Electric Circuits

Objective: This course provides the students with basic concepts and methods for analyzing linear electric circuits in sinusoidal steady state as well as obtaining its transient response. Upon completion of this course, students should be able to:

1. Understand the basic concepts of electric circuit.
2. Analyze DC and AC linear circuit in steady-state.
3. Analyze the transient response of linear circuit.
3. Analyze linear circuit in the frequency domain.

Contents

Chapter 1: Basic Concepts

1.1 Current, Voltage, Power

Electric current is the time rate of change of charge: $i = \frac{dq}{dt}$ [A]

The voltage between two points is the work needed to move a unit charge from one point to the other: $u = \frac{dw}{dq}$ [V]

Power is the time rate of expending or absorbing energy: $p = ui$ [W]

Passive sign convention: if the current enters through the positive polarity of the voltage then $p = ui$, otherwise $p = -ui$.

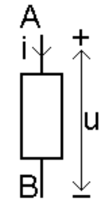


Fig.1.1: Two-terminal device

Example: Consider the two-terminal device in Fig.1.1 with $i = 2\sin(\pi t/2)$ [A] and $u = 4\cos(\pi t/2)$ [V] $\Rightarrow$ $p = ui = 8\sin(\pi t/2)\cos(\pi t/2) = 4\sin(\pi t)$ [W]. Fig.1.2 plots i(t), u(t) and p(t). Observe that

- $0 < t < 2$: $i > 0$, the current i is flowing from A to B, i.e. i follows the direction of the arrow.
- $2 < t < 4$: $i < 0$, the current i is flowing from B to A, i.e. i is opposite to the direction of the arrow.
- $0 < t < 1$: $u > 0$, the potential at the (+) sign terminal is higher than the potential at (-) sign terminal.
- $1 < t < 3$: $u < 0$, the potential at the (-) sign terminal is higher than the potential at (+) sign terminal.
- $0 < t < 1$: $p > 0$, the device absorbs the electric power.
- $1 < t < 2$: $p < 0$, the device supplies the electric power.

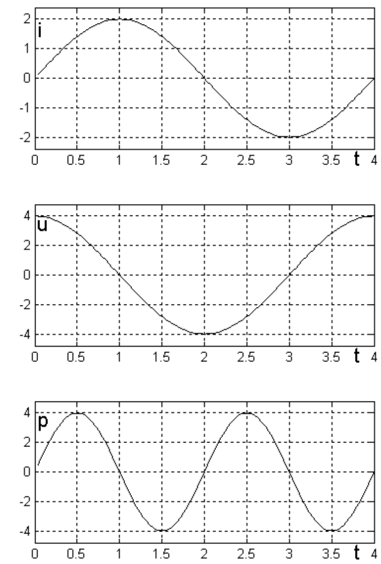


Fig.1.2

Definition

- Two or more elements are in series if they are cascaded and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

1.2 Kirchhoff's laws

Notation

- A branch is a simple path composed of two-terminal devices connected in series
- A node is the point of connection between branches.
- A loop is any closed path in a circuit.
- A mesh is a loop of planar circuit which does not contain any other loops within it.

A circuit is said to be a planar circuit if it is possible to draw its diagram on a plane surface in such a way that no branch passes over or under any other branch.

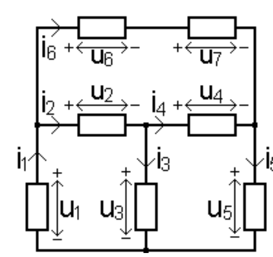


Fig.1.3

Kirchhoff's laws

- KCL: *The algebraic sum of all currents entering a node is zero*, $\sum_{node} \pm i_k = 0$
- KVL: *The algebraic sum of all voltages around a loop is zero*, $\sum_{loop} \pm u_k = 0$

Theorem: *The algebraic sum of power in a circuit, at any instant of time, must be zero*: $\sum p_k = 0$.

1.3 Basic Elements

1) Resistor

$u = Ri$

R: resistance [Ω]

$G = 1/R$: (reciprocal of resistance is conductance) [S]

$p = ui = Ri^2$

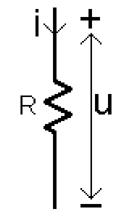


Fig. 1.3.1: Resistor

2) Independent sources

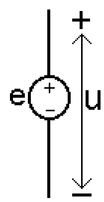


Fig. 1.3.2: Ideal Independent Voltage Source

$u = e$

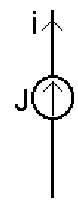


Fig.1.3.3: Ideal Independent Current Source

$i = J$

3) Dependent sources

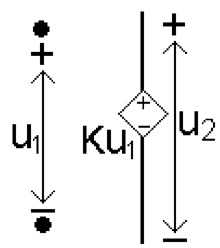


Fig. 1.3.4: Voltage Controlled Voltage Source

$u_2 = Ku_1$

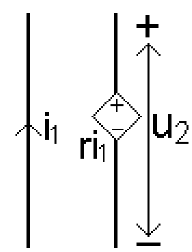


Fig.1.3.5: Current Controlled Voltage Source

$u_2 = ri_1$

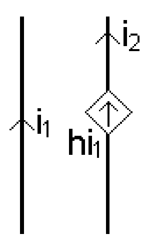


Fig. 1.3.6: Current Controlled Current Source

$i_2 = hi_1$

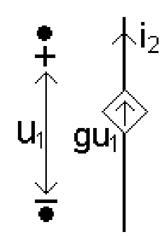


Fig.1.3.7: Voltage Controlled Current Source

$i_2 = gu_1$

4) Operational Amplifier (Op Amp)

$u = 0$

$i_1 = i_2 = 0$

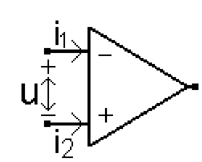


Fig. 1.3.8: Op Amp

5) Inductor

$$u = L\frac{di}{dt}$$

C: inductance [H]

$$E_L = \frac{1}{2}Li^2$$

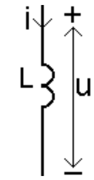


Fig. 1.3.9: Inductor

6) **Capacitor**

$$i = C\frac{du}{dt}$$

C: capacitance [F]

$$E_C = \frac{1}{2}Cu^2$$

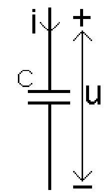


Fig. 1.3.10: Capacitor

7) **Magnetically coupled circuits**

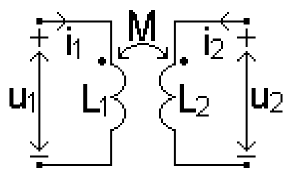


Fig. 1.3.11: Mutual Inductance

$$\begin{cases} u_1 = L_1\dfrac{di_1}{dt} + M\dfrac{di_2}{dt} \\ u_2 = L_2\dfrac{di_2}{dt} + M\dfrac{di_1}{dt} \end{cases}$$

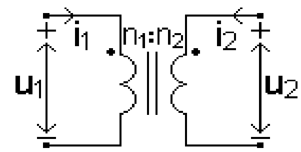


Fig.1.3.12: Ideal Transformer

$$\begin{cases} \dfrac{u_1}{u_2} = \dfrac{n_1}{n_2} \\ \dfrac{i_1}{i_2} = -\dfrac{n_2}{n_1} \end{cases}$$

The product of the magnetic flux and the number of turns of a coil is called the 'flux linkage' of the coil. The dots in Fig 1.3.11 and 1.3.12 indicate the winding polarity of the coils.

1.4 Practical Applications

1) **Voltage Divider and Current Divider**

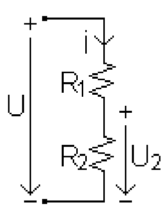


Fig.1.4.1

Voltage divider

$$U_2 = \frac{R_2}{R_1 + R_2}U$$

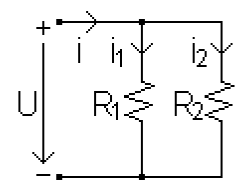


Fig.1.4.2

Current divider

$$i_2 = \frac{R_1}{R_1 + R_2}i$$

2) **A Two-Stage Op Amp Circuit**: Find v in term of e_1 and e_2 (Fig.1.4.3)

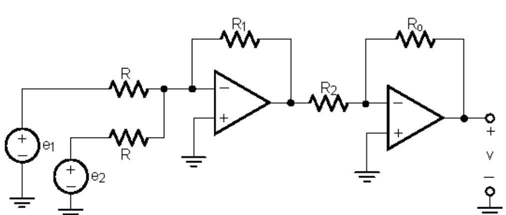


Fig.1.4.3: A summing amplifier cascaded with an inverting amplifier

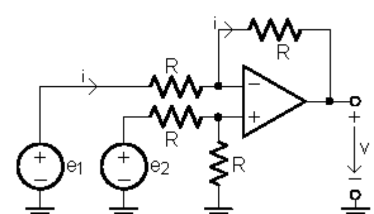


Fig.1.4.4: A difference amplifier

3) **A Difference Amplifier**: Find v in term of e_1 and e_2 (Fig.1.4.4)

$$V_+ = V_- = \frac{e_2}{2}$$

$$i = \frac{1}{R}(e_1 - V_-) = \frac{1}{R}(e_1 - \frac{e_2}{2})$$

$$V = V_- - Ri = \frac{e_2}{2} - (e_1 - \frac{e_2}{2}) = e_2 - e_1$$

4) An Instrumentation Amplifier: Find v in term of v_1 and v_2 (Fig.1.4.5)

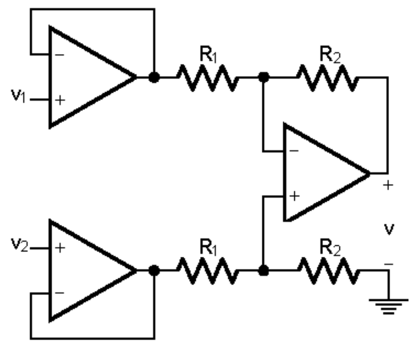


Fig.1.4.5: An instrumentation amplifier

Instrumentation amplifier is particularly well suited to applications where the input voltage signal is very small (for example, on the order of millivolts), such as that produced by thermocouples or strain gauges, and where a significant common-mode noise signal of several volts may be present. If components of the instrumentation amplifier are fabricated all on the same silicon “chip,” then it is possible to obtain well-matched device characteristics and to achieve precise ratios for the two sets of resistors.

5) Nonlinear component: Diode and Zener Diode

6) A Reliable Voltage Source: Find v in Fig.1.4.6

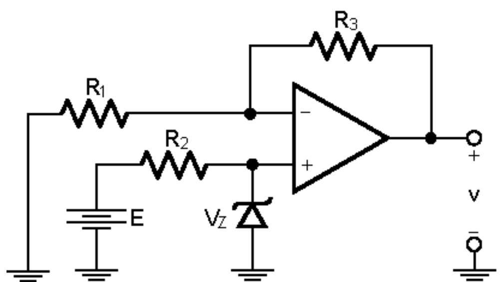


Fig.1.4.6

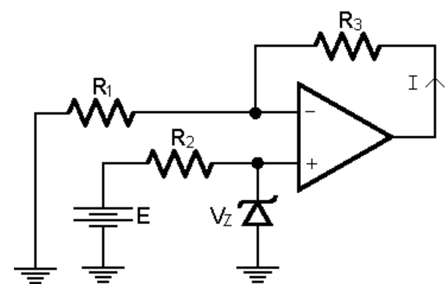


Fig.1.4.7

7) A Reliable Current Source: Find I in Fig.1.4.7

8) Comparator

Chapter 2: Sinusoidal Circuit Analysis

2.1 Phasor

1) Complex Number

$c = a + jb$ rectangular form

$= |c|\angle\varphi$: polar form

$a = \mathrm{Re}\{c\} = |c|\cos(\varphi)$ real part

$b = \mathrm{Im}\{c\} = |c|\sin(\varphi)$ imaginary part

$|c| = \sqrt{a^2 + b^2}$ length (magnitude, absolute value, or modulus)

$\varphi = \tan^{-1}(\frac{b}{a})$ angle

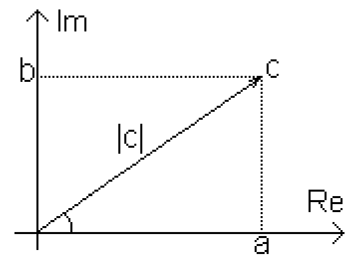


Fig.2.1.1: Complex number

Example: addition, subtraction, multiplication, division, reciprocation

- Find the polar form of the following complex number: 2+2j, 2j, -2+2j, -2, -2-2j, -2j, 2-2j, 2.
- Find the rectangular form of the following complex number: $2\angle 30°$, $2\angle -30$, $2\angle 120°$, $2\angle -120°$.
- Let $c_1 = 2+3j$, $c_2 = 6+8j$, $c_3 = 8\angle 30°$, $c_4 = 6\angle 60°$, find c_1+c_2, c_1-c_2, c_3+c_4, c_3-c_4, c_1c_2, c_1/c_2, c_3c_4, c_3/c_4.

2) **Sinusoid** is a collective term referring to both sine and cosine functions

$x(t) = X_m\cos(\omega t+\varphi)$

X_m: peak amplitude, ω: angular frequency (radian frequency) [rad/sec], $\omega = 2\pi f = 2\pi/T$, f: frequency [Hz], $T = 1/f$: period [s], t: time [seconds], φ: initial phase (phase offset, phase shift, phase factor) [radians], $\omega t+\varphi$: instantaneous phase [radians].

The rms value of a periodic function is defined as the square root of the mean value of the squared function. With sinusoidal function: $X_m = X_{RMS}\sqrt{2}$.

