Intraday Liquidity and Money Market Dislocations*

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Abstract

This paper investigates the pricing of repurchase agreements (repos) within the new post-crisis regulatory framework. We find that new liquidity regulation prevents banks from using intraday credit provisions from the Fed. As a consequence, reserve-rich banks—rather than the Fed—are the marginal provider of liquidity to money markets. In this new regime, intraday liquidity can suddenly become scarce and constrain the supply of repo, leading to sharp increases in repo rates. These spikes in repo rates are more likely when the supply of Treasury debt financed by shadow banks is large and settlement volumes are high.

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1 Introduction

The market for repurchase agreements (repos)—a market for short-term collateralized loans with an estimated outstanding size over \$15 trillion (ICMA, 2020)—is a prime source of funding for many financial institutions, including insurance companies, asset managers, and institutional investors. For this reason, lasting disruptions in this market have the potential to amplify negative financial shocks and cause severe adverse effects to the economy, as observed during the 2008-2009 financial crisis (Brunnermeier, 2009; Gorton, 2009).

In recent years, repo rates have been characterized by increased volatility and sporadic spikes with a high prevalence on quarter-ends. Notably, on December 31, 2018, September 16, 2019, and March 17, 2020, US dollar repo rates jumped up by around, respectively, 260, 380, and 60 bps (see Figure 1). These sudden increases in repo rates surprised many market participants and prompted strong reactions from the US Federal Reserve (the Fed). While the Fed does not directly target repo rates,[1] a spike in repo rates reflects a low pass-through of its monetary policy to firms at the core of the financial system with pressing liquidity needs. Hence, the Fed intervened forcefully in September 2019 to reverse the tapering of its balance sheet—adding more than \$400 bn of reserves to the banking sector through the resurrection of its repo and bill purchase operations.

As of today, it remains an open question as to why sudden pressures arise in this market and whether they reflect structural issues. Explanations commonly put forward points to the emergence of intermittent shortages of *intraday* liquidity with important consequences for financial stability and the conduct of monetary policy (Pozsar, 2019). According to new liquidity regulations banks have to create a buffer of liquid assets that can be used to meet outflows in stressed scenarios. In particular, central bank reserves have a unique attribute in a liquidity stress: Unlike Treasuries, which need to be monetized first, reserves *are* money and can be directly used to settle a debt or a transaction. If reserves play a special role in banks' strategy to meet their intraday liquidity needs and fulfill their regulatory requirements, fluctuations in the quantity of reserves available to banks could be responsible for recurrent surges in repo rates. Yet, despite the importance of the repo market for the implementation of monetary policy and the functioning of financial markets, the academic literature lacks a theoretical framework connecting these elements.

[1] The Fed officially targets the fed funds rate—an interbank uncollateralized index rate which accounts for a smaller set of financial institutions with fading volumes and relevance since the 2008 financial crisis. The fed funds market also displayed increased volatility on September 16, 2019, but to lower levels.

To fill this gap and answer these questions, we propose a new macro-banking model extending traditional theories of monetary policy implementation in the interbank fed funds market (Poole, 1968) to include a repo market in which leveraged shadow banks (i.e. hedge funds and securities dealers) trade liquidity with banks. The model features two main elements. First, banks are subject to an intraday stress-test buffer requiring them to hold on to a portion of reserves *at any point of the day* and in proportion to their expected gross outflows. Second, both the (interbank) fed funds market and the (bank-shadow bank) repo market are assumed to ultimately settle in central bank reserves. That is, the action of lending in those markets triggers an outflow of reserves for banks.

In the model, households allocate their wealth between deposits with traditional banks and repos with shadow banks, according to their relative preferences for the two liquid assets.[2] These preferences are subject to shocks with implications for the funding of banks and shadow banks. When households rebalance their portfolio from repo to deposit, banks end up with a surplus of fund whereas shadow banks have a deficit. Hence, banks have the option to invest this surplus in repos with shadow banks. As long as the repo rate is weakly above their valuation for reserves, banks benefit from swapping reserves for repos. In a frictionless world, traditional banks elastically lend in repo markets and thereby prevent repo rates from rising far above the interest on reserves. In contrast, binding intraday constraints may change traditional banks' cost and ability to redeploy funds in the repo market and pressure rates upward.

Our first theoretical finding is that the introduction of an intraday liquidity requirement creates a strict constraint on the amount that banks can lend in the repo market. Once a threshold is reached, the repo rate rapidly increases above the discount window rate as banks lacking intraday liquidity cannot take advantage of arbitrage spreads between fed funds and repo rates. This outcome contrasts with the dynamics of the pre-regulatory reform era. Before the introduction of liquidity regulations, banks could always lend in repo markets—even with a negative intraday balance of reserves—by drawing from an overdraft at the Fed. As liquidity regulations now require banks to hold a positive amount of reserves at any point during the day, banks stop lending when hitting their constraints and the supply of repo its elasticity. In this regime, there is no injection of intraday

[2] In reality, households only invest in repo markets indirectly through money market funds. For simplicity, we omit to model these institutions and assume that households can directly invest in repos. Shadow banks should therefore be interpreted in the paper's context as leveraged institutions borrowing in the repo market such as relative value funds, hedge funds, and securities broker-dealers.

reserves at the margin by the Fed and trades have to settle using the fixed amount of reserves available. This inelasticity can explain the strong non-linear forces observed in repo markets.

In the model, repo rates are determined by the interaction between an inelastic demand for repo financing from shadow banks holding Treasury securities and a sometimes-constrained repo supply from banks. Various factors affect the balance between these two forces and, hence, the probability of hitting banks' intraday limits. First, the size of the central bank balance sheet plays a crucial role in determining the total supply of reserves available to banks for settlement. An economy with more reserves can sustain larger repo demand shocks before generating a surge in repo rates.

An essential contribution of this paper is to point out to fiscal policy as an important driver of money market imbalances. In the model, fiscal policy can affect both the demand and supply of repos through three channels. On the supply side, the issuance of new Treasury debt and payment of household tax liabilities have the side effects of draining reserves away from banks' accounts to the Treasury account at the central bank. These operations withdraw from the pool of reserves that banks can use. The issuance of new Treasury debt also has a second effect on banks' ability to lend in repo markets. Because larger outflows are anticipated on issuance days, stress-tests require banks to hold more reserves, which means that fewer reserves are available for money market lending.

On the demand side, a surge in the quantity of Treasury debt outstanding leads to an increase in the demand for repos. Mechanically, an expansion of public debt generates an increase in households' tax liabilities and assets to invest in today. When households invest a large proportion of these assets in deposits with banks while shadow banks hold a large proportion of the newly issued T-bonds, the financial system increases its reliance on bank-to-shadow bank repo lending. This increase in repo volumes eventually pushes banks' repo supply closer to their constraints and increases the probability of hitting them. We document that this pattern is observed in the run-up to the September 2019 spike. Banks then increased their repo lending of around $200 billion, while non-banks increased their repo borrowing by a similar magnitude to absorb a significant portion of newly issued Treasuries.

The model also features striking implications for the relationship between the repo and Treasury markets. In our dynamic model, shadow banks anticipate the probability of banks hitting their constraints and repo market dislocating. To compensate for heightened liquidity risk in disrupted repo markets, shadow banks require larger excess returns for holding Treasury securities. Therefore, the model predicts that when the economy moves closer to intraday constraints, Treasury yields increase ahead of a repo market disruption. In other words, Treasuries' "moneyness" highly depends on its pledgeability as collateral in repo markets. With disrupted repo markets, Treasuries lose this property, and their yields adjust to reflect increased liquidity risk for shadow banks. This prediction of the model was not observed in September 2019 when Treasury yields remained stable by the time of the Fed's intervention but could be observed in March 2020 when the Covid-crisis hit financial markets. A potential reconciliation for these diverging patterns would be that the September events took markets by surprise, whereas the repo market's fragility was expected, and therefore priced, by March 2020, precisely because it was revealed seven months earlier[3].

Our analysis interprets the sharp increase in rates in September 2019 as a combination of two slow-moving factors and two triggers. First, the supply of reserves decreased gradually from August 2014 to September 2019, thereby pushing banks closer to their intraday liquidity limits. Second, the structural demand for repo increased due to an increase in dealers' holdings of Treasuries. Finally, the combination of new Treasury issuance and tax settlement further decreased the supply of reserves available to banks to the point of reaching their intraday liquidity constraint. There was no possible adjustment in banks' balance sheets at this stage, and repo rates surged above the discount window rates.

Related Literature This paper contributes to the literature linking the pricing of money market assets to post-crisis regulation. Bech and Klee (2011) propose a model of post-crisis fed funds market and argue that banks' limits to arbitrage are responsible for fed funds rates trading below the interest on reserves. Bech and Keister (2017) study the impact of the Basel III liquidity coverage ratio on interbank interest rates in a model of monetary policy implementation. Andersen, Duffie, and Song (2019) demonstrate that the funding value adjustments of major dealers are debt overhang costs to their shareholders. Duffie and Krishnamurthy (2016) show that the current setting of U.S. dollar money markets—with frictions associated with imperfect competition, regulation, infrastructure, and other forms of institutional segmentation within money markets—limits the pass-through effectiveness

[3]Besides this liquidity risk channel, the spike in Treasury yields can also attributed to the strong selling pressure from foreign investors and exacerbated by leveraged cash-bond future arbitrage strategies. See He, Nagel and Song (2020); Schrimpf et al (2020): and Di Maggio (2020).

of the Federal Reserve's monetary policy. On the empirical side, Anbil and Senyuz (2018) and Munyan (2017) look at tri-party repos data and show that some foreign banks engage in window dressing behavior (deleveraging at month-ends) in response to the Basel III implementation of the leverage ratio requirement. In its focus, the paper is close to the empirical work of Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin (2020), Correa, Du, and Liao (2020) and Avalos, Ehlers, and Eren (2019) exploring potential explanations to the September 2019 repo rate spike. These studies point out to the role of large global banks with balance sheet constraints as the supplier of liquidity to dollar money markets. Yang (2020) proposes a theoretical model in which repo spikes appear as a consequence of strategic complementarity in intraday payment timing among banks. The main innovation of this paper is to propose a general equilibrium model of post-crisis repo markets and highlight the role of intraday liquidity requirements as the critical regulation to understand observed repo spikes.

Our paper also relates to the literature on the implementation of monetary policy following the seminal work from Poole (1968), adapted to dynamic OTC markets by Afonso and Lagos (2015), and a macroeconomic framework by Bianchi and Bigio (2014) and Piazzesi and Schneider (2018). These models capture fed funds market dynamics with banks exchanging scarce reserves to mitigate their risk of borrowing at the discount window. Nesting the model of Poole (1968), our paper extends his approach to the consequences of reserves scarcity—within the new regulatory framework—to the repo market in which non-bank institutions are also trading. In this regard, the paper also relates to Pozsar, Adrian, Ashcraft, and Boesky (2013), Chernenko and Sunderam (2014), Sunderam (2015), and Adrian and Ashcraft (2016) which document the critical role of these institutions collectively referred to as "shadow banks" in creating liquidity.

Last, the paper is linked to the literature on the repo market and its central role in financial stability. Gorton and Metrick (2012) document a rise in margin requirements in the repo market during the 2008-2009 crisis and argue that the repo market suffered a run, which amplified the financial crisis. Krishnamurthy, Nagel, and Orlov (2014) find that—despite being quantitatively small—the 2008 contraction in repo mainly affected key dealer banks with significant exposures to private sector securities, which then had knock-on effects on security markets, and led these dealer banks to resort to the Fed's emergency lending programs.

2 Descriptive Analysis

In this section, we discuss four sets of facts at the core of our analysis: the recurrence of repo spikes, the introduction of new regulations, the tapering of the Fed's balance sheet, and the increase in shadow banks' Treasuries repo financing.

Recurring Spikes in Repo Rates In a repo transaction, an institution sells an asset to another institution at a given price and commits to repurchase the same asset from the second party at a different—typically lower—price at a future date. Although a repo is structured legally as a sale and repurchase of securities, it behaves economically like a collateralized loan. If the seller defaults before the maturity of the repo expire, the buyer retains the asset, which acts as collateral to mitigate the credit risk that the buyer/lender has on the seller/borrower. For this reason, the return on the transaction generated by the difference in prices is referred to as the *repo rate*.

[Figure 1 about here]

Figure 1 displays the time series of the repo rates from January 2010 to January 2020. Repo rates have been characterized by increasing volatility, with peaks culminating at more than 275 bps in December 2018 and more than 400 bps in September 2019 above the interest on reserves. One can also see that spikes of lower intensity are present throughout the rest of the series and tend to be located at fixed intervals corresponding to quarter-ends. Notably, a spike of more than 70 bps could already be observed in September 2016. Nevertheless, we also note that many quarter-ends do not display any sign of pressure on rates.

[Figures 2 and 3 about here]

Figures 2 and 3 show that banks—although typically lending more when the GCF-IOR spread is higher—actually decreased their lending volumes on the week of September 16, 2019, despite the GCF spreads moving to historically high levels.

New Regulation In the aftermath of the 2008 financial crisis, policymakers worldwide introduced new regulations to address vulnerabilities in the financial system. Notably, following the Basel Committee—which introduced a set of rules collectively referred to as Basel III—tighter capital and liquidity regulations were introduced. In the debates that have followed the September events, three types of regulation have been incriminated as possible drivers of the spike.

First, capital regulation, such as the Supplementary Leverage Ratio (SLR), compels banks to hold a given share of their liabilities in equity to protect debtors and depositors. The supplementary leverage ratio generally applies to financial institutions with more than $250 billion in total consolidated assets. It requires them to hold a minimum ratio of 3%, measured against their total leverage exposure, with more stringent requirements for the largest and most systemic financial institutions. As argued by Duffie (2018), the introduction of the SLR entails that the space on the balance sheets of major dealer banks is today more expensive than before the 2008 financial crisis. Accordingly, large dealer-banks have increased intermediation spreads[4], resulting in a hike in funding costs for other institutions.

Second, the 2008 financial crisis illustrated how quickly market liquidity could evaporate and prompted regulators to introduce additional liquidity regulations. The Liquidity Coverage Ratio (LCR) builds on traditional liquidity coverage methodologies used internally by banks to assess exposure to contingent liquidity events. It ensures that banks hold enough liquid assets to cover their total net cash outflows over the next 30 calendar days. In Basel III lingo, banks have to hold adequate stock of unencumbered high-quality liquid assets (HQLAs)—i.e., cash or assets that can be converted into cash quickly through sales with no significant loss of value. HQLAs include Level 1 assets, which can be held without limit or haircut, and Level 2 assets, which cannot exceed 40% of the liquidity reserve capped at a maximum of 40% and receive a 15% haircut. In practice, Level 1 HQLAs include both reserves at the Fed and US Treasuries, whereas Level 2 HQLAs include mortgage-backed securities guaranteed by Government-Sponsored Enterprise. The ratio of a bank's total amount of HQLAs divided by expected net outflows has to be over 100% to satisfy the LCR.

Third, the LCR is complemented by various internal Liquidity Stress Tests (LSTs)—

[4] A usual measure of repo intermediation cost is the spread between tri-party repo rates in which dealers borrow from money funds and the inter-dealer GCF repo rate.

following Regulation YY and resolution planning rules—which require banks to hold enough cash to face intraday liquidity outflows[5]. As mentioned previously, according to the LCR, all categories of Level 1 HQLAs—such as reserves and Treasuries—are treated as substitutes. However, according to the vice-chair for the supervision of the Federal Reserve, Randal K. Quarles, it may, in practice, be difficult to liquidate a large stock of Treasury securities to meet large *day one* outflows (Quarles, 2020). Within Regulation YY's enhanced prudential standards and Resolution Liquidity Adequacy and Positioning (RLAP), large firms are required to conduct internal liquidity stress tests, and supervisors expect firms to ensure that their liquidity buffers can cover estimated day-one outflows without reliance on the Federal Reserve. For institutions with large intraday outflows such as clearing banks or banks with a broker-dealer subsidiary, these liquidity buffers can be sizable—i.e., in 2008-2009, several firms experienced outflows exceeding tens of billions of dollars in a single day. Therefore, these firms are required to hold large quantities of reserves at the Fed—rather than Treasuries—as a complementary liquidity buffer to their LCR.[6]

The notion that the September spike may be linked to intraday liquidity issues and its regulation is shared by prominent actors of money markets. Already in May 2019, Zoltan Pozsar (2019) argued that intraday liquidity had been much scarcer than commonly admitted, and the repo market was likely to face severe disruptions going forward. Moreover, when asked about how the Fed would adjust to the September events, its chairman, Jerome Powell, pointed out that: *"It used to be a common thing for banks to have intraday liquidity from the Fed, what is called "daylight overdrafts." That's something we can look at. Also, there are just a few technical things that we can look at that would perhaps make the liquidity that we have—which we think is ample in the financial system—move more freely*

[5]For example, back in 2013 the Bank for International Settlement released guidance for banks on the specific monitoring tools for intraday liquidity management, explicitly indicating the banks need to monitor their daily maximum intraday liquidity usage, available intraday liquidity at the start of each day as well as their total gross payments throughout the day, although these were provided at the time only in the form of guiding principles (BCBS248).

[6]Before September 2019, the supervisory guidelines for banks' internal liquidity stress tests were kept secret, and it was unclear if regulators were indeed drawing a distinction between reserves and cash-equivalents such as Treasuries. Vice-chairman Randal K. Quarles recently clarified this point in a series of declarations. For instance, on February 6, 2020, he mentions that *"supervisors expect firms to estimate day-one outflows and to ensure that their liquidity buffers can cover those outflows without reliance on the Federal Reserve. For firms with large day-one outflows, reserves can meet this need most clearly."* In the same speech, he opens the door for taking discount window liquidity as part of the stress-test so that: *"[i]f firms could assume that this traditional form of liquidity provision from the Fed was available in their stress-planning scenarios, the liquidity characteristics of Treasury securities could be the same as reserves, and both assets would be available to meet same-day needs."* Quarles (2020).

and be more liquid, if you will.".

