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Summary of: "The Stability of Highly Pathogenic Avian Influenza Epidemic Model with Saturated Contact Rate"

Variables and Explanations

R_0 - reproduction number, the number of cases one case generates on average over the course of its infectious period in an otherwise uninfected population
human: S - susceptible, I - infected, R - recovery, b - birth rate, alpha - natural mortality, epsilon - illness mortality
avian: X - susceptible poultry, Y - infected poultry, c - birth rate, d - natural mortality, m - illness mortality, omega - infection rate of susceptible to infected
beta - infected poultry of the infection rate of susceptible humans? (infection rate of infected poultry to susceptible humans?)
gamma - recovery rate that infects individuals through treatment?(recovery rate of infected individuals undergoing treatment?)
delta - effects of infectious disease when contact rate of the disease is saturated
disease-free equilibrium (E_0) - no infected, all susceptible
endemic equilibrium (E_+) - disease is always present without reintroduction

Processes and Definitions

R_0 <= 1 - the disease will go extinct over time (not found in the population)
R_0 > 1 disease remains endemic
endemic - the disease will be found in the population
contact rate of disease is saturated - all infected are in contact with all susceptible?
Jacobian matrix - matrix of all first-order partial derivatives of a vector-valued function
characteristic equation - equation solved to find a matrix's eigenvalues
lyapunov function - scalar function that may be used to prove the stability of an equilibrium of an ODE
Routh-Hurwitz stability criterion - a mathematical test that is a necessary and sufficient condition for the stability of a linear time invariant control system
LaSalle Invariance principle - a criterion for the asymptotic stability of an autonomous (possibly nonlinear) dynamical system

This paper begins by providing some background into the Avian Influenza Virus. The paper identifies several types, all denoted with the patter H(number)N(number) (e.g. H7N2), and gives some more human infection information on the types: H7N9 and H5N1. People contract the virus primarily from contact with infected live or dead poultry, poultry parts or poultry feces. It is not clear if human to human transmission is possible. The purpose of this research paper is to "propose a Avian influenza model with saturated contact rate", and from the abstract, it appears that the most useful information gleaned from the model are the disease free equilibrium and endemic equilibrium.

The authors first set out on identifying and explaining several variables that I have mentioned above, and develop an SIR model as shown in system 1.1. In section 2, the authors prove the existence of two equilibrium points E_0 and E_+ for system 2.1. When R_0 <= 1, the equilibrium point is E_0. When R_0 > 1, the equilibrium point is E_+. They present a flow diagram in figure 1 which has a point that I didn't find clear. The arrows interconnecting Y, S and I indicate that something can be passed from S to Y, but I realized that their intended meaning is that Y and S both contribute to I.

In section 3 and 4, the authors show the stability of E_0 and E_+. Both sections begin the same, by forming a Jacobian matrix of system 2.1 and adjusting that matrix to reflect the correct state of either E_0 or E_+. Next, they find the characteristic equations of the adjusted Jacobian matrix and use those equations to show that E_0 is locally asymptotically stable when $R_0 <= 1$ and E_+ is locally asymptotically stable when $R_0 > 1$. Next, they use two Lyapunov functions based on system 2.1 to show E_0 and E_+ are globally asymptotically stable. Additionally, in finding the characteristic equation of the Jacobian matrix for E_+, the authors use something called a "Harwize" criterion (which I found should actually read "Hurwitz", maybe this is a translation error?).

In section 5, the authors present numerical simulations to validate their findings. At first glance, it seemed strange to me that they only use one set of parameters for the variables listed above to validate E_0 and E_+. However, I realized that if the theorems developed in sections 2, 3 and 4 are correct, then all simulations would yield a result that indicate that both E_0 and E_+ are globally asymptotically stable. The graphs displayed could be more clear. It is difficult to differentiate between X and Y, and the values of on the Y-axis don't represent real life population values (unless the authors intend for these to represent thousands or millions of individuals).

Ultimately, the authors suggest solutions that will cause the listed variables to change in such a way that yields an $R_0 <= 1$ which in turn drives system 2.1 to E_0. Admittedly, I do not follow all of the math involved in this paper, so I can't comment on the correctness of their processes in verifying the model. It does seem to be an acceptable model, as it shows that it is certainly possible to eradicate the disease from the population, assuming the required parameters can be reached. Real world evidence would further validate the model, but it may be difficult to come by a large amount of acceptable data, as Avian Influenza doesn't seem to be particularly wide spread. I had two small issues with the paper. The writing in places was a bit confusing, but for the most part acceptable. Also, R and R_0 at first glance seemed to have a relationship but didn't in actuality. Maybe different symbols for these variables would be appropriate?