Introduction

The spread of distress and spillover across institutions causes the rise of systemic risk which is the risk of the entire financial system collapsing. Therefore a systemic risk measure was developed to capture the spread of financial distress across institutions. The most common risk measure used by financial institutions is value at risk (VaR) but a single institution risk measure doesn't capture it's contributions to the overall systemic risk. Hence, $\Delta CoVaR$ is developed to capture the institution's contribution to the overall systemic risk as well as tail dependency and spillover. Forward $\Delta CoVaR$ was developed to capture the build-up of systemic risk in the run-up phase. It does that by regressing time-varying $\Delta CoVaR$ on lagged institutional characteristics (i.e size, leverage, and maturity mismatch) and conditioning variables (i.e. market volatility and fixed income spreads). Estimation of $\Delta CoVaR$ shows that institutional characteristics across financial institutions like higher leverage and valuations and more maturity mismatch and larger size contribute the most to the overall systemic risk.

Section 2: Literature review

2.1 Theoretical Background on Systemic Risk

Spillovers can cause institutions to take more risks and leverages during the run-up phase of a crisis. Spillovers occur in the form of externality. The outcome of externality will not be constrained by Pareto efficiency discovered in Bhattacharya and Gale (1987) work in a banking context and Stiglitz (1982) and Geanakoplos and Polemarchakis (1986) work on a general equilibrium incomplete market setting. These externality effects are also studied in Caballero and Krishnamurthy (2004) work on international finance.

Procyclicality is the tendency of financial institutions following a trend that occurs during an economic cycle. Procyclicality occurs when the risk is low in a boom phase and high in a crisis. This is called the "Volatility paradox" which Brunnerimrier and Sannikov (2014) came up with. During a volatility paradox, financial institutions are willing to take more risk and leverage because volatility and risk measure is low which in return makes the financial system more vulnerable during a crisis. Brunnermeier and Pedersen (2009) and Borio (2004) address margin/haircut spiral and procyclicality and discuss how to address them.

2.2 Other Systemic Risk Measures

There are other systemic risk measures besides $\Delta CoVaR$. For instance, Huang, Zhou, and Zhu (2010) developed a system risk measure that estimated the prices of insurance in case of systemic financial distress from credit default swaps (CDS) prices. While Acharya, Pedersen, Philippon, and Rcihardson (2010) developed a systemic risk measure that calculated the expected shortfall of high-frequency marginal. Their systemic risk measure addresses the question on which financial institutions are more exposed during a crisis which is similar to what $Exposure - \Delta CoVaR$ address. Different conditions of $\Delta CoVaR$ can address different questions. Other works of literature have expanded the use of $\Delta CoVaR$ in more financial sectors. For instance, Adams, Fuss, and Gropp (2010) used it to study risk spillovers between financial sectors. Others like Lehar (2005) and Gray, Merton, and Bodie (2007), use contingent claims to study systemic risk.

2.3 The Econometrics of Tail Risk and Contagion

$\Delta CoVaR$ measures are related to tail risk and volatility models as shown in various literature. Engle and Manganelli (2004) developed $CAViaR$ using quantile regression with the GARCH model to capture the tail behavior of asset returns that vary in time. $\Delta CoVaR$ is also related to volatility spillovers and contagion shown in Classens and Forbes (2001). The common approach to analyze for volatility spillovers is to use the multivariate GARCH process and another approach is the multivariate extreme value theory. But Danielsson and de Vries (2000) believe that extreme value theory only works well for very low quantiles. Meanwhile, Hartman, Straetmans, and de Vries (2004) create a contagion measure that can capture extreme events.

Section 3: $CoVaR$ methodology

3.1 Definition of $CoVaR$

VaR_q^i is the value at risk and defined as the maximum loss of an institution i at confidence level $q\%$, $\Pr(X^i \leq VaR_q^i) = q\%$ quantile. X^i is the loss of the institution i which is defined as the "return loss" in this paper. VaR_q^i is usually positive when $q > 50$ and higher risks are equivalent to higher VaR_q^i.

$CoVaR_q^{j|C(X^i)}$ is the conditional value at risk and defined as the VaR of institution j or financial system conditioning on some event C of institution i and implicitly defined as Pr $(X^j|C(X^i) \leq CoVaR_q^{j|C(X^i)}) = q\%$ quantile. X^j is the return loss of institution j or the financial system and $C(X^i)$ is some event C of institution i.

$\Delta CoVaR_q^{j|i}$ is defined as the difference between the $CoVaR$ of the financial system j conditional on institution i being in distress and the $CoVaR$ of the financial system j conditional on institution i being in the median state. Its equation is

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|X^i=VaR_q^i} - CoVaR_q^{j|X^i=VaR_{50}^i} .$$

From this, institutions' i contribution to financial system j can be captured. In dollar terms,

$$\Delta^{\$} CoVaR_q^{j|i} = {}^{\$}Size^i \cdot \Delta CoVaR_q^{j|i} .$$

Where ${}^{\$}Size^i$ is the size of institution i.

$CoVaR$ is estimated by conditioning on some event C of an institution. Event C can be institution i loss greater than or equal to VaR_q^i with the likelihood of $(1-q)\%$. The likelihood of the conditioning event does not take institution i risk taking into account in order to prevent less risky institutions from having a higher $CoVaR$ when the conditioning event is more extreme. Which is also why we condition on a quantile instead of a particular return level i.

$\Delta CoVaR$ calculates the increase in $CoVaR$ as the conditioning event is the change from institution i in a median state (VaR_{50}^i) to the distressed state (VaR_q^i). $\Delta^{\$} CoVaR$ includes the size of institution i to compare the institution's contributions to systemic risk across institutions. Size is captured with the institution's market equity. $CoES$ is the conditional expected shortfall. The expected shortfall is the expected loss conditioning on VaR. $CoES_q^{j|i}$ is the expected loss of institution j conditioning on its losses exceeding $CoVaR_q^{j|i}$. $\Delta CoES_q^{j|i}$ is defined as $CoES_q^{j|i} - CoES_{50}^{j|i}$. Some advantage that $CoES$ have over VaR are subadditivity and it takes the distribution within the tail into consideration.

3.2 the Economics of Systemic Risk

The time-series aspect of systemic risk is systemic risk buildings up during the boom phase when volatility and measured risk is low. The buildup of systemic risk during this phase will lead to a "volatility paradox." Therefore, "$forward - \Delta CoVaR$" was developed to capture the buildup of systemic risk. It estimates the relationship between the current institution's characteristics and future tail dependency which implicitly defined as $\Delta CoVaR_{q,t}^{j|i}$. The time-series aspect of $\Delta CoVaR_{q,t}^{j|i}$ is represented by t.

