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## 1 Some definition

```
#include <bits/stdc++.h>
#include <random>
#include <chrono>
#include <ctime>

#define N
#define matrix_size 2
#define mod 1000000007LL
#define eps 1e-8
#define base 137 // Or 37

#define cross(A, B) (A.x * B.y - A.y * B.x)
#define dot(A, B) (A.x * B.x + A.y * B.y)
```

```
#define ccw(A, B, C) (-(A.x * (C.y - B.y) + B.x * (A.y - C.y) + C.x * (B.y - A.y))) // positive when ccw
#define CROSS(a, b, c, d) (a * d - b * c)
```

## 2 Data structure

### 2.1 Mo's algorithm

$$O(N * \sqrt{N} + Q * \sqrt{N})$$

```
S = sqrt(N);
bool cmp(Query A, Query B) // compare 2 queries
{
    if (A.l / S != B.l / S) {
        return A.l / S < B.l / S;
    }
    return A.r < B.r;
}
```

### 2.2 Set and map

Use `set.lower_bound()` instead of `lower_bound(set.begin(), set.end())` for better performance

The same is true for `map`

### 2.3 BIT

```
void update(int x, int val)
{
    for (; x <= n; x += x & ~x) BIT[x] = min(BIT[x], val);
}

int get(int x)
{
    int res = 1e9;
    for (; x > 0; x -= x & ~x) res = min(res, BIT[x]);
    return res;
}
```

### 2.4 IT2D

```
int Max[4096][4096];

struct dir {
    int ll, rr, id;
    dir(int L, int R, int X)
    { ll=L, rr=R, id=X; }
    dir left() const
    { return dir(ll, (ll+rr)/2, id*2); }
    dir right() const
    { return dir((ll+rr)/2+1, rr, id*2+1); }
    inline bool irrelevant(int L, int R) const
    { return ll>R || L>rr || L>R; }
};

void maximize(int &a, int b)
{ a=max(a, b); }

void maximize(const dir &dx, const dir &dy, int x, int y, int k, bool only_y) {
    if (dx.irrelevant(x, x) || dy.irrelevant(y, y)) return;
    maximize(Max[dx.id][dy.id], k);
    if (!only_y && dx.ll != dx.rr) {
        maximize(dx.left(), dy, x, y, k, false);
        maximize(dx.right(), dy, x, y, k, false);
    }
    if (dy.ll != dy.rr) {
        maximize(dx, dy.left(), x, y, k, true);
        maximize(dx, dy.right(), x, y, k, true);
    }
}

int max_range(const dir &dx, const dir &dy, int lx, int rx, int ly, int ry) {
    if (dx.irrelevant(lx, rx) || dy.irrelevant(ly, ry))
        return 0;
    if (lx<=dx.ll && dx.rr<=rx) {
        if (ly<=dy.ll && dy.rr<=ry) return Max[dx.id][dy.id];
        int Max1 = max_range(dx, dy.left(), lx, rx, ly, ry);
        int Max2 = max_range(dx, dy.right(), lx, rx, ly, ry);
        return max(Max1, Max2);
    } else {
        int Max1 = max_range(dx.left(), dy, lx, rx, ly, ry);
        int Max2 = max_range(dx.right(), dy, lx, rx, ly, ry);
        return max(Max1, Max2);
    }
}
```

}

## 3 Graph

### 3.1 Dinic

```
namespace Dinic // really fast, O(n^2 m) or O(sqrt(n)m) if
bipartite
{
    vector<int> adj[N];
    long long c[N][N], f[N][N];
    int s = 0, t = 0, d[N], ptr[N];
    bool BFS()
    {
        queue<int> q;
        memset(d, -1, sizeof(d));
        d[s] = 0; q.push(s);
        while (!q.empty())
        {
            int u = q.front(); q.pop();
            for (int v : adj[u])
            {
                if (d[v] == -1 && c[u][v] > f[u][v])
                {
                    d[v] = d[u] + 1;
                    q.push(v);
                }
            }
        }
        return d[t] != -1;
    }
    long long DFS(int x, long long delta)
    {
        if (x == t) return delta;
        for (; ptr[x] < adj[x].size(); ++ptr[x]) // Skip
the used edge
        {
            int y = adj[x][ptr[x]];
            if (d[y] == d[x] + 1 && c[x][y] > f[x][y])
            {
                long long push = DFS(y, min(delta, c[x][y]
- f[x][y]));
                if (push)
                {
                    f[x][y] += push;

