A Review of the Altman Z-Score

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Abstract

Predicting bankruptcy of businesses, or at the very least, poor financial health, is of great concern to a large number of entities. Edward Altman is one of the pioneers in bankruptcy prediction, with his Altman Z-Score. He utilized a discriminant analysis approach in 1968 to develop his Z-Score model, and continued (and continues) to adapt and upgrade it over the years. This paper discusses 2-group linear multivariate discriminant analysis and addresses its use and results in a few of Altman's Z-Score model adaptations.

Introduction and History

Determining whether, or not, a business firm is on a path towards bankruptcy (i.e. its financial health) is a natural concern for a large number of entities including credit analysts, equity analysts and the firm itself, just to name a few. Banks don't want to lend money to failing firms, nor do investors want to invest in such firms. However, asking "if" a firm will fail is not the only concern. The "when" and "why" are also important. Will bankruptcy occur in 1, 2 or 5 years? Will it occur because the firm overestimated inventory requirements?

Any process of determining the health of a firm requires metrics that demonstrate aspects of the firm's health. Financial ratio analysis is a classic, common, but relatively simple, quantitative method of determining a firm's health. Financial ratios are accounting, economic and market data that are scaled to allow comparisons among firms. For example, a firm with $1 million in cash but an imminent debt payment of $10 million is not as healthy as another firm with $1 million in cash but an imminent debt payment of $500 thousand.

Developed in 1968, the Altman Z-Score (not to be confused with the standard z-score) is one of the first multivariate statistical attempts to provide insight into the questions of "if", "when" and "why" in regard to firm failure. The Z-Score was a novel application of Fisher's Discriminant Analysis (DA) technique [**11**]. Altman [**1**] notes that previous statistical work addressed the relationship among financial ratios, financial health and bankruptcy prediction. Beaver's paper [**10**] is one such example. Also, DA had been utilized previously in finance for consumer credit evaluation and investment classification [**1**].

Since 1968, the Z-Score has been utilized and modified by practitioners and academics all over the world. Altman has continued to adapt and update his model using old and new statistical methods and financial data. This paper explains DA, particularly in a multivariate and 2 group setting, and how Altman, and others, have used DA to develop various "Z-Score" models.

Discriminant Analysis

Discriminant Analysis is a statistical technique that analyzes 2 or more variables shared by data that fall into 2 or more groups. The first goal is to determine if the group means are significantly (statistically) different by postulating: the null hypothesis is that the group means are equal, and the alternate hypothesis is that they are not equal. The second goal is to predict to which group an observation is most likely to belong, if the null hypothesis is rejected. Since the Z-Score is only concerned with classifying bankrupt and non-bankrupt groups, this section will outline multivariate 2-group DA (henceforth referred to as MDA). Relevant information from [**12**], [**13**] and [**14**] is summarized in this section. Many online and print sources also provide the same information.

MDA is computationally similar to MANOVA and shares the same set of assumptions.

1. The data come from multivariate normal distributions, though this can usually be violated without greatly affecting significance tests.
2. Variances and co-variances for variables are homogeneous across groups. The Box M test and Bartlett's test, among others, are methods of determining whether this assumption holds or not. If the assumption holds, use a linear discriminant function. If not, use a quadratic discriminant function.
3. Correlations between means and variances can decrease the validity of significance tests. This is an issue caused by extreme outliers distorting means and variances for some variables for one group.
4. Variance and co-variance matrices are not ill-conditioned due to redundant variables. In other words, the rows and columns of the matrices are linearly independent, thus the matrices are invertible.

