CoVaR

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Introduction

- Systemic risk: the risk of the entire financial system collapse
  - Cause by spread of distress
  - Spillovers
- Systemic risk measure capture the spread of the financial distress across institutions
  - ΔCoVaR - captures tail dependency and negative spillovers
    - Institutional characteristics: size, leverage, and maturity mismatch
    - Conditioning variables: market volatility and fixed income spreads
  - Forward ΔCoVaR - captures the buildup of systemic risk
- Theoretical predictions: higher leverage, more maturity mismatch, larger size, and higher valuations forecast higher systemic risk contributions

Background on Systemic Risk

- Bhattacharya and Gale (1987)
  - Spillovers in the form of externalities
- Allen, Babus, and Carletti (2010)
  - Network effects can lead to spillover effects
- Brunnermeier and Sannikov (2014)
  - "Volatility paradox"
- Brunnermeier and Pedersen (2009) and Adrian and Boyarchenko (2012)
  - The margin/haircut spiral

Other Systemic Risk Measures

- Huang, Zhou, and Zhu (2010)
    - Systemic risk indicator for credit default swap (CDS)
- Acharya, Pedersen, Philippon, and Richardson (2010)
    - High frequency marginal expected shortfall similar to "*Exposure-ΔCoVaR*"
    - Changing the condition can address different questions
- Billio, Getmansky, Lo, and Pelizzon (2010)
    - A systemic risk measure that relies on Granger causality
- Lehar (2005) and Gray, Merton, and Bodie (2007)
    - Uses contingent claims analysis to measure systemic risk

Study of Tail Risk and Contagion

- Engle and Manganelli (2004)
  - Develop *CAViaR* to capture the time varying tail behavior
- Methods to test volatility spillovers
  - Estimate multivariate GARCH processes
  - Multivariate extreme value theory

Definition of *ΔCoVaR*

- Recall that VaR_q^i is implicitly defined as the $q\%$ quantile, i.e.,

$$\Pr\left(X^i \leq VaR_q^i\right) = q\%,$$

- **Definition 1** *We denote by $CoVaR_q^{j|\mathbb{C}(X^i)}$ the VaR of institution j (or the financial system) conditional on some event $\mathbb{C}\left(X^i\right)$ of institution i. That is, $CoVaR_q^{j|\mathbb{C}(X^i)}$ is implicitly defined by the $q\%$-quantile of the conditional probability distribution:*

$$\Pr\left(X^j|\mathbb{C}\left(X^i\right) \leq CoVaR_q^{j|\mathbb{C}(X^i)}\right) = q\%.$$

Definition of *ΔCoVaR* cont...

- *We denote institution i's contribution to j by*

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|X^i=\mathrm{VaR}_q^i} - CoVaR_q^{j|X^i=\mathrm{VaR}_{50}^i},$$

and in dollar terms

$$\Delta^{\$}CoVaR_q^{j|i} = {}^{\$}\mathrm{Size}^i \cdot \Delta CoVaR_q^{j|i}.$$

Definition of *ΔCoVaR* cont...

- Conditioning
  - Condition on some event C that is equally likely among all institutions
  - Condition on a quantile level instead of a particular return level
- *ΔCoVaR*
  - Increase in *CoVaR* as one change conditioning events from the median state return of institution i to the distress state
- *Δ$^{\$}$CoVaR*
  - Includes the size of the institution i
  - Market equity of the institution
- *CoES*
  - Co-expected shortfall $CoES_q^{j|i}$
  - $\Delta CoES_q^{j|i} = CoES_q^{j|i} - CoES_{50}^{j|i}.$

Economics of Systemic Risk

- Time-series
  - System risk buildup when measured risk is low
  - Lead to a volatility paradox
  - *Forward - ΔCoVaR* capture buildup systemic risk
- Cross-sectional
  - Spillover effects
    - Indirect: price effects
    - Direct: contractual links
  - The first component of systemic risk
  - Contemporaneous *ΔCoVaRi* captures spillover and common exposure effects
    - How much an institution contributes to systemic risk?

