Mục lục

1 Some definition

```
#include <bits/stdc++.h>
#include <random>
#include <chrono>
#include <ctime>

#define cross(A, B) (A.x * B.y - A.y * B.x)
```

```
#define dot(A, B) (A.x * B.x + A.y * B.y)
#define ccw(A, B, C) (-(A.x * (C.y - B.y) + B.x * (A.y - C.
    y) + C.x * (B.y - A.y))) // positive when ccw
#define CROSS(a, b, c, d) (a * d - b * c)

#define LL(x) (x << 1)
#define RR(x) ((x << 1) + 1)

using namespace std;
const int N = 1000005;
const int M = 30000;

const int Bases = 2;
const long long base[] = {137, 37};
const long long mod = 1000000007LL;

long long addi(long long a, long long b, long long m = mod)
     { a += b; if (a < 0) a += m; if (a >= m) a -= m; return
     a; }
long long subt(long long a, long long b, long long m = mod)
     { a -= b; if (a < 0) a += m; if (a >= m) a -= m; return
     a; }
long long mult(long long a, long long b, long long m = mod)
     { return a * b % m; }
long long power(long long a, long long b, long long m = mod
    )
{
     long long tmp = 1;
     for (; b > 0; b >>= 1)
     {
          if (b & 1LL) tmp = mult(tmp, a, m);
          a = mult(a, a, m);
     }
     return tmp;
}
long long inv(long long a, long long m = mod) { return
    power(a, m - 2, m); }
```

2 Data structure

2.1 Mo's algorithm

$$O(N * \sqrt{N} + Q * \sqrt{N})$$

```
S = sqrt(N);
bool cmp(Query A, Query B) // compare 2 queries
{
  if (A.l / S != B.l / S) {
    return A.l / S < B.l / S;
  }
  return A.r < B.r;
}
```

2.2 Set and map

Use set.lower_bound() instead of lower_bound(set.begin(), set.end()) for better performance
The same is true for map

2.3 BIT

```
void update(int x, int val)
{
  for (; x <= n; x += x & ~x) BIT[x] = min(BIT[x], val);
}

int get(int x)
{
  int res = 1e9;
  for (; x > 0; x -= x & ~x) res = min(res, BIT[x]);
  return res;
}
```

2.4 IT2D

```
int Max[4096][4096];

struct dir {
  int ll, rr, id;
  dir (int L, int R, int X)
    { ll=L, rr=R, id=X; }
  dir left() const
    { return dir(ll, (ll+rr)/2, id*2); }
  dir right() const
    { return dir((ll+rr)/2+1, rr, id*2+1); }
  inline bool irrelevant(int L, int R) const
    { return ll>R || L>rr || L>R; }
```

```
};

void maximize(int &a, int b)
  { a=max(a, b); }

void maximize(const dir &dx, const dir &dy, int x, int y,
    int k, bool only_y) {
  if (dx.irrelevant(x, x) || dy.irrelevant(y, y)) return;
  maximize(Max[dx.id][dy.id], k);
  if (!only_y && dx.ll != dx.rr) {
    maximize(dx.left(), dy, x, y, k, false);
    maximize(dx.right(), dy, x, y, k, false);
  }
  if (dy.ll != dy.rr) {
    maximize(dx, dy.left(), x, y, k, true);
    maximize(dx, dy.right(), x, y, k, true);
  }
}

int max_range(const dir &dx, const dir &dy, int lx, int rx,
     int ly, int ry) {
  if (dx.irrelevant(lx, rx) || dy.irrelevant(ly, ry))
    return 0;
  if (lx<=dx.ll && dx.rr<=rx) {
    if (ly<=dy.ll && dy.rr<=ry) return Max[dx.id][dy.id];
    int Max1 = max_range(dx, dy.left(), lx, rx, ly, ry);
    int Max2 = max_range(dx, dy.right(), lx, rx, ly, ry);
    return max(Max1, Max2);
  } else {
    int Max1 = max_range(dx.left(), dy, lx, rx, ly, ry);
    int Max2 = max_range(dx.right(), dy, lx, rx, ly, ry);
    return max(Max1, Max2);
  }
}
```