```

```
                f[y][x] -= push;
                return push;
            }
        }
        return 0;
    }
    long long maxFlow(int x, int y) // From x to y
    {
        long long flow = 0;
        s = x; t = y;
        while (BFS())
        {
            memset(ptr, 0, sizeof(ptr));
            while (long long tmp = DFS(s, 1e9))
                flow += 1LL * tmp;
        }
        return flow;
    }
};
```

### 3.2 Mincost

```
int calc(int x, int y) { return (x >= 0) ? y : 0 - y; }

bool findpath()
{
    for (int i = 1; i <= n; i++) { trace[i] = 0; d[i] = inf;
    }
    q.push(n); d[n] = 0;
    while (!q.empty())
    {
        int u = q.front();
        q.pop();
        inq[u] = false;
        for (int i = 0; i < adj[u].size(); i++)
        {
            int v = adj[u][i];
            if (c[u][v] > f[u][v] && d[v] > d[u] + calc(f[u][v],
cost[u][v]))
            {
                trace[v] = u;
                d[v] = d[u] + calc(f[u][v], cost[u][v]);
                if (!inq[v])
                {

```

```

        inq[v] = true;
        q.push(v);
    }
}
}
}
return d[t] != inf;
}

void incflow()
{
    int v = t, delta = inf;
    while (v != n)
    {
        int u = trace[v];
        if (f[u][v] >= 0)
            delta = min(delta, c[u][v] - f[u][v]);
        else
            delta = min(delta, 0 - f[u][v]);
        v = u;
    }
    v = t;
    while (v != n)
    {
        int u = trace[v];
        f[u][v] += delta;
        f[v][u] -= delta;
        v = u;
    }
}

```

### 3.3 HLD

```

void DFS(int x, int pa)
{
    DD[x] = DD[pa] + 1; child[x] = 1; int Max = 0;
    for (int i = 0; i < DSK[x].size(); i++)
    {
        int y = DSK[x][i].fi;
        if (y == pa) continue;
        p[y] = x;
        d[y] = d[x] + DSK[x][i].se;
        DFS(y, x);
        child[x] += child[y];
        if (child[y] > Max)

```

```

    {
        Max = child[y];
        tree[x] = tree[y];
    }
}
if (child[x] == 1) tree[x] = ++nTree;
}

void init()
{
    nTree = 0;
    DFS(1, 1);
    DD[0] = long(1e9);
    for (int i = 1; i <= n; i++) if (DD[i] < DD[root[tree[i]]])
        root[tree[i]] = i;
}

int LCA(int u, int v)
{
    while (tree[u] != tree[v])
    {
        if (DD[root[tree[u]]] < DD[root[tree[v]]]) v = p[root[tree[v]]];
        else u = p[root[tree[u]]];
    }
    if (DD[u] < DD[v]) return u; else return v;
}

```

### 3.4 Cu khp

Nút  $u$  là khp: if ( $\text{low}[v] \neq \text{num}[u]$ )  $\text{arti}[u] = \text{arti}[u] \text{ --- } p[u] \neq -1 \text{ --- }$   
 $\text{child}[u] \neq 2$ ;  
 Cnh  $u, v$  là cu khi  $\text{low}[v] \neq \text{num}[v]$

### 3.5 Monotone chain

```

void convex_hull (vector<pt> & a) {
    if (a.size() == 1) { // Only 1 point
        return;
    }

    // Sort with respect to x and then y
    sort(a.begin(), a.end(), &cmp);

