ACM Notebook team HCMUS-KMN

KMN

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1 Derivatives and integrals

$$\frac{d}{dx}\ln u = \frac{u'}{u} \qquad \frac{d}{dx}\frac{1}{u} = -\frac{u'}{u^2}$$

$$\frac{d}{dx}\sqrt{u} = \frac{u'}{2\sqrt{u}}$$

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2 Sum

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3+2^3+3^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4+2^4+3^4+\cdots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

3 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots,\ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots,\ (-1 < x \le 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots,\ (-1 \le x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots,\ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots,\ (-\infty < x < \infty)$$

4 Trigonometric

$$
\begin{aligned}
\sin(v+w) &= \sin v \cos w + \cos v \sin w \\
\cos(v+w) &= \cos v \cos w - \sin v \sin w \\
\tan(v+w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\
\sin v + \sin w &= 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2} \\
\cos v + \cos w &= 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}
\end{aligned}
$$

$$
\begin{aligned}
a \cos x + b \sin x &= r \cos(x - \phi) \\
a \sin x + b \cos x &= r \sin(x + \phi)
\end{aligned}
$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

5 Some define

```cpp
#include <bits/stdc++.h>

#define maxN
#define matrix_size 2
#define base 1000000007LL
#define eps 1e-8

#define cross(A,B) (A.x*B.y-A.y*B.x)
#define dot(A,B) (A.x*B.x+A.y*B.y)
#define ccw(A,B,C) (-(A.x*(C.y-B.y) + B.x*(A.y-C.y) + C.x*(B.y-A.y))) // positive
     when ccw
#define CROSS(a,b,c,d) (a*d - b*c)
```

6 Geometry

```cpp
struct line
{
    double a,b,c;
    line() {}
    line(double A,double B,double C):a(A),b(B),c(C){}
    line(Point A,Point B)
    {
        a=A.y-B.y; b=B.x-A.x; c=-a*A.x-b*A.y;
    }
};

Point intersect(line AB,line CD)
{
    AB.c=-AB.c; CD.c=-CD.c;
    double D=CROSS(AB.a,AB.b,CD.a,CD.b);
    double Dx=CROSS(AB.c,AB.b,CD.c,CD.b);
    double Dy=CROSS(AB.a,AB.c,CD.a,CD.c);
    if (D==0.0) return Point(1e9,1e9);
    else return Point(Dx/D,Dy/D);
}
```

7 Dinic

```
bool BFS()
{
    queue<int> q;
    for (int i=1; i<=n; i++) d[i]=0,Free[i]=true;
    q.push(s);
    d[s]=1;
    while (!q.empty())
    {
        int u=q.front(); q.pop();
        for (int i=0; i<DSK[u].size(); i++)
        {
            int v=DSK[u][i].fi;
            if (d[v]==0 && DSK[u][i].se>f[u][v])
            {
                d[v]=d[u]+1;
                q.push(v);
            }
        }
    }
    return d[t]!=0;
}

int DFS(int x,int delta)
{
    if (x==t) return delta;
    Free[x]=false;
    for (int i=0; i<DSK[x].size(); i++)
    {
        int y=DSK[x][i].fi;
        if (d[y]==d[x]+1 && f[x][y]<DSK[x][i].se && Free[y])
        {
            int tmp=DFS(y,min(delta,DSK[x][i].se-f[x][y]));
            if (tmp>0)
            {
                f[x][y]+=tmp; f[y][x]-=tmp; return tmp;
            }
        }
    }
    return 0;
}
```

8 Mincost

```
int calc(int x,int y){ return (x>=0) ? y : 0-y; }

bool findpath()
{
    for (int i=1; i<=n; i++){ trace[i]=0; d[i]=inf; } q.push(n); d[n]=0;
    while (!q.empty())
    {
        int u=q.front(); q.pop(); inq[u]=false;
        for (int i=0; i<DSK[u].size(); i++)
        {
            int v=DSK[u][i];
            if (c[u][v]>f[u][v] && d[v]>d[u]+calc(f[u][v],cost[u][v]))
            {
                trace[v]=u;
                d[v]=d[u]+calc(f[u][v],cost[u][v]);
                if (!inq[v])
                {
                 inq[v]=true;
                    q.push(v);
                }
            }
        }
    }
    return d[t]!=inf;
}

void incflow()
{
    int v=t,delta=inf;
    while (v!=n)
    {
        int u=trace[v];
        if (f[u][v]>=0) delta=min(delta,c[u][v]-f[u][v]);
        else delta=min(delta,0-f[u][v]);
        v=u;
    }
    v=t;
    while (v!=n)
    {
        int u=trace[v];
        f[u][v]+=delta; f[v][u]-=delta;
        v=u;
    }
}
```

