ECE100
Homework-4

Total Points: 100

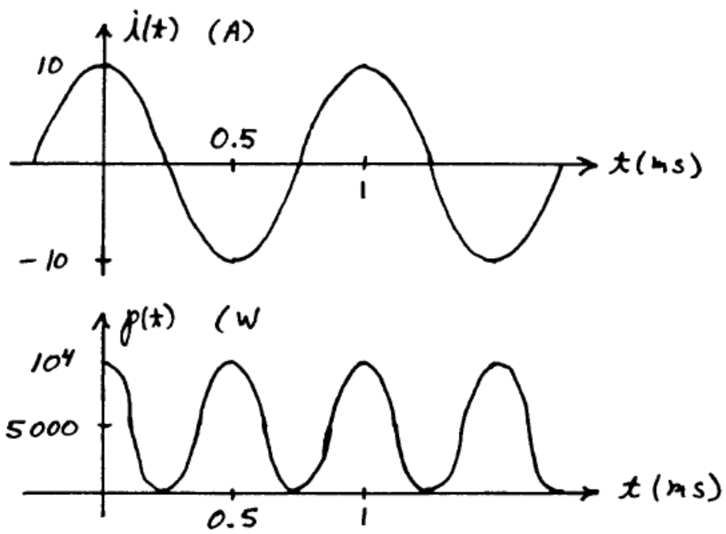
Submit your work in a pdf file electronically in the CCLE website before May 9th 11:59 pm. Late homework will not get credit!

1. A current i(t) = 10 cos(2000 πt) flows through a 100 Ω resistance. Sketch i(t) and power p(t) to scale versus time. Find the average power delivered to the resistance.

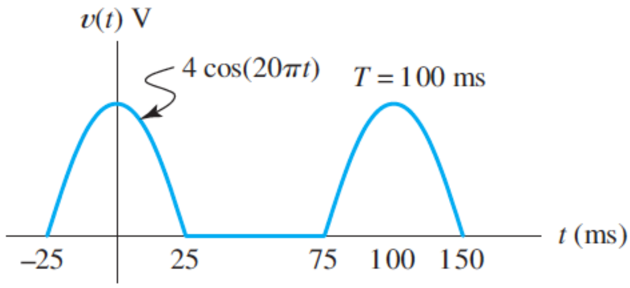
$$i(t) = 10\cos(2000\pi t)\ A$$

$$p(t) = Ri^2(t) = 10^4\cos^2(2000\pi t) = 5000[1+\cos(4000\pi t)]\ W$$

$$P_{avg} = R(I_{rms})^2 = 100\left(10/\sqrt{2}\right)^2 = 5000\ W$$



2. Calculate the rms value of the half-wave rectified sinusoidal wave shown in Figure below

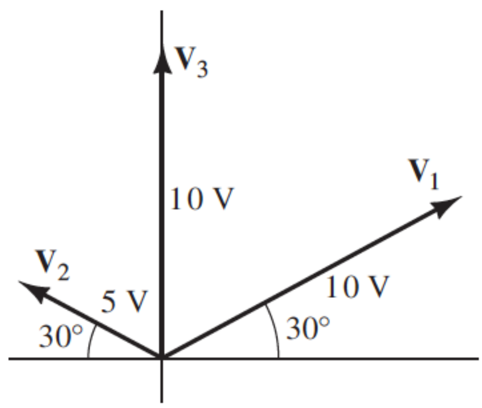


The limits on the integral don't matter as long as they cover one period

$$V_{rms} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} v^2(t)dt} = \sqrt{10\int_{-0.025}^{0.025}[4\cos(20\pi t)]^2 dt} = \sqrt{10\int_{-0.025}^{0.025}[8+8\cos(40\pi t)]dt}$$

$$V_{rms} = \sqrt{\left(80t + \frac{80}{40\pi}\sin(40\pi t)\right)\Bigg|_{t=-0.025}^{t=0.025}} = \sqrt{4} = 2 \text{ V}$$

3. Consider the phasors shown in Figure below. The frequency of each signal is f = 200Hz. Write a time-domain expression for each voltage in the form $V_m \cos(\omega t + \theta)$. State the phase relationships between pairs of these phasors



$\omega = 2\pi f = 400\pi$

$v_1(t) = 10\cos(400\pi t + 30^\circ)$
$v_2(t) = 5\cos(400\pi t + 150^\circ)$
$v_3(t) = 10\cos(400\pi t + 90^\circ)$

$v_1(t)$ lags $v_2(t)$ by 120°
$v_1(t)$ lags $v_3(t)$ by 60°
$v_2(t)$ leads $v_3(t)$ by 60°

4. (a) The current and voltage for a certain circuit element is shown in Figure - A. Determine the nature and value of the element.