2-Group linear MDA seeks to find the coefficients a_i in Eq. 1 (below) that maximizes the scatter between classes (SBC) (Eq. 2) and minimizes the scatter within classes (SWC) (Eq. 3). Eq. 4 shows the F statistic in matrix form. In Eq. 5, the goal is to minimize F. The coefficients a_i are given by the eigenvector associated with the dominant eigenvalue λ. In the following equations, g refers to group 1, h refers to group 2, n_g is the number of observations in g, n_h is the number of observations in h, and k is the number of variables (i.e. dimensions). For example, x_{1g2} is the value for the 1st variable and the 2nd observation in g. Matrices W and B both have dimensions (n_g+n_h, k).

$$discriminant\ line: a_0 + a_1x_1 + a_2x_2 + \cdots + a_kx_k = 0 \quad Eq. 1$$

$$SBC = s_B^2 = n_g\left(\sum_{j=1}^{k} a_j(\bar{x}_{gj} - \bar{x}_j)\right)^2 + n_h\left(\sum_{j=1}^{k} a_j(\bar{x}_{hj} - \bar{x}_j)\right)^2 \quad Eq. 2$$

$$SWC = s_W^2 = \sum_{i=1}^{n_g} (\sum_{j=1}^{k} a_j(x_{gji} - \bar{x}_{gj}))^2 + \sum_{i=1}^{n_h} (\sum_{j=1}^{k} a_j(x_{hji} - \bar{x}_{hj}))^2 \quad Eq. 3$$

$$W = \begin{bmatrix} x_{1g1} - \bar{x}_{1g} & x_{2g1} - \bar{x}_{2g} & \cdots & x_{kg1} - \bar{x}_{kg} \\ x_{1g2} - \bar{x}_{1g} & x_{2g2} - \bar{x}_{2g} & \cdots & x_{kg2} - \bar{x}_{kg} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1gn_g} - \bar{x}_{1g} & x_{2gn_g} - \bar{x}_{2g} & \cdots & x_{kgn_g} - \bar{x}_{kg} \\ x_{1h1} - \bar{x}_{1h} & x_{2h1} - \bar{x}_{2h} & \cdots & x_{kh1} - \bar{x}_{kh} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1hn_h} - \bar{x}_{1h} & x_{2hn_h} - \bar{x}_{2h} & \cdots & x_{khn_h} - \bar{x}_{kh} \end{bmatrix} \quad B = \begin{bmatrix} \bar{x}_{1g} - \bar{x}_1 & \cdots & \bar{x}_{kg} - \bar{x}_k \\ \vdots & \ddots & \vdots \\ \bar{x}_{1g} - \bar{x}_1 & \cdots & \bar{x}_{kg} - \bar{x}_k \\ \bar{x}_{1h} - \bar{x}_1 & \cdots & \bar{x}_{kh} - \bar{x}_k \\ \vdots & \ddots & \vdots \\ \bar{x}_{1h} - \bar{x}_1 & \cdots & \bar{x}_{kh} - \bar{x}_k \end{bmatrix} \quad V = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix}$$

$$F = \frac{s_W^2}{S_B^2} = \frac{V^t U_w V}{V^t U_B V} \ where\ U_W = W^t W \ and\ U_B = B^t B \quad Eq. 4$$

$$\frac{dF}{dV} = \frac{2(U_w V(V^t U_B V) - (V^t U_w V) U_B V)}{(V^t U_B V)^2} = 0 \rightarrow \ U_w V(V^t U_B V) = (V^t U_w V) U_B V \rightarrow U_B V$$

$$= \frac{(V^t U_B V)}{(V^t U_w V)} U_w V, \frac{(V^t U_B V)}{(V^t U_w V)} \ is\ a\ scalar\ so\ U_W^{-1} U_B V = \lambda V \quad Eq. 5$$

Z-Score for Manufacturers

In sections III and IV of [**1**], Altman discusses the development and results of the Z-Score model. He gathered a training sample of 66 U.S. manufacturing corporations with asset sizes ranging from $1 million to $25 million. Of these, 33 had declared bankruptcy under Chapter X of the National Bankruptcy Act during the period 1946-1965. The other 33 firms were still operating. The asset size range was limited since bankruptcy among larger firms was rare. This choice kept the bankrupt (Group 1) and non-bankrupt (Group 2) samples relatively homogeneous with regard to asset size.