```

```

pt p1 = a[0], p2 = a.back();

vector<pt> up, down;
up.push_back(p1);
down.push_back(p1);

for (size_t i=1; i<a.size(); ++i) {
    // Add to the upper chain

    if (i==a.size()-1 || cw(p1, a[i], p2)) {
        while (up.size()>=2 && !cw(up[up.size()-2], up[up.size()-1], a[i]))
            up.pop_back();
        up.push_back(a[i]);
    }

    // Add to the lower chain
    if (i==a.size()-1 || ccw(p1, a[i], p2)) {
        while (down.size()>=2 && !ccw(down[down.size()-2], down[down.size()-1], a[i]))
            down.pop_back();
        down.push_back(a[i]);
    }
}

// Merge 2 chains
a.clear();
for (size_t i=0; i<up.size(); ++i)
    a.push_back(up[i]);
for (size_t i=down.size()-2; i>0; --i)
    a.push_back(down[i]);
}

```

## 3.6 MST

Prim: remember to have visited array

## 3.7 HopcroftKarp

```

namespace HopcroftKarp // O(sqrt(n) * m)
{
    vector<int> adj[N]; int match[N], d[N];
    bool BFS()
    {
        queue<int> q;

```

```

memset(d, -1, sizeof(d));
for (int i = 1; i <= n; ++i) if (!match[i])
{
    d[i] = 0;
    q.push(i);
}
bool flag = false;
while (!q.empty())
{
    int u = q.front(); q.pop();
    for (int v : adj[u])
    {
        if (match[v] == 0)
        {
            flag = true;
            continue;
        }
        if (d[match[v]] == -1)
        {
            d[match[v]] = d[u] + 1;
            q.push(match[v]);
        }
    }
}
return flag;
}

bool DFS(int x)
{
    for (int y : adj[x])
    {
        if (match[y] == 0 || (d[match[y]] == d[x] + 1
&& DFS(match[y])))
        {
            match[y] = x;
            match[x] = y;
            return true;
        }
    }
    d[x] = -1;
    return false;
}

long long maxMatching() // From x to y
{
    long long matching = 0;
    while (BFS())

```

```

    {
        for (int i = 1; i <= n; ++i) if (!match[i] &&
DFS(i))
            ++matching;
    }
    return matching;
};

```

### 3.8 Hungarian

```

struct Hungarian {
    long c[N][N], fx[N], fy[N], d[N];
    int mx[N], my[N], trace[N], arg[N];
    queue<int> q;
    int start, finish, n, m;
    const long inf = 1e18;

    void Init(int _n, int _m) {
        n = _n, m = _m;
        FOR(i, 1, n) {
            mx[i] = my[i] = 0;
            FOR(j, 1, n) c[i][j] = inf;
        }
    }

    void addEdge(int u, int v, long cost) { c[u][v] = min(c[u][v], cost); }

    inline long getC(int u, int v) { return c[u][v] - fx[u] - fy[v]; }

    void initBFS() {
        while (!q.empty()) q.pop();
        q.push(start);
        FOR(i, 0, n) trace[i] = 0;
        FOR(v, 1, n) {
            d[v] = getC(start, v), arg[v] = start;
        }
        finish = 0;
    }

    void findAugPath() {
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            FOR(v, 1, n) if (!trace[v]) {

```

```

                long w = getC(u, v);
                if (!w) {
                    trace[v] = u;
                    if (!my[v]) { finish = v; return; }
                    q.push(my[v]);
                }
                if (d[v] > w) { d[v] = w; arg[v] = u; }
            }
        }
    }

    void subX_addY(){
        long delta = inf;
        FOR(v, 1, n) if (trace[v] == 0 && d[v] < delta) delta = d[v];
        fx[start] += delta;
        FOR(v, 1, n) if (trace[v]) {
            int u = my[v];
            fy[v] -= delta, fx[u] += delta;
        } else d[v] -= delta;

        FOR(v, 1, n) if (!trace[v] && !d[v]) {
            trace[v] = arg[v];
            if (!my[v]) { finish = v; return; }
            q.push(my[v]);
        }
    }

    void Enlarge() {
        do {
            int u = trace[finish], nxt = mx[u];
            mx[u] = finish, my[finish] = u, finish = nxt;
        } while (finish);
    }

    long minCost() {
        FOR(u, 1, n) {
            fx[u] = c[u][1];
            FOR(v, 1, n) fx[u] = min(fx[u], c[u][v]);
        }
        FOR(v, 1, n) {
            fy[v] = c[1][v] - fx[1];
            FOR(u, 1, n) fy[v] = min(fy[v], c[u][v] - fx[u]);
        }
    }
}