(b) Repeat for Figure B

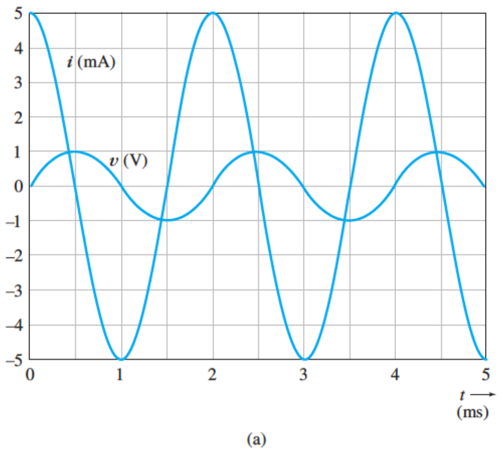


Figure- A

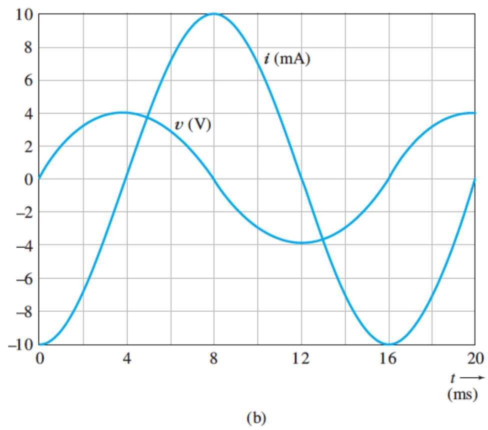
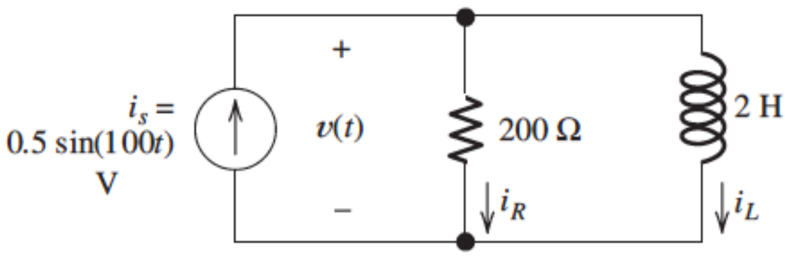


Figure- B

(a) From the plot, we see that $T = 2$ ms, so we have $f = 1/T = 500$ Hz and $\omega = 1000\pi$. Also, we see that the current leads the voltage by 0.5 ms or 90°, so we have a capacitance. Finally, $1/\omega C = V_m / I_m = 200\ \Omega$, from which we find that $C = 1.592\ \mu\text{F}$.

(b) From the plot, we see that $T = 16$ ms, so we have $f = 1/T = 62.5$ Hz and $\omega = 125\pi$. Also, we see that the current lags the voltage by 4 ms or 90°, so we have an inductance. Finally, $\omega L = V_m / I_m = 400\ \Omega$, from which we find that $L = 1.018$ H.

5. Find the phasors for the voltage and the currents of the circuit shown in Figure below. Construct a phasor diagram showing Is, V, I_R, and I_L. What is the phase relationship between V and Is?