3 Graph

3.1 Dinic

```
namespace Dinic // really fast, O(n^2 m) or O(sqrt(n)m) if
   bipartite
{
    vector<int> adj[N];
    long long c[N][N], f[N][N];
    int s = 0, t = 0, d[N], ptr[N];
```



```
    bool BFS()
    {
        queue<int> q;
        memset(d, -1, sizeof(d));
        d[s] = 0; q.push(s);
        while (!q.empty())
        {
            int u = q.front(); q.pop();
            for (int v : adj[u])
            {
                if (d[v] == -1 && c[u][v] > f[u][v])
                {
                    d[v] = d[u] + 1;
                    q.push(v);
                }
            }
        }
        return d[t] != -1;
    }
    long long DFS(int x, long long delta)
    {
        if (x == t) return delta;
        for (; ptr[x] < adj[x].size(); ++ptr[x]) // Skip
the used edge
        {
            int y = adj[x][ptr[x]];
            if (d[y] == d[x] + 1 && c[x][y] > f[x][y])
            {
                long long push = DFS(y, min(delta, c[x][y]
- f[x][y]));
                if (push)
                {
                    f[x][y] += push;
                    f[y][x] -= push;
                    return push;
                }
            }
        }
        return 0;
    }
    long long maxFlow(int x, int y) // From x to y
    {
        long long flow = 0;
        s = x; t = y;
        while (BFS())
```

```
        {
            memset(ptr, 0, sizeof(ptr));
            while (long long tmp = DFS(s, 1e9))
                flow += 1LL * tmp;
        }
        return flow;
    }
};
```

3.2 Mincost

```
int calc(int x, int y) { return (x >= 0) ? y : 0 - y; }

bool findpath()
{
  for (int i = 1; i <= n; i++) { trace[i] = 0; d[i] = inf;
   }
  q.push(n); d[n] = 0;
  while (!q.empty())
  {
    int u = q.front();
    q.pop();
    inq[u] = false;
    for (int i = 0; i < adj[u].size(); i++)
    {
      int v = adj[u][i];
      if (c[u][v] > f[u][v] && d[v] > d[u] + calc(f[u][v],
   cost[u][v]))
      {
        trace[v] = u;
        d[v] = d[u] + calc(f[u][v], cost[u][v]);
        if (!inq[v])
        {
          inq[v] = true;
          q.push(v);
        }
      }
    }
  }
  return d[t] != inf;
}

void incflow()
{
  int v = t, delta = inf;
  while (v != n)
  {
    int u = trace[v];
    if (f[u][v] >= 0)
      delta = min(delta, c[u][v] - f[u][v]);
    else
      delta = min(delta, 0 - f[u][v]);
    v = u;
  }
  v = t;
  while (v != n)
  {
    int u = trace[v];
    f[u][v] += delta;
    f[v][u] -= delta;
    v = u;
  }
}
```

3.3 HLD

```
void DFS(int x,int pa)
{
  DD[x]=DD[pa]+1; child[x]=1; int Max=0;
  for (int i=0; i<DSK[x].size(); i++)
  {
    int y=DSK[x][i].fi;
    if (y==pa) continue;
    p[y]=x;
    d[y]=d[x]+DSK[x][i].se;
    DFS(y,x);
    child[x]+=child[y];
    if (child[y]>Max)
    {
      Max=child[y];
      tree[x]=tree[y];
    }
  }
  if (child[x]==1) tree[x]=++nTree;
}

void init()
{
  nTree=0;
  DFS(1,1);
```

```
  DD[0]=long(1e9);
  for (int i=1; i<=n; i++) if (DD[i]<DD[root[tree[i]]])
   root[tree[i]]=i;
}

int LCA(int u,int v)
{
  while (tree[u]!=tree[v])
  {
    if (DD[root[tree[u]]]<DD[root[tree[v]]]) v=p[root[tree[
   v]]];
    else u=p[root[tree[u]]];
  }
  if (DD[u]<DD[v]) return u; else return v;
}
```

3.4 Tarjan

If u is articulation:
if (low[v] >= num[u]) arti[u] = arti[u] or p[u] != -1 or child[u] >= 2;
If (u, v) is bridge: $low[v] >= num[v]$

3.5 Monotone chain

```
void convex_hull (vector<pt> & a) {
  if (a.size() == 1) { // Only 1 point
    return;
  }

  // Sort with respect to x and then y
  sort(a.begin(), a.end(), &cmp);

  pt p1 = a[0],  p2 = a.back();

  vector<pt> up, down;
  up.push_back (p1);
  down.push_back (p1);

  for (size_t i=1; i<a.size(); ++i) {
    // Add to the upper chain

    if (i==a.size()-1 || cw (p1, a[i], p2)) {
      while (up.size()>=2 && !cw (up[up.size()-2], up[up.
   size()-1], a[i]))
        up.pop_back();
      up.push_back (a[i]);
    }

    // Add to the lower chain
    if (i==a.size()-1 || ccw (p1, a[i], p2)) {
      while (down.size()>=2 && !ccw (down[down.size()-2],
   down[down.size()-1], a[i]))
        down.pop_back();
      down.push_back (a[i]);
    }
  }

  // Merge 2 chains
  a.clear();
  for (size_t i=0; i<up.size(); ++i)
    a.push_back (up[i]);
  for (size_t i=down.size()-2; i>0; --i)
    a.push_back (down[i]);
}
```

