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# 1 Some definition

```
#include <bits/stdc++.h>

#define N
#define matrix_size 2
#define mod 1000000007LL
#define eps 1e-8
#define base 137 // Or 37

#define cross(A,B) (A.x*B.y-A.y*B.x)
#define dot(A,B) (A.x*B.x+A.y*B.y)
#define ccw(A,B,C) (-(A.x*(C.y-B.y) + B.x*(A.y-C.y) + C.x*(B.y-A.y))) // positive when ccw
#define CROSS(a,b,c,d) (a*d - b*c)
```

# 2 Graph

#### 2.1 Dinic

```
bool BFS()
  queue < int > q;
  for (int i=1; i<=n; i++) d[i]=0,Free[i]=</pre>
   true;
  q.push(s);
  d[s]=1;
  while (!q.empty())
      int u=q.front(); q.pop();
      for (int i=0; i<DSK[u].size(); i++)</pre>
      {
           int v=DSK[u][i].fi;
           if (d[v] == 0 && DSK[u][i].se>f[u][v
   ])
           {
               d[v]=d[u]+1;
               q.push(v);
      }
  return d[t]!=0;
int DFS(int x,int delta)
  if (x==t) return delta;
  Free[x]=false;
  for (int i=0; i<DSK[x].size(); i++)</pre>
      int y=DSK[x][i].fi;
      if (d[y] == d[x] + 1 && f[x][y] < DSK[x][i].
   se && Free[y])
           int tmp=DFS(y,min(delta,DSK[x][i].
   se-f[x][y]));
           if (tmp>0)
```

```
f[x][y]+=tmp; f[y][x]-=tmp;
return tmp;
}
}
return 0;
}
```

#### 2.2 Mincost

```
int calc(int x,int y){ return (x>=0) ? y : 0-
   y; }
bool findpath()
  for (int i=1; i<=n; i++){ trace[i]=0; d[i]=</pre>
   inf; } q.push(n); d[n]=0;
  while (!q.empty())
    int u=q.front(); q.pop(); inq[u]=false;
    for (int i=0; i<DSK[u].size(); i++)</pre>
      int v=DSK[u][i];
      if (c[u][v]>f[u][v] && d[v]>d[u]+calc(f
   [u][v],cost[u][v]))
      {
        trace[v]=u;
        d[v]=d[u]+calc(f[u][v],cost[u][v]);
        if (!inq[v])
           inq[v]=true;
          q.push(v);
        }
      }
    }
  }
  return d[t]!=inf;
void incflow()
  int v=t,delta=inf;
  while (v!=n)
    int u=trace[v];
    if (f[u][v]>=0) delta=min(delta,c[u][v]-f
   [u][v]);
    else delta=min(delta,0-f[u][v]);
    v=u:
  }
  v=t;
  while (v!=n)
    int u=trace[v];
    f[u][v]+=delta; f[v][u]-=delta;
  }
```

## 2.3 HLD

```
void DFS(int x,int pa)
  DD[x]=DD[pa]+1; child[x]=1; int Max=0;
  for (int i=0; i<DSK[x].size(); i++)</pre>
    int y=DSK[x][i].fi;
    if (y==pa) continue;
    p[y]=x;
    d[y]=d[x]+DSK[x][i].se;
    DFS(y,x);
    child[x]+=child[y];
    if (child[y]>Max)
    {
      Max=child[y];
      tree[x]=tree[y];
    }
  }
  if (child[x]==1) tree[x]=++nTree;
void init()
  nTree=0;
  DFS(1,1);
  DD[0] = long(1e9);
  for (int i=1; i<=n; i++) if (DD[i]<DD[root[</pre>
   tree[i]]]) root[tree[i]]=i;
int LCA(int u,int v)
  while (tree[u]!=tree[v])
    if (DD[root[tree[u]]] < DD[root[tree[v]]])</pre>
   v=p[root[tree[v]]];
    else u=p[root[tree[u]]];
  if (DD[u]<DD[v]) return u; else return v;</pre>
```

