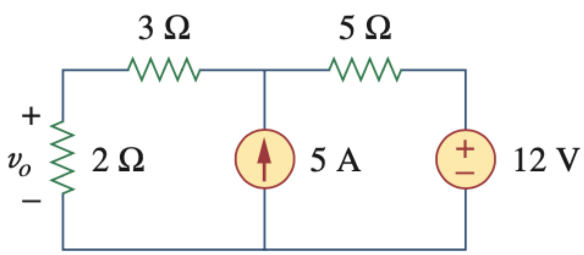
ECE 100 (Spring 2021) - Homework #5 (not graded)

This will serve as Midterm Exam preparation questions.

Due Date: Not graded

Problem 1: Solve following circuit 3 different ways

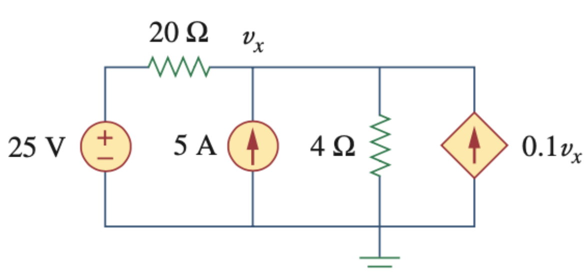


(a) Solve using KCL/KVL
(b) Solve using Source Transformations: **this can be done by converting the 3+2=5ohm resistors in parallel with a 5A current source into a voltage source of V=25V in series with a 5-ohm resistor**
(c) Solve using Superposition:
   (i) Zero out voltage source (convert to short-circuit)
   (ii) Zero out current source (convert to open-circuit)

Solution: $v_o = v_o' + v_o'' = 7.4V$

Problem 2: Superposition

Use superposition to find node voltage, v_x.

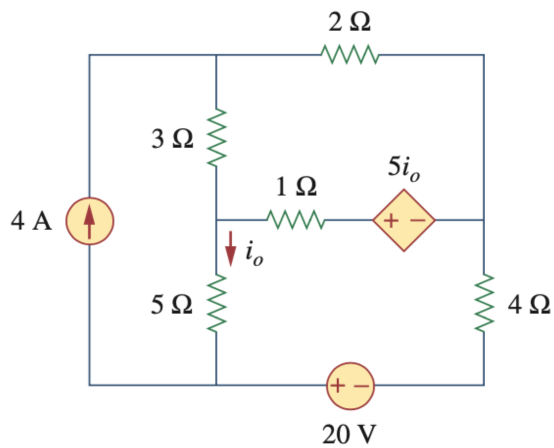


Note: only zero out independent sources (you cannot remove the dependent source in this circuit).

Solution: $v_x = v_x' + v_x'' = 31.25V$

Problem 3: Superposition

Use superposition to find current, i_o.



Solution:

The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

$$i_o = i'_o + i''_o \quad \textbf{(4.4.1)}$$

where i'_o and i''_o are due to the 4-A current source and 20-V voltage source respectively. To obtain i'_o, we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i'_o. For loop 1,

$$i_1 = 4 \text{ A} \quad \textbf{(4.4.2)}$$

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \quad \textbf{(4.4.3)}$$

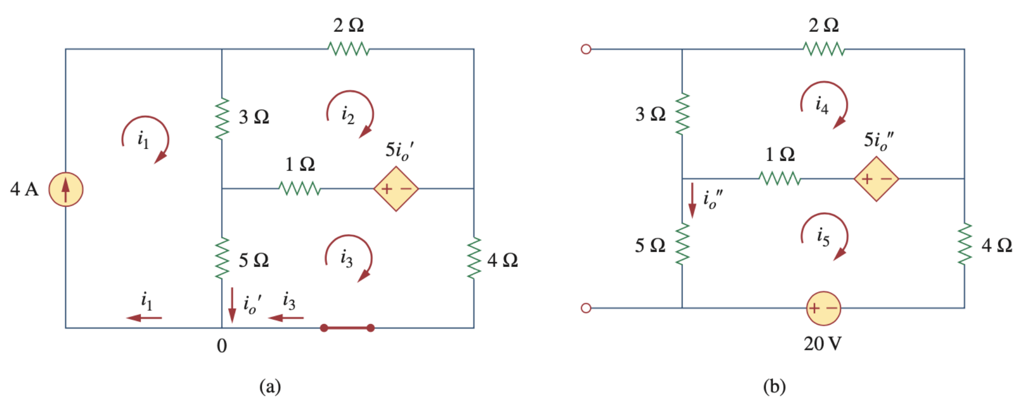


Figure 4.10

For Example 4.4: Applying superposition to (a) obtain i'_o, (b) obtain i''_o.