9 HLD

```
void DFS(int x,int pa)
{
    DD[x]=DD[pa]+1; child[x]=1; int Max=0;
    for (int i=0; i<DSK[x].size(); i++)
    {
        int y=DSK[x][i].fi;
        if (y==pa) continue;
        p[y]=x;
        d[y]=d[x]+DSK[x][i].se;
        DFS(y,x);
        child[x]+=child[y];
        if (child[y]>Max)
        {
            Max=child[y];
            tree[x]=tree[y];
        }
    }
    if (child[x]==1) tree[x]=++nTree;
}

void init()
{
    nTree=0;
    DFS(1,1);
    DD[0]=long(1e9);
    for (int i=1; i<=n; i++) if (DD[i]<DD[root[tree[i]]]) root[tree[i]]=i;
}

int LCA(int u,int v)
{
    while (tree[u]!=tree[v])
    {
        if (DD[root[tree[u]]]<DD[root[tree[v]]]) v=p[root[tree[v]]];
        else u=p[root[tree[u]]];
    }
    if (DD[u]<DD[v]) return u; else return v;
}
```

10 Cầu khớp

Nút u là khớp: if (low[v] >= num[u]) arti[u] = arti[u] || p[u] != -1 || child[u] >= 2;
Cạnh u, v là cầu khi low[v] >= num[v]

11 Monotone chain

```
void convex_hull (vector<pt> & a) {
  if (a.size() == 1) { // ỉch có 1 để im
    return;
  }

  // Sort with respect to x and then y
  sort(a.begin(), a.end(), &cmp);

  pt p1 = a[0],  p2 = a.back();

  vector<pt> up, down;
  up.push_back (p1);
  down.push_back (p1);

  for (size_t i=1; i<a.size(); ++i) {
    // Add to the upper chain

    if (i==a.size()-1 || cw (p1, a[i], p2)) {
      while (up.size()>=2 && !cw (up[up.size()-2], up[up.size()-1], a[i]))
        up.pop_back();
      up.push_back (a[i]);
    }

    // Add to the lower chain
    if (i==a.size()-1 || ccw (p1, a[i], p2)) {
      while (down.size()>=2 && !ccw (down[down.size()-2], down[down.size()-1], a[i]))
        down.pop_back();
      down.push_back (a[i]);
    }
  }

  // Merge 2 chains
  a.clear();
  for (size_t i=0; i<up.size(); ++i)
    a.push_back (up[i]);
  for (size_t i=down.size()-2; i>0; --i)
    a.push_back (down[i]);
}
```

12 MST

Prim: remember to have visited array

13 Bignum mul

```cpp
string mul(string a,string b)
{
    int m=a.length(),n=b.length(),sum=0;
    string c="";
    for (int i=m+n-1; i>=0; i--)
    {
        for (int j=0; j<m; j++) if (i-j>0 && i-j<=n) sum+=(a[j]-'0')*(b[i-j-1]-'0'
    );
        c=(char)(sum%10+'0')+c;
        sum/=10;
    }
    while (c.length()>1 && c[0]=='0') c.erase(0,1);
    return c;
}
```

14 Aho Corasick

```cpp
struct Node {
    int nxt[26], go[26];
    bool leaf;
    long long val, sumVal;
    int p;
    int pch;
    int link;
};

Node t[N];
int sz;

void New(Node &x, int p, int link, int pch) {
    x.p = p;
    x.link = link;
    x.pch = pch;
    x.val = 0;
    x.sumVal = -1;
    memset(x.nxt, -1, sizeof (x.nxt));
    memset(x.go, -1, sizeof (x.go));
}

void AddString(const string &s, int val) {
    int v = 0;
    for (char c : s) {
        int id = c - 'A';
        if (t[v].nxt[id] == -1) {
            New(t[sz], v, -1, id);
            t[v].nxt[id] = sz++;
        }
        v = t[v].nxt[id];
    }
    t[v].leaf = true;
    t[v].val = val;
```

```
}

int Go(int u, int c);

int Link(int u) {
    if (t[u].link == -1) {
        if (u == 0 || t[u].p == 0) t[u].link = 0;
        else t[u].link = Go(Link(t[u].p), t[u].pch);
    }
    return t[u].link;
}

int Go(int u, int c) {
    if (t[u].go[c] == -1) {
        if (t[u].nxt[c] != -1) t[u].go[c] = t[u].nxt[c];
        else t[u].go[c] = (u == 0 ? 0 : Go(Link(u), c));
    }
    return t[u].go[c];
```