3.6 MST

Prim: remember to have visited array

3.7 HopcroftKarp

```
namespace HopcroftKarp // O(sqrt(n) * m)
{
    vector<int> adj[N]; int match[N], d[N];
    bool BFS()
    {
        queue<int> q;
        memset(d, -1, sizeof(d));
        for (int i = 1; i <= n; ++i) if (!match[i])
        {
            d[i] = 0;
            q.push(i);
        }
        bool flag = false;
        while (!q.empty())
        {
            int u = q.front(); q.pop();
            for (int v : adj[u])
            {
```

```
                if (match[v] == 0)
                {
                    flag = true;
                    continue;
                }
                if (d[match[v]] == -1)
                {
                    d[match[v]] = d[u] + 1;
                    q.push(match[v]);
                }
            }
        }
        return flag;
    }
    bool DFS(int x)
    {
        for (int y : adj[x])
        {
            if (match[y] == 0 || (d[match[y]] == d[x] + 1
    && DFS(match[y])))
            {
                match[y] = x;
                match[x] = y;
                return true;
            }
        }
        d[x] = -1;
        return false;
    }
    long long maxMatching() // From x to y
    {
        long long matching = 0;
        while (BFS())
        {
            for (int i = 1; i <= n; ++i) if (!match[i] &&
    DFS(i))
                ++matching;
        }
        return matching;
    }
};
```

3.8 Hungarian

```
struct Hungarian {
long  c[N][N], fx[N], fy[N], d[N];
int mx[N], my[N], trace[N], arg[N];
queue<int> q;
int start, finish, n, m;
const long inf = 1e18;

void Init(int _n, int _m) {
  n = _n, m = _m;
  FOR(i, 1, n) {
    mx[i] = my[i] = 0;
    FOR(j, 1, n) c[i][j] = inf;
  }
}
void addEdge(int u, int v, long cost) { c[u][v] = min(c[u
 ][v], cost); }
inline long getC(int u, int v) { return c[u][v] - fx[u] -
  fy[v]; }

void initBFS() {
  while (!q.empty()) q.pop();
  q.push(start);
  FOR(i, 0, n) trace[i] = 0;
  FOR(v, 1, n) {
    d[v] = getC(start, v), arg[v] = start;
  }
  finish = 0;
}

void findAugPath() {
  while (!q.empty()) {
    int u = q.front();
    q.pop();
    FOR(v, 1, n) if (!trace[v]) {
      long w = getC(u, v);
      if (!w) {
        trace[v] = u;
        if (!my[v]) { finish = v; return; }
        q.push(my[v]);
      }
      if (d[v] > w) { d[v] = w; arg[v] = u; }
    }
  }
}

void subX_addY(){
```

```
    long delta = inf;
    FOR(v, 1, n) if (trace[v] == 0 && d[v] < delta) delta =
    d[v];
    fx[start] += delta;
    FOR(v, 1, n) if (trace[v]) {
      int u = my[v];
      fy[v] -= delta, fx[u] += delta;
    } else d[v] -= delta;

    FOR(v, 1, n) if (!trace[v] && !d[v]) {
      trace[v] = arg[v];
      if (!my[v]) { finish = v; return; }
      q.push(my[v]);
    }
  }

  void Enlarge() {
    do {
      int u = trace[finish], nxt = mx[u];
      mx[u] = finish, my[finish] = u, finish = nxt;
    } while (finish);
  }

  long minCost() {
    FOR(u, 1, n) {
      fx[u] = c[u][1];
      FOR(v, 1, n) fx[u] = min(fx[u], c[u][v]);
    }
    FOR(v, 1, n) {
      fy[v] = c[1][v] - fx[1];
      FOR(u, 1, n) fy[v] = min(fy[v], c[u][v] - fx[u]);
    }

    FOR(u, 1, n) {
      start = u;
      initBFS();
      while (finish == 0) {
        findAugPath();
        if (!finish) subX_addY();
      }
      Enlarge();
    }

    int res = 0;
    FOR(i, 1, n) res += c[i][mx[i]];
    return res;
  }
};
```

4 String

4.1 Aho Corasick

```
struct Node
{
  int nxt[26], go[26];
  bool leaf;
  long long val, sumVal;
  int p;
  int pch;
  int link;
};

Node t[N];
int sz;

void New(Node &x, int p, int link, int pch)
{
  x.p = p;
  x.link = link;
  x.pch = pch;
  x.val = 0;
  x.sumVal = -1;
  memset(x.nxt, -1, sizeof(x.nxt));
  memset(x.go, -1, sizeof(x.go));
}

void AddString(const string &s, int val)
{
  int v = 0;
  for (char c : s)
  {
    int id = c - 'A';
    if (t[v].nxt[id] == -1)
    {
      New(t[sz], v, -1, id);
      t[v].nxt[id] = sz++;
    }
    v = t[v].nxt[id];
  }
```

```
    t[v].leaf = true;
    t[v].val = val;
}

int Go(int u, int c);

int Link(int u)
{
    if (t[u].link == -1)
    {
        if (u == 0 || t[u].p == 0)
            t[u].link = 0;
        else
            t[u].link = Go(Link(t[u].p), t[u].pch);
    }
    return t[u].link;
}

int Go(int u, int c)
{
    if (t[u].go[c] == -1)
    {
        if (t[u].nxt[c] != -1)
            t[u].go[c] = t[u].nxt[c];
        else
            t[u].go[c] = (u == 0 ? 0 : Go(Link(u), c));
    }
    return t[u].go[c];
}
```