From the banking side, when asked why JP Morgan did not lend in repo when spreads were large in September despite holding large amount of reserves, its chairman, James Dimon, answered: *"[...] we have $120 billion in our checking account at the Fed, and it goes down to $60 billion and then back to $120 billion during the average day. But we believe the requirement under CLAR and resolution and recovery is that we need enough in that account, so if there's extreme stress during the course of the day, it doesn't go below zero. If you go back to before the crisis, you'd go below zero all the time during the day. So the question is, how hard is that as a red line?"*

Scarcer and More Volatile Reserves

[Figures 4 and 5 about here]

Figure 4 displays the evolution of the liabilities on the Fed's balance sheet. Responding to the financial turmoil, the Fed started in 2008 to purchase large amounts of long-term securities—a policy instrument commonly referred to as Quantitative Easing (QE). As a consequence, the volume of reserves held by banks[7] had increased by a factor of 40, leading to a doubling of the total size of their combined balance sheet.

In November 2010, the Fed announced the second round of QE, and further increased its balance sheet by $600 billion by the end of the second quarter of 2011. The third round of QE was then announced in September 2012, leading to an additional increase in the Fed's balance sheet of over $1.5 trillion by 2015. Until October 2017, the Fed then kept its balance sheet at a stable size. At this juncture, it started to "normalize" its balance sheet by not reinvesting a part of maturing securities. The intention was to reduce the balance sheet at an initial of pace $30 billion per month, then adjusted to $15 billion per month, up to September 2019.

At this point, reserves were anticipated to "*likely still be somewhat above the level of reserves necessary to efficiently and effectively implement monetary policy. In that case, the Committee [anticipated] that it [would] likely hold the size of the SOMA portfolio roughly constant for a time. During such a period, persistent gradual increases in currency and*

[7]In the US, only a restricted set of institutions—including depository institutions, Federal Home Loan Banks and Government-sponsored Enterprises—have an account at the Fed and can directly hold reserves.

other non-reserve liabilities would be accompanied by corresponding gradual declines in reserve balances to a level consistent with efficient and effective implementation of monetary policy." Board of Governors of the Federal Reserve System (2019)[8] In September 2019—as repo rates suddenly hiked to more than 600bps—the Fed reversed course and started to increase its balance sheet again, initially by lending in the repo market and eventually through direct purchases of Treasury bills.

This evolution in the Fed's balance sheet's size was also accompanied by a change in the *composition* of its liabilities. After the 2008 financial crisis, the Fed started offering overnight liquidity services to a growing number of non-bank institutions—ultimately leading to a further reduction in the supply of reserves to banks (Pozsar, 2017).

First, the US Treasury no longer keeps its cash balances with private banks but rather with the Fed at its Treasury General Account (TGA). These balances can run as high as $400 billion and are highly volatile, in particular during tax payment and debt issuance periods, as well as around quarter-end and year-end. For instance, in September 2019, TGA balances increased by more than $150 billion within two weeks, thereby removing a similar amount of reserves from the stock available to banks.

Second, the Fed allowed some foreign central banks to move their short-term balances to its balance sheet within the Foreign Reverse Repo Facility (FRRP). In September 2019, the FRRP reached a peak of $300 billion.

Third, in 2014, the Fed let money market funds access its balance sheet through the Domestic Reverse Repo Facility (DRRP). The DRRP became an important element in the Fed's monetary policy implementation strategy by creating a floor under which money market funds would not lend to the repo market. The DRRP was heavily used by money market funds, which would deposit on average $150 billion with the Fed per day up to 2018, when a large supply of Treasury bills pushed repo rates above the DRRP rate.

Importantly, these different items on the liability side of the Fed's balance sheet are not under its direct control. The Fed decides how much it remunerates the facility, but it does not set limits for the quantities that institutions can deposit at the facilities. When

[8] Moreover, based on the 2018 Senior Financial Officer Survey, the Fed estimated a measure of total demand for reserves in the banking system, or the "lowest comfortable level of reserves" banks were requiring, to guarantee a sufficient amount of reserves were available to meet system demand. Source: Keating, Thomas, Francis Martinez, Luke Pettit, Marcelo Rezende, Mary-Frances Styczynski, and Alex Thorp (2019). "Estimating System Demand for Reserve Balances Using the 2018 Senior Financial Officer Survey," FEDS Notes. Washington: Board of Governors of the Federal Reserve System, April 9, 2019.

taken in combination—a gradual reduction in balance sheet size and the introduction of volatile facilities to non-bank institutions—these different factors have made the quantity of reserves available to banks gradually scarcer and more volatile. In September 2019, the quantity of reserves hit bottom after two years of gradual reduction and a sudden surge in TGA balances. Figure 5 shows two additional elements on the distribution of reserves holdings between banks. First, the distribution of holdings is highly skewed, with the first largest holder—JPMorgan—accounts for almost two-thirds of the reserves holdings of the next ten largest holders in the third quarter of 2018. Second, the figure shows that the loss of reserves in the months ahead of September 2019 was also skewed. JPMorgan's reserves portfolio dropped from $274 billion to less than $120 billion between 2018 Q3 and 2019 Q2 while the ten other banks in the sample dropped less than $70 billion in reserves.

Treasury Supply, Shadow Banks Balance Sheets, and Repo Volumes Finally, we document a sharp rise in outstanding stocks of Treasuries available to the public, i.e., net of Fed holdings, accompanied by an increase in repo volumes from banks to non-banks in the run-up of September 2019.

[Figures 6, 7, and 8 about here]

Figure 6 plots the net supply of Treasuries outstanding between January 2010 and September 2019. The shaded area represents the portion of these Treasuries held by the Fed and, therefore, not available to the public. One can see a large increase in the amount of Treasuries available to the public between 2017 and 2019 brought about by two factors. First, and as already mentioned, in Q4 2017, the Fed started to unwind its Treasury portfolio. Second, since the beginning of the Trump administration, the US has run larger deficits than in previous years.

Figure 7 shows that domestic banks and non-banks have largely absorbed this increase in Treasury securities supply. Figure 8 shows that a large portion of the rise in Treasury holdings from non-banks has been financed in the repo market by banks. Such an increase appears to be associated with a significant increase in repo lending from banks, presumably from shadow banks borrowing to finance their larger Treasury inventory.

3 Empirical Motivation

In this section, we present motivating evidence that money market spreads tend to be larger on days with high settlement volumes, hinting at a role for intraday liquidity scarcity. This observation complements the results of Duffie et al. (2021), documenting a strong relationship between late payment timing—an indicator of intraday liquidity scarcity—and money market spreads.

3.1 Data Description

As our main dependent variable, we use the Treasury General Collateral Rate, as collected from the Depository Trust & Clearing Corporation (DTCC), over the interest paid on excess reserves (IOER), from FRED. We use the first-difference in the spread to account for serial correlation of the dependent variable in the regressions. As additional dependent variables, we also include the MBS general collateral rate (MBSGCF) spread (also from DTCC), the Secured Overnight Financing Rate (SOFR, from the New York Fed), and the JPY-USD FX swap implied dollar funding rate from Correa et al. (2020), all in excess of the IOER. All regressions are run over the post-regulation period and before the September 2019 repo spike, i.e., from 2015-12-15 to 2019-08-31.

As independent variables, we include the daily change in central bank reserves available to banks and a setof calendar and event dummies to measure the effect on days associated with large settlement and payment volumes, as used in the literature. The difference in Reserves is measured with the weekly Total Reserve Balances at Federal Reserve Banks from the Saint-Louis Fed (Fred). As the variable is available only on a Wednesday-level basis, we construct the daily series by subtracting the cumulative change in the Treasury General Account (from the Daily Treasury Statement from the Bureau of Fiscal Services) from each Wednesday observation to fill the missing observations.

Top P10 Bills Issuance Days and Top P10 Coupons Issuance Days are dummy variables equal to 1 on days with Bills or Coupons Issuance in the highest 10% percentile. The threshold values are 113 $bn for Bills Issuance and 26 $bn for Coupon Issuance. The issuance series are collected from the DTS website. We include quarter- and month-starts and ends calendar effects to measure days of high expected payment flow as standard in the literature[9]. Finally, we include dummies for days corresponding to corporate tax payment deadlines and separate dummies for each of the two days before and after the tax deadlines. Major tax deadlines occur regularly in the middle of the month. If any of these days fall outside of intra-week days or on business holidays (according to the SIFMA calendar), then the first business day after is used. For example, 09/15/2019 was a Sunday. Therefore the tax deadline dummy is equal to 1 on 09/16/2019.

3.2 Discussion

[Table 1 about here]

In our main specification in column (4) of Table 1, we regress changes in the spread between the Treasury General Collateral repo rate and the interest paid on excess reserves on changes in the supply of reserves and our four categories dummy variables for large settlement and payment days, introduced gradually. Regressions display significant positive spread variations on the three types of dummies.

Across all regressions, a positive change in reserves supply is associated with a lower spread. In the first regression, the effect is statistically significant although the coefficient is reduced and is no longer significant once we introduce the quarterly and monthly calendar effects. This reduction of significance is due mostly to short-term variations in the supply of reserves coming from movement in the Treasury General Account at the end of the months. Both Quarter End and Month End dummies are associated with larger spreads, which are reversed at the end of the next period[10]. This result is consistent with the work of Correa et al. (2020).

In all four specifications of the main regression, we see a positive and significant positive effect on Treasury issuance days. This effect is stronger for days of large coupon issuance as compared to bill issuance. This result is consistent with the analysis of Pozsar (2019), who

[9] see Judson and Klee (2010); Furfine (2001); Carpenter and Demiralp (2006), Ashcraft et al.(2011), McAndrews, and Kroeger (2016); Bech, Martin, and McAndrews (2012). These earlier studies used calendar fixed effects to proxy for high expected daily payment activity and volatility in the federal funds rate to identify banks' demand for reserves and study the liquidity channel on the federal funds rate and/or repo markets.

[10] In unreported results we estimated the regressions also over the pre-regulatory period and find no evidence of the quarter-end effects.

argues that high bill supply drives higher demand for overnight general collateral repos around its trend. In contrast, chunky coupon supply days tend to cause episodic spikes in overnight general collateral repo volumes.

The regression analysis in specification (4) also reveals greater spread changes on days preceding corporate tax deadlines. The effect is statistically significant at 1% level and appears to be reversed by the same magnitude on the day after the tax deadline. Tax deadlines are associated with two important types of flows in money markets. Firstly, as discussed in Greenwood et al. (2015), the Treasury strategically increases issuance of short-term Bills ahead of corporate tax deadlines, repaying the debt rapidly after the deadlines. Secondly, corporations and money market funds pull out cash from banks ahead of the corporate tax deadlines to make the payments to the Treasury, decreasing the supply of reserves in the system. Both channels imply a correlation between tax deadlines and Δ Reserves as this increase in the TGA balance sterilizes reserves from the pool available to banks.

Regressions (5), (6), and (7) provide a similar analysis for other funding rates. The result of regression (4) mostly carries over to the MBS collateral repo spread in (5) and to the SOFR spread in (6). The JPY-USD FX-spread appears to react most strongly to Quarter End and Month End.

The overall results concur with our earlier discussion on the effect of reserve balances, high payment flows and Treasury issuance on repo rates, in line with the simultaneous occurrence of these events leading to the September 2019 spike. While these channels have likely contributed to explaining the higher repo rates, the magnitude of the predicted increase falls well behind of the observed 600bps spike. Most strikingly, these effects were in part already anticipated to the extent that they reflect known calendar effects or given that new Treasury issuance amounts are announced ahead of the settlement days. This further suggests than an unobserved trigger interacting with these factors determined the unexpected and abnormal spike in the repo market. While a reduced-form empirical approach warrants a deeper analysis of the intra-day dynamics, in the next section we present a theoretical framework and illustrate how the confluence of these channels may trigger banks' intra-day liquidity constraints and can generate highly non-linear movements in repo rates.

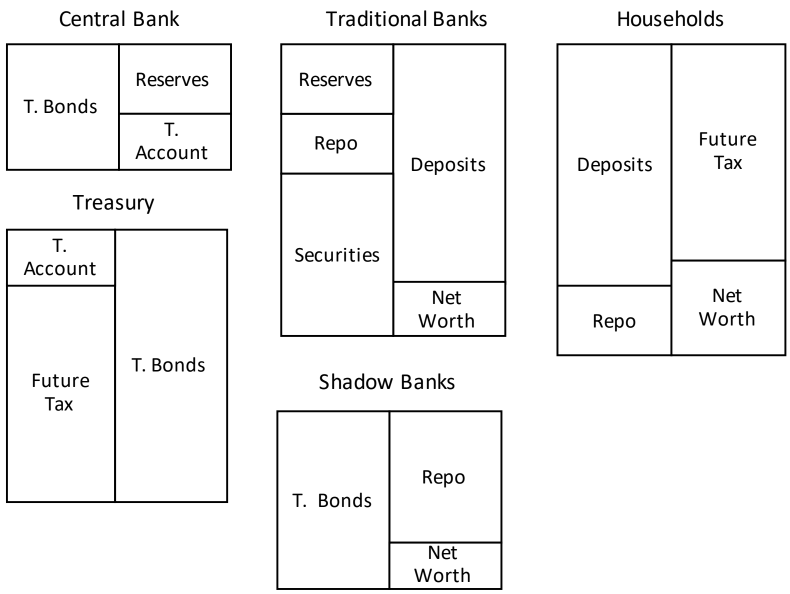

Figure 9: Agent's Balance Sheets

4 Model

4.1 Environment

Informally, the model is a general equilibrium extension of Poole (1968) featuring shadow banks and a repo market. Formally, we let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions and assume that all stochastic processes are adapted. The economy is populated by a continuum of traditional banks, shadow banks, and households, as well as a treasury and a central bank. Time is discrete and infinite. Any period has two stages: the morning and the afternoon. The repo market takes place in the afternoon and explicitly settles in central bank reserves.

Assets and Market Structures Figure 9 introduces the model's market structure through an example of sectors' balance sheets. There are two goods in positive supply: a securitized productive capital k and a final consumption good y produced by capital. The Treasury issues T-bonds b against future households tax liabilities τ; the central bank holds some of the outstanding T-bonds by issuing reserves m to the traditional banking sector

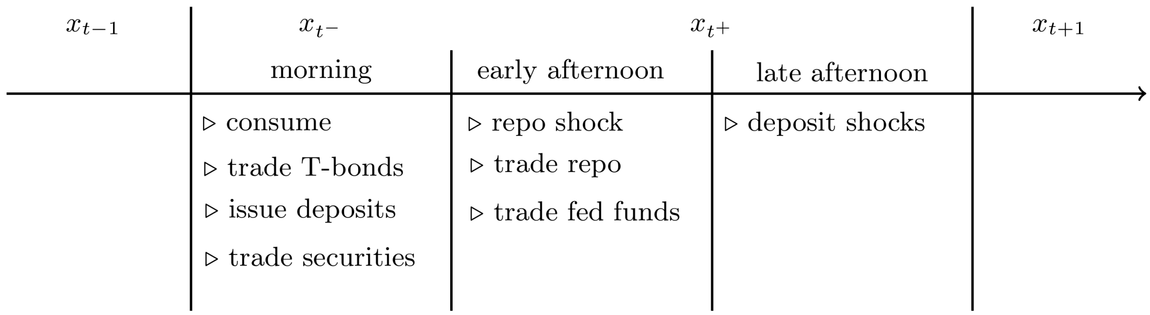

Figure 10: Model Timeline

and the Treasury. Households and shadow banks cannot hold reserves. Rather, households hold their wealth in deposits d issued by traditional banks and repurchase agreements (repo) p issued by traditional and shadow banks. Traditional banks holds securitized capital k valued at price q, reserves at the central bank m that can be traded as fed funds f, and repos p. Shadow banks finance a portfolio of T-bonds b with repo p from both banks and households. Reserves m are the unit of account, while output y is the numeraire. All rates and asset prices are expressed in real terms.

Timing and State Space Figure 10 summarizes the timing of the economy. Subscript t^- indicates variables and prices set in the morning, while the subscript t^+ indicates variables and prices set in the afternoon. X_{t^-} represents the state space of the economy in the morning of time t, while X_{t^+} represents the state space in the afternoon. Uppercase variables designate aggregate variables. Markets for the consumption good, T-bonds, securitized capital, deposits, and bank equity clear in the morning. The repo and fed funds markets clear in the afternoon. The model features two types of shocks. First, an aggregate shock $\alpha^h_{t^+}$ to households' preference for liquid assets (deposits and repo) occurs in the afternoon and results in an aggregate deposit flow. Second, as in Poole (1968), deposits are subject to idiosyncratic shocks in the late afternoon.

Preferences and Technology Traditional banks, shadow banks, and households are risk neutral. Additionally, households also value liquidity services:

$$U^h(c^h_{t^+}, d^h_{t^+}, p^h_{t^+}, \alpha^h_{t^+}) = c^h_{t^-} + \varphi \min\left\{\frac{d^h_{t^+}}{\alpha^h_{t^+} K_{t^-}}, \frac{p^h_{t^+}}{(1-\alpha^h_{t^+})K_{t^-}}\right\}^{1-\gamma} / 1-\gamma.$$

The parameter φ governs the weight of liquidity services relative to the numeraire consumption. The preference for liquidity has decreasing returns parameterized by γ. We scale liquid assets by the aggregate supply of capital so that the utility derived from holding liquid assets does not depend on the size of the economy. In the afternoon, the preference shock $\alpha^h_{t^+}$ determines the weight that each type of liquidity receives in generating aggregate liquidity benefits and is uniformly distributed between 0 and 1. The Leontief aggregator implies that the relative demand of deposits and repo is such that $(1-\alpha^h_{t^+})d^h_{t^+} = \alpha^h_{t^+} p^h_{t^+}$.

