Pstat 172A Review Notes

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Insurance

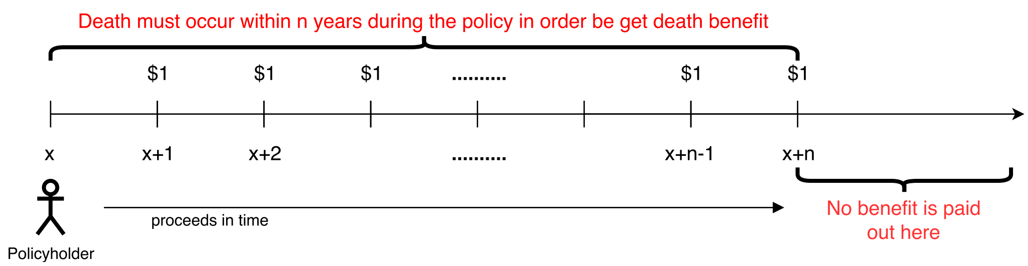
Insurance can be categorized based on the timing of death benefits.
(a) If death benefit is paid out at the end of year of death → discrete insurance.
(b) If death benefit is paid out at the moment of death → continuous insurance.

1 Discrete insurance

1.1 n-year term insurance

Assume n-year term life insurance is sold to (x)

- Assume death benefit is \$1.
- Death benefit is paid out at the end of year of death if death occurs within n years (or death occurs before age $x + n$).
- If (x) is alive beyond age $x + n$, no death benefit is paid out.



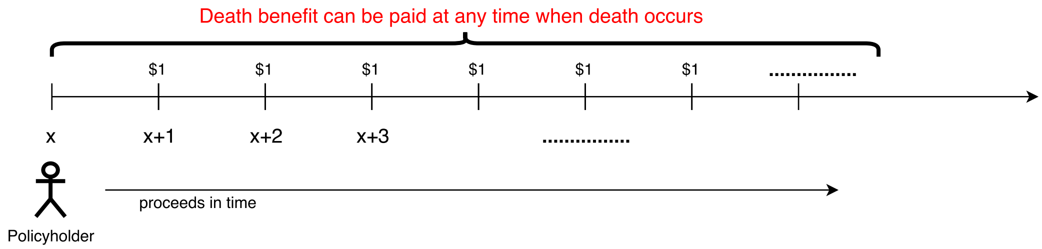
Let Z be the random present value of death benefit for n-year term insurance. Note Z is random because its value depends on the event of death while death happens randomly. We are interested in the expectation of Z, or the mean of death benefit over this type of insurance policy.
Notation: $E(Z) = A^1_{x:\overline{n}|}$
Formula:

$$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} {}_{k|}q_x \text{ and } {}^2A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{2(k+1)} {}_{k|}q_x$$

1.2 Whole life insurance

Assume whole life insurance is sold to (x)

- Assume death benefit is \$1
- Death benefit is paid out at the end of year death. In this case, policy duration $n = \infty$ or $n = \omega - x - 1$



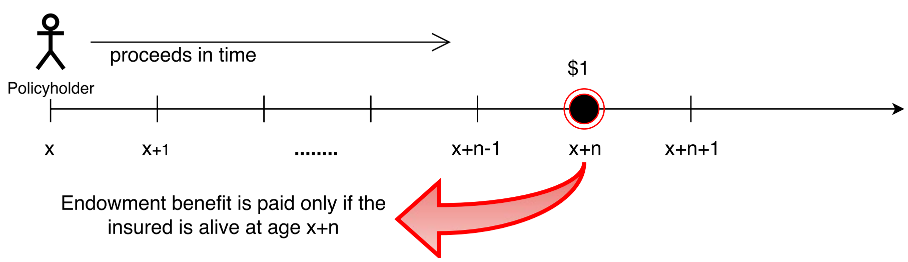
Notation: $E(Z) = A_x$
Formulas:

$$E(Z) = A_x = \sum_{k=0}^{\infty=\omega-x-1} v^{k+1}\,{}_{k|}q_x \text{ and } {}^2A_x = \sum_{k=0}^{\infty=\omega-x-1} v^{2(k+1)}\,{}_{k|}q_x$$

1.3 n-year pure endowment

Assume n-year pure endowment is sold to x

- Assume benefit is \$1
- The insurer pays the benefit at time n (or age $x+n$) if (x) survives to that point. It is irrelevant how long x survives beyond age $x+n$, all that matters is whether or not x survives to time n.