4.2 Manacher

```
void init() {
    cnt = 0;
    t[0] = '~';
    for (int i = 0; i<n; i++) {
        t[++cnt] = '#';t[++cnt] = s[i];
    }
    t[++cnt] = '#'; t[++cnt] = '-';
}

void manacher() {
    int n = cnt - 2;
    int r = 1; int C = 1;
    int ans = 0;
    for (int i = 2; i<n; i++) {
        int i_mirror = C * 2 - i;
        z[i] = (r > i) ? min(z[i_mirror], r - i) : 0;
        while (t[i + z[i] + 1] == t[i - z[i] - 1]) z[i]++;
        if (i + z[i] > r) {
            C = i;
            r = i + z[i];
        }
    }
}
```

4.3 Suffix Array

```
struct SuffixArray {
    string s;
    int n;
    vector<int> SA, RA, tempSA, tempRA, LCP;
    int L[N];

    void reset(string st) {
        s = st;
        RA.clear();
        s.push_back('$');
        n = s.size();
        RA.resize(n + 1, 0);
        SA = RA, tempSA = tempRA = LCP = RA;
    }

    void BuildSA() {
        REP(i, n) SA[i] = i, RA[i] = s[i];
        for (int k = 1; k < n; k <<= 1) {
            radix_sort(k);
            radix_sort(0);
            tempRA[SA[0]] = 0;
            for (int i = 1, r = 0; i < n; ++i) {
                if (getRA(SA[i - 1]) != getRA(SA[i]) || getRA(SA[i
    - 1] + k) != getRA(SA[i] + k)) ++r;
                tempRA[SA[i]] = r;
            }
            REP(i, n) RA[i] = tempRA[i];
            if (RA[SA[n - 1]] == n - 1) break;
        }
    }

    void BuildLCP() {
```

```
    // kasai
    REP(i, n) RA[SA[i]] = i;
    int k = 0;
    REP(i, n) {
      if (RA[i] == n - 1) {
        k = 0; continue;
      }
      int j = SA[RA[i] + 1];
      while (i + k < n && j + k < n && s[i + k] == s[j + k
   ]) ++k;
      LCP[RA[i]] = k;
      if (k) k--;
    }
  }
private:
  inline int getRA(int i) { return (i < n ? RA[i] : 0); }
  void radix_sort(int k) {
    memset(L, 0, sizeof L);
    REP(i, n) L[getRA(i + k)]++;
    int p = 0;
    REP(i, N) {
      int x = L[i];
      L[i] = p;
      p += x;
    }
    REP(i, n) {
      int &x = L[getRA(SA[i] + k)];
      tempSA[x++] = SA[i];
    }
    REP(i, n) SA[i] = tempSA[i];
  }
};
```

4.4 Z function

```
vector<int> Zfunc(int n, vector<int> &a) {
  vector<int> z(n);
  z[0] = n;
  int l = 0, r = 0;
  FOR(i, 1, n - 1) {
    z[i] = (i <= r ? min(r - i + 1, z[i - l]) : 0);
    while (i + z[i] < n && a[z[i]] == a[i + z[i]]) ++z[i];
    if (i + z[i] > r) {
      r = i + z[i] - 1;
      l = i;
    }
  }
  return z;
}
```

4.5 KMP

```
// SUBSTR spoj
string s, t;int pos[N];
void build()
{
  pos[0] = -1;
  int pre = -1, cur = 0;
  while (cur < t.length())
  {
    while (pre >= 0 && t[cur] != t[pre])
    {
      pre = pos[pre];
    }
    pos[++cur] = ++pre;
  }
}

int main()
{
  cin >> s; cin >> t;
  build();
  int cur = 0;
  for (int i = 0; i < (int)s.length(); ++i)
  {
    while (cur >= 0 && s[i] != t[cur])
    {
      cur = pos[cur];
    }
    ++cur;
    if (cur == (int)t.length())
    {
      cout << i - (int)t.length() + 2 << ' ';
      cur = pos[cur];
    }
  }

  return 0;
}
```

