters into a new liquid stage for the next period.

The Liquid Stage In the liquid stage, all banks can trade assets without frictions. Holding risky securities s_t exposes banks to *aggregate* risk realizing in the liquid stage. I write the return received from holding securities during the liquid stage as

$$r_t^s = \mu_t^s s_t \Delta t + \sigma_t^s s_t \varepsilon_t^\ell \sqrt{\Delta t},$$

where ε_t^ℓ is binomial stochastic variable distributed with even probabilities:

$$\varepsilon_t^\ell = \begin{cases} +1 & \text{with } p = 1/2, \\ -1 & \text{with } p = 1/2. \end{cases}$$

The law of motion for the wealth of banks in the liquid stage can therefore be written as

$$\Delta^\ell n_t = \left(\mu_t^s q_t s_t + r_t^m m_t + r_t^f f_t + r_t^b b_t - r_t^d d_t - c_t n_t + \mu_t^\tau n_t \right) \Delta t + \left(\sigma_t^s s_t + \sigma_t^\tau n_t \right) \varepsilon_t^\ell \sqrt{\Delta t}. \quad (33)$$

Bankers face a portfolio choice problem with four different assets: securities portfolio s_t , treasury bills b_t , central bank reserves m_t , interbank lending f_t , and deposits d_t . In equation (33), r_t^b is the interest rate on interbank lending, r_t^m the interest rate paid by the central bank on its reserves, r_t^b the interest rate paid by the government on T-bills, and r_t^d the interest rate on deposits. Banks also choose their consumption rate c_t as a fraction of their wealth and receive a flow of transfers per unit of wealth of $\tau_t = \mu_t^\tau \Delta t + \sigma_t^\tau \varepsilon_t^\ell \sqrt{\Delta t}$ from the central bank.

The Illiquid Stage Each individual banks is subject to an *idiosyncratic* deposit shock:

$$\Delta^i d_t = \sigma_t^d d_t \varepsilon_t^i \sqrt{\Delta t}$$

where ε_t^i is a binomial stochastic variable distributed with even probabilities:

$$\varepsilon_t^i = \begin{cases} +1 & \text{with } p = 1/2, \\ -1 & \text{with } p = 1/2. \end{cases}$$

In the illiquid period, interbank loans f_t cannot be contracted. The balance sheet constraint of the bank imposes that the flow of deposits is matched with an equivalent

flow of securities, treasury bills b_t , and/or central bank reserves m_t . That is,

$$\Delta^i s_t + \Delta^i m_t + \Delta^i f_t + \Delta^i b_t = \Delta^i d_t.$$

The flows of assets $\Delta^i s_t$, $\Delta^i b_t$, and $\Delta^i m_t$ are chosen by deficit banks in order to minimize the net cost of transactions. To simplify the model, I assume that the costs of trading illiquid assets are fixed exogenously¹⁶ and transferred from deficit to surplus banks. I capture these costs with parameters λ^s , λ^m , λ^f , and λ^b . Surplus banks do not face liquidity constraints and take these opportunities to purchase these assets at a discounted price as given. Because the policy functions are linear in the agents' wealth, the distribution of these flows do not impact the recursive competitive equilibrium.

We can then write the net impact of the cost of the deposit shock on an individual bank's wealth as

$$\Delta^i n_t = \lambda^s \Delta^i s_t + \lambda^m \Delta^i m_t + \lambda^f \Delta^i f_t + \lambda^b \Delta^i b_t.$$

Substituting for the balance sheet constraint, we have:

$$\Delta^i n_t = \lambda^m m_t + \lambda^f f_t + \lambda^b \Delta^i b_t + \lambda^s (\Delta^i d_t - \Delta^i m_t - \Delta^i f_t - \Delta^i b_t),$$

which can be rewritten as :

$$\Delta^i n_t = \lambda^s \left(\Delta^i d_t - \frac{\lambda^s - \lambda^m}{\lambda^s} \Delta^i m_t - \frac{\lambda^s - \lambda^f}{\lambda^s} \Delta^i f_t - \frac{\lambda^s - \lambda^b}{\lambda^s} \Delta^i b_t \right). \quad (34)$$

¹⁶I do not provide a micro-foundation for the cost of fire sale, but I refer to the large literature in which it arises either as a consequence of shift in bargaining power under a strong selling pressure (see [Brunnermeier and Pedersen, 2005](#); [Duffie and Strulovici, 2012](#); [Duffie, Gârleanu, and Pedersen, 2005, 2007](#)) or asymmetry of information (see [Malherbe, 2014](#); [Wang, 1993](#)). The intuition is that using reserves or other liquid money market assets have a negligible cost compared with having to sell risky securities. The intuition for including short-maturity loans as liquid assets is that, if the illiquid stage lasts for a longer period than the maturity of the short-term loan, the bank will be able to use the funds lent at the due date, thereby creating a liquidity component of the term structure as modeled by [Acharya and Skeie \(2011\)](#) and documented empirically by [Greenwood et al. \(2015\)](#).