# 2.4 Cầu khớp

Nút u là khớp: if (low[v] >= num[u]) arti[u] = arti[u] || p[u] != -1 || child[u] >= 2; Cạnh u, v là cầu khi low[v] >= num[v]

### 2.5 Monotone chain

```
void convex_hull (vector<pt> & a) {
  if (a.size() == 1) { // Only 1 point
    return;
}

// Sort with respect to x and then y
  sort(a.begin(), a.end(), &cmp);

pt p1 = a[0], p2 = a.back();

vector<pt> up, down;
  up.push_back (p1);
```

```
down.push_back (p1);
for (size_t i=1; i<a.size(); ++i) {</pre>
 // Add to the upper chain
 if (i==a.size()-1 || cw (p1, a[i], p2)) {
    while (up.size()>=2 && !cw (up[up.size
 ()-2], up[up.size()-1], a[i]))
      up.pop_back();
    up.push_back (a[i]);
 // Add to the lower chain
 if (i==a.size()-1 || ccw (p1, a[i], p2))
    while (down.size()>=2 && !ccw (down[
 down.size()-2], down[down.size()-1], a[i
      down.pop_back();
    down.push_back (a[i]);
 }
}
// Merge 2 chains
a.clear();
for (size_t i=0; i<up.size(); ++i)</pre>
  a.push_back (up[i]);
for (size_t i=down.size()-2; i>0; --i)
  a.push_back (down[i]);
```

#### 2.6 MST

Prim: remember to have visited array

# 3 String

#### 3.1 Aho Corasick

```
struct Node
 int nxt[26], go[26];
 bool leaf;
 long long val, sumVal;
 int p;
 int pch;
  int link;
};
Node t[N];
int sz;
void New(Node &x, int p, int link, int pch)
  x.p = p;
  x.link = link;
  x.pch = pch;
  x.val = 0;
  x.sumVal = -1;
```

```
memset(x.nxt, -1, sizeof(x.nxt));
  memset(x.go, -1, sizeof(x.go));
void AddString(const string &s, int val)
  int v = 0;
  for (char c : s)
    int id = c - 'A';
    if (t[v].nxt[id] == -1)
      New(t[sz], v, -1, id);
      t[v].nxt[id] = sz++;
    v = t[v].nxt[id];
 }
  t[v].leaf = true;
  t[v].val = val;
int Go(int u, int c);
int Link(int u)
 if (t[u].link == -1)
    if (u == 0 || t[u].p == 0)
      t[u].link = 0;
      t[u].link = Go(Link(t[u].p), t[u].pch);
 }
  return t[u].link;
int Go(int u, int c)
 if (t[u].go[c] == -1)
  {
    if (t[u].nxt[c] != -1)
      t[u].go[c] = t[u].nxt[c];
      t[u].go[c] = (u == 0 ? 0 : Go(Link(u),
   c));
 }
 return t[u].go[c];
```

### 3.2 Manacher

```
void init() {
  cnt = 0;
  t[0] = '~';
  for (int i = 0; i<n; i++) {
    t[++cnt] = '#';t[++cnt] = s[i];
  }
  t[++cnt] = '#'; t[++cnt] = '-';
}
void manacher() {
  int n = cnt - 2;</pre>
```

```
int r = 1; int C = 1;
int ans = 0;
for (int i = 2; i < n; i + +) {
   int i_mirror = C * 2 - i;
   z[i] = (r > i) ? min(z[i_mirror], r - i)
   : 0;
   while (t[i + z[i] + 1] == t[i - z[i] -
   1]) z[i] + +;
   if (i + z[i] > r) {
      C = i;
      r = i + z[i];
   }
}
```