Each period, the stock of capital produces a units of consumption good. All units of capital are pooled in an economy-wide diversified vehicle in quantity K_{t^-} with price q_{t^-}. The expected return on this securitized capital is given by:

$$1 + r^k_{t^-} = \mathbb{E}_{t^-}\left[\frac{a + q_{t^-+1}}{q_{t^-}}\right].$$

Intraday Flows We specify the timing of intraday flows based on agents' portfolio holdings. These flows will turn out to be crucial for the model's dynamics. In the afternoon, the repo market opens and banks have the option to lend repo to shadow banks. We assume that repos do not roll over and require new settlements every day. This operation triggers an outflow of reserves. Hence from the point of view of banks' balance sheets, repo lending amounts to swapping a quantity p_{t^+} of reserves into repo. Shadow bankers use these repos to net their daylight overdrafts positions o_{t^-} at the clearing house division of traditional banks. The intraday laws of motion for deposits d_{t^+}, overdrafts o_{t^+}, and reserves m_{t^+} are given by:

$$\begin{aligned} d_{t^+} &= d_{t^-} + \Delta d_{t^+}, \\ o_{t^+} &= o_{t^-} - \Delta o_{t^+}, \\ m_{t^+} &= m_{t^-} + \Delta d_{t^+} + \Delta o_{t^+} - p_{t^+}. \end{aligned}$$

Note that we do not impose a nonnegativity constraint on m_{t^-}. We interpret this ability for reserves to become negative in the morning as a temporary intraday overdraft provided by the central bank.[11] This overdraft plays an important role in our results.

Late Afternoon Deposit Shocks At the end of the afternoon, traditional banks are subject to a deposit shock Δd_{t^+} that results in reserve transfers. These shocks are meant to capture that demandable deposits have stochastic maturity from the point of view of banks, and therefore carries liquidity risk. Banks with net deposit outflows late in the day have to transfer reserves to other banks and may end up with less reserves than required. As in Poole (1968), this feature generates a motive in banks for holding reserves as buffer. Moreover, shadow banks run overdrafts at their clearing banks to temporarily fund their investments before borrowing in repo markets and generate additional intraday flows.

We specify the intraday inflow and outflow of deposits $\Delta d^i_{t^+}$ and $\Delta d^o_{t^+}$ such that $\Delta d_{t^+} = \Delta d^i_{t^+} - \Delta d^o_{t^+}$. The intraday outflow of deposits is defined as:

$$\Delta^o d_{t^+} = d_{t^-} \mu^o_{t^+} + d_{t^-} \varepsilon^o_{t^+},$$

where μ^o_t is the average outflow per unit of deposit[12] and $\varepsilon^o_{t^+}$ is an idiosyncratic shock distributed according to a uniform distribution truncated over $[0, \sigma]$ where $\sigma < 1$. Similarly, the intraday inflow of deposits is defined as:

$$\Delta^i d_{t^+} = d_{t^-} \mu^i_{t^+} + d_{t^-} \varepsilon^i_{t^+},$$

where $\mu^i_{t^+}$ is the average inflow per unit of deposit[13] and $\varepsilon^o_{t^+}$ is an idiosyncratic shock

[11] In the US, the Fed allows qualifying banks to overdraw on their Federal Reserve accounts in order to make payments via the payment system Fedwire. Banks must ensure that their accounts are not in a negative position at the end of the day, otherwise they would need to borrow with penalty at the discount window.

[12] The average outflow of deposits is given by:

$$\mu^o_{t^+} = \frac{\max\{0, -\Delta D^h_{t^+}\}}{D_{t^-}}.$$

See Appendix B for more details.

[13] The average inflow of deposits is given by:

$$\mu^i_{t^+} = \frac{\max\{0, \Delta D^h_{t^+}\}}{D_{t^-}}.$$

See Appendix B for more details.

distributed according to a uniform distribution truncated over $[0, \sigma]$. Note that the average outflow $\mu^o_{t^+}$ is not equal to the average inflow $\mu^i_{t^+}$ as the aggregate demand for deposits changes in the afternoon following the household preference shock.

Bank Regulation To explore the role of regulation in the repo market, we posit that traditional banks are subject to three regulatory constraints corresponding to existing rules on banks' liquidity management. First, to nest Poole's (1968) model, we assume banks have to satisfy a required reserve ratio on deposits. At the end of the afternoon, the reserve requirement is met if

$$m_{t^+} \geq \chi^m d_{t^-}, \tag{RR}$$

where χ^m is set by the regulator. If the reserve requirement is not satisfied, the bank has to borrow reserves at the discount window at additional cost $r^w > 0$.[14]

Second, when choosing their asset portfolio, banks have to meet the following requirement to satisfy LCR:

$$\rho k_{t^-} + m_{t^-} - \chi^m d_{t^-} \geq \chi^d d_{t^-}, \tag{LCR}$$

where the left-hand side is the total HQLA portfolio, composed of the fraction capital that meets the LCR requirements ρ, the collateral assets received or used in the repo transactions, and reserves in excess of the reserve requirement. On the right-hand side, $\chi^d < 1$ is the estimated outflow of deposits for a thirty-day period.[15]

Finally, according to the intraday liquidity stress-test requirement (LST), banks have to hold enough liquid assets to cover their intraday outflows of deposit without relying on expected inflows. As only reserves can be used to meet the first day outflows of liquidity stress, the bank is compliant with LST if:

$$m_{t^-} - p_{t^+} - f_{t^+} - \max\left\{0, \chi^d d_{t^-} - \rho k_{t^-} - p_{t^+} - f_{t^+}\right\} \geq \chi^m d_{t^-} + \Delta^o d_{t^+}. \tag{LST}$$

That is, banks need to have a buffer of reserves to cover daylight outflows of deposits in excess of what is required in HQLA by LCR and used in repo transactions. These

[14]Thus, the discount window rate is given by $r^m_{t^-} + r^w$.

[15]See LCR 40 note 40.48 for assumption on σ for repo.

restrictions are set in the morning using stress test model internal to the bank such that this requirement is breached during the day with at most ζ percent chances.[16]

4.2 Agents' Problem

Traditional Banks In the morning, traditional banks consume, issue deposits d_{t^-}, invest in capital k_{t^-} and in reserves m_{t^-} and open credit lines in the form of daylight overdrafts o_{t^-} through its clearing house to shadow bankers. In the afternoon, they choose how much to lend in the repo market. The problem of traditional banks can be written in recursive form as:

$$V\big(n_{t^-}, p_{t^+-1}; X_{t^-}\big) = \max_{c_{t^-} \le n_{t^-}, k_{t^-} \ge 0, d_{t^-} \ge 0, m_{t^-}, o_{t^-} \ge 0} \mathbb{E}_{t^-}\left[\max_{p_{t^+}, f_{t^+}} \mathbb{E}_{t^+}\left\{c_{t^-} + \beta V\big(n_{t^-+1}, p_{t^+}; X_{t^-+1}\big)\right\}\right], \tag{1}$$

subject to the morning balance sheet constraint:

$$k_{t^-} + m_{t^-} + o_{t^-} = n_{t^-} - c_{t^-} + d_{t^-},$$

where returns on their portfolio are such that:

$$\begin{aligned} n_{t^-+1} = {} & n_{t^-} - c_{t^-} + k_{t^-} r^k_{t^-} + m_{t^+} r^m_{t^-} + o_{t^-} r^o_{t^-} + o_{t^+}\lambda + p_{t^+} r^p_{t^+} + f_{t^+} r^f_{t^+} - d_{t^+} r^d_{t^-} \\ & - (\chi d_{t^-} - m_{t^+}) r^w \mathbb{1}\{m_{t^+} < \chi^m d_{t^-}\}. \end{aligned}$$

The parameter β is the state price density for future dividends. The variables $r^k_{t^-}$, $r^m_{t^-}$, $r^o_{t^-}$, $r^d_{t^-}$, $r^p_{t^+}$, and $r^f_{t^+}$ are the returns (or interest rates) on capital, reserves, overdrafts, deposits, repo and fed funds, respectively. If at the end of the day the stock of reserves falls short

[16] For ease of exposition, we simplified the LST constraint in the main text. The full constraint that we use in the model is given by:

$$\begin{aligned} & m_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\} \\ & - \max\big\{0, \chi^d d_{t^-} - \rho k_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\}\big\} \ge \chi^m d_{t^+} + \Delta^o d_{t^+}. \end{aligned}$$

The first maximum operators enforce that the bank cannot rely on money markets in the afternoon ($\Delta p_{t^-} < 0$ and/or $f_{t^+} < 0$) to satisfy LST in the morning. The second maximum operators ensure that excess LCR liquidity is left unchanged. Indeed, if the bank borrows in repo markets ($p_{t^+} < 0$), it does not affect the LCR requirements since the additional collateral requirements are compensated with additional borrowed reserves. Similarly, if the bank borrows in fed funds markets ($f_{t^+} < 0$), it does not affect the LCR requirements since the additional reserves are compensated by cash outflows on the following day.

of the requirements from (RR), banks have to pay the discount window penalty r^w. We leave out a collateral requirement constraint for repos as it does not play any role in our analysis.

Shadow Banks In the morning, shadow banks consume and purchase treasuries $\underline{b}_{t^-}$ with an overdraft at a clearing house, a division of a traditional bank. Clearing banks hold deposits to fund these overdrafts in the morning. In the afternoon, shadow bankers transfer funds raised in repo markets to the clearing banks to net their position (see Appendix B). When still running an overdraft at the end of the day ($\underline{o}_{t^+} \geq 0$), they incur expensive overnight credit at a penalty cost $\lambda > 1/\beta - 1$ with their clearing bank.[17] This charge represents the opportunity cost for traditional banks to provide overnight short-term funds while they are constraint by (LST).

The problem of shadow banks can be written in recursive form as:

$$\underline{V}\big(\underline{n}_{t^-}; X_{t^-}\big) = \max_{\underline{c}_{t^-} \leq \underline{n}_{t^-}, \underline{b}_{t^-} \geq 0, \underline{o}_{t^-} \geq 0} \mathbb{E}_{t^-}\left[\max_{\underline{p}_{t^+}} \mathbb{E}_{t^+}\left\{\underline{c}_{t^-} + \beta \underline{V}\big(\underline{n}_{t^-+1}; X_{t^-+1}\big)\right\}\right], \tag{2}$$

subject to the morning balance sheet constraint:

$$\underline{b}_{t^-} = \underline{n}_{t^-} - \underline{c}_{t^-} + \underline{o}_{t^-},$$

where returns on their portfolio are such that:

$$\underline{n}_{t^-+1} = \underline{n}_{t^-} - \underline{c}_{t^-} + \underline{b}_{t^-} r^b_{t^-} + \underline{d}_{t^-} r^d_{t^-} - \underline{o}_{t^-} r^o_{t^-} - \underline{p}_{t^+} r^p_{t^+} - \lambda \max\{0, \underline{o}_{t^+}\}.$$

All variables have a similar interpretation as for banks—with an lower bar notation to designate variables specific to shadow banks. The intraday laws of motion for bonds $\underline{b}_{t^+}$ and repo $\underline{p}_{t^+}$ are given by:

$$\underline{b}_{t^-} = (1-\kappa)\underline{b}_{t^--1} + \Delta \underline{b}_{t^-},$$
$$\underline{o}_{t^+} = \underline{o}_{t^-} - \underline{p}_{t^+},$$

[17] Daylight overdrafts at the Bank of New York Mellon Corporation cost 60 bps per annum per minute for less than \$5 billion, and 120 bps per annum per minute for amounts greater than \$5 billion—all collateralized. Furthermore, the pricing of and reputational risk around daylight overdrafts are a strong deterrent for dealers to use overdrafts frequently and liberally. The daylight overdrafts turn into overnight general collateral (GC) repo if they are not paid back by "sunset". See Pozsar (2019).

where κ is the fraction of Treasuries that matures each period.

Central Bank The central bank controls the supply of liquid assets available to the banking sector by swapping reserves for T-bonds (and conversely) through open market operations. In other words, the central bank decides on the stock of reserves $M_{t\text{-}}$ and the amount of T-bonds removed from the market and held by the central bank $\overline{B}_{t\text{-}}$ subject to the balance sheet constraint:

$$\overline{B}_{t\text{-}} = M_{t\text{-}}.$$

The upper bar differentiates the central bank's holdings of T-bonds $\overline{B}_{t\text{-}}$ from the bonds issued by the Treasury $B_{t\text{-}}$. For simplicity, we assume that the central bank operates with zero net worth and transfers all seigniorage and discount window revenues to the Treasury.

Treasury The Treasury issues T-bonds against future tax liabilities of other agents and has access to reserves through its treasury account at the central bank. The issuance of T-bonds follows an exogenous stochastic process:

$$B_t = (1 - \kappa^b) B_{t\text{-}-1} + \Delta B_{t\text{-}}.$$

Lump-sum tax policies allow the Treasury to issue T-bonds. Correspondingly, the net present value of future tax liabilities must equal the outstanding amount of T-bonds minus the quantity of reserves in the treasury account: $T_{t\text{-}} + G_{t\text{-}} = B_{t\text{-}}$. The Treasury raises taxes $\tau_{t\text{-}}$ from households each period to balance its budget:

$$\tau_{t\text{-}} T_{t\text{-}} = r^b_{t\text{-}} B_{t\text{-}} + r^m_{t\text{-}} M_{t\text{-}} - r^m_{t\text{-}} G_{t\text{-}} - r^b_{t\text{-}} \overline{B}_{t\text{-}} - r^w M^{w+}_{t\text{-}},$$

where $M^{w+}_{t\text{-}}$ is the quantity of reserves borrowed at the discount window at the end of the day. Finally, we assume that the quantity of reserves in the treasury account $G_{t\text{-}}$ corresponds to the net inflow of reserves due to the issuance of T-bonds and tax payments:

$$G_{t\text{-}} = \max\{\Delta B_{t\text{-}} + \tau_{t\text{-}} T_{t\text{-}}, 0\}.$$

Because the central bank provides liquidity to the economy, reserves earn a liquidity premium $r^m_{t\text{-}} < r^b_{t\text{-}}$. Thus, the discounted value of future taxes is lower than the tax liability and the value of liquidity insurance benefit of reserves L_{t^-} is given by:

$$L_{t^-} = T_{t^-} - \mathbb{E}_{t^-}\left[\sum_{j=t+1}^{\infty} \beta^{j-t}\tau_{j^-}T_{t^-}\right].$$

Implicitely, these benefits are transferred to households in the form of tax rebates.

Households Households are risk neutral, maximize their lifetime utility of consumption, and discount the future at rate β. They also get utility from holding liquid assets such as deposits and repos. They also have to pay taxes $\tau^h_{t^-}$ to the Treasury where $t^h_{t^-}$ is the net present value of future tax liabilities. The problem of the households can be written in recursive form as:

$$V^h\big(n^h_{t^-}, p^h_{t^+-1}; X_{t^-}\big) = \max_{c^h_{t^-} \leq n^h_{t^-}, d^h_{t^-} \geq 0} \mathbb{E}_{t^+}\left[\max_{p^h_{t^+} \geq 0} \mathbb{E}_{t^-}\left\{U^h(c^h_{t^-}, d^h_{t^+}, p^h_{t^+}, \alpha^h_{t^+}) + \beta V^h\big(n^h_{t^-+1}, p^h_{t^+}; X_{t^-+1}\big)\right\}\right] \tag{3}$$

subject to the balance sheet constraint:

$$d^h_{t^-} + (1-\kappa^p)p^h_{t^+-1} + \ell^h_{t^-} = n^h_{t^-} - c^h_{t^-} + t^h_{t^-},$$

where returns on their portfolio are such that:

$$n^h_{t^-+1} = n^h_{t^-} - c^h_{t^-} + d^h_{t^+}r^d_{t^-} + p^h_{t^+}r^p_{t^+} - t^h_{t^-}\tau_{t^-}.$$

The intraday laws of motion for the stock of deposits is given by:

$$d^h_{t^+} = d^h_{t^-} - \Delta p^h_{t^+}$$

The relative demand for repo $p^h_{t^+}$ depends on the liquidity preference shock $\alpha^h_{t^+}$ realized in the afternoon. Thus, this shock generates uncertainty for the aggregate supply of short-term fund to shadow banks. The value of tax rebates from the treasury $\ell^h_{t^-}$ is non-tradable.

4.3 Equilibrium

Definition 1. *Given an initial allocation of all asset variables at $t = 0$, monetary policy decisions $\{M_{t^-} : t \geq 0\}$, fiscal policy decisions $\{B_{t^-}, \tau_{t^-} : t \geq 0\}$, and household's liquidity preference shocks $\{\alpha^h_{t^+} : t \geq 0\}$, a* ***sequential equilibrium*** *is a set of adapted stochastic processes for (i) prices $\{r^k_{t^-}, r^m_{t^-}, r^o_{t^-}, r^b_{t^-}, r^d_{t^-}, r^p_{t^+}, r^f_{t^+} : t \geq 0\}$; (ii) individual controls for traditional banks $\{c_{t^-}, k_{t^-}, m_{t^-}, o_{t^-}, d_{t^-}, p_{t^+}, f_{t^+}, : t \geq 0\}$, (iii) shadow banks $\{\underline{c}_{t^-}, \underline{b}_{t^-}, \underline{o}_{t^-}, \underline{p}_{t^+} : t \geq 0\}$; (iv) households $\{c^h_{t^-}, d^h_{t^-}, p^h_{t^+} : t \geq 0\}$; (v) value of liquidity services and tax liabilites $\{\ell^h_{t^-}, L^h_{t^-}, t^h_{t^-}, T_{t^-} : t \geq 0\}$; and (vi) agents' net worth $\{n^h_{t^-}, n_{t^-}, \underline{n}_{t^-} : t \geq 0\}$ such that:*

1. *Agents solve their respective problems defined in equations (1), (2), and (3).*

2. *Morning markets clear:*

   (a) *capital:* $\int_0^1 k_{t^-}(i)di = K_{t^-}$,

   (b) *deposits:* $\int_0^1 d^h_{t^-}(h)dh = \int_0^1 d_{t^-}(i)di$,

   (c) *T-bonds:* $\int_0^1 \underline{b}_{t^-}(j)dj + \overline{B}_{t^-} = B_{t^-}$,

   (d) *reserves:* $\int_0^1 m_{t^-}(i)di + G_{t^-} = M_{t^-}$,

   (e) *overdrafts:* $\int_0^1 o_{t^-}(i)di = \int_0^1 \underline{o}_{t^-}(j)dj$,

   (f) *output:* $\int_0^1 c^h_{t^-}(h)dh + \int_0^1 c_{t^-}(i)di + \int_0^1 \underline{c}_{t^-}(j)dj = aK_{t^-}$,

3. *Afternoon markets clear:*

   (g) *repo:* $\int_0^1 p_{t^+}(i)di + \int_0^1 p^h_{t^+}(h)dh = \int_0^1 \underline{p}_{t^+}(j)dj$,

   (h) *fed funds:* $\int_0^1 f_{t^+}(i)di = 0$,

4. *The aggregation of liquidity services and tax liabilities are consistent:*

$$\int_0^1 \ell^h_{t^-}(h)dh = L_{t^-},$$
$$\int_0^1 t^h_{t^-}(h)dh = T_{t^-}.$$

5 Theoretical Analysis

In this section, we study the implications of the intraday liquidity constraint. Then, we derive asset pricing equations for each asset in the economy. These equations depend on the constraints banks face, which in turn depends on aggregate portfolio choices and intraday flows. As a benchmark, we first derive the model's implications for an economy in which only the reserve requirement is sometimes binding. We then study the dynamics of an economy in which the intraday liquidity constraint can also bind. All proofs are relagated to Appendix A.

