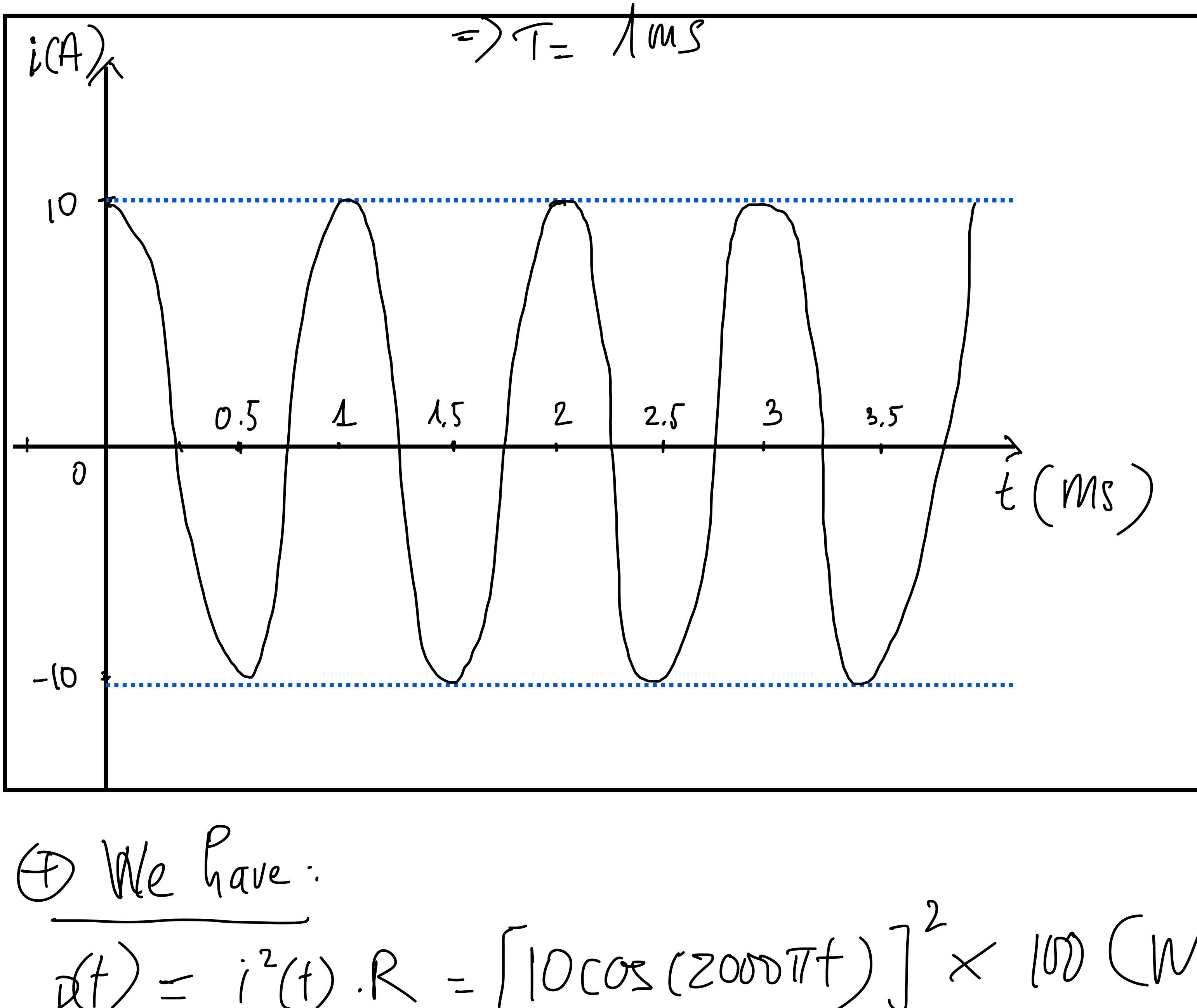
Home Work 6:

Nhat Ho

Problem 1: Given $i(t) = 10\cos(2000\pi t)$ flow through a $100\,\Omega$ resistance

* Sketch $i(t)$. $\quad \omega = 2000\pi = 2\pi f \Rightarrow f = 1000\ Hz$

$\Rightarrow T = 1ms$



⊕ We have:

$$p(t) = i^2(t) \cdot R = \left[10\cos(2000\pi t)\right]^2 \times 100\ (W)$$

$$= 10000\cos^2(2000\pi t)\ (W)$$

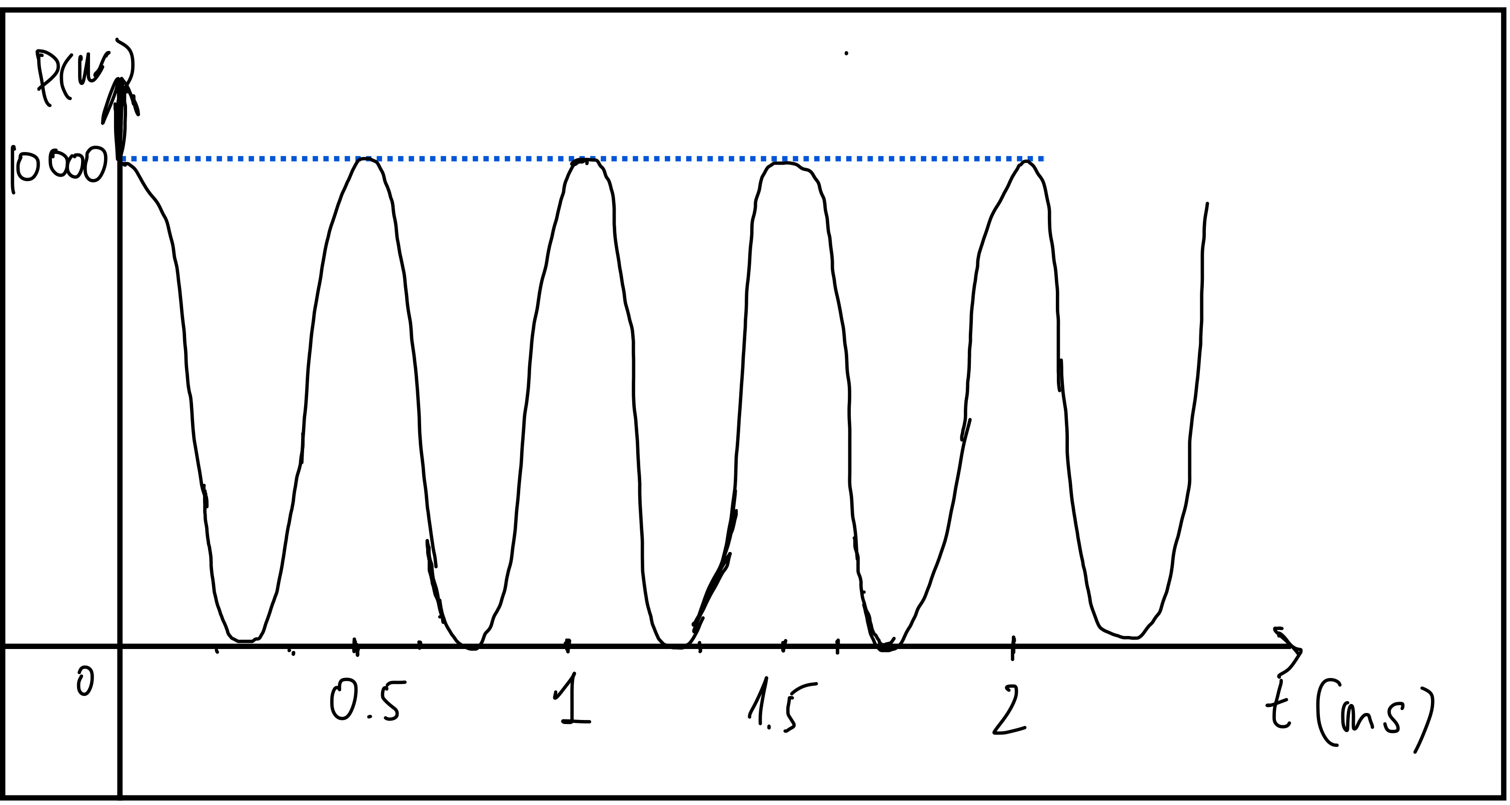
Sketch p(t):