The cross-sectional aspect of systemic risk is the spillover effects that cause the first distress shocks in the crisis. The $\Delta CoVaR^i$ measures tail dependency as well as captures the spillover effect and institutions contribute to the overall financial system in distress. As volatility increases, margin/haircut spiral arises and forces market participants to sell at a discounted price. Then it will increase the price impact on the market. Therefore a lot of risks are buildup during the run-up phase in the first component of systemic risk.

3.3 Tail Dependency versus Causality

$\Delta CoVaR_q^{j|i}$ measure tail dependency but doesn't correctly capture the spillover effects or externalities due to several reasons. One of these reasons is other institutions can reposition themselves in order to reduce the impact of spillovers therefore spillover effects can’t be observed in equilibrium. Another reason is $\Delta CoVaR_q^{j|i}$ capture common exposure to external macroeconomic risk factors.

Causality can be shown with the following model. There’s a simple stylized financial system that has two groups of institutions which are type i and type j. There are two latent independent risk factors, ΔZ^i, and ΔZ^j. We assume that due to the spillover effects, institution type i is directly exposed to the shock processes ΔZ^i and indirectly expose to ΔZ^j and visa versa for institution type j. The process of returns generating from assumed data for institution type i $(-X_{t+1}^i = \Delta N_{t+1}^i / N_t^i)$ and type j $(-X_{t+1}^j = \Delta N_{t+1}^j / N_t^j)$ will be

$$-X_{t+1}^i = \bar{\mu}^i(\cdot) + \bar{\sigma}^{ii}(\cdot)\Delta Z_{t+1}^i + \bar{\sigma}^{ij}(\cdot)\Delta Z_{t+1}^j$$
$$-X_{t+1}^j = \bar{\mu}^j(\cdot) + \bar{\sigma}^{jj}(\cdot)\Delta Z_{t+1}^j + \bar{\sigma}^{ji}(\cdot)\Delta Z_{t+1}^i$$

Inside of $(\cdot)$ are the following state variable ($M_t, L_t^i, L_t^j, N_t^i, N_t^j$) where M_t is the state of the macro-economy, L_t^i, L_t^j are the leverage and liquidity mismatch of institution type i and j and N_t^i, N_t^j are the net worth levels of institution type i and j. The geometric drift and volatility loading are functions of these state variables. Leverage L_t^i are a choice variable for institution type i and if L_t^i increase then the loading of its own risk factors ΔZ_{t+1}^i will also increase. Similarly, exposure of institution type i to the others latent risk factor, ΔZ_{t+1}^j, due to spillover captures by $\bar{\sigma}^{ij}(\cdot)$ can also increase its own leverage L_t^i as well as others leverage L_t^j.

Since the two shock processes, Z_{t+1}^i and Z_{t+1}^j are unobservable then the data generating process equation can be reduced to the following forms:

$$-X_{t+1}^i = \mu^i(\cdot) - \sigma^{ij}(\cdot)X_{t+1}^j + \sigma^{ii}(\cdot)\Delta Z_{t+1}^i$$
$$-X_{t+1}^j = \mu^j(\cdot) - \sigma^{ji}(\cdot)X_{t+1}^i + \sigma^{ji}(\cdot)\Delta Z_{t+1}^j$$

Where ΔZ_{t+1}^j is replaced with $-X_{t+1}^j$ and ΔZ_{t+1}^i is replaced with $-X_{t+1}^i$. If shock factor $\Delta Z_{t+1}^i <$ 0 then $-X_{t+1}^i$ will be lower by the product of volatility loading and shock factor $\sigma_t^{ii}\Delta Z_{t+1}^i$. The first round of spillovers effect will also reduce other institution’s return $-\Delta X_{t+1}^j$ by $\sigma_t^{ji}\sigma_t^{ii}\Delta Z_{t+1}^i$. due to the second round of spillovers effect, lowering $-\Delta X_{t+1}^j$ will also lower $-\Delta X_{t+1}^i$ by $\sigma_t^{ij}\sigma_t^{ji}\sigma_t^{ii}\Delta Z_{t+1}^i$. The reduction will continue through the third, fourth, and etc. rounds of spillover effects until a fixed point is reached. Then the volatility loading of the primitive data generating process will be $\overline{\sigma}_t^{ii} = \sum_{n=0}^{\infty} (\sigma_t^{ij}\sigma_t^{ji})^n \sigma_t^{ii} = \frac{\sigma_t^{ii}}{1-\sigma_t^{ij}\sigma_t^{ji}}$, $\overline{\sigma}_t^{ij} = \sum_{n=0}^{\infty} (\sigma_t^{ij}\sigma_t^{ji})^n \sigma_t^{ij}\ \sigma_t^{jj} = \frac{\sigma_t^{ij}\sigma_t^{jj}}{1-\sigma_t^{ij}\sigma_t^{ji}}$. $\overline{\sigma}_t^{jj}$ and $\overline{\sigma}_t^{ji}$ can be obtained by replacing i with j and visa versa. From these, we can link the reduced form σ s to the primitive $\overline{\sigma}$ s.

3.4 CoVaR, Exposure-CoVaR, Network-CoVaR

There are different types of $\Delta CoVaR$. For instance, $Network - \Delta CoVaR$ is simply defined as $\Delta CoVaR_q^{j|i}$ where subscripts i and j refers to individual institutions. When the subscript j is referred to the financial system instead of individual institutions then it's denoted by $\Delta CoVaR_q^{system|i}$. $\Delta CoVaR_q^{j|i}$ and $\Delta CoVaR_q^{system|i}$ is directional. Directional means that the financial system conditional on institution i , $\Delta CoVaR_q^{system|i}$, is not equivalent to the institution i conditional on the financial system, $\Delta CoVaR_q^{i|system}$. The direction of the conditioning is important because conditions can be changed to address different questions. In this paper, the conditioning direction is $\Delta CoVaR_q^{system|i}$ which addresses how risky the financial system is when institution i is in a state of distress compared to when it's in a normal state. This can be expressed as follows

$$\Delta CoVaR_q^{system|i} = \Delta CoVaR_q^{system|X^i=VaR_q^i} - \Delta CoVaR_q^{system|X^i=VaR_{50}^i}$$

Sometimes it is useful to compute the opposite conditioning, $\Delta CoVaR_q^{j|system}$, to address which institutions are more at risk when a financial crisis occurs. In this paper, $\Delta CoVaR_q^{j|system}$ is defined as $Exposure\ \Delta CoVaR$ to measure individual institution's exposure to systemic risk. The condition direction of $Exposure\ \Delta CoVaR$ is also important. For example, if the overall financial system is in distress then the conditional institutions are more likely to expose to risk therefore their $Exposure\ \Delta CoVaR$ is high. But if we condition on this institution being in distress at the same time, we would not get a high $Exposure\ \Delta CoVaR$ for the overall financial system. This shows that we wouldn't get the same result if we change our direction on conditioning. In this case, $Exposure\ \Delta CoVaR$ will be misleading regarding systematicness.

