Enhanced study of complex systems by unveiling hidden symmetries with Dynamical Visibility

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Introduction

Nature presents complex systems all around us, for example, animal populations, neurons in our brain, the climate, COVID-19 infection rate. Our society also provides a wide variety of them, more even as technology advances rapidly: 5G technology, power grids, airports, or social networks. Characterizing and understanding complex dynamics is a relevant challenge today.

Photonic neurons are diode lasers with optical feedback and modulation, that can deliver optical spikes, replicating the behavior of other complex system's dynamics, such as neuronal activity [1], we perform experiments and analyze the complexity of these photonic neurons.

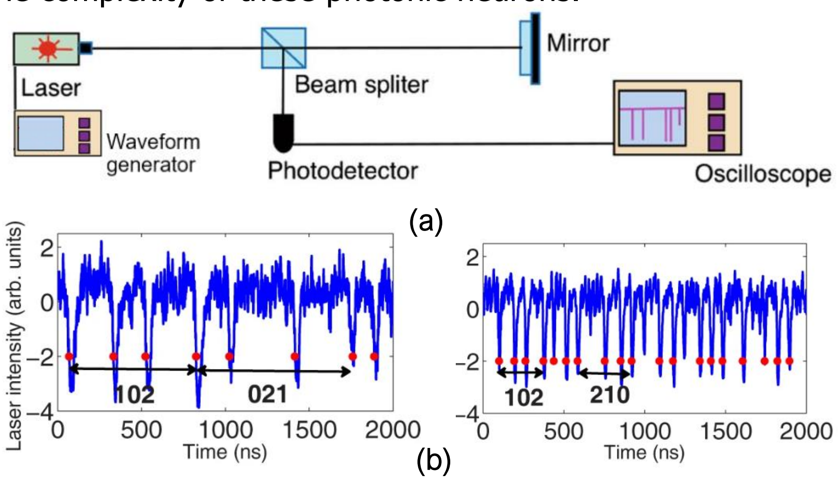

Fig.1. (a) Experiment set up **(b)** Time trace of spikes in the system

Analysis method

- We analyze the time series of the optical spikes of the photonic neurons for 10 different frequencies.
- We construct ordinal patterns, also known as words [1].
- We then plot the words probabilities as a function of modulation amplitude. The further spread out the words are, the more deterministic the dynamics is.

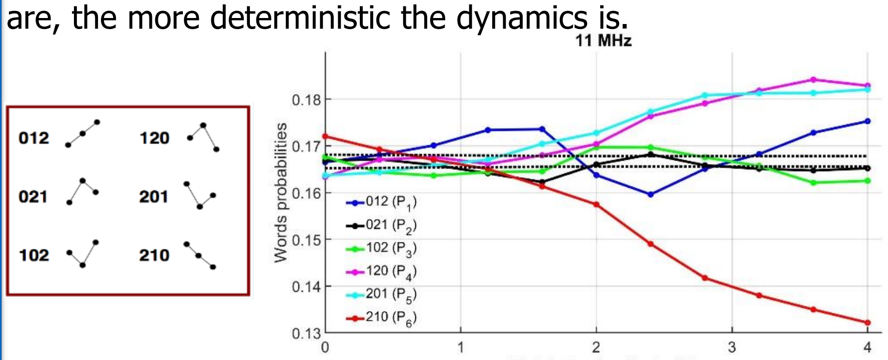

Fig.2. (a) Ordinal Patterns and words **(b)** Words probability vs Modulation Amplitude

Dynamical Visibility

- We develop a new technique, Dynamical Visibility, to unveil the hidden symmetry of the system.

$$V_\alpha = 1 - \left|\frac{\omega_1 - \omega_\alpha}{\omega_1 + \omega_\alpha}\right| \qquad V_\beta = 1 - \left|\frac{\omega_6 - \omega_\beta}{\omega_6 + \omega_\beta}\right|$$

$$V_\delta = 1 - \left|\frac{P_2 - P_3}{P_2 + P_3}\right| - \left|\frac{P_4 - P_5}{P_4 + P_5}\right|$$

$$V_\rho = 1 - |P_1 - P_6| - |P_2 - P_4| - |P_3 - P_5|$$

where $\omega_1 = \left|P_1 - \frac{1}{6}\right|$ $\qquad \omega_\alpha = \left|(P_2 - \frac{1}{6}) + (P_3 - \frac{1}{6})\right|$

$\omega_6 = \left|P_6 - \frac{1}{6}\right|$ $\qquad \omega_\beta = \left|(P_4 - \frac{1}{6}) + (P_5 - \frac{1}{6})\right|$

P_i: probability of word i

Results

Words probabilities and visibility

- As the words clusters spread out, transition, or create a certain structure, visibility can reflect those behaviors by indicating the strength of the symmetry.

