AMS 513 Spring 2019, Stony Brook University – Instructor: Pinezich

Computer Project 2 DUE: Wednesday February 27 by midnight.

This project has two parts. The first part is analytical; in the second part you will write a Mathematica notebook related to Part 1. The following are acceptable formats to write up your Part 1 solution: (1) Within the Mathematica notebook of Part 2; (2) A latex document output in PDF; (3) a Word document (or Word converted to PDF); (4) Neatly printed by hand and scanned into electronic format. All material should be submitted on the BlackBoard.

Part 1

A nickel, a dime and a quarter are tossed (with values 5, 10, 25, resp.). Let X be the random variable given by the sum of the coin values that land with a head. Denoting a three-toss outcome as $\omega = (\omega_N \omega_D \omega_Q)$, X can be written

$$X(\omega) = 5 \times \mathbb{I}_{\{\omega_N = H\}} + 10 \times \mathbb{I}_{\{\omega_D = H\}} + 25 \times \mathbb{I}_{\{\omega_Q = H\}}.$$

Let A be the event that exactly two heads occur in the outcome.

(a) Given that the coins are all fair, what is the expected value of X given that A has occurred, that is, $\mathbb{E}(X|A)$?

(b) Suppose the probability of a head is given by $p_N = \frac{4}{5}$, $p_D = \frac{1}{2}$, $p_Q = \frac{1}{4}$, what is $\mathbb{E}(X|A)$?

(c) With the probabilities from (b), what is the generalized conditional expectation $\mathbb{E}(X|\sigma(A))$?

Part 2

Create a Mathematica notebook that estimates the conditional expectations in Part 1 (a), (b), (c). For (c) you need to estimate the conditional expectations on the atoms of $\sigma(A)$. Here are some hints:

- Construct a list of three tosses using the `RandomVariate` function with the appropriate distribution.

- Define a function `ifTwoHeads` that returns the value $X(\omega)$ of a three-toss sequence $\omega = \omega_N \omega_D \omega_Q$ if there are exactly two heads in it. Use the `Function` command to do this.

- Make a list `lotsOfTrials` of three-toss sequences using the `Table` function, for some number N of trials.

- Use the `Map` function to apply `ifTwoHeads` to `lotsOfTrials`.

You will then have use a few more steps to derive the numerical answers for (a), (b), and (c). Write your code in a way that makes it easy to change the probability values, and the conditions, as needed.