3.5 properties of $\Delta CoVaR$

The Properties of $\Delta CoVaR$ are the clone property, systemic as part of a herd, and endogeneity of systemic risk. The clone property is when one large individually systemic institution is split into n smaller institution clones, the $CoVaR$ of the large institution should be the same as the $CoVaR$ s of n clones. In other words, conditioning on the distress of a large institution is equivalent to conditioning on one of the n clones.

The other property is systemic as part of a herd. This property is connected to the clone property which can be shown in the following example. If a larger systemic institution is split into n clones then each clone is a systemic as part of a herd. If one of these institutions falls into distress and the distress is caused by a common factor then other institutions will also be in distress which causes a systemic crisis. The $\Delta CoVaR$ for each clone will be the same for the large institution.

The last property is the endogeneity of systemic risk property. The $\Delta CoVaR$ of each institution are endogenous and depended on the other institution's risk-taking. Therefore, a regulatory framework needs to be imposed to force institutions to lower their leverage and liquidity mismatch (L^i) in order to lower the reduced form $\sigma^{i\cdot}(\cdot)$ and primitive $\bar{\sigma}^{i\cdot}(\cdot)$ which captures the spillover effects.

Section 4: $\Delta CoVaR$ Estimation

This section begins by discussing alternative approaches to $\Delta CoVaR$ estimation. This is followed by a more detailed discussion on the quantile regression approach taken in the paper, and how quantile regression is applied in both contemporaneous and time-varying contexts. The time-varying approach is further discussed, beginning with the lagged state variables used and continuing with a review of the summary statistics, in sample and out of sample experiments and robustness of $\Delta CoVaR$ estimation to shorter time horizons.

4.1 Alternative Empirical Approaches

The paper mentions several methods for $\Delta CoVaR$ estimation that are found in the literature. The approach taken in the paper is quantile regression. Others have used a multivariate-GARCH approach which has the advantage of being able "to capture dynamic evolution of systemic risk contributions", and the appendix of the paper discusses a bivariate-GARCH approach. Copulas are another approach that allows for "estimation of the whole joint distribution including fat tails and heteroskedasticity". Maximum likelihood and distributional assumptions (e.g. multivariate Student-T) can be used "to calculate the joint distribution of CoVaR across firms". For extreme tail estimation (>99%), extreme value theory is applied in another paper.

4.2 Quantile Regression of Contemporaneous $\Delta CoVaR$

In this simplified $\Delta CoVaR$ estimation, the weekly market equity losses of the financial sector (X^{System}) are modeled as dependent on the weekly market equity losses of a particular institution (X^i) for the same week. Since CoVaR is a tail measure at a specified quantile, quantile regression is applied instead of least squares regression. The resulting $\Delta CoVaR$ estimation equations are given as follows:

$$CoVaR_q^{System|X^i} = \hat{X}_q^{System|X^i} = \hat{\alpha}_q^i + \hat{B}_q^i X^i$$

$$CoVaR_q^i = CoVaR_q^{System|X^i=VaR_q^i} = \hat{\alpha}_q^i + \hat{B}_q^i VaR_q^i$$

$$\Delta CoVaR_q^i = CoVaR_q^i - CoVaR_q^{System|X^i=VaR_{50}^i} = \hat{B}_q^i (VaR_q^i - VaR_{50}^i)$$

4.3 Quantile Regression of Time-Varying $\Delta CoVaR$

This section expands on the previous section by introducing a time varying component, a necessary step for predicting future values of $\Delta CoVaR$. Here, both the financial sector losses and institution losses are dependent on a set of lagged macro state variables (M_{t-1}). The seven state variables chosen are "well known to capture time variation in conditional moments of asset returns and are liquid and tractable" and are as follows: change in three month US treasury yield, change in slope of the yield curve, TED spread, change in spread between Moody's Baa bonds and ten year US treasury, weekly S&P 500 returns, weekly real estate sector return in excess of the financial sector return and equity volatility (22 day rolling standard deviation). Quantile regression is applied to regression equations:

$$X_t^i = \alpha_q^i + \gamma_q^i M_{t-1} + \epsilon_{q,t}^i$$

$$X_t^{system|i} = \alpha_q^{system|i} + \gamma_q^{system|i} M_{t-1} + B_q^{system|i} X_t^i + \epsilon_{q,t}^{system|i}$$

Yielding the following prediction equations:

$$VaR_{q,t}^i = \hat{\alpha}_q^i + \hat{\gamma}_q^i M_{t-1}$$

$$CoVaR_{q,t}^i = \hat{\alpha}_q^{system|i} + \hat{\gamma}_q^{system|i} M_{t-1} + \hat{B}_q^{system|i} VaR_{q,t}^i$$

$$\Delta CoVaR_{q,t}^i = CoVaR_{q,t}^i - CoVaR_{50,t}^i = \hat{B}_q^{system|i} (VaR_{q,t}^i - VaR_{50,t}^i)$$

4.4 Δ*CoVaR* Summary Statistics

1823 financial institutions are included with dates ranging from 1971 Q1 to 2013Q2. All of the institutions have at least 260 weeks of equity return data, with some spanning the entire period. The results are for the 99% quantile VaR and Δ*CoVaR*, but the analysis also applies similarly to the 95% quantile, which isn't given. In Appendix 1, a table of the average t-statistics for the regression coefficients across all institutions is given. Note that VaR^{System} refers to the quantile regression where institution losses $X^i$are replaced by financial sector losses X^{System}. For Δ*CoVaR*, the coefficients for the lagged change in three month US treasury yield and change in slope of the yield curve are not significant at 99% ($|t| > 2.576$) but the others clearly are. For example, lower market returns are significantly associated with higher Δ*CoVaR*, which is expected since higher losses typically happen in times of market distress.

4.5 Δ*CoVaR* vs VaR

In Appendix 2 figure 1, the time series averages for the Δ*CoVaR* and VaR for each institution are plotted. The plots slightly suggests a negative relationship (i.e. companies with high average VaR pose little systemic risk) which could be explained by small but risky companies that can fail without spillover. It also hints that the really systemically important companies are on average less risky (lower VaR) and that regulations based on institutions in isolation may not be sufficient. In figure 2, the average of the 50 largest institutions' time series are plotted. Here, a clearer relationship is present, most noticeably with the Δ*CoVaR* (red) spikes being accompanied by spikes in VaR (gray).