Since $\cos^2(2000\pi t) = \dfrac{1+\cos(4000\pi t)}{2}$

$\Rightarrow 10000\cos^2(2000\pi t) = 5000(1+\cos(4000\pi t))$

$\Rightarrow \omega = 4000\pi = 2\pi f \Rightarrow f = 2000\,Hz$

$\Rightarrow T = 0.5\,ms$

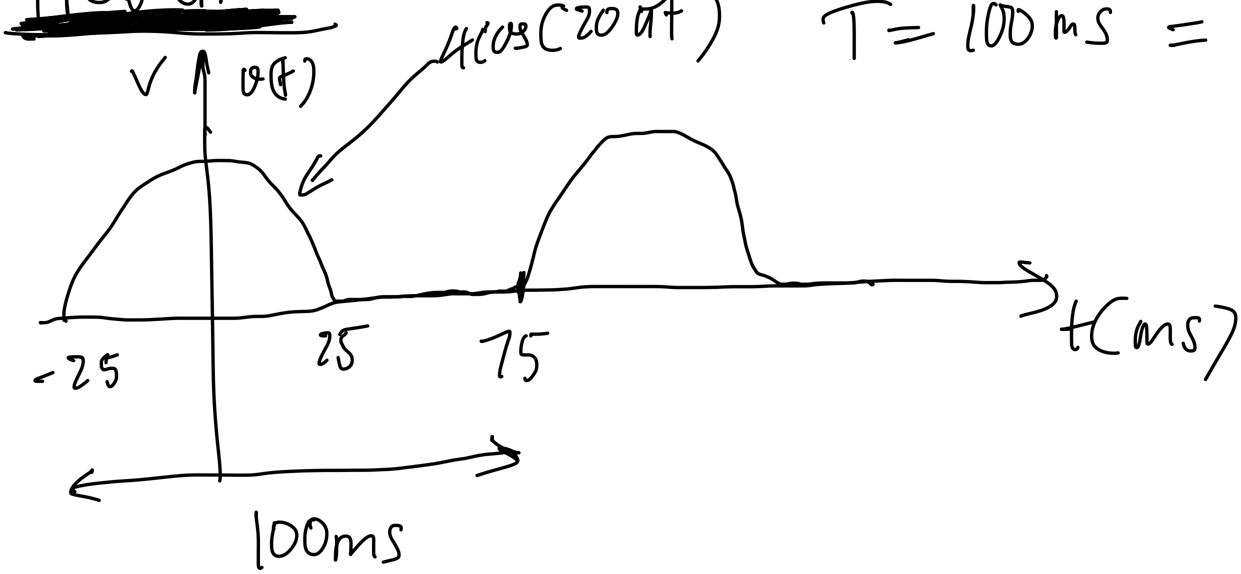


(*) Average power: We have $I_{rms} = \dfrac{I_m}{\sqrt{2}} = \dfrac{10}{\sqrt{2}}(A)$

$\Rightarrow P_{avg} = I_{rms}^2 \times R = \dfrac{10^2}{2} \times 100 = \boxed{5000\,(W)}$

Problem 2:

$T = 100\,ms =$



We have: $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$

$$\Rightarrow V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt$$

Check the period from $-25 \to 75$, we have:

$$v(t) = \begin{cases} 4\cos(20\pi t), & -25 \le t \le 25 \text{ (ms)} \\ 0, & 25 \le t \le 75 \text{ (ms)} \end{cases}$$

$$\Rightarrow V_{rms}^2 = \frac{1}{0.1}\left[\int_{-0.025}^{0.025} [4\cos(20\pi t)]^2\,dt\right]$$

$$= 160 \int_{0.025}^{0.025} \cos^2(20\pi t)\, dt \,.$$

Also, $\displaystyle\int_{-0.025}^{0.025} \cos^2(20\pi t)\, dt = \int_{-0.025}^{0.025} \frac{1+\cos(40\pi t)}{2}\, dt$

$$= \int_{-0.025}^{0.025} \frac{1}{2}\, dt + \frac{1}{2}\int_{-0.025}^{0.025} \cos(40\pi t)\, dt$$

$$= \frac{1}{2}(0.025 + 0.025) + \frac{1}{2}\cdot\frac{1}{40\pi} \sin(40\pi t)\Big|_{-0.025}^{0.025}$$

$$= 0.025 + \frac{1}{80\pi}\left[\sin\pi + \sin\pi\right]$$

$$= 0.025 \,.$$

$$\Rightarrow V_{rms}^2 = 160 \times 0.025 = 4$$

$$\Rightarrow \boxed{V_{rms} = \sqrt{4} = 2(V)}$$

Problem 3: Given:

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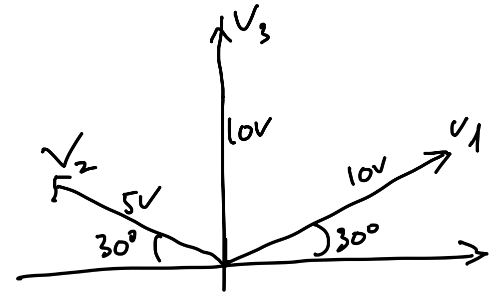