```

```

FOR(u, 1, n) {
    start = u;
    initBFS();
    while (finish == 0) {
        findAugPath();
        if (!finish) subX_addY();
    }
    Enlarge();
}

int res = 0;
FOR(i, 1, n) res += c[i][mx[i]];
return res;
}
};

```

## 4 String

### 4.1 Aho Corasick

```

struct Node
{
    int nxt[26], go[26];
    bool leaf;
    long long val, sumVal;
    int p;
    int pch;
    int link;
};

Node t[N];
int sz;

void New(Node &x, int p, int link, int pch)
{
    x.p = p;
    x.link = link;
    x.pch = pch;
    x.val = 0;
    x.sumVal = -1;
    memset(x.nxt, -1, sizeof(x.nxt));
    memset(x.go, -1, sizeof(x.go));
}

void AddString(const string &s, int val)

```

```

{
    int v = 0;
    for (char c : s)
    {
        int id = c - 'A';
        if (t[v].nxt[id] == -1)
        {
            New(t[sz], v, -1, id);
            t[v].nxt[id] = sz++;
        }
        v = t[v].nxt[id];
    }
    t[v].leaf = true;
    t[v].val = val;
}

int Go(int u, int c);

int Link(int u)
{
    if (t[u].link == -1)
    {
        if (u == 0 || t[u].p == 0)
            t[u].link = 0;
        else
            t[u].link = Go(Link(t[u].p), t[u].pch);
    }
    return t[u].link;
}

int Go(int u, int c)
{
    if (t[u].go[c] == -1)
    {
        if (t[u].nxt[c] != -1)
            t[u].go[c] = t[u].nxt[c];
        else
            t[u].go[c] = (u == 0 ? 0 : Go(Link(u), c));
    }
    return t[u].go[c];
}

```

### 4.2 Manacher

```

void init() {

```

```

cnt = 0;
t[0] = '~';
for (int i = 0; i < n; i++) {
    t[++cnt] = '#'; t[++cnt] = s[i];
}
t[++cnt] = '#'; t[++cnt] = '-';
}

void manacher() {
    int n = cnt - 2;
    int r = 1; int C = 1;
    int ans = 0;
    for (int i = 2; i < n; i++) {
        int i_mirror = C * 2 - i;
        z[i] = (r > i) ? min(z[i_mirror], r - i) : 0;
        while (t[i + z[i] + 1] == t[i - z[i] - 1]) z[i]++;
        if (i + z[i] > r) {
            C = i;
            r = i + z[i];
        }
    }
}

```

### 4.3 Suffix Array

```

struct SuffixArray {
    string s;
    int n;
    vector<int> SA, RA, tempSA, tempRA, LCP;
    int L[N];

    void reset(string st) {
        s = st;
        RA.clear();
        s.push_back('#');
        n = s.size();
        RA.resize(n + 1, 0);
        SA = RA, tempSA = tempRA = LCP = RA;
    }

    void BuildSA() {
        REP(i, n) SA[i] = i, RA[i] = s[i];
        for (int k = 1; k < n; k <= 1) {
            radix_sort(k);
            radix_sort(0);

```

```

        tempRA[SA[0]] = 0;
        for (int i = 1, r = 0; i < n; ++i) {
            if (getRA(SA[i - 1]) != getRA(SA[i]) || getRA(SA[i - 1] + k) != getRA(SA[i] + k)) ++r;
            tempRA[SA[i]] = r;
        }
        REP(i, n) RA[i] = tempRA[i];
        if (RA[SA[n - 1]] == n - 1) break;
    }
}

void BuildLCP() {
    // kasai
    REP(i, n) RA[SA[i]] = i;
    int k = 0;
    REP(i, n) {
        if (RA[i] == n - 1) {
            k = 0; continue;
        }
        int j = SA[RA[i] + 1];
        while (i + k < n && j + k < n && s[i + k] == s[j + k]) ++k;
        LCP[RA[i]] = k;
        if (k) k--;
    }
}

private:
inline int getRA(int i) { return (i < n ? RA[i] : 0); }
void radix_sort(int k) {
    memset(L, 0, sizeof L);
    REP(i, n) L[getRA(i + k)]++;
    int p = 0;
    REP(i, N) {
        int x = L[i];
        L[i] = p;
        p += x;
    }
    REP(i, n) {
        int &x = L[getRA(SA[i] + k)];
        tempSA[x++] = SA[i];
    }
    REP(i, n) SA[i] = tempSA[i];
}
};