Tail Dependency versus Causality

- $\Delta CoVaR_q^{j|i}$ is a statistical tail-dependency measure
    - Does not “correctly” capture spillover effects or externalities
- Specific model to show causality
    - Two groups: institutions of type i and type j
    - Two latent independent risk factors: ΔZ^i and ΔZ^j.
    - The generating process of returns for institutions of type i $-X_{t+1}^i = \Delta N_{t+1}^i / N_t^i$
        - $$-X_{t+1}^i = \overline{\mu}^i(\cdot) + \overline{\sigma}^{ii}(\cdot)\Delta Z_{t+1}^i + \overline{\sigma}^{ij}(\cdot)\Delta Z_{t+1}^j$$ $$-X_{t+1}^j = \overline{\mu}^j(\cdot) + \overline{\sigma}^{jj}(\cdot)\Delta Z_{t+1}^j + \overline{\sigma}^{ji}(\cdot)\Delta Z_{t+1}^i.$$
        -
    - $(\cdot)$ = $(M_t, L_t^i, L_t^j, N_t^i, N_t^j)$ $\longrightarrow$ state of macro-economy, leverage and liquidity mismatch, net worth levels
    - Geometric drift and volatility loadings are functions of these state variables

Tail Dependency versus Causality Cont…

- Cause and Effect
  - Leverage increases the loading on its latent risk factor ΔZ^i_{t+1}.
  - Exposure of institution type i to ΔZ^j_{t+1} due to spillover, $\sigma^{ij}(\cdot)$, increase in its own leverage as well as the others leverage.
- Reduced formula equations
  - $$\begin{aligned} -X^i_{t+1} &= \mu^i(\cdot) - \sigma^{ij}(\cdot) X^j_{t+1} + \sigma^{ii}(\cdot)\Delta Z^i_{t+1}, \\ -X^j_{t+1} &= \mu^j(\cdot) - \sigma^{ji}(\cdot) X^i_{t+1} + \sigma^{jj}(\cdot)\Delta Z^j_{t+1}. \end{aligned}$$
  - There's a distress shock $\Delta Z^i_{t+1} < 0$ $\longrightarrow$ lowers $-X^i_{t+1}$ by $\sigma^{ii}_t \Delta Z^i_{t+1}$
  - First rounds of spillover effects reduce $-\Delta X^j_{t+1}$ by $\sigma^{ji}_t \sigma^{ii}_t \Delta Z^i_{t+1}$.
  - Second rounds of spillover effects reduce $-\Delta X^i_{t+1}$ by $\sigma^{ij}_t \sigma^{ji}_t \sigma^{ii}_t \Delta Z^i_{t+1}$

Tail Dependency versus Causality Cont...

- Argument will continue until a fixed point is reached
  - $$\bar{\sigma}_t^{ii} = \sum_{n=0}^{\infty}(\sigma_t^{ij}\sigma_t^{ji})^n\sigma_t^{ii} = \frac{\sigma_t^{ii}}{1-\sigma_t^{ij}\sigma_t^{ji}}$$
  - $$\bar{\sigma}_t^{ij} = \sum_{n=0}^{\infty}(\sigma_t^{ij}\sigma_t^{ji})^n\sigma_t^{ij}\sigma_t^{jj} = \frac{\sigma_t^{ij}\sigma_t^{jj}}{1-\sigma_t^{ij}\sigma_t^{ji}}$$
  - $\bar{\sigma}_t^{jj}$ and $\bar{\sigma}_t^{ji}$ Can be obtain by replacing i with j and vice versa