15 Manacher

```
void init() {
    cnt = 0;
    t[0] = '~';
    for (int i = 0; i<n; i++) {
        t[++cnt] = '#';t[++cnt] = s[i];
    }
    t[++cnt] = '#'; t[++cnt] = '-';
}

void manacher() {
    int n = cnt - 2;
    int r = 1; int C = 1;
    int ans = 0;
    for (int i = 2; i<n; i++) {
        int i_mirror = C * 2 - i;
        z[i] = (r > i) ? min(z[i_mirror], r - i) : 0;
        while (t[i + z[i] + 1] == t[i - z[i] - 1]) z[i]++;
        if (i + z[i] > r) {
            C = i;
            r = i + z[i];
        }
    }
}
```

16 Miller Rabin

```
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383          6 :  pirmes <= 13
// n < 2^64                       7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, -n2] if
// you want to use magic.
long long power(long long x, long long p, long long mod) {
    long long s = 1, m = x;
    while (p) {
        if (p & 1) s = mult(s, m, mod);
        p >>= 1;
        m = mult(m, m, mod);
    }
    return s;
}
bool witness(long long a, long long n, long long u, int t) {
    long long x = power(a, u, n);
    for (int i = 0; i < t; i++) {
        long long nx = mult(x, x, n);
        if (nx == 1 && x != 1 && x != n - 1) return 1;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(long long n, int s = 100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if (n < 2) return 0;
    if (!(n & 1)) return n == 2;
    long long u = n - 1;
    int t = 0;
    // n-1 = u*2^t
    while (!(u & 1)) {
        u >>= 1;
        t++;
    }
    while (s--) {
        long long a = randll() % (n - 1) + 1;
        if (witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

17 Chinese Remainer

```
// Solve linear congruences equation:
// - a[i] * x = b[i] MOD m[i] (mi don't need to be co-prime)
// Tested:
// - https://open.kattis.com/problems/generalchineseremainder
bool linearCongruences(const vector<ll> &a, const vector<ll> &b,
        const vector<ll> &m, ll &x, ll &M) {
    ll n = a.size();
    x = 0; M = 1;
    REP(i, n) {
        ll a_ = a[i] * M, b_ = b[i] - a[i] * x, m_ = m[i];
        ll y, t, g = extgcd(a_, m_, y, t);
        if (b_ % g) return false;
        b_ /= g; m_ /= g;
        x += M * (y * b_ % m_);
        M *= m_;
    }
    x = (x + M) % M;
    return true;
}
```

18 Extended Euclid

```
// other pairs are of the form:
```

```
// x' = x + k(b / gcd)
// y' = y - k(a / gcd)
// where k is an arbitrary integer.
// to minimize, set k to 2 closest integers near -x / (b / gcd)
// the algo always produce one of 2 small pairs.
int extgcd(int a, int b, int &x, int &y) {
    int g = a; x = 1; y = 0;
    if (b != 0) g = extgcd(b, a % b, y, x), y -= (a / b) * x;
    return g;
}
```

19 FFT

```
typedef complex<double> ComplexType;

const double PI = acos(-1);
const ComplexType I(0.0, 1.0);
// ceil(log2(n)) + 1
const int MAX2N = (1 << 15);

ComplexType root_unity[MAX2N + 1];