4.6 Hash

```
long long POW[Bases][N];

struct Hash
{
  long long a[Bases];
  Hash operator+(const Hash& src)
  {
    Hash tmp;
    for (int i = 0; i < Bases; ++i) tmp.a[i] = addi(a[i], src.a[i]);
    return tmp;
  }
  Hash operator-(const Hash& src)
  {
    Hash tmp;
    for (int i = 0; i < Bases; ++i) tmp.a[i] = subt(a[i], src.a[i]);
    return tmp;
  }
  Hash operator*(int x)
  {
    Hash tmp;
    for (int i = 0; i < Bases; ++i) tmp.a[i] = mult(a[i], POW[i][x]);
    return tmp;
  }
  Hash operator+(char c)
  {
    Hash tmp;
    for (int i = 0; i < Bases; ++i) tmp.a[i] = addi(a[i], c);
    return tmp;
  }
  bool operator==(const Hash& src)
  {
    for (int i = 0; i < Bases; ++i) if (a[i] != src.a[i]) return false;
    return true;
  }
};

bool operator<(const Hash& a, const Hash& b)
{
  for (int i = 0; i < Bases; ++i)
    if (a.a[i] < b.a[i]) return true;
    else if (a.a[i] > b.a[i]) return false;
    return false;
}

Hash hash1[N], hash2[N];
void initHash(int n)
{
  for (int j = 0; j < Bases; ++j) POW[j][0] = 1;
  for (int j = 0; j < Bases; ++j) for (int i = 1; i <= n; ++i) POW[j][i] = mult(POW[j][i - 1], base[j]);
}

void calcHash(int n)
{
  for (int j = 0; j < Bases; ++j) hash1[0].a[j] = 0;
  for (int i = 1; i <= n; ++i) hash1[i] = hash1[i - 1] * 1 + (s[i] - 'a');
}

void calcHashRev(int n)
{
  for (int j = 0; j < Bases; ++j) hash2[j].a[n + 1] = 0;
  for (int i = n; i >= 0; --i) hash2[i] = hash2[i + 1] * 1 + (s[i] - 'a');
}

Hash getHash(int l, int r) { return hash1[r] - hash1[l - 1] * (r - l + 1); }
Hash getHashRev(int l, int r) { return hash2[l] - hash2[r + 1] * (r - l + 1); }
```

4.7 Hash 2D

$$H[i][j] = H[i-1][j] * p + H[i][j-1] * q - H[i-1][j-1] * p * q + s[i][j] \quad (1)$$

$$Hash(a,b)(x,y) = H[x][y] - H[a-1][y] * p^{x-a+1} - H[x][b-1] * q^{y-b+1} + H[a-1][b-1] * p^{x-a+1} * q^{y-b+1} \quad (2)$$

10

5 Math

5.1 Invert of 2x2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

5.2 Derivatives and integrals

$$\frac{d}{dx}\ln u = \frac{u'}{u} \qquad \frac{d}{dx}\frac{1}{u} = -\frac{u'}{u^2}$$

$$\frac{d}{dx}\sqrt{u} = \frac{u'}{2\sqrt{u}}$$

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\mathrm{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

5.3 Sum

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3+2^3+3^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4+2^4+3^4+\cdots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

5.4 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \ (-1 < x \le 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, \ (-1 \le x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, \ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, \ (-\infty < x < \infty)$$

5.5 Trigonometric

$$\sin(v+w) = \sin v\cos w + \cos v\sin w$$

$$\cos(v+w) = \cos v\cos w - \sin v\sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v\tan w}$$

$$\sin v + \sin w = 2\sin\frac{v+w}{2}\cos\frac{v-w}{2}$$

$$\cos v + \cos w = 2\cos\frac{v+w}{2}\cos\frac{v-w}{2}$$

$$a\cos x + b\sin x = r\cos(x-\phi)$$
$$a\sin x + b\cos x = r\sin(x+\phi)$$

where $r = \sqrt{a^2+b^2}, \phi = \text{atan2}(b,a)$.

5.6 Number Theory

$$a + b = a \oplus b + 2 \times (a \wedge b)$$
$$(a \div b)\%c = a \times b^{c-2}$$

5.7 Gaussian elimination

```
// Gauss-Jordan elimination.
// Returns: number of solution (0, 1 or INF)
//   When the system has at least one solution, ans will
    contains
//   one possible solution
// Possible improvement when having precision errors:
//   - Divide i-th row by a(i, i)
//   - Choosing pivoting row with min absolute value (
    sometimes this is better that maximum, as implemented
    here)
// Tested:
// - https://open.kattis.com/problems/equationsolver
// - https://open.kattis.com/problems/equationsolverplus
int gauss (vector < vector<double> > a, vector<double> &
    ans) {
  int n = (int) a.size();
  int m = (int) a[0].size() - 1;

  vector<int> where (m, -1);
  for (int col=0, row=0; col<m && row<n; ++col) {
    int sel = row;
    for (int i=row; i<n; ++i)
      if (abs (a[i][col]) > abs (a[sel][col]))
        sel = i;
    if (abs (a[sel][col]) < EPS)
      continue;
    for (int i=col; i<=m; ++i)
      swap (a[sel][i], a[row][i]);
    where[col] = row;

    for (int i=0; i<n; ++i)
      if (i != row) {
        double c = a[i][col] / a[row][col];
        for (int j=col; j<=m; ++j)
          a[i][j] -= a[row][j] * c;
      }
    ++row;
  }

  ans.assign (m, 0);
  for (int i=0; i<m; ++i)
    if (where[i] != -1)
      ans[i] = a[where[i]][m] / a[where[i]][i];
  for (int i=0; i<n; ++i) {
    double sum = 0;
    for (int j=0; j<m; ++j)
      sum += ans[j] * a[i][j];
    if (abs (sum - a[i][m]) > EPS)
      return 0;
  }

  // If we need any solution (in case INF solutions), we
    should be
  // ok at this point.
  // If need to solve partially (get which values are fixed
    /INF value):
//  for (int i=0; i<m; ++i)
//    if (where[i] != -1) {
//      REP(j,n) if (j != i && fabs(a[where[i]][j]) > EPS)
    {
//        where[i] = -1;
//        break;
//      }
//    }
  // Then the variables which has where[i] == -1 --> INF
    values

  for (int i=0; i<m; ++i)
    if (where[i] == -1)
      return INF;
  return 1;
}
```

