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PSTAT 172A - Winter 2019

Review some important notations:

1. T_0, T_x, and the relationship between T_0 and T_x ($T_x = T_0 - x$); K_x = completed (integer) number of years until (x)'s death
2. $S_x(t) = P(T_x > t) := {}_tp_x$; $F_x(t) = P(T_x \le t) := {}_tq_x$; l_x, d_x, and their relationship with ${}_tp_x$ and ${}_tq_x$
3. **Factorization of survival probability**: ${}_tp_x = {}_np_x({}_{t-n}p_{x+n})$ and **deferred mortality**: ${}_{t|u}q_x = {}_tp_x({}_uq_{x+t}) = {}_{t+u}q_x - {}_tq_x$
4. Force of mortality (hazard rate) μ_x= instantaneous rate of mortality at age x given alive at age x: $\mu_x = -\frac{d}{dx}lnS_0(x)$. Hence, relationship between survival function and force of mortality: $S_0(x) = {}_xp_0 = e^{-\int_0^x \mu_t dt}$. Generalization: force of morality t years after age x: $\mu_{x+t} = \frac{f_x(t)}{1-F_x(t)} = -\frac{d}{dt}lnS_0(x+t) \to f_x(t) = {}_tp_x(\mu_{x+t})$. Some handy formulas involving T_x and μ_x:
   (a) ${}_np_x = e^{-\int_x^{x+n} \mu_s ds} = e^{-\int_0^n \mu_{x+t} dt}$. **Note**: μ_{x+t} is also denoted as $\mu_x(t)$.
   (b) ${}_nq_x = P[0 \le T_x \le n] = \int_0^n f_x(s)ds = \int_0^n {}_sp_x\mu_{x+s}ds$ and ${}_{t|u}q_x = \int_t^{t+u} {}_sp_x\mu_{x+s}ds$.
5. (a) $\mathring{e}_x = E[T_x] = \int_0^\infty t{}_tp_x\mu_{x+t}dt = \int_0^\infty {}_tp_xdt$. **Note**: replace ∞ by $\omega - x$ if ω is the upper age limit. Similarly, $E[T_x^2] = \int_0^\infty t^2{}_xp_t\mu_{x+t}dt = \int_0^\infty 2t{}_tp_xdt$. Hence: $Var[T_x] = E[T_x^2] - E[T_x]^2$.
   (b) n-year term expectation of life for (x): $\mathring{e}_{x:\overline{n|}} = \int_0^n {}_tp_xdt = E[min\{T_x, n\}] = n{}_np_x + \int_0^n t{}_tp_x\mu_{x+t}dt$
   (c) **Recursive relationship**: $\mathring{e}_x = \mathring{e}_{x:\overline{n|}} + {}_np_x\mathring{e}_{x+n}$ and $\mathring{e}_x = \int_0^1 {}_xp_tdt + p_x\mathring{e}_{x+1}$
6. (a) $e_x = E[K_x] = \sum_{k=0}^\infty kP[K_x = k] = \sum_{k=0}^k k{}_{k|}q_x = \sum_{k=1}^\infty {}_kp_x$. **Note**: replace ∞ by $\omega - x - 1$ if ω is the upper age limit. $E[K_x^2] = \sum_{k=0}^\infty k^2{}_{k|}q_x$.
   (b) n-year term curtate expectation: $e_{x:\overline{n|}} = \sum_{k=1}^n {}_kp_x$
   (c) **Recursive relationship**: $e_x = e_{x:\overline{n|}} + {}_np_xe_{x+n}$ and $e_x = p_x + p_xe_{x+1}$

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HOMEWORK 1

Problem 6
Given: (a) ${}_3p_{70} = .95$ (b) ${}_2p_{71} = .96$ (c) $\int_{71}^{75} \mu_xdx = .107$. Calculate ${}_5p_{70}$.

Problem 16
(a) Show that $\mathring{e}_x \le \mathring{e}_{x+1} + 1$ (b) Show that $\mathring{e}_x \ge e_x$ (c) Explain why $\mathring{e}_x \approx e_x + 1/2$.
(d) Is $\mathring{e}_x$ always a non-increasing function of x?

Problem 17
A subgroup of lives is subject to twice the normal force of mortality, i.e. $\mu'_x = 2\mu_x$ where the prime indicates the rate for the subgroup. Express q'_x in terms of q.

Problem 18
You are given $p_{30} = .95$ for a standard insured with a force of mortality of μ_{30+t}, $0 \le t \le 1$. For a preferred insured, force of mortality is $\mu_{30+t} - c$, $0 \le t \le 1$. Find c such that the probability that (30) will die within one year is 25% lower for a preferred life than for a standard insured.

Supplemental problem 1
Using $S_0(x) = 1 - \frac{x^2}{100}$ for $0 \le x \le 10 = \omega$. Find the mean, variance, median and the mode of T_4.

Supplemental problem 2
You are given: (a) $l_x = (100 - x)^{1/2}$, $0 \le x \le 100$ (b) $\mathring{e}_{36:\overline{28|}} = 24.67$. Calculate $\int_0^{28} t{}_tp_{36}\mu_{36+t}dt$.