// DONT FORGET TO CALL INIT!
void init_fft() {
    for (int i = 0; i <= MAX2N; ++i)
        root_unity[i] = exp(2 * PI * i / MAX2N * -I);
}

void fft(vector<ComplexType>& a, const vector<int>& p) {
    int n = a.size();
    vector<ComplexType> b(n);
    for (int i = 0; i < n; ++i)
        b[i] = a[p[i]];
    copy(b.begin(), b.end(), a.begin());
    for (int m = 1, t = MAX2N / 2; m < n; m *= 2, t /= 2)
        for (int i = 0; i < n; i += m * 2)
            for (int j = 0; j < m; ++j) {
                int u = i + j, v = i + j + m;
                a[v] *= root_unity[j * t];
                ComplexType tmp = a[u] - a[v];
                a[u] += a[v];
                a[v] = tmp;
            }
}

vector<long long> polymul(const vector<int>& a, const vector<int>& b) {
    int n = max(a.size(), b.size());
    if (__builtin_popcount(n) != 1) n = 1 << (32 - __builtin_clz(n));
    n *= 2;
    vector<ComplexType> pa(n), pb(n);
    copy(a.begin(), a.end(), pa.begin());
    copy(b.begin(), b.end(), pb.begin());

    vector<int> p(n);
    for (int i = 1; i < n; ++i)
        if (i & 1) p[i] = p[i - 1] + n / 2;
        else p[i] = p[i / 2] / 2;

    fft(pa, p), fft(pb, p);
    transform(pa.begin(), pa.end(), pb.begin(), pa.begin(), multiplies<
   ComplexType>());
    // inverse FFT
    for_each(pa.begin(), pa.end(), [](ComplexType &c) { c = conj(c); });
    fft(pa, p);

    vector<long long> res(n);
    transform(pa.begin(), pa.end(), res.begin(), [&](auto c) { return lround(c.
   real() / n); });
    return res;
}
```

20 Hungarian

```
struct Hungarian {
    long  c[N][N], fx[N], fy[N], d[N];
    int mx[N], my[N], trace[N], arg[N];
    queue<int> q;
    int start, finish, n, m;
    const long inf = 1e18;

    void Init(int _n, int _m) {
        n = _n, m = _m;
        FOR(i, 1, n) {
            mx[i] = my[i] = 0;
            FOR(j, 1, n) c[i][j] = inf;
        }
    }
    void addEdge(int u, int v, long cost) { c[u][v] = min(c[u][v], cost); }
    inline long getC(int u, int v) { return c[u][v] - fx[u] - fy[v]; }

    void initBFS() {
        while (!q.empty()) q.pop();
        q.push(start);
        FOR(i, 0, n) trace[i] = 0;
        FOR(v, 1, n) {
            d[v] = getC(start, v), arg[v] = start;
        }
        finish = 0;
    }

    void findAugPath() {
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            FOR(v, 1, n) if (!trace[v]) {
                long w = getC(u, v);
                if (!w) {
                    trace[v] = u;
                    if (!my[v]) { finish = v; return; }
                    q.push(my[v]);
                }
                if (d[v] > w) { d[v] = w; arg[v] = u; }
            }
        }
    }

    void subX_addY(){
        long delta = inf;
        FOR(v, 1, n) if (trace[v] == 0 && d[v] < delta) delta = d[v];
        fx[start] += delta;
        FOR(v, 1, n) if (trace[v]) {
            int u = my[v];
            fy[v] -= delta, fx[u] += delta;
        } else d[v] -= delta;

        FOR(v, 1, n) if (!trace[v] && !d[v]) {
            trace[v] = arg[v];
            if (!my[v]) { finish = v; return; }
            q.push(my[v]);
        }
    }

    void Enlarge() {
        do {
            int u = trace[finish], nxt = mx[u];
            mx[u] = finish, my[finish] = u, finish = nxt;
        } while (finish);
    }

    long minCost() {
        FOR(u, 1, n) {
            fx[u] = c[u][1];
            FOR(v, 1, n) fx[u] = min(fx[u], c[u][v]);
        }
        FOR(v, 1, n) {
            fy[v] = c[1][v] - fx[1];
            FOR(u, 1, n) fy[v] = min(fy[v], c[u][v] - fx[u]);
        }

        FOR(u, 1, n) {
            start = u;
            initBFS();
            while (finish == 0) {
                findAugPath();
                if (!finish) subX_addY();
            }
            Enlarge();
        }

        int res = 0;
        FOR(i, 1, n) res += c[i][mx[i]];
        return res;
    }
};
```

21 PollardRho

```
// does not work when n is prime
long long modit(long long x, long long mod) {
    if (x >= mod) x -= mod;
    //if(x<0) x+=mod;
    return x;
}
long long mult(long long x, long long y, long long mod) {
    long long s = 0, m = x % mod;
    while (y) {
        if (y & 1) s = modit(s + m, mod);
```