Different ΔCoVaR

- Network - *ΔCoVaR*
    - Subscripts j and i in $\Delta CoVaR_q^{j|i}$ refer to individual institutions
    - Tail dependency can be studied across the financial network
- Exposure - *ΔCoVaR*
    - A measure of an individual institution's exposure to the systemic risk
    - $\Delta CoVaR^{j|system}$ Institutions j increase in value at risk when a financial crisis occurs
- Direction of conditioning
    - $\Delta CoVaR_q^{j|i}$ is directional
    - $\Delta CoVaR_q^{system|i} \neq \Delta CoVaR_q^{i|system}$

$$\Delta CoVaR_q^{system|i} = CoVaR_q^{system|X^i=VaR_q^i} - CoVaR_q^{system|X^i=VaR_{50}^i}$$

Properties of *ΔCoVaR*

- Clone property
  - Splitting one large institution into n clones
  - *ΔCoVaR/CoVaR* of the large institution = *ΔCoVaR/CoVaR* of the n clones
- Systemic as Part of a Herd
  - If one institution falls into distress then other institutions will also be in distress
    - Distress caused by a common factor
  - Connects with the clone property
    - Each clone is a systemic as part of a herd
- Endogeneity of Systemic Risk
  - Each institution's *ΔCoVaR* depends on other institutions risk taking
  - Institutions lowering their leverage and liquidity mismatch would lower $\sigma^{i\cdot}(\cdot)$ and $\bar{\sigma}^{i\cdot}(\cdot)$ which captures the spillover effects

ΔCoVaR Estimation

- Alternative estimation approaches
- Data overview
- Contemporaneous ΔCoVaR estimation - Quantile Regression
    - Summary statistics and analysis on time varying ΔCoVaR
    - Relationship between ΔCoVaR and VaR
    - Out of sample estimates of ΔCoVaR
    - Robustness of ΔCoVaR to shorter time horizons
- Forward $\Delta^{\$}$CoVaR overview
    - Forward $\Delta^{\$}$CoVaR Predictors
    - Predictive power of Forward $\Delta^{\$}$CoVaR

Alternative Estimation Approaches

- This paper uses Quantile Regression from Koenker and Basset
- Appendix provides a bivariate GARCH framework
- Multivariate GARCH
- Copulas allow for estimation of joint distribution including fat tails and heteroskedacity
- Bayesian quantile regression
- Maximum likelihood estimation with distributional assumptions (e.g. Student-T)

Data Overview

- Publicly available data for 1823 publicly traded financial institutions
- Split into four sectors: commercial banks, security broker-dealers (including investment banks), insurance companies and real estate companies
- Daily market equity data
- Quarterly balance sheet data
- Data range of 1971Q1 - 2013Q2 covers six recessions and several financial crises. One section extends this back to 1926
- Specific data used will be described in each section

Quantile Regression - Contemporaneous

- Weekly market equity q%-quantile loss for financial sector dependent on weekly market equity loss of institution i
- Daily market equity data is available but weekly values used

$$\hat{X}_q^{System|X^i} = \hat{\alpha}_q^i + \hat{B}_q^i X^i$$

$$CoVaR_q^{System|X^i} = \hat{X}_q^{System|X^i}$$

$$CoVaR_q^i = CoVaR_q^{System|X^i=VaR_q^i} = \hat{\alpha}_q^i + \hat{B}_q^i VaR_q^i$$

$$\Delta CoVaR_q^i = CoVaR_q^i - CoVaR_q^{System|X^i=VaR_{50}^i} = \hat{B}_q^i (VaR_q^i - VaR_{50}^i)$$

Lagged Macro State Variables

- Denoted by M_{t-h} in the remainder of the paper with h > 1
- These are well known to capture time variation in conditional moments of asset returns, liquid and easily tractable
- Change in three month treasury yield
- Change in slope of treasury yield curve
- Short term TED spread for short term liquidity risk
- Change in spread between Moody's Baa bonds and ten year treasury
- Weekly S&P 500 return
- Weekly real estate return in excess of market financial sector index return
- Equity volatility (22 day rolling standard deviation)