4.6 In Sample and Out of Samples Estimates of Δ*CoVaR*

In this section, the intent is to compare in-sample (blue) and out-of-sample (red) Δ*CoVaR* estimation during the 2007-2009 financial crisis. Out-of-sample estimation is performed using expanding windows. In Appendix 3, time series plots of Δ*CoVaR* for Lehman Brothers, Bank of America, JP Morgan and Goldman Sachs during the period are given. Typically, in-sample Δ*CoVaR* estimates are greater than out-of-sample estimates, though much less so for Goldman. JPM and BofA are relatively low throughout the crisis, indicating they were contributing less to systemic risk, while Goldman reaches the highest Δ*CoVaR* after Lehman's bankruptcy,

4.7 Historical $\Delta CoVaR$ - Robustness to Shorter Time Horizons

In the previous sections, the data ranged from 1971Q1 - 2013Q2. While this is a long period of time, financial crises are rare tail events, which makes estimation or prediction related to these events difficult. So, it is important to consider the effects of a longer time horizon on $\Delta CoVaR$ estimation. Among the institutions considered, four have data going back to 1926Q3. In Appendix 4, blue is the average of the four estimated 95%-$\Delta CoVaR$ time series using data going back to 1926Q3, while red is the average using data going to 1971Q1. There is a high correlation (96%) between the two series, indicating the $\Delta CoVaR$ is robust to shorter time horizons, though the longer series almost consistently estimates a higher $\Delta CoVaR$. Qualitatively, it appears the Great Depression, maybe surprisingly, was associated with lower $\Delta CoVaR$ than more recent crises. However, these companies all survived that crisis and all crises since then, so this analysis is subject to survivorship bias.

Section 5: Forward-$\Delta CoVaR$ Estimation

This research and paper are from the US Federal Reserve, which has the dual mandate of maintaining price stability and maximum sustainable employment. Naturally, questions about financial system risk and related regulation are relevant to the dual mandate, and Forward-$\Delta CoVaR$ is one tool to address these questions. Forward-$\Delta CoVaR$ is an extension to the work in the previous section, and it addresses the volatility paradox and the issue of tail risk measurement accuracy. In the following sections, 95% and 99% quantiles and time horizons h = 1,4,8 quarters are considered.

5.1.1 $\Delta CoVaR$ Predictors

For the time-varying $\Delta CoVaR$ estimation approach, a set of lagged state variables are employed first to estimate institution losses, and then alongside the estimated institution loss, the financial system loss (i.e. CoVaR). However, since large losses are rare tail events (with a low number of observations), the paper suggests more accurate $\Delta CoVaR$ prediction can be achieved by regressing $\Delta^{\$}$CoVaR against both the lagged macro state variables and four lagged institution-specific balance sheet variables X_{t-h}^{i} that are frequently and reliably measured. These are as follows: ratio of market value assets to market equity (leverage), ratio of book assets to short term debt less short term investments less cash (maturity mismatch), ratio of log of market equity to log of average market equity across all firms (size) and the number of consecutive quarters in the top decile of market-to-book ratio across firms (boom). For bank holding companies (BHC), more specific balance sheet items are available that can replace maturity mismatch. Asset items include loans, loan-loss allowances, intangible loss allowances, intangible assets and trading assets. Liability items include interest-bearing core deposits, non-interest-bearing deposits, large time deposits and demand deposits. All of the items are expressed as a % of total book assets.

5.1.2 Analysis of Regression Results for $\Delta CoVaR$ Predictors

The predictors described in the previous sub section are a good start, but it is important to determine if they are actually statistically significant. First, quarterly $\Delta CoVaR$ is estimated as in the previous section for all institutions for 1971Q1-2013Q2. Next, these $\Delta CoVaR$ estimates are converted $\Delta^{\$}$CoVaR and regressed against the lagged macro and institution variables for all institutions, both quantiles and each time horizon. The regression and forward prediction equations are as follows:

$$\Delta^{\$}CoVaR_{q,t}^{i} = a + cM_{t-h} + bX_{t-h}^{i} + \eta_t^i$$

$$\Delta_h^{Fwd}CoVaR_{q,t}^{i} = \hat{a} + \hat{c}M_{t-h} + \hat{b}X_{t-h}^{i}$$

In Appendix 5, the average regression coefficients and Newey-West standard errors across all institutions, along with the significance levels for the coefficients are given. The real estate excess return coefficient (Housing) is not typically significant at 10% or greater. Maturity mismatch varies from 90% to 99% significant. Otherwise, the macro and institution variables are typically 99% significant (***), indicating they have good predictive power for the Forward-$\Delta CoVaR$.

The 95% quantile 2-year leverage of 14.573 indicates that for each basis point increase in leverage, $\Delta^{\$}CoVaR_{95,t}^{i}$ increases on average 14.573 basis points, so an institution wishing to decrease its $\Delta^{\$}CoVaR_{95,t}^{i}$ could consider decreasing its leverage. The other institution specific variables could also be considered. This is important because it shows large size companies (i.e. "too big to fail") don't necessarily have to decrease their size to decrease their $\Delta^{\$}CoVaR_{95,t}^{i}$., though size does have the largest contribution to $\Delta^{\$}CoVaR_{95,t}^{i}$ at 1054.993. It also shows the concept of "systemic as part of a herd", where for example a large number of smaller but highly leveraged companies contribute in aggregate a lot to $\Delta^{\$}CoVaR_{95,t}^{i}$. So, regulations focused solely on large companies may not adequately address system risk.

In Appendix 6, the same approach is applied to bank holding companies for 95% quantile Forward-$\Delta CoVaR$ across each horizon, with maturity mismatch replaced by the specific asset variables in three scenarios and by the specific liability variables in the other three scenarios. The same analysis applies here as well. Trading assets are the only consistently 99% significant asset variable, which is understandable since trading is a risky activity. Non-interest bearing deposits are easily taken out of the institution, leaving the bank with less cash on hand, which explains why it is positive and 99% significant in all cases. In contrast, core deposits and time deposits are not as quickly and easily removed, which is why they are negative and 99% significant.

5.2 Counter Cyclicality of Contemporaneous $\Delta CoVaR$ and Forward-$\Delta CoVaR$

The figure in Appendix 7 plots the average contemporaneous $\Delta CoVaR$ and average 2 year Forward-$\Delta CoVaR$ over the 50 largest institutions, starting in 1971Q1. The Forward-$\Delta CoVaR$ is predicted in-sample up to 2002Q1 and out-of-sample thereafter. In the figure, it is typically clear that the two measures are negatively correlated, which is the idea of the countercyclical volatility paradox where systemic risk builds in the financial system in times of low volatility. The point here is that regulations based on contemporaneous measures will not be adequate in times of low volatility, so a forward looking measure should be employed.

5.3 Cross-Sectional Predictive Power of Forward-$\Delta CoVaR$

In this section, the predictive power of 95% quantile Forward-$\Delta CoVaR$ during the 2007-2009 financial crisis is analyzed. First, Forward-$\Delta CoVaR$ for each time horizon and all BHCs up to 2006Q4 is calculated, providing predictions for $\Delta CoVaR$ through 2008Q4. Next, the realized

Δ*CoVaR* for each time horizon and all BHCs is calculated for each quarter during the period 2007Q1 to 2008Q4. Finally, a regression of the realized Δ*CoVaR*s on the Forward-Δ*CoVaR*s is performed for matching quarters (e.g. realized Δ*CoVaR* for 2008Q4 is regressed on the 2 year Forward-Δ*CoVaR* at 2006Q4. In Appendix 8, the table contains the regression results for five quarter and horizon scenarios. The coefficients are all 99% significant and the R^2 values range from 17.8% to 78.9%, indicating that Forward-Δ*CoVaR* is associated with the realized Δ*CoVaR* and that it is explaining a good amount of the variance in the realized Δ*CoVaR*. So, Forward-Δ*CoVaR* indeed has predictive power.