Notation: $E(Z) = A_{x:\overset{\;1}{\overline{n|}}}$
Formulas:

$$E(Z) = A_{x:\overset{\;1}{\overline{n|}}} = v^n\,{}_np_x \text{ and } {}^2A_{x:\overset{\;1}{\overline{n|}}} = v^{2n}\,{}_np_x$$

1.4 n-year endowment insurance

Assume n-year endowment insurance is sold to (x)

- Assume benefit (either death benefit or pure endowment benefit) is $1
- Death benefit is paid out if death occurs within the next n years (or before age $x+n$) or if (x) is alive at age $x+n$

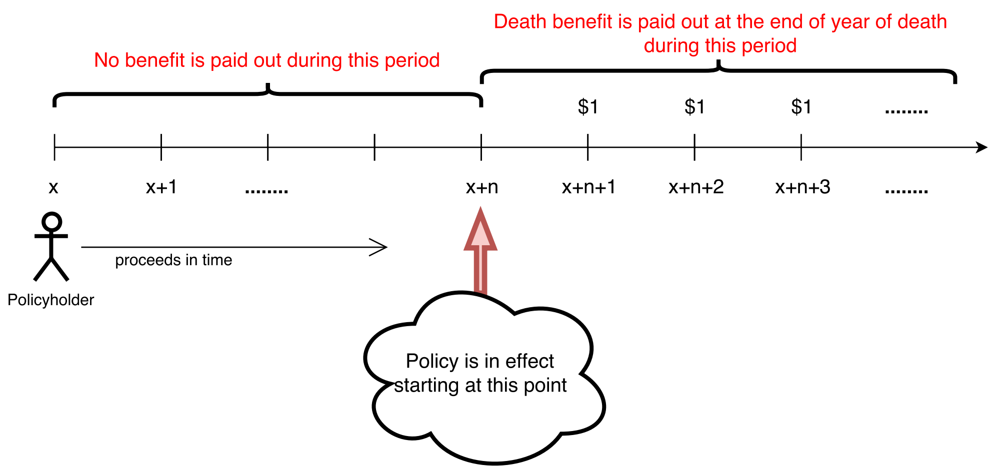
Notation: $E(Z) = A_{x:\overline{n|}}$
Formulas:

$$E(Z) = A_{x:\overline{n|}} = A^{1}_{x:\overline{n|}} + A_{x:\overset{\;\;1}{\overline{n|}}} \text{ and } {}^{2}A_{x:\overline{n|}} = {}^{2}A^{1}_{x:\overline{n|}} + {}^{2}A_{x:\overset{\;\;1}{\overline{n|}}}$$

1.5 n-year deferred insurance

Assume n-year deferred insurance is sold to (x)

- Assume death benefit is $1
- Death benefit is paid out if only the insured survives to age $x+n$ and after that death occurs.



Notation: $E(Z) = {}_{n|}A_x$
Formula:

$$E(Z) = {}_{n|}A_x = A_x - A^{1}_{x:\overline{n|}} = v^n \, {}_np_x \, A_{x+n}$$