Quantile Regression - Time Varying

- Estimate VaR and ΔCoVaR as a function of lagged state variables
- Quantile regression equations

$$X_t^i = \alpha_q^i + \gamma_q^i M_{t-1} + \epsilon_{q,t}^i$$

$$X_t^{system|i} = \alpha_q^{system|i} + \gamma_q^{system|i} M_{t-1} + B_q^{system|i} X_t^i + \epsilon_{q,t}^{system|i}$$

- Prediction equations

$$VaR_{q,t}^i = \hat{\alpha}_q^i + \hat{\gamma}_q^i M_{t-1}$$

$$CoVaR_{q,t}^i = \hat{\alpha}_q^{system|i} + \hat{\gamma}_q^{system|i} M_{t-1} + \hat{B}_q^{system|i} VaR_{q,t}^i$$

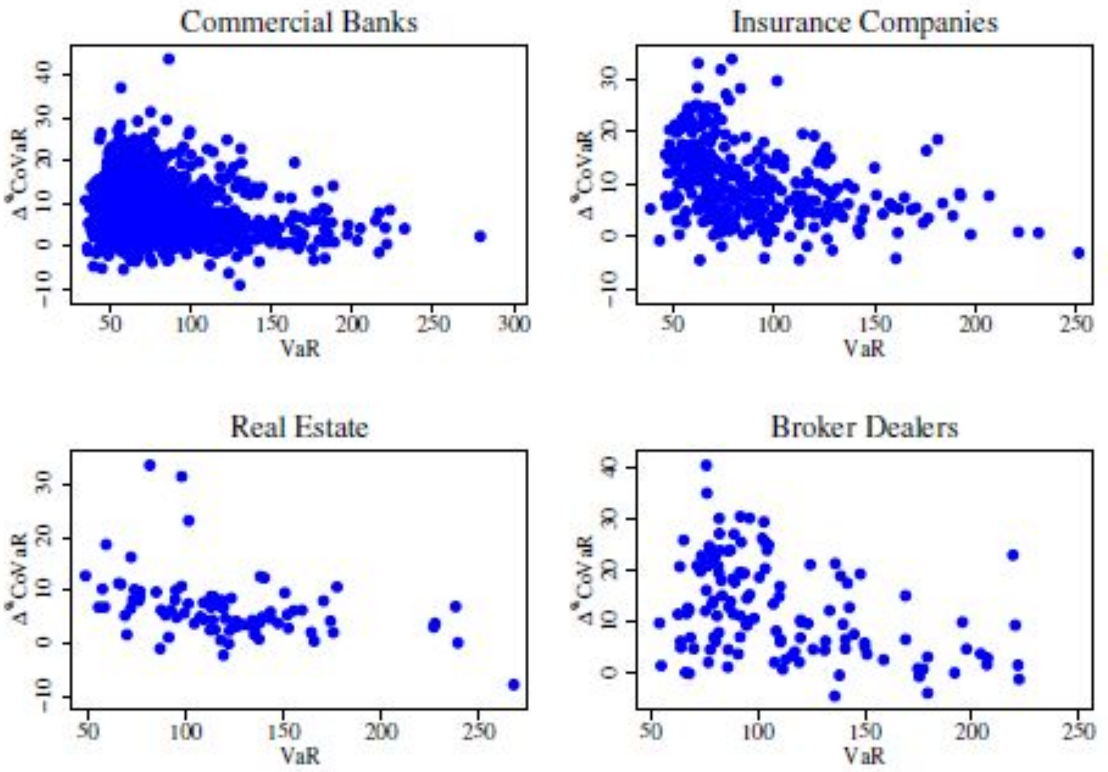
$$\Delta CoVaR_{q,t}^i = CoVaR_{q,t}^i - CoVaR_{50,t}^i = \hat{B}_q^{system|i} (VaR_{q,t}^i - VaR_{50,t}^i)$$