5.8 Geometry

```
struct line
{
  double a,b,c;
  line() {}
  line(double A,double B,double C):a(A),b(B),c(C){}
  line(Point A,Point B)
  {
    a=A.y-B.y; b=B.x-A.x; c=-a*A.x-b*A.y;
  }
};

Point intersect(line AB,line CD)
{
  AB.c=-AB.c; CD.c=-CD.c;
  double D=CROSS(AB.a,AB.b,CD.a,CD.b);
  double Dx=CROSS(AB.c,AB.b,CD.c,CD.b);
  double Dy=CROSS(AB.a,AB.c,CD.a,CD.c);
  if (D==0.0) return Point(1e9,1e9);
  else return Point(Dx/D,Dy/D);
}
```

5.9 Miller Rabin

```
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383        6 :  pirmes <= 13
// n < 2^64                     7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2,  n 2 ] if
// you want to use magic.
long long power(long long x, long long p, long long mod) {
  long long s = 1, m = x;
  while (p) {
    if (p & 1) s = mult(s, m, mod);
    p >>= 1;
    m = mult(m, m, mod);
  }
  return s;
}
bool witness(long long a, long long n, long long u, int t)
   {
  long long x = power(a, u, n);
  for (int i = 0; i < t; i++) {
    long long nx = mult(x, x, n);
    if (nx == 1 && x != 1 && x != n - 1) return 1;
    x = nx;
  }
  return x != 1;
}
bool miller_rabin(long long n, int s = 100) {
  // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if (n < 2) return 0;
  if (!(n & 1)) return n == 2;
  long long u = n - 1;
  int t = 0;
  // n-1 = u*2^t
  while (!(u & 1)) {
    u >>= 1;
    t++;
  }
  while (s--) {
    long long a = randll() % (n - 1) + 1;
    if (witness(a, n, u, t)) return 0;
  }
  return 1;
}
```

5.10 Chinese Remainer

```
// Solve linear congruences equation:
// - a[i] * x = b[i] MOD m[i] (mi don't need to be co-prime
   )
// Tested:
// - https://open.kattis.com/problems/
   generalchineseremainder
bool linearCongruences(const vector<ll> &a, const vector<ll
   > &b,
    const vector<ll> &m, ll &x, ll &M) {
  ll n = a.size();
  x = 0; M = 1;
  REP(i, n) {
    ll a_ = a[i] * M, b_ = b[i] - a[i] * x, m_ = m[i];
    ll y, t, g = extgcd(a_, m_, y, t);
    if (b_ % g) return false;
    b_ /= g; m_ /= g;
    x += M * (y * b_ % m_);
    M *= m_;
```

```
  }
  x = (x + M) % M;
  return true;
}
```

5.11 Extended Euclid

```
// other pairs are of the form:
// x' = x + k(b / gcd)
// y' = y - k(a / gcd)
// where k is an arbitrary integer.
// to minimize, set k to 2 closest integers near -x / (b /
   gcd)
// the algo always produce one of 2 small pairs.
int extgcd(int a, int b, int &x, int &y) {
  int g = a; x = 1; y = 0;
  if (b != 0) g = extgcd(b, a % b, y, x), y -= (a / b) * x;
  return g;
}
```