Section 6: Conclusion

Δ*CoVaR* is a tail risk measure designed to indicate the level of systemic risk that financial institutions pose to the greater financial system. While it is a tail risk measure where direct observations are rare (i.e. financial crises), the paper shows that there are more frequent and easily obtained variables that are statistically significant for estimating Δ*CoVaR* or predicting Forward-Δ*CoVaR*. The analyses provide an explanation for the concepts of "too big to fail", "volatility paradox" and "systemic as part of a herd" and provide guidance on how regulation should address these issues.

Section 7: Simulation

Due to the high number of analyses performed on a large amount of data over a long time period, it is not possible to recreate any of the paper's results. However, enough public and free data is available to perform a limited analysis on the Δ*CoVaR* time series of 8 large financial institutions over the most recent 10 years. Daily data for the macro variables and institution returns is available through the US Federal Reserve and Yahoo Finance going back a few decades. Daily returns for the S&P 500 Financial Sector and Real Estate Sector were only available from S&P going back 10 years, but weekly returns were not available. After cleaning the data and matching the dates for all data, the daily data were converted to weekly data on a 5 day basis, instead of a "business week" basis since it is difficult to adjust for weeks less than 5 days. The simulation is performed in Python using Pandas and statsmodel libraries. The program calculates the contemporaneous and time-varying Δ*CoVaR* regressions as described in section 4, outputs to a text file the regression results for each institution and displays a time series plot of the time-varying Δ*CoVaR*s for each institution.

The time series plot is given in Appendix 9. There is no direct analog for this plot in the paper, but it can still provide some insight into these institutions' recent contribution to system risk. JPM and BAC are the largest companies in the group, so it is reasonable to expect they have higher Δ*CoVaR*. The first spike around observation 80 is mid to late 2011, when there was an approximately 10% drop in the S&P500. The most recent spike around observation 500 is expected due to the crisis brought on by COVID-19. Interestingly, it appears that this event has a relatively outsized effect on BAC's Δ*CoVaR*.

Appendix 1

	VaR^{system}	VaR^{i}	$\Delta CoVaR^{i}$
Three month yield change (lag)	(1.95)	(-0.26)	(2.10)
Term spread change (lag)	(1.73)	(-0.04)	(1.72)
TED spread (lag)	(6.87)	(1.97)	(8.86)
Credit spread change (lag)	(5.08)	(-0.28)	(4.08)
Market return (lag)	(-16.98)	(-3.87)	(-18.78)
Real estate excess return (lag)	(-3.78)	(-1.86)	(-4.41)
Equity volatility (lag)	(12.81)	(7.47)	(15.81)
Market equity oss X^{i}			(7.38)
Pseudo-R^2	39.94%	21.23%	43.42%