Altman gathered the income statements and balance sheets of the 66 companies. He compiled a list of 22 financial ratios and categorized them into 5 ratio groups: liquidity, profitability, leverage, solvency and activity ratios (see Appendix A for explanations). The 22 ratios were chosen on the basis of their popularity in the literature and relevancy to bankruptcy.

After running numerous MDAs on varying sets of the 22 ratios, Altman chose a discriminant function (Eq. 6) consisting of 5 variables (see Appendix B for the details) to best discriminate between bankrupt and non-bankrupt firms. The 5 variables are denoted X_1, X_2, X_3, X_4 and X_5.

$$Z = .012X_1 + .014X_2 + .033X_3 + .006X_4 + .999X_5 \quad Eq. 6$$

To determine which discriminant function was chosen, Altman utilized 4 observations:

1. The statistical significance of the discriminant function and relative contribution of each independent variable (i.e. ratio).
2. Inter-correlations between the ratios.
3. Predictive accuracy of the discriminant function.
4. Personal judgment.

For each of the 5 chosen variables, Altman ran an F test, with degrees of freedom 1 (2 group means - 1) and 60 (closest DoF to 66 - 2 on an F distribution table), to test the discriminating ability of each variable on a univariate basis. For Group 1, the data come from the most recent pre-bankruptcy financial statements. The F test for each variable relates the difference between the group means of the data to the group variability of the data. The results are shown in table 1, columns 1-4.

Variable	**Group 1 Mean**	**Group 2 Mean**	**F Ratio**	**Scaled Coefficient**	**Discriminating Power Rank**
X_1	-6.1%	41.4%	32.6	3.29	5
X_2	-62.6%	35.5%	58.86	6.04	4
X_3	-31.8%	15.3%	26.56	9.89	1
X_4	40.1%	247.7%	33.26	7.42	3
X_5	150.0%	190%	2.84	8.41	2

Table 1 - Tests of Individual Variable Significance and Relative Discriminating Power

Variables X_1, X_2, X_3, X_4 are all significant at the 0.001 level, indicating a significant difference for each variable between groups. X_5 is not significant at the 0.05 level in this test.

Altman determined the discriminating power of each variable in Eq. 6 by scaling the coefficients by the standard deviation of the associated variable. Table 1, columns 5 and 6, show the results. Interestingly, X_5 has the second-most discriminating power in the multivariate setting, despite it not being significant in a univariate setting. The F-value (as discussed in the "discriminant analysis" section) associated with Eq. 6 is 20.7, which is significant at the 0.01 level for degrees of freedom 5 (5 coefficients + 1 intercept - 1) and 60.

To determine the predictive accuracy of Eq. 6, Altman conducted a series of 6 tests. The results are listed in Table 2.

	Bankrupt			**Non-Bankrupt**		
Test	**# Samples**	**Correct**	**Incorrect**	**# Samples**	**Correct**	**Incorrect**
1	33	31 (94%)	2 (6%)	33	32 (97%)	1 (3%)
2	32	23 (72%)	9 (28%)	33	31 (94%)	2 (6%)
4	25	24 (96%)	1 (4%)			
5				66	52 (79%)	14 (21%)
6a	29	14 (48%)	15 (52%)			
6b	28	8 (29%)	20 (71%)			
6c	25	9 (36%)	16 (64%)			

Table 2 - Results of Tests 1,2,4,5,6

Test 1 was run on the training data. As expected, since Eq. 6 was determined using the training, the results were good for both bankrupt and non-bankrupt groups.

Test 2 was run on the same firms as in Test 1, but the data were from 2 years prior to bankruptcy. One of the bankrupt firms was not included in Test 2, for an unknown reason. The results were decent for the bankrupt group and good for the non-bankrupt group, considering Eq. 6 was determined using data from a different time period. It is likely that the business climate had not changed much over the time period.