5.12 FFT

```
namespace FFT
{
  struct cd
  {
    double real, img;
    cd(double x = 0, double y = 0) : real(x), img(y) {}
    cd operator+(const cd& src) { return cd(real + src.real
    , img + src.img); }
    cd operator-(const cd& src) { return cd(real - src.real
    , img - src.img); }
    cd operator*(const cd& src) { return cd(real * src.real
     - img * src.img, real * src.img + src.real * img); }
  };
  cd conj(const cd& x) { return cd(x.real, -x.img); }
  const int MaxN = 1 << 15;
  const double PI = acos(-1);
  cd w[MaxN]; int rev[MaxN];

  void initFFT()
  {
    for (int i = 0; i < MaxN; ++i)
      w[i] = cd(cos(2 * PI * i / MaxN), sin(2 * PI * i /
    MaxN));
  }
  void FFT(vector<cd>& a)
  {
    int n = a.size();
    for (int i = 0; i < n; ++i)
      if (rev[i] < i) swap(a[i], a[rev[i]]);

    for (int len = 2; len <= n; len <<= 1)
      for (int i = 0; i < n; i += len)
        for (int j = 0; j < (len >> 1); ++j)
        {
          cd u = a[i + j], v = a[i + j + (len >> 1)] * w[
   MaxN / len * j];
          a[i + j] = u + v;
          a[i + j + (len >> 1)] = u - v;
        }
  }
  void calcRev(int n)
  {
    rev[0] = 0;
    for (int i = 1; i < n; ++i)
      if (i & 1) rev[i] = rev[i - 1] + (n >> 1);
      else rev[i] = rev[i >> 1] >> 1;
  }
  vector<long long> polymul(const vector<int>& a, const
   vector<int>& b)
  {
    int n = a.size() + b.size() - 1;
    if (__builtin_popcount(n) != 1) n = 1 << (32 -
   __builtin_clz(n));

    vector<cd> pa(a.begin(), a.end()); pa.resize(n);
    vector<cd> pb(b.begin(), b.end()); pb.resize(n);

    calcRev(n); // Doesn't need to call multiple times

    FFT(pa); FFT(pb);
    for (int i = 0; i < n; ++i) pa[i] = conj(pa[i] * pb[i])
   ;
    FFT(pa);
    //output of pa will be conj of the real answer
    vector<long long> res(n);
    for (int i = 0; i < n; ++i) res[i] = llround(pa[i].real
    / n);
    return res;
```

```
  }
};
```

5.13 PollardRho

```
// does not work when n is prime
long long modit(long long x, long long mod) {
  if (x >= mod) x -= mod;
  //if(x<0) x+=mod;
  return x;
}
long long mult(long long x, long long y, long long mod) {
  long long s = 0, m = x % mod;
  while (y) {
    if (y & 1) s = modit(s + m, mod);
    y >>= 1;
    m = modit(m + m, mod);
  }
  return s;
}
long long f(long long x, long long mod) {
  return modit(mult(x, x, mod) + 1, mod);
}
long long pollard_rho(long long n) {
  if (!(n & 1)) return 2;
  while (true) {
    long long y = 2, x = random() % (n - 1) + 1, res = 1;
    for (int sz = 2; res == 1; sz *= 2) {
      for (int i = 0; i < sz && res <= 1; i++) {
        x = f(x, n);
        res = __gcd(abs(x - y), n);
      }
      y = x;
    }
    if (res != 0 && res != n) return res;
  }
}
```

6 Theorem

6.1 Fermat's little theorem

If p is a prime number, then for any number a, $a^p - a$ is an integer multiple of p

$$a^p \equiv a \ (mod\ p)$$

If a is not divisible by p

$$a^{p-1} \equiv 1 \ (mod\ p)$$

6.2 Euler's theorem

If a and n are coprime, then

$$a^{\phi(n)} \equiv 1 \ (mod\ n)$$

6.3 Euler's totient function

The number of coprime $\leq n$

$$\phi(n) = n \prod (1 - \frac{1}{p})$$

With p is the prime divided by n

6.4 Goldbach's conjecture

Every even number greater than 2 is the sum of 2 primes. $\leq 4 * 10^{18}$

6.5 Dirichlet

Given n holes and $n+1$ pigeons to distribute evenly, then at least 1 hole must have 2 pigeons

6.6 Pythagorean triple

$$a = m^2 - n^2,\ b = 2mn,\ c = m^2 + n^2$$

where m and n are positive integer with $m > n$, and with m and n are coprime and not both odd.

15

6.7 Legendre's formula

Factor $n!$

$$v_p(n!) = \sum_{i=1}^{\infty} \left\lfloor \frac{n}{p^i} \right\rfloor$$

With p is prime

6.8 Stirling's approximation

$$n! \approx \sqrt{2\pi n} * (\frac{n}{e})^n$$

7 Other

7.1 Matrix

```
struct matrix
{
  static const int MATRIX_SIZE = 2;
  long long a[MATRIX_SIZE][MATRIX_SIZE];
  matrix()
  {
    for (int i = 0; i < MATRIX_SIZE; ++i)
      for (int j = 0; j < MATRIX_SIZE; ++j)
        a[i][j] = 0;
  }
  matrix(bool x) : matrix()
  {
    for (int i = 0; i < MATRIX_SIZE; ++i) a[i][i] = 1;
  }
};

matrix matmul(const matrix& a, const matrix& b, long long m = mod)
{
  int n = a.MATRIX_SIZE;
  matrix res;
  for (int ii = 0; ii < n; ++ii) for (int jj = 0; jj < n; ++jj)
  {
    res.a[ii][jj] = 0;
    for (int kk = 0; kk < n; ++kk)
      res.a[ii][jj] = addi(res.a[ii][jj], mult(a.a[ii][kk], b.a[kk][jj], m), m);
  }
  return res;
}

matrix matpow(const matrix& a, long long n, long long m = mod)
{
  if (n == 0) return matrix(true);
  matrix tmp = matpow(a, n >> 1, m);
  return (n & 1) ? matmul(matmul(tmp, tmp, m), a, m) : matmul(tmp, tmp, m);
}
```