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \qquad \textbf{(4.4.4)}$$

But at node 0,

$$i_3 = i_1 - i'_o = 4 - i'_o \qquad \textbf{(4.4.5)}$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i'_o = 8 \qquad \textbf{(4.4.6)}$$

$$i_2 + 5i'_o = 20 \qquad \textbf{(4.4.7)}$$

which can be solved to get

$$i'_o = \frac{52}{17}\text{ A} \qquad \textbf{(4.4.8)}$$

To obtain i''_o, we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i''_o = 0 \qquad \textbf{(4.4.9)}$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_o = 0 \qquad \textbf{(4.4.10)}$$

But $i_5 = -i''_o$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i''_o = 0 \qquad \textbf{(4.4.11)}$$

$$i_4 + 5i''_o = -20 \qquad \textbf{(4.4.12)}$$

which we solve to get

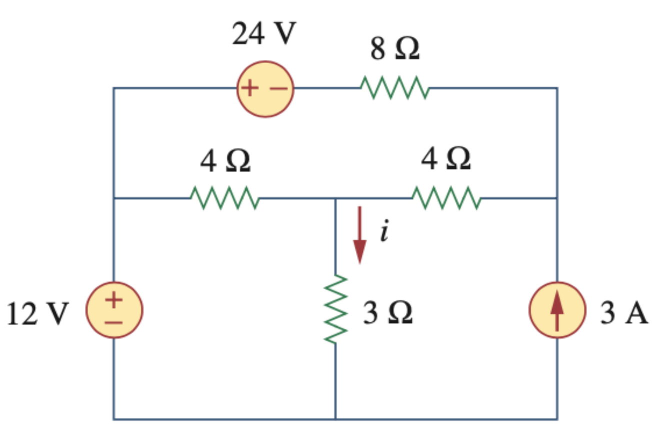
$$i''_o = -\frac{60}{17}\text{ A} \qquad \textbf{(4.4.13)}$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706\text{ A}$$

Problem 4: Superposition

Use superposition to find current, i



Solution:

In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1, consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4/16 = 3\ \Omega$. Thus,

$$i_1 = \frac{12}{6} = 2\text{ A}$$

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis gives

$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \qquad \textbf{(4.5.1)}$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b \qquad \textbf{(4.5.2)}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis gives

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1 \qquad \textbf{(4.5.3)}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1 \qquad \textbf{(4.5.4)}$$

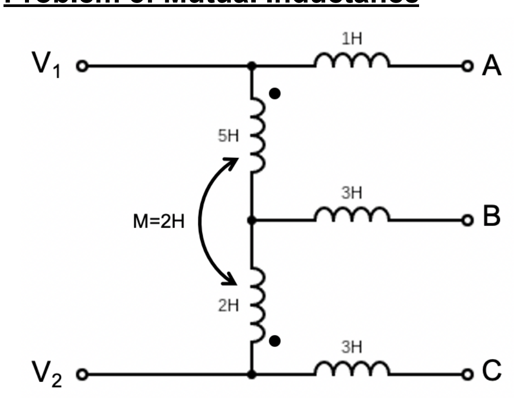
Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

$$i_3 = \frac{v_1}{3} = 1\text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2\text{ A}$$

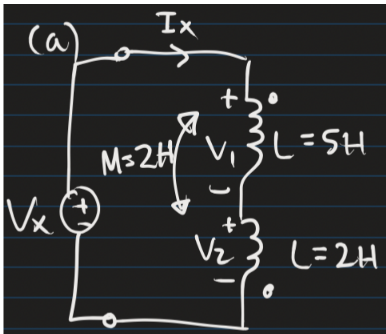
<u>Problem 5: Mutual Inductance</u>



Hint: Coupling, with mutual inductance M=2H, is shown between 5H & 2H inductors. There is no coupling to any of the other branches.

(a) Find the equivalent inductance, L_{eq}, seen between terminals V_1 and V_2.