Moreover, a second type of liquidity friction constrains the amount of asset that can be sold by deficit banks during the time interval Δt . A deficit bank can only decrease its asset holdings and only up to a certain threshold. In order to converge to a Brownian motion in the continuous time approximation, this amount is proportional to $\sqrt{\Delta t}$. For example, a deficit bank cannot sell more than a fraction $\delta^s \sqrt{\Delta t}$ of its risky securities over the interval Δt . I write these constraints as

$$0 \geq \Delta^i s_t \geq -\delta^s s_t \sqrt{\Delta t}. \quad (35)$$

$$0 \geq \Delta^i m_t \geq -\delta^m m_t \sqrt{\Delta t}, \quad (36)$$

$$0 \geq \Delta^i f_t \geq -\delta^f f_t \sqrt{\Delta t}, \quad (37)$$

$$0 \geq \Delta^i b_t \geq -\delta^b b_t \sqrt{\Delta t}, \quad (38)$$

The optimization problem of deficit banks in the illiquid stage amounts to the static¹⁷ minimization of their losses under the liquidity constraints:

$$\min_{\Delta^i s_t, \Delta^i m_t, \Delta^i f_t, \Delta^i b_t} \Delta^i n_t$$

where $\Delta^i n_t$ is given by (34), $\Delta^i d_t = -\sigma_t^d \sqrt{\Delta t}$ and subject to the liquidity frictions (35), (36), (37), and (38).

I first consider the case in which liquid assets are not sufficient for a deficit bank to cover its funding needs; that is, $\sigma_t^d d_t > \delta^m m_t + \delta^f f_t + \delta^b b_t$. As using risky securities s_t is the most costly asset, deficit banks always first use their liquid assets m_t , b_t and f_t and only then resort to selling securities in order to settle remaining due debt

¹⁷The problem is static, as banks are able to fully readjust their balance sheets at the beginning of the next period.

positions. Hence, the optimal portfolio adjustments are given by:

$$\begin{aligned}\Delta^i s_t &= \Delta^i d_t + \Delta^i m_t + \Delta^i f_t + \Delta^i b_t, \\ \Delta^i m_t &= -\delta^m m_t \sqrt{\Delta t}, \\ \Delta^i f_t &= -\delta^f f_t \sqrt{\Delta t}, \\ \Delta^i b_t &= -\delta^b b_t \sqrt{\Delta t}.\end{aligned}$$

Intuitively, in order to avoid having to fire-sale illiquid securities at a cost λ^s , deficit banks mobilizes as much as they can from their other (more liquid) asset holdings. Note that all losses from a deficit bank are gained by a surplus bank. Therefore, assuming that $\sigma_t^d d_t > \delta^m m_t + \delta^f f_t + \delta^b b_t$, the law of motion of bank wealth in the illiquid stage can be written as

$$\Delta^i n_t = \lambda^s \left(\sigma_t^d d_t - \theta^m m_t - \theta^f f_t - \theta^b b_t \right) \varepsilon_t^i \sqrt{\Delta t}.$$

where $\theta^j \equiv \frac{\lambda^j - \lambda^s}{\lambda^s} \delta^j$ for $j \in \{m, f, b\}$ is defined as the liquidity index of a given asset taking into account the liquidity frictions on prices and on quantities.

Let's now consider the case in which liquidity is sufficient to cover a negative funding shock: $\sigma_t^d d_t \leq \delta^m m_t + \delta^f f_t + \delta^b b_t$. In this case, the deficit bank does not have to pay any securities fire-sale cost but still has to cover the cost of using liquid assets. Computing this cost requires to know which assets have been used. Using a similar logic as previously, the deficit bank will always first use the less costly assets. This unnecessarily complicates the problem, as it creates multiple kinks in the liquidity risk function. In order to keep the model tractable in its continuous-time approximation, I make the following technical assumption.