The sine and cosine function are very closely related and can be made equivalent simply by adjusting their initial phase. In calculus, the sine and cosine functions are derivatives of one other.

3) Phasor

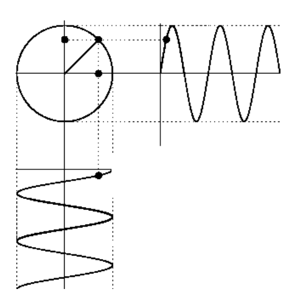
$x(t) = X_m\cos(\omega t+\varphi)$ $\leftrightarrow$ $\dot{X} = X_m\angle\varphi$, $\dot{X}_{RMS} = \frac{X_m}{\sqrt{2}}\angle\varphi$

$$x(t) = \mathrm{Re}\{\dot{X}e^{j\omega t}\} = \mathrm{Re}\{\dot{X}_{RMS}\sqrt{2}e^{j\omega t}\}$$

$x(t) = X_m\sin(\omega t+\varphi)$ $\leftrightarrow$ $\dot{X} = X_m\angle\varphi$, $\dot{X}_{RMS} = \frac{X_m}{\sqrt{2}}\angle\varphi$

$$x(t) = \mathrm{Im}\{\dot{X}e^{j\omega t}\} = \mathrm{Im}\{\dot{X}_{RMS}\sqrt{2}e^{j\omega t}\}$$

Consider a vector of length X_m, rotating in a counterclockwise direction at a steady speed ω, the vector tracing a circle with a radius equal to its length. Projecting the vector onto the x- and y-axes allows us to determine its coordinates in the xy-plane. (The x- and y-axis are the horizontal and vertical lines intersecting at the circle's centre.)



Properties

- Linearity: $x(t) = a_1x_1(t) + a_2x_2(t) \quad \Rightarrow \quad \dot{X} = a_1\dot{X}_1 + a_2\dot{X}_2$

- Derivative: $y(t) = \dfrac{dx}{dt} \quad \Rightarrow \quad \dot{Y} = j\omega\dot{X}$

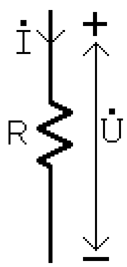
3) Complex Form of Basic Laws

a) Kirchhoff's laws

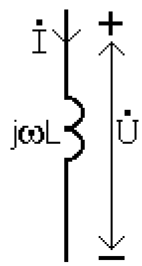
KCL: $\sum\limits_{node} \pm \dot{I}_k = 0$

KVL: $\sum\limits_{loop} \pm \dot{U}_k = 0$

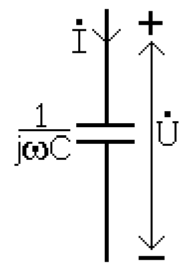
b) Ohm's laws



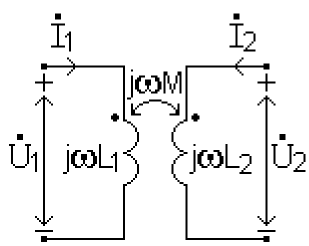
$\dot{U} = R\dot{I}$



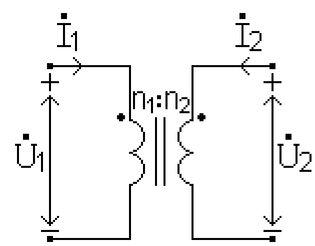
$\dot{U} = j\omega L\dot{I}$



$\dot{U} = \dfrac{\dot{I}}{j\omega C}$



$$\begin{cases} \dot{U}_1 = j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2 \\ \dot{U}_2 = j\omega M\dot{I}_1 + j\omega L_2\dot{I}_2 \end{cases}$$



$$\begin{cases} \dfrac{\dot{U}_1}{\dot{U}_2} = \dfrac{n_1}{n_2} \\ \dfrac{\dot{I}_1}{\dot{I}_2} = -\dfrac{n_2}{n_1} \end{cases}$$

Fig.2.1.2

Rule: *The voltage due to the current I has plus sign where the current I enters.*

Example:

Given the circuit in Fig.2.1.3 with

$e = 100\cos(1000t)$V,

$R = 100\Omega$, $L = 100$mH, $C = 10\mu$F.

Find $i_1(t)$, $i_2(t)$, $i(t)$, $u(t)$.

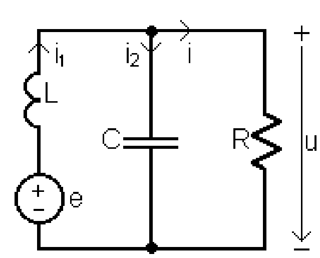
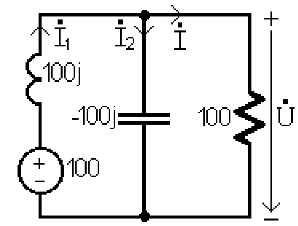


Fig.2.1.3



Solution:

$$\dot{U}\left(\frac{1}{100j}+\frac{1}{-100j}+\frac{1}{100}\right)=\frac{100}{100j}=-j$$

$\Rightarrow\ \dot{U} = -100j = 100\angle\text{-}90° \quad \Rightarrow\ u(t) = 100\cos(1000t - 90°)$ V

$\dot{I} = \dfrac{-100j}{100} = 1\angle\text{-}90° \quad \Rightarrow\ i(t) = \cos(1000t - 90°)$ A

$\dot{I}_2 = \dfrac{-100j}{-100j} = 1\angle 0° \quad \Rightarrow\ i_2(t) = \cos(1000t)$ A

$\dot{I}_1 = 1 - j = \sqrt{2}\angle\text{-}45° \quad \Rightarrow\ i_1(t) = \sqrt{2}\cos(1000t - 45°)$ A

Example A: Phasor Diagrams

The circuit in Fig.2.1.4A is in sinusoidal steady-state with RMS values

$U_1 = U_2 = 50$V

$I = I_1 = I_2 = 1$A.

Sketch the phasors of U, U_1,U_2, I, I_1, I_2.

Deduce the impedance of the two-pole.

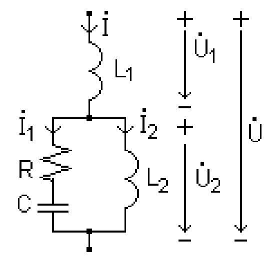


Fig.2.1.4A

Example B:

The circuit in Fig.2.1.4B is operating in sinusoidal steady-state with $I_R = I_L = 1$A, $I_C = 2$A, $U = 50$V (RMS values). Sketch the phasors of the voltage and currents. Find J.

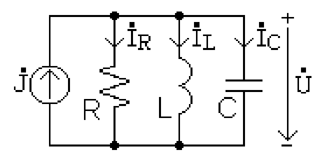


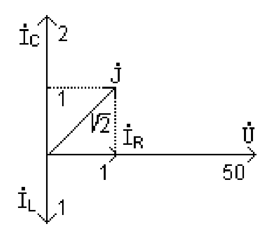
Fig.2.1.4B

Let $\dot{U}_R = \dot{U}_C = 100\angle 0°$ V $\Rightarrow$ $\dot{I}_R = 1\angle 0°$ A

$\dot{I}_L = 1\angle\text{-}90°$ A

$\dot{I}_C = 2\angle 90°$ A

KCL $\Rightarrow$ $\dot{J} = \dot{I}_R + \dot{I}_L + \dot{I}_C = \sqrt{2}\angle 45°$ A $\Rightarrow J = \sqrt{2}$ A



2.2 Impedance and Admittance

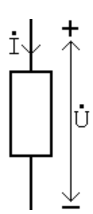


Fig.2.2.1

Consider a two-terminal circuit (one-port network) without independent source. The impedance **Z** of the circuit is the ratio of the phasor voltage $\dot{U}$ to the phasor current $\dot{I}$, measured in ohms

$$Z = \frac{\dot{U}}{\dot{I}} = R + jX = z\angle\varphi,$$ Z: impedance, R: resistance, X: reactance [Ω]

The reciprocal of impedance is admittance

$$Y = \frac{1}{Z} = G + jB = y\angle\alpha,$$ Y: admittance, G: conductance, B = susceptance.[S]

X > 0: the impedance is inductive (or lagging since current lags voltage)

X < 0: the impedance is capacitive (or leading since current leads voltage)

Example: Given the circuit in Fig.2.2.1 (the phase-shifting bridge circuit) where $e = E_m \sin(\omega t)$, $R = 1/\omega C$, $R_{POT} >> R$ and can be considered as open circuit. Show that the circuit provides an output voltage V_O with a variable phase shift from -45^o (lagging) to $+45^o$ (leading), depending on the position of the potentiometer wiper.

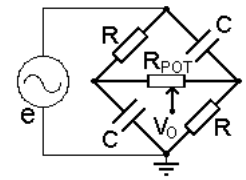


Fig.2.2.1

2.3 Circuit transformation

1) Impedances in series: The equivalent impedance of any number of impedances connected in series is the sum of the individual impedances (Fig.2.3.1)
2) Impedances in parallel: The equivalent impedance of two parallel impedances is equal to the product of their impedances divided by their sum (Fig.2.3.2)

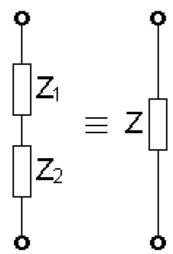


Fig.2.3.1

$$Z = Z_1 + Z_2$$

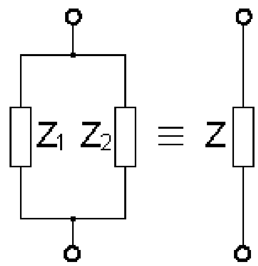


Fig.2.3.2

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

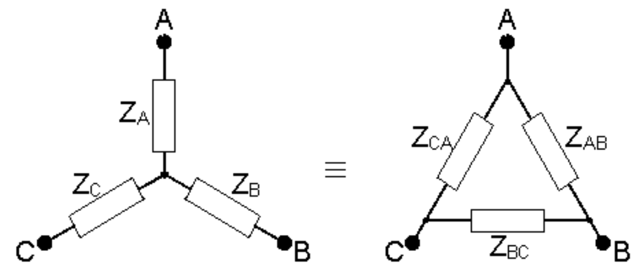


Fig. 2.3.3

$$Z_A = \frac{Z_{AB} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}, \quad Z_{AB} = Z_A + Z_B + \frac{Z_A Z_B}{Z_C}$$

3) Wye-Delta Conversion (Fig. 2.3.3)
   - Each impedance in the Δ network is the sum of all possible products of Y impedances taken two at a time, divided by the opposite Y impedance.
   - Each impedance in the Y network is the product of the impedances in the two adjacent Δ branches, divided by the sum of the three Δ impedances.
4) Voltage sources in series: Fig.2.3.4
5) Current sources in parallel: Fig.2.3.5
6) Conversion of a practical voltage source to a practical current source and vise-versa: Fig.2.3.6

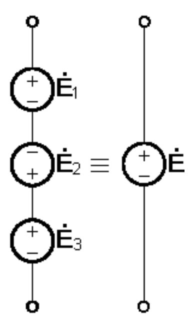


Fig.2.3.4: $\dot{E} = \dot{E}_1 - \dot{E}_2 + \dot{E}_3$

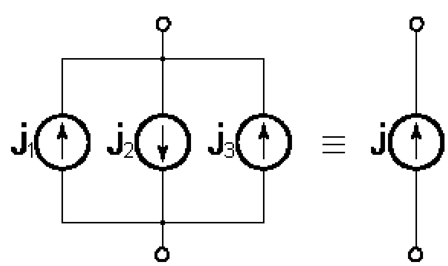


Fig.2.3.5: $\dot{J} = \dot{J}_1 - \dot{J}_2 + \dot{J}_3$

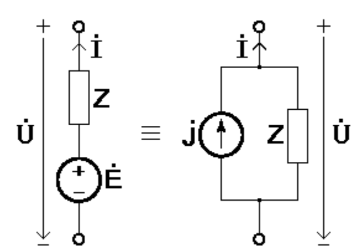


Fig. 2.3.6: $\dot{E} = Z\dot{J}$

Example: Using circuit transformation, find the current I of the circuit in Fig.2.3.7

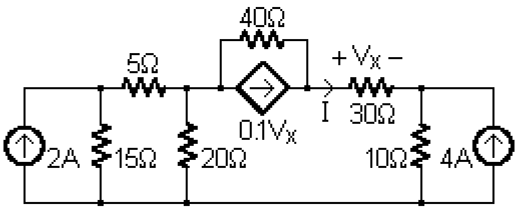


Fig.2.3.7

Solution:

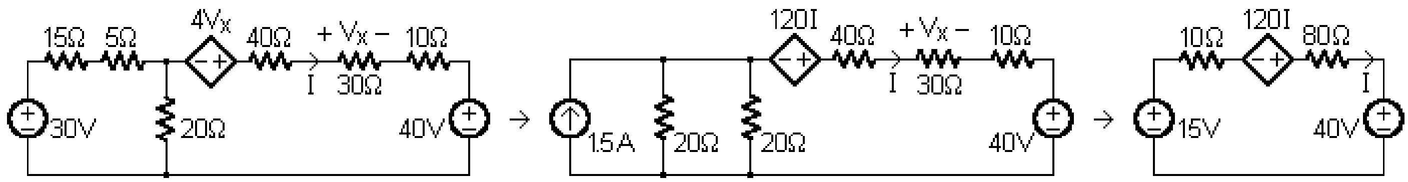


Fig.2.3.8

KVL: -15 + 90I - 120I + 40 = 0 ⇒ I = -5/6 A

Example: Given the circuit in Fig.2.3.9 with E = 100 V, J = 1 A, $\beta = 3$, $R_1 = 30\ \Omega$, $R_2 = 40\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 20\ \Omega$, find the current i_1, i_2, i_3, i_4.