```



## 4.4 Z function

```
vector<int> Zfunc(int n, vector<int> &a) {
    vector<int> z(n);
    z[0] = n;
    int l = 0, r = 0;
    FOR(i, 1, n - 1) {
        z[i] = (i <= r ? min(r - i + 1, z[i - l]) : 0);
        while (i + z[i] < n && a[z[i]] == a[i + z[i]]) ++z[i];
        if (i + z[i] > r) {
            r = i + z[i] - 1;
            l = i;
        }
    }
    return z;
}
```

## 4.5 KMP

```
// SUBSTR spoj
string s, t; int pos[N];
void build()
{
    pos[0] = -1;
    int pre = -1, cur = 0;
    while (cur < t.length())
    {
        while (pre >= 0 && t[cur] != t[pre])
        {
            pre = pos[pre];
        }
        pos[++cur] = ++pre;
    }
}

int main()
{
    cin >> s; cin >> t;
    build();
    int cur = 0;
    for (int i = 0; i < (int)s.length(); ++i)
    {
        while (cur >= 0 && s[i] != t[cur])
        {
            cur = pos[cur];
        }
    }
}
```

```
    }
    ++cur;
    if (cur == (int)t.length())
    {
        cout << i - (int)t.length() + 2 << ' ';
        cur = pos[cur];
    }
}

return 0;
}
```

## 4.6 Hash 2D

$$H[i][j] = H[i-1][j] * p + H[i][j-1] * q - H[i-1][j-1] * p * q + s[i][j] \quad (1)$$

$$Hash(a, b)(x, y) = H[x][y] - H[a-1][y] * p^{x-a+1} - H[x][b-1] * q^{y-b+1} + H[a-1][b-1] * p^{x-a+1} * q^{y-b+1} \quad (2)$$

## 5 Math

### 5.1 Derivatives and integrals

$$\begin{aligned} \frac{d}{dx} \ln u &= \frac{u'}{u} & \frac{d}{dx} \frac{1}{u} &= -\frac{u'}{u^2} \\ \frac{d}{dx} \sqrt{u} &= \frac{u'}{2\sqrt{u}} \\ \frac{d}{dx} \sin x &= \cos x & \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \cos x &= -\sin x & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1+x^2} \\ \int \tan ax &= -\frac{\ln|\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

## 5.2 Sum

$$\begin{aligned} 1 + 2 + 3 + \dots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \dots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \dots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \end{aligned}$$

## 5.3 Series

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty) \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1) \\ \sqrt{1+x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty) \end{aligned}$$

## 5.4 Trigonometric

$$\begin{aligned} \sin(v+w) &= \sin v \cos w + \cos v \sin w \\ \cos(v+w) &= \cos v \cos w - \sin v \sin w \\ \tan(v+w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2} \\ a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where  $r = \sqrt{a^2 + b^2}$ ,  $\phi = \text{atan2}(b, a)$ .

## 5.5 Number Theory

$$\begin{aligned} a + b &= a \oplus b + 2 \times (a \wedge b) \\ (a \div b) \% c &= a \times b^{c-2} \end{aligned}$$

## 5.6 Gaussian elimination

```
// Gauss-Jordan elimination.
// Returns: number of solution (0, 1 or INF)
//   When the system has at least one solution, ans will
//   contains
//   one possible solution
// Possible improvement when having precision errors:
//   - Divide i-th row by a(i, i)
//   - Choosing pivoting row with min absolute value (
//   sometimes this is better than maximum, as implemented
//   here)
// Tested:
// - https://open.kattis.com/problems/equationsolver
// - https://open.kattis.com/problems/equationsolverplus
```

```

int gauss (vector < vector<double> > a, vector<double> &
ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }

    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    // If we need any solution (in case INF solutions), we
    // should be
    // ok at this point.
    // If need to solve partially (get which values are fixed
    // /INF value):
    // for (int i=0; i<m; ++i)
    //     if (where[i] != -1) {