# 3.3 Suffix Array

```
struct SuffixArray {
  string s;
  int n:
 vector<int> SA, RA, tempSA, tempRA, LCP;
 int L[N];
 void reset(string st) {
    s = st;
   RA.clear();
   s.push_back('$');
   n = s.size();
   RA.resize(n + 1, 0);
   SA = RA, tempSA = tempRA = LCP = RA;
 }
 void BuildSA() {
    REP(i, n) SA[i] = i, RA[i] = s[i];
    for (int k = 1; k < n; k <<= 1) {</pre>
     radix_sort(k);
      radix_sort(0);
      tempRA[SA[O]] = 0;
      for (int i = 1, r = 0; i < n; ++i) {</pre>
        if (getRA(SA[i - 1]) != getRA(SA[i])
   || getRA(SA[i - 1] + k) != getRA(SA[i] +
   k)) ++r;
        tempRA[SA[i]] = r;
      REP(i, n) RA[i] = tempRA[i];
      if (RA[SA[n-1]] == n-1) break;
 }
 void BuildLCP() {
    // kasai
    REP(i, n) RA[SA[i]] = i;
    int k = 0;
    REP(i, n) {
      if (RA[i] == n - 1) {
        k = 0; continue;
      int j = SA[RA[i] + 1];
      while (i + k < n \&\& j + k < n \&\& s[i + k])
   k] == s[j + k]) ++k;
      LCP[RA[i]] = k;
```

```
if (k) k--;
    }
  }
private:
  inline int getRA(int i) { return (i < n ?</pre>
   RA[i] : 0); }
  void radix_sort(int k) {
    memset(L, 0, sizeof L);
    REP(i, n) L[getRA(i + k)]++;
    int p = 0;
    REP(i, N) {
      int x = L[i];
      L[i] = p;
      p += x;
    REP(i, n) {
      int &x = L[getRA(SA[i] + k)];
      tempSA[x++] = SA[i];
    REP(i, n) SA[i] = tempSA[i];
  }
};
```

#### 3.4 Z function

```
vector<int> Zfunc(int n, vector<int> &a) {
  vector<int> z(n);
  z[0] = n;
  int l = 0, r = 0;
  FOR(i, 1, n - 1) {
    z[i] = (i <= r ? min(r - i + 1, z[i - 1])
        : 0);
    while (i + z[i] < n && a[z[i]] == a[i + z
      [i]]) ++z[i];
    if (i + z[i] > r) {
        r = i + z[i] - 1;
        l = i;
      }
  }
  return z;
}
```

#### 3.5 KMP

```
// SUBSTR spoj
string s, t;int pos[N];
void build()
{
   pos[0] = -1;
   int pre = -1, cur = 0;
   while (cur < t.length())
   {
      while (pre >= 0 && t[cur] != t[pre])
      {
        pre = pos[pre];
      }
      pos[++cur] = ++pre;
   }
}
int main()
```

```
{
  cin >> s; cin >> t;
  build();
  int cur = 0;
  for (int i = 0; i < (int)s.length(); ++i)
  {
    while (cur >= 0 && s[i] != t[cur])
    {
       cur = pos[cur];
    }
    ++cur;
    if (cur == (int)t.length())
    {
       cout << i - (int)t.length() + 2 << ' ';
       cur = pos[cur];
    }
}
return 0;
}</pre>
```

## 3.6 Hash 2D

$$H[i][j] = H[i-1][j] * p + H[i][j-1] * q - H[i-1][j-1] * p * q + s[i][j]$$
 (1)

$$Hash(a,b)(x,y) = H[x][y] - H[a-1][y] * p^{x-a+1}$$
$$- H[x][b-1] * q^{y-b+1}$$
$$+ H[a-1][b-1] * p^{x-a+1} * q^{y-b+1}$$
(2)

# 4 Math

# 4.1 Derivatives and integrals

$$\frac{d}{dx} \ln u = \frac{u'}{u} \quad \frac{d}{dx} \frac{1}{u} = -\frac{u'}{u^2}$$

$$\frac{d}{dx} \sqrt{u} = \frac{u'}{2\sqrt{u}}$$

$$\frac{d}{dx} \sin x = \cos x \quad \frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx} \cos x = -\sin x \quad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx} \tan x = 1 + \tan^2 x \quad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \quad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \quad \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