Test 3 is an interesting process that addresses concerns about search bias, that the model is effective for the training sample but not as effective for the population. The search bias may have been introduced during the process of reducing the number of potential variables from 22 to 5. The training sample is labeled 1 to 33 for each group and 16 firms are chosen as the new training sample (the remaining 17 are the secondary sample) using 5 different methods: random sampling, odd numbered firms, firms 2-18, firms 1-16 and firms 17-32. MDA is conducted for each new training sample. The resulting 5 discriminant functions had the following percentage of correct classifications on the secondary sample: 91.2, 91.2, 97, 97, 91.2 and average of 93.5. T-tests (Eq. 7) for each method were significant at the 0.001 level, indicating that the model has discriminating power for other training samples. The search bias is likely not significant.

$$t = \frac{proportion\ correct - 0.5}{\sqrt{\frac{0.5(1-0.5)}{34}}} \quad Eq.7$$

Test 4 was conducted on a secondary sample of bankrupt firms with characteristics similar to the firms in the training sample. Correct classification of 96% of the secondary sample is better than 94% of the training sample. Altman postulates that this is caused by a lack of upward bias in the training sample, or that Eq. 6 is less than optimal. There may be a better equation to model the phenomenon.

Test 5 was conducted on a secondary sample of non-bankrupt firms, but they were chosen under different criteria than the non-bankrupt firms from the previous tests. 33 were chosen from 1958 and 33 were chosen from 1961; these years were chosen due to weak GDP growth in those years. All of the firms are manufacturing firms that suffered losses (i.e. negative net income) in at least one of the years, with 65% suffering losses 2 or 3 years previously. These firms are chosen without regard to asset size (whereas asset sizes of the training firms ranged from \$1 to \$25 million). 79% correct classification is a good result indicative of the discriminating power of Eq. 6, particularly considering that the firms in the sample displayed some signs of heading towards bankruptcy.

Test 6 was similar to Test 2, but extended to 3(a), 4(b) and 5(c) years prior to bankruptcy. Not all of the firms were included in the samples, as some did not exist in that year, or data was otherwise not available. As expected, the predictive power of Eq. 6 generally decreased as the number of years before bankruptcy increased.

With regard to applications of the Z-Score model, Altman notes that the model correctly classified all firms with a Z-Score of less than 1.81 as bankrupt. It also correctly classified all firms with a score of greater than 2.99 as non-bankrupt. He calls the area between 1.81 and 2.99 a "zone of ignorance", as this is the range of values with which misclassifications occur in the model.

Z-Score for Railroads

In 1973, Altman applied a similar methodology to the development of a Z-Score model for the railroad industry [**2**]. He notes that bankruptcy prediction in the railroad industry may be more amenable to DA (than the manufacturing industry) since the railroad firms are more homogeneous. The firms chosen either had $5 million or more in revenue (i.e. Class 1) or had filed for bankruptcy under Section 77 of the Bankruptcy Act. Again, financial ratios from all 5 categories (Appendix A) were used as the variables. The bankrupt sample consisted of 21 firms that declared bankruptcy between 1939 and 1970, but 2 approaches were applied to form a non-bankrupt group. The first, and primary, approach used proportionally weighted yearly railroad industry financial ratio averages as the data. The secondary approach used individual firms selected from the same years as the bankrupt firms.

For the primary approach, Altman chose 14 ratios initially (using the same reasoning as before), and narrowed the list down to 7 (see Appendix B) in the final discriminant function (Eq.

8). Eq. 8 was determined using stepwise MDA. For the 14 initial ratios, an F-ratio measuring the significance of the mean values for each ratio between the two groups was determined. The ratio with the greatest F-ratio was chosen, and from there other variables were included such that the F-value for MDA was maximized.

$$Z = 0.2003Y_1 + .2070Y_2 + 0.0059X_2 + 0.0647Y_3 + 0.1040Y_4 + 0.0885Y_5 + 0.0688X_3 \quad Eq. 8$$

Many of the same tests were run on the railroad Z-Score model as on the manufacturing Z-Score model. Both Tests 1 and 2 resulted in only 1 misclassified bankrupt firm and no misclassifications of the industry averages. Test 3 was run as before with the following percentage of correct classifications on a secondary sample of 20 observations, respectively: 90, 85, 95 (even railroads + #1) , 65, and 83. A secondary sample of 50 railroads was chosen by randomly selecting a year between 1946 and 1969 and then randomly selecting a Class 1 railroad from that year. Of these 50, 6 were classified as bankrupt. 2 actually went bankrupt, 1 discontinued operations and 2 other were acquired by other railroads that eventually went bankrupt.