To streamline our analysis, we make some restrictions. First, we restrict the analysis to monetary and fiscal policies that are constant over time. That is, $M_{t^-} = M$ and $B_{t^-} = B$ for all t. Consequently, $T_{t^-} = T$ for all t as well. Second, we limit the size of feasible fiscal policies such that the balance sheet of the Treasury cannot be larger that the total value of productive capital: $B < q_{t^-}$. Third, in Assumption 1, we restrict the parameter space such that (i) the price of capital is constant over time and (ii) $\phi^p_{t^-} = 1$ is never an equilibrium. Fourth, we restrict the analysis to the set of equilibria where there are enough reserves such that LST does not constrain the issuance of deposits by traditional banks.[18]

Assumption 1 (Parameter Space). *The set of parameters is such that* $\rho > \frac{a}{1/\beta - 1}$ *and* $\chi^d < 1/2$.

5.1 Solving

Liquidity Stress Test Constraint The first step of the analysis is to decompose banks' intraday liquidity constraint according to the composition of HQLAs available. First, we define the size of the reserve buffer required to satisfy the LST requirement in equation (LST) as:

$$\Phi^d_{t^-} \equiv \sigma \Phi^{-1}\left((1-\zeta)\Phi\left(\sigma^{-1}\right) + \zeta \Phi(0)\right) + \mu^o_{t^-},$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. The intraday liquidity constraint can take two forms depending on whether reserves have

[18] This is a limit case in which LST is so restrictive that the central bank loses the control of the interest on reserves as at the margin the interest on reserves is not pinned down by the quantity of reserves but by the tightness of LST and $r^m_{t^-} + \phi^m_{t^-} r^w < r^k$.

to be used to satisfy the liquidity coverage ratio LCR. In the situation in which reserves are needed to satisfy LCR, they are required to comply with:

$$m_{t^-} - \chi d_{t^-} + \varphi k_{t^-} + (1-\kappa^p) p_{t^+-1} \geq \Phi^d_{t^-} d_{t^-}. \tag{LST-}$$

As this constraint does not involve the issuance of new repos in the afternoon, it is only binding in the morning—for this reason, we refer to this constraint as (LST-). Similarly, if there is enough eligible collateral such that reserves do not need to be used to satisfy LCR, traditional banks need to satisfy:

$$m_{t^-} - \max\{0, \Delta p_{t^+}\} \geq \Phi^d_{t^-} d_{t^-}. \tag{LST+}$$

In that case, the constraint applies to the issuance of repo in the afternoon and, hence, denoted LST+.[19] LST is binding if and only if either LST- or LST+ binds.

We denote the morning probability that a repo spike occurs in the afternoon by:

$$\phi^p_{t^-} \equiv \mathbb{P}_{t^-}\left[\Delta p^h_{t^+} < \underline{o}_{t^-} + \Phi^d_{t^-} d_{t^-} - m_{t^-}\right].$$

In short, a repo spike occurs when the supply of repo by households is lower than the minimum required to satisfy LST+ and net the shadow bankers' overdrafts. For notational convenience, if in the afternoon a repo spike occurs, we set $\phi^p_{t^+} = 1$ such that $\phi^p_{t^-} = \mathbb{P}_{t^-}\left[\phi^p_{t^+} = 1\right]$.

Bond and Capital Markets Shadow banks invest in capital and Treasury bonds until the return on their portfolio equals the expected cost of funds in the repo market:

$$r^b_{t^-} = \mathbb{E}_{t^-}[r^p_{t^+}]. \tag{4}$$

As traditional bankers have the same discount factor as shadow bankers, the return on capital is equal to the return on T-bonds, on the condition that LST- does not bind. That

[19] Note that since the targets are set in the morning, we assume that banks do not update their forecast of aggregate deposit flows by observing the repo market rate in the afternoon. This assumption is consistent with liquidity management practice of the banks and simplifies the analysis.

is,

$$r^b_{t^-} = r^k_{t^-}.$$

If LST- binds, there is a collateral premium on capital and reserves and $r^k_{t^-} < r^b_{t^-}$.

Reserves By holding reserves, traditional banks get the interest on reserves, do not have to borrow at the discount window, and in case there is a repo spike, they can lend an additional units of repo. Thus, we get that:

$$r^m_{t^-} + \phi^m_{t^-} r^w + \phi^p_{t^-}(\mathbb{E}_{t^-}[r^p_{t^+}|\phi^p_{t^+} = 1] - r^m_{t^-} - \phi^{m|p}_{t^+} r^w) = r^k_{t^-},$$

where $\phi^m_{t^-} = \mathbb{P}_{t^-}\left[m_{t^-} + \Delta d_{t^+} - \Delta p_{t^+} < \chi d_{t^-}\right]$ is the probability of having to borrow at the discount window conditional on the information available in the morning and $\phi^{m|p}_{t^+} = \mathbb{P}_{t^+}\left[\Phi^p_{t^-} + \Delta d_{t^+} < \chi d_{t^-}\right]$ is the probability of having to borrow at the discount window when constrained by LST+. Thus, the liquidity premium on reserves $r^k_{t^-} - r^m_{t^-}$ depends on the tightness of the reserve requirement and the probability of a repo spike.

5.2 A Benchmark Without Regulation

We first look at a benchmark case with the reserve requirement as the only binding constraint. This case corresponds to the setting studied in Poole (1968) extended to include a repo market. We study how the repo market and the fed funds market interact with each other. In this benchmark case, banks act as arbitrageurs between these two markets and ensure the two rates are equal at all times. Consequently, changes in the repo demand from shadow banks also affect the demand curve for reserves in the fed funds market, which the central bank needs to take into account to meet its target.

We first derive the demand for repo financing. Shadow banks fund themselves in the repo market unless the repo rate is higher than the daylight overdraft cost. That is,

$$\underline{p}_{t^+} = \begin{cases} \underline{k}_{t^-} + \underline{b}_{t^-} - \underline{e}_{t^-} & \text{if} \quad r^p_{t^+} < \lambda, \\ (0, \underline{k}_{t^-} + \underline{b}_{t^-} - \underline{e}_{t^-}) & \text{if} \quad r^p_{t^+} = \lambda, \\ 0 & \text{if} \quad r^p_{t^+} > \lambda. \end{cases}$$

When the repo rate $r^p_{t^+}$ is below the daylight overdraft cost λ, shadow banks will finance all their portfolio with repos. When the repo rate is above λ, they prefer to pay the daylight overdraft cost on their entire portfolio, and their repo demand falls to zero. On the edge, when the repo rate is precisely equal to λ, shadow banks are indifferent between repo funding and overdrafts such that quantities are determined by the households' demand elasticity for repo. Proposition 1 characterizes the main properties of the unconstrained equilibrium for money market rates.

Proposition 1. *In an economy in which LST never binds, the repo rate is always equal to the fed funds rate, and both of these rates are bounded by the interest on reserves below and the discount window rate above:*

$$r^m_{t^-} \leq r^f_{t^+} = r^p_{t^+} \leq r^m_{t^-} + r^w.$$

This proposition unfolds directly from the first-order conditions for reserves and repo holdings:

$$r^m_{t^-} + r^w \phi^m_{t^+} = r^f_{t^+} = r^p_{t^+}, \tag{5}$$

where $\phi^m_{t^+} \equiv \mathbb{P}_{t^+}\left[m_{t^-} + \Delta d_{t^+} - \Delta p_{t^+} < \chi d_{t^-}\right]$ is the probability of having to borrow at the discount window conditional on available information in the afternoon. According to equation (5), banks' first-order conditions translate into a no-arbitrage condition between repos and fed funds rates. As $\phi^m_{t^+}$ is between zero and one, both rates are contained within the bounds of the interest rate on reserve $r^m_{t^-}$ and the discount window rate $r^m_{t^-} + r^w$ when LST+ is not binding. When unconstrained, banks will supply any quantity necessary for rates to adjust to the interest rate paid on reserves plus a liquidity premium on reserves—the marginal benefit of lowering the probability of resorting to the discount window. This outcome corresponds to the traditional result of Poole (1968) for the fed funds market; here extended to the repo market.

[Figure 11 about here]

Figure 11 plots the equilibrium in the two markets. The upper left panel corresponds to the traditional corridor diagram and shows the fed funds rate $r^f_{t^+}$ as a function of the quantity of reserves. The downward blue line is the demand schedule for reserves, which is a negative function of the fed funds rate. When the fed funds rate is high, banks want to lend more in fed funds (and repo) markets and therefore have less precautionary demand for reserves. At the top, this demand schedule is bounded by the discount window rate as this rate is the worst rate possible when facing a liquidity shock. At the bottom, it is bounded by the interest on reserves: the value of holding reserves when there are no liquidity benefits. The vertical red line represents the supply of reserves as provided by the central bank.

The upper right panel displays the simultaneous equilibrium in the repo market. The repo supply schedule in red is an increasing function of the repo rate $r^p_{t^+}$—higher repo rates incentivize banks to lend more in repo markets at the cost of resorting to the discount window more often. The blue line represents the excess repo demand from shadow banks: the inelastic shadow bank demand to fund their overdraft position minus the supply by the household sector with elasticity ζ. The lower the repo rates, the less the cash balance households allocate to repos with shadow banks—hence, the larger the excess demand for repo from shadow banks. When rates reach , shadow banks prefer to pay the daily overdraft cost rather than borrow in repo markets. This outside option, therefore, caps the repo rate.

The two lower panels (b) and (c) illustrate how the equilibrium adjusts to a negative shock to households' preference for repos. As households want to hold less repo, shadow banks' excess demand for repos is larger—resulting in the blue line moving rightward in the right panel. As a consequence, banks want to lend more in repo markets to potentially benefit from higher rates. Doing so increases banks' liquidity risk and, hence, precautionary motives for holding reserves. The demand for reserves, therefore, increases in the left panel as well. Panel (c) shows that, to keep the fed funds rate on target, the central bank needs to increase the supply of reserves to banks, allowing these to lend in the repo market without increasing their liquidity risk.

In this benchmark economy, banks always act as arbitrageurs between money market segments. In doing so, they play the essential role of transporting the liquidity from the market for reserves into the broader repo market. A key assumption that makes these arbitrage always possible—however large the repo market—is that banks can go to a negative reserves balance within the day through their overdraft at the central bank. For large amounts of repos to be settled, banks may have to run a negative overdraft balance during the day.

5.3 Intraday Liquidity Regulation

In this section, we characterize an economy that is subject to the LST+ constraint. We first show that when the constraint is binding, the repo rate disconnects from the fed funds rate and jumps above the opportunity cost of reserves. The model is qualitatively similar to an economy in which banks cannot run intraday overdrafts at the central bank. We then show that the probability that the system hits the constraint and generate a repo spike depends both on monetary policy and fiscal policy, and has consequences for T-bond yields.

Money Markets This first section explores the implications of LST+ for money markets. LST+ proves critical in preventing banks from intermediating liquidity from reserves markets to repo markets, resulting in spikes in repo rates. When LST+ binds for traditional banks, quantities of repo supplied are constrained and traditional banks cannot take advantage of high repo rates. In this case, the market is rationed, shadow banks are short of funds, the repo rate jumps above the opportunity cost of reserves, potentially up to the overnight credit cost λ, and the results from Proposition 1 does not hold anymore.

Proposition 2. *When LST+ is binding, that is $\phi^p_{t^+} = 1$:*

- *the repo rate jumps above the discount window rate;*
- *there are no transactions in the fed funds market.*

Proposition 2 describes the effect of the LST+ constraint—when binding—on money market rates. When LST+ binds for traditional banks, quantities of repo supplied are constrained and traditional banks cannot take advantage of high repo rates. Banks would like to lend more in repo market but cannot because they don't have enough reserves to settle the corresponding transactions. Because the liquidity risk of shadow banks is not bounded by the discount window spread r^w—to which they don't have access—but rather by their overnight credit cost λ, the repo rate can jump above the discount window rate. Finally, the repo rate disconnects from the fed funds rate. As no bank is able to lend reserves, no transaction occurs in the fed funds market and the fed funds rate is never observed above the discount window rate.

Proposition 3. *An economy in which LST+ is sometimes binding, that is $\phi^p_{t^-} > 0$,*

- *is characterized by jumps in repo rates and;*
- *is qualitatively equivalent to an economy without intraday overdrafts at the central bank.*

Proposition 3 characterizes an economy in which LST+ is sometimes binding, depending on shock realizations and agents' portfolio choices. When the LST+ constraint binds, the repo rate spikes up to the rate required to incentivize households to provide more cash in the money markets or until it reaches λ. Such dynamics also characterize an alternative setting in which banks would not have access to intraday overdrafts at the central bank. In such an economy, banks would stop lending after having exhausted all reserves in their balance sheets. In both case, there is a hard constraint on the amount banks can lend in repo markets.

[Figure 12 about here]

Figure 12 illustrates this result. The upper panels represent an economy without the LST constraint. The left panel (a) illustrates a possible equilibrium for the repo market when the supply of reserves available is larger than the quantity of repo used to lend in the repo market. In this example, the repo demand meets repo supply—and determines a repo rate—inside the IOR-DW corridor. The right panel (b) displays an example of equilibrium with a larger repo demand. The quantity of reserves available is not large enough for banks to be able meet this repo demand. Hence, some banks end up with negative intraday reserve balances—an overdraft—at the central bank. These temporary intraday loans of reserves from the central bank allow banks to fully meet the demand but increase the probability of having to borrow at the discount window at the end of the day.

Lower panel (c) proposes an example in an economy in which banks are subject to the LST constraint and repo demand is low. In this case, as in panel (a), there are enough reserves available relative to demand for the market to clear with repo rates inside the corridor. Panel (d) plots an economy with the LST constraint and a large repo demand. Despite a larger total amount of reserves than for the upper panels (a) and (b), there are not enough reserves available to meet the repo demand because a large portion is used and *locked in* to meet LST. In contrast to panel (c), banks cannot extend their repo lending beyond the reserves they have available through an overdraft because doing so would require to first fully deplete their reserve account, which is not possible under LST.

In that sense, LST is responsible for removing the elasticity provided by intraday overdrafts at the Fed. LST therefore creates an inelastic kink in the repo supply at the locus where all available reserves have already been used. When the repo demand exceeds the quantity of available reserves, scarcity arises in the repo market and repo rates spike up to shadow banks' overnight credit charge λ.

Fiscal Policy This second section explores the implication of fiscal policy on money markets in an economy with LST with comparative statics. Larger T-bond stock and issuance is found to increase the probability of seeing a surge in repo rates through three distinct channels.

Proposition 4. *In an economy in which LST+ is sometimes binding, the short-term impact of an increase in the quantity of T-bonds results in an increase of the probability of a repo spike through three channels:*

- *a larger quantity of T-bonds outstanding increases the demand for shadow bank repo financing;*
- *a larger treasury account decreases the supply of reserves available to banks;*
- *a larger spot issuance of T-bonds increases the settlement needs for reserves.*

There are three channels through which the issuance of new Treasuries influences the probability of a spike. First, an increase in the supply of T-bonds generates a larger need for excess repo financing from shadow banks. This feature arises in the model as a consequence of all T-bonds having to be held by shadow banks whereas households wants to invest a part of their tax liabilities in deposits. With a large excess demand for repo, the economy moves closer to hitting the LST constraint and the probability of a spike increases. Second, by increasing the size of the treasury account on a given date, Treasury debt issuance withdraws from the volume of reserves available to banks to settle their repo lending. Third, by increasing predicted reserves outflows for traditional banks as shadow bankers make larger purchases, new Treasury debt issuance also increases the quantity of reserves needed under LST. These two latter channels shift the inelastic kink in the repo supply to the left and increase the chance of a repo spike.

Monetary Policy We then explore how monetary policy can affect money market conditions. We find that it reduces the probability of a repo spike in two ways.

Proposition 5. *In an economy in which LST+ is sometimes binding, the short-term impact of an increase in the central bank portfolio results in a decreases of the probability of a repo spike through two channels:*

- *a lower quantity of T-bonds have to be absorbed by shadow banks;*
- *a larger quantity of reserves allows traditional banks to lend more in the money markets.*

When the central bank increases its balance sheet, it removes T-bonds from shadow banks' balance sheets and adds reserves to banks' balance sheets. Both sides decrease the probability of a spike. More reserves means that banks have more room before hitting LST. Less T-bonds means less excess repo demand from shadow banks and less reserves required for settlement.

Treasury Yields Last, we characterize in Proposition 6 how T-bond yields are affected by an increase in the probability of a repo spike.

Proposition 6. *In an economy in which the LST is sometimes bindings, an increase in the probability of a repo spike is associated with an increase in T-bond yields.*

In our dynamic setting, when the repo market is more likely to be disrupted, shadow banks require a larger return to hold T-bonds to compensate for increased liquidity risk. This property of the model directly unfolds from equation 4.

Long-Term Long-term means including the impact of the change of policy on the net worth of agents, i.e. changes in profitability and risk of different financial activities gives incentives to agents to increase or decrease their net worth.

Proposition 7. *If the equilibrium wealth share of households is perfectly inelastic to changes in policy decisions (M, B) and the repo rate can jump to a rate sufficiently high, $\lambda > \underline{\lambda}$, then the long-term impact of a change in policy decision M or B dampens the short-term impact.*

6 Conclusion

This paper investigates the role of regulation in explaining observed spikes in the repo market. Our analysis points to intraday liquidity requirements as constraining the supply of repo when the supply of reserves is low enough. Our model helps in explaining the observed pattern of mostly quiet money markets with sometimes extreme dislocations. We highlight that various factors, including Treasury debt issuance and tax payment, may simultaneously reduce the supply of reserves and making the intraday liquidity requirement scarcer, thereby leading to a sharp increase in the probability of a repo spike.

