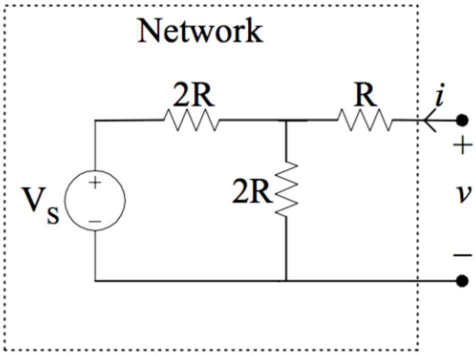
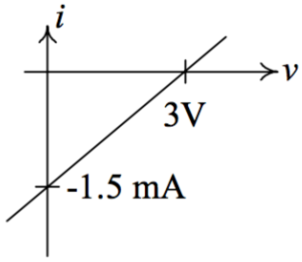
ECE100
Homework-4

Total Points: 100

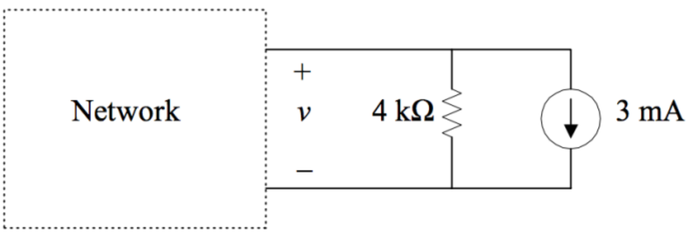
Submit your work in a pdf file electronically in the CCLE website before April 25th 11:59 pm. Late homework will not get credit!

1. A network that is implemented with three resistors and a voltage source as shown below. Its terminal characteristics are also given graphically below.





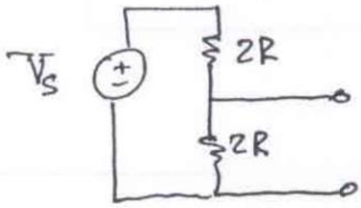
(a) From the graphical data given above, determine numerical values for the parameters of the Thevenin equivalent of the network.
(b) Determine numerical values for the parameters V_S and R that characterize the implementation of the network shown above.
(c) The network is connected to an external current source and resistor as shown below. Determine the value of its terminal voltage v given the external connection.



$$V_{TH} = \text{Open-circuit } (i=0) \text{ voltage} = 3V$$

$$R_{TH} = -\frac{\text{Open-circuit voltage}}{\text{Short-circuit } (v=0) \text{ Current}} = \frac{3V}{1.5mA} = 2k\Omega$$

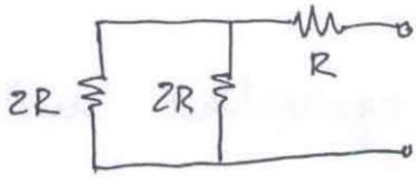
Open circuit voltage $\Rightarrow$



$V_{oc} = \frac{V_s}{2}$

$\Rightarrow V_s = 2\, V_{TH} = 6\ V$

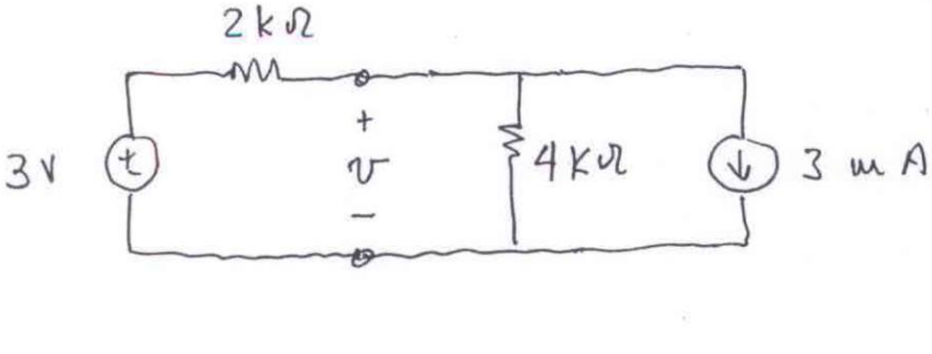
$R_{TH} =$



$= 2R = 2\,k\Omega$

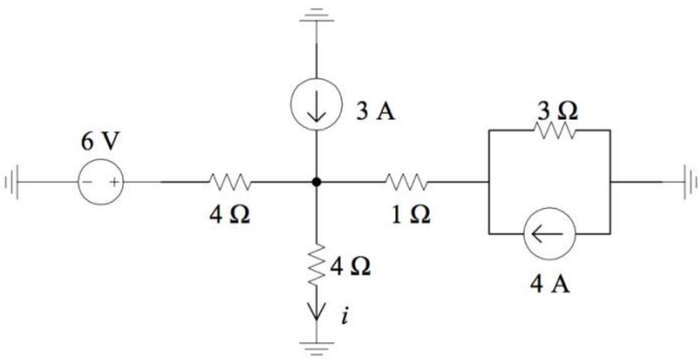
$\Rightarrow \quad R = 1\,k\Omega$

Use the Thevenin equivalent and superpositon.



