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Summary of: "A Markov Chain Model for the Probability of Precipitation Occurrence in Intervals of Various Length"

**Variables and Equations**

$P_n$  - probability of precipitation occurrence in an interval of  $n$  days

$P_{n-1}$  - probability of precipitation occurrence in an interval of  $n-1$  days

$P_1$  - probability of a day being wet

$p_{n-1}$  - conditional probability of a day being wet following a period of  $n-1$  dry days

$\tau_1$  - conditional probability that a day will be wet if the previous day was wet

Eq. 2:  $P_n = (1 - p_{n-1}) P_{n-1} + p_{n-1}$

Eq. 3:  $P_n = (1 - p_1) P_{n-1} + p_1$

Eq. 4:  $P_n = 1 - (1 - P_1)(1 - p_1)^{n-1}$

Eq. 5:  $P_1 = p_1 (1 - \tau_1 + p_1)^{-1}$

This paper discusses "the probability of precipitation occurrence in intervals of various length at Denver, Colorado", and is an extension of previous work done by Topil. The goal of this paper is to show that the probability distributions as determined by Topil's empirical work are "closely approximated by a simple Markov chain model." Similar work by researchers in Tel Aviv show that the goal is a reasonable assumption.

This paper studies two models. The first is a random model, "in which the probability of a day being wet is independent of what occurred on any preceding day." The second is a Markov chain model, in which "the probability of precipitation on any day depends on whether or not precipitation fell the preceding day." The random model is given by Eq. 2, with  $p_{n-1} = p_1 = P_1$ . The Markov chain model is given by Eq. 3 and Eq. 4. Both are equivalent, but are convenient for different cases. Eq. 3 appears less computationally intensive, which may have been a consideration in 1963 and preceding years.  $p_1$  and  $\tau_1$  are estimated from Topil's empirical data (see tables 1 and 2), and  $P_1$  is determined by substituting  $p_1$  and  $\tau_1$  into Eq. 5.

For comparison of the two models and the empirical data, two definitions of "wet" are given. One definition states that a day is wet if there is a trace or more of precipitation. The other states that a day is wet if there 0.01 inch or more of precipitation. Figures 1 and 2 show the number of days ( $n$ ) vs.  $P_n$ , with each figure using data as determined for each definition of "wet". Plotted in each are Topil's observed relative frequencies as determined from his empirical data, and the Markov chain model curves are overlaid. Figure 3 shows  $P_n$  (Markov) vs.  $P_n$  (Observed) and  $P_n$  (Random) vs.  $P_n$  (Observed), where Observed refers to Topil's observed relative frequencies (for each definition of "wet"). The paper does not provide any quantitative validation of either model, but Figure 3 shows visually that the Markov chain model is superior to the random model and fits the data well.

The concluding remarks indicate that this paper is essentially a quick glance into the convenience of using a Markov chain to model precipitation occurrence. The brevity of the abstract, small number of resources, lack of model validation and brevity of the paper overall also indicate this. The author is a member of the US Weather Bureau, so this paper may have originally been a proposal to superiors, or maybe the standards of what is considered acceptable have increased.