Appendix 2

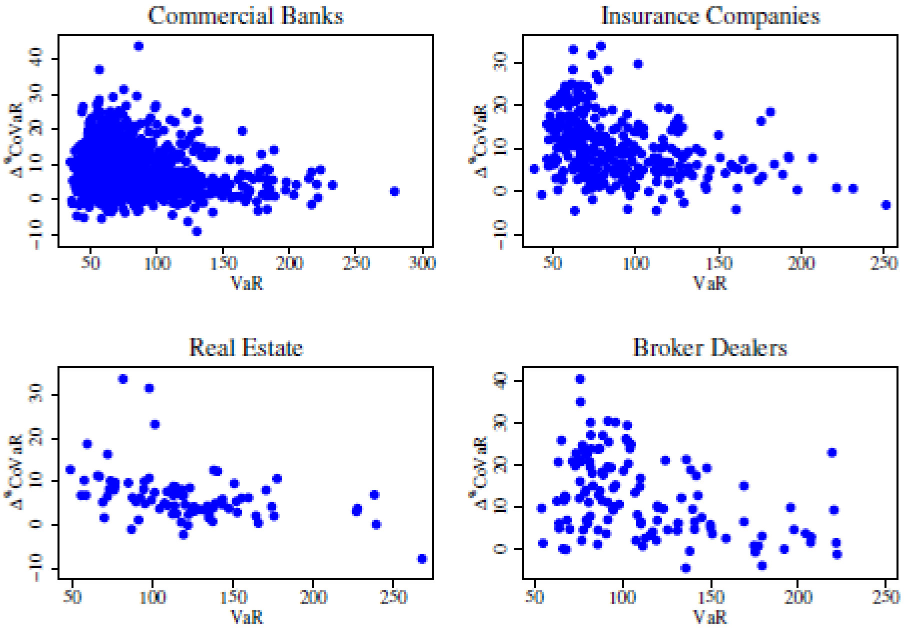


Figure 1

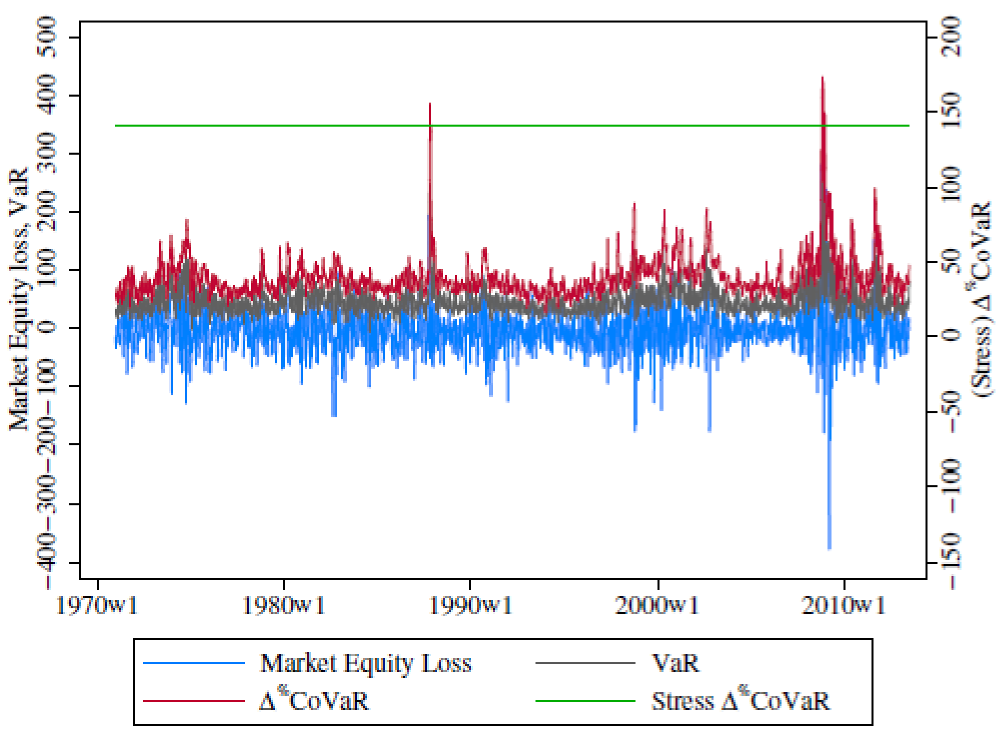
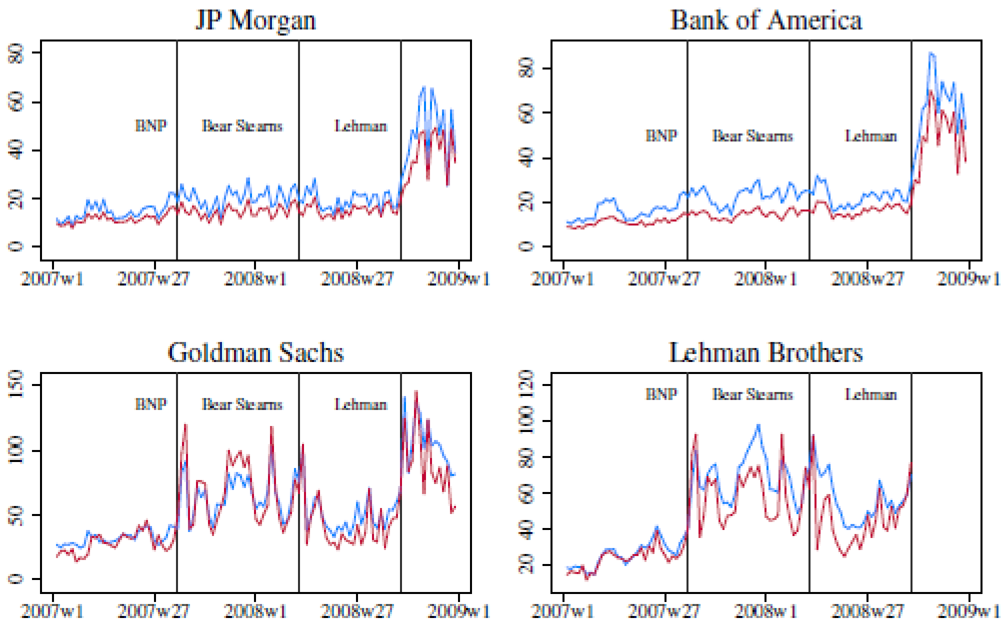
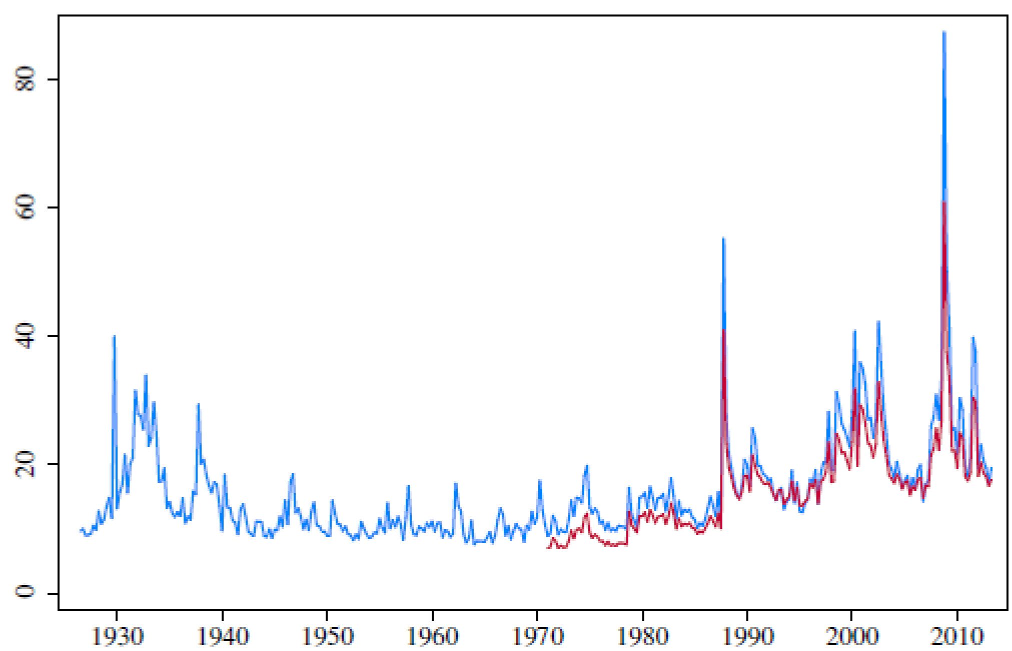