```cpp
        y >>= 1;
        m = modit(m + m, mod);
    }
    return s;
}
long long f(long long x, long long mod) {
    return modit(mult(x, x, mod) + 1, mod);
}
long long pollard_rho(long long n) {
    if (!(n & 1)) return 2;
    while (true) {
        long long y = 2, x = random() % (n - 1) + 1, res = 1;
        for (int sz = 2; res == 1; sz *= 2) {
            for (int i = 0; i < sz && res <= 1; i++) {
                x = f(x, n);
                res = __gcd(abs(x - y), n);
            }
            y = x;
        }
        if (res != 0 && res != n) return res;
    }
}
```

22 Suffix Array

```cpp
struct SuffixArray {
    string s;
    int n;
    vector<int> SA, RA, tempSA, tempRA, LCP;
    int L[N];

    void reset(string st) {
        s = st;
        RA.clear();
        s.push_back('$');
        n = s.size();
        RA.resize(n + 1, 0);
        SA = RA, tempSA = tempRA = LCP = RA;
    }

    void BuildSA() {
        REP(i, n) SA[i] = i, RA[i] = s[i];
        for (int k = 1; k < n; k <<= 1) {
            radix_sort(k);
            radix_sort(0);
            tempRA[SA[0]] = 0;
            for (int i = 1, r = 0; i < n; ++i) {
                if (getRA(SA[i - 1]) != getRA(SA[i]) || getRA(SA[i - 1] + k) !=
    getRA(SA[i] + k)) ++r;
                tempRA[SA[i]] = r;
            }
            REP(i, n) RA[i] = tempRA[i];
            if (RA[SA[n - 1]] == n - 1) break;
        }
    }

    void BuildLCP() {
        // kasai
        REP(i, n) RA[SA[i]] = i;
        int k = 0;
        REP(i, n) {
            if (RA[i] == n - 1) {
                k = 0; continue;
            }
            int j = SA[RA[i] + 1];
            while (i + k < n && j + k < n && s[i + k] == s[j + k]) ++k;
            LCP[RA[i]] = k;
            if (k) k--;
        }
    }
private:
    inline int getRA(int i) { return (i < n ? RA[i] : 0); }
    void radix_sort(int k) {
        memset(L, 0, sizeof L);
        REP(i, n) L[getRA(i + k)]++;
        int p = 0;
        REP(i, N) {
            int x = L[i];
            L[i] = p;
            p += x;
        }
        REP(i, n) {
            int &x = L[getRA(SA[i] + k)];
            tempSA[x++] = SA[i];
        }
        REP(i, n) SA[i] = tempSA[i];
    }
};
```

23 Z function

```
vector<int> Zfunc(int n, vector<int> &a) {
    vector<int> z(n);
    z[0] = n;
    int l = 0, r = 0;
    FOR(i, 1, n - 1) {
        z[i] = (i <= r ? min(r - i + 1, z[i - l]) : 0);
        while (i + z[i] < n && a[z[i]] == a[i + z[i]]) ++z[i];
        if (i + z[i] > r) {
            r = i + z[i] - 1;
            l = i;
        }
    }
    return z;
}
```

24 Catalan

$\frac{(2n)!}{(n+1)!n!} = \prod_{k=2}^{n} \frac{n+k}{k}$

25 Sieve

for (int j = i; j * i <= lim; ++j) notPrime[j * i] = true

26 Prime under 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

27 Pascal triangle C(n,k)=number from line 0, column 0

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
```

28 Fibo

0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181 6765

29 Tips

1. Giả sử nó là số nguyên tố đi. Giả sử nó liên quan tới số nguyên tố đi.
2. Giả sử nó là số có dạng 2^n đi.
3. Giả sử chọn tối đa là 2, 3 số gì là có đáp án đi.
4. Có liên quan gì tới Fibonacci hay tam giác pascal?
5. Dãy này đơn điệu không em ei? Hay tổng của 2,3 số fibonacci?
6. q <= 2
7. Sort lại đi, biết đâu thấy điều hay hơn?
8. Chia nhỏ ra xem.
9. Bỏ hết những thằng ko cần thiết ra
10. Áp đại data struct nào đấy vô

11. khóc
12. Cầu nguyện
13. Random shuffe để ac
14. Xoay mảng 45 độ