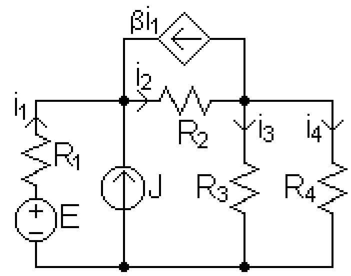


Fig.2.3.9

Solution:

Using circuit transformations, we get the equivalent circuit in Fig.2.3.10 with $R = R_3 \,//\, R_4$

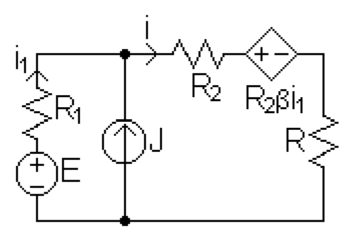


Fig.2.3.10

KCL: $i_1 + J - i = 0$

KVL: $-E + R_1 i_1 + R_2 i + R_2 \beta i_1 + R i = 0$

⇒ $-E + R_1 i_1 + (R+R_2)(i_1 + J) + R_2 \beta i_1 = 0$

⇒ $i_1 = \dfrac{E-(R+R_2)J}{R_1+R+R_2+\beta R_2} = 0.25\text{A}$

KCL: $i_2 = (1+\beta)i_1 + J = 2\text{A}$

⇒ $i_3 = i_4 = i/2 = (i_1 + J)/2 = 0.625\text{A}$

2.4 Power

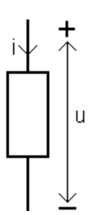


Fig.2.4.1

Consider a two-terminal circuit in sinusoidal steady-state

$$u(t) = U\sqrt{2}\sin(\omega t + \varphi_u) \quad \leftrightarrow \quad \dot{U} = U\angle\varphi_u$$

$$i(t) = I\sqrt{2}\sin(\omega t + \varphi_i) \quad \leftrightarrow \quad \dot{I} = I\angle\varphi_i$$

Let $\varphi = \varphi_u - \varphi_i$ (phase of voltage relative to current)

⇒ $u(t) = U\sqrt{2}\sin(\omega t + \varphi_i + \varphi) = U\sqrt{2}\cos(\varphi)\sin(\omega t + \varphi_i) + U\sqrt{2}\sin(\varphi)\cos(\omega t + \varphi_i)$

1) Instantaneous Power

$$p(t) = u(t)i(t) = 2UI\cos(\varphi)\sin^2(\omega t + \varphi_i) + 2UI\sin(\varphi)\cos(\omega t + \varphi_i)\sin(\omega t + \varphi_i)$$

$$= UI\cos(\varphi)[1 - \cos(2\omega t + 2\varphi_i)] + UI\sin(\varphi)\sin(2\omega t + 2\varphi_i) = p_1(t) + p_2(t)$$

$$p_1(t) = UI\cos(\varphi)[1 - \cos(2\omega t + 2\varphi_i)]$$

$p_2(t) = UI\sin(\varphi)\sin(2\omega t + 2\varphi_i)$

2) Active Power, Reactive Power, Complex Power, Apparent Power

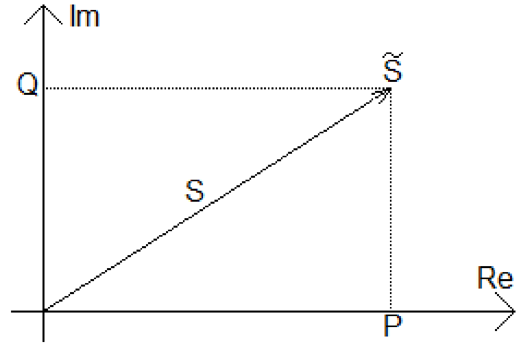


Fig.2.4.2: Power Triangle

Active Power (Real Power, Average Power)

$$P = \frac{1}{T}\int_0^T p(t)dt = UI\cos(\varphi) \text{ [W]}$$

Reactive Power (Amplitude of $p_2(t)$)

$$Q = UI\sin(\varphi) \text{ [VAr]}$$

Complex Power: $\tilde{S} = P + jQ = UI\angle\varphi = \dot{U}\,\dot{I}^*$

Apparent Power: $S = |\tilde{S}|$ [VA]

3) Two terminal without independent source

$\dot{U} = Z\dot{I} \Rightarrow \tilde{S} = ZI^2 = RI^2 + jXI^2 = zI^2\angle\varphi$

$\Rightarrow P = RI^2, Q = XI^2, S = zI^2, \tilde{S} = ZI^2$

4) The Power Factor

The ratio of active power to apparent power is called the power factor. For two components transmitting the same amount of active power, the one with lower power factor will have higher circulating currents due to energy that returns to the source from energy storage in the load. These higher currents produce higher losses and reduce overall transmission efficiency. A lower power factor circuit will have a higher apparent power and higher losses for the same amount of active power. The power factor is 1.0 when the voltage and current are in phase. It is zero when the current leads or lags the voltage by 90 degrees. When the voltage and current are 180 degrees out of phase, the power factor is negative one, and the component is feeding energy into the circuit. Power factors are usually stated as "leading" or "lagging" to show the sign of the phase angle of current with respect to voltage. Voltage is designated as the base to which current angle is compared, meaning that we think of current as either "leading" or "lagging" voltage.

Example: The active power is 700 W and the phase angle between voltage and current is 45.6°. The power factor is cos(45.6°) = 0.7. The apparent power is then: 700/cos(45.6°) = 1000 VA. A power factor of 0.7 means that only 70 percent of the total current supplied is actually doing work; the remaining 30 percent is reactive and has to be made up by the utility. Usually, utilities do not charge consumers for the reactive power losses as they do no real work for the consumer. However, if there are inefficiencies at the customer's load that causes the power factor to fall below a certain level, utilities may charge customers in order to cover an increase in their power plant fuel use and their worse line and plant capacity.

5) Balancing Power Delivered with Power Absorbed in an AC Circuit

Theorem: $\sum_{\text{circuit}} \tilde{S}_k = 0 \Rightarrow \sum_{\text{circuit}} P_k = 0$ and $\sum_{\text{circuit}} Q_k = 0$

Corollary: The complex power of a two-terminal circuit is equal to the sum of complex power of all components in the two-terminal circuit.

6) Maximum Power Transfer

7) Wattmeter

Fig.2.4.3 presents an electrodynamic wattmeter which consists of a pair of fixed coils, known as current coils, and a movable coil known as voltage coil. The current coils are connected in series with the load, while the voltage coil is connected in parallel. The voltage coil carries a pointer (needle) that moves over a scale to indicate the measurement. The current in the current coil generates an electromagnetic field around the coil. The strength of this field is proportional to the current and in phase with it. The voltage coil has, as a general rule, a high-value resistor connected in series with it to reduce the current that flows through it. The result of this arrangement is that the deflection of the pointer is proportional to the average instantaneous product of voltage and current, thus measuring true power, P = VIcos(φ).

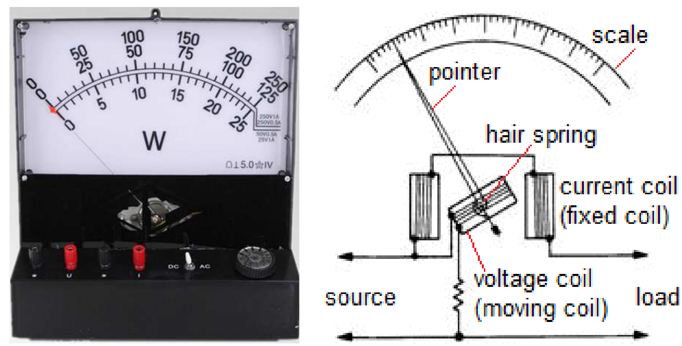


Fig.2.4.3

2.5 Resonance

Resonance occurs when the impedance is purely resistive, thus the voltage and the current are in phase. Condition of resonance:

$$\text{Im}\{Z\} = 0 \text{ or } \text{Im}\{Y\} = 0$$

At resonance, the reactive energy in the circuit oscillates between the inductor and the capacitor. The "sharpness" of the resonance is measured quantitatively by the quality factor Q.

$$Q = \frac{2\pi E_P}{E_D}$$

Where E_P is the peak energy stored in the circuit at resonance and E_D is the energy dissipated by the circuit in one period at resonance.

2.6 Two-port Networks

Two terminals constitute a port if the current entering one terminal equals the current emerging from the other. A two-port network is an electrical network with two separate ports.

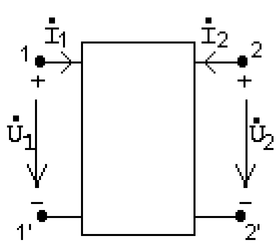


Fig. 2.6.1

Impedance parameters

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

Admittance parameters

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

Hybrid parameters

$$\begin{cases} \dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2 \end{cases}$$

Transmission parameters

$$\begin{cases} \dot{U}_1 = A_{11}\dot{U}_2 - A_{12}\dot{I}_2 \\ \dot{I}_1 = A_{21}\dot{U}_2 - A_{22}\dot{I}_2 \end{cases}$$

Inverse transmission parameters

$$\begin{cases} \dot{U}_2 = B_{11}\dot{U}_1 - B_{12}\dot{I}_1 \\ \dot{I}_2 = B_{21}\dot{U}_1 - B_{22}\dot{I}_1 \end{cases}$$

Inverse hybrid parameters

$$\begin{cases} \dot{I}_1 = G_{11}\dot{U}_1 + G_{12}\dot{I}_2 \\ \dot{U}_2 = G_{21}\dot{U}_1 + G_{22}\dot{I}_2 \end{cases}$$

Chapter 3: Methods of Analysis

3.1 Nodal Analysis

1. Designate a reference node and label the nodal voltages. The number of terms in the nodal equations can be minimized by selecting the reference node with the greatest number of branches connected to it.
2. Write a KCL equation for each of the non-reference nodes. Use KVL to express the branch currents in terms of node voltages.
3. Solve the resulting equations to obtain the unknown node voltages.

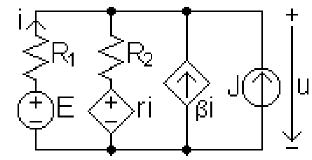


Fig.3.1.1A

Example A: Using nodal analysis, find the voltage u of the circuit in Fig.3.1.1A with E = 50 V, J = 5 A, R_1 = 100 Ω, R_2 = 25 Ω, r = 100 Ω, β = 3.

Solution: Nodal equation

$$u\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{E}{R_1}+\frac{ri}{R_2}+\beta i+J=\frac{E}{R_1}+J+\left(\frac{r}{R_2}+\beta\right)i$$

KVL: $i=\dfrac{E-u}{R_1}$

$$\Rightarrow \quad u\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{E}{R_1}+J+\left(\frac{r}{R_2}+\beta\right)\frac{E-u}{R_1}=\left(1+\frac{r}{R_2}+\beta\right)\frac{E}{R_1}+J-\left(\frac{r}{R_2}+\beta\right)\frac{u}{R_1}$$

$$\Rightarrow \quad u\left(\frac{1+\dfrac{r}{R_2}+\beta}{R_1}+\frac{1}{R_2}\right)=\left(1+\frac{r}{R_2}+\beta\right)\frac{E}{R_1}+J$$

$$\Rightarrow \quad u = 75\text{V}$$

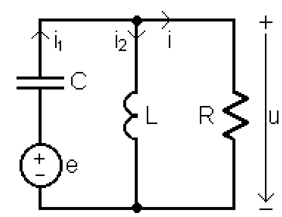


Fig.3.1.1B

Example B: Given the circuit in Fig.3.1.1B with

e = 100cos(1000t)V,

R = 100Ω, L = 100mH, C = 10μF.

Find i(t), u(t).

Nodal analysis

$$\dot{U}\left(\frac{1}{100j}+\frac{1}{-100j}+\frac{1}{100}\right)=-\frac{100}{100j}=j \qquad \Rightarrow \quad \dot{U}=100j=100\angle 90^\circ$$

$$\Rightarrow \quad u(t) = 100\cos(1000t+90^\circ)\ \text{V} \qquad \Rightarrow \quad i(t)=\cos(1000t+90^\circ)\ \text{A}$$

Super-node: Consider the circuit in Fig.3.1.2. The branch connecting node 2 and node 3 has only a voltage source. There is no way by which we can express the current as a function of the voltage, for the definition of a voltage source is exactly that the voltage is independent of the current. An easy method is to treat node 2, node 3, and the voltage source together as a sort of super-node (indicated by the region enclosed by the broken line) and apply KCL "the total current entering the closed surface is zero".

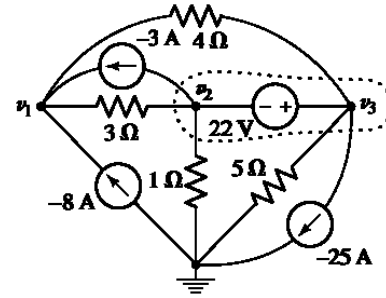


Fig.3.1.2

The KCL equation at node 1: $-8 - 3 = (v_1 - v_2)/3 + (v_1 - v_3)/4$
The KCL equation at the 2-3 super-node: $3 + 25 = (v_2 - v_1)/3 + (v_3 - v_1)/4 + v_3/5 + v_2/1$
Utilize the fact that there is a 22 V voltage source between nodes 2 and 3: $v_2 - v_3 = -22$

Solving these equations, the solution for v_1 is 1.071 V

3.2 Mesh Analysis

Basic mesh analysis procedure (for planar circuit)

1. Assign mesh currents to the meshes.
2. Apply KVL to each of the meshes. Use KCL law to express the voltages in terms of the mesh currents.
3. Solve the resulting equations to get the mesh currents.