$\mathbf{I}_s = 0.5\angle -90^\circ$ A

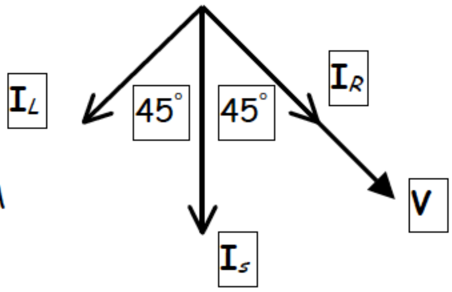
$\mathbf{V} = \mathbf{I}_s \dfrac{1}{1/200 + 1/j200}$

$= 70.71\angle -45^\circ$ V

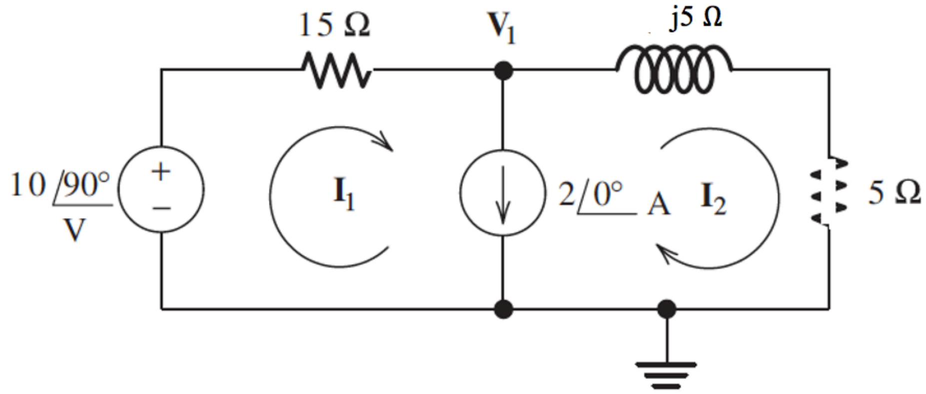
$\mathbf{I}_R = \mathbf{V}/R = 0.3536\angle -45^\circ$ A

$\mathbf{I}_L = \mathbf{V}/j\omega L = 0.3536\angle -135^\circ$ A

$\mathbf{V}$ leads $\mathbf{I}_s$ by 45°



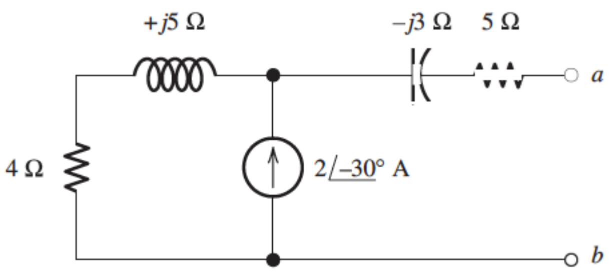
6. Solve for the node voltage shown in Figure



The KCL equation is $\dfrac{\mathbf{V}_1 - j10}{15} + \dfrac{\mathbf{V}_1}{5 + j5} + 2 = 0$. Solving, we find

$\mathbf{V}_1 = -10.58 - j2.35 = 10.84\angle -167.47^\circ$ V.

7. Find the Thevenin voltage, Thevenin impedance, and Norton current for the two terminal circuit shown in Figure



Under open-circuit conditions, we have

$$\mathbf{V}_t = \mathbf{V}_{ab-oc} = (4 + j5)2\angle -30^\circ = 11.92 + j4.66 = 12.80\angle 21.34^\circ \text{ V}$$

With the source zeroed, we look back into the terminals and see

$$Z_t = 5 - j3 + j5 + 4 = 9 + j2\,\Omega$$

Next, the Norton current is

$$\mathbf{I}_n = \frac{\mathbf{V}_t}{Z_t} = 1.389\angle 8.81^\circ \text{ A}$$

8. A balanced wye-connected three-phase source has line-to-neutral voltages of 440V rms. Find the rms line-to-line voltage magnitude. If this source is applied to a wye-connected load composed of three 30 Ω resistances, find the rms line-current magnitude and the total power delivered.