## 4.2 Sum

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^{2} + 2^{2} + 3^{2} + \dots + n^{2} = \frac{n(2n+1)(n+1)}{6}$$

$$1^{3} + 2^{3} + 3^{3} + \dots + n^{3} = \frac{n^{2}(n+1)^{2}}{4}$$

$$1^{4} + 2^{4} + 3^{4} + \dots + n^{4} = \frac{n(n+1)(2n+1)(3n^{2} + 3n - 1)}{30}$$

#### 4.3 Series

$$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^{2}}{2} + \frac{x^{3}}{3} - \frac{x^{4}}{4} + \dots, (-1 < x \le 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^{2}}{8} + \frac{2x^{3}}{32} - \frac{5x^{4}}{128} + \dots, (-1 \le x \le 1)$$

$$\sin x = x - \frac{x^{3}}{3!} + \frac{x^{5}}{5!} - \frac{x^{7}}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^{2}}{2!} + \frac{x^{4}}{4!} - \frac{x^{6}}{6!} + \dots, (-\infty < x < \infty)$$

# 4.4 Trigonometric

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$

$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2\sin \frac{v+w}{2}\cos \frac{v-w}{2}$$

$$\cos v + \cos w = 2\cos \frac{v+w}{2}\cos \frac{v-w}{2}$$

$$a\cos x + b\sin x = r\cos(x-\phi)$$

$$a\sin x + b\cos x = r\sin(x+\phi)$$

where 
$$r = \sqrt{a^2 + b^2}$$
,  $\phi = \operatorname{atan2}(b, a)$ .

## 4.5 Geometry

```
struct line
{
   double a,b,c;
   line() {}
   line(double A,double B,double C):a(A),b(B),
        c(C){}
   line(Point A,Point B)
   {
        a=A.y-B.y; b=B.x-A.x; c=-a*A.x-b*A.y;
   }
};

Point intersect(line AB,line CD)
{
   AB.c=-AB.c; CD.c=-CD.c;
   double D=CROSS(AB.a,AB.b,CD.a,CD.b);
   double Dx=CROSS(AB.c,AB.b,CD.c,CD.b);
   double Dy=CROSS(AB.a,AB.c,CD.a,CD.c);
   if (D==0.0) return Point(1e9,1e9);
   else return Point(Dx/D,Dy/D);
}
```

# 4.6 Miller Rabin

```
// n < 4,759,123,141
                            3: 2, 7, 61
// n < 1,122,004,669,633
                            4: 2, 13, 23,
   1662803
// n < 3,474,749,660,383
                                  6: pirmes
    <= 13
// n < 2^64
// 2, 325, 9375, 28178, 450775, 9780504,
   1795265022
// Make sure testing integer is in range [2,
   -n2] if
// you want to use magic.
long long power(long long x, long long p,
   long long mod) {
 long long s = 1, m = x;
 while (p) {
    if (p & 1) s = mult(s, m, mod);
   p >>= 1;
   m = mult(m, m, mod);
 return s;
bool witness(long long a, long long n, long
   long u, int t) {
 long long x = power(a, u, n);
 for (int i = 0; i < t; i++) {</pre>
   long long nx = mult(x, x, n);
   if (nx == 1 && x != 1 && x != n - 1)
  return 1;
    x = nx;
 }
 return x != 1;
bool miller_rabin(long long n, int s = 100) {
 // iterate s times of witness on n
 // return 1 if prime, 0 otherwise
```

```
if (n < 2) return 0;
if (!(n & 1)) return n == 2;
long long u = n - 1;
int t = 0;
// n-1 = u*2^t
while (!(u & 1)) {
  u >>= 1;
  t++;
}
while (s--) {
  long long a = randll() % (n - 1) + 1;
  if (witness(a, n, u, t)) return 0;
}
return 1;
}
```

