CSC135 Lu

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Test 2.

1) Formal definition of Context Free Grammar $G = (V, T, S, P)$.

T - a set of terminal symbols

V - a set of nonterminal symbols

S - a element of V, starting symbol.

P - a set of production rules with certain format restrictions.

the grammar is said to be context-free if all productions in P have the form:

$$\{ A \to x \ ; \text{where } A \in V \text{ and } x \in (V \cup T)^* \}$$

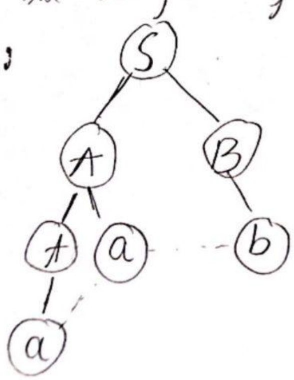
2) a) $S \to AB \mid aaB$

$A \to a \mid Aa$

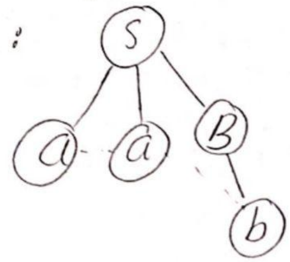
$B \to b$

Let pick the string aab. If there are two ways to derive the string using parsing trees, the grammar is ambiguous.

way 1:



way 2:



We have the same result which is the string aab for both ways. Thus, the given grammar is ambiguous.

2)b) Find a CFG for the following Language : $L = \{a^n b^m : n \neq m\}$

We have 2 cases : (1) $n > m$

(2) $n < m$

$S \rightarrow S_1 S_2 \mid S_2 S_3 \mid S_2$

$S_1 \rightarrow a S_1 \mid a$ $\qquad$ $S_3 \rightarrow S_3 b \mid b$

$S_2 \rightarrow a S_2 b \mid \lambda$

3) $L = \{a^n b^{n+3} : n \geq 0\}$.

Proof: Assume L is a regular language, By the Pumping Lemma, There exist an integer m in pumping lemma

Let $w = a^m b^{m+3} \in L$, $|w| = 2m+3 > m$

By Pumping Lemma, $w = xyz$.

where $1 \leq |xy| \leq m$; $1 \leq |y| \leq m$; $|y| = k$, $k = 1, 2 \ldots m$.

$y = a^k$

$xy^iz \in L$ where $i = 0, 1, 2, \ldots$

Let $i = 0 \Rightarrow w_0 = xy^0z = xz = a^{m-k} b^{m+3} \notin L$

because $k = 1, 2 \ldots m$.

Let $i = 2 \Rightarrow w_2 = xy^2z = a^{m-k} a^{2k} b^{m+3}$

$= a^{m+k} b^{m+3} \notin L$

because $m = 1, 2, \ldots m \Rightarrow a^{m+k} b^{m+3}$

$\neq a^m b^{m+3}$

Contradiction $\Rightarrow$ L cannot be regular

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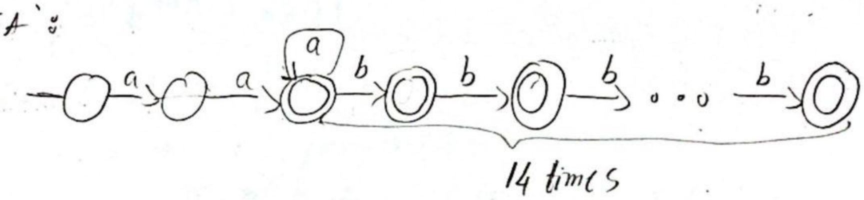
4)

Prove $L = \{a^n b^k ;\ n \geq 2 ;\ K \leq 14\}$ is regular.

If there is a RE or FA that is contructed from this Language, then the language is regular.

RE: $aaa^*(b^0 + b^1 + b^2 + \ldots + b^{14})$.

FA:



Chapter 4:

3) $L = \{w : n_a(w) = n_b(w)\}$ is not regular.

Assume L is regular.

Let m be a constant in the pumping lemma, we can choose

$$w = a^m b^m \in L, \ |w| \geq m, \ m \geq 0$$

By Pumping Lemma,

For all possible x, y, z with $w = xyz$; $|xy| \leq m$. $|y| \geq 1$. The are following case

$xy^iz \in L$ when $i = 0, 1, 2, \ldots$

Case 1: $x = a^{m-r}, y = a^r, z = b^m, r \geq 1$. We let $i = 2$, $xy^2z = a^{m-r}a^r a^r b^m = a^{m+r}b^m \notin L$

because $m + r \neq m$ since $r \neq 0$ $(r \geq 1)$

$\Rightarrow$ contradicted

$\Rightarrow$ L is not regular

Since $L^* = L$, which is also not regular

4) a) $L = \{a^n b^l a^k : k \geq n + l\}$

Assume, L is regular, By pumping lemma, there is an integer m in the pumping lemma, we can choose,

$$w = a^m b^m a^{2m}, \ |w| \geq m, \ m \geq 0$$

By Pumping Lemma

For all possible x, y, z with $w = xyz$; $|xy| \leq m$, $|y| \geq 1$. The are following case

$xy^iz \in L, i = 0, 1, 2 \ldots$

Case 1: $x = a^{m-r}, y = a^r, z = b^m a^{2m}$

Let $i = 2 \Rightarrow xy^2z = xyyz = a^{m-r}a^r a^r b^m a^{2m}$

$= a^{m+r} b^m a^{2m}$.

$2m \geq m + r + m$. ?

no.

$\Rightarrow$ contradicted.

$\Rightarrow$ L is not regular

d)

$L = \{ a^n b^l : n \leq l \}$. Assume L is regular, By Pumping Lemma, there is an integer m in the pumping lemma we can choose

$$w = a^m b^m \in L, \; m \geq 0$$

For all possible x, y, z with $w = x, y, z$, $|xy| \leq m$; $|y| \geq 1$
$xy^iz \in L$ when $i = 0, 1, 2 \ldots$

Case 1: $x = a^{m-r}$, $y = a^r$, $z = b^m$. Let $i = 2 \Rightarrow$

$$xy^2z = a^{m-r}(a^r)^2 b^m = a^{m+r} b^m$$

$$m + r > m \quad \notin L.$$

$\Rightarrow$ Contradiction

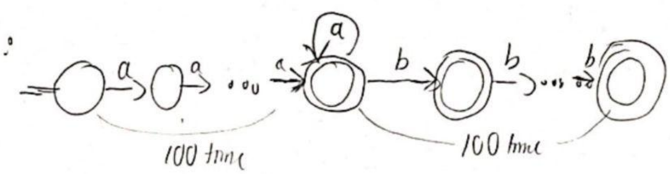
$\Rightarrow$ L is not regular.

15) f) $L = \{ a^n b^l, \; n \geq 100, \; l \leq 100 \}$.

This is regular. Because we can construct RE or FA:

RE: $a^{100} a^* (b^0 + b^1 + \ldots + b^{100})$.

FA:



g). $L = \{ a^n b^l : |n - l| = 2 \}$ in the pumping Lemma.

Assume L is regular. By pumping lemma, there is an integer m

We have: $n - l = 2$ or $n - l = -2$.

Let choose $w = a^{m+2} b^m$

For all possible x, y, z with $w = xyz$, $|xy| \leq m$, $|y| \geq 1$.

$x = a^{m-k}$, $y = a^k$, $z = a^2 b^m$. Let $i = 0$

$\Rightarrow xy^0z = xz = a^{m-k} a^2 b^m \notin L \Rightarrow$ contradiction $\Rightarrow L$ is not regular

15) a) $L = \{ a^n b^l c^k : n + l + k > 5 \}$

Let L is regular.

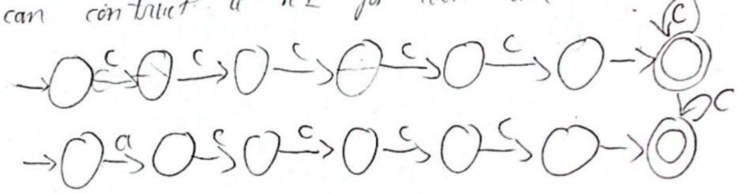
"If we put this in to different cases". For examples:

$w =$ $n = 0,\ l = 0,\ k \geq 5$

$n = 1,\ l = 0,\ k > 4$

...

we can contruct a RE for each case.



...

b) $L = \{ a^n b^l a^k : n > 5,\ l > 3,\ k \leq l \}$.

Assume L is regular., By Pumping Lemma., There is an integer m.

[illegible] m the Pumping Lemma. We can choose

$w = a^6 b^m a^m,\ m > 3$

For all possible x, y, z: $w = xyz$; $|xy| \leq m$, $|y| \geq 1$. Then are following case:

Case 1: $x = a^6 b^{m-r-6}$, $y = a^r$, $z = b^6 a^{m-r}$, $r \geq 1$, $i = 5$

$\Rightarrow xy^0z = a^6 b^{m-r-6}\, b^6 a^m = a^6 b^{m-r} a^m \notin L$

By Pumping Lemma. $\Rightarrow L$ is not regular.

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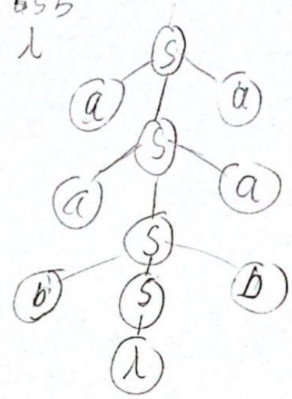
Chap 5

2) $G = (\{S\}, \{a, b\}, S, P)$

$S \to aSa$ $\Rightarrow S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa$

$S \to bSb$

$S \to \lambda$



3) $w = abbbaabbaba$

$S \to abB \to abbbAa \to abbbaaBba \to abbbaabbAaba \to abbbaabbaba$

7) a) $L = \{a^n b^m, n \leq m + 3\}$

$S \to aA \mid aaA \mid aaaA \mid A$

$A \to aAb \mid Ab \mid \lambda$

d) $L = \{a^n b^m, 2n \leq m \leq 3n\}$

$S \to aSbb \mid aSbbb \mid \lambda$

f) $L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$

$S \to aSb \mid SS \mid S_1$

$S_1 \to aS_1 \mid \lambda$

8) (a) $L = \{a^n b^m c^k : n = m \text{ or } m < k\}$ with $n, m, k \geq 0$

$S \Rightarrow A \mid B$

$A \Rightarrow A_1 A_2 \quad \mid A_1 \rightarrow aA_1 b \mid \lambda \quad A_2 \rightarrow A_2 c \mid \lambda$

$B \rightarrow B_1 B_2 \quad \mid B_1 \rightarrow aB_1 \mid \lambda \quad B_2 \rightarrow bB_2 c \mid B_2 c \mid \lambda$

(b) $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow S_3 S_4$

$S_3 \rightarrow aS_3 b \mid \lambda$

$S_4 \rightarrow cS_4 \mid \lambda$

$S_2 \rightarrow S_5 S_6$

$S_5 \rightarrow aA \mid \lambda$

$S_6 \rightarrow bS_6 c \mid B_b \mid B_c$

$B_b \rightarrow bB_b \mid b$

$B_c \rightarrow cB_c \mid c$

(d) $L = \{a^n b^m c^k : n + 2m = k\}$

$S \rightarrow aSc \mid B$

$B \rightarrow bBcc \mid \lambda$

(h) $L = \{a^n b^m c^k : k \geq 3\}$

$S \Rightarrow S_1 S_2$

$S_1 \rightarrow aS_1 \mid S_1 b \mid \lambda$

$S_2 \rightarrow S_2 c \mid ccc$

5.2)

6) $S \to AB \mid aaB$
$A \to a \mid Aa$
$B \to b$

Let "aab" is a string for this grammar.
There are two ways to derive the string:

- $S \Rightarrow aaB \Rightarrow aab$.
- $S \Rightarrow AB \Rightarrow Aab \Rightarrow aab$

$\Rightarrow$ This grammar is ambigous

10) $\Sigma = \{a, b\} \Rightarrow a+b,\ a.b,\ a^*,\ (a)$

$G \to S \to S + S_1 \mid S_1$
$S_1 \to S_1 \cdot S_2 \mid S_2$
$S_2 \to S_2^* \mid S_3$
$S_3 \to a \mid b \mid \lambda \mid (S) \mid \phi$

13) $S \to aSbS \mid bSaS \mid \lambda$

Let "abab" is a string for this grammar
There are 2 ways to derive this string:

$S \Rightarrow aSbS \Rightarrow abaSbS \Rightarrow abab$
$S \Rightarrow aSbS \Rightarrow abSaSb \Rightarrow abab$

$\Rightarrow$ ambigous

or