Example: Using mesh analysis, find the current i of the circuit in Fig.2 with E = 150 V, J = 4 A, $R_1 = 75\ \Omega$, $R_2 = 25\ \Omega$, $R_3 = 25\ \Omega$, $\beta = 3$, $r = 50\ \Omega$.

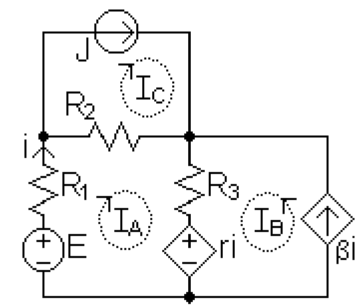


Fig.3.2.1

Solution: Mesh currents $i = I_A$, $\beta i = I_B$, $J = I_C$

KVL: $I_A(R_1+R_2+R_3) + I_BR_3 - I_CR_2 + ri - E = 0$

$\Rightarrow \quad i(R_1+R_2+R_3) + \beta iR_3 - JR_2 + ri - E = 0$

$$\Rightarrow \quad i = \frac{E + R_2J}{R_1 + R_2 + (1+\beta)R_3 + r} = 1\text{A}$$

Super-mesh: Consider the circuit in Fig.3.2.2. There is a 7 A independent current source is in the common boundary of two meshes. A "super-mesh" is created from these two meshes (the current source is in the interior of the super-mesh). The number of meshes is thus reduced by 1 for each current source present. If the current source lies on the perimeter of the circuit, then the single mesh in which it is found is ignored. Kirchhoff's voltage law is thus applied only to those meshes or super-meshes in the reinterpreted network.

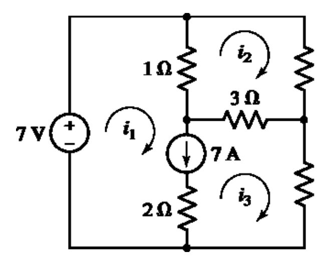


Fig.3.2.2

Applying KVL about the super-mesh: $-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0 \;\Rightarrow\; i_1 - 4i_2 + 4i_3 = 7$
and around mesh 2: $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \;\Rightarrow\; -i_1 + 6i_2 - 3i_3 = 0$
Finally, the independent source current is related to the mesh currents: $i1 - i3 = 7$

$\Rightarrow \quad i_1 = 9$ A, $i_2 = 2.5$ A, and $i_3 = 2$ A

Nodal vs. mesh analysis: a comparison

With nodal analysis, a circuit with N nodes will lead to at most (N − 1) KCL equations. Each super-node defined will further reduce this number by 1. With mesh analysis, a circuit with M meshes will lead to at most M KVL equations. Each super-mesh will reduce this number by 1. The approach that will result in the smaller number of simultaneous equations can be chosen.

3.3 Thevenin's and Norton's Theorems

Thevenin's theorem: a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source E_{Th} in series with an impedance Z_{Th}, where E_{Th} is the open-circuit voltage at the terminals and Z_{Th} is the input or equivalent impedance at the terminals when the independent sources are turned off.

Norton's theorem: a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with an impedance Z_{Th}, where I_N is the short-circuit current through the terminals and Z_{Th} is the input or equivalent impedance at the terminals when the independent sources are turned off.

3.4 Superposition Principle

Superposition Theorem: The total current (voltage) in any part of a linear circuit equals the algebraic sum of the currents (voltages) produced by each independent source separately. To evaluate the separate currents to be combined, replace all other independent voltage sources by short circuits and all other independent current sources by open circuits.

Example: Given the circuit in Fig. 3.4.1 with

$e = 100\cos(1000t)$ V,
$E = 200$ V (DC source),
$R = 100\ \Omega$,
$L = 100$ mH,
$C = 10\ \mu$F.

Find $i_1(t)$, $i_2(t)$, $i(t)$

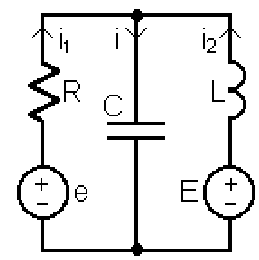
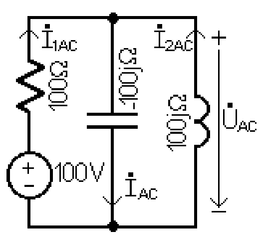


Fig. 3.4.1



a) AC component

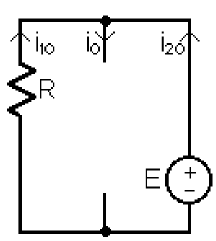
$$\dot{U}_{AC}\left(\frac{1}{100j}+\frac{1}{-100j}+\frac{1}{100}\right)=\frac{100}{100}=1$$

$$\Rightarrow \dot{U}_{AC}=100\text{ V} \quad \Rightarrow \quad u_{AC}(t)=100\cos(1000t)\text{ V}$$

$$\Rightarrow \dot{I}_{AC}=\frac{100}{-100j}=1\angle 90°\text{ A} \quad \Rightarrow \quad i_{AC}(t)=\cos(1000t+90°)\text{ A}$$

$$\dot{I}_{2AC}=\frac{-100}{100j}=1\angle 90°\text{ A} \quad \Rightarrow \quad i_{2AC}(t)=\cos(1000t+90°)\text{ A}$$

$$\dot{I}_{1AC}=\dot{I}_{AC}-\dot{I}_{2AC}=0\text{ A} \quad \Rightarrow \quad i_{1AC}(t)=0\text{ A}$$



b) DC component

DC steady-state $\Rightarrow$ L short circuit, C open circuit

$\Rightarrow$ $u_0 = 200$ A
$i_0 = 0$ A
$i_{10} = -2$ A
$i_{20} = 2$ A

c) Superposition

$u(t) = 200 + 100\cos(1000t)$ V
$i(t) = \cos(1000t + 90°)$ A
$i_1(t) = -2$ A
$i_2(t) = 2 + \cos(1000t + 90°)$ A

Example: Given the circuit in Fig.3.4.2 with

$R_1 = R_2 = 10\ k\Omega$,

$C_1 = C_2 = 100\ \mu F$.

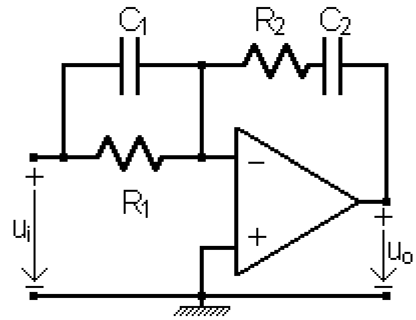
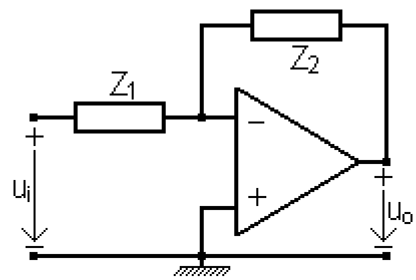


Fig.3.4.2

a) Find the voltage transfer function $K_U = \dfrac{\dot{U}_o}{\dot{U}_i}$ as function of ω.

b) Find $u_o(t)$ if $u_i(t) = 2\cos(0{,}5t) + 2\sin(t) + 2\cos(2t)$ V

Solution



$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1},\quad Z_2 = \frac{1 + j\omega C_2 R_2}{j\omega C_2}$$

$$K_U(j\omega) = \frac{\dot{U}_o}{\dot{U}_i} = -\frac{Z_2}{Z_1} = -\frac{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)}{j\omega C_2 R_1}$$

$$= -\frac{(1 + j\omega)^2}{j\omega}$$

Find $u_o(t)$ as $u_i(t) = 2\cos(0{,}5t) + 2\sin(t) + 2\cos(2t)$ V

$$K_U(0{,}5j) = -\frac{(1 + 0{,}5j)^2}{0{,}5j} = -2{,}5\angle{-37°},\quad K_U(j) = -\frac{(1 + j)^2}{j} = -2,\quad K_U(2j) = -\frac{(1 + 2j)^2}{2j} = -2{,}5\angle 37°$$

$\Rightarrow \quad u_o(t) = -5\cos(0{,}5t - 37°) - 4\sin(t) - 5\cos(2t + 37°)$ V

Example: Given the circuit in Fig.3.4.3 with

$R = 10\ k\Omega$,

$C = 100\ \mu F$.

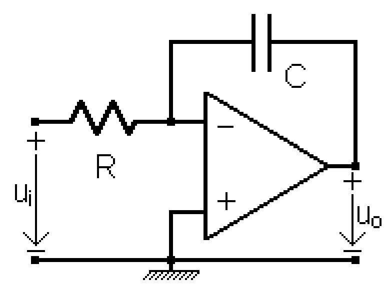


Fig.3.4.3

a) Find the voltage transfer function $K_U = \dfrac{\dot{U}_o}{\dot{U}_i}$ as function of ω.

b) Find $u_o(t)$ if $u_i(t) = 2\cos(0{,}5t) + 2\sin(t) + 2\cos(2t)$ V

Solution

$$Z_1 = R,\quad Z_2 = \frac{1}{j\omega C} \Rightarrow K_U(j\omega) = \frac{\dot{U}_o}{\dot{U}_i} = -\frac{Z_2}{Z_1} = -\frac{1}{j\omega CR} = j\frac{1}{\omega}$$

$K_U(0{,}5j) = 2j = 2\angle 90°$, $K_U(j) = j = 1\angle 90°$, $K_U(2j) = 0{,}5j = 0{,}5\angle 90°$

$u_o(t) = 2\cos(0{,}5t + 90°)$ V $+ \sin(t + 90°) + 0{,}5\cos(2t + 90°)$ V

3.5 Fourier Analysis

$x(t)$ has period T → fundamental frequency $\omega_o = \dfrac{2\pi}{T}$

1) Trigonometric Fourier Series

$$x(t) = X_o + \sum_{k=1}^{+\infty}\left(S_k \sin(k\omega_o t) + C_k \cos(k\omega_o t)\right)$$

$$X_o = \frac{1}{T}\int_0^T x(t)dt,\qquad S_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_o t)dt,\qquad C_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_o t)dt$$

2) **Exponential Fourier Series**

$$x(t) = \sum_{k=-\infty}^{+\infty} \dot{X}_k e^{k\omega_o t} = X_o + \sum_{k=1}^{+\infty} 2|\dot{X}_k|\cos(k\omega_o t+\varphi_k)$$

$$\dot{X}_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt = |\dot{X}_k| \angle\varphi_k$$

Parseval theorem

3) **Frequency spectrum** $|\dot{X}_k| \angle\varphi_k$

Example: The circuit in Fig.3.5.1 has R = 1 Ω, L = 1 H. e(t) is a periodic voltage source with period $T_o = 2\pi$ s. Expand e(t) into Fourier series. Find i(t).

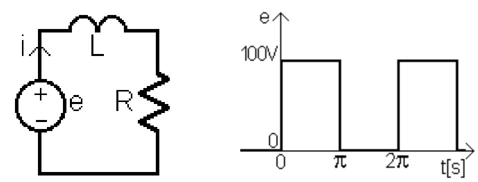


Fig.3.5.1

Solution:

$$e(t) = 50 + \sum_{k=1}^{\infty} \frac{100}{k\pi}\left(1-(-1)^k\right)\sin(kt) \text{ V}$$

DC component: $I_o = 50$ A

The kth harmonic $i_k(t) = \dfrac{100\left(1-(-1)^k\right)}{k\pi\sqrt{1+k^2}}\sin(kt - \text{arctg}(k))$ A

$$\Rightarrow i(t) = 50 + \sum_{k=1}^{\infty} \frac{100\left(1-(-1)^k\right)}{k\pi\sqrt{1+k^2}}\sin(kt - \text{arctg}(k)) \text{ A}$$

Example: The circuit in Fig.3.5.2 has R = 100 Ω, C = 10 μF. e is a periodic voltage source with period $T_o = 2\pi$ s. Expand e(t) into Fourier series. Find an expression of u(t).

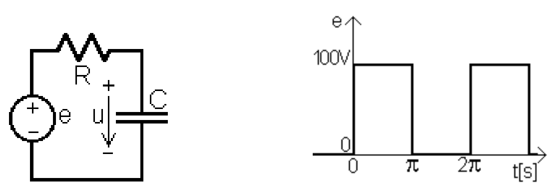
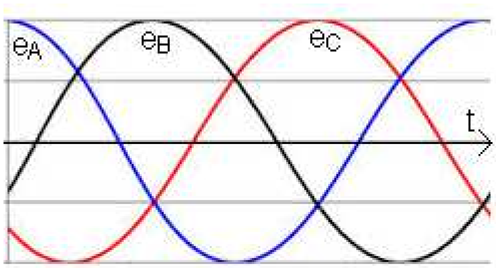


Fig.3.5.2

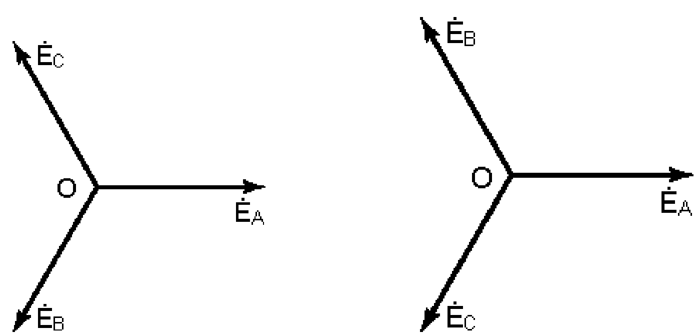
Solution: $e(t) = 50 + \sum_{k=1}^{\infty} \dfrac{100}{k\pi}\left(1-(-1)^k\right)\sin(1000kt)$ V

3.6 Three-Phase Circuit



Balanced Positive Sequence

Fig.3.6.1: Three-Phase Waveform



Balanced Positive Sequence Balanced Negative Sequence

Fig.3.6.2: Three-Phase Phasor

Note that a system of three unbalanced phasors can be resolved in the following three symmetrical components:

- Positive Sequence: A balanced three-phase system with the same phase sequence as the original sequence.
- Negative sequence: A balanced three-phase system with the opposite phase sequence as the original sequence.
- Zero Sequence: Three phasors that are equal in magnitude and phase.