$f = 200 Hz$

Since $\omega = 2\pi f \Rightarrow \omega = 2\pi \times 200 = 400\pi$ (rad/s)

From the diagram, we see that the V_1 is 30° leading with the reference line & has $V_m = 10$ V

$\Rightarrow V_1 = V_m \cos(\omega t + \theta) \Leftrightarrow$

$\boxed{V_1 = 10\cos(400\pi t + 30°)}$ (V)

For the V_3, it has $V_m = 10$ V & $\theta = 90°$ from the diagram

$\Rightarrow \boxed{V_3 = 10\cos(400\pi t + 90°)}$ (V)

For the V_2, it has $V_m = 5$V & the degree of it will be equal $180° - 30° = 150°$ lead the reference line

$\Rightarrow V_2 = \boxed{5\cos(400\pi t + 150°)\ (V)}$. Also, from the diagram, we have:

$$\boxed{\begin{array}{l} V_1 \text{ lags } V_3 \text{ by } 60° \\ V_1 \text{ lags } V_2 \text{ by } 120° \\ V_3 \text{ lags } V_2 \text{ by } 60° \end{array}}$$

Problem 4:

a) The current and voltage for a certain circuit element is shown in Figure A of the.
Determine the nature and value of the element

From the figure A, we know the period of of current & voltage wave form is

$T = 2ms \Rightarrow f = \frac{1}{T} = \frac{1}{2\times10^{-3}} = 500\,Hz$

$\Rightarrow \omega = 2\pi f = 2\times\pi\times500 = 1000\pi$ (rad/s)

Also we have the peak value of current

$I_m = 5(mA) = 5\times10^{-3}(A)$

$\Rightarrow i(t) = I_m \cos(\omega t + \theta)$ (A)

$\Rightarrow i(t) = 5\times10^{-3}\cos(1000\pi t + \theta)$ (A)

Since the current gets maximum value at $t=0$ (from the figure) $\Rightarrow 5\times10^{-3}\cos(\theta) = I_m$

$\Rightarrow$ the phase angle of current is $\theta = 0°$

$\Rightarrow i(t) = 5 \times 10^{-3} \cos(1000\pi t)$ (A)

$\Leftrightarrow I = 5 \times 10^{-3} \angle 0°$ (A) (1)

* Also, the maximum of voltage at 0.5ms

$\Rightarrow$ the different time between the current & voltage is 0.5ms. From the figure we can have $\theta_i - \theta_v = \theta = \omega t$ with $t = 0.5$ms as their different time.

$\Rightarrow 0 - \theta_v = 1000\pi \times 0.5 \times 10^{-3} = \frac{\pi}{2}$

$\Rightarrow \theta_v = -\frac{\pi}{2} = -90°$

Also, the peak value of voltage $V_m = 1V$

$\Rightarrow v(t) = V_m \cos(\omega t + \theta) = 1 \cos(1000\pi t - 90°)$ (V)

$\Rightarrow V = 1 \angle -90°$ (V) (2)

From ① & ②, We have:

$$V = 1 \angle -90^\circ \ (V)$$

$$I = 5\times10^{-3} \angle 0^\circ \ (A)$$

$\Rightarrow$ Voltage lags the current by 90°

$\Rightarrow$ the component is $\boxed{\text{pure capacitance}}$

$$\Rightarrow Z_C = \frac{V_C}{I_C} \Rightarrow \frac{1}{\omega C} \angle -90^\circ = \frac{1 \angle -90^\circ}{5\times10^{-3} \angle 0^\circ} = 200 \angle -90^\circ$$

$$\Rightarrow \frac{1}{\omega C} = 200 \Rightarrow C = \frac{1}{\omega \times 200}$$

$$\Rightarrow C = \frac{1}{1000\pi \times 200} = 1.59 \times 10^{-6} \ F$$

$$\Leftrightarrow \boxed{C = 1.59 \ \mu F}$$

b) Repeat for b.

From the figure B of homework, we have the period time of both current and voltage is 16 ms $\Rightarrow T = 16 ms \Rightarrow f = \frac{1}{16 \times 10^{-3}}$

$\Rightarrow f = 62.5\ Hz \Rightarrow \omega = 2\pi f = 2 \times 62.5\pi$

$\Rightarrow \omega = 125\pi$ (Rad/s).

Also we have the peak value of voltage.

$V_m = 4V \Rightarrow v(t) = V_m \cos(\omega t + \theta)$ (V)

$\Rightarrow v(t) = 4 \cos(125\pi t + \theta)$ (V)

Since $v(t)$ gets maximum value V_m at $t = 4ms$

$\Rightarrow \cos(125\pi \times 4 \times 10^{-3} + \theta) = 1$

$\Rightarrow \cos(0.5\pi + \theta) = 1$

$\Rightarrow \theta = -\pi/2$

$\Rightarrow v(t) = 4\cos(125\pi t - \frac{\pi}{2})$ V

$\Leftrightarrow V = 4 \angle -90°$ (V) 1

+ From the figure B, we know the voltage gets maximum value at $t = 4ms$ while the current gets maximum value at $t = 8ms$

$\Rightarrow$ the different time between them $\Delta t = 4ms$

$$\Rightarrow \theta_v - \theta_i = \theta = \omega . \Delta t$$

$$\Rightarrow -90^\circ - \theta_i = 125\pi \times 4 \times 10^{-3} = \frac{\pi}{2}$$

$$\Rightarrow \theta_i = \frac{-\pi}{2} - \frac{\pi}{2} = -\pi .$$

Also $I_m = 10mA$

$$\Rightarrow i(t) = I_m \cos(\omega t + \theta)$$

$$\Leftrightarrow i(t) = 10 \times 10^{-3} \cos(125\pi t - \pi) \ (A)$$

$$\Rightarrow I = 10^{-2} \angle -\pi \ (A) \quad (2)$$

From (1) & (2), we have:

$$\left. \begin{aligned} V &= 4 \angle -90^\circ \ (V) \\ I &= 10^{-2} \angle -180^\circ \ (A) \end{aligned} \right\} \Rightarrow \text{voltage leads current by } 90^\circ$$