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Appendices

A Omitted Derivations and Proofs

State Variables and Recursive Formulation As a consequence of the risk-neutrality of preferences and linearity of technology, agents of a same type are indifferent between portfolio allocations in proportion of their net worth. Hence, we only have to track the distribution of wealth between types and not within types to pin down market prices. As the whole economy scales with the quantity of capital. Without loss of generality, we set the quantity of capital to 1.

Thus, the two state variables of the economy are the quantity of wealth in the hands of the traditional and shadow banking sectors n_{t^-} and $\underline{n}_{t^-}$. Thus, the wealth in the hands of households is simply given by: $n^h_{t^-} = q_{t^-} \times 1 - n_{t^-} - \underline{n}_{t^-}$.

From here on, we characterize the economy as a recursive Markov equilibrium.

Definition 2 (Markov Equilibrium). *Given the policy parameters M and B, a Markov equilibrium in $X_{t^-} = (n_{t^-}, \underline{n}_{t^-})$ and $X_{t^+} = (X_{t^-}, \alpha^h_{t^+})$ is a set of functions $g_{t^-} = g(X_{t^-})$ and $g_{t^+} = g(X_{t^+})$ for (i) prices $\{r^k_{t^-}, r^m_{t^-}, r^o_{t^-}, r^b_{t^-}, r^d_{t^-}, r^p_{t^+}, r^f_{t^+}\}$; (ii) individual controls for traditional banks $\{c_{t^-}, k_{t^-}, m_{t^-}, o_{t^-}, d_{t^-}, p_{t^+}, f_{t^+}\}$, (iii) shadow banks $\{\underline{c}_{t^-}, \underline{b}_{t^-}, \underline{o}_{t^-}, \underline{p}_{t^+}\}$; (iv) households $\{c^h_{t^-}, d^h_{t^-}, p^h_{t^+}\}$; (v) value of liquidity services $\{\ell^h_{t^-}, L^h_{t^-}\}$ such that:*

1. *Agents solve their respective problems defined in equations (1), (2), and (3).*
2. *The balance sheet constraint of the central bank and the treasury are satisfied.*
3. *Morning markets clear:*

   (a) *capital:* $\int_0^1 k_{t^-}(i)di = K_{t^-}$,

   (b) *deposits:* $\int_0^1 d^h_{t^-}(h)dh = \int_0^1 d_{t^-}(i)di$,

   (c) *T-bonds:* $\int_0^1 \underline{b}_{t^-}(j)dj + \overline{B}_{t^-} = B$,

   (d) *reserves:* $\int_0^1 m_{t^-}(i)di + G_{t^-} = M$,

   (e) *overdrafts:* $\int_0^1 o_{t^-}(i)di = \int_0^1 \underline{o}_{t^-}(j)dj$,

   (f) *output:* $\int_0^1 c^h_{t^-}(h)dh + \int_0^1 c_{t^-}(i)di + \int_0^1 \underline{c}_{t^-}(j)dj = aK_{t^-}$,

4. *Afternoon markets clear:*

$$\begin{aligned}
&\text{(g) repo:} && \textstyle\int_0^1 p_{t^+}(i)di + \int_0^1 p^h_{t^+}(h)dh = \int_0^1 \underline{p}_{t^+}(j)dj,\\
&\text{(h) fed funds:} && \textstyle\int_0^1 f_{t^+}(i)di = 0,
\end{aligned}$$

5. *The laws of motion for the state variables n_{t^-} and $\underline{n}_{t^-}$ are consistent with equilibrium functions and demographics,*

6. *The aggregation of liquidity services and tax liabilities are consistent:*

$$\int_0^1 \ell^h_{t^-}(h)dh = L_{t^-},$$
$$\int_0^1 t^h_{t^-}(h)dh = T.$$

Traditional Bankers We can write the problem of traditional bankers in recursive form as:

$$V\left(n_{t^-}; X_{t^-}\right) = \max_{c_{t^-} \leq n_{t^-}, k_{t^-} \geq 0, m_{t^-} \geq 0, o_{t^-} \geq 0, d_{t^-} \geq 0} \mathbb{E}_{t^-}\left[\max_{p_{t^+}, f_{t^+}} \mathbb{E}_{t^+}\left\{c_{t^-} + \beta V\left(n_{t^-+1}; X_{t^-+1}\right)\right\}\right]$$

subject to the morning balance sheet constraint:

$$k_{t^-} + m_{t^-} + o_{t^-} = n_{t^-} - c_{t^-} + d_{t^-},$$

where returns on their portfolio are such that:

$$\begin{aligned}
n_{t^-+1} = n_{t^-} - c_{t^-} + k_{t^-} r^k_{t^-} + m_{t^+} r^m_{t^-} + o_{t^-} r^o_{t^-} + o_{t^+}\lambda + p_{t^+} r^p_{t^+} + f_{t^+} r^f_{t^+}\\
- d_{t^+} r^d_{t^-} - (\chi^m d_{t^-} - m_{t^+}) r^w \mathbb{1}\{m_{t^+} < \chi^m d_{t^-}\}.
\end{aligned}$$

The LST constraint can be written as:

$$\begin{aligned}
&\mathbb{P}\big(m_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\}\\
&- \max\left\{0, \chi^d d_{t^-} - \rho k_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\}\right\} \geq \chi^m d_{t^-} + \Delta^o d_{t^+}\big) = \zeta.
\end{aligned}$$

With further algebra, we get

$$
\begin{aligned}
&m_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\} \\
&- \max\left\{0, \chi^d d_{t^-} - \rho k_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\}\right\} \geq \Phi^d_{t^-} d_{t^-},
\end{aligned}
$$

where the size of the reserve buffer is defined as:

$$
\Phi^d_{t^-} \equiv \chi^m + \sigma\left(1 - \zeta\right) + \mu^o_{t^-},
$$

where $F(\cdot)$ is the distribution of the outflow deposit shock $\varepsilon^o_{t^+}$. This constraint can be split into three constraints depending on whether p_{t^+} and $\chi d_{t^-} - \rho k_{t^-} - p_{t^+}$ are positive:

$$
\begin{aligned}
\lambda^\ell_{t^-} :& \quad m_{t^-} \geq \Phi^d_{t^-} d_{t^-}, \\
\lambda^m_{t^-} :& \quad m_{t^-} - \chi^d d_{t^-} + \rho k_{t^-} \geq \Phi^d_{t^-} d_{t^-}, \\
\lambda^p_{t^+} :& \quad m_{t^-} - p_{t^+} \geq \Phi^d_{t^-} d_{t^-}.
\end{aligned}
$$

The LST constraint is satisfied if and only if all of these constraints are satisfied. The lambdas denote the corresponding Lagrangian multipliers. Note that since $\Phi^d_{t^-} > 0$, LCR is always satisfied if LST is satisfied. The constraint $\lambda^m_{t^-}$ corresponds to LST- in the main text while $\lambda^\ell_{t^-}$ and $\lambda^p_{t^+}$ correspond to LST+.

We set $f_{t^+} = 0$ in the $\lambda^p_{t^+}$ constraint. This is without loss of generality. Indeed, as we show in the proof of Proposition 2, if $\lambda^p_{t^+} > 0$ then $f_{t^+} = 0$.

Capital is chosen in order to satisfy the balance sheet constraint. The first-order condition with respect to consumption c_{t^-} is given by:

$$
1 = \beta \mathbb{E}_{t^-}[V'(\underline{n}_{t^-+1})(1 + r^k_{t^-})] + \rho \lambda^m_{t^-}.
$$

Together with the envelope condition:

$$
V'(\underline{n}_{t^-}) = \beta \mathbb{E}_{t^-}[V'(\underline{n}_{t^-+1})(1 + r^k_{t^-})] + \rho \lambda^m_{t^-},
$$

we get:

$$
1 = \beta(1 + r^k_{t^-}) + \rho \lambda^m_{t^-}. \tag{6}
$$

The first-order condition with respect to reserves m_{t^-} is given by:

$$r^m_{t^-} + \phi^m_{t^-} r^w + \lambda^\ell_{t^-} + (1-\rho)\lambda^m_{t^-} + \mathbb{E}_{t^-}\left[\lambda^p_{t^+}\right] = r^k_{t^-}, \tag{7}$$

where

$$\phi^m_{t^-} = \mathbb{P}_{t^-}\left[m_{t^-} + \Delta d_{t^+} - p_{t^+} - f_{t^+} + o_{t^-} < \chi^m d_{t^-}\right].$$

The first-order condition with respect to deposits d_{t^-} is given by:

$$\begin{aligned} r^d_{t^-} - r^k_{t^-} + \mu^d_{t^-}(r^d_{t^-} - r^m_{t^-}) = (\mu^m_{t^-} - \phi^m_{t^-}\chi^m)r^w - \lambda^p_{t^-}\Phi^p_{t^-} - \mathbb{E}_{t^-}[\lambda^p_{t^+}]\Phi^p_{t^-} \\ - (\chi^d + \Phi^p_{t^-})\lambda^m_{t^-} + \rho\lambda^m_{t^-}, \end{aligned} \tag{8}$$

where

$$\mu^m_{t^-} = \mathbb{E}_{t^-}\left[\Delta d_{t^+}/d_{t^-} \mathbb{1}\left\{m_{t^-} + \Delta d_{t^+} - p_{t^+} - f_{t^+} + o_{t^-} < \chi^m d_{t^-}\right\}\right].$$

The first-order condition with respect to repo transactions p_{t^+} is given by:

$$r^p_{t^+} = r^m_{t^-} + \phi^m_{t^+} r^w + \lambda^p_{t^+}, \tag{9}$$

where

$$\phi^m_{t^+} = \mathbb{P}_{t^+}\left[m_{t^-} + \Delta d_{t^+} - p_{t^+} - f_{t^+} + o_{t^-} < \chi^m d_{t^-}\right].$$

The first-order condition with respect to fed fund transactions f_{t^+} is given by:

$$r^f_{t^+} = r^p_{t^+}. \tag{10}$$

Last, the first-order condition with respect to daily overdrafts o_{t^-} is given by:

$$r^o_{t^-} + \phi^p_{t^-}\lambda + (1-\phi^p_{t^-})(r^m_{t^-} + \phi^m_{t^-} r^w) = r^k_{t^-}. \tag{11}$$

Shadow Banks We can write the problem of traditional bankers in recursive form as:

$$\underline{V}\big(\underline{n}_{t^-};X_{t^-}\big) = \max_{\underline{c}_{t^-}\leq \underline{n}_{t^-},\underline{b}_{t^-}\geq 0,\underline{o}_{t^-}\geq 0} \mathbb{E}_{t^-}\left[\max_{\Delta \underline{p}_{t^+}} \mathbb{E}_{t^+}\left\{\underline{c}_{t^-} + \beta \underline{V}\big(\underline{n}_{t^-+1};X_{t^-+1}\big)\right\}\right]$$

subject to the morning balance sheet constraint:

$$\underline{b}_{t^-} = \underline{n}_{t^-} - \underline{c}_{t^-} + \underline{o}_{t^-}, \tag{12}$$

where returns on their portfolio are such that:

$$\underline{n}_{t^-+1} = \underline{n}_{t^-} - \underline{c}_{t^-} + \underline{b}_{t^-} r^b_{t^-} - \underline{o}_{t^-} r^o_{t^-} - \Delta \underline{p}_{t^+} r^p_{t^+} - \lambda \max\{0, \underline{o}_{t^-} - \Delta \underline{p}_{t^+}\}.$$

Daily overdrafts are chosen in order to satisfy the balance sheet constraint. If the repo rate is below the overnight credit charge λ, shadow bankers choose to fund the entirety of their daily overdrafts with repo. When the repo rate reaches λ, they are indifferent. That is,

$$\Delta \underline{p}_{t^+} = \begin{cases} \underline{o}_{t^-} & \text{if } r^p_{t^+} < \lambda, \\ [0, \underline{o}_{t^-}] & \text{if } r^p_{t^+} = \lambda. \end{cases} \tag{13}$$

Implicitly, we omit the possibility for $p_{t^+} < 0$ and $r^p_{t^+} > \lambda$ as it will never be an equilibrium outcome. Thus, we can rewrite the law of motion of wealth as:

$$\underline{n}_{t^-+1} = (\underline{n}_{t^-} - \underline{c}_{t^-})(1 + r^o_{t^-} + r^p_{t^+}) + \underline{b}_{t^-}(r^b_{t^-} - r^o_{t^-} - r^p_{t^+}).$$

The first-order condition with respect to consumption $\underline{c}_{t^-}$ is given by:

$$1 = \beta \mathbb{E}_{t^-}[\underline{V}'(\underline{n}_{t^-+1})(1 + r^o_{t^-} + r^p_{t^+})]$$

Together with the envelope condition:

$$\underline{V}'(\underline{n}_{t^-}) = \beta \mathbb{E}_{t^-}[\underline{V}'(\underline{n}_{t^-+1})(1 + r^o_{t^-} + r^p_{t^+})],$$

we get:

$$1 = \beta \mathbb{E}_{t^-}[1 + r^o_{t^-} + r^p_{t^+}]. \tag{14}$$

Lastly, the first-order condition with respect to bonds $\underline{b}_{t^-}$ is given by:

$$r^b_{t^-} = \mathbb{E}_{t^-}[r^o_{t^-} + r^p_{t^+}]. \tag{15}$$

Households We can write the problem of households in recursive form as:

$$V^h\big(n^h_{t^-}; X_{t^-}\big) = \max_{c^h_{t^-} \leq n^h_{t^-}, d^h_{t^-} \geq 0} \mathbb{E}_{t^-}\left[\max_{p^h_{t^+} \geq 0} \mathbb{E}_{t^-}\left\{U^h(c^h_{t^+}, d^h_{t^+}, p^h_{t^+}, \alpha^h_{t^+}) + \beta V^h\big(n^h_{t^-+1}; X_{t^-+1}\big)\right\}\right]$$

subject to the morning balance sheet constraint:

$$d^h_{t^-} + \ell^h_{t^-} = n^h_{t^-} + t^h_{t^-} - c^h_{t^-} \tag{16}$$

where returns on their portfolio are such that:

$$n^h_{t^-+1} = n^h_{t^-} - c^h_{t^-} + d^h_{t^+} r^d_{t^-} + p^h_{t^+} r^p_{t^+} - t^h_{t^-}\tau_{t^-}.$$

The Leontief aggregator implies that the relative demand of deposits and repo is such that:

$$(1 - \alpha^h_{t^+}) d^h_{t^+} = \alpha^h_{t^+} p^h_{t^+}.$$

Using the law of motion for deposits and repos, the utility function simplifies to:

$$U^h_{t^-} = c^h_{t^-} + \varphi \frac{(d^h_{t^-})^{1-\gamma}}{1-\gamma}.$$

Deposits are chosen in order to satisfy the balance sheet constraint. The first-order condition for $c^h_{t^-}$ is given by:

$$1 - \varphi(d^h_{t^-})^{-\gamma} = \beta \mathbb{E}_{t^-}[V^{h,\prime}(n_{t^-+1})(1 + r^d_{t^-})].$$

Together with the envelope condition:

$$V^{h,\prime}(n^h_{t^-}) = \varphi(d^h_{t^-})^{-\gamma} + \beta \mathbb{E}_{t^-}[V^{h,\prime}(\underline{n}_{t^-+1})(1 + r^d_{t^-})],$$

we get:

$$1 - \varphi(d^h_{t^-})^{-\gamma} = \beta(1 + r^d_{t^-}). \tag{17}$$

For the tax rebates to be fairly priced and consistent with aggregate quantities, the following pricing equation needs to be satisfied:

$$(1/\beta - 1)L_{t^-} = (1/\beta - 1)T - \tau_{t^-}T.$$

Equilibrium Because of the risk-neutrality of preferences, given the monetary and fiscal policies and the state variables $n^h_{t^-}$, n_{t^-}, and $\underline{n}_{t^-}$, we have a multiplicity of equilibria. This multiplicity in equilibria arises because the quantity of risk in the economy is not pinned down by concave stochastic discount factors. For example, as long as the expected returns on bonds and repo are identical, $r^b_{t^-} = \mathbb{E}_{t^-}[r^p_{t^+}]$, shadow bankers are indifferent between allocations with different risk of repo spike $\phi^p_{t^-}$ and consumption $\underline{c}_{t^-}$. However, once consumption is chosen, there is a unique equilibrium as the demand for deposits (as a function of the deposit interest rate $r^d_{t^-}$) by households is monotonically increasing and the supply of deposits by the traditional bankers is monotonically decreasing. Also note that since $\ell^h_{t^-}$ is non-tradable, it must be that $n^h_{t^-} \geq \ell^h_{t^-}$. Thus, the quantity that matters for the economy and the probability of a spike is the wealth of the agents post-consumption and net of the value of liquidity services defined as:

$$\begin{aligned}
\widetilde{n}^h_{t^-} &\equiv n^h_{t^-} - c^h_{t^-} - \ell^h_{t^-}, \\
\widetilde{n}_{t^-} &\equiv n_{t^-} - c_{t^-}, \\
\underline{\widetilde{n}}_{t^-} &\equiv \underline{n}_{t^-} - \underline{c}_{t^-}.
\end{aligned}$$

In short, given the state $(n^h_{t^-}, n_t, \underline{n}_{t^-})$, the agents are indifferent between any consumption profiles $(c_t, \underline{c}_t, c^h_t)$ that would lead to one of the multiple equilibria $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, \underline{\widetilde{n}}_{t^-})$. However, given $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, \underline{\widetilde{n}}_{t^-})$, there is a unique set of equilibrium morning prices and allocations, and given $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, \underline{\widetilde{n}}_{t^-}, \alpha_{t^+})$, there is a unique set of equilibrium afternoon prices and allocations. Thus, in the following sections we solve for the unique set of morning and afternoon prices and allocations given $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, \underline{\widetilde{n}}_{t^-}, \alpha_{t^+})$. This also implies that the constraints on consumption, $c^h_{t^-} \leq n^h_{t^-} + \ell^h_{t^-}$, $c_{t^-} \leq n_{t^-}$, $c^h_{t^-} \leq n^h_{t^-}$, are never binding as long as there exists at least one equilibrium with positive post-consumption wealth $(\widetilde{n}^h_{t^-} \geq 0, \widetilde{n}_{t^-} \geq 0, \underline{\widetilde{n}}_{t^-} \geq 0)$. Finally, since $\widetilde{n}^h_{t^-} + \widetilde{n}_{t^-} + \widetilde{\underline{n}}_{t^-} = q^k_{t^-}$, we only need to characterize the equilibria given $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, \alpha_{t^+})$.