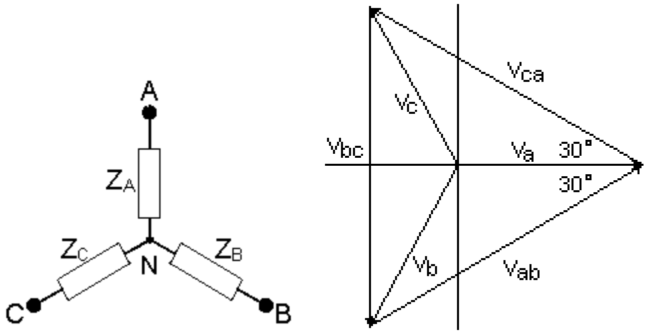


Fig.3.6.3: Relation between line voltage and phase voltage in balanced positive sequence wye configuration

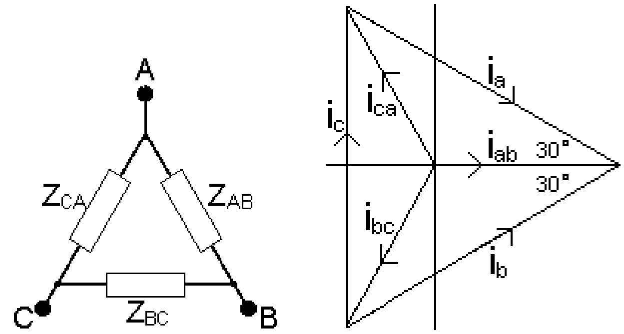


Fig.3.6.4: Relation between line current and phase current in balanced positive sequence delta configuration

Example: The three-phase load in Fig. 3.6.5 is connected to a three phase balanced positive sequence voltage source with phase voltage U_p = 220 V, Z_d = 1 Ω, Z_1 = 10-10j Ω and Z_2 = 30+30j Ω.

Find I_A, I_{A1}, I_{A2}, I_{AB}.

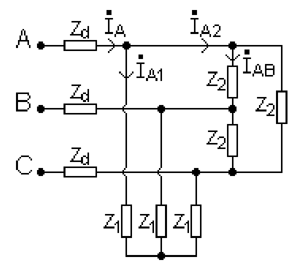
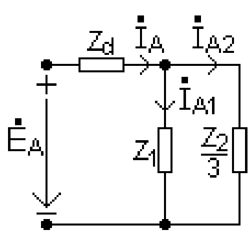


Fig. 3.6.5

Solution



Phase A

$$Z = Z_1 // \frac{Z_2}{3} = 10\ \Omega$$

$$\dot{I}_A = \frac{\dot{E}_A}{Z_d + Z} \Rightarrow I_A = \frac{220}{11} = 20\text{ A}$$

$$\dot{I}_{A1} = \frac{Z\dot{I}_A}{Z_1} \Rightarrow I_{A1} = \frac{200}{10\sqrt{2}} = 10\sqrt{2}\text{ A}$$

$$\dot{I}_{A2} = \frac{3Z\dot{I}_A}{Z_2} \Rightarrow I_{A2} = \frac{200}{10\sqrt{2}} = 10\sqrt{2}\text{ A}$$

$$I_{AB} = \frac{I_{A2}}{\sqrt{3}} = 10\sqrt{\frac{2}{3}}\text{ A}$$

3.7 Computer-Aided Circuit Analysis

A powerful computer software package known as PSpice is commonly employed for rapid analysis of circuits, and the schematic capture tools are typically integrated with either a printed circuit board or integrated circuit layout tool. SPICE (Simulation Program with Integrated Circuit Emphasis) is now an industry standard. Depending on the type of circuit application being considered, there are now several companies offering variations of the basic SPICE package.

Although computer-aided analysis is a relatively quick means of determining voltages and currents in a circuit, we should be careful not to allow simulation packages to completely replace traditional “paper and pencil” analysis. There are several reasons for this. First, in order to design we must be able to analyze. Overreliance on software tools can inhibit the development of necessary analytical skills, similar to introducing calculators too early in grade school. Second, it is virtually impossible to use a complicated software package over a long period of time without making some type of data-entry error. If we have no basic intuition as to what type of answer to expect from a simulation, then there is no way to determine whether or not it is valid. Thus, the generic name really is a fairly accurate description: computer-aided analysis. Human brains are not obsolete. Not yet, anyway.

Chapter 4: Transient Analysis

4.1 Introduction

1) The Source-Free RL Circuit

Example: Consider the circuit in Fig.4.1.1. The switch K is closed for very long time and suddenly open at t = 0. Find i?

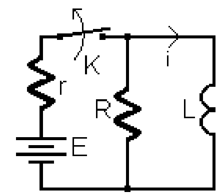


Fig.4.1.1

t < 0: K is closed for very long time. The circuit is in DC steady-state. The inductor acts as a short circuit and $i = I_o = E/r$

t > 0: K is open. The Kirchhoff's voltage law yields

$$L\frac{di}{dt} + Ri = 0 \Rightarrow i(t) = I_o e^{-t/\tau}$$

where $\tau = L/R$ is the time constant of LR circuit and I_o is the initial condition

2) The Source-Free RC Circuit

Example: Consider the circuit in Fig.4.1.2. The switch K is closed for very long time and suddenly open at t = 0. Find u?

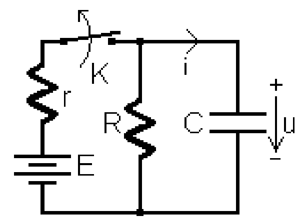


Fig.4.1.2

t < 0: K is closed for very long time. The circuit is in DC steady-state. The capacitor acts as an open circuit and $u = U_o = RE/(r+R)$

t > 0: K is open. The Kirchhoff's voltage law yields

$$RC\frac{du}{dt} + u = 0 \Rightarrow u(t) = U_o e^{-t/\tau}$$

where $\tau = RC$ is the time constant of RC circuit and U_o is the initial condition

3) The Lossless LC Circuit

Example: Consider the circuit in Fig.4.1.3. The switch K is closed for very long time and suddenly open at t = 0. Find u and i?

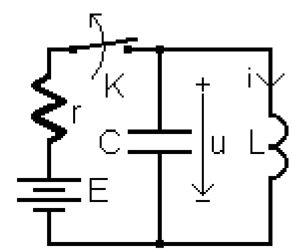


Fig.4.1.3

t < 0: K is closed for very long time. The circuit is in DC steady-state. The capacitor acts as an open circuit and the inductor acts as a short circuit, i = E/r and u = 0.

t > 0: K is open: $i = -C\frac{du}{dt}$ and $u = L\frac{di}{dt} \Rightarrow \frac{d^2u}{dt^2} + \frac{1}{LC}u = 0$

The characteristic equation (or auxiliary equation): $p^2 + \frac{1}{LC} = 0 \Rightarrow p = \pm j\sqrt{\frac{1}{LC}}$: undamped response.

4) The Source-Free Parallel RLC Circuit

Example: Consider the circuit in Fig.4.1.4. The switch K is closed for very long time and suddenly open at t = 0. Find u and i?

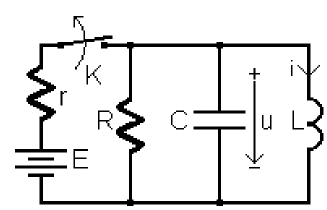


Fig.4.1.4

t < 0: K is closed for very long time. The circuit is in DC steady-state. The capacitor acts as an open circuit and the inductor acts as a short circuit, $i = I_o = E/r$ and u = 0.

t > 0: K is open: $\frac{u}{R} + C\frac{du}{dt} + i = 0$ and $u = L\frac{di}{dt}$

$$\Rightarrow \frac{1}{R}\frac{du}{dt} + C\frac{d^2u}{dt^2} + \frac{u}{L} = 0 \Rightarrow \frac{d^2u}{dt^2} + \frac{1}{RC}\frac{du}{dt} + \frac{1}{LC}u = 0$$

The characteristic equation: $p^2 + \frac{1}{RC}p + \frac{1}{LC} = 0 \Rightarrow p = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$

p: complex frequencies, $\omega_o = \sqrt{\frac{1}{LC}}$: resonant frequency, $\alpha = \frac{1}{2RC}$: exponential damping coefficient

$\alpha > \omega_o \Rightarrow$ p are real numbers: overdamped response

$\alpha < \omega_o \Rightarrow$ p are complex numbers: underdamped response

$\alpha = \omega_o$: critically damped response

5) The Source-Free Series RLC Circuit

Example: Consider the circuit in Fig.4.1.5. The switch K is closed for very long time and suddenly open at t = 0. Find u and i?

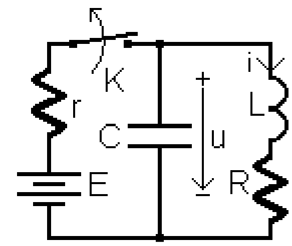


Fig.4.1.5

t < 0: K is closed for very long time. The circuit is in DC steady-state. The inductor acts as a short circuit and the capacitor acts as an open circuit, $i = I_o = E/(r+R)$ and $u = U_o = RI_o = RE/(r+R)$.

t > 0: K is open: $i = -C\frac{du}{dt}$ and $u = Ri + L\frac{di}{dt} \Rightarrow u = -RC\frac{du}{dt} - LC\frac{d^2u}{dt^2}$

$$\Rightarrow \frac{d^2u}{dt^2} + \frac{R}{L}\frac{du}{dt} + \frac{1}{LC}u = 0$$

The characteristic equation: $p^2 + \frac{R}{L}p + \frac{1}{LC} = 0 \Rightarrow p = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$

p: complex frequencies, $\omega_o = \sqrt{\frac{1}{LC}}$: resonant frequency, $\alpha = \frac{R}{2L}$: exponential damping coefficient

$\alpha > \omega_o \Rightarrow$ p are real numbers: overdamped response

$\alpha < \omega_o \Rightarrow$ p are complex numbers: underdamped response

$\alpha = \omega_o$: critically damped response

4.2 Classical Method

Example A: Consider the circuit in Fig.A with E = 100 V, r = 10 Ω, R = 100 Ω, L = 0.1 H. Initially the switch K is closed for very long time and suddenly open at t = 0. Find u(t) and i (t).

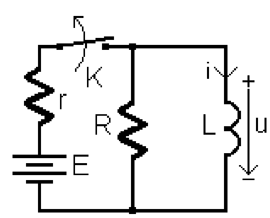


Fig.A

t < 0: K is closed for very long time $\Rightarrow$ the circuit is operating in DC steady-state $\Rightarrow$ the inductor acts as a short circuit

$\Rightarrow$ $u = 0$ V

$i = E/r = 10$ A

t > 0: K is open

KVL: $L\frac{di}{dt} + Ri = 0$

The characteristic equation: $pL + R = 0 \quad \Rightarrow \quad p = -\frac{R}{2L} = -1000$

$\Rightarrow \quad i(t) = Ce^{-1000t}$ [A]

Initial conditions:

$i(0) = 10 = C$

$\Rightarrow \quad i(t) = 10e^{-1000t}$ [A]

$\Rightarrow \quad u(t) = L\frac{di}{dt} = -1000e^{-1000t}$ [V]

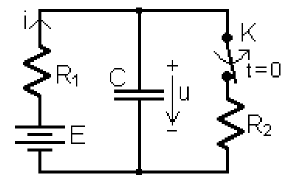


Fig.B

Example B: Consider the circuit in Fig.B with E = 100 V, $R_1 = R_2 = 100\ \Omega$, C = 10 μF. Initially the switch K is closed for very long time and suddenly open at t = 0. Find u(t).

t < 0: K is closed for very long time ⇒ the circuit is operating in DC steady-state ⇒ the capacitor acts as an open circuit.

$\Rightarrow \quad i = E/(R_1+R_2) = 0.5$ [A] $\quad \Rightarrow \quad u = R_2 i = 50$ [V]

<u>t > 0</u>: K is open

KVL: $R_1 i + u = E$

$i = C\frac{du}{dt}$

$\Rightarrow \quad R_1 C\frac{du}{dt} + u = E$

The natural response:
Characteristic polynomial (of the homogeneous differential equation)

$R_1Cp + 1 = 0 \quad \Rightarrow \quad p = -\frac{1}{R_1C} = -1000$

$\Rightarrow \quad u_N(t) = A_1e^{-1000t}$ [V]

The forced response (steady-state response)
(the response of the circuit a long time after switching)**:**

$u_{SS} = E = 100$ V

The complete response of the circuit is the sum of the natural response and the forced response:

$u(t) = u_{SS}(t) + u_N(t) = 100 + A_1e^{-1000t}$ [V]

Initial conditions:

$u(0) = 50 = 100 + A_1 \Rightarrow A_1 = -50$

$\Rightarrow \quad u(t) = 100 - 50e^{-1000t}$ [V]

Example: The circuit in Fig.4.2.1 has $e(t) = 100\sin(1000t)$ V, $R_1 = R_2 = 100\Omega$, L = 0.1 H, C = 10 μF. For t < 0, the switch K is closed. For t > 0, the switch K is open. Find i(t).