```

```

//         REP(j,n) if (j != i && fabs(a[where[i]][j]) > EPS)
//         {
//             where[i] = -1;
//             break;
//         }
//     }
// Then the variables which has where[i] == -1 --> INF
// values

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

## 5.7 Geometry

```

struct line
{
    double a,b,c;
    line() {}
    line(double A,double B,double C):a(A),b(B),c(C){}
    line(Point A,Point B)
    {
        a=A.y-B.y; b=B.x-A.x; c=-a*A.x-b*A.y;
    }
};

Point intersect(line AB,line CD)
{
    AB.c=-AB.c; CD.c=-CD.c;
    double D=CROSS(AB.a,AB.b,CD.a,CD.b);
    double Dx=CROSS(AB.c,AB.b,CD.c,CD.b);
    double Dy=CROSS(AB.a,AB.c,CD.a,CD.c);
    if (D==0.0) return Point(1e9,1e9);
    else return Point(Dx/D,Dy/D);
}

```

## 5.8 Miller Rabin

```

// n < 4,759,123,141          3 : 2, 7, 61
// n < 1,122,004,669,633      4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383      6 : pirmses <= 13
// n < 2^64                   7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022

```

```
// Make sure testing integer is in range [2, n 2 ] if
// you want to use magic.
long long power(long long x, long long p, long long mod) {
    long long s = 1, m = x;
    while (p) {
        if (p & 1) s = mult(s, m, mod);
        p >>= 1;
        m = mult(m, m, mod);
    }
    return s;
}

bool witness(long long a, long long n, long long u, int t)
{
    long long x = power(a, u, n);
    for (int i = 0; i < t; i++) {
        long long nx = mult(x, x, n);
        if (nx == 1 && x != 1 && x != n - 1) return 1;
        x = nx;
    }
    return x != 1;
}

bool miller_rabin(long long n, int s = 100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if (n < 2) return 0;
    if (!(n & 1)) return n == 2;
    long long u = n - 1;
    int t = 0;
    // n-1 = u*2^t
    while (!(u & 1)) {
        u >>= 1;
        t++;
    }
    while (s--) {
        long long a = randll() % (n - 1) + 1;
        if (witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

## 5.9 Chinese Remainer

```
// Solve linear congruences equation:
// -  $a[i] * x = b[i] \text{ MOD } m[i]$  (mi don't need to be co-prime)
)
```

```
// Tested:
// - https://open.kattis.com/problems/generalchineseremainder
bool linearCongruences(const vector<ll> &a, const vector<ll>
    > &b,
    const vector<ll> &m, ll &x, ll &M) {
    ll n = a.size();
    x = 0; M = 1;
    REP(i, n) {
        ll a_ = a[i] * M, b_ = b[i] - a[i] * x, m_ = m[i];
        ll y, t, g = extgcd(a_, m_, y, t);
        if (b_ % g) return false;
        b_ /= g; m_ /= g;
        x += M * (y * b_ % m_);
        M *= m_;
    }
    x = (x + M) % M;
    return true;
}
```

## 5.10 Extended Euclid

```
// other pairs are of the form:
//  $x' = x + k(b / \text{gcd})$ 
//  $y' = y - k(a / \text{gcd})$ 
// where k is an arbitrary integer.
// to minimize, set k to 2 closest integers near  $-x / (b / \text{gcd})$ 
// the algo always produce one of 2 small pairs.
int extgcd(int a, int b, int &x, int &y) {
    int g = a; x = 1; y = 0;
    if (b != 0) g = extgcd(b, a % b, y, x), y -= (a / b) * x;
    return g;
}
```

