Approximation Proof (Nathan Hanzel)

Step 1: Fractional Knapsack

x_i is the fractional value $\left(\sum_{i=n}^{n} p_i x_i\right)$ $x_i \in [0,1]$

OPT for relaxed problem is OPT' (it will be AT LEAST as good because fractional profits allow for more flexibility)

Step 2:

Assume sorted order by profit/size ratio

if our problem is made up of p profits and s sizes, assume $p_1 \cdots p_{k-1}, p_k$ and k is the first item that can't fit with the rest

so we take a fraction

some fraction

$$p_1 + p_2 \cdots + p_{k-1} + \alpha p_k \geq OPT'$$

since the fraction lets us get right up to the OPTIMAL (or above), never below since p_k would fit the the capacity $p_1 + p_2 \cdots + \alpha p_k < OPT'$ would never happen

Step 3:

So we know $OPT \leq OPT'$, therefore

$$p_1 + p_2 \cdots + \alpha p_k \geq OPT.$$

Then take our greedy algorithm...

$p_1 + p_2 \cdots p_{k-1}$ or most profitable p_k

One of these is half (at least) of OPT because

$$p_1 + p_2 \cdots + \alpha p_k \geq OPT$$

and either

$\{p_1 + p_2 \cdots + p_{k-1}\}$ or $\{p_k\}$ has to be at least half of OPT

(if they are both less than $.5 \cdot OPT$, then they aren't a true sum of OPT)

Example:

$p_1 = 60$ $s_1 = 10$ $(p/s = 6)$

$p_2 = 100$ $s_2 = 20$ $(p/s = 5)$

$p_3 = 120$ $s_3 = 30$ $(p/s = 4)$

$B = 50$ (capacity)

Item 1 & 2 fit in sack

$p_1 \& 2 = 160 \leftarrow$ Winner

$p_3 = 120$

Optimal w/ fractions

$100 + (2/3)(120) = 180$

$8/9$ is more than $1/2$