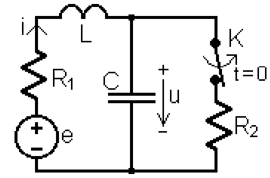
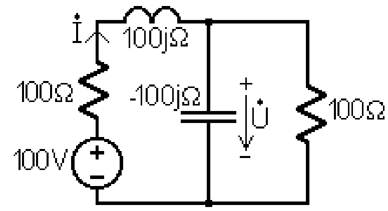


Fig.4.2.1

t < 0: K is closed, the circuit is in sinusoidal steady state.



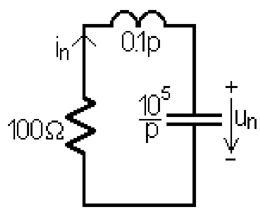
$$-100j \,//\, 100 = \frac{-100j(100)}{100-100j} = 50 - 50j\ \Omega$$

$$\dot{I} = \frac{100}{100+100j+50-50j} = \frac{2}{3+j} = \frac{2}{\sqrt{10}}\angle{-18.44°}$$

$$\Rightarrow i(t) = \frac{2}{\sqrt{10}}\sin(1000t - 18.44°)\text{ A}$$

$$\dot{U} = 50(1-j)\dot{I} = 50\sqrt{2}\angle{-45°}\frac{2}{\sqrt{10}}\angle{-18.44°} = \frac{100}{\sqrt{5}}\angle{-63.44°} \quad \Rightarrow \quad u(t) = \frac{100}{\sqrt{5}}\sin(1000t - 63.44°)\text{ V}$$

t > 0: K is open, the circuit is in transient state



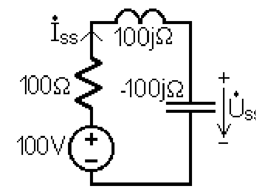
The natural response: Characteristic Polynomial (of the Homogeneous Differential Equation)

$$100 + 0.1p + \frac{10^5}{p} = 0 \Rightarrow p^2 + 10^3p + 10^6 = 0$$

$$\Rightarrow p = -500 \pm j500\sqrt{3} \Rightarrow i_n(t) = A_1e^{-500t}\cos(500\sqrt{3}\,t + A_2)\text{ A}$$

The forced response or steady-state response

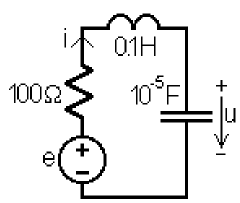
(the response of the circuit a long time after switching):



$$\dot{I}_{SS} = \frac{100}{100+100j-100j} = 1\angle 0° \qquad \Rightarrow \qquad i_{SS}(t) = \sin(1000t)\text{ A}$$

The complete response of the circuit is the sum of the natural response and the forced response:

$i(t) = i_{XL}(t) + i_{TD}(t) = \sin(1000t) + A_1e^{-500t}\cos(500\sqrt{3}\,t + A_2)$ A



Initial conditions: i(0+), i'(0+)

t = 0−: the instant prior to t = 0

t = 0+: the instant immediately after switching

The current flowing through an inductor is continuous

$$i_L(0-) = i(0-) = \frac{2}{\sqrt{10}}\sin(-18.44°) = -0.2 = i_L(0+) = i(0+)$$

The voltage across a capacitor is continuous

$$u_C(0-) = u(0-) = \frac{100}{\sqrt{5}}\sin(-63.44°) = -40 = u_C(0+) = u(0+)$$

KVL: $-e + 100i + 0.1i' + u = 0 \quad \Rightarrow \quad i'(0+) = 10(e(0+) - 100i(0+) - u(0+)) = 600$

$i(0+) = A_1\cos(A_2) = -0.2$

$$i'(0+) = 1000 - 500A_1\cos(A_2) - 500\sqrt{3}A_1\sin(A_2) = 600 \Rightarrow A_1\sin(A_2) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \operatorname{tg}(A_2) = -\frac{5}{\sqrt{3}} \Rightarrow A_2 = -71°$$

$$\Rightarrow A_1 = -0.6$$

$i(t) = \sin(1000t) - 0.6e^{-500t}\cos(500\sqrt{3}\,t - 71°)$ A

4.3 Circuit Analysis using the Laplace Transform

1) The Laplace Transform

Definition $X(s) = \int_0^{+\infty} x(t)e^{-st}dt$

$$x(t) = \frac{1}{j2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$$

Properties of the Laplace transform

Linearity	$y(t) = a_1x_1(t) + a_2x_2(t)$	$\Rightarrow$	$Y(s) = a_1X_1(s) + a_2X_2(s)$
Differentiation	$y(t) = \frac{dx}{dt}$ $y(t) = \frac{d^nx}{dt^n}$	$\Rightarrow$ $\Rightarrow$	$Y(s) = sX(s) - x(0)$ $Y(s) = s^nX(s) - s^{n-1}x(0) - s^{n-2}x'(0) - \ldots - x^{(n-1)}(0)$
Integration	$y(t) = \int_0^t x(\tau)d\tau$	$\Rightarrow$	$Y(s) = \frac{X(s)}{s}$
Initial value theorem	$\lim_{s\to\infty} sX(s) = x(0^+)$		
Final value theorem	$\lim_{s\to 0} sX(s) = x(+\infty)$		
Shift in time	$y(t) = x(t-T)u(t-T)$ $T > 0$	$\Rightarrow$	$Y(s) = e^{-Ts}X(s)$
Complex shifting	$y(t) = e^{-at}x(t)$	$\Rightarrow$	$Y(s) = X(s+a)$
Real convolution	$z(t) = x(t)*y(t)$ $x(t)*y(t) = \int_{-\infty}^{+\infty} x(\tau)y(t-\tau)d\tau$	$\Rightarrow$	$Z(s) = X(s)Y(s)$

Laplace transform table

x(t)	X(s)	x(t)	X(s)
$\delta(t)$	1	1	$\frac{1}{s}$
t	$\frac{1}{s^2}$	$\sin(\omega_o t)$	$\frac{\omega_o}{s^2+\omega_o^2}$
t^n	$\frac{n!}{s^{n+1}}$	$\cos(\omega_o t)$	$\frac{s}{s^2+\omega_o^2}$
e^{-at}	$\frac{1}{s+a}$	$e^{-at}\sin(\omega_o t)$	$\frac{\omega_o}{(s+a)^2+\omega_o^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$e^{-at}\cos(\omega_o t)$	$\frac{s+a}{(s+a)^2+\omega_o^2}$

where $\delta(t)$ is the unit impulse: $\delta(t) = \begin{cases} +\infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$

$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$

2) Basic circuit laws

Example: The circuit in Fig.4.3.1 has
E = 20V (DC)
R = 20Ω, C = 0,05F.
The capacitor C is initially uncharged.
At t = 0, the switch K is closed. Determine an expression for the voltage u and the current i.

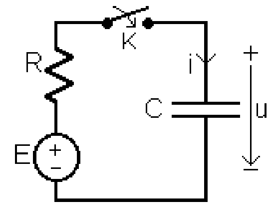
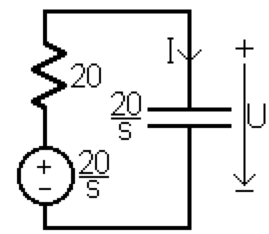


Fig.4.3.1

Solution

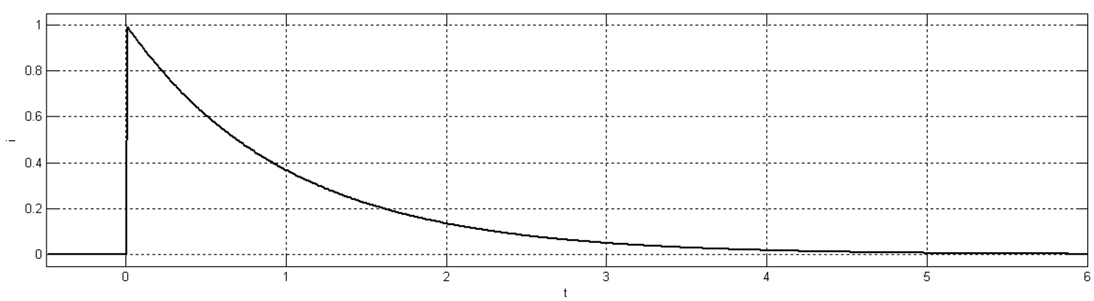
<u>**t < 0**</u>: K is open and C is uncharged $\Rightarrow$ i(t) = 0A, u(t) = 0V

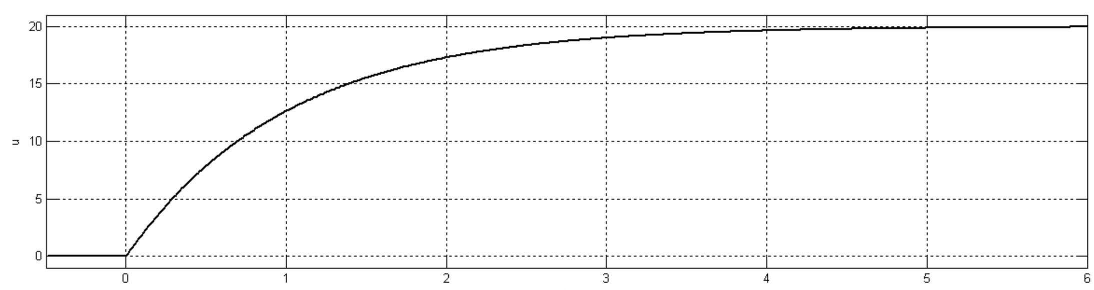
<u>**t > 0**</u>: K is closed



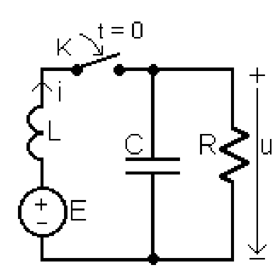
$$I(s) = \frac{\frac{20}{s}}{20 + \frac{20}{s}} = \frac{1}{s+1} \qquad \Rightarrow \qquad i(t) = e^{-t}$$

$$U(s) = \frac{20}{s(s+1)} = \frac{20}{s} - \frac{20}{s+1} \qquad \Rightarrow \qquad u(t) = 20(1 - e^{-t})$$



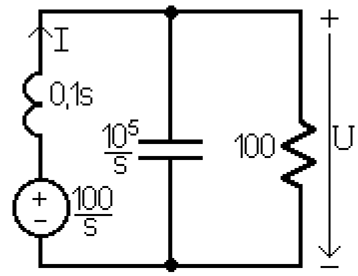


Example: The circuit in Fig.4.3.2 has E = 100V (DC), R = 100 Ω, L = 100 mH, C = 10 μF. The switch K is open for very long time and is closed at t = 0. Using the Laplace transform, find u(t) and i(t).



<u>**t < 0**</u>: K is open $\Rightarrow$ **u = 0 V**
i = 0 A

Fig.4.3.2



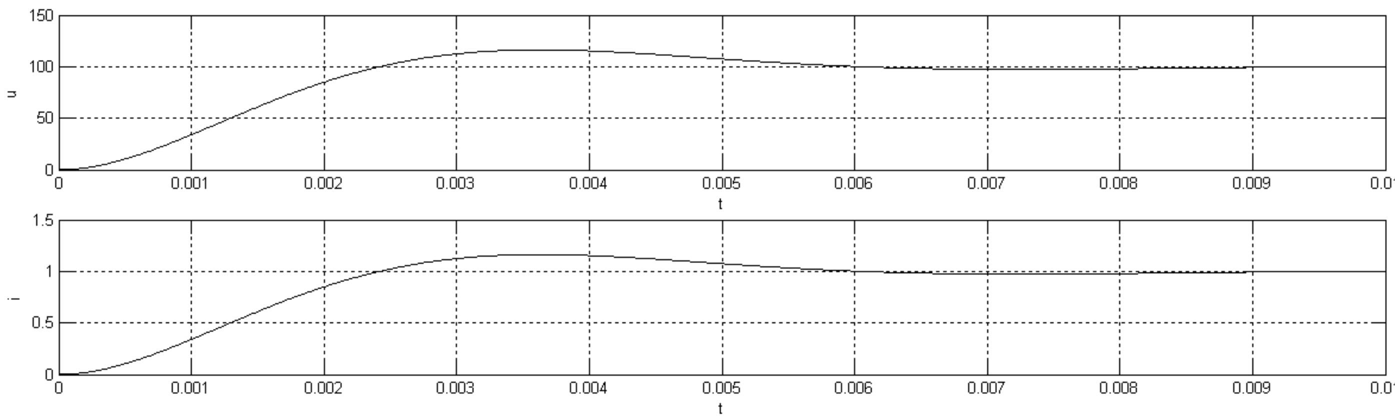
t > 0: $U\left(\frac{10}{s}+\frac{s}{10^5}+\frac{1}{100}\right) = \frac{10^3}{s^2}$

$\Rightarrow \quad U = \frac{10^8}{s(s^2+10^3 s+10^6)} = \frac{100}{s} - \frac{100(s+500)+5x10^4}{(s+500)^2+(500\sqrt{3})^2}$

$\Rightarrow \quad u(t) = 100 - 100e^{-500t}\cos(500\sqrt{3}\,t) - \frac{100}{\sqrt{3}}e^{-500t}\sin(500\sqrt{3}\,t)$ V

$I = \frac{(s+10^3)U}{10^5} = \frac{10^3(s+10^3)}{s(s^2+10^3 s+10^6)} = \frac{1}{s} - \frac{s}{s^2+10^3 s+10^6} = \frac{1}{s} - \frac{s+500-500}{(s+500)^2+(500\sqrt{3})^2}$

$\Rightarrow \quad i(t) = 1 - e^{-500t}\cos(500\sqrt{3}\,t) + \frac{1}{\sqrt{3}}e^{-500t}\sin(500\sqrt{3}\,t)$ A



Example 2: The circuit in Fig.4.3.2 has R = 100 Ω, L = 100 mH. Using the Laplace transform, find i(t).