(a)



$$V_x = V_1 + V_2$$

$$V_1 = 5\frac{di}{dt} - 2\frac{di}{dt} = 3\frac{di}{dt}$$

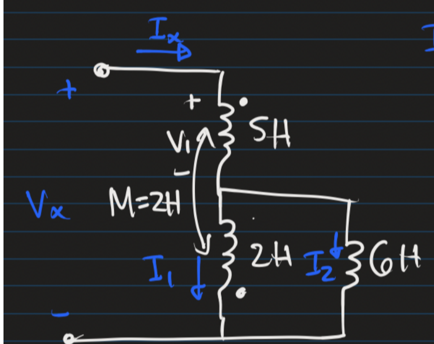
$$V_2 = 2\frac{di}{dt} - 2\frac{di}{dt} = 0\frac{di}{dt}$$

$$V_x = V_1 + V_2 = \underbrace{3}\frac{di}{dt}$$

$$\boxed{L_{eq} = 3H}$$

(b) Find the equivalent inductance, Leq, seen between terminals V1 and V2 if nodes B & C are connected together.

(b) B & C connected:



$$I_x = I_1 + I_2 \qquad V_x = V_1 + V_2$$

$$(1)\quad V_1 = 5\frac{dI_x}{dt} - 2\frac{dI_1}{dt}$$

$$(2)\quad V_2 = 2\frac{dI_1}{dt} - 2\frac{dI_x}{dt} = 6\frac{dI_2}{dt}$$

$$\frac{dI_2}{dt} = \frac{d}{dt}\left[I_x - I_1\right]$$

$$\rightarrow (2):\quad 2\frac{dI_1}{dt} - 2\frac{dI_x}{dt} = 6\left[\frac{d}{dt}I_x - \frac{d}{dt}I_1\right]$$

$$\rightarrow 8\frac{dI_x}{dt} = 8\frac{dI_1}{dt} \quad \Rightarrow \quad \underline{\frac{dI_x}{dt} = \frac{dI_1}{dt}}$$

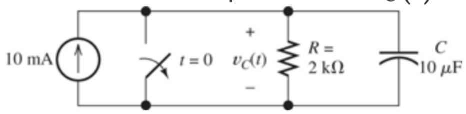
$$V_x = V_1 + V_2 = \left[5\frac{dI_x}{dt} - 2\frac{dI_1}{dt}\right] + \left[2\frac{dI_1}{dt} - 2\frac{dI_x}{dt}\right]$$

$$V_x = \underbrace{\left[5 - 2 + 2 - 2\right]}_{3}\frac{dI_x}{dt}$$

$$\Rightarrow \boxed{L_{eq} = 3H}$$

Problem 6

P4.13. Derive an expression for $v_C(t)$ in the circuit of **Figure P4.13** and sketch $v_C(t)$ to scale versus time.



Prior to $t = 0$, we have $v_C(t) = 0$ because the switch is closed. After $t = 0$, we can write the following KCL equation at the top node of the circuit:

$$\frac{v_C(t)}{R} + C\frac{dv_C(t)}{dt} = 10 \text{ mA}$$

Multiplying both sides by R and substituting values, we have

$$0.02\frac{dv_C(t)}{dt} + v_C(t) = 20 \qquad (1)$$

The solution is of the form

$$v_C(t) = K_1 + K_2\exp(-t/RC) = K_1 + K_2\exp(-50t) \qquad (2)$$

Using Equation (2) to substitute into Equation (1), we eventually obtain

$$K_1 = 20$$

The voltage across the capacitance cannot change instantaneously, so we have

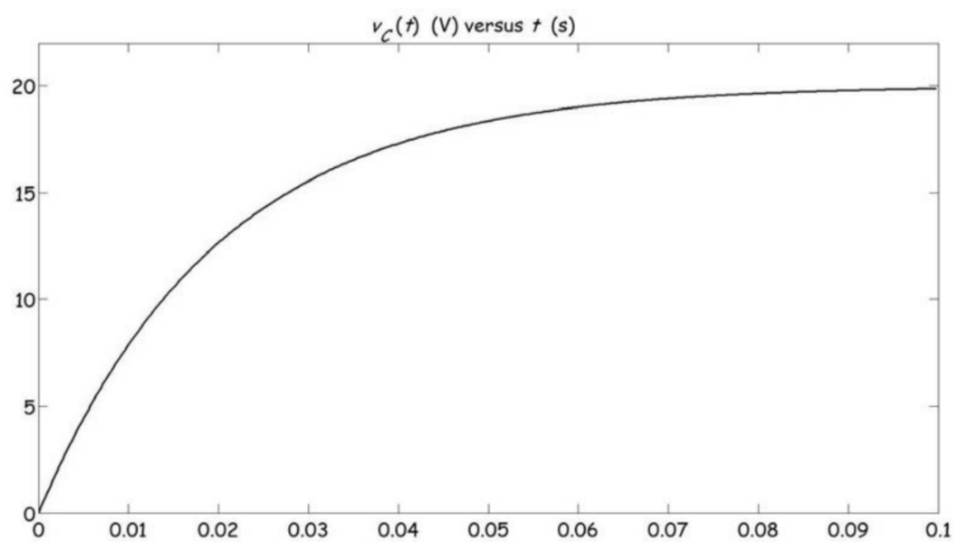
$$v_C(0+) = v_C(0-) = 0$$

$$v_C(0+) = 0 = K_1 + K_2$$

Thus, $K_2 = -K_1 = -20$, and the solution is

$$v_C(t) = 20 - 20\exp(-50t) \text{ for } t > 0$$

The sketch should resemble the following plot:



<u>**Problem 7**</u>

P4.18. Consider the circuit shown in **Figure P4.18**. Prior to $t = 0,\ v_1 = 100\ \mathrm{V}$, and $v_2 = 0$.

a. Immediately after the switch is closed, what is the value of the current [i.e., what is the value of $i(0+)$]?
b. Write the KVL equation for the circuit in terms of the current and initial voltages. Take the derivative to obtain a differential equation.
c. What is the value of the time constant in this circuit?
d. Find an expression for the current as a function of time.
e. Find the value that v_2 approaches as t becomes very large.

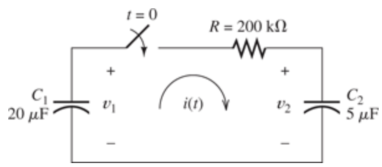


Figure P4.18

(a) The voltages across the capacitors cannot change instantaneously.
Thus, $v_1(0+) = v_1(0-) = 100\ \mathrm{V}$ and $v_2(0+) = v_2(0-) = 0$. Then, we can write

$$i(0+) = \frac{v_1(0+) - v_2(0+)}{R} = \frac{100 - 0}{200 \times 10^3} = 0.5\ \mathrm{mA}$$

(b) Applying KVL, we have

$$-v_1(t) + Ri(t) + v_2(t) = 0$$

$$\frac{1}{C_1}\int_0^t i(t)dt - 100 + Ri(t) + \frac{1}{C_2}\int_0^t i(t)dt + 0 = 0$$

Taking a derivative with respect to time and rearranging, we obtain

$$\frac{di(t)}{dt} + \frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)i(t) = 0 \qquad (1)$$

(c) The time constant is $\tau = R\dfrac{C_1C_2}{C_1 + C_2} = 800\ \mathrm{ms}$.

(d) The solution to Equation (1) is of the form

$$i(t) = K_1 \exp(-t/\tau)$$

However, $i(0+) = 0.5\ \mathrm{mA}$, so we have
$K_1 = 0.5\ \mathrm{mA}$ and $i(t) = 0.5\exp(-1.25t)\ \mathrm{mA}$.

(e) The final value of $v_2(t)$ is

$$v_2(\infty) = \frac{1}{C_2}\int_0^\infty i(t)dt + v_2(0+)$$

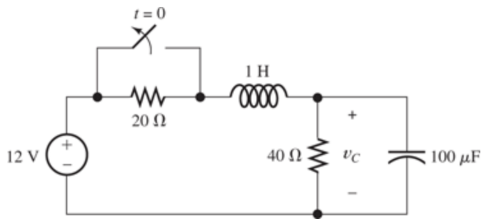
$$= 0.2 \times 10^6 \int_0^t 0.5 \times 10^{-3} \exp(-1.25t)dt + 0$$

$$= -80\exp(-1.25t)\Big|_0^\infty$$

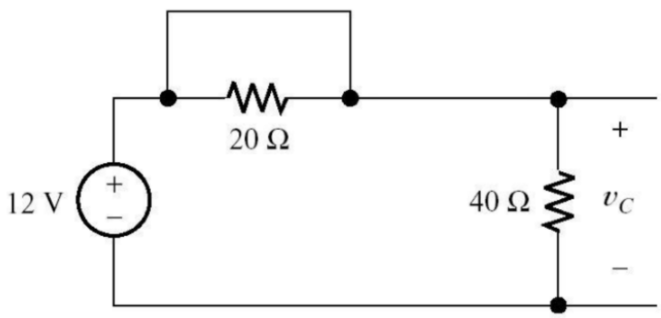
$$= 80\ \mathrm{V}$$

Problem 8

P4.24. The circuit shown in **Figure P4.24** has been set up for a long time prior to $t = 0$ with the switch closed. Find the value of v_C prior to $t = 0$. Find the steady-state value of v_C after the switch has been opened for a long time.