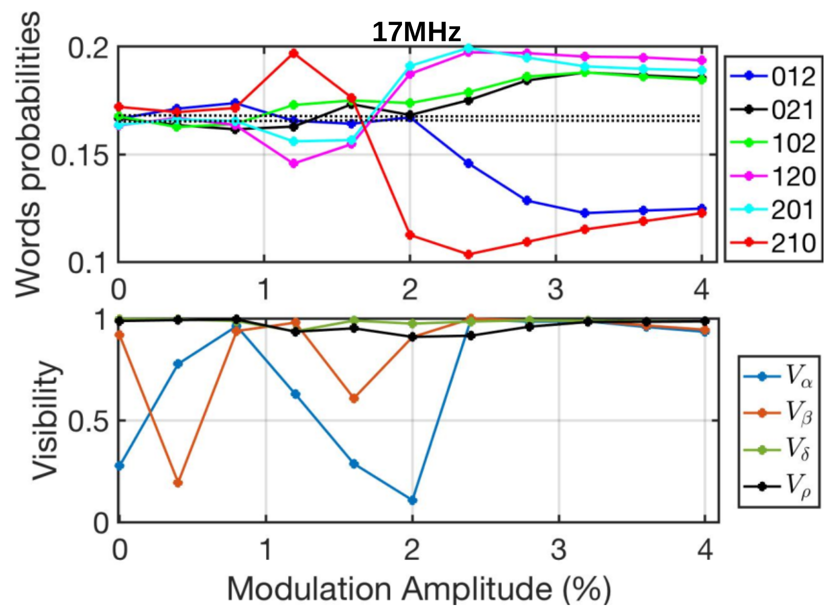

Fig.3. Words probabilities and Dynamical Visibility for the photonic neuron (top) at 17 MHz and Visibility (bottom).

References

[1] C. Bandt & B. Pompe, "Permutation entropy: a natural complexity measure for time series". Phys. Rev. Lett. **88**, 174102 (2002).

Logistic map

- The logistic map is an iterative map that displays regular and chaotic dynamics:

$$x_{n+1} = r x_n (1 - x_n)$$

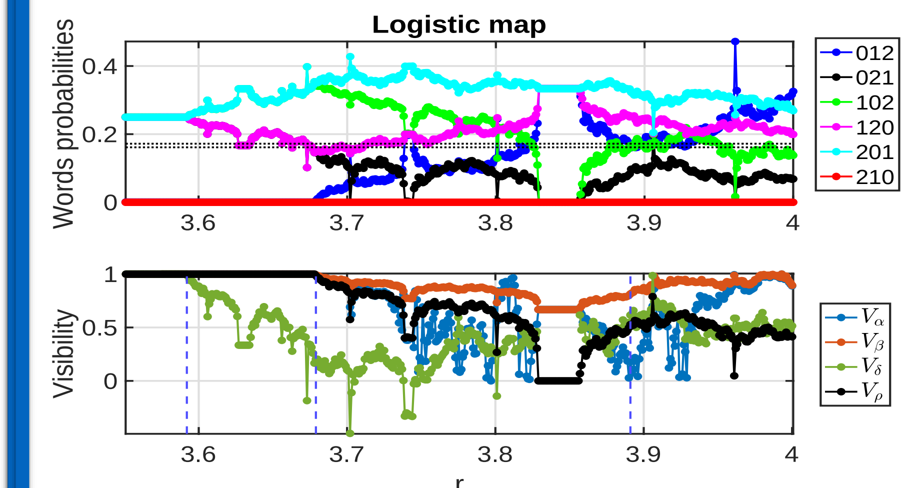

Fig.4. The logistic map shows the symmetry breaks in the words' clusters

Circle map

- We used the mathematical model of a circle map to validate our technique through the equation below:

$$\phi(i+1) = \phi(i) + \rho + \frac{K}{2\pi}\left[\sin(2\pi\phi(i)) + \alpha_c \sin(4\pi\phi(i))\right] + \beta_c \xi(i)$$

where $\beta = 0.02;\ \alpha = -0.2, \rho = -0.145$

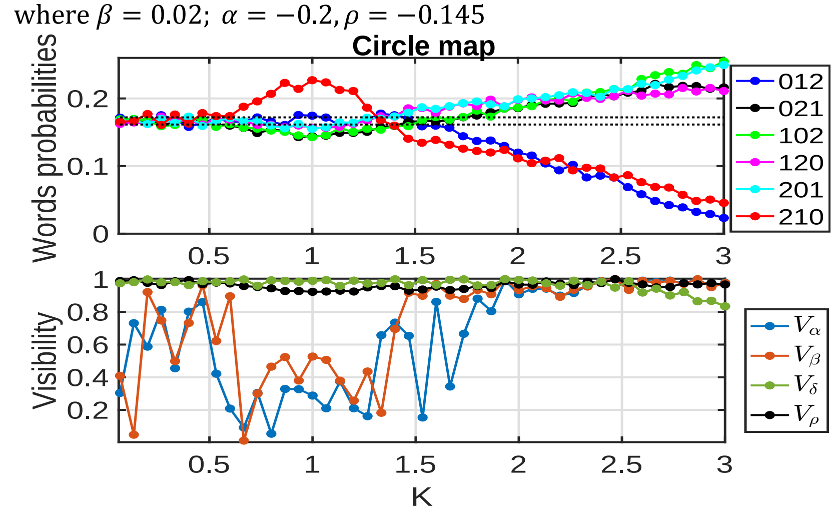

Fig.5. 17MHz graphs are modeled with the circle map parameters

Conclusions

- We introduce a novel measure to extract the hidden dynamical structure in chaotic systems.
- Dynamic Visibility unveils hidden symmetries and temporal correlations.
- DV characterizes regions based on their dynamics.
- Circle map is a good model to simulate a photonic neuron.