Because the probability of having to borrow at the discount window is increasing in deposits d_{t^-} and decreasing in reserves m_{t^-} and overdrafts o_{t^-}, all banks will choose the same ratio of deposits to reserves and overdrafts. Since they are risk-neutral, the distribution of wealth across traditional banks does not matter for their liquidity risk management decisions. Furthermore, none of the constraint can be binding for one bank without being binding for all banks, otherwise the constraint bank would have incentives to change its allocation of consumption and portfolio to relax the constraint as its marginal valuation would not equal the market price. Thus, it is without loss of generality that we solve for a representative traditional bank. The same argument applies to shadow banks and households.

Lemma 1. *Given Assumption 1, we get that* $q_{t^-} = q = \frac{a}{1/\beta - 1}$*,* $\lambda^m_{t^-} = 0$*, and* $\phi^p_{t^-} < 1$ *for all equilibria* $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-})$*.*

Proof. We guess and verify that if $\rho > \frac{a}{1/\beta - 1}$, then $q_{t^-} = \frac{a}{1/\beta - 1}$ and $\lambda^m_{t^-} = 0$.

First, since $1 = \beta(1 + r^k_{t^-})$ and $1 + r^k_{t^-} = \mathbb{E}_{t^-}\left[\frac{a + q_{t^-+1}}{q_{t^-}}\right]$, we have that $q_{t^-} = q = \frac{a}{1/\beta - 1}$.

Second, we have that $\rho > q > \chi^d(q + (1-\kappa)B) > \chi^d d_{t^-}$ since $B < q$, $\chi^d < 1/2$, $d_{t^-} = \widetilde{n}^h_{t^-} + (1-\kappa)B$, and $\widetilde{n}^h_{t^-} \leq q$. Thus, $m_{t^-} - \chi^d d_{t^-} + \rho > \Phi^d_{t^-} d_{t^-}$ and $\lambda^m_{t^-} = 0$ as otherwise $m_{t^-} < \Phi^d_{t^-} d_{t^-}$.

Finally, since $q_{t^-} = \frac{a}{1/\beta - 1}$, we have that $r^k_{t^-} - 1/\beta - 1$. However, if $\phi^p_{t^-} = 1$, the first-order condition with respect to reserves m_{t^-} in equation (7) implies that $r^k_{t^-} = \lambda$, which is a contradiction. □

As mentioned in the main text, we restrict the analysis to the set of equilibria such that the quantity of reserves available jointly with LST are never constraining the issuance of deposits by traditional banks in the morning with condition C5. That is, the pair $(n^h_{t^-}, n_{t^-})$ and the set of parameters are such that $\lambda^\ell_{t^-} = 0$. When $\lambda^\ell_{t^-} > 0$, the equilibrium is a limit case in which LST is so restrictive that the central bank loses the control of the interest on reserves as at the margin the interest on reserves is not pinned down by the quantity of reserves but by the tightness of LST: $r^m_{t^-} + \phi^m_{t^-} r^w < r^k$.

Thus, from the shadow bankers' first-order conditions for consumption and bonds in equations (14) and (15), we have:

$$r^b_{t^-} = \mathbb{E}_{t^-}[r^o_{t^-} + r^p_{t^+}] = 1/\beta - 1.$$

Using the first-order condition of traditional bankers for reserves m_{t^-} in equation (7), we get:

$$r^m_{t^-} + \phi^m_{t^-} r^w + \mathbb{E}_{t^-}\left[\lambda^p_{t^+}\right] = r^k_{t^-}.$$

Using the first-order condition of traditional bankers with respect to repo transactions p_{t^+} in equation (9) yields:

$$r^p_{t^+} = r^m_{t^-} + \phi^m_{t^+} r^w + \lambda^p_{t^+},$$

When $\lambda^p_{t^+} > 0$, LST+ is binding, $r^p_{t^+} = \lambda$, and $\lambda^p_{t^+} = \lambda - r^m_{t^-} - \phi^m_{t^+} r^w$. When $\lambda^p_{t^+} = 0$, $r^p_{t^+} = r^m_{t^-} + \phi^m_{t^+} r^w$. Thus,

$$\mathbb{E}_{t^-}\left[r^p_{t^+}\right] = \phi^p_{t^-}\lambda + (1 - \phi^p_{t^-})(r^m_{t^-} + \phi^m_{t^-} r^w) = r^k_{t^-},$$

and

$$(1 - \phi^p_{t^-})(r^m_{t^-} + \phi^m_{t^-} r^w) + \phi^p_{t^-}\lambda = r^k_{t^-}.$$

Thus, if $\phi^p_{t^-} < 1$,

$$r^m_{t^-} = \frac{1/\beta - 1 - \phi^p_{t^-}\lambda}{1 - \phi^p_{t^-}} - \phi^m_{t^-} r^w.$$

From the first-order condition of traditional bankers with respect to daily overdrafts o_{t^-} in equation (11), we get:

$$r^o_{t^-} = r^k_{t^-} - \phi^p_{t^-}\lambda - (1 - \phi^p_{t^-})(r^m_{t^-} + \phi^m_{t^-} r^w) = 0.$$

Thus, at equilibrium, the following is indeed satisfied:

$$r^b_{t^-} = r^k_{t^-}.$$

Combining the first-order condition of households and traditional bankers with respect to deposits $d^h_{t^-}$ and d_{t^-} in equations (17) and (8) yields:

$$\frac{1-\varphi(d^h_{t^-})^{-\gamma}}{\beta}-1=\frac{r^k_{t^-}+\mu^d_{t^-}r^m_{t^-}+(\mu^m_{t^-}-\phi^m_{t^-}\chi^m)r^w-\phi^p_{t^-}(\lambda-r^m_{t^-}-\phi^m_{t^-}r^w)\Phi^d_{t^-}}{1+\mu^d_{t^-}}. \tag{18}$$

This equation pins down the aggregate quantity of deposits at equilibrium. In the following paragraph, we can characterize $\phi^m_{t^-}$, $\phi^m_{t^+}$, $\mu^m_{t^-}$, and $\phi^p_{t^-}$.

Firstly, note that from the accounting identities for deposits flows and overdrafts, we have that:

$$\mu^d_{t^+}d_{t^-}=\mu^i_{t^+}d_{t^-}-\mu^o_{t^+}d_{t^-}=-p^h_{t^+}$$

and

$$\Delta \underline{p}_{t^+}=\Delta o_{t^+}=p_{t^+}+p^h_{t^+}.$$

This means that at the equilibrium, the end-of-the-day stock of reserves is simply given by:

$$m_{t^+}=m_{t^-}+\Delta d_{t^+}+\Delta o_{t^+}-p_{t^+}=m_{t^-}=M-\kappa B.$$

The probabilities of having to borrow at the discount window $\phi^m_{t^-}$ and $\phi^m_{t^+}$ are given by:

$$\phi^m_{t^-}=\phi^m_{t^+}=\max\left\{0,\frac{\chi^m-\frac{m_{t^-}}{d_{t^-}}+\sigma}{2\sigma}\right\},$$

and the expected size of the reserve gap:

$$\begin{aligned}\mu^m_{t^-}&=\mathbb{E}_{t^-}\left[\Delta d_{t^+}/d_{t^-}\mathbb{1}\left\{m_{t^-}+\Delta d_{t^+}-p_{t^+}+o_{t^-}<\chi^m d_{t^-}\right\}\right]\\&=\phi^m_{t^-}\mu^d_{t^-}+\frac{\left(\max\left\{-\sigma,\chi^m-\frac{m_{t^-}}{d_{t^-}}\right\}\right)^2-\sigma^2}{4\sigma}.\end{aligned}$$

Finally, the traditional bankers are constrained by LST+ when:

$$m_{t^-}-p_{t^+}\leq \Phi^d_{t^-}d_{t^-}.$$

That is, when the supply of repo by the households is too low:

$$p^h_{t^+} \leq o_{t^-} + \Phi^d_{t^-} d_{t^-} - m_{t^-}.$$

Given the distributional assumption on α_{t^+}, the probability of a spike is then given by:

$$\phi^p_{t^-} = \mathbb{P}\left[p^h_{t^+} \leq o_{t^-} + \Phi^d_{t^-} d_{t^-} - m_{t^-}\right] \tag{19}$$

$$= \max\left\{0, \min\left\{1, \frac{o_{t^-} + \Phi^d_{t^-} d_{t^-} - m_{t^-}}{d_{t^-}}\right\}\right\}, \tag{20}$$

and $\mu^d_{t^-} = -0.5$ and $\mu^o_{t^-} = 0.5$. Thus, $\Phi^d_{t^-} = \Phi^d = \chi^m + \sigma(1-\zeta) + 0.5$.

Using the morning balance sheet constraints of shadow bankers and households in equations (12) and (16), we can characterize d_{t^-} and o_{t^-} in terms of $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-})$:

$$d_{t^-} = T + \widetilde{n}^h_{t^-},$$
$$o_{t^-} = B - (q - \widetilde{n}_{t^-} - \widetilde{n}^h_{t^-}).$$

Thus, from equation (18), we can define the object $\mathcal{D}$:

$$\mathcal{D}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B) \equiv \frac{1-\varphi(d_{t^-})^{-\gamma}}{\beta} - \frac{r^k - 0.5 r^m_{t^-} + (\mu^m_{t^-} - \phi^m_{t^-}\chi) r^w - \phi^p_{t^-}(\lambda - r^m_{t^-} - \phi^m_{t^-} r^w)\Phi^d}{0.5} - 1. \tag{21}$$

Thus, the necessary and sufficient conditions for the pair $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-})$ to be an equilibrium are given by:

$$\widetilde{n}_{t^-} \geq 0 \tag{C1}$$
$$\widetilde{n}^h_{t^-} \geq 0, \tag{C2}$$
$$q - \widetilde{n}_{t^-} - \widetilde{n}^h_{t^-} \geq 0, \tag{C3}$$
$$q - \widetilde{n}^h_{t^-} - \widetilde{n}_{t^-} \leq B - M, \tag{C4}$$
$$\widetilde{n}^h_{t^-} \leq (M - \kappa B)/\Phi^d - (1-\kappa_b)B, \tag{C5}$$
$$\mathcal{D}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B) = 0. \tag{C6}$$

C4 arises from the restriction that $\underline{o}_{t^-}$. C5 was set previously such that $\lambda^\ell_{t^-} = 0$. The set of conditions C1 to C5 represent a polygon in the space $(\widetilde{n}_{t\text{-}}, \underline{n}_{t\text{-}})$. Define $\mathcal{C}$ the set of pairs $(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}})$ that satisfy C1 to C5 and $\mathcal{E}$ the set of equilibrium pairs $(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}})$ that satisfy C1 to C6. Lemma 2 provides the necessary and sufficient conditions for the existence of the set of equilibrium pairs $\mathcal{E}$.

Lemma 2. *Given $\widetilde{n}^h_{t\text{-}} \in \mathcal{C}$, if and only if $\mathcal{D}(\widetilde{n}^h_{t\text{-}}, \max\{0, q + M - B - \widetilde{n}^h_{t\text{-}}\}, M, B) > 0 \geq \mathcal{D}(\widetilde{n}^h_{t\text{-}}, q - \widetilde{n}^h_{t\text{-}}, M, B)$, there exists one unique $\widetilde{n}^h_{t\text{-}}$ such that the pair $(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}})$ is an equilibrium with $\phi^p_{t\text{-}} > 0$.*

Also, given $\widetilde{n}^h_{t\text{-}} \in \mathcal{C}$, if and only if $\mathcal{D}(\widetilde{n}^h_{t\text{-}}, \max\{0, q + M - B - \widetilde{n}^h_{t\text{-}}\}, M, B) = 0$, then for all $\widetilde{n}_{t\text{-}} \leq \overline{n}(\widetilde{n}^h_{t\text{-}}) \in \mathcal{C}$, the pair $(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}})$ is an equilibrium with $\phi^p_{t\text{-}} = 0$.

Proof. First, we derive the following partial derivative:

$$\frac{\partial \mathcal{D}(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}}, M, B)}{\partial \widetilde{n}_{t\text{-}}} = (1 - 2\phi^p_{t\text{-}}\Phi^d)\frac{\partial r^m_{t\text{-}}}{\partial \widetilde{n}_{t\text{-}}} + 2\Phi^d(\lambda - r^m_{t\text{-}} - \phi^m_{t\text{-}} r^w)\frac{\partial \phi^p_{t\text{-}}}{\partial \widetilde{n}_{t\text{-}}},$$

where

$$\frac{\partial r^m_{t\text{-}}}{\partial \widetilde{n}_{t\text{-}}} = \frac{1/\beta - 1 - \lambda}{(1 - \phi^p_{t\text{-}})^2}\frac{\partial \phi^p_{t\text{-}}}{\partial \widetilde{n}_{t\text{-}}} \leq 0,$$

and

$$\frac{\partial \phi^p_{t\text{-}}}{\partial \widetilde{n}_{t\text{-}}} = \frac{1}{d_{t\text{-}}} > 0 \qquad \text{if } \phi^p_{t\text{-}} > 0 \text{ and 0 otherwise.}$$

With further algebra we get:

$$\frac{\partial \mathcal{D}(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}}, M, B)}{\partial \widetilde{n}_{t\text{-}}} = \frac{1/\beta - 1 - \lambda}{(1 - \phi^p_{t\text{-}})^2}\frac{1 - 2\Phi^d}{d_{t\text{-}}} \geq 0, \tag{22}$$

since $1/\beta - 1 - \lambda < 0$ and $1 - 2\Phi^d < 0$. Define $\overline{n}(\widetilde{n}^h_{t\text{-}})$ such that $\phi^p(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}}, M, B) = 0$ for all $\widetilde{n}_{t\text{-}} \leq \overline{n}_{t\text{-}}$ and $\phi^p(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}}) > 0$ for all $\widetilde{n}_{t\text{-}} > \overline{n}_{t\text{-}}$. That is,

$$\overline{n}(\widetilde{n}^h_{t\text{-}}) = 2M - (1 + \Phi^d)B - (1 - \Phi^d)\kappa B + q - (1 + \Phi^d)\widetilde{n}^h_{t\text{-}}.$$

Thus, if $\widetilde{n}_{t\text{-}} \leq \overline{n}_{t\text{-}}$, then $\frac{\partial \mathcal{D}(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}}, M, B)}{\partial \widetilde{n}_{t\text{-}}} = 0$ and if $\widetilde{n}_{t\text{-}} > \overline{n}_{t\text{-}}$, then $\frac{\partial \mathcal{D}(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}}, M, B)}{\partial \widetilde{n}_{t\text{-}}} > 0$. $\square$

Proof of Proposition 1 If $\phi^p_{t^-} = 0$, from equations (7), (10), (9) we get:

$$r^m_{t^-} + \phi^m_{t^-} r^w = r^f_{t^+} = r^p_{t^+} = 1/\beta - 1.$$

Thus, $r^m_{t^-} \leq r^f_{t^+} = r^p_{t^+} \leq r^m_{t^-} + r^w$ as $\phi^m_{t^-} \in [0, 1]$. $\square$

Proof of Proposition 2 First, we show that if LST is binding then $f_{t^+} = 0$ for all traditional banks. As $\lambda^m_{t^-} = 0$, the constraint is given by:

$$m_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\} \geq \Phi^d d_{t^-}.$$

Assume that it is binding and that $f_{t^+} > 0$ for a traditional bank. From the market clearing condition, it must mean that $f_{t^+} < 0$ for another traditional bank. Because m_{t^-} and d_{t^-} are identical across banks for the first-order conditions (7) and (8) to hold, it means that the constraint is not binding for the bank with $f_{t^+} < 0$ and the first-order condition for repo (7) cannot hold. Thus, it must be that if LST is binding, then $f_{t^+} = 0$.

Second, $\phi^p_{t^+} = 1$, from equations (13) and for the repo market to clear with limited supply, $r^p_{t^+}$ must be equal to λ such that shadow bankers are indifferent between paying the overnight credit charge and paying the repo rate. Furthermore,

$$r^m_{t^-} + r^w = \frac{1/\beta - 1 - \phi^p_{t^-}\lambda}{1 - \phi^p_{t^-}} + (1 - \phi^m_{t^-}) r^w < 1/\beta - 1 + (1 - \phi^m_{t^-}) r^w < 1/\beta - 1 + r^w < \lambda.$$

Thus, $r^p_{t^+} = \lambda > r^m_{t^-} + r^w$. $\square$

Proof of Proposition 3 If $\phi^p_{t^-} > 0$, the repo rate has a probability $1 - \phi^p_{t^-}$ to be equal to $r^m_{t^-} + r^w$, which is a continuously decreasing function of $\phi^p_{t^-}$, and a probability $\phi^p_{t^-}$ to be equal to λ.

If intraday overdrafts at the central bank are not permitted, it means that the stock of reserves has to stay positive at all times during the day. That is,

$$m_{t^-} - \max\{0, p_{t^+}\} - \max\{0, f_{t^+}\} \geq 0.$$

That constraint is equivalent to LST when $\lambda^m_{t^-} = 0$ and $\Phi^d = 0$. $\square$

With this characterization of $\mathcal{E}$, we can derive comparative statics with respect to monetary and fiscal policies. To do so, we first study short-term impact of a change in policy then long-term impact. We define the short-term impact as the partial derivative of endogenous variables, such as the probability of a spike $\phi^p_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)$, with respect to the policy decisions (M, B). The long-term impact is then the total derivative of endogenous variables with respect to the policy decisions (M, B).