ΔCoVaR Summary Statistics - 99% Quantile

- Average t-statistics for regression coefficients across all institutions

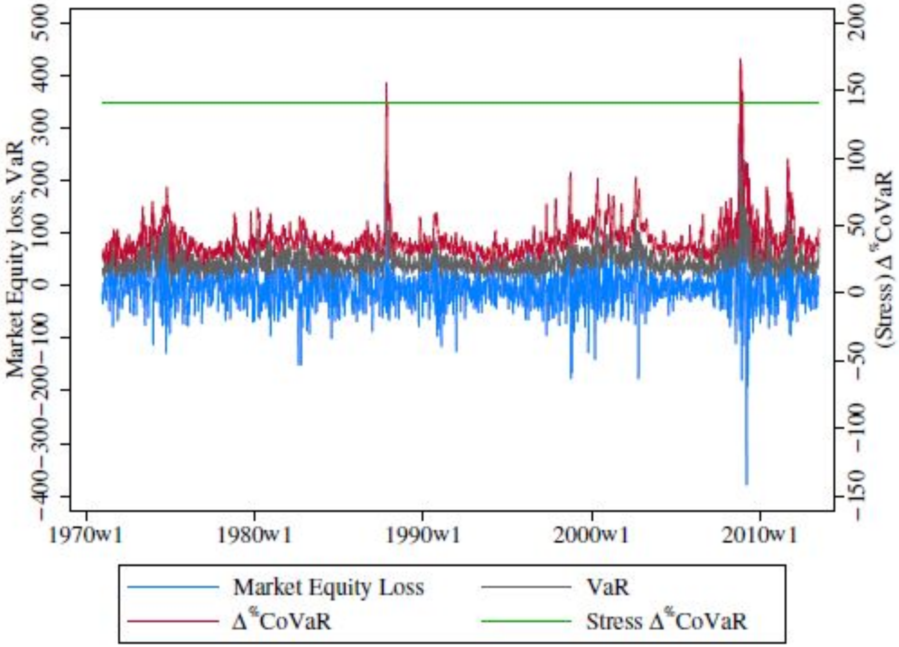
	VaR^{system}	VaR^i	$\Delta CoVaR^i$
Three month yield change (lag)	(1.95)	(-0.26)	(2.10)
Term spread change (lag)	(1.73)	(-0.04)	(1.72)
TED spread (lag)	(6.87)	(1.97)	(8.86)
Credit spread change (lag)	(5.08)	(-0.28)	(4.08)
Market return (lag)	(-16.98)	(-3.87)	(-18.78)
Real estate excess return (lag)	(-3.78)	(-1.86)	(-4.41)
Equity volatility (lag)	(12.81)	(7.47)	(15.81)
Market equity oss X^i			(7.38)
Pseudo-R^2	39.94%	21.23%	43.42%