## 5.11 FFT

```
namespace FFT
{
    struct cd
    {
        double real, img;
        cd(double x = 0, double y = 0) : real(x), img(y) {}
        cd operator+(const cd& src) { return cd(real + src.
            real, img + src.img); }
    }
```

```

    cd operator-(const cd& src) { return cd(real - src.
real, img - src.img); }
    cd operator*(const cd& src) { return cd(real * src.
real - img * src.img, real * src.img + src.real * img);
}
};
cd conj(const cd& x) { return cd(x.real, -x.img); }
const int MaxN = 1 << 15;
const double PI = acos(-1);
cd w[MaxN]; int rev[MaxN];

void initFFT()
{
    for (int i = 0; i < MaxN; ++i)
        w[i] = cd(cos(2 * PI * i / MaxN), sin(2 * PI *
i / MaxN));
}
void FFT(vector<cd>& a)
{
    int n = a.size();
    for (int i = 0; i < n; ++i)
        if (rev[i] < i) swap(a[i], a[rev[i]]);

    for (int len = 2; len <= n; len <<= 1)
        for (int i = 0; i < n; i += len)
            for (int j = 0; j < (len >> 1); ++j)
            {
                cd u = a[i + j], v = a[i + j + (len >>
1)] * w[MaxN / len * j];
                a[i + j] = u + v;
                a[i + j + (len >> 1)] = u - v;
            }
}
void calcRev(int n)
{
    rev[0] = 0;
    for (int i = 1; i < n; ++i)
        if (i & 1) rev[i] = rev[i - 1] + (n >> 1);
        else rev[i] = rev[i >> 1] >> 1;
}
vector<long long> polymul(const vector<int>& a, const
vector<int>& b)
{
    int n = a.size() + b.size() - 1;
    if (__builtin_popcount(n) != 1) n = 1 << (32 -

```

```

__builtin_clz(n));

    vector<cd> pa(a.begin(), a.end()); pa.resize(n);
    vector<cd> pb(b.begin(), b.end()); pb.resize(n);

    calcRev(n); // Doesn't need to call multiple times

    FFT(pa); FFT(pb);
    for (int i = 0; i < n; ++i) pa[i] = conj(pa[i] * pb
[i]);
    FFT(pa);
    //output of pa will be conj of the real answer
    vector<long long> res(n);
    for (int i = 0; i < n; ++i) res[i] = llround(pa[i].
real / n);
    return res;
}
};

```

## 5.12 PollardRho

```

// does not work when n is prime
long long modit(long long x, long long mod) {
    if (x >= mod) x -= mod;
    //if(x<0) x+=mod;
    return x;
}
long long mult(long long x, long long y, long long mod) {
    long long s = 0, m = x % mod;
    while (y) {
        if (y & 1) s = modit(s + m, mod);
        y >>= 1;
        m = modit(m + m, mod);
    }
    return s;
}
long long f(long long x, long long mod) {
    return modit(mult(x, x, mod) + 1, mod);
}
long long pollard_rho(long long n) {
    if (!(n & 1)) return 2;
    while (true) {
        long long y = 2, x = random() % (n - 1) + 1, res = 1;
        for (int sz = 2; res == 1; sz *= 2) {
            for (int i = 0; i < sz && res <= 1; i++) {

```

```

        x = f(x, n);
        res = __gcd(abs(x - y), n);
    }
    y = x;
}
if (res != 0 && res != n) return res;
}
}

```

## 6 Theorem

### 6.1 Fermat's little theorem

If  $p$  is a prime number, then for any number  $a$ ,  $a^p - a$  is an integer multiple of  $p$

$$a^p \equiv a \pmod{p}$$

If  $a$  is not divisible by  $p$

$$a^{p-1} \equiv 1 \pmod{p}$$

### 6.2 Euler's totient function

The number of coprime  $\leq n$

$$\phi(n) = n \prod \left(1 - \frac{1}{p}\right)$$

With  $p$  is the prime divided by  $n$

### 6.3 Dirichlet

Given  $n$  holes and  $n + 1$  pigeons to distribute evenly, then at least 1 hole must have 2 pigeons

### 6.4 Pythagorean triple

$$a = m^2 - n^2, b = 2mn, c = m^2 + n^2$$

where  $m$  and  $n$  are positive integer with  $m > n$ , and with  $m$  and  $n$  are coprime and not both odd.