Assumption 1 (Costless Liquidity Absent Fire-Sale Risk). *When there is no fire-sale risk, $\sigma_t^d d_t \leq \delta^m m_t + \delta^f f_t + \delta^b b_t$, there is no cost of mobilizing liquid assets : $\lambda^m = \lambda^b = \lambda^f = 0$.*

When assumption 1 holds, the threshold at which banks do not have to fire sale securities corresponds to the threshold at which liquidity risk is nil and the law of

motion for the wealth of banks is given by

$$\Delta^i n_t = 0.$$

Continuous-Time Approximation We can combine the law of motion of both stages to get

$$\begin{aligned}\Delta n_t &= \Delta^\ell n_t + \Delta^i n_t \\ &= \left(\mu_t^s q_t s_t + r_t^m m_t + r_t^f f_t + r_t^b b_t - r_t^d d_t - c_t n_t + \mu_t^\tau n_t \right) \Delta t + \left(\sigma_t^s s_t + \sigma_t^\tau n_t \right) \varepsilon_t^\ell \sqrt{\Delta t} \\ &\quad + \lambda^s \max \left\{ \sigma_t^d d_t - \theta^m m_t - \theta^f f_t - \theta^b b_t, 0 \right\} \varepsilon_t^i \sqrt{\Delta t}.\end{aligned}$$

Finally, the limit when Δt tends to 0 is given by

$$\begin{aligned}dn_t &= \left(\mu_t^s q_t s_t + r_t^m m_t + r_t^f f_t + r_t^b b_t - r_t^d d_t - c_t n_t + \mu_t^\tau n_t \right) dt + \left(\sigma_t^s s_t + \sigma_t^\tau n_t \right) dZ_t \\ &\quad + \lambda^s \max \left\{ \sigma_t^d d_t - \theta^m m_t - \theta^f f_t - \theta^b b_t, 0 \right\} \lambda^s d\tilde{Z}_t,\end{aligned}$$

where Z_t is an aggregate Brownian motion and \tilde{Z}_t is an idiosyncratic Brownian motion.

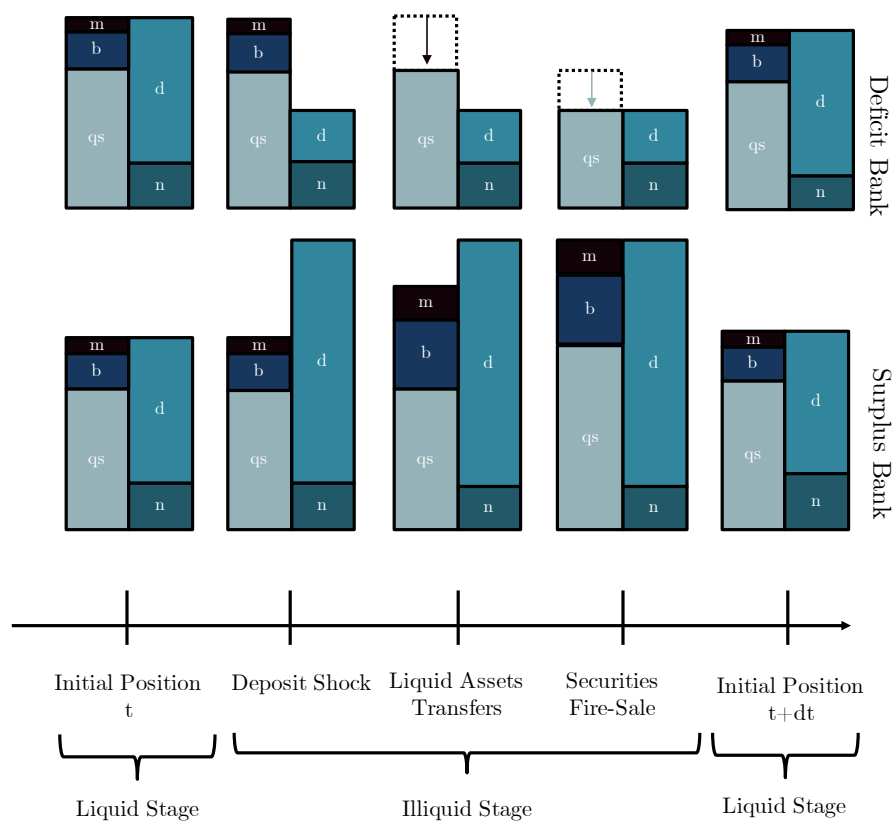


Figure 13: Sketch of Balance-Sheet Adjustments in the Discrete-Time Model

C Shadow Rate

The procedure to compute the shadow rate and the z-spread are similar to [Lenel et al. \(2019\)](#) and [Greenwood et al. \(2015\)](#). More precisely, I use the estimates of the affine term structure model of [Gürkaynak et al. \(2007\)](#) at the 3-month maturity and recover the model implied shadow rate in the following equation:

$$g_t(0.25, 0) = \beta_0 + \beta_1 \exp(-0.25/\tau_1) + \beta_2(0.25/\tau_1) \exp(-0.25/\tau_1) + \beta_3(0.25/\tau_2) \exp(-0.25/\tau_2),$$

where the six parameters $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ are the estimated [Gürkaynak et al. \(2007\)](#) and made available by the Federal Reserve Board on an updated basis.

D Additional Tables

This section collects additional empirical exercises on pre- and post-crisis money markets.

3m T-bill-Repo Spread	(1) daily	(2) daily	(3) weekly	(4) weekly	(5) monthly	(6) monthly
$(\text{T-bill}/\text{GDP})_d$	-216.7* (-2.04)	-63.61 (-0.84)				
VIX_d	0.644** (2.66)	0.977*** (5.97)				
Fed Funds_d		7.933*** (12.02)				
$(\text{T-bill}/\text{GDP})_w$			-221.0* (-2.30)	-65.02 (-0.92)		
VIX_w			0.655** (2.98)	1.000*** (6.53)		
Fed Funds_w				7.998*** (13.19)		
$(\text{T-bill}/\text{GDP})_m$					-221.2* (-2.02)	-62.22 (-0.80)
VIX_m					0.666** (2.65)	1.033*** (6.13)
Fed Funds_m						7.984*** (11.66)
Intercept	33.24*** (3.93)	-17.59* (-2.52)	33.43*** (4.32)	-18.11** (-2.77)	33.22*** (3.80)	-18.94* (-2.60)
Observations	4362	4362	915	915	212	212
Sample: 1991-2008						

Table 3: Liquidity Premia and T-bill Supply: 1991-2008 This table reproduces the results of Nagel (2016) for the pre-crisis sample. The table reports daily, weekly and monthly regressions for the spread between the 3-month T-bill rate and the 3-month General Collateral Repo rate on the ratio of outstanding T-bills to GDP, the volatility index VIX, and the effective fed funds rate. Newey-West standard errors with 3-month maximum lag are reported in parenthesis.

	(1)	(2)	(3)	(4)	(5)	(6)
3m T-bill-Repo Spread	daily	daily	weekly	weekly	monthly	monthly
(T-bill/GDP) _d	-336.2*** (-5.14)	-369.2*** (-5.91)				
VIX _d	0.0611 (0.51)	0.169 (1.17)				
Fed Funds _d		3.385* (2.33)				
(T-bill/GDP) _w			-353.3*** (-5.90)	-390.3*** (-6.73)		
VIX _w			0.0753 (0.64)	0.195 (1.36)		
Fed Funds _w				3.509** (2.70)		
(T-bill/GDP) _m					-357.4*** (-5.08)	-395.4*** (-5.78)
VIX _m					0.0711 (0.50)	0.189 (1.08)
Fed Funds _m						3.164* (2.26)
Intercept	43.98*** (6.54)	43.99*** (6.78)	45.39*** (7.49)	45.52*** (7.79)	45.85*** (6.34)	46.20*** (6.62)
Observations	2185	2185	456	456	106	106
Sample: 2010-2018						

Table 4: Liquidity Premia and T-bill Supply: 2010-2018 This table extends the regressions of Nagel (2016) for the post-crisis sample. The table reports daily, weekly and monthly regressions for the spread between the 3-month T-bill rate and the 3-month General Collateral Repo rate on the ratio of outstanding T-bills to GDP, the volatility index VIX, and the effective fed funds rate. Newey-West standard errors with 3-month maximum lag are reported in parenthesis.

	(1)	(2)	(3)
	Repo-IOR Spread	T-bill-IOR Spread	Reverse Repo Volumes
Adj. T-bills	0.329 (0.00569)	0.299 (0.00612)	-2.596 (0.0787)
	-0.642 (0.00851)	-0.658 (0.00931)	4.665 (0.118)
N	2250	2250	1249

Table 5: Tobit Model Regression This table reports the coefficients from the estimations of equations (29), (30), and (31), using the measure of T-bill supply adjusted for the effect of the money market reform. Reported standard errors are obtained with the bootstrapping method from Flood (1985).