Gross Premium

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In previous section, we look at net premium (or benefit premium) where the value of net premium is determined to be the cost of the insurance product. Recall the equivalence principle to calculate net premium:

$$\text{APV of premiums (at } t = 0) = \text{APV of all benefits (at } t = 0)$$

In practice, when a policy is issued, many types of expenses can occur besides the benefits. We now learn how to calculate a more reasonable price of premium to cover the cost of these expenses. Premium that covers all of the expenses for an issued insurance product is called gross premium.

1 Gross premium

1.1 Expenses

Expenses can be fixed or variable:

- **Fixed**: some expenses are fixed irrespective of the type of volume of the business you write.
- **Variable**: expenses are varied with business written (number of policies, size of the policies, premiums, etc.)

Initial/ acquisition expenses
It is related to new business or when the policy is issued.
Maintenance/ renewal expenses
After the policy is issued, there are continuing administrative expenses and commission.
Settlement/ termination expenses
When death occurs, there are costs to finalize and disburse benefit payments.

1.2 Equivalence principle

Use the equivalence principle to calculate the gross premium such that:

APV of gross premiums = APV of all benefits + APV of all expenses

2 Gross future loss

Recall: if ${}_0L$ is the loss-at-issue random variable, then:

$$ {}_0L = \text{PV of benefits} \; - \; \text{PV of net premiums} $$

Also, the net premium is calculated using equivalence principle is the same as calculating the expectation of ${}_0L$ and set it equal to 0.

$$ \begin{aligned} \mathbb{E}({}_0L) = \text{APV of benefits} \; - \; \text{APV of net premiums} = 0 \\ \implies \text{APV of benefits} = \text{APV of net premiums} \end{aligned} $$

In this chapter, we can also calculate the loss at issue taking into account the gross premiums and all the expenses associated with an insurance policy. Such a loss at issue is called **expense augmented loss/ gross future loss**.

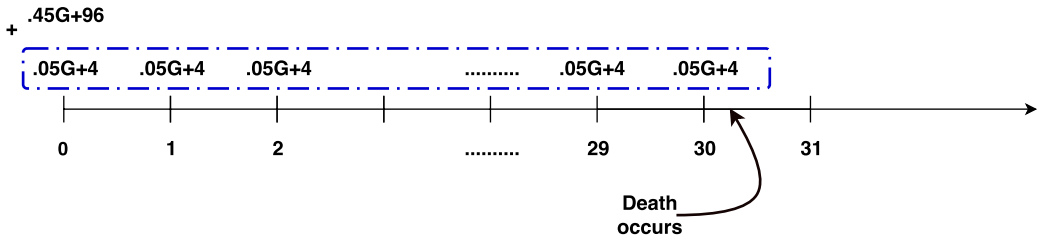
Actuarial notation: ${}_0L_e$ or ${}_0L^g$.
Equivalence principle:

${}_0L_g$ = PV of benefits + PV of expenses - PV of gross premiums

Example: For a fully discrete whole life insurance of $\$1,000$ on (45), premium of \$22 are payable annually. Given:
(i) $v = 0.96$
(ii) Initial expenses are 50% of first year premium plus \$100
(iii) Renewal expenses are 50% of renewal premium plus \$4, and are paid at the beginning of each year.
Calculate the gross future loss if the policyholder dies at $t = 30.2$.
Solution:

1. Fully discrete whole life $\rightarrow$ benefits are paid at the end of the year of death. Since the policyholder dies at $t = 30.2$, death benefit is made at $t = 31$.

2. Let G be the gross premium for the insurance policy ($G = 22$):
   (a) Renewal expenses are 5% of renewal premium plus \$4: $e_t = .05G + 4$ where $t = 1, 2, ...30$.
   (b) Initial expenses are 50% of first year premium plus \$100: $e_0 = .5G + 100 = (.05G + 4) + (.45G + 96) = e_t + (0.45G + 96)$. We have a diagram to describe the flows of expenses over the years:



3. Using equivalence principle for gross premium:

$$
\begin{aligned}
{}_0L_g &= \text{PV of benefits } + \text{ PV of expenses } - \text{ PV of gross premiums} \\
&= 1,000v^{31} + \Big((.05G+4)\ddot{a}_{\overline{31}|} + (.45G+96)\Big) - G\ddot{a}_{\overline{31}|} \\
&= 84.692
\end{aligned}
$$

In the above equation, $G = 22$ and $\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$.