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PSTAT 172A - Winter 2019

<u>**Review some important notations**</u>:

1. **Select and ultimate mortality**
   (a) $[x]$ - Individual is select at age x. Square brackets denote underwritten or otherwise a preferred risk. Opposite of select is ultimate. It is often the case that the select group has a better health than the ultimate group. Hence: $q_{[x]} < q_x$.
   (b) $q_{[x]+k}$ denotes the one year mortality probability for an individual at exact age $x+k$ who passed a selection procedure k years earlier at age x. For positive selection (such as medical examination): $q_{[x]+k} < q_{x+k}$, for negative selection (such as disability): $q_{[x]+k} > q_{x+k}$.
   (c) The select period is the smallest n for which the select mortality probability are equal to the ultimate mortality probability: $q_{[x]+n} = q_{x+n}$.
   (d) Note: ${}_kp_{[x]} = \frac{l_{[x]+k}}{l_{[x]}}$ (not $\frac{l_{[x+k]}}{l_{[x]}}$), ${}_{n|m}q_{[x]} = \frac{l_{[x]+n} - l_{[x]+n+m}}{l_{[x]}}$
2. Some important assumptions on the distribution of T_x
   (a) **Uniform distribution (De Moivre's Law)**: Under this model, $\forall x \in [0, \omega] : T_x \sim \text{Uniform}[0, \omega - x]$

   i. $f_x(t) = \frac{1}{\omega - x}$
   ii. $l_x = k(\omega - x)$
   iii. ${}_np_x = \frac{l_{x+n}}{l_x} = 1 - \frac{n}{\omega - x}$
   iv. $\mu_y = \frac{1}{\omega - y}$, for $0 \le y \le \omega$
   v. $\mathring{e}_x = E[T_x] = \frac{\omega - x}{2}$
   vi. $Var[T_x] = \frac{(\omega - x)^2}{12}$
   vii. $\mathring{e}_{x:\overline{n}|} = n - \frac{n^2}{2(\omega - x)}$
   viii. $e_x = \frac{\omega - x - 1}{2}$

   (b) **Uniform distribution of deaths (UDD)**: Under this assumption, l_{x+t} is a linear function for t where $0 \le t \le 1$: $l_{x+t} = l_x - td_x \rightarrow {}_tp_x = 1 - tq_x$, or ${}_tq_x = tq_x$. Following formulas are true for $0 \le t \le 1$:

   i. $\mu_{x+t} = \frac{f_x(t)}{S_x(t)} = \frac{\frac{d}{dt}F_x(t)}{1 - {}_tq_x} = \frac{\frac{d}{dt}(tq_x)}{1 - tq_x} = \frac{q_x}{1 - tq_x}$
   ii. $f_x(t) = {}_tp_x \mu_{x+t} = q_x$
   iii. ${}_tp_{x+s} = \frac{l_x - (t+s)d_x}{l_x - sd_x} = \frac{1 - (t+s)q_x}{1 - sq_x}$ if $0 \le s + t \le 1$
   iv. ${}_tq_{x+s} = 1 - {}_tp_{x+s} = \frac{tq_x}{1 - sq_x}$ if $0 \le s + t \le 1$

   (c) **Constant force of mortality (CFM)**: Under this assumption, l_{x+t} has the form: $l_{x+t} = (l_x)^{1-t}(l_{x+1})^t = l_x(p_x)^t$ for $0 \le t \le 1 \rightarrow {}_tp_x = (p_x)^t$. Following formulas are true for $0 \le t \le 1$:

   i. $\mu_{x+t} = \frac{-\frac{d}{dt}{}_tp_x}{{}_tp_x} = \frac{-\frac{d}{dt}(p_x)^t}{(p_x)^t} = \frac{-ln p_x p_x^t}{p_x^t} = -ln(p_x)$
   ii. ${}_tp_x = {}_tp_{x+s} = e^{-t\mu_x} = (p_x)^t$ if $0 \le t + s \le 1$

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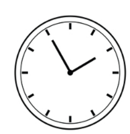
HOMEWORK 3 (LIFE TABLE)

<u>**Problem 10**</u>
Given $l_x = 1000$, $l_{x+1} = 900$, and $l_{x+2} = 800$.
Find ${}_{0.8}q_{x+0.7}$ under: (a) UDD (b) CFM.



<u>**Problem 9**</u>
You are given a life table with a one-year select period:

x	$l_{[x]}$	$d_{[x]}$
85	1000	100
86	850	100

Given also that $\mathring{e}_{[85]} = 5.556$, and that deaths are uniformly distributed over the year of age. Find $\mathring{e}_{[86]}$.

<u>**Problem 14**</u>
If death are uniformly distributed over each year of age and $\mu_{45.5} = 0.5$, calculate $\mathring{e}_{45:\overline{1}|}$.

<u>**Problem 19**</u>
A disease has a constant force of mortality μ. Historically 10% of all people with the disease die within 20 years. A more virulent strain of the disease is encountered with a constant force of mortality 2μ. What is the probability of an individual who has the new strain of the disease dying within 20 years?

<u>**Problem 11**</u>
On a certain life table, the average force of mortality between ages 90 and 92 is 0.2. If $q_{90} = .17$, find q_{91}.