Figure 2

Appendix 3



Appendix 4



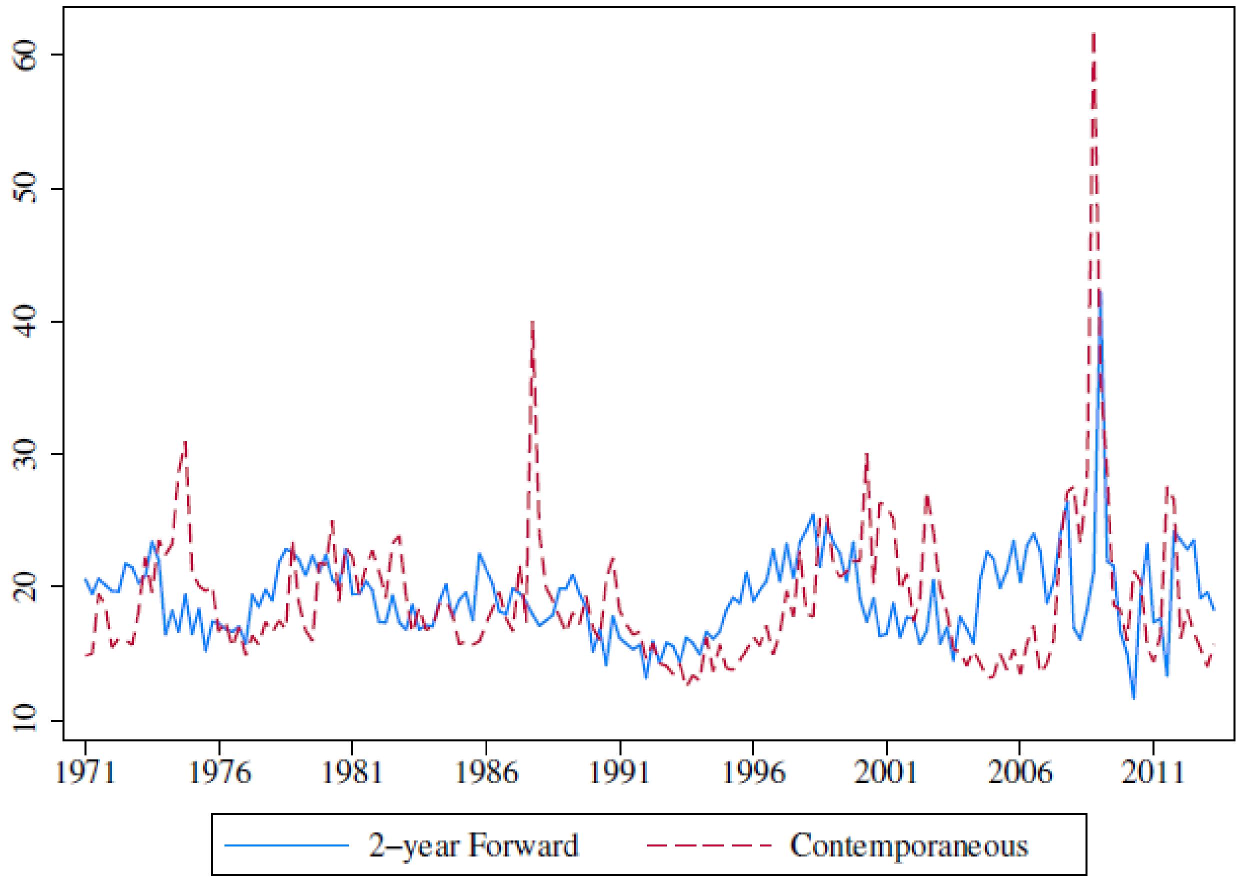
Appendix 5

	Panel A: $\Delta^{\$}CoVaR^{i}_{95,t}$			Panel B: $\Delta^{\$}CoVaR^{i}_{99,t}$		
	2 Year	1 Year	1 Quarter	2 Year	1 Year	1 Quarter
VaR	7.760***	8.559***	9.070***	2.728***	3.448***	4.078***
	(9.626)	(10.566)	(11.220)	(7.484)	(9.443)	(10.225)
Leverage	14.573***	13.398***	13.272***	17.504***	15.958***	15.890***
	(5.946)	(6.164)	(6.317)	(5.854)	(6.299)	(6.627)
Size	1,054.993***	1,014.396***	990.862***	1,238.674***	1,195.072***	1,170.075***
	(22.994)	(23.420)	(23.630)	(27.549)	(28.243)	(28.603)
Maturity mismatch	7.306**	5.779**	4.559*	9.349***	7.918**	6.358**
	(2.187)	(1.968)	(1.760)	(2.725)	(2.537)	(2.225)
Boom	154.863***	160.391***	151.389***	155.184***	169.315***	165.592***
	(4.161)	(4.431)	(4.414)	(3.653)	(3.962)	(3.908)
Equity volatility	74.284	67.707	135.484***	212.860***	203.569***	286.300***
	(1.317)	(1.346)	(2.889)	(3.211)	(3.478)	(4.960)
Three month yield change	-111.225***	-144.750***	-127.052***	-75.877***	-123.081***	-111.545***
	(-5.549)	(-6.686)	(-7.059)	(-3.390)	(-5.518)	(-5.748)
TED spread	-431.094***	-187.730**	-232.091***	-436.214***	-175.884**	-196.513**
	(-6.989)	(-2.403)	(-3.207)	(-6.282)	(-2.163)	(-2.488)
Credit spread change	-145.121***	-165.345***	-78.447**	-147.680***	-172.642***	-102.399**
	(-3.319)	(-3.959)	(-2.036)	(-2.659)	(-3.336)	(-2.023)
Term spread change	-275.593***	-243.509***	-187.183***	-251.197***	-236.494***	-179.935***
	(-7.570)	(-9.017)	(-8.500)	(-6.678)	(-8.094)	(-7.216)
Market return	88.971***	30.799	-97.783***	97.187***	33.065	-111.336***
	(3.881)	(1.401)	(-4.327)	(3.735)	(1.381)	(-4.469)
Housing	27.373*	32.940**	17.800	4.517	15.249	6.644
	(1.845)	(2.479)	(1.156)	(0.269)	(0.992)	(0.390)
Foreign FE	-439.424**	-424.325**	-405.148**	-828.557***	-811.836***	-788.096***
	(-2.376)	(-2.378)	(-2.295)	(-4.492)	(-4.579)	(-4.532)
Insurance FE	-724.971***	-681.143***	-649.836***	-435.109***	-408.868***	-391.193***
	(-7.610)	(-7.629)	(-7.639)	(-4.086)	(-4.114)	(-4.138)
Real Estate FE	-50.644	-42.466	-24.328	66.136	80.733	98.908
	(-0.701)	(-0.647)	(-0.395)	(0.794)	(1.067)	(1.387)
Broker Dealer FE	128.640	99.435	84.613	396.346**	343.386**	310.082**
	(0.850)	(0.695)	(0.612)	(2.500)	(2.311)	(2.187)
Others FE	-373.424***	-388.902***	-381.562***	-209.309***	-235.844***	-240.875***
	(-5.304)	(-5.934)	(-6.121)	(-2.727)	(-3.338)	(-3.586)
Constant	4,608.697***	4,348.786***	3,843.332***	5,175.855***	4,970.410***	4,443.515***
	(16.292)	(17.542)	(19.802)	(18.229)	(19.768)	(21.566)
Observations	79,317	86,474	91,750	79,317	86,474	91,750
Adjusted R^2	24.53%	24.36%	24.35%	26.89%	26.75%	26.76%