Proof of Proposition 4 Given equation (19), we get that if $\phi^p_{t^-} > 0$:

$$\frac{\partial \phi^p_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial B} = \frac{1}{d_{t^-}} \frac{\partial \underline{o}_{t^-}}{\partial B} - \frac{1}{d_{t^-}} \frac{\partial m_{t^-}}{\partial B} - \frac{\underline{o}_{t^-} - m_{t^-}}{(d_{t^-})^2} \frac{\partial d_{t^-}}{\partial B}.$$

The first term represents the change in the demand for shadow bank repo financing:

$$\frac{\partial \underline{o}_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial B} = \frac{\partial (B - M - (q - \widetilde{n}^h_{t^-} - \widetilde{n}_{t^-}))}{\partial B} = 1.$$

The second term represents the change in the supply of reserves available to traditional banks:

$$\frac{\partial \underline{m}_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial B} = \frac{\partial (M - \kappa B)}{\partial B} = -\kappa.$$

The third term represents the change in the quantity of deposits which directly impacts the he settlement needs for reserves to satisfy the LST constraint $\Phi^d d_{t^-}$:

$$\frac{\partial d_{t^-}}{\partial B} = \frac{\partial ((1-\kappa) B + \widetilde{n}^h_{t^-})}{\partial B} = 1 - \kappa.$$

Summing over the three terms gives:

$$\frac{\partial \phi^p_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial B} = \frac{(1+\kappa)\widetilde{n}^h_{t^-} + (1-\kappa) 2M + (1-\kappa)\underline{\widetilde{n}}_{t^-}}{(d_{t^-})^2} \geq 0,$$

which is positive. □

Proof of Proposition 5 Given equation (19), we get that if $\phi^p_{t^-} > 0$:

$$\frac{\partial \phi^p_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial M} = \frac{1}{d_{t^-}} \frac{\partial \underline{o}_{t^-}}{\partial M} - \frac{1}{d_{t^-}} \frac{\partial m_{t^-}}{\partial M}.$$

The first term represents the change in the quantity of T-bonds that have to be absorbed by shadow banks:

$$\frac{\partial \underline{o}_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial M} = \frac{\partial(B - M - (q - \widetilde{n}^h_{t^-} - \widetilde{n}_{t^-}))}{\partial M} = -1.$$

The second term represents the change in the quantity of reserves available to traditional banks to lend in repo markets:

$$\frac{\partial \underline{m}_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial M} = 1.$$

Summing over the two terms gives:

$$\frac{\partial \phi^p_{t^-}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B)}{\partial M} = -\frac{2}{d_{t^-}} < 0,$$

which is strictly negative. $\square$

Since we have multiple equilibria, to study the long-term dynamics, we need to impose an additional restriction $\mathcal{G}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B) = 0$ to pin down the direction of the change of the equilibrium pair $(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B) \in \mathcal{E}$. The mapping g is equivalent to add additional economic forces that would pin down the relative shares of the traditional and shadow banking sectors. For example, a model in which the equilibrium wealth in the shadow banking sector is a constant fraction η of the equilibrium wealth of the traditional banking sector would yield $\mathcal{G}(\widetilde{n}^h_{t^-}, \widetilde{n}_{t^-}, M, B) = \frac{q - \widetilde{n}^h_{t^-} - \widetilde{n}_{t^-}}{\widetilde{n}_{t^-}} - \eta$.

Using the implicit function theorem, we can write:

$$\frac{\partial \mathbf{n}(\mathbf{P})}{\partial B} = -\left[\frac{\partial f_i(\mathbf{n}(\mathbf{P}), \mathbf{P})}{\partial n_j}\right]^{-1} \left[\frac{\partial \mathbf{f}(\mathbf{n}(\mathbf{P}), \mathbf{P})}{\partial B}\right],$$

where

$$\mathbf{f}(\mathbf{n}, \mathbf{P}) = \begin{bmatrix} \mathcal{D}(\mathbf{n}, \mathbf{P}) \\ \mathcal{G}(\mathbf{n}, \mathbf{P}) \end{bmatrix}, \qquad \mathbf{n}(\mathbf{P}) = \begin{bmatrix} \widetilde{n}^h(\mathbf{P}) \\ \widetilde{n}(\mathbf{P}) \end{bmatrix}, \qquad \mathbf{P} = \begin{bmatrix} B & M \end{bmatrix}.$$

Since

$$\left[\frac{\partial f_i(\mathbf{n}(\mathbf{P}),\mathbf{P})}{\partial n_j}\right]^{-1} = \begin{bmatrix} \mathcal{D}_{\widetilde{n}^h} & \mathcal{D}_{\widetilde{n}} \\ \mathcal{G}_{\widetilde{n}^h} & \mathcal{G}_{\widetilde{n}} \end{bmatrix}^{-1} = \frac{1}{\mathcal{D}_{\widetilde{n}^h}\mathcal{G}_{\widetilde{n}} - \mathcal{D}_{\widetilde{n}}\mathcal{G}_{\widetilde{n}^h}} \begin{bmatrix} \mathcal{G}_{\widetilde{n}} & -\mathcal{D}_{\widetilde{n}} \\ -\mathcal{G}_{\widetilde{n}^h} & \mathcal{D}_{\widetilde{n}^h} \end{bmatrix},$$

we get:

$$\frac{\partial \mathbf{n}(\mathbf{P})}{\partial B} = -\frac{1}{\mathcal{D}_{\widetilde{n}^h}\mathcal{G}_{\widetilde{n}} - \mathcal{D}_{\widetilde{n}}\mathcal{G}_{\widetilde{n}^h}} \begin{bmatrix} \mathcal{G}_{\widetilde{n}} & -\mathcal{D}_{\widetilde{n}} \\ -\mathcal{G}_{\widetilde{n}^h} & \mathcal{D}_{\widetilde{n}^h} \end{bmatrix} \begin{bmatrix} \mathcal{D}_B \\ \mathcal{G}_B \end{bmatrix},$$

$$\frac{\partial \mathbf{n}(\mathbf{P})}{\partial M} = -\frac{1}{\mathcal{D}_{\widetilde{n}^h}\mathcal{G}_{\widetilde{n}} - \mathcal{D}_{\widetilde{n}}\mathcal{G}_{\widetilde{n}^h}} \begin{bmatrix} \mathcal{G}_{\widetilde{n}} & -\mathcal{D}_{\widetilde{n}} \\ -\mathcal{G}_{\widetilde{n}^h} & \mathcal{D}_{\widetilde{n}^h} \end{bmatrix} \begin{bmatrix} \mathcal{D}_M \\ \mathcal{G}_M \end{bmatrix},$$

where we use the following notation for partial derivatives: $\partial A/\partial x = A_x$. Thus,

$$\frac{\partial \widetilde{n}^h(\mathbf{P})}{\partial B} = -\frac{\mathcal{D}_B\mathcal{G}_{\widetilde{n}} - \mathcal{D}_{\widetilde{n}}\mathcal{G}_B}{\mathcal{D}_{\widetilde{n}^h}\mathcal{G}_{\widetilde{n}} - \mathcal{D}_{\widetilde{n}}\mathcal{G}_{\widetilde{n}^h}}, \qquad \frac{\partial \widetilde{n}(\mathbf{P})}{\partial B} = -\frac{\mathcal{D}_B\mathcal{G}_{\widetilde{n}^h} - \mathcal{D}_{\widetilde{n}^h}\mathcal{G}_B}{\mathcal{D}_{\widetilde{n}}\mathcal{G}_{\widetilde{n}^h} - \mathcal{D}_{\widetilde{n}^h}\mathcal{G}_{\widetilde{n}}},$$

$$\frac{\partial \widetilde{n}^h(\mathbf{P})}{\partial M} = -\frac{\mathcal{D}_M\mathcal{G}_{\widetilde{n}} - \mathcal{D}_{\widetilde{n}}\mathcal{G}_M}{\mathcal{D}_{\widetilde{n}^h}\mathcal{G}_{\widetilde{n}} - \mathcal{D}_{\widetilde{n}}\mathcal{G}_{\widetilde{n}^h}}, \qquad \frac{\partial \widetilde{n}(\mathbf{P})}{\partial M} = -\frac{\mathcal{D}_M\mathcal{G}_{\widetilde{n}^h} - \mathcal{D}_{\widetilde{n}^h}\mathcal{G}_M}{\mathcal{D}_{\widetilde{n}}\mathcal{G}_{\widetilde{n}^h} - \mathcal{D}_{\widetilde{n}^h}\mathcal{G}_{\widetilde{n}}}.$$

Furthermore,

$$\begin{aligned} \mathcal{D}_B = {} & \frac{(1-\kappa)\varphi\gamma}{\beta}(d_{t\text{-}})^{-\gamma-1} + 2r^w\frac{M-\kappa B}{d_{t\text{-}}}\frac{M(1-\kappa)+\kappa\widetilde{n}^h_{t\text{-}}}{2\sigma d^2_{t\text{-}}} \\ & + \left(1-2\Phi^d\right)\frac{1/\beta-1-\lambda}{(1-\phi^p_{t\text{-}})^2}\frac{1+\kappa+\Phi^d(1-\kappa)-\phi^p_{t\text{-}}(1-\kappa)}{d_{t\text{-}}}, \end{aligned}$$

$$\mathcal{D}_M = -2r^w\frac{M-\kappa B}{d_{t\text{-}}}\frac{1}{2\sigma d_{t\text{-}}} - \left(1-2\Phi^d\right)\frac{1/\beta-1-\lambda}{(1-\phi^p_{t\text{-}})^2}\frac{2M}{d_{t\text{-}}},$$

$$\mathcal{D}_{\widetilde{n}} = \left(1-2\Phi^d\right)\frac{1/\beta-1-\lambda}{(1-\phi^p_{t\text{-}})^2}\frac{1}{d_{t\text{-}}} > 0.$$

We can use these results to understand if long-term dynamics alleviate or dampen the short-term impact given a mapping $\mathcal{G}(\widetilde{n}^h_{t\text{-}}, \widetilde{n}_{t\text{-}}, M, B)$.

Proof of Proposition 7 If $\mathcal{G}(\mathbf{n}, \mathbf{P}) = \widetilde{n}^h_{t\text{-}} - \eta$, then:

$$\frac{d\phi^p(\mathbf{n}, \mathbf{P})}{dB} = \frac{\partial\phi^p(\mathbf{n}, \mathbf{P})}{\partial B} + \frac{\partial\phi^p(\mathbf{n}, \mathbf{P})}{\partial\widetilde{n}_{t\text{-}}} \frac{\partial\widetilde{n}_{t\text{-}}(\mathbf{P})}{\partial B},$$

and

$$\frac{\partial\widetilde{n}_{t\text{-}}(\mathbf{P})}{\partial B} = -\frac{\mathcal{D}_B}{\mathcal{D}_{\widetilde{n}}}.$$

Define $\underline{\lambda}^B(\mathbf{n}, \mathbf{P})$ such that:

$$\begin{aligned} 0 = &\frac{(1-\kappa)\varphi\gamma}{\beta}(d_{t\text{-}})^{-\gamma-1} + 2r^w \frac{M - \kappa B}{d_{t\text{-}}} \frac{M(1-\kappa) + \kappa\widetilde{n}^h_{t\text{-}}}{2\sigma d^2_{t\text{-}}} \\ &+ \left(1 - 2\Phi^d\right) \frac{1/\beta - 1 - \underline{\lambda}^B(\mathbf{n}, \mathbf{P})}{(1-\phi^p_{t\text{-}})^2} \frac{1 + \kappa + \Phi^d(1-\kappa) - \phi^p_{t\text{-}}(1-\kappa)}{d_{t\text{-}}}. \end{aligned}$$

Thus, if $\lambda > \underline{\lambda}^B(\mathbf{n}, \mathbf{P})$, then $\mathcal{D}_B > 0$ and

$$\frac{d\phi^p(\mathbf{n}, \mathbf{P})}{dB} - \frac{\partial\phi^p(\mathbf{n}, \mathbf{P})}{\partial B} < 0.$$

Similarly, we have that:

$$\frac{d\phi^p(\mathbf{n}, \mathbf{P})}{dM} = \frac{\partial\phi^p(\mathbf{n}, \mathbf{P})}{\partial M} + \frac{\partial\phi^p(\mathbf{n}, \mathbf{P})}{\partial\widetilde{n}_{t\text{-}}} \frac{\partial\widetilde{n}_{t\text{-}}(\mathbf{P})}{\partial M},$$

and

$$\frac{\partial\widetilde{n}(\mathbf{P})}{\partial M} = -\frac{\mathcal{D}_M}{\mathcal{D}_{\widetilde{n}}}.$$

Define $\underline{\lambda}^M(\mathbf{n}, \mathbf{P})$ such that:

$$0 = -2r^w \frac{M - \kappa B}{d_{t\text{-}}} \frac{1}{2\sigma d_{t\text{-}}} - \left(1 - 2\Phi^d\right) \frac{1/\beta - 1 - \underline{\lambda}^M(\mathbf{n}, \mathbf{P})}{(1-\phi^p_{t\text{-}})^2} \frac{2M}{d_{t\text{-}}}.$$

Thus, if $\lambda > \underline{\lambda}^M(\mathbf{n}, \mathbf{P})$, then $\mathcal{D}_M < 0$ and

$$\frac{d\phi^p(\mathbf{n}, \mathbf{P})}{dM} - \frac{\partial\phi^p(\mathbf{n}, \mathbf{P})}{\partial M} > 0.$$

Both statements are true if $\lambda > \underline{\lambda}(\mathbf{n}, \mathbf{P}) \equiv \max\{\underline{\lambda}^B(\mathbf{n}, \mathbf{P}), \underline{\lambda}^M(\mathbf{n}, \mathbf{P})\}$.

B Descriptions of the Flows

TBC

C Figures

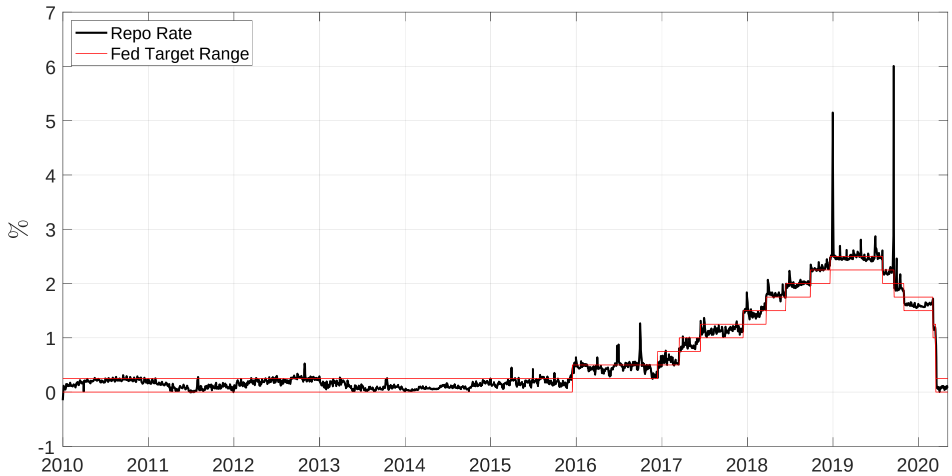


Figure 1: Repo Rates Spiking. The figure displays the time series for the DTCC GCF Repo Index tracking the average daily interest rate paid for the most-traded GCF Repo contracts for US Treasury General Collateral along with the lower and upper bounds for the Fed's fed fund rate target range.
Source: Depository Trust & Clearing Corporation and Federal Reserve Economic Data.

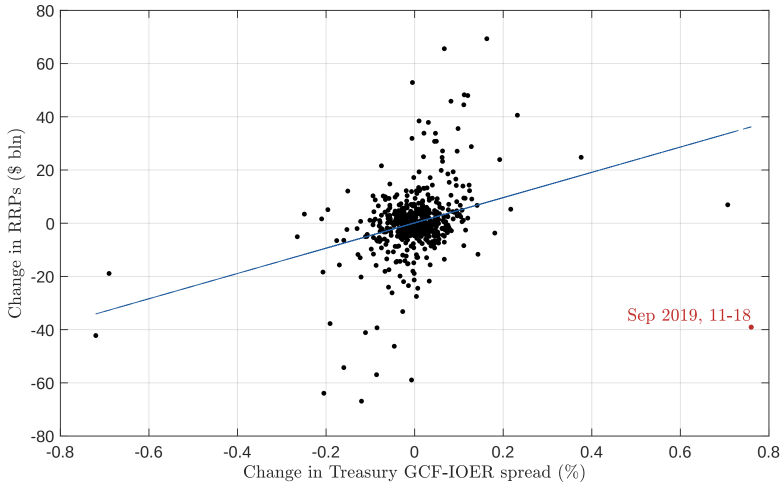


Figure 2: Relationship Between Treasury GCR-IOER Spreads and Banks' Repo lending. The figure provides a scatter plot representation of the relationship between week-to-week differences in the Treasury GCR-IOER spread, and aggregate banks reverse repo (lending) positions along with an OLS regression line computed excluding the four outliers. The sample period is 2010/01/01 to 2019/09/30.
Source: Depository Trust & Clearing Corporation

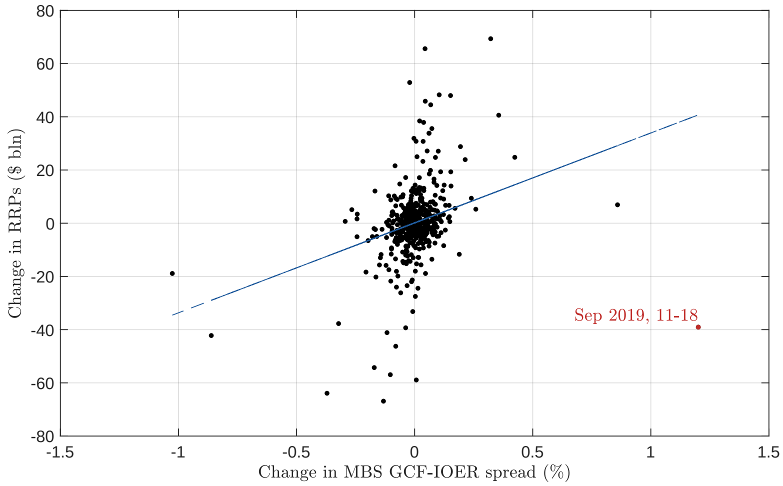


Figure 3: Relationship Between MBS GCR-IOER Spreads and Banks' Repo Lending. The figure provides a scatter plot representation of the relationship between week-to-week differences in the MBS GCR-IOER spread, and aggregate banks reverse repo (lending) positions along with an OLS regression line computed excluding the four outliers. The sample period is 2010/01/01 to 2019/09/30.