ΔCoVaR vs VaR Across Institutions



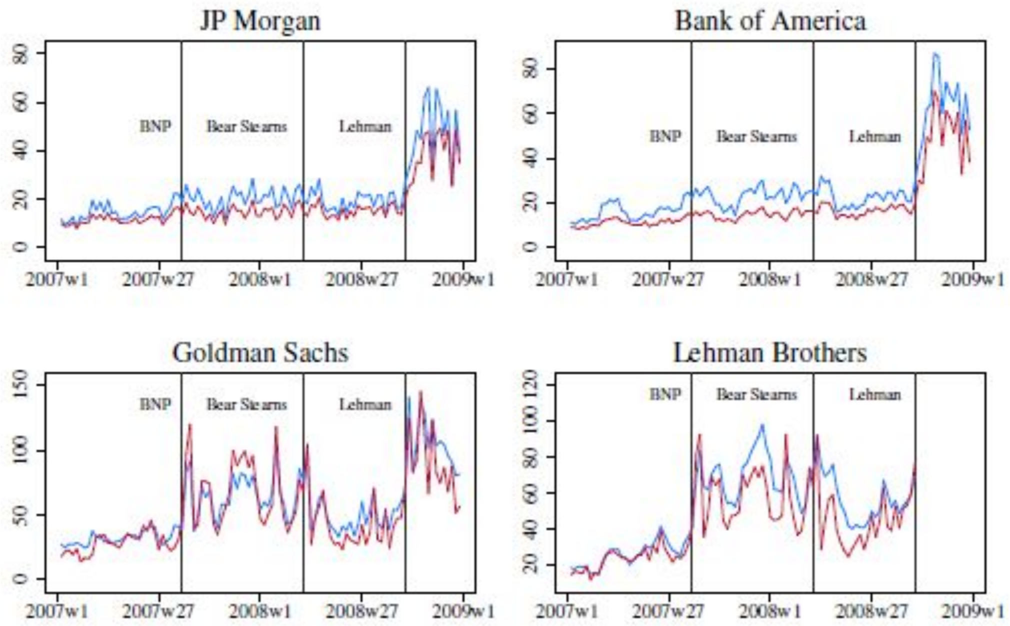
ΔCoVaR vs VaR Over Time

- Average VaR^i and $\Delta CoVaR^i$ over 50 largest financial institutions



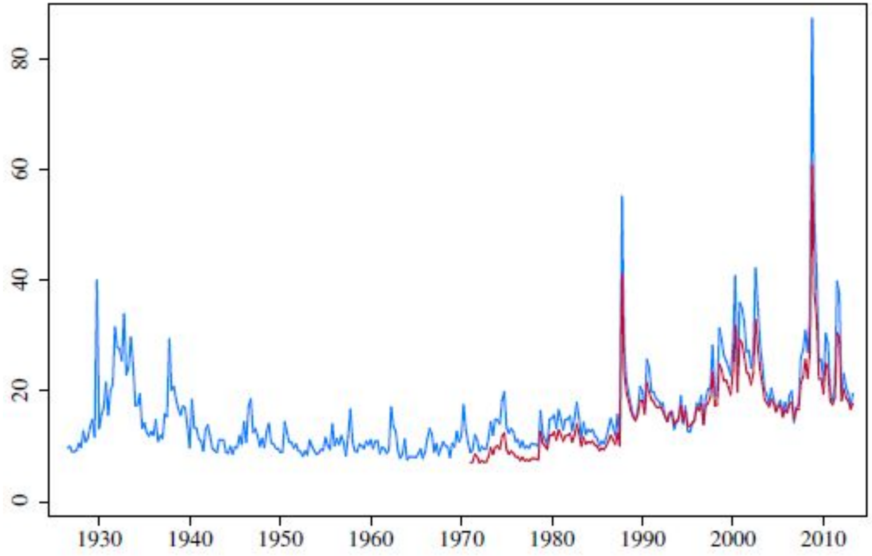
In Sample vs Out of Sample ΔCoVaR

- 95% quantile. Out of sample estimated on expanding windows.



Robustness of ΔCoVaR to Shorter Time Horizon

- Average 95% ΔCoVaR for four companies with data going back to 1926Q3
- 96% correlation



Lagged Institution Variables

- Denoted by X^i_{t-h} in the following slides
- Leverage (market value of assets / market equity)
- Maturity mismatch (book assets / (short term debt minus short term investments minus cash))
  - The following replace maturity mismatch for bank holding companies (BHC))
  - Assets (as % of total book assets): loan-loss allowances, intangible loss allowances, intangible assets and trading assets
  - Liabilities (as % of total book assets): interest bearing core deposits, non interest bearing deposits, large time deposits and demand deposits
- Size (log market equity / log average market equity)
- Boom (# of consecutive quarters in the top 10% of market to book equity)

Forward-$\Delta^{\$}$CoVaR

- $\Delta^{\$}$CoVaR normalized by quarterly average market equity across firms
- Using lagged macro and institution characteristics to predict $\Delta^{\$}$CoVaR
- Forecast horizon h = 1,4,8 quarters
- Multiple regression equation. Applied for 99% and 95% quantiles.