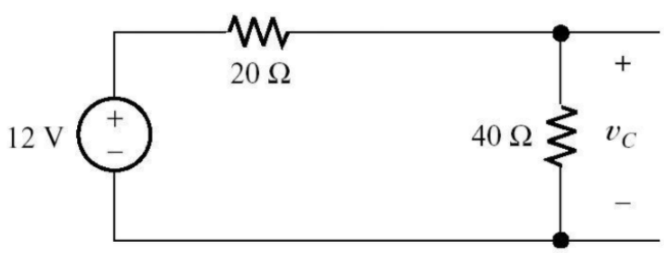


Prior to $t = 0$, the steady-state equivalent circuit is:



and we see that $v_c = 12\text{ V}$.

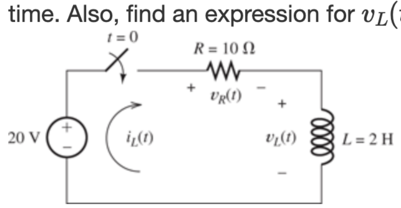
A long time after $t = 0$, the steady-state equivalent circuit is:



and we have $v_c = 12\dfrac{40}{40+20} = 8\text{ V}$.

Problem 9

P4.38. For the circuit shown in **Figure P4.38**, find an expression for the current $i_L(t)$ and sketch it to scale versus time. Also, find an expression for $v_L(t)$ and sketch it to scale versus time.



The solution is of the form

$$i(t) = K_1 + K_2 \exp(-Rt/L)$$

At $t = 0+$, we have

$$i(0+) = 0 = K_1 + K_2$$

and at $t = \infty$, we have

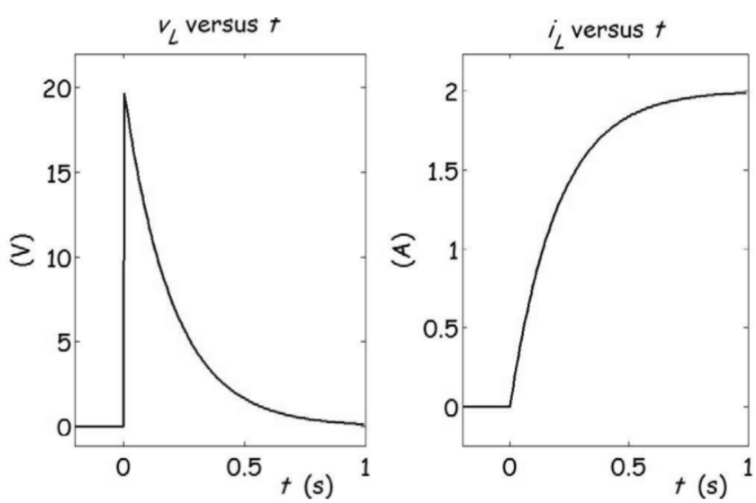
$$i(\infty) = (20\text{ V})/(10\ \Omega)$$
$$= 2.0 = K_1$$

The time constant is $\tau = L/R = 200\text{ ms}$. Thus, the solution is

$$i(t) = 2 - 2\exp(-5t)$$

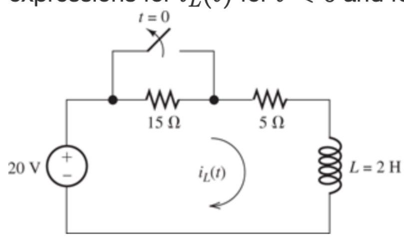
The voltage across the inductor is

$$v_L(t) = L\frac{di(t)}{dt} = 20\exp(-5t)$$



Problem 10

P4.39. The circuit shown in **Figure P4.39** is operating in steady state with the switch closed prior to $t = 0$. Find expressions for $i_L(t)$ for $t < 0$ and for $t \geq 0$. Sketch $i_L(t)$ to scale versus time.



In steady state, the