

Week-2 Practical  
CP2410: Algorithm and Data Structure  
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GitHub: <https://github.com/minhquan0902/CP2410--Practicals.git>

## Task 1:

(R-3.8) Order the following functions by asymptotic growth rate.  $4n \log n + 2n^2$ ,  $10^2 \log n$ ,  $3n + 100 \log n$ ,  $4n \log n + 2n$ ,  $2^{10}$ ,  $2^{(\log n)}$ ,  $3n + 100 \log n$ ,  $4n$ ,  $2^n$ ,  $n^2 + 10n$ ,  $n^3$ ,  $n \log n$

Solution:

Order from least to greatest

$2^{10}$ ,  $2^{(\log n)}$ ,  $3n + 100 \log n$ ,  $4n$ ,  $n \log n$ ,  $4n \log n + 2n$ ,  $n^2 + 10n$ ,  $n^3$ ,  $2^n$

Explanation:

We have:

In the sequence below, if a function  $f(n)$  precedes a function  $g(n)$ , we say that  $f(n)$  is asymptotically better than  $g(n)$ .  $1$ ,  $\lg n$ ,  $n$ ,  $n \lg n$ ,  $n^2$ ,  $n^3$ ,  $2^n$

$2^{10}$  is constant value

$2^{(\log n)}$  is linear  $O(n)$  by the definition of log

$3n + 100 \log n$  is  $O(n)$

$4n$  is also  $O(n)$  and larger than  $3n + 100 \log n$  because the  $4n$  term is larger than the  $3n$  term

$n \log n$  is  $O(n \log n)$

$4n\log n + 2n$  is also  $O(n\log n)$  because the  $n\log n$  term dominates the  $2n$  term

$N^2 + 10n$  is  $O(n^2)$

$N^3$  is  $O(n^3)$

$2^n$  is  $O(2^n)$  – exponential

**Task 2:** 2. (R-3.2) The number of operations executed by algorithms A and B is  $8n \log n$  and  $2n^2$ , respectively. Determine  $n_0$  such that A is better than B for  $n \geq n_0$ .

Algorithm A:  $8n\log n$

Algorithm B:  $2n^2$

The graphs describing the behavior of these algorithms start out with A higher (slower) than B, and eventually cross. After the point where they cross, B is always higher than A. Therefore, we need to find the point where they cross, that is the value where:

$$8n\log n = 2n^2.$$

Applying algebra, we get:

$$8n\log n = 2n^2$$

$$\Leftrightarrow 4n\log n = n^2$$

$$\Leftrightarrow 4\log n = n$$

$$\Leftrightarrow 4 = n/\log n$$

Solve for  $n$  we have  $n=16$  since  $4 = 16/\log_2(16) = 16/4 = 4$

Thus  $n_0 = 17$ , since for all  $n \geq 17$ , A will be faster than B (at 16 they're equal.)

**Task 3:** (R-3.9) Show that if  $d(n)$  is  $O(f(n))$ , then  $a \cdot d(n)$  is  $O(f(n))$ , for any constant  $a > 0$ .

given that  $d(n) = O(f(n))$

now  $a \cdot d(n) = O(a \cdot f(n))$  where  $a$  is any constant

now, considering Big O notation rule,  $O(kn)$  is  $O(n)$  where  $k$  is constant

- ⇒  $O(a \cdot f(n))$  is also  $O(f(n))$
- ⇒ Proved

**Task 4:** See the following functions from ch03/exercises.py in the sample code. For each of example1, to example5, determine the running time, in big Oh notation, of the function in terms of n.

```
def example1(s):  
    n = len(s)  
    total = 0  
    for j in range(n):  
        total += s[j]  
    return total
```

there are 2 operations before the loop, the loop runs based off of n, overall runtime is  $O(n)$

```
def example2(S):  
    n = len(S)  
    total = 0  
    for j in range(0, n, 2):  
        total += S[j]  
    return total
```

2 operations before the loop, the loop runs  $n / 2$  times, Overall runtime =  $O(n)$

```
def example3(S):  
    n = len(S)  
    total = 0  
    for j in range(n):  
        for k in range(1 + j):  
            total += S[k]  
    return total
```

2 operations before the loop, inner loop runs  $1 + 2 + n$  times Overall run time is  $O(n^2)$

```
def example4(S):  
    n = len(S)  
    prefix = 0  
    total = 0  
    for j in range(n):  
        prefix += S[j]  
        total += prefix  
    return total
```

3 operations before the loop, loop runs n times, Overall run time is  $O(n)$

```
def example5(A, B):  
    n = len(A)  
    count = 0  
    for i in range(n):  
        total = 0  
        for j in range(n):  
            for k in range(1 + j):  
                total += A[k]  
            if B[i] == total:  
                count += 1  
    return count
```

2 operations before the loop. outer loop runs n times, Overall run time is  $O(n^3)$