$$\Delta^{\$}CoVaR^{i}_{q,t} = a + cM_{t-h} + bX^{i}_{t-h} + \eta^{i}_{t}$$

- Prediction equation

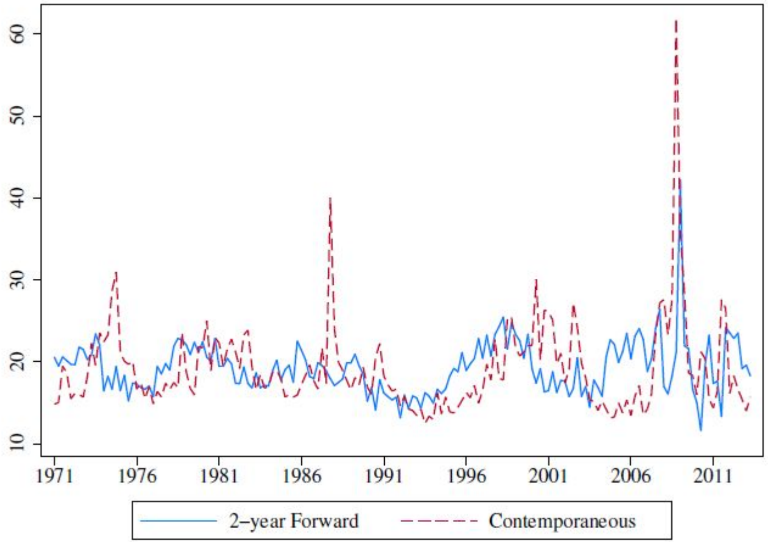
$$\Delta^{Fwd}_{h}CoVaR^{i}_{q,t} = \hat{a} + \hat{c}M_{t-h} + \hat{b}X^{i}_{t-h}$$

Forward $\Delta^{\$}$CoVaR Statistics

- Across all firms - 95% and 99% quantiles
  - Leverage, size and boom are 99% significant across all horizons and quantiles
  - Maturity mismatch is at least 90% significant in all cases
  - All have ~25% Adjusted-R^2
  - “Too Big To Fail” suggests size alone is enough but smaller firms failing en masse (i.e. systemic as part of a herd) is also a major consideration
- Bank holding companies - 95%
  - Multiple regression run with maturity mismatch replaced with asset variables
  - Trading assets 99% significant across all horizons
  - Multiple regression run with maturity mismatch replaced with liability variables
  - Interest bearing deposits, non interest bearing deposits and large time deposits are all 99% significant
  - Adjusted-R^2 ranges from 28% to 36%

ΔCoVaR and 2 Year Forward-ΔCoVaR

- Quarterly average of 50 largest financial institutions as of 2007Q1
- Forward-ΔCoVaR estimated in sample up to 2002Q1
- Forward-ΔCoVaR at any time is the prediction for 2 years ahead



Predictive Power of Forward-ΔCoVaR

- Use 2008Q4 2 Year Forward 95% quantile as an example
- Calculate Forward-ΔCoVaR using data up to 2006Q4 for BHC
- Calculate ΔCoVaR at 2008Q4 for BHC
- Regress ΔCoVaR on Forward-ΔCoVaR

	Crisis $\Delta CoVaR$				
	2008Q4	2008Q4	2008Q4	2007Q4	2007Q1
2Y *Forward*-$\Delta CoVaR$ (2006Q4)	1.206***				
1Y *Forward*-$\Delta CoVaR$ (2007Q4)		0.664***			
1Q *Forward*-$\Delta CoVaR$ (2008Q3)			1.708***		
1Y *Forward*-$\Delta CoVaR$ (2006Q4)				0.848***	
1Q *Forward*-$\Delta CoVaR$ (2006Q4)					0.541***
Constant	13.08***	18.51***	2.409***	4.505***	2.528***
Observations	378	418	430	428	461
R^2	36.6 %	17.8 %	78.9 %	49.6 %	55.5%

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