Appendix 6

	Panel A: BHC Liability Variables			Panel B: BHC Asset Variables		
	2 Year	1 Year	1 Quarter	2 Year	1 Year	1 Quarter
VaR	9.995***	11.068***	11.699***	4.251**	6.402***	7.715***
	(4.774)	(5.099)	(5.310)	(2.211)	(3.305)	(3.977)
Leverage	56.614***	44.044***	38.639***	40.779***	29.967***	23.919***
	(9.654)	(9.683)	(9.340)	(6.476)	(6.103)	(5.532)
Size	1,457.875***	1,394.324***	1,360.318***	1,203.344***	1,141.656***	1,107.145***
	(13.539)	(13.781)	(13.850)	(13.069)	(13.352)	(13.591)
Boom	88.923	108.619	90.283	124.161*	143.028**	123.243**
	(1.238)	(1.617)	(1.560)	(1.758)	(2.179)	(2.209)
Equity volatility	92.400	-55.501	48.969	292.221***	90.937	165.272
	(0.821)	(-0.527)	(0.436)	(2.672)	(0.887)	(1.521)
Three month yield change	-541.105***	-512.923***	-442.339***	-357.807***	-343.193***	-278.257***
	(-6.815)	(-6.442)	(-7.344)	(-5.567)	(-5.145)	(-5.585)
TED Spread	-709.255***	-230.846	-448.387*	-622.720***	-142.805	-369.578*
	(-3.784)	(-0.830)	(-1.926)	(-3.688)	(-0.548)	(-1.716)
Credit spread change	-338.997***	-268.226**	-136.785	-208.571*	-113.161	35.318
	(-2.860)	(-2.082)	(-1.132)	(-1.879)	(-0.935)	(0.297)
Term spread change	-838.872***	-681.318***	-556.294***	-640.835***	-505.287***	-391.500***
	(-6.863)	(-7.575)	(-8.060)	(-6.521)	(-6.886)	(-6.718)
Market return	-16.289	-61.080	-205.718***	25.380	-34.474	-188.620***
	(-0.329)	(-1.156)	(-3.698)	(0.541)	(-0.706)	(-3.601)
Housing	88.442***	116.009***	69.066**	64.023**	95.015***	51.096*
	(2.853)	(4.091)	(2.297)	(2.236)	(3.687)	(1.795)
Core deposits	-50.915***	-53.888***	-53.439***			
	(-8.005)	(-8.217)	(-8.244)			
Non-interest deposits	51.610***	46.680***	43.844***			
	(3.840)	(4.278)	(4.519)			
Time deposits	-68.087***	-65.152***	-62.553***			
	(-8.347)	(-8.123)	(-8.064)			
Demand deposits	-13.782	-14.811	-16.195			
	(-0.929)	(-1.254)	(-1.511)			
Total loans				9.419**	6.568*	3.811
				(2.285)	(1.792)	(1.106)
Loan loss reserves				-133.352	-131.401	-72.177
				(-1.086)	(-1.263)	(-0.776)
Intanglible assets				48.721	43.285	41.082
				(0.884)	(0.874)	(0.915)
Trading assets				576.060***	565.476***	549.158***
				(5.332)	(5.636)	(6.088)
Constant	10,820.323***	10,077.831***	9,222.897***	5,825.307***	5,127.386***	4,404.582***
	(10.244)	(10.388)	(11.572)	(7.899)	(7.849)	(8.573)
Observations	25,578	28,156	30,128	25,481	28,060	30,030
Adjusted R^2	28.94%	28.17%	28.13%	36.29%	35.47%	35.41%

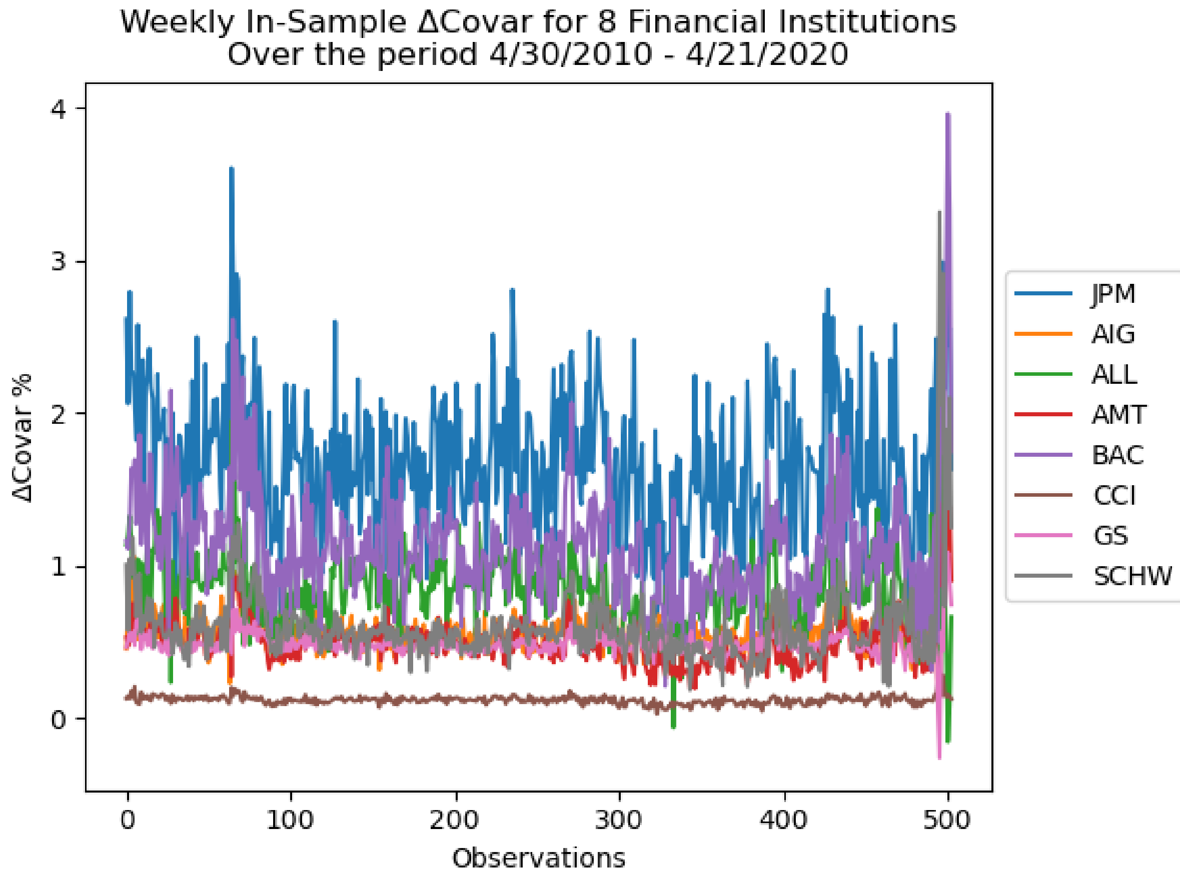
Appendix 7



Appendix 8

	Crisis $\Delta CoVaR$				
	2008Q4	2008Q4	2008Q4	2007Q4	2007Q1
2Y *Forward*-$\Delta CoVaR$ (2006Q4)	1.206***				
1Y *Forward*-$\Delta CoVaR$ (2007Q4)		0.664***			
1Q *Forward*-$\Delta CoVaR$ (2008Q3)			1.708***		
1Y *Forward*-$\Delta CoVaR$ (2006Q4)				0.848***	
1Q *Forward*-$\Delta CoVaR$ (2006Q4)					0.541***
Constant	13.08***	18.51***	2.409***	4.505***	2.528***
Observations	378	418	430	428	461
R^2	36.6 %	17.8 %	78.9 %	49.6 %	55.5%

Appendix 9



Weekly In-Sample ΔCovar for 8 Financial Institutions
Over the period 4/30/2010 - 4/21/2020
ΔCovar %
Observations
0
1
2
3
4
0
100
200
300
400
500
JPM
AIG
ALL
AMT
BAC
CCI
GS
SCHW

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