2 Continuous insurance

Type	EZ	$E(Z^2)$
n-year term	$\bar{A}^1_{x:\overline{n}\vert} = \int_0^n b_t v^t {}_tp_x \mu_{x+t} dt \big(= \int_0^n b_t e^{-\delta} {}_tp_x \mu_{x+t} dt \big)$	${}^2\bar{A}^1_{x:\overline{n}\vert} = \int_0^n b_t^2 v^{2t} {}_tp_x \mu_{x+t} dt$
whole life	$\bar{A}_x = \int_0^\infty b_t v^t {}_tp_x \mu_{x+t} dt$	${}^2\bar{A}_x = \int_0^\infty b_t^2 v^{2t} {}_xp_t \mu_{x+t} dt$
n-year pure endowment	$\bar{A}_{x:\overline{n}\vert}^{\ \ 1} = E v^n {}_np_x$	${}^2\bar{A}_{x:\overline{n}\vert}^{\ \ 1} = E^2 v^{2t} {}_np_x$
n-year endowment	$\bar{A}_{x:\overline{n}\vert} = \bar{A}^1_{x:\overline{n}\vert} + \bar{A}_{x:\overline{n}\vert}^{\ \ 1}$	${}^2\bar{A}_{x:\overline{n}\vert} = {}^2\bar{A}^1_{x:\overline{n}\vert} + {}^2\bar{A}_{x:\overline{n}\vert}^{\ \ 1}$
n-year deferred	${}_{n\vert}\bar{A}_x = \bar{A}_x - \bar{A}^1_{x:\overline{n}\vert} = v^n {}_np_x \bar{A}_{x+n}$	${}^2_{n\vert}\bar{A}_x = v^{2n} {}_np_x {}^2\bar{A}_{x+n}$

*Note: b_t is the death benefit at time t and E is the endowment benefit.

3 Insurance formulas under special assumptions

3.1 Under linear assumption (DeMoivre's Assumption)

Insurance formulations under $S_0(t) = 1 - \frac{t}{\omega}$ (De Moivre)

$$A^{1}_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot a_{\overline{n}|} = \frac{1}{\omega - x} \cdot \frac{1 - v^n}{i}$$

$$A_x = \frac{1}{\omega - x} \cdot a_{\overline{\omega - x}|} = \frac{1}{\omega - x} \cdot \frac{1 - v^{\omega - x}}{i}$$

$$^2A^{1}_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot {}^2a_{\overline{n}|} = \frac{1}{\omega - x} \cdot \frac{1 - v^{2n}}{2i + i^2}$$

$$^2A_x = \frac{1}{\omega - x} \cdot {}^2a_{\overline{\omega - x}|} = \frac{1}{\omega - x} \cdot \frac{1 - v^{2(\omega - x)}}{2i + i^2}$$

$$\overline{A}^{1}_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot \overline{a}_{\overline{n}|} = \frac{1}{\omega - x} \cdot \frac{1 - e^{-\delta n}}{\delta}$$

$$\overline{A}_x = \frac{1}{\omega - x} \cdot \overline{a}_{\overline{\omega - x}|} = \frac{1}{\omega - x} \cdot \frac{1 - e^{-\delta(\omega - x)}}{\delta}$$

$$^2\overline{A}^{1}_{x:\overline{n}|} = \frac{1}{\omega - x} \cdot {}^2\overline{a}_{\overline{n}|} = \frac{1}{\omega - x} \cdot \frac{1 - e^{-2\delta n}}{2\delta}$$

$$^2\overline{A}_x = \frac{1}{\omega - x} \cdot {}^2\overline{a}_{\overline{\omega - x}|} = \frac{1}{\omega - x} \cdot \frac{1 - e^{-2\delta(\omega - x)}}{2\delta}$$

3.2 Under constant force of mortality

Insurance formulations under constant force assumption

$$A^{1}_{x:\overline{n}|} = \frac{q(1 - v^n p^n)}{q + i} \quad , \quad A_x = \frac{q}{q + i} \quad , \quad {}^2A_x = \frac{q}{q + 2i + i^2}$$

$$\overline{A}^{1}_{x:\overline{n}|} = \frac{\mu(1 - e^{-n(\mu + \delta)})}{\mu + \delta} \quad , \quad \overline{A}_x = \frac{\mu}{\mu + \delta} \quad , \quad {}^2\overline{A}_x = \frac{\mu}{\mu + 2\delta}$$