$\Rightarrow$ the component is $\boxed{\text{pure inductance}}$

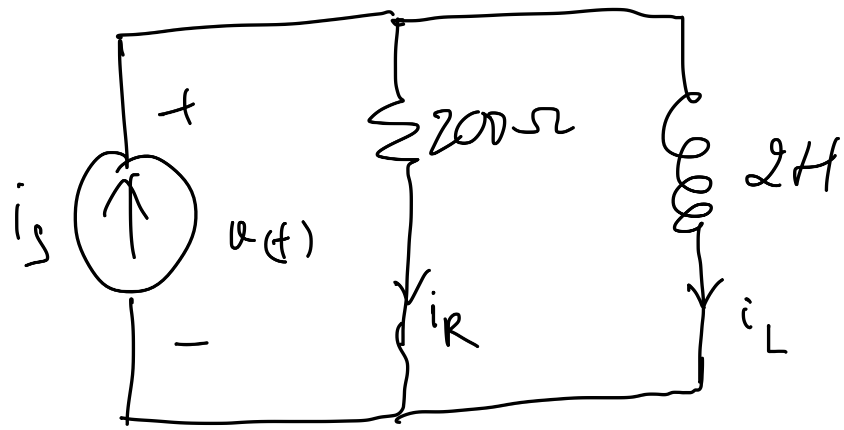
$$\Rightarrow Z_L = \frac{V_L}{I_L} = \frac{4\angle -90^\circ}{10^{-2}\angle -180^\circ} = j\omega L$$

$$\Rightarrow j\omega L = 400\angle 90^\circ \Rightarrow \omega L = 400$$

$$\Rightarrow L = \frac{400}{\omega} = \frac{400}{125\pi} = 1.02\,H$$

$$\Rightarrow \boxed{L \simeq 1.02\,H}$$

Problem 5: Given $i_s = 0.5\sin(100t)$ (A)



We have:

$w = 100$ (rad/s)

$\Rightarrow Z_L = jwL$

$= j\,100 \times 2$

$= 200j$

Since $(R \parallel H) \Rightarrow Z_{eq} = \dfrac{R \times Z_L}{R + Z_L} = \dfrac{200 \times 200j}{200 + 200j}$

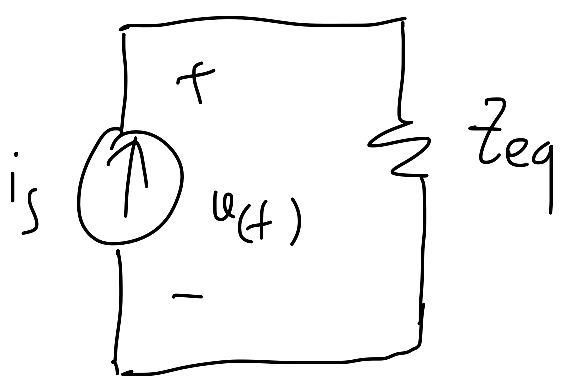
$= \dfrac{200j}{1+j} = 100 + 100j$ (Ω)

$\Rightarrow$ new circuit:

$i_s(t) = 0.5\sin(100t)$ (A)

$= 0.5\cos(100t - \pi/2)$

$= 0.5\angle -90°$ (A)



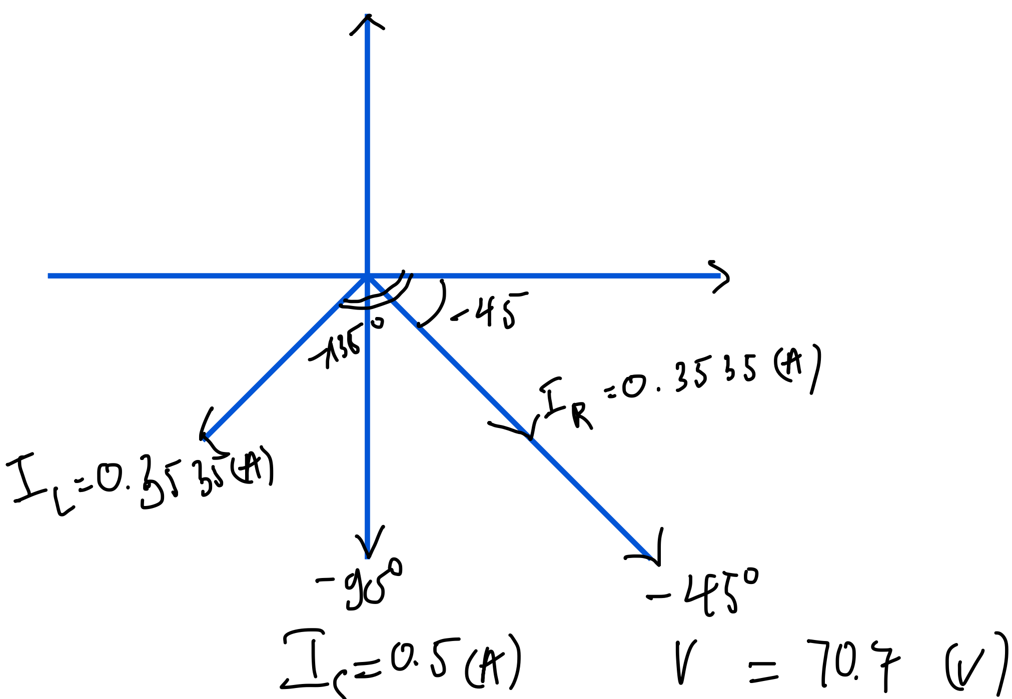
$\Rightarrow v(t) = i_s(t) \times Z_{eq} = (100 + 100j)\; 0.5\angle -90°$

$$v(t) = 50 - 50j = \boxed{70.7\angle -45^\circ} \ (V)$$

$$\Rightarrow i_R = \frac{v(t)}{R} = \frac{70.7\angle -45}{200}$$

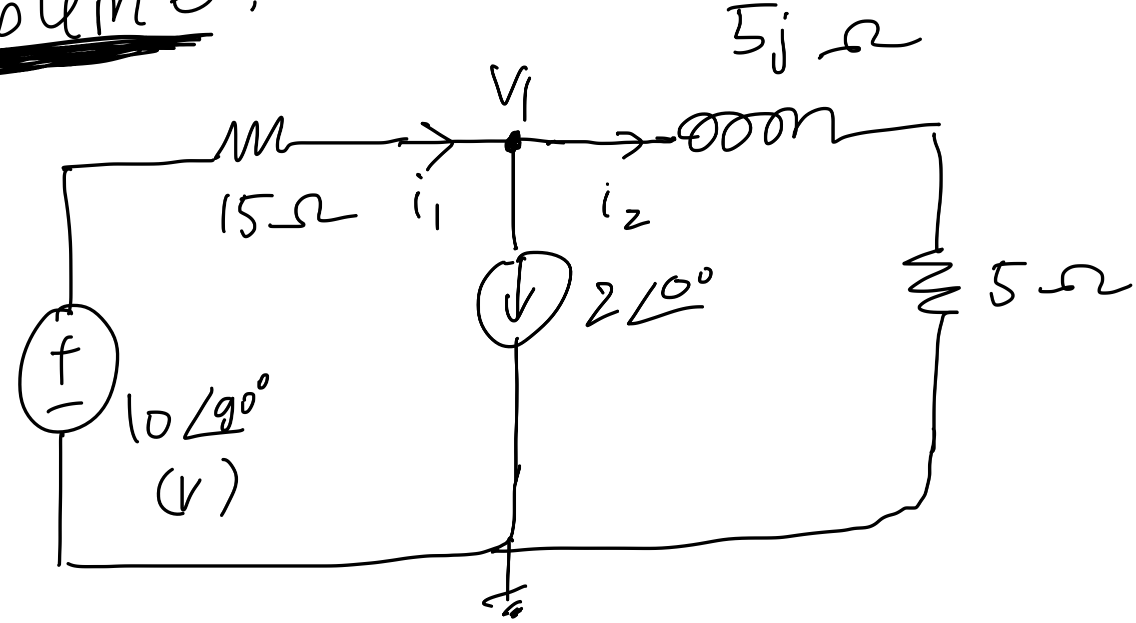
$$\Rightarrow i_R = \boxed{0.3535\angle -45^\circ} \ (A)$$