For the secondary approach, Altman used the same 7 variables but calculated new coefficients using the stepwise MDA process. The means for each of the variables between the non-bankrupt groups in each approach were typically similar, but the spread of the data was higher in this approach. Tighter spreads help in discrimination, and the Z-Score calculated with this approach (13.0) was lower than the Z-Score calculated in the first approach (20.1). Both are significant at the 0.01 level though.

ZETA for Manufacturers and Retailers

Developed in 1977, ZETA [**3**] is an updated but proprietary version of the original Z-Score. 5 relevant developments had occurred since 1968 that warrant the effort to create an improved Z-Score model. The average asset size of bankrupt firms increased, so this study samples firms ranging from $20 million to $100 million in asset size. More recent firms were sampled for the study, with 50 of 53 bankrupt firms failing during the period 1970-1977. With certain adjustments, ZETA includes retail firms in the current population of manufacturing firms. Recent and upcoming changes to financial reporting and accounting standards were considered, as were recent development in DA.

For this study, an initial pool of 27 variables was decreased to 7: X_3, A_1, A_2, X_2, A_3, A_4 and A_5 (see Appendix B). Training data for the 53 bankrupt firm was from 1-year prior to bankruptcy. 58 non-bankrupt firms were chosen from the same years and industries and as the bankrupt firms. 6 methods, including stepwise MDA, were used to determine which of the 27 variables best discriminated between bankrupt and non-bankrupt groups. Altman notes that in other applications of the 6 methods, the results did not clearly indicate a dominant variable set. However, X_2, A_1 and A_4 were consistently relatively important across the tests, so Altman did not have that issue in this study. Since ZETA is proprietary, the coefficients are not included in the paper.

Altman considered both linear and quadratic MDA. The Box H_1 (i.e. Box M) Test indicates that a quadratic MDA is appropriate for the data, but Altman decided to take the linear approach for the following reasons.

1. He argues that the linear and quadratic versions both had similar predictive power for the training data (both at 92.8% correctly classified), but the linear version had better predictive power for secondary data (discussed in the next paragraph).
2. The 6 methods of determining the relative importance of the data are based on the linear version.
3. There is also a concern that the 35 quadratic parameters could be more sensitive to outliers than the 7 linear parameters.

As before, there are concerns about upward bias on the classification of training data, but Altman notes that this isn't of great concern since the results of the Lachenbruch validation test are close to the results on the training data. Secondary data consists of data gathered for the same firms as the training data but from previous years. Table 3 shows ZETA's predictive power on the secondary data. In comparison to the original Z-Score [**1**], ZETA is far superior at classifying firms using data from the previous 2-5 years.

	Lachenbruch	**2-Years**	**3-Years**	**4-Years**	**5-Years**
Linear	91.0%	89.0%	83.5%	79.8%	76.8%
Quadratic	86.5%	84.7%	78.9%	74.0%	69.7%

Table 3 - Percentage of Correctly Classified Firms

Another validation test was performed in a manner similar to that of Test 3, discussed in the "Z-Score for Manufacturers" section. In this study, 10 new training samples are gathered by randomly selecting half of the samples from the original training data and secondary data for each year. The other half is the new secondary data. For each of the training samples, a new linear discriminant function is calculated and the other half of the data from that year are classified. This process is performed twice. In both replications, the classification results were similar to those of the first linear ZETA model, so search bias is not a concern.