Source: Depository Trust & Clearing Corporation and Federal Reserves H8 Bank's Balance Sheet Reports

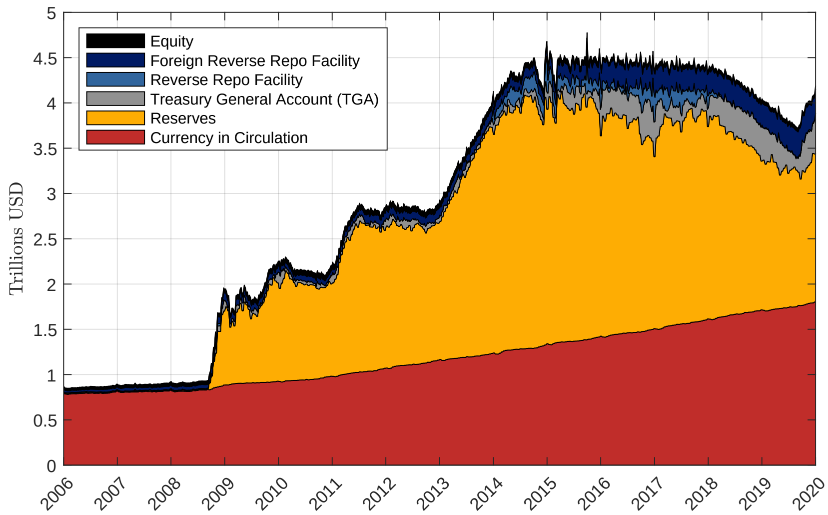


Figure 4: Evolution of the Liability Side of the Fed's Balance Sheet. The figure displays the large increase in Fed's reserves following QE waves, followed by a reduction driven by the tapering of Fed's portfolio and the growth of other liability items.
Source: Federal Reserve Economic Data

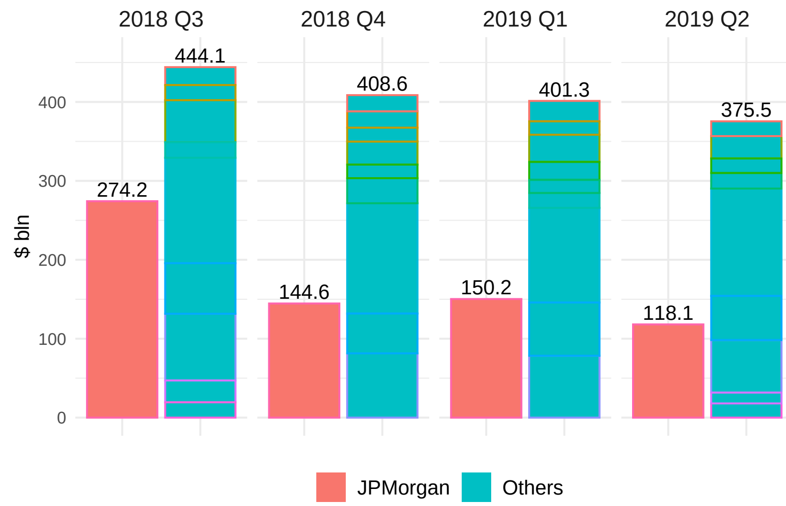


Figure 5: Evolution of the Distribution of Reserves. The figure shows the distribution of central bank reserves between JP Morgan, the larger holder, and the next ten largest holders.
Source: Bank Call Reports

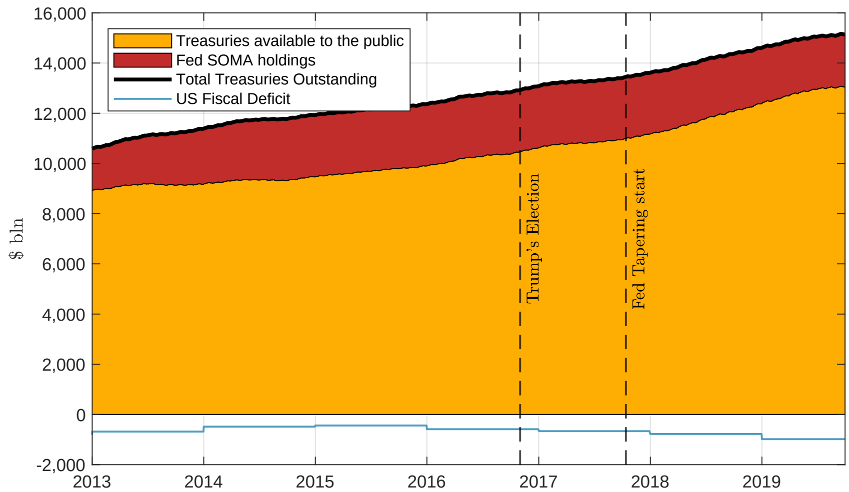


Figure 6: Evolution of Total Outstanding and Fed Holdings of Treasuries. The figure displays the acceleration in the growth the supply of Treasury securities available to the public after the Trump election and the start of the tapering of the Fed balance sheet. The series was smoothed using an expanding window mean. The sample is between January 2013 and October 2019.
Source: Federal Reserve Economic Data and CRSP Treasury Data

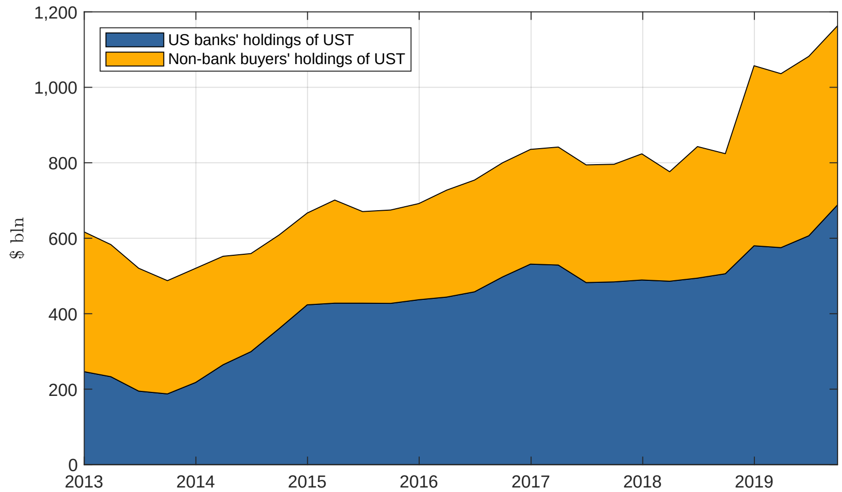


Figure 7: Evolution of Domestic Banks and Non-banks Treasury Holdings. The figure shows the increase in US bank and non-bank portfolio of Treasury securities, with an acceleration in 2019. Non-bank holdings include hedge funds and security brokers-dealers. The sample is between January 2013 and October 2019.
Source: Federal Reserve's Z1 Financial Accounts

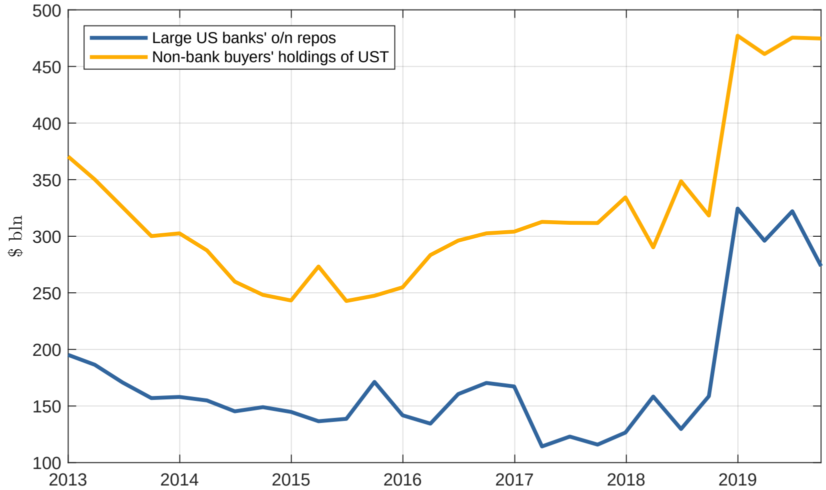
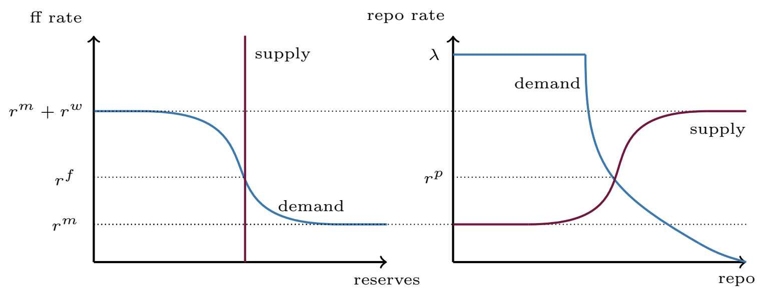


Figure 8: Evolution of Non-bank Treasury Holdings and Bank Repo. The figure shows a sharp increase in both non-bank holdings of Treasury securities and repo lending from banks. Non-bank holdings include hedge funds and security brokers-dealers. The sample is between January 2013 and October 2019.
Source: Federal Reserve's Z1 Financial Accounts and H8 Banks Balance Sheet Reports

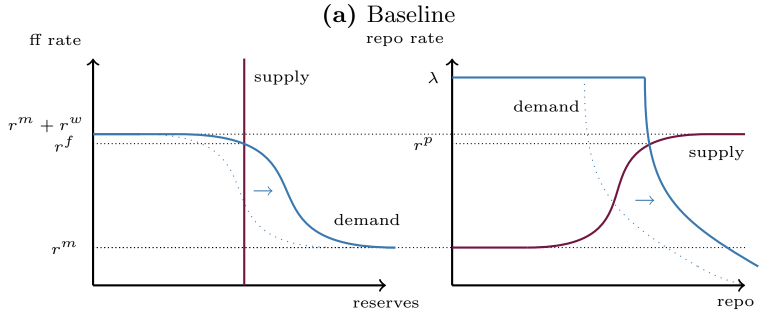
	Δ TGCF-IOER				Δ MBSGCF-IOER	Δ SOFR-IOER	Δ JPYUSD FX-IOER
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Δ Reserves ($bln)	-0.0233**	-0.00955	-0.00617	-0.0104	-0.00987	-0.00400	-0.174*
	(0.009)	(0.007)	(0.006)	(0.007)	(0.007)	(0.004)	(0.088)
Top P10 Bills Issuance Days	4.076***	3.414***	3.458***	3.372***	2.885***	1.827***	-1.107
	(0.564)	(0.507)	(0.511)	(0.502)	(0.498)	(0.301)	(8.077)
Top P10 Coupons Issuance Days	9.966***	8.725***	7.731***	9.556**	7.290***	4.044***	16.67
	(2.837)	(1.595)	(2.011)	(3.138)	(1.879)	(0.892)	(25.610)
Quarter Start		-27.45*	-21.52	-22.28	-20.98**	-4.470	-254.7*
		(12.426)	(12.575)	(12.844)	(6.889)	(3.488)	(100.626)
Quarter End		27.83	24.77	24.77	21.09	5.953	352.4**
		(15.355)	(15.846)	(15.879)	(11.058)	(3.378)	(115.701)
Month Start			-5.592***	-5.832***	-8.287***	-5.190***	-27.05*
			(1.196)	(1.263)	(1.424)	(1.016)	(12.242)
Month End			3.773*	2.147	5.231*	4.857***	23.22
			(1.876)	(2.746)	(2.028)	(1.169)	(23.879)
Tax Deadline Days -2				0.788	0.687	0.733	3.725
				(0.722)	(0.703)	(0.426)	(4.911)
Tax Deadline Days -1				2.769***	2.016*	1.095**	5.404
				(0.721)	(0.908)	(0.414)	(4.725)
Tax Deadline Days				-5.187	-3.259	-0.110	-10.67
				(3.184)	(2.005)	(1.060)	(28.960)
Tax Deadline Days +1				-2.976**	-2.692**	-1.080	-19.17
				(0.908)	(0.872)	(0.562)	(12.263)
Tax Deadline Days +2				-2.709*	-3.450**	-2.238*	3.717
				(1.101)	(1.213)	(0.871)	(7.809)
Constant	-1.543***	-1.326***	-1.193***	-1.087***	-0.813***	-0.618***	-3.100
	(0.329)	(0.206)	(0.195)	(0.202)	(0.202)	(0.135)	(2.584)
N	928	928	928	928	926	926	827
Adj.R^2	0.07	0.23	0.24	0.24	0.37	0.35	0.34

Heteroskedasticity-robust standard errors in parenthesis. * p<0.05, ** p<0.01, *** p<0.001

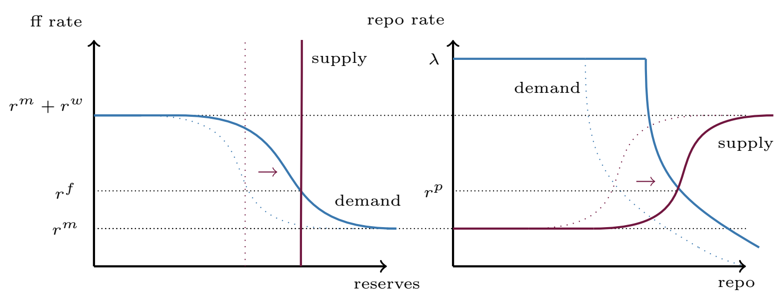
Table 1: Regression Models of Changes in Spreads The dependent variables are the first-difference of the Treasury GCF, MBS GCF, SOFR and JPY-USD FX implied dollar rate over the IOER. Heteroskedasticity-robust standard errors are reported in parenthesis. The sample period is 2015-12-15 to 2019-08-31.



(a) Baseline

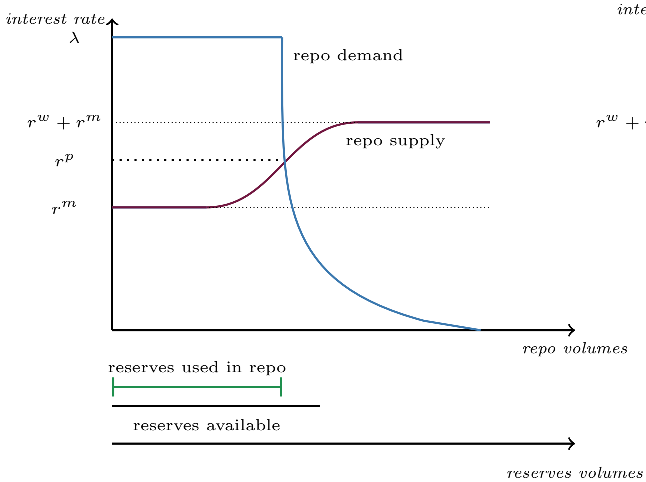


(b) Repo Supply Shock

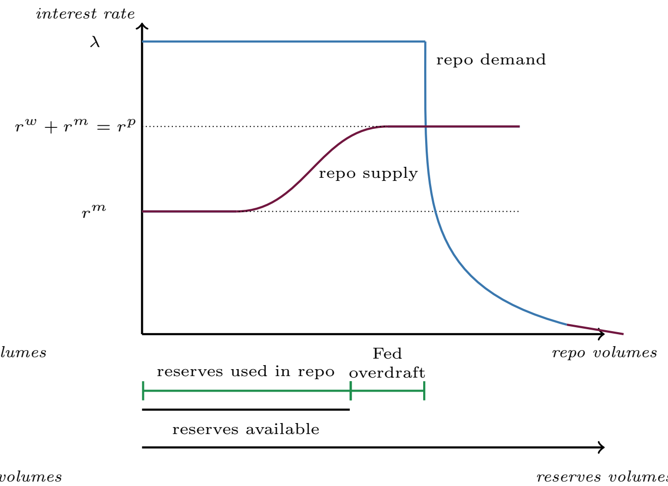


(c) Reserves Supply Reaction

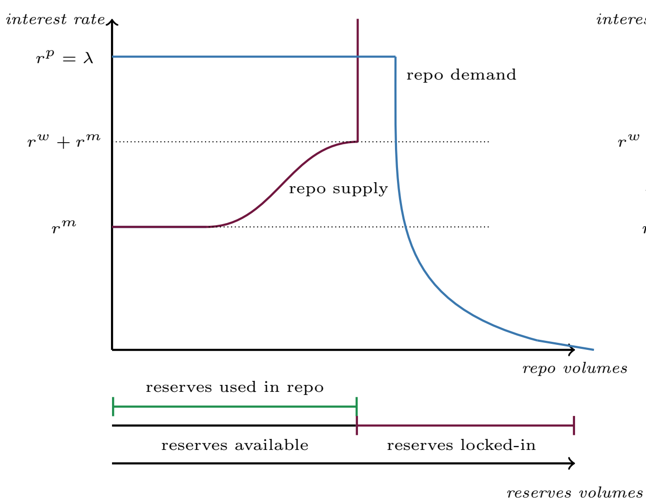
Figure 11: Integrated Money Markets Benchmark



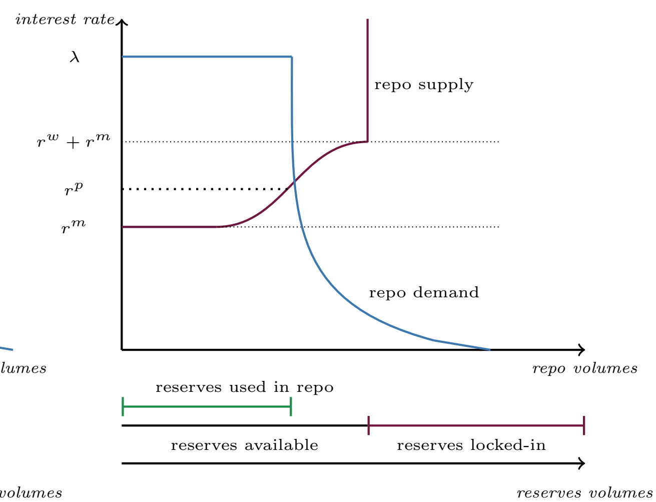
(a) No LST and Abundant Reserves



(b) No LST and CB Overdraft



(a) LST and Abundant Reserves



(b) LST and Scarce Reserves

Figure 12: Repo Markets with and without LST