$$i_L = \frac{v(t)}{Z_L} = \frac{70.7\angle -45^\circ}{200j} = \boxed{0.3535\angle -135^\circ} \ (A)$$



Relationship between V & I_s:
Voltage V leads the current I_s by 45°.

Problem 6:



Apply KCL at V_1: $i_1 = 2\angle 0° + i_2$

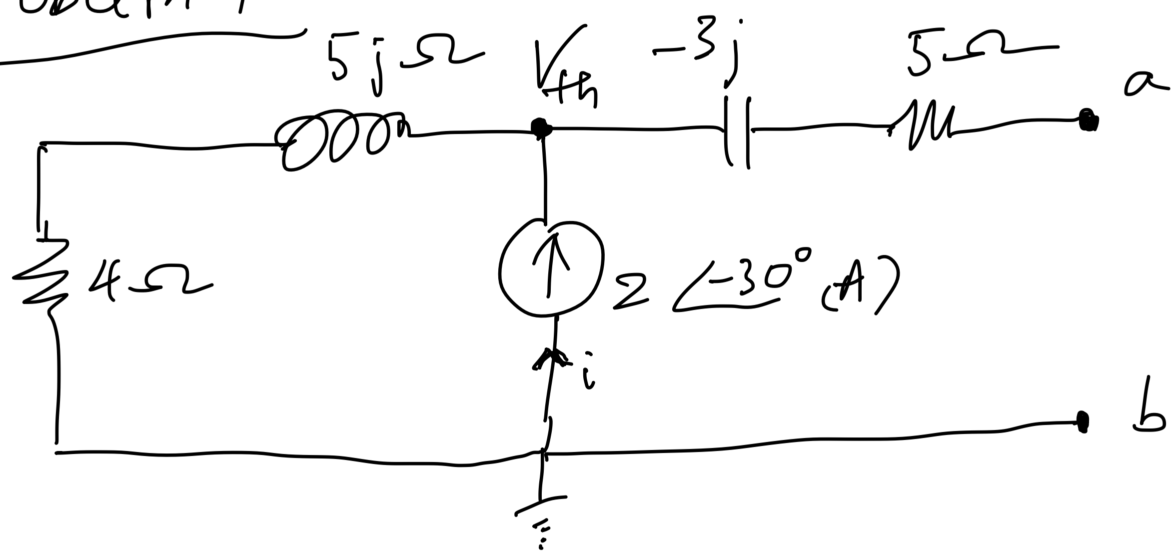
$$\Rightarrow \frac{10\angle 90° - V_1}{15\ \Omega} = 2\angle 0° + \frac{V_1}{5j+5}$$

$$\Rightarrow \frac{10\angle 90°}{15} - 2\angle 0° = \frac{V_1}{5j+5} + \frac{V_1}{15}$$

$$\Rightarrow -2 + \frac{2}{3}j = \left(\frac{1}{5j+5} + \frac{1}{15}\right) V_1 =$$

$$\boxed{\begin{array}{l} \Rightarrow V_1 = -10.588 - 2.353j \ \text{(V)} \\ \text{or } V_1 = 10.847\angle -167.47° \ \text{(V)} \end{array}}$$

Problem 7:

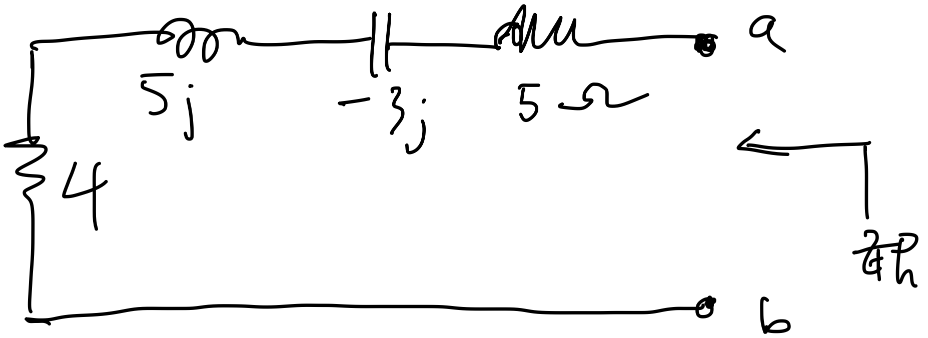


Find V_{th}: We have $V_{th} = i \times (5j + 4)\,\Omega$

$$\Rightarrow V_{th} = 2\angle{-30^\circ} \times (4 + 5j)$$

$$\Rightarrow \boxed{V_{th} = 12.806 \angle{21.34^\circ}\ (V)}$$

Find Z_{th}: We have new circuit



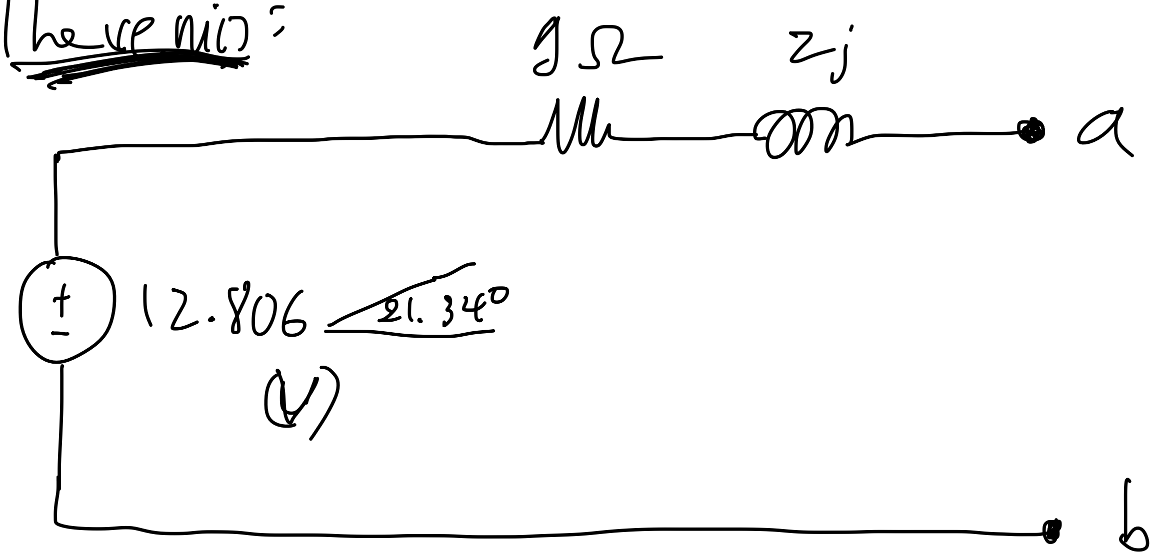
$$\Rightarrow Z_{th} = 4 + 5j - 3j + 5 = \boxed{9 + 2j\ (\Omega)}$$

$$\Rightarrow I_{Norton} = \frac{V_{th}}{Z_{th}} = \frac{12.806\angle 21.34^\circ}{9+2j}$$

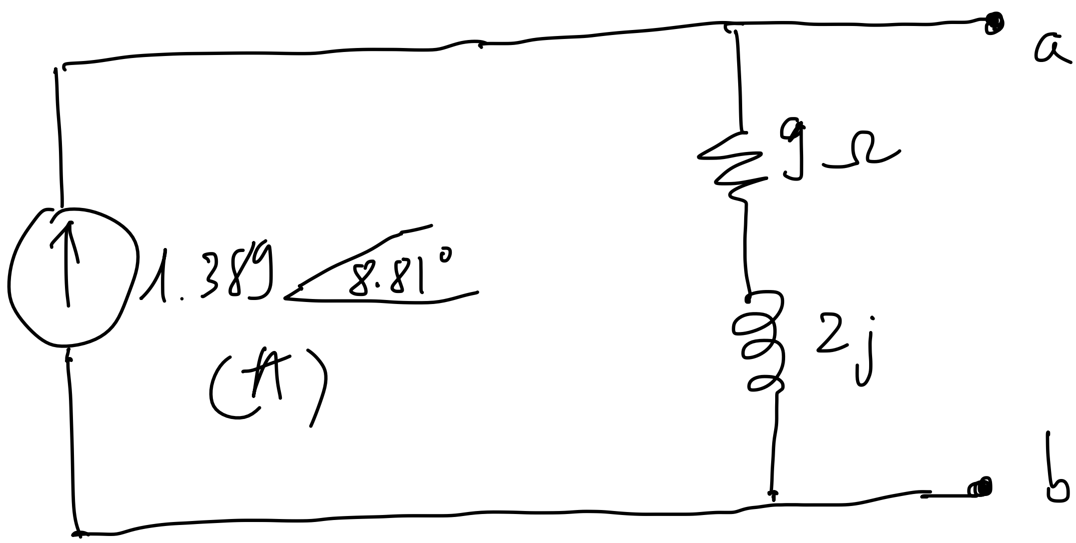
$$\Rightarrow \boxed{I_N = 1.389\angle 8.81^\circ \ (A)}$$

We have:

Thevenin:

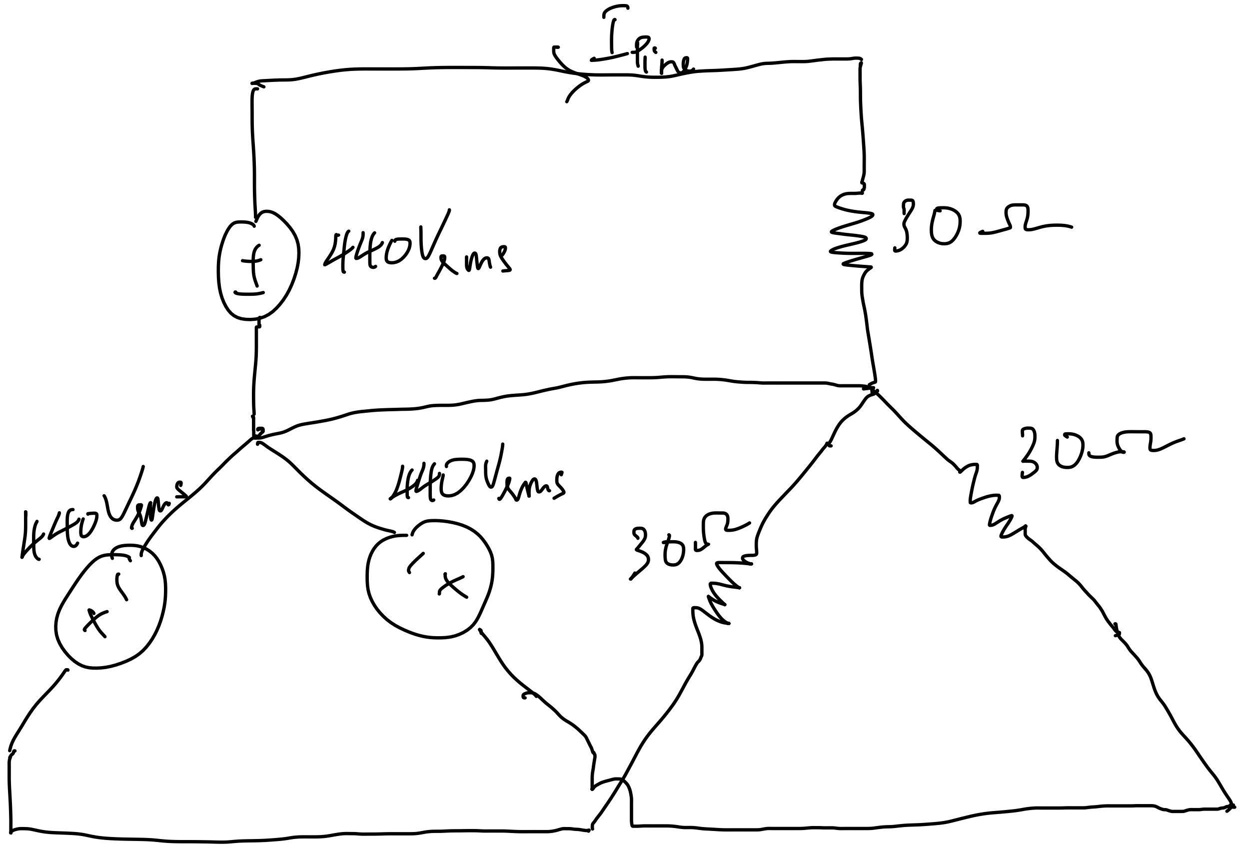


Norton:



Problem 8:

We have the circuit:



Given $V_{an} = 440 V_{rms}$

In connected star, $V_{line-line} = \sqrt{3}\, V_{line-neutral} = \sqrt{3} V_{an}$

$\Rightarrow V_{line-line} = \sqrt{3} \times 440 V_{rms} = \boxed{726.1 \ V_{rms}}$

Also $I_{line} = \dfrac{V_{line-neutral}}{R} = \dfrac{440 V_{rms}}{30 \Omega} = 14.67 \ (A_{rms})$

$\Rightarrow \boxed{I_{line} = 14.67 \ (A \ rms)}$

The power from one voltage source:

$$P_1 = \frac{V_{rms}^2}{R} = \frac{440^2}{30\,\Omega} = 6453.33\,(W)$$

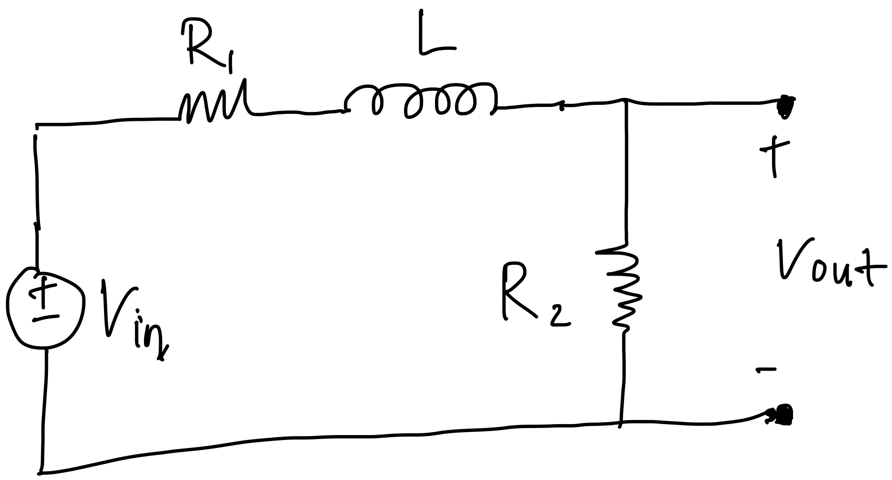
$$\Rightarrow P_1 = 6.453\,(kW)$$

Since this is an balanced wye-connected 3phases

$$\Rightarrow P_{total} = P_1 + P_2 + P_3 = 3P_1 = \boxed{19.36\,(kW)}$$

Problem 9 :

a) Derive an expression for the transfer function $H(f) = \frac{V_{out}}{V_{in}}$. Find an expression for the half power frequency



We have : $Z_L = j\omega L$

Using voltage divider , $V_{out} = \frac{R_2}{R_1+R_2+j\omega L} V_{in}$

$$\Rightarrow H(f) = \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1+R_2+j\omega L} = \frac{R_2}{(R_1+R_2)\left(1+\frac{j\omega L}{R_1+R_2}\right)}$$

$$= \frac{R_2}{R_1+R_2} \times \frac{1}{1+j\frac{\omega L}{R_1+R_2}}$$

Check for: $\frac{\omega L}{R_1+R_2} = \frac{2\pi f L}{R_1+R_2} = \frac{f}{\frac{R_1+R_2}{2\pi L}}$

$$\Rightarrow H(f) = \frac{R_2}{R_1+R_2} \times \frac{1}{1+j\frac{f}{\left(\frac{R_1+R_2}{2\pi L}\right)}}$$

$$\Rightarrow \boxed{f_B = \frac{R_1+R_2}{2\pi L}} \quad (1)$$

then, we have:

$$\boxed{H(f) = \frac{R_2}{R_1+R_2} \times \frac{1}{1+j\left(\frac{f}{f_B}\right)}}$$

b) Given $R_1 = 50\,\Omega$, $R_2 = 50\,\Omega$, $L = 15\,\mu H$

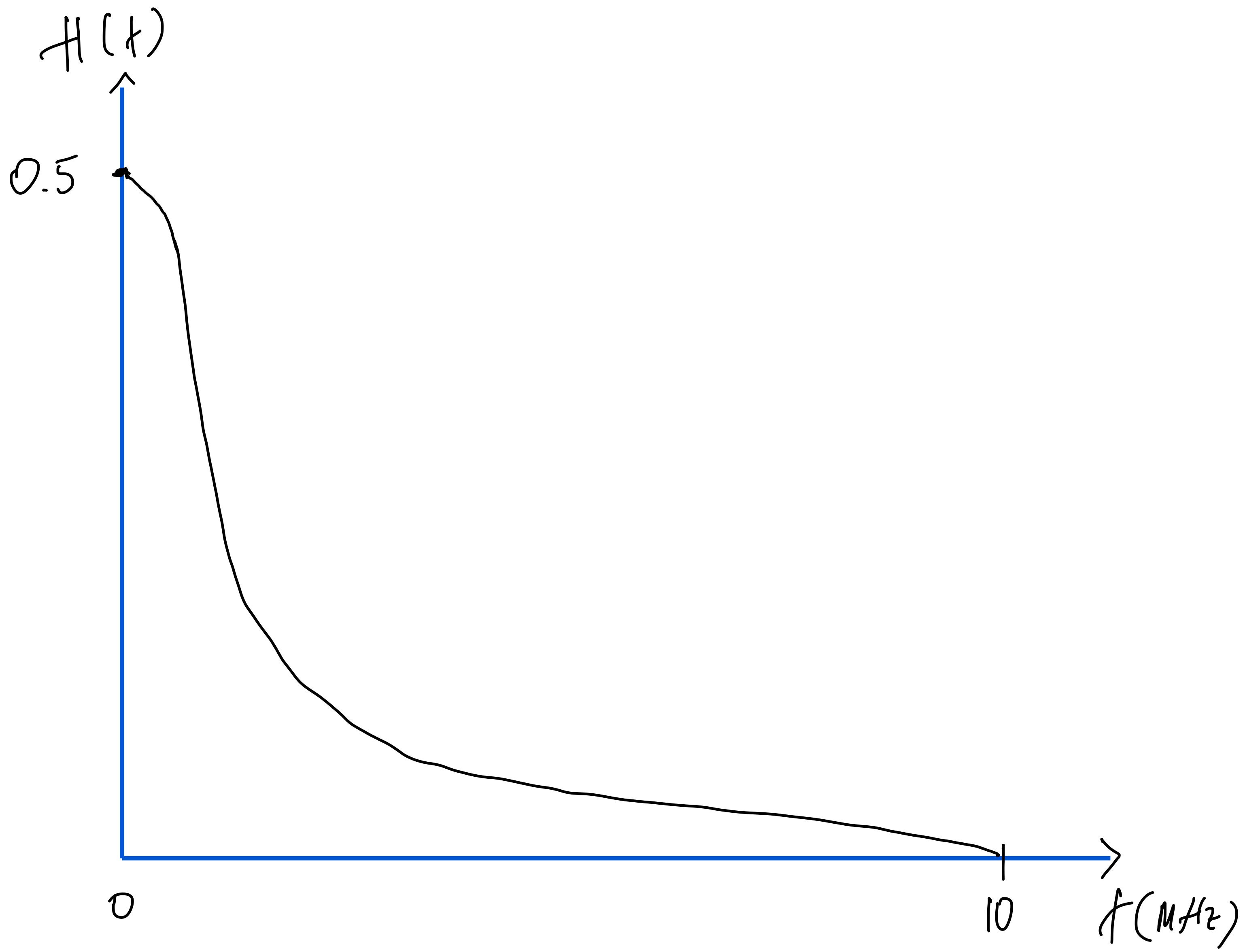
Sketch the magnitude of the transfer functions versus frequency