$$v = \frac{2}{3} \cdot 3V - \frac{4k\Omega \cdot 2k\Omega}{6k\Omega} \cdot 3mA = -2\,V$$

2. Determine the current i in the network below.



Use superposition.

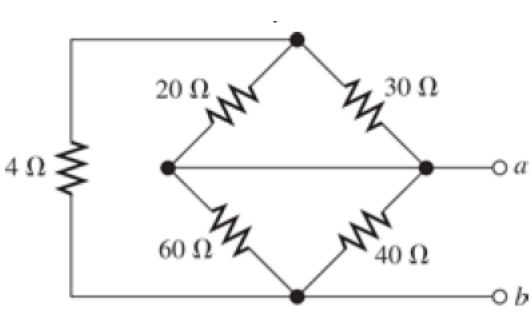
6V Source ⇒ $i = \frac{1}{2} \cdot \frac{6V}{6\Omega} = \frac{1}{2} A$

3A Source ⇒ $i = \frac{1}{3} \cdot 3A = 1 A$

4A Source ⇒ $i = \frac{1}{2} \cdot \frac{1}{2} \cdot 4A = 1 A$

Total $i = 2\frac{1}{2}$ A

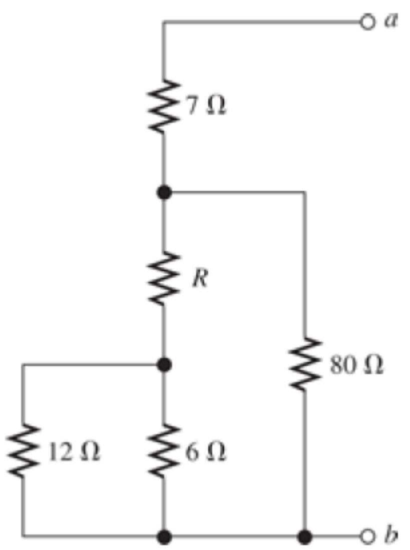
3. Find the equivalent resistance between terminals a and b in Figure below



The 20-Ω and 30-Ω resistances are in parallel and have an equivalent resistance of R_{eq1} = 12 Ω. Also the 40-Ω and 60-Ω resistances are in parallel with an equivalent resistance of R_{eq2} = 24 Ω. Next we see that R_{eq1} and the 4-Ω resistor are in series and have an equivalent resistance of R_{eq3} = 4 + R_{eq1} = 16 Ω. Finally R_{eq3} and R_{eq2} are in parallel and the overall equivalent resistance is

$$R_{ab} = \frac{1}{1/R_{eq1} + 1/R_{eq2}} = 9.6\,\Omega$$

4. The equivalent resistance between terminals a and b in Figure below is 23 ohms. Determine the value of R.



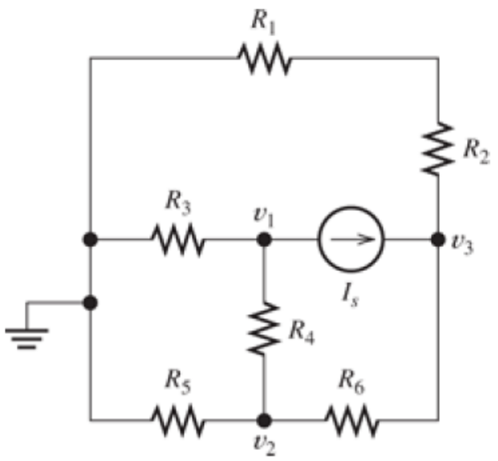
R + (6 || 12) = R + 4

(R+ 4) || 80 = 23 – 7 =16

⇨ R+ 4 =20

⇨ R = 16 ohms

5. Given R1=15 Ω, R2=5 Ω, R3=20 Ω, R4=10 Ω, R5=8 Ω, R6=4 Ω and Is=5 A, solve for the node voltages shown in Figure.



Writing KCL equations at nodes 1, 2, and 3, we have

$$\frac{v_1}{R_3} + \frac{v_1 - v_2}{R_4} + I_s = 0$$

$$\frac{v_2 - v_1}{R_4} + \frac{v_2 - v_3}{R_6} + \frac{v_2}{R_5} = 0$$

$$\frac{v_3}{R_1 + R_2} + \frac{v_3 - v_2}{R_6} = I_s$$

In standard form, we have:

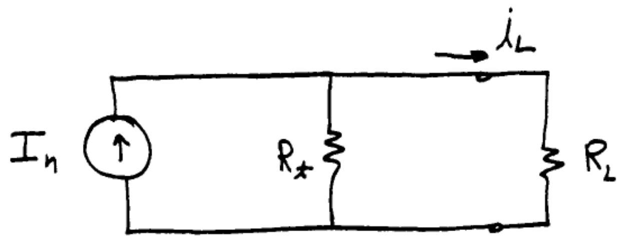
$$0.15v_1 - 0.10v_2 = -5$$

$$-0.10v_1 + 0.475v_2 - 0.25v_3 = 0$$

$$-0.25v_2 + 0.30v_3 = 5$$

$v_1 = -30.56\ \text{V} \qquad v_2 = 4.167\ \text{V} \qquad v_3 = 20.14\ \text{V}$

6. Starting from the Norton equivalent circuit (current source I_n in parallel with R_t) with a resistive load attached (R_L), find an expression for the power delivered to the load in terms of I_n, R_t and R_L. Assuming that I_n, R_t are fixed values and that R_L is variable, show that maximum power is delivered for $R_L = R_t$. Find an expression for maximum power delivered to the load in terms of and I_n, R_t.



By the current division principle:

$$i_L = I_n \frac{R_t}{R_L + R_t}$$

The power delivered to the load is

$$P_L = (i_L)^2 R_L = (I_n)^2 \frac{(R_t)^2 R_L}{(R_L + R_t)^2}$$

Taking the derivative and setting it equal to zero, we have

$$\frac{dP_L}{dR_L} = 0 = (I_n)^2 \frac{(R_t)^2 (R_t + R_L)^2 - 2(R_t)^2 R_L (R_t + R_L)}{(R_t + R_L)^4}$$

which yields $R_L = R_t$.

The maximum power is $P_{L\max} = (I_n)^2 R_t / 4$.

7. A 100 μF capacitance is initially charged to 1000 V. At t=0 it is connected to a 1-kΩ resistance. At what time t_2 has 50 percent of the initial energy stored in the capacitance been dissipated in the resistance?

The initial energy is

$$W_1 = \frac{1}{2} C (V_i)^2 = \frac{1}{2} 100 \times 10^{-6} \times 1000^2 = 50 \text{ J}$$

At $t = t_2$, half of the energy remains, and we have $25 = \frac{1}{2} C [v(t_2)]^2$, which yields $v(t_2) = 707.1$ V. The voltage across the capacitance is given by

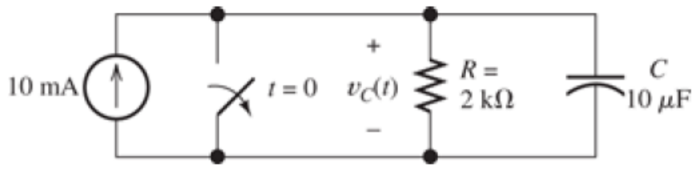
$$v_C(t) = V_i \exp(-t/RC) = 1000 \exp(-10t) \text{ for } t > 0$$

Substituting, we have $707.1 = 1000 \exp(-10t_2)$. Solving, we obtain

$$\ln(0.7071) = -10t_2$$

$$t_2 = 0.03466 \text{ seconds}$$

8. Derive an expression for $v_c(t)$ in the circuit below and sketch $v_c(t)$ to scale versus time. (Note that switch was closed before t=0 and becomes an open after t=0)



Prior to $t = 0$, we have $v_C(t) = 0$ because the switch is closed. After $t = 0$, we can write the following KCL equation at the top node of the circuit:

$$\frac{v_C(t)}{R} + C\frac{dv_C(t)}{dt} = 10\text{ mA}$$

Multiplying both sides by R and substituting values, we have

$$0.02\frac{dv_C(t)}{dt} + v_C(t) = 20 \qquad (1)$$

The solution is of the form

$$v_C(t) = K_1 + K_2\exp(-t/RC) = K_1 + K_2\exp(-50t) \qquad (2)$$

Using Equation (2) to substitute into Equation (1), we eventually obtain

$$K_1 = 20$$

The voltage across the capacitance cannot change instantaneously, so we have

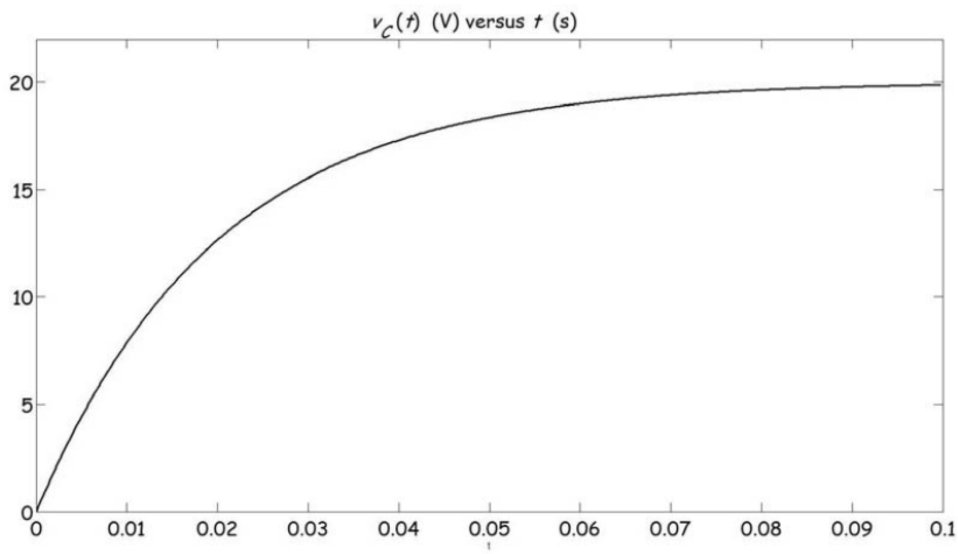
$$v_C(0+) = v_C(0-) = 0$$

$$v_C(0+) = 0 = K_1 + K_2$$

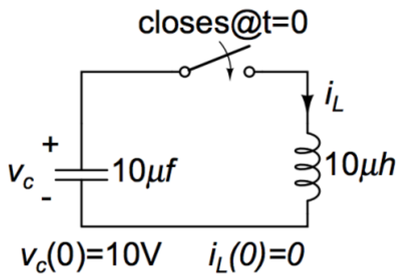
Thus, $K_2 = -K_1 = -20$, and the solution is

$$v_C(t) = 20 - 20\exp(-50t) \text{ for } t > 0$$

The sketch should resemble the following plot:



9. Determine the maximum value of I_L (10 points)

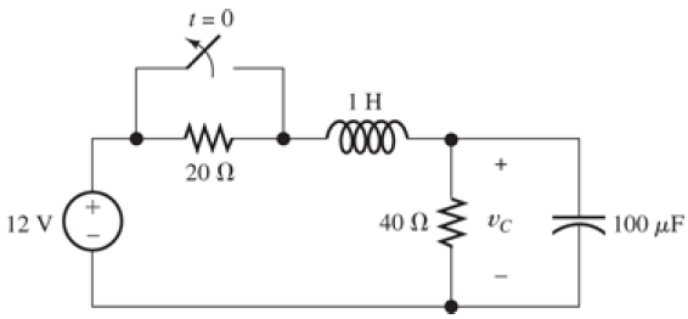


Energy oscillates between L and C.

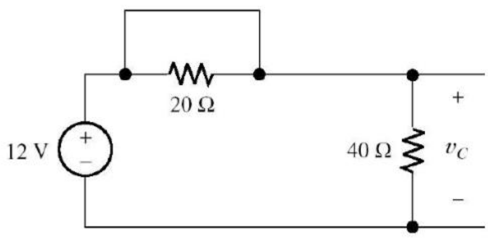
$0.5\ C\ V^2 = 0.5\ L\ I^2$

Plugging in values, I = 10 A

10. The circuit shown in Figure has been set up for a long time prior to t=0 with the switch closed. Find the value of v_C prior to t=0. Find the steady-state value of v_C after the switch has been opened for a long time. (15 points)

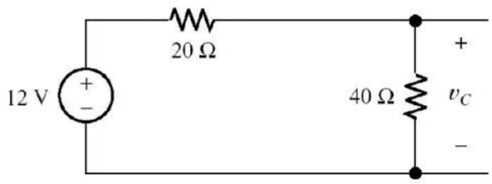


Prior to $t = 0$, the steady-state equivalent circuit is:



and we see that $v_c = 12\text{ V}$.

A long time after $t = 0$, the steady-state equivalent circuit is:



and we have $v_c = 12\dfrac{40}{40+20} = 8\text{ V}$.