Optimization of Robot Scoring for 2013 FIRST Robotics Competition

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Abstract

Each year the FIRST Robotics Competition challenges teams from across the world to develop a game-playing robot that is capable of outperforming the other teams. For 2013, we developed disc shooting robot. In this paper, we detail our endeavor to develop a model to maximize our robot's scoring potential.

I. Introduction

The FIRST Robotics Competition is an international robotics competition that develops a new game every year and challenges each team to build a robot that is the best at the game. The teams, consisting of high school students and their mentors, are given six weeks to develop and build their robots. At the end of the six weeks, teams meet in a series of competitions to determine whose robot is the best at the game.

For 2013, FIRST developed a game entitled ULTIMATE ASCENT, the goal of which is to score the most points in 120 seconds by shooting discs from the robot into any combination of three goals, each of which has a different point value and shot difficulty. Additionally, robots can block opposing robots from shooting. Using the mathematical tools of discrete probabilistic modeling and discrete model optimization, along with our observations, we aim to develop a strategy that maximizes our points scored.

II. Methodology

First, we must list and explain some variables, so future discussion on the problem will be clear. There are three positions on the playing field: Close position (c), Mid position (m) and 3/4 position (t). For clarity, there is a reload position, but we make the simplifying assumption that this position will only be used for reloading; no shooting or blocking will take place from/at this position. There are three goals that can be shot at from the c, m, and t positions: High (H, worth 3 points), Middle (M, worth 2 points) and Low (L, worth 1 point). Also, three important assumptions are made about the robot: 1) it always starts at the Close position, 2) it always starts with 4 discs loaded, and can carry a max of 4 discs, and 3) it shoots 1 disc per second, with a disc considered "live" if it is shot before time runs out.

We split the problem into three tasks. First, we developed a Markov chain model of the robot (see Appendix A) and solved the transition matrix to determine the long term probability for each state (see Appendices B1 and B2). Table 1 describes the states. The number(s) associated with each state indicate the time spent in each state (weight of each state). These values were determined through observation. We made the simplifying assumption that the robot always starts in the shooting state.

S - shooting state, 4	TR - travel to reload state, c:10, m:5, t:3	BR - state of being blocked on the way to reload, 5
R - reload state, 3	TS - travel to shoot state, c:10, m:5, t:3	BS - state of being blocked on the way to shoot, 5

Table 1 : Markov chain states

Next, we developed an objective function and optimization constraints to maximize our points scored. This task did not include factoring in the effects of blocking. Third, we combined the tasks 1 and 2 to maximize our score, with the effects of blocking included.

III. Data and Analysis

	One-way time to travel to reload point	Probability of defense blocking (P_B)	Probability of High goal shot success	Probability of Middle goal shot success	Probability of Low goal shot success
Close	10 seconds	0.7	$H_c = 0.8$	$M_c = 0.9$	$L_c = 0.99$
Mid-Field	5 seconds	0.5	$H_m = 0.6$	$M_m = 0.8$	$L_m = 0.85$
3/4 Field	3 seconds	0.2	$H_t = 0.3$	$M_t = 0.5$	$L_t = 0.5$

Table 2: Markov chain model and optimization model data

For task 1, the process of determining the long term probabilities for each state is detailed in Appendices B1 and B2. First, note that in the long term, the probability of transitioning to any state from any other state at time *n* is the same as making the same transition at time *n+1*. Figure 4 in Appendix B1 uses this fact, and the data listed in Table 2, to analytically determine the long term probability of being in each state, before state weights are included. Figures 5, 6 and 7 show the long term probabilities, with the state weights included. Table 3 lists the time spent in each state for a 120 second game.

	S	TR	BR	R	TS	BS
Close	14.12 (15)	35.29	12.35	10.59	35.29	12.35
Mid	21.82 (22)	27.27	13.64	16.36	27.27	13.64
3/4	32	24	8	24	24	8

Table 3: Time (seconds) spent in each state during a 120 second game

For task 2, we developed an objective function and optimization constraints to maximize our points scored, without the effects of blocking included in the calculations. Eq. 1, Eq. 2 and Eq. 3 detail the process for developing the objective function.

