Lab 05 –
Continuous Probability Distribution

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Required libs: math, numpy, matplotlib.

```
import matplotlib.pyplot as plt
import math
```

Provide the *generator_data* function as follows:

```
def generator_data(a, b, size):
    n = (b-a)/(size-1)
    result = []
    s = a;
    while s < b:
        result.append(s)
        s = s+n;
    if len(result) < size:
        result.append(b)
    return result
```

Let X be a continuous random variable in [4, 6]. Create an array of 100 continuous real numbers in [4, 6] and save to variable X.

```
X = generator_data(a, b, 100)
print(X)
```

1 Uniform distribution

The random variable X that has a constant even distribution will receive values on [a, b]. The probability that X receives any value of the range (a, b) is equal to $\frac{1}{b-a}$. The notation X is evenly distributed on (a, b) is X ~ U (a, b).

The probability density function of a continuous uniform distribution is determined by the formula:

$$p(x) = \frac{1}{b-a} \text{ với } x \in (a, b) \quad (1)$$

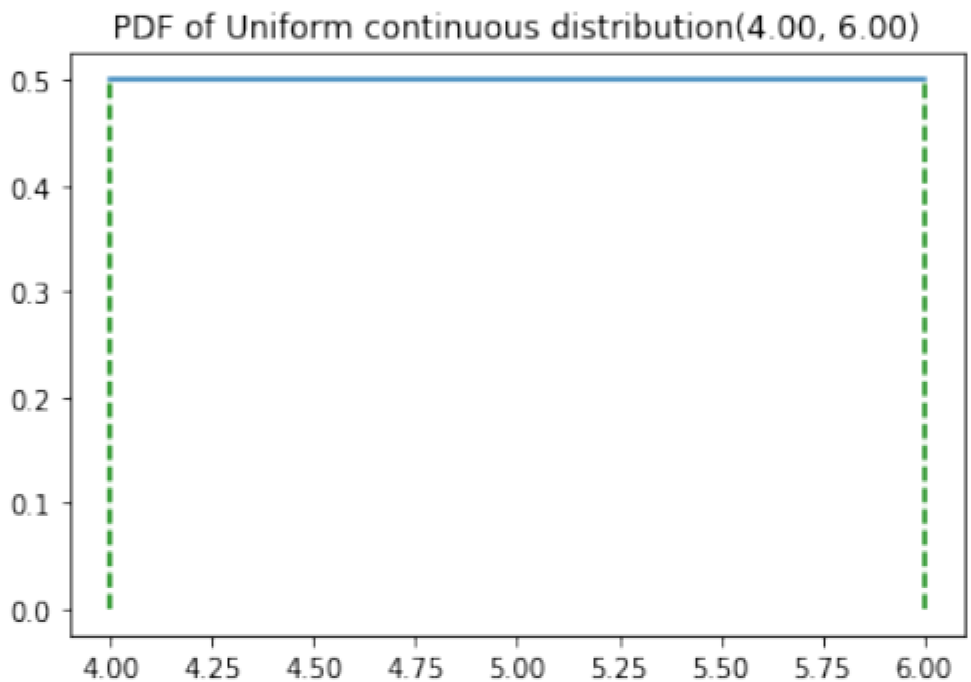
Write the probability density function of continuous distribution:

```
def pmf_uniform_cont(a, b):
    # your code
```

Using the above pmf_uniform_cont function to graph the relationship between the random variable X and the corresponding probability. The horizontal axis represents the value of the random variable X, the vertical axis represents the corresponding probability p (x).

```
def plot_pmf_uniform_cont(a, b):
    '''
    Plot the probability mass function of Uniform(a, b)
    '''
    X = generator_data(a, b, 100)
    if b != a:
        P = [pmf_uniform_cont(a, b) for x in X]

    plt.plot(X, P, '-')
    plt.plot([a, a], [0, 1/(b-a)], 'g--')
    plt.plot([b, b], [0, 1/(b-a)], 'g--')

    plt.title('PDF of Uniform continuous distribution(%0.2f, %0.2f)'
        %(a,b))

    plt.show()

plot_pmf_uniform_cont(4, 6)
```

Implementation results are as follows:



2 Normal distribution

Normal distribution (also known as Gaussian distribution) is an important probability distribution, with applications in many areas.

The probability density function of the normal distribution is determined by the formula:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{2}$$

The cumulative distribution function of the normal distribution is determined by the formula:

$$\varphi(x) = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right] \tag{3}$$

Where μ is the mean (expected) and σ is the standard deviation.

Write the probability density function and the cumulative distribution function of the normal distribution:

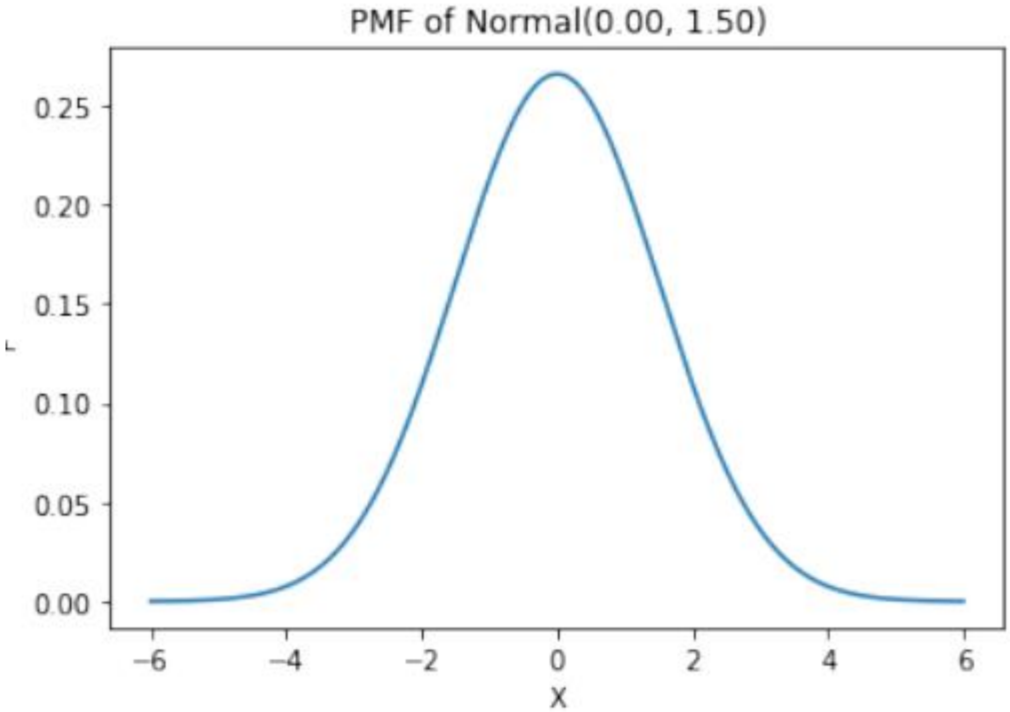
```python
import math
def pmf_normal(x, mu, sigma):
    # your code

def cdf_normal(x, mu, sigma):
    # your code
```

Use the pmf_normal above function to plot a graph representing the relationship between random variable X (following the normal distribution with two parameters μ and σ) and the corresponding probability.

The horizontal axis represents the value of the random variable X, the vertical axis represents the corresponding probability p (x).

```python
def plot_pmf_normal(mu, sigma):
    '''
    Plot the probability mass function of Normal(mu, sigma)
    '''
    X = generator_data(mu - 4*sigma, mu + 4*sigma, 1000)
    P_normal = [pmf_normal(x, mu, sigma) for x in X]
    plt.plot(X, P_normal, '-')
    plt.title('PMF of Normal(%.2f, %.2f)' %(mu,sigma))
    plt.xlabel('X')
    plt.ylabel('P')
    plt.show()

plot_pmf_normal(0, 1.5)
```

The result is:

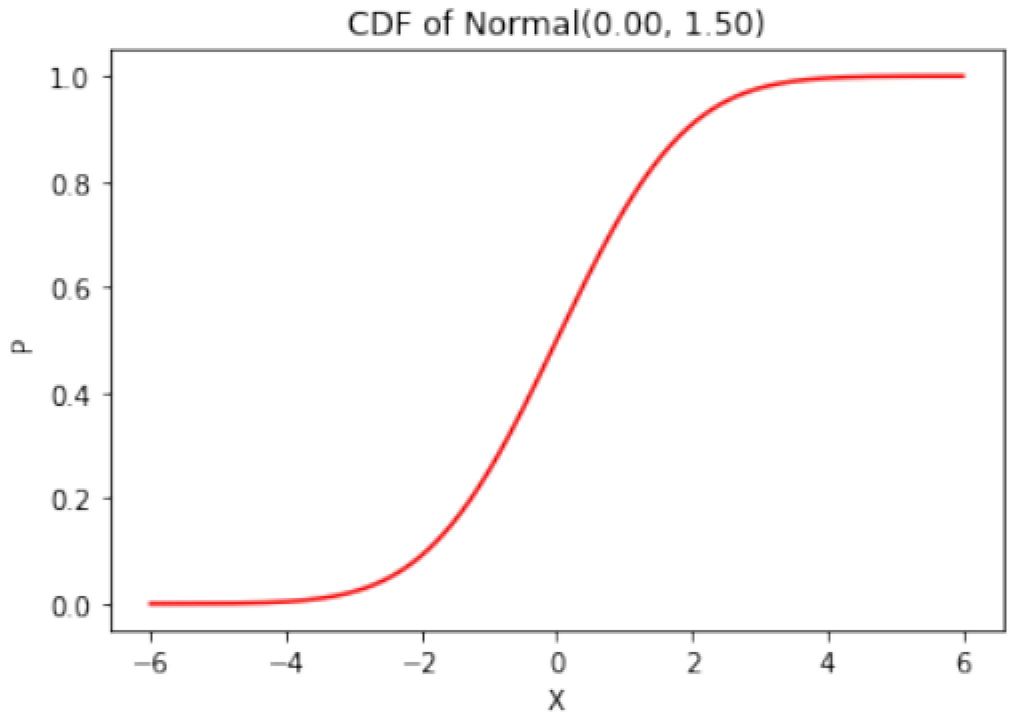


3 Exercises

1. Use the cdf_normal function above to plot a graph representing the relationship between random variable X (following the normal distribution with two parameters μ and σ) and the probability of cumulative distribution. The horizontal axis represents the value of the random variable X, the vertical axis represents the probability φ(x), respectively.

```
def plot_cdf_normal(mu, sigma):
    '''
    Plot the Cumulative distribution function of Normal(mu,
       sigma)
    '''
    # your code

plot_cdf_normal(0, 1.5)
```

Reference answers:



2. The time to produce a product of workshop A is a random variable that follows the normal distribution with parameters $\mu = 10$ and $\sigma = 1$ (minute units). Use the functions in the exercise to calculate the probability that the product will be generated in the period from 9 minutes to 12 minutes. (Answer: 0.8185)