## 4.7 Chinese Remainer

```
// Solve linear congruences equation:
// - a[i] * x = b[i] MOD m[i] (mi don't need
  to be co-prime)
// Tested:
// - https://open.kattis.com/problems/
   generalchineseremainder
bool linearCongruences(const vector<11> &a,
   const vector<ll> &b,
    const vector<11> &m, 11 &x, 11 &M) {
  ll n = a.size();
  x = 0; M = 1;
  REP(i, n) {
    ll a_{-} = a[i] * M, b_{-} = b[i] - a[i] * x,
   m_{-} = m[i];
   11 y, t, g = extgcd(a_, m_, y, t);
    if (b_ % g) return false;
    b_ /= g; m_ /= g;
    x += M * (y * b_  % m_);
    M *= m_{;}
 }
 x = (x + M) \% M;
 return true;
```

## 4.8 Extended Euclid

```
// other pairs are of the form:
// x' = x + k(b / gcd)
// y' = y - k(a / gcd)
// where k is an arbitrary integer.
// to minimize, set k to 2 closest integers
    near -x / (b / gcd)
// the algo always produce one of 2 small
    pairs.
int extgcd(int a, int b, int &x, int &y) {
    int g = a; x = 1; y = 0;
    if (b != 0) g = extgcd(b, a % b, y, x), y
        -= (a / b) * x;
    return g;
}
```

#### 4.9 FFT

```
typedef complex<double> ComplexType;
const double PI = acos(-1);
const ComplexType I(0.0, 1.0);
// \text{ceil}(\log 2(n)) + 1
const int MAX2N = (1 << 15);
ComplexType root_unity[MAX2N + 1];
// DONT FORGET TO CALL INIT!
void init_fft() {
  for (int i = 0; i <= MAX2N; ++i)</pre>
    root_unity[i] = exp(2 * PI * i / MAX2N *
}
void fft(vector < ComplexType > & a, const vector
   \langle int \rangle \& p)  {
  int n = a.size();
  vector < ComplexType > b(n);
  for (int i = 0; i < n; ++i)</pre>
    b[i] = a[p[i]];
  copy(b.begin(), b.end(), a.begin());
  for (int m = 1, t = MAX2N / 2; m < n; m *=</pre>
   2, t /= 2)
    for (int i = 0; i < n; i += m * 2)</pre>
      for (int j = 0; j < m; ++j) {
        int u = i + j, v = i + j + m;
        a[v] *= root_unity[j * t];
        ComplexType tmp = a[u] - a[v];
        a[u] += a[v];
        a[v] = tmp;
      }
vector<long long> polymul(const vector<int>&
   a, const vector < int > & b) {
  int n = max(a.size(), b.size());
  if (__builtin_popcount(n) != 1) n = 1 <<</pre>
   (32 - __builtin_clz(n));
  n *= 2;
  vector < ComplexType > pa(n), pb(n);
  copy(a.begin(), a.end(), pa.begin());
  copy(b.begin(), b.end(), pb.begin());
  vector<int> p(n);
  for (int i = 1; i < n; ++i)</pre>
    if (i & 1) p[i] = p[i - 1] + n / 2;
    else p[i] = p[i / 2] / 2;
  fft(pa, p), fft(pb, p);
  transform(pa.begin(), pa.end(), pb.begin(),
    pa.begin(), multiplies < ComplexType > ());
  // inverse FFT
  for_each(pa.begin(), pa.end(), [](
   ComplexType &c) { c = conj(c); });
  fft(pa, p);
  vector<long long> res(n);
  transform(pa.begin(), pa.end(), res.begin()
```

```
, [&](auto c) { return lround(c.real() /
    n); });
  return res;
}
```