7.2 Bignum mul

```
string mul(string a,string b)
{
  int m=a.length(),n=b.length(),sum=0;
  string c="";
  for (int i=m+n-1; i>=0; i--)
  {
    for (int j=0; j<m; j++) if (i-j>0 && i-j<=n) sum+=(a[j]-'0')*(b[i-j-1]-'0');
    c=(char)(sum%10+'0')+c;
    sum/=10;
  }
  while (c.length()>1 && c[0]=='0') c.erase(0,1);
  return c;
}
```

7.3 Random

```
// Random using mt19937
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

// For random long long
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());

// Random shuffle using mt19937 as the generator
shuffle(a.begin(), a.end(), rng);
```

```
// Random range
int random(int a, int b)
{
  return uniform_int_distribution<int>(a, b)(rng);
}
```

7.4 Builtin bit function

```
__builtin_popcount(x); // number of bit 1 in x
__builtin_popcountll(x); // for long long
__builtin_clz(x); // number of leading 0
__builtin_clzll(x); // for long long
__builtin_ctz(x); // number of trailing 0
__builtin_ctzll(x); // for long long

(x & ~x) : the smallest bit 1 in x
floor(log2(x)) : 31 - __builtin_clz(x | 1);
floor(log2(x)) : 63 - __builtin_clzll(x | 1);
```

7.5 Pythagorean triples

c under 100 there are 16 triples: (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25) (20, 21, 29) (12, 35, 37) (9, 40, 41) (28, 45, 53) (11, 60, 61) (16, 63, 65) (33, 56, 65) (48, 55, 73) (13, 84, 85) (36, 77, 85) (39, 80, 89) (65, 72, 97)

$100 \le c \le 300$: (20, 99, 101) (60, 91, 109) (15, 112, 113) (44, 117, 125) (88, 105, 137) (17, 144, 145) (24, 143, 145) (51, 140, 149) (85, 132, 157) (119, 120, 169) (52, 165, 173) (19, 180, 181) (57, 176, 185) (104, 153, 185) (95, 168, 193) (28, 195, 197) (84, 187, 205) (133, 156, 205) (21, 220, 221) (140, 171, 221) (60, 221, 229) (105, 208, 233) (120, 209, 241) (32, 255, 257) (23, 264, 265) (96, 247, 265) (69, 260, 269) (115, 252, 277) (160, 231, 281) (161, 240, 289) (68, 285, 293)

7.6 Sieve

```
// faster for > 1e6
void sieve_new()
{
  for (int i = 2; i <= 1000000; ++i)
  {
    if (!notPrime[i]) prime.push_back(i);
    for (int j = 0; i * prime[j] <= 1000000 && j < prime.
   size(); ++j) {
      notPrime[i * prime[j]] = true;
      if (i % prime[j] == 0) break;
    }
  }
}
//
void sieve_old()
{
  for (long long i = 2; i <= 1000000; ++i)
  if (!notPrime[i]) {
    prime.push_back(i);
    for (long long j = i; j * i <= 1000000; ++j)
      notPrime[i * j] = true;
  }
}
```

7.7 Catalan

$$\frac{(2n)!}{(n+1)!\,n!} = \prod_{k=2}^{n} \frac{n+k}{k}$$

7.8 Prime under 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

7.9 Pascal triangle

C(n,k)=number from line 0, column 0

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

7.10 Fibo

0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181 6765

8 Tips

- Giả sử nó là số nguyên tố đi. Giả sử nó liên quan tới số nguyên tố đi.
- Giả sử nó là số có dạng 2^n đi.
- Giả sử chọn tối đa là 2, 3 số gì là có đáp án đi.
- Có liên quan gì tới Fibonacci hay tam giác pascal?
- Dãy này đơn điệu không em ei? Hay tổng của 2, 3 số fibonacci?
- Chia nhỏ ra xem.
- Random shuffe để AC
- Xoay mảng 45 độ
- Tạo đỉnh ảo cho đồ thị (vd như Kruskal)
- Tìm t thỏa điều kiện nào đó thì chặt
- Merge set thì phải merge từ set nhỏ sang lớn ko thì TLE
- Xử lý ma trận cũng giống xử lý số bình thường, các phép nhân chia mod đều như cũ