We use these values to plug into (1), then

$$f_B = \frac{50+50}{2\pi \times 15 \times 10^{-6}} = 1.06 \times 10^{6}\ Hz$$

$$= 1.06\ MHz$$

$$\Rightarrow H(f) = \frac{1}{1+j\frac{f}{1.06\times10^{6}}} \times \frac{50}{50+50} = \frac{0.5}{1+j\frac{f}{1.06\times10^{6}}}$$



Problem 10:

a) Given $R = 5k\Omega$, $L = 50\mu H$, $C = 200pF$

$\Rightarrow$ Resonant frequency: $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{50\times10^{-6}\times 200\times10^{-12}}} = 1.592\times10^{6}\ Hz$$

$$\Rightarrow \boxed{f_0 = 1.592\ MHz}$$

* Quality factor:

$$Q_p = 2\pi f_0 CR = 2\pi\times 1.592\times10^{6}\times 200\times10^{-12}\times 5\times10^{3}$$

$$\Rightarrow \boxed{Q_p = 10}$$

$\Rightarrow$ the bandwidth: $B = \dfrac{f_0}{Q_p} = \dfrac{1.592\times10^{6}}{10}$

$$= 1.592\times10^{5}\ (Hz) \Rightarrow \boxed{B = 159.2\ (kHz)}$$

b) Dertermine L & C, given $R = 1K\Omega$, $f_0 = 10MHz$

$B = 500KHz$

We have: $B = \dfrac{f_0}{Q_p} \Rightarrow Q_p = \dfrac{f_0}{B} = \dfrac{10\times10^6}{500\times10^3}$

$\Rightarrow Q_p = 20.$

Also, $Q_p = 2\pi f_0 CR \Rightarrow C = \dfrac{Q_p}{2\pi f_0 R}$

$= \dfrac{20}{2\pi\times10\times10^6\times1000} = 3.183\times10^{-10}(F)$

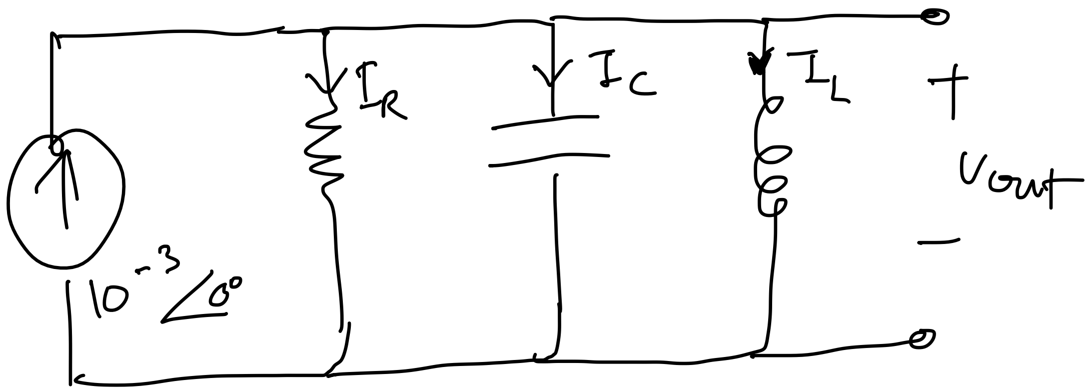
$\Rightarrow \boxed{C = 0.3183\,(nF)}$

Besides, $Q_p = \dfrac{R}{2\pi f_0 L} \Rightarrow L = \dfrac{R}{2\pi f_0 Q_p}$

$= \dfrac{1000}{2\pi\times10^7\times20} = 0.796\times10^{-6}(H)$

$\Rightarrow \boxed{L = 0.796\,(\mu H)}$

We have : $I = 10^{-3} = 10^{-3}\angle 0^\circ$



At resonance, $V_{out} = IR = 10^{-3}\angle 0^\circ \times 1000$

$\Rightarrow V_{out} = 1\angle 0^\circ$ (V)

$$\Rightarrow I_R = \frac{V_{out}}{R} = \frac{1\angle 0^\circ}{1000} = 10^{-3}\angle 0^\circ \text{ (A)}$$

$$\Rightarrow \boxed{I_R = 1\angle 0^\circ \text{ (mA)}}$$

$$\oplus\ I_C = \frac{V_{out}}{Z_C} = \frac{V_{out}}{\frac{-j}{2\pi f_0 C}} = \frac{j V_{out}}{\frac{1}{2\pi f_0 C}}$$

$$= j\, V_{out}\, 2\pi f_0 C = j.1\angle 0^\circ \times 2\pi \times 10^7 \times 0.3183 \times 10^{-9}$$

$$\simeq 0.02\angle 90^\circ \text{ (A)}$$

$$\Rightarrow \boxed{I_C \simeq 20 \angle 90^\circ \ (mA)}$$

$$\oplus \ I_L = \frac{V_{out}}{Z_L} = \frac{V_{out}}{j2\pi f_0 L} = \frac{1\angle 0^\circ}{j2\pi \times 10^7 \times 0.796 \times 10^{-6}}$$

$$= 0.02 \angle -90^\circ \ (A)$$

$$\Rightarrow \boxed{I_L = 20 \angle -90 \ (mA)}$$

$\Rightarrow$ The phasor diagram :