6.28 Tornadoes hit a region according to a Poisson process with $\lambda = 2$. The number of insurance claims filed after any tornado has a Poisson distribution with mean 30. The number of tornadoes is independent of the number of insurance claims. Find the expectation and standard deviation of the total number of claims filed by time t.

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Let $N(t)$ denote the number of tornado arrivals by time t. Tornadoes occur according to a Poisson process with rate $\lambda = 2$, thus the mean of $N(t)$ following a Poisson distribution is:

$$\mathrm{E}[N(t)] = \lambda t = 2t. \tag{1}$$

Let X_i denote the number of claims filed for the i-th tornado. This number has a Poisson distribution with mean 30, so $\mathrm{E}[X_i] = 30$. And let $Y(t)$ be the sum of the claims from all tornadoes up to time t:

$$Y(t) = \sum_{i=1}^{N(t)} X_i.$$

To find the expectation $\mathbb{E}[Y(t)]$, we use the linearity of expectation:

$$\mathrm{E}[Y(t)] = \mathrm{E}\left[\sum_{i=1}^{N(t)} X_i\right] = \sum_{i=1}^{N(t)} \mathrm{E}[X_i]. \tag{2}$$

However, since $N(t)$ is a random variable, we need to consider the expected number of terms in the sum. The expected number of tornadoes by time t is $\mathrm{E}[N(t)] = 2t$ (equation (1)). Therefore, we can rewrite the expectation in equation (2) as:

$$\mathrm{E}[Y(t)] = \mathrm{E}[N(t)] \cdot \mathrm{E}[X_i] = 2t \cdot 30 = 60t.$$

For a Poisson-distributed random variable X_i with mean 30, the variance is also 30. So, $\mathrm{Var}(X_i) = 30$. Since $N(t)$ and X_i are independent, the variance and standard deviation of $Y(t)$ is:

$$\mathrm{Var}(Y(t)) = \mathrm{E}[N(t)] \cdot \mathrm{Var}(X_i) = 2t \cdot 30 = 60t.$$

$$\mathrm{SD}(Y(t)) = \sqrt{\mathrm{Var}(Y(t))} = \sqrt{60t}.$$