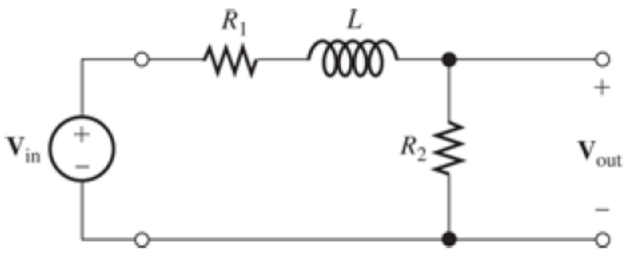
$$V_L = \sqrt{3} \times V_Y = \sqrt{3} \times 440 = 762.1 \text{ V rms}$$

$$I_L = \frac{V_Y}{R} = \frac{440}{30} = 14.67 \text{ A rms}$$

$$P = 3V_Y I_L \cos(\theta) = 3 \times 440 \times 14.67 \times \cos(0)$$
$$= 19.36 \text{ kW}$$

9. (a) Derive an expression for the transfer function H(f) = Vout/Vin for the circuit shown in Figure. Find an expression for the half-power frequency.

   (b) Given R1 = 50 Ω, R2 = 50 Ω , and L = 15 μH, sketch the magnitude of the transfer function versus frequency



(a) Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_2}{R_1 + R_2 + j2\pi fL} = \frac{R_2/(R_1+R_2)}{1 + j2\pi fL/(R_1+R_2)} = \frac{R_2/(R_1+R_2)}{1 + j(f/f_B)}$$

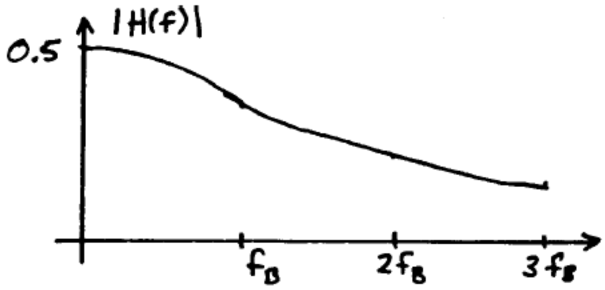
where $f_B = (R_1 + R_2)/(2\pi L)$

(b) Evaluating for the component values given, we have:

$$f_B = 1.061\text{ MHz}$$

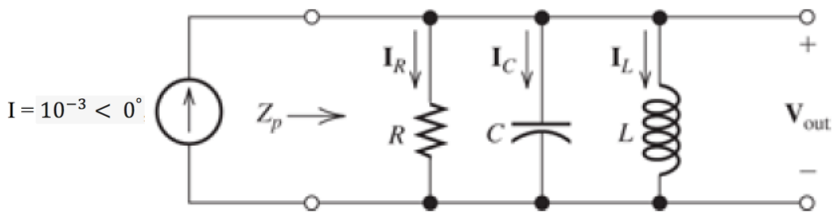
$$H(f) = \frac{0.5}{1 + j(f/f_B)}$$

A sketch of the transfer function magnitude is:



10. (a) A parallel resonant circuit has R = 5 k Ω, L = 50 μH, and C = 200 pF. Determine the resonant frequency, quality factor, and bandwidth.

(b) Consider the parallel resonant circuit shown in Figure below. Determine the L and C values, given R = 1 k Ω, f_0 = 10 MHz, and bandwidth B = 500 kHz. If the source current I = $10^{-3} < 0^{\circ}$, draw a phasor diagram showing the currents through each of the elements in the circuit at resonance.

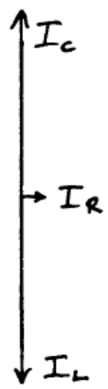


(a)

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.592\text{ MHz}$$

$$Q_p = \frac{R}{2\pi f_0 L} = 10.00 \qquad B = \frac{f_0}{Q_p} = 159.2\text{ kHz}$$

(b)



$$Q_p = \frac{f_0}{B} = 20$$

$$C = \frac{Q_p}{2\pi f_0 R} = 318.3\ \text{pF}$$

$$L = \frac{R}{2\pi f_0 Q_p} = 0.7958\ \mu\text{H}$$

$$\mathbf{I} = \mathbf{I}_R = 1\angle 0^\circ\ \text{mA}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{j2\pi f_0 L} = \frac{R\mathbf{I}}{j2\pi f_0 L} = 20\angle -90^\circ\ \text{mA}$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{1/(j2\pi f_o C)} = \frac{R\mathbf{I}}{1/(j2\pi f_0 C)} = 20\angle +90^\circ\ \text{mA}$$