Homework 4

Nathaniel Hamovitz
Math 108B, Sung, F22

due 2022-11-10

1. Fix a field k. Given k-vector spaces U, V, and W, and k-linear maps $f : U \mapsto V$ and $G : V \mapsto W$, the diagram

$$0 \longrightarrow U \longrightarrow^f$$

Axler 2e, Exercise 7.4 Suppose $P \in \mathcal{L}(V)$ is such that $P^2 = P$. Prove that P is an orthogonal projection iff P is self-adjoint.

Proof. □

Axler 2e, Exercise 7.21 Prove or give a counterexample: if $S \in \mathcal{L}(V)$ and there exists an orthogonal basis $(e_1, \ldots, e_n)$ of V such that $\|Se_j\| = 1$ for each e_j, then S is an isometry.

Proof. □

Axler 2e, Exercise 7.23 Define $T \in \mathcal{L}\left(\mathbb{F}^3\right)$ by

$$T(z_1, z_2, z_3) = (z_3, 2_z1, 3z_2).$$

Find (explicitly) an isometry $S \in \mathcal{L}\left(\mathbb{F}^3\right)$ such that $T = S\sqrt{T^*T}$.

Consider. □

Axler 2e, Exercise 7.29 Suppose $T \in \mathcal{L}(V)$. Prove that $\dim \operatorname{range} T$ equals the number of nonzero singular values of T.

Proof. □

Axler 2e, Exercise 7.30 Suppose $S \in \mathcal{L}(V)$. Prove that S is an isometry iff all the singular values of S equal 1.

Proof. □