## 6.5 Legendre's formula

Factor  $n!$

$$v_p(n!) = \sum_{i=1}^{\infty} \left\lfloor \frac{n}{p^i} \right\rfloor$$

With  $p$  is prime

## 7 Other

### 7.1 Bignum mul

```

string mul(string a, string b)
{
    int m=a.length(),n=b.length(),sum=0;
    string c="";
    for (int i=m+n-1; i>=0; i--)
    {
        for (int j=0; j<m; j++) if (i-j>0 && i-j<=n) sum+=(a[j]
        ]-'0')*(b[i-j-1]-'0');
        c=(char)(sum%10+'0')+c;
        sum/=10;
    }
    while (c.length()>1 && c[0]=='0') c.erase(0,1);
    return c;
}

```

### 7.2 Random

```

// Random using mt19937
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
count());

// For random long long
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch
().count());

// Random shuffle using mt19937 as the generator
shuffle(a.begin(), a.end(), rng);

```

```
// Random range
int random(int a, int b)
{
    return uniform_int_distribution<int>(a, b)(rng);
}
```

### 7.3 Builtin bit function

```
__builtin_popcount(x); // number of bit 1 in x
__builtin_popcountll(x); // for long long
__builtin_clz(x); // number of leading 0
__builtin_clzll(x); // for long long
__builtin_ctz(x); // number of trailing 0
__builtin_ctzll(x); // for long long
```

```
(x & ~x) : the smallest bit 1 in x
floor(log2(x)) : 31 - __builtin_clz(x | 1);
floor(log2(x)) : 63 - __builtin_clzll(x | 1);
```

### 7.4 Pythagorean triples

c under 100 there are 16 triples: (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25) (20, 21, 29) (12, 35, 37) (9, 40, 41) (28, 45, 53) (11, 60, 61) (16, 63, 65) (33, 56, 65) (48, 55, 73) (13, 84, 85) (36, 77, 85) (39, 80, 89) (65, 72, 97)

$100 \leq c \leq 300$ : (20, 99, 101) (60, 91, 109) (15, 112, 113) (44, 117, 125) (88, 105, 137) (17, 144, 145) (24, 143, 145) (51, 140, 149) (85, 132, 157) (119, 120, 169) (52, 165, 173) (19, 180, 181) (57, 176, 185) (104, 153, 185) (95, 168, 193) (28, 195, 197) (84, 187, 205) (133, 156, 205) (21, 220, 221) (140, 171, 221) (60, 221, 229) (105, 208, 233) (120, 209, 241) (32, 255, 257) (23, 264, 265) (96, 247, 265) (69, 260, 269) (115, 252, 277) (160, 231, 281) (161, 240, 289) (68, 285, 293)

### 7.5 Sieve

```
for (int j = i; j * i != lim; ++j) notPrime[j * i] = true
```

### 7.6 Catalan

$$\frac{(2n)!}{(n+1)!n!} = \prod_{k=2}^n \frac{n+k}{k}$$

### 7.7 Prime under 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

### 7.8 Pascal triangle

$C(n,k)$ =number from line 0, column 0

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
```

### 7.9 Fibo

0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181 6765

## 8 Tips

- Test kĩ trc khi np. Code nhìn úng cha chc úng âu
- Test conner case
- Có overflow ko?
- c kĩ mô t test
- Gi s nó là s nguyên t i. Gi s nó liên quan ti s nguyên t i.
- Gi s nó là s có dng  $2^n$  i.
- Gi s chn ti a là 2, 3 s gì là có áp án i.
- Có liên quan gì ti Fibonacci hay tam giác pascal?

- Dãy này n iu không em ei? Hay tng ca 2,3 s fibonacci?
  - $q \leq 2$
  - Sort li i, bit âu thy iu hay hn?
  - Chia nh ra xem.
- B ht nhng thng ko cn thit ra
  - Áp i data struct nào y vô
  - Random shuffe AC
  - Xoay mng 45

**Keep Smiling**  
Gotta solve them all