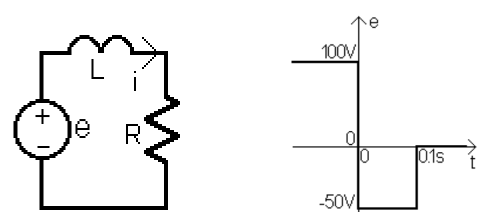
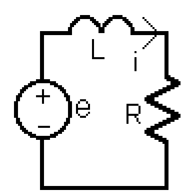


Fig.4.3.2

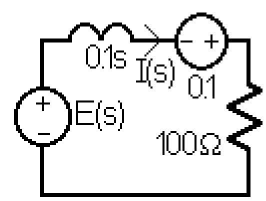


t < 0:

The circuit is in DC steady state with e = 100V

$\Rightarrow \quad i = 1A$

t > 0:



$e(t) = -50(1(t) - 1(t-0.1)) \Rightarrow E(s) = -\frac{50}{s}\left(1-e^{-0.1s}\right)$

$$I(s) = \frac{0.1-\frac{50}{s}\left(1-e^{-0.1s}\right)}{0.1s+100} = \frac{1}{s+1000} - \frac{500\left(1-e^{-0.1s}\right)}{s(s+1000)} = \frac{1}{s+1000} - \frac{500}{s(s+1000)} + \frac{500e^{-0.1s}}{s(s+1000)}$$

$$\frac{1}{s+1000} \leftrightarrow e^{-1000t}$$

$$\frac{500}{s(s+1000)} = \frac{0.5}{s} - \frac{0.5}{s+1000} \leftrightarrow 0.5 - 0.5e^{-1000t}$$

$$\frac{500e^{-0.1s}}{s(s+1000)} \leftrightarrow [0.5 - 0.5e^{-1000t + 100}]1(t-0.1)$$

$$i(t) = -0.5 + 1.5e^{-1000t} + [0.5 - 0.5e^{-1000t + 100}]1(t-0.1)$$

Exercises

1.1 Given the circuit in Fig.E1.1 with E = 60 V, J = 1 A, $R_1 = R_2 = R_3 = 10\ \Omega$, $R_4 = 30\ \Omega$. Find I_0, I_1, I_2, I_3, I_4.

1.2 Given the circuit in Fig.E1.2 with E = 20 V, J = 2 A, $R_0 = 5\ \Omega$, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$, $R_4 = 10\ \Omega$. Find I_0, I_1, I_2, I_3, I_4.

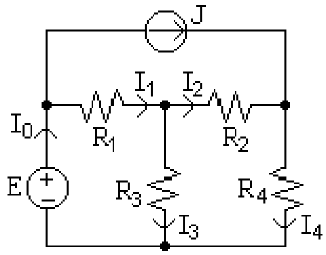


Fig.E1.1

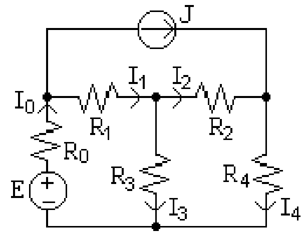


Fig.E1.2

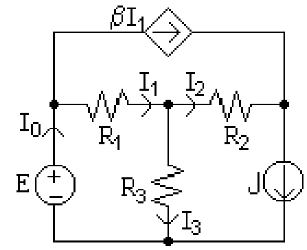


Fig.E1.3

1.3 Given the circuit in Fig.E1.3 with E = 50 V, J = 1 A, $\beta = 2$, $R_1 = R_2 = R_3 = 10\ \Omega$. Find I_0, I_1, I_2, I_3.

1.4 Given the circuit in Fig.E1.4 with E = 20 V, J = 2 A, $\beta = 2$, $R_0 = 5\ \Omega$, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$. Find I_0, I_1, I_2, I_3.

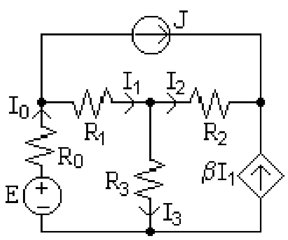


Fig.E1.4

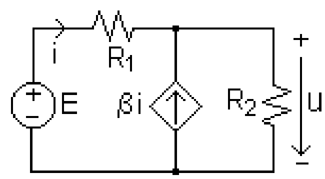


Fig.E1.5

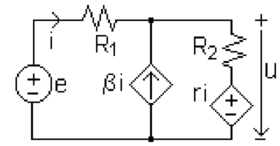


Fig.E1.6

1.5 Given the circuit in Fig.E1.5 with E = 20 V, $R_1 = 1\ \text{k}\Omega$, $R_2 = 10\ \Omega$, $\beta = 99$. Find u and i.

1.6 Given the circuit in Fig.E1.6 with e = 20sin(10t) V, $R_1 = 100\ \Omega$, $R_2 = 25\ \Omega$, $\beta = 3$, $r = 10\ \Omega$. Find u and i.

1.7 Given the circuit in Fig E1.7 with R = 0.1 Ω, L = 0.1 H. Find u if $i = 10^3 t$ A

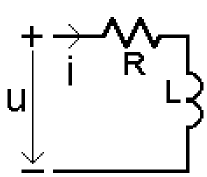


Fig.E1.7

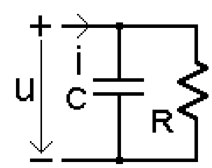


Fig.E1.8

1.8 Given the circuit in Fig E1.8 with R = 10 kΩ, C = 100 μF. Find i if $u = 10^5 t$ V

2.1 Given the circuit in Fig.E2.1 with e(t) = 50sin(100t) V, R = 10 Ω, L = 1 H, C = 1000 μF, $\beta = 19$. Find u(t) and i(t).
Solution: i(t) = 0.5sin(100t) A; $u(t) = 50\sqrt{2}\sin(100t - 45^o)$ V

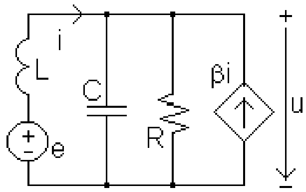


Fig.E2.1

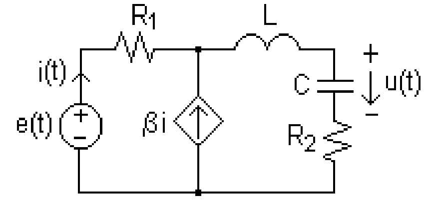


Fig.E2.2

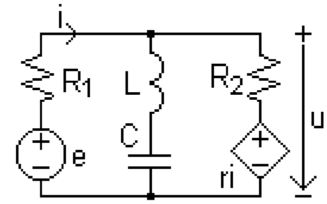


Fig.E2.3

2.2 Given the circuit in Fig.E2.2 with e(t) = 20sin(1000t) V, R_1 = 1 kΩ, R_2 = 10 Ω, L = 30 mH, C = 100 μF, β = 99. Find u(t) and i(t).

Solution: $i(t) = \dfrac{1}{100\sqrt{2}}\sin(1000t - 45^\circ)$ A; $u(t) = 5\sqrt{2}\sin(1000t - 135^\circ)$ V

2.3 Given the circuit in Fig.E2.3 with e(t) = 60sin(1000t) V, $R_1 = R_2 = r$ = 30 Ω, L = 30 mH, C = 50 μF. Find u(t) and i(t).

Solution: $i(t) = \dfrac{2\sqrt{5}}{3}\sin(1000t - 27^\circ)$ A; $u(t) = 20\sqrt{2}\sin(1000t + 45^\circ)$ V

2.4 The circuit in Fig.E2.4 is in sinusoidal steady-state with $U = U_L$ = 100V, $I_C = I_R$ = 1A (RMS values). Sketch the phasors of the voltages and currents. Find R, L, C and the input impedance of the one-port network if f = 50 Hz.

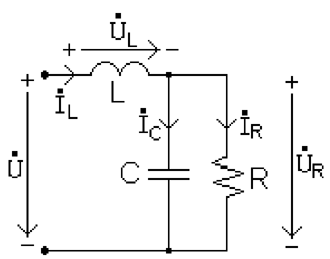


Fig.E2.4

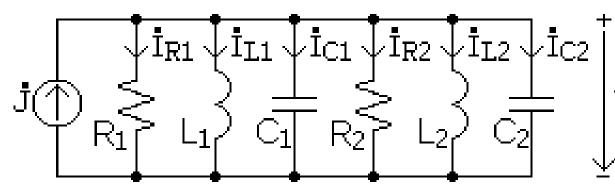


Fig.E2.5

2.5 The circuit in Fig.E2,5 is in sinusoidal steady-state with $I_{R1} = I_{L1}$ = 1A, $I_{C1} = I_{C2} = I_{R2}$ = 2A, I_{L2} = 3A, U = 50V (RMS values). Sketch the phasors of the voltages and currents. Find J and the complex power of the current source. Show that the sum of the complex powers for the six passive elements is equal to the complex power of the source.

Solution: $\dot{J} = 3\angle 0^\circ$ A

2.6 Find the resonant frequencies of the one-port networks in Fig.E2.6.

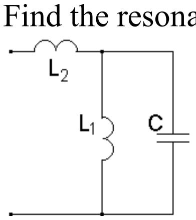


Fig.E2.6A

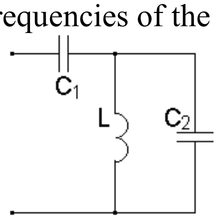


Fig.E2.6B

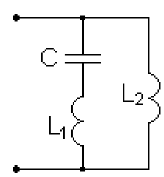


Fig.E2.6C

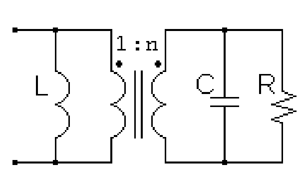


Fig.E2.6D

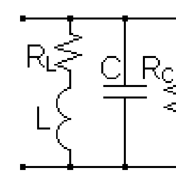


Fig.E2.6E

Solution:

Fig.E2.6A: $\omega_1 = \dfrac{1}{\sqrt{L_1C}}$, $\omega_2 = \sqrt{\dfrac{1}{L_1C} + \dfrac{1}{L_2C}}$; Fig.E2.6B: $\omega_1 = \dfrac{1}{\sqrt{LC_2}}$, $\omega_2 = \sqrt{\dfrac{1}{L(C_1 + C_2)}}$

Fig.E2.6C: $\omega_1 = \dfrac{1}{\sqrt{L_1C}}$, $\omega_2 = \sqrt{\dfrac{1}{(L_1 + L_2)C}}$; Fig.E2.6D: $\omega = \dfrac{1}{n\sqrt{LC}}$; Fig.E2.6E: $\omega = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R_L}{L}\right)^2}$

2.7 The circuit in Fig.E2.7 is in sinusoidal steady-state with U = 220 V (RMS), f = 50 Hz. The load Z_L is inductive with active power P_L = 10 kW and $\cos(\varphi_L) = 0.707$. Find the capacitance C such that the one-port network is inductive with $\cos(\varphi) = 0.95$. Find I, I_L, I_C.

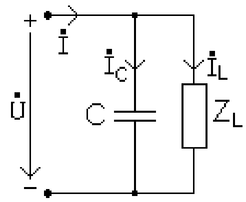


Fig.E2.7

2.8 Given the circuit in Fig.E2.8 with $\dot{E}$ = 20 V (RMS), ω = 500 rad/s, L_1 = 200 mH; C_1 = 10 μF, C_2 = 10 μF; R_2 = 200 Ω, the two-port network has $Z = \begin{bmatrix} 200 & 0 \\ 50 & 200j \end{bmatrix}$ Ω. Find $\dot{I}_1$, $\dot{I}_2$, $\dot{U}_1$, $\dot{U}_2$.

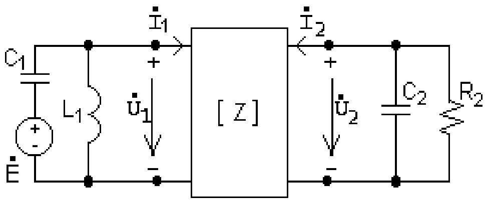


Fig.E2.8

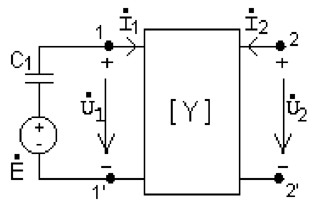


Fig.E2.9

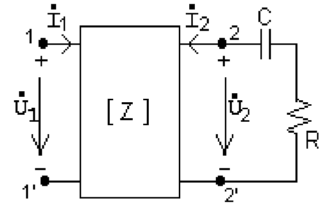


Fig.E2.10

2.9 Given the circuit in Fig.E2.9 with $\dot{E}$ = 100 V (RMS), ω = 500 rad/s, C_1 = 20 μF, the two-port has $Y = \begin{bmatrix} 0{,}002(1-2j) & 0 \\ 0{,}2 & 0{,}01(1+j) \end{bmatrix}$ S. Find the Thevenin equivalent circuit and the maximum active power of the one-port network.