$$score = 3H + 2M + L \qquad (Eq. 1)$$

$$score = 2.4Hc + 1.8Hm + 0.9Ht + 1.8Mc + 1.6Mm + Mt + 0.99Lc + 0.85Lm + 0.5Lt \quad \text{Eq. 2}$$

$$score = 2.4Hc + 1.8Hm + Mt \qquad (Eq. 3)$$

Eq. 1 is a general equation for determining score when the shooting percentage is 100%. Eq. 2 includes the shooting probabilities as listed in Table 2 and shows the result of multiplying those probabilities by the points per successful shot, giving us the effective score per shot (whether successful or unsuccessful). Since the number of discs that can be shot is dependent on location and independent of which goal is shot at, we simplify Eq. 2 to Eq. 3 to choose which goal gives us the highest effective score per shot at each location. Appendix C details this process and the process of determining the optimization constraint given by Ineq. 1.

$$6Hc + \frac{30}{7}Hm + \frac{10}{3}Mt \leq 120 \qquad (Ineq. 1)$$

We choose Eq. 3 as the objective function, and along with Ineq. 1 as the optimization constraint, the online simplex method tool (1), gives the following result:

$$max\ score = 2.4 * 0 + 1.8 * 28 + 1.0 * 0 = 50.4 \text{ long term average score} \qquad (Eq. 4)$$

IV. Results

$$8Hc + \frac{60}{11}Hm + \frac{15}{4}Mt \leq 120 \qquad (Ineq. 2)$$

For task 3, we use the results of task 1 (see Table 3) to form Ineq. 2 (see Appendix C). With the objective function (Eq. 3) and optimization constraint (Ineq. 2), we use the online simplex method tool (1) to calculate maximum score, with blocking included:

$$max\,score = 2.4 * 0 + 1.8 * 22 + 1.0 * 0 = 39.6 \text{ long term average score} \qquad (Eq.5)$$

V. Conclusions

Comparing the result of Eq. 4 (50.4 long term average score) with the result of Eq. 5 (39.6 long term average score), we see that blocking causes approximately a 21% drop in long term average scoring. If future observations and models prove that a score of 39.6 is sufficient to win the competition, then this is an acceptable loss. However, it is likely that this will not be the case, so it is necessary to make improvements, the most obvious of which is improving 3/4-field shooting accuracy. Improving this statistic will allow us to shoot from that position, which will both decrease travel time and time spent blocked, allowing for more time spent shooting.

Since it may not be clear, it is important to point out that our model limits us to taking all of our shots at any one of the three positions: Close, Mid-Field or 3/4-Field. It may be the case that a higher score can be achieved by taking shots at a combination of the three positions, but such a model will be more difficult to develop. Also, (2) describes other scoring opportunities and rules that will affect our optimal score. Due to limited time, we were not able to include these considerations in our model.