Related Work

Developed in 1983, the Z''-Score (Eq. 9) model can be applied to manufacturing and non-manufacturing firms, as well as public and private firms. [**6**], [**8**] and [**9**] provide details on the use of the Z''-Score in an Emerging Markets Scoring (EMS) model, but the details of its development are not given. The goal of the EMS model is to assign to firms in emerging markets a bond rating that can be compared to U.S. corporate bonds. The Z''-Score is the first of 6 steps in the model.

$$Z'' = 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05B_1 + 3.25 \quad Eq. 9$$

In [**4**], Altman utilizes quadratic DA and 2-Group DA to "develop a system for identifying serious financial problems in savings and loan associations." The resulting model is a "12-variable econometric model that is both accurate and practical for at least three semi-annual periods preceding the serious problem data."

In [**5**], Altman and La Fleur use the Z-Score [**1**] to aid in returning GTI, a manufacturing firm, from near bankruptcy to financially healthy. As the CEO of GTI, La Fleur was in a position to direct company policy in a direction that would increase the financial ratios relevant to the Z-Score, and thus the Z-Score itself.

In the wake of the Enron and WorldCom crashes and upcoming implementation of Basel II, credit risk models gained increased importance. Altman [**7**] compares multiple Z-Score models to KMV's EDF model with regard to bankruptcy prediction.

[**15**] details application of the Z-Score methodology to bankruptcy prediction in China. This is an interesting case since Chinese accounting and financial reporting standards are different than those in the US. So, direct application of a model developed with data from non-Chinese firms may not be warranted, and a new Z-Score model must be created.

Appendix A - Explanations

Liquidity: In business, this has two definitions: 1) how quickly an asset can be exchanged for cash without significant loss of value and 2) measures of the ability of a firm to satisfy its short-term (less than 1 year) obligations. In this paper, liquidity refers to explanation 2.

Profitability: Measures of how well the firm is utilizing its assets and minimizing expenses to generate net income (i.e. revenue after expenses including taxes are deducted).

Leverage: Measures of a firm's debt load.

Solvency: Measures comparing assets to liabilities.

Activity/Efficiency: Measures of how well the firm is utilizing its assets.

Appendix B - Z-Score Financial Ratios and Other Measures

Many of these ratios provide information for multiple categories. The most relevant are given.

$X_1 = \frac{working\ capital}{total\ assets}$. A liquidity ratio.

$X_2 = \frac{retained\ earnings}{total\ assets}$. A profitability-over-time ratio.

$X_3 = \frac{earnings\ before\ interest\ and\ taxes}{total\ assets}$. A profitability ratio.

$X_4 = \frac{market\ value\ of\ equity}{book\ value\ of\ total\ debt}$. A solvency ratio.

$X_5 = \frac{sales}{total\ assets}$. An activity ratio.

$Y_1 = \frac{cash\ flow}{fixed\ charges}$. A solvency and leverage ratio.

$Y_2 = \frac{transportation\ expenses}{operating\ revenue}$. A liquidity ratio.

$Y_3 = 3\ year\ growth\ rate\ in\ operating\ revenue$. Not technically a ratio but measures profitability.

$Y_4 = \frac{earnings\ after\ taxes}{operating\ revenue}$. A profitability ratio.

$Y_5 = \frac{operating\ expenses}{operating\ revenue}$. A liquidity ratio.

$A_1 = normalized\ 10\ year\ standard\ error\ of\ estimate\ of\ X_3$. A stability of profitability measure.

$A_2 = log_{10} \frac{earnings\ before\ interest\ and\ taxes}{total\ interest\ payments}$. A leverage measure.

$A_3 = \frac{current\ assets}{current\ liabilities}$. A liquidity and solvency ratio.

$A_4 = \frac{market\ value\ of\ equity}{total\ capital}$. A solvency and leverage ratio.

$A_5 = log_{10}(total\ assets)$.

$B_1 = \frac{book\ value\ of\ equity}{total\ liabilities}$. A solvency ratio.

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