## 4.10 Hungarian

```
struct Hungarian {
  long c[N][N], fx[N], fy[N], d[N];
  int mx[N], my[N], trace[N], arg[N];
  queue < int > q;
  int start, finish, n, m;
  const long inf = 1e18;
  void Init(int _n, int _m) {
    n = _n, m = _m;
    FOR(i, 1, n) {
      mx[i] = my[i] = 0;
      FOR(j, 1, n) c[i][j] = inf;
  }
  void addEdge(int u, int v, long cost) { c[u
   ][v] = min(c[u][v], cost); }
  inline long getC(int u, int v) { return c[u
   ][v] - fx[u] - fy[v]; }
  void initBFS() {
    while (!q.empty()) q.pop();
    q.push(start);
    FOR(i, 0, n) trace[i] = 0;
    FOR(v, 1, n) {
      d[v] = getC(start, v), arg[v] = start;
    finish = 0;
  void findAugPath() {
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      FOR(v, 1, n) if (!trace[v]) {
        long w = getC(u, v);
        if (!w) {
          trace[v] = u;
          if (!my[v]) { finish = v; return; }
          q.push(my[v]);
        }
       if (d[v] > w) \{ d[v] = w; arg[v] = u;
    }
      }
    }
  void subX_addY(){
    long delta = inf;
    FOR(v, 1, n) if (trace[v] == 0 \&\& d[v] <
   delta) delta = d[v];
    fx[start] += delta;
    FOR(v, 1, n) if (trace[v]) {
      int u = my[v];
      fy[v] -= delta, fx[u] += delta;
```

```
} else d[v] -= delta;
  FOR(v, 1, n) if (!trace[v] && !d[v]) {
    trace[v] = arg[v];
    if (!my[v]) { finish = v; return; }
    q.push(my[v]);
}
void Enlarge() {
    int u = trace[finish], nxt = mx[u];
    mx[u] = finish, my[finish] = u, finish
  } while (finish);
long minCost() {
 FOR(u, 1, n) {
    fx[u] = c[u][1];
    FOR(v, 1, n) fx[u] = min(fx[u], c[u][v])
 ]);
  FOR(v, 1, n) {
    fy[v] = c[1][v] - fx[1];
    FOR(u, 1, n) fy[v] = min(fy[v], c[u][v]
  - fx[u]);
  }
  FOR(u, 1, n) {
    start = u;
    initBFS();
    while (finish == 0) {
      findAugPath();
      if (!finish) subX_addY();
    }
    Enlarge();
  int res = 0;
  FOR(i, 1, n) res += c[i][mx[i]];
  return res;
}
```

#### 4.11 PollardRho

```
// does not work when n is prime
long long modit(long long x, long long mod) {
  if (x >= mod) x -= mod;
  //if(x<0) x+=mod;
  return x;
}
long long mult(long long x, long long y, long long mod) {
  long long s = 0, m = x % mod;
  while (y) {
    if (y & 1) s = modit(s + m, mod);
      y >>= 1;
      m = modit(m + m, mod);
  }
}
```

```
return s;
}
long long f(long long x, long long mod) {
  return modit(mult(x, x, mod) + 1, mod);
long long pollard_rho(long long n) {
  if (!(n & 1)) return 2;
  while (true) {
    long long y = 2, x = random() % (n - 1) +
    1, res = 1;
    for (int sz = 2; res == 1; sz *= 2) {
      for (int i = 0; i < sz && res <= 1; i</pre>
   ++) {
        x = f(x, n);
        res = \_gcd(abs(x - y), n);
      y = x;
    if (res != 0 && res != n) return res;
}
```

## 5 Theorem

#### 5.1 Fermat's little theorem

If p is a prime number, then for any number a,  $a^p - a$  is an integer multiple of p

$$a^p \equiv a \pmod{p}$$

If a is not divisible by p

$$a^{p-1} \equiv 1 \pmod{p}$$

#### 5.2 Euler's totient function

The number of coprime  $\leq n$ 

$$\phi(n) = n \prod (1 - \frac{1}{p})$$

With p is the prime divided by n

#### 5.3 Dirichlet

Given n holes and n+1 pigeons to distribute evenly, then at least 1 hole must have 2 pigeons

# 5.4 Pythagorean triple

$$a = m^2 - n^2$$
,  $b = 2mn$ ,  $c = m^2 + n^2$ 

where m and n are positive integer with m > n, and with m and n are coprime and not both odd.

# 6 Other

# 6.1 Bignum mul

```
string mul(string a,string b)
{
   int m=a.length(),n=b.length(),sum=0;
   string c="";
   for (int i=m+n-1; i>=0; i--)
   {
      for (int j=0; j<m; j++) if (i-j>0 && i-j
      <=n) sum+=(a[j]-'0')*(b[i-j-1]-'0');
      c=(char)(sum%10+'0')+c;
      sum/=10;
   }
   while (c.length()>1 && c[0]=='0') c.erase
      (0,1);
   return c;
}
```

#### 6.2 Random

```
// Random using mt19937
mt19937 rng(chrono::steady_clock::now().
    time_since_epoch().count());

// For random long long
mt19937_64 rng(chrono::steady_clock::now().
    time_since_epoch().count());

// Random shuffle using mt19937 as the
    generator
shuffle(a.begin(), a.end(), rng);

// Random range
int random(int a, int b)
{
    return uniform_int_distribution<int>(a, b)(
        rng);
}
```

### 6.3 Builtin bit function

```
__builtin_popcount(x); // number of bit 1 in
    x
__builtin_popcountll(x); // for long long
__builtin_clz(x); // number of leading 0
__builtin_clzll(x); // for long long
__builtin_ctz(x); // number of trailing 0
__builtin_ctzll(x); // for long long

(x & ~x) : the smallest bit 1 in x
floor(log2(x)) : 31 - __builtin_clz(x | 1);
floor(log2(x)) : 63 - __builtin_clzll(x | 1);
```

# 6.4 Pythagorean triples

c under 100 there are 16 triples: (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25) (20, 21, 29) (12, 35, 37) (9, 40, 41) (28, 45, 53) (11, 60, 61) (16, 63, 65) (33, 56, 65) (48, 55,

73) (13, 84, 85) (36, 77, 85) (39, 80, 89) (65, 72, 97)

 $\begin{array}{l} 100 \leq c \leq 300; \; (20,\,99,\,101) \; (60,\,91,\,109) \; (15,\,112,\\ 113) \; (44,\,117,\,125) \; (88,\,105,\,137) \; (17,\,144,\,145) \; (24,\,143,\\ 145) \; (51,\,140,\,149) \; (85,\,132,\,157) \; (119,\,120,\,169) \; (52,\,165,\\ 173) \; (19,\,180,\,181) \; (57,\,176,\,185) \; (104,\,153,\,185) \; (95,\,168,\\ 193) \; (28,\,195,\,197) \; (84,\,187,\,205) \; (133,\,156,\,205) \; (21,\,220,\\ 221) \; (140,\,171,\,221) \; (60,\,221,\,229) \; (105,\,208,\,233) \; (120,\,209,\,241) \; (32,\,255,\,257) \; (23,\,264,\,265) \; (96,\,247,\,265) \; (69,\,260,\,269) \; (115,\,252,\,277) \; (160,\,231,\,281) \; (161,\,240,\,289) \; (68,\,285,\,293) \end{array}$ 

#### 6.5 Sieve

for (int j = i; j \* i <= lim; ++j) notPrime[j \* i] = true

#### 6.6 Catalan

$$\frac{(2n)!}{(n+1)! \, n!} = \prod_{k=2}^{n} \frac{n+k}{k}$$

#### 6.7 Prime under 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

## 6.8 Pascal triangle

#### 6.9 Fibo

 $0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89\ 144\ 233\ 377\ 610\ 987\ 1597$   $2584\ 4181\ 6765$ 

# 7 Tips

- Test kĩ trước khi nộp. Code nhìn đúng chưa chắc đúng đâu
- Test conner case

- Giả sử nó là số nguyên tố đi. Giả sử nó liên quan tới số nguyên tố đi.
- Giả sử nó là số có dạng  $2^n$  đi.
- Giả sử chọn tối đa là 2, 3 số gì là có đáp án đi.
- Có liên quan gì tới Fibonacci hay tam giác pascal?
- Dãy này đơn điệu không em ei? Hay tổng của 2,3 số fibonacci?
- $q \le 2$
- Sort lại đi, biết đâu thấy điều hay hơn?

- Chia nhỏ ra xem.
- Bỏ hết những thẳng ko cần thiết ra
- Áp đại data struct nào đấy vô
- khóc
- Cầu nguyện
- Random shuffe để ac
- Xoay mảng 45 độ

# Keep Smilling

Gotta solve them all