VI. References

1. http://www.zweigmedia.com/RealWorld/simplex.html

2. 2013_Game_Manual_03_The_Game.pdf

Appendix A - State Diagrams

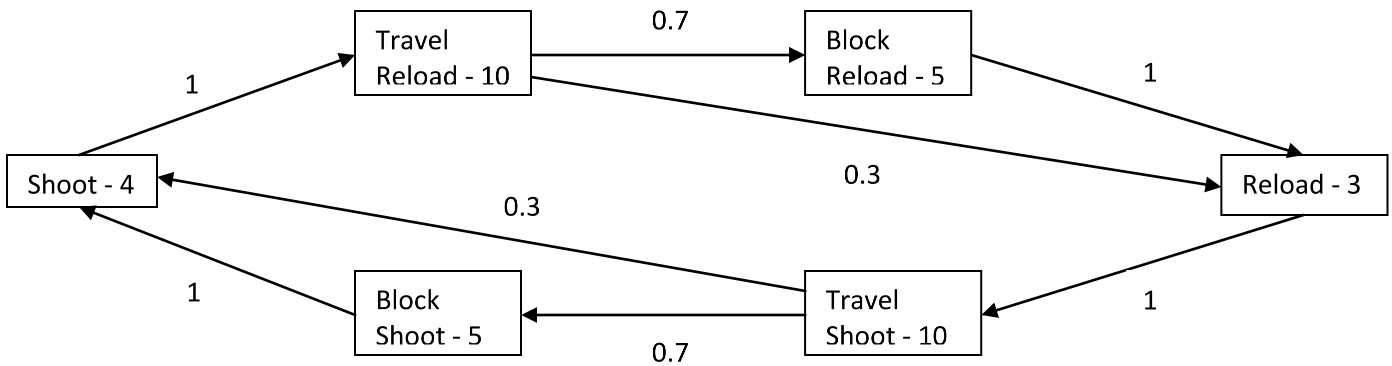

Fig. 1. Weighted Close Position Markov Chain Diagram

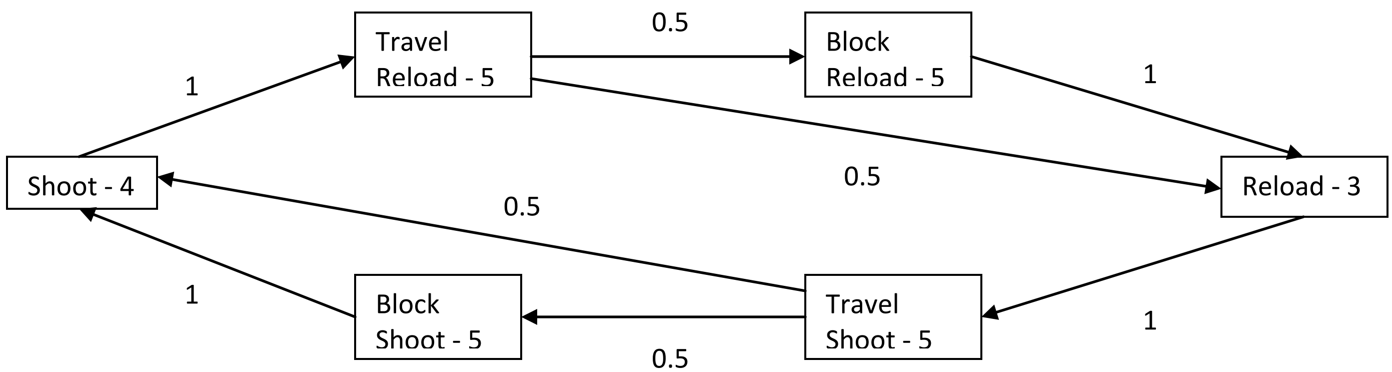

Fig. 2. Weighted Mid Position Markov Chain Diagram

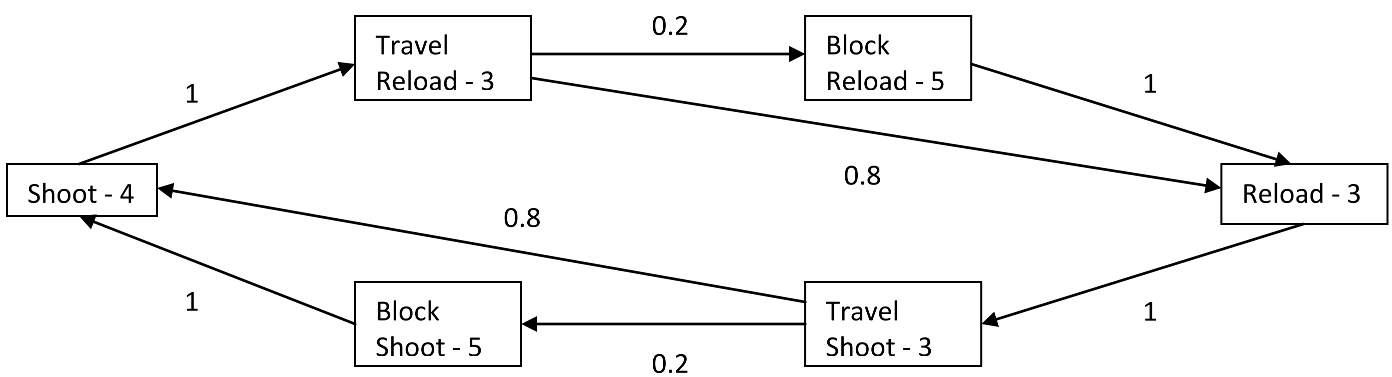

Fig. 3. Weighted 3/4 Position Markov Chain Diagram

Appendix B1 - Markov Chain Solutions

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1-P_B & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_B & 0 & 0 & 0 & 0 \\ 0 & 1-P_B & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_B & 0 \end{bmatrix} * \begin{bmatrix} S_n \\ TR_n \\ BR_n \\ R_n \\ TS_n \\ BS_n \end{bmatrix} = \begin{bmatrix} S_n \\ TR_n \\ BR_n \\ R_n \\ TS_n \\ BS_n \end{bmatrix}$$

Long Term Analytical Solution Equation

Close: $P_B = 0.7$
Mid: $P_B = 0.5$
3/4: $P_B = 0.2$

Fig. 4 - Long Term Analytical Solution Equation

$$BS_n * \begin{bmatrix} 10/7 \\ 10/7 \\ 1 \\ 10/7 \\ 10/7 \\ 1 \end{bmatrix} \xrightarrow{\text{Including Weights}} BS_n * \begin{bmatrix} 4*10/7 \\ 10*10/7 \\ 5*1 \\ 3*10/7 \\ 10*10/7 \\ 5*1 \end{bmatrix}$$

$1 = (40/7 + 100/7 + 5 + 30/7 + 100/7 + 5) * BS_n$

$BS_n = 7/340$

$$\begin{bmatrix} S = 2/17 \\ TR = 5/17 \\ BR = 7/68 \\ R = 3/34 \\ TS = 5/17 \\ BS = 7/68 \end{bmatrix}$$

Fig. 5 - Close Position Analytical Solution

Appendix B2 - Markov Chain Solutions

$$BS_n * \begin{bmatrix} 2 \\ 2 \\ 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} \xrightarrow{\text{Including Weights}} BS_n * \begin{bmatrix} 4*2 \\ 5*2 \\ 5*1 \\ 3*2 \\ 5*2 \\ 5*1 \end{bmatrix}$$

$$1 = (8+10+5+6+10+5) * BS_n$$

$$BS_n = 1/44$$

$$\begin{bmatrix} S = 2/11 \\ TR = 5/22 \\ BR = 5/44 \\ R = 3/22 \\ TS = 5/22 \\ BS = 5/44 \end{bmatrix}$$

Fig. 6 - Mid Position Analytical Solution

$$BS_n * \begin{bmatrix} 5 \\ 5 \\ 1 \\ 5 \\ 5 \\ 1 \end{bmatrix} \xrightarrow{\text{Including Weights}} BS_n * \begin{bmatrix} 4*5 \\ 3*5 \\ 5*1 \\ 3*5 \\ 3*5 \\ 5*1 \end{bmatrix}$$

$$1 = (20+15+5+15+15+5) * BS_n$$

$$BS_n = 1/75$$

$$\begin{bmatrix} S = 4/15 \\ TR = 3/15 \\ BR = 1/15 \\ R = 3/15 \\ TS = 3/15 \\ BS = 1/15 \end{bmatrix}$$

Fig. 7 - 3/4 Position Analytical Solution

Appendix C - Max Shots at Each Location

<u>Without Blocking</u>

Max Shots Close Position

1. Shoot 4 discs - 4 sec
2. Travel to reload - 10 sec
3. Reload - 3 sec
4. Travel to shoot - 10 sec
5. R: Repeat 1-4 for 120 sec

27R = 120
R = 4.44: 4 * 4 = 16 discs shot during full completions of R
0.44 * 27 = 11.88 > 4: 4 discs shot during 5th R
16 + 4 = 20 discs shot

120/20 = 6 seconds per shot

Max Shots Mid-Field Position

1. Travel Close to Mid - 5 sec
2. Shoot 4 discs - 4 sec
3. Travel to reload - 5 sec
4. Reload - 3 sec
5. Travel to shoot - 5 sec
6. R: Repeat 2-5 for 115 sec

17R + 5 = 120
R = 6.76: 6 * 4 = 24 discs shot during full completions of R
0.76 * 17 = 12.92 > 4: 4 discs shot during 7th R
24 + 4 = 28 discs shot

120/28 = 30/7 seconds per shot

Max Shots 3/4-Field Position

1. Travel Close to 3/4 - 7 sec
2. Shoot 4 discs - 4 sec
3. Travel to reload - 3 sec
4. Reload - 3 sec
5. Travel to shoot - 3 sec
6. R: Repeat 2-5 for 113 sec

13R + 7 = 120
R = 8.69: 8 * 4 = 32 discs shot during full completions of R
0.69 * 13 = 8.97 > 4: 4 discs shot during 9th R
32 + 4 = 36 discs shot

120/36 = 10/3 seconds per shot

<u>With Blocking</u>

	Close	Mid-Field	3/4-Field
Time spent Shooting (S)	14.12 (15)*	21.82 (22)*	32

Max Shots Close Position

120/15 = 8 seconds per shot

Max Shots Mid-Field Position

120/22 = 60/11 seconds per shot

Max Shots 3/4-Field Position

120/32 = 15/4 seconds per shot

*We round up due to the assumption that a disc is still live if it is shot before time runs out.