2.10 Given the circuit in Fig.E2.10 with ω = 1000 rad/s, R = 100 Ω, C = 20 μF, the two-port has $Z = \begin{bmatrix} 100 & 0 \\ 50 & 100(1+j3) \end{bmatrix}$ Ω. Find $K_U = \dot{U}_2/\dot{U}_1$ and $K_I = \dot{I}_2/\dot{I}_1$.

2.11 Find the transfer function $H(j\omega) = \dfrac{V_o}{V_i}$ of the filters in Fig.E2.11 where R = 10 kΩ, C = 1 μF. Sketch the magnitude and the phase of H(jω) versus ω. Find the cut-off frequency ω_C.

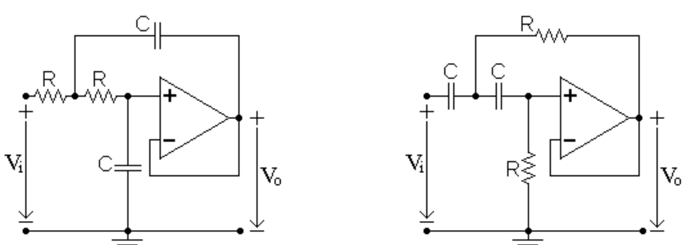


Fig.E2.11

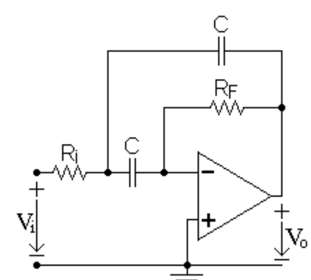


Fig.E2.12

2.12 Find the transfer function $H(j\omega) = \dfrac{V_o}{V_i}$ of the filter in Fig.E2.12 where $R_F = 10\ k\Omega$, $R_i = 10\ k\Omega$, $C = 1\ \mu F$. Sketch the magnitude and the phase of $H(j\omega)$ versus ω. Find the cut-off frequencies ω_C.

2.13 The RL load in Fig.E2.13 is compensated by adding the shunt capacitance C so that the power factor of the combined (compensated) circuit is exactly unity. How is C related to R, L, and ω in that case?

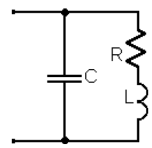
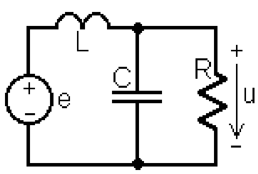


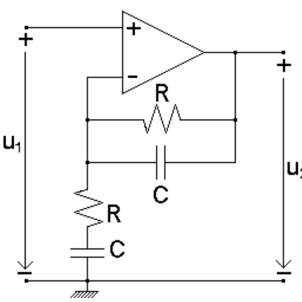
Fig.E2.13

3.1 Given the circuit in Fig.E3.1 with $R = 100\ \Omega$, $L = 100$ mH, $C = 10\ \mu F$.

a) Find the voltage transfer function $K_u(j\omega) = \dfrac{\dot{U}}{\dot{E}}$

b) Find u(t) if e(t) = 100 + 100sin(500t) + 100sin(1000t) V.





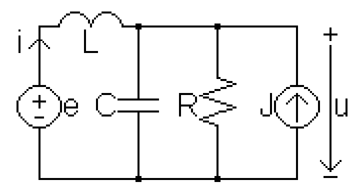


Fig.E3.1 Fig.E3.2 Fig.E3.3

3.2 Given the circuit in Fig.E3.2 with $R = 10\ k\Omega$, $C = 1\ \mu F$.

a) Find the voltage transfer function $K_u(j\omega) = \dfrac{\dot{U}_2}{\dot{U}_1}$

b) Find $u_2(t)$ if $u_1(t) = 100 + 100\sin(500t) + 100\sin(1000t)$ V.

3.3 Given the circuit in Fig.E3.3 with e(t) = 100 + 100sin(1000t) V, J = 2 + 2cos(1000t) A, $R = 100\ \Omega$, L = 100 mH, $C = 10\ \mu F$. Find u(t) and i(t).

3.4 The circuit in Fig.E3.4 has $R = 1\Omega$, L = 1H, C = 1F. J is a periodic current source with period $T_o = 2\pi$ [s]. Expand J(t) into Fourier series. Find an expression of u(t) and i(t).

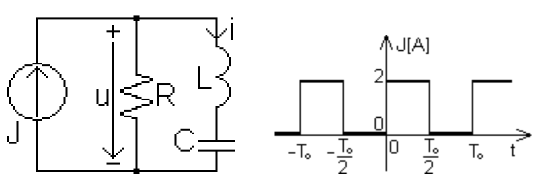


Fig.E3.4

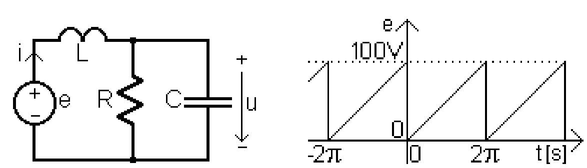
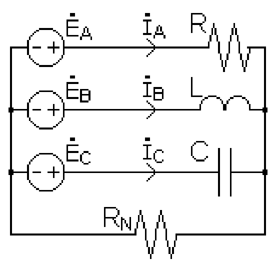


Fig.E3.5

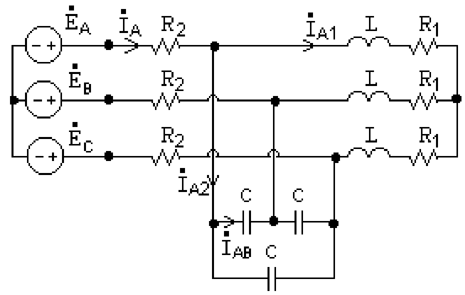
3.5 The circuit in Fig.E3.5 has $R = 1\Omega$, L = 1H, C = 1F. e is a periodic voltage source with period $T_o = 2\pi$ [s]. Expand e(t) into Fourier series. Find an expression of u(t) and i(t).

3.6 Given the three-phase circuit in Fig.3.6 with $R = \omega L = 1/(\omega C) = 100\ \Omega$. The three-phase source is balanced with line voltage $U_L = 380$ V (RMS). Find the voltage U_N between the two neutral points as function of R_N. Find R_N such that $U_N = U_d/10$. With this value of R_N, sketch the phasor diagram of the circuit.

3.7 Given the three-phase circuit in Fig.3.7 with $R_1 = \omega L = 100\ \Omega$, $1/\omega C = 600\ \Omega$, $R_2 = 20\ \Omega$. The three-phase source is balanced (+) sequence with phase voltage $V_P = 220$ V (RMS). Find I_A, I_{A1}, I_{A2} and I_{AB}.



Fig,3,6



Fig,3,7

4.1 Given the circuit in Fig.E4.1 with $e = 100\cos(1000t)$ V, $R = 100\ \Omega$, $L = 100$ mH, $C = 10\ \mu F$. The switch is open for very long time and is closed at $t = 0$. Find $u(t)$ and $i(t)$.

4.2 Given the circuit in Fig.E4.2 with $R = 100\Omega$, $L = 100$ mH, $C = 10\ \mu F$. For $t < 0$, K is open, the capacitor is uncharged and $i(t) = 0$ A. At $t = 0$, K is closed. Find $u(t)$, $i(t)$, $i_1(t)$ and $i_2(t)$ in two cases:

a) $e = 100$ V (DC) b) $e = 100\cos(1000t)$ V c) $e = 100 + 100\cos(1000t)$ V

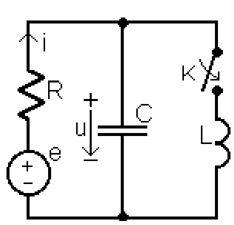


Fig.E4.1

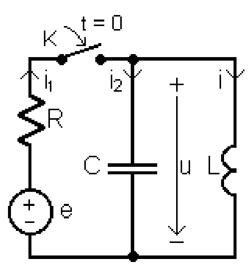


Fig.E4.2

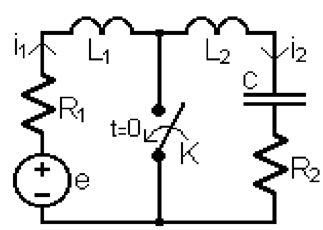


Fig.E4.3

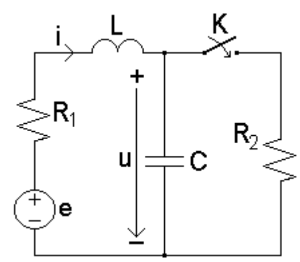


Fig.E4.4

4.3 Given the circuit in Fig.E4.3 with $e(t) = 100\sin(1000t)$ V, $R_1 = R_2 = 100\ \Omega$, $L_1 = 200$ mH, $L_2 = 100$ mH, $C = 10\ \mu F$. The switch K is open for very long time and is closed at $t = 0$. Find $i_1(t)$ and $i_2(t)$.

4.4 Given the circuit in Fig.E4.4 with $R_1 = R_2 = 100\ \Omega$, $L = 100$ mH, $C = 10\ \mu F$. The switch K is open for very long time and is closed at $t = 0$. Find $u(t)$ and $i(t)$ in two cases:

a) $e = 100$ V (DC) b) $e = 100\cos(1000t)$ V c) $e = 100 + 100\cos(1000t)$ V

4.5 Given the circuit in Fig.E4.5 with $R = 100\ \Omega$, $L = 100$ mH, $C = 10\ \mu F$. Find $u(t)$ and $i(t)$.

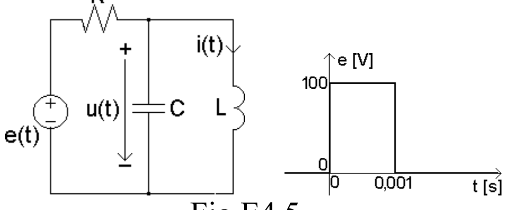


Fig.E4.5

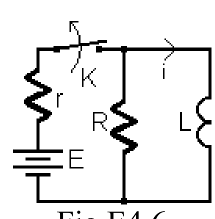


Fig.E4.6

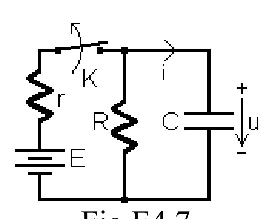


Fig.E4.7

4.6 Consider the circuit in Fig.E4.6 with E = 24 V, r = 100Ω, R = 1 kΩ, L = 100 mH. The switch K is closed for very long time and suddenly open at t = 0. Find the current i.

4.7 Consider the circuit in Fig.E4.7 with E = 24 V, r = 2 kΩ, R = 10 kΩ, C = 10 μF. The switch K is closed for very long time and suddenly open at t = 0. Find the voltage u.

5.1 Consider the circuit in Fig.E.1 with $e(t) = 20\sin(100\pi t)$ V, R = 1 kΩ, $V_{Z1} = V_{Z2} = 10$V. Find and sketch the waveform of the voltage u(t).

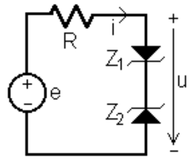


Fig.E5.1

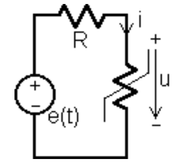


Fig.E5.2

5.2 Consider the circuit in Fig.E5.2 with $e(t) = 40 + \sin(100\pi t)$ V, R = 2 Ω, the Volt-Ampere characteristics of the nonlinear resistor is given by $u = i^3/50$. Linearize the circuit at the DC operating point and find i(t).

6.1 The lossless transmission line in Fig.E6.1 has length d = 20 m, $L_0 = 2.5$ μH/m and $C_0 = 1$ nF/m. The source is sinusoidal with amplitude E = 50 V, frequency f = 1 MHz. $R_1 = 25$ Ω. $R_2 = 100$ Ω. Find and sketch the distribution of the voltage and the current along the line.

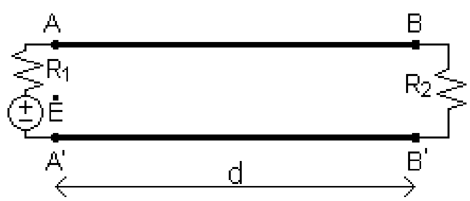


Fig.E6.1

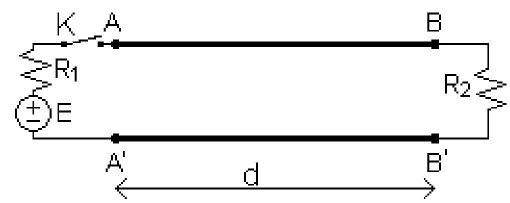


Fig.E6.2

6.2 The lossless transmission line in Fig.E6.2 has length d = 20 m, $L_0 = 2.5$ μH/m and $C_0 = 1$ nF/m. The DC voltage source has E = 50 V. $R_1 = 50$ Ω. $R_2 = 100$ Ω. The switch K is initially open and is closed at t = 0. Find and sketch the distribution of the voltage and the current along the line at times $t_1 = 0.6$ μs, $t_2 = 1.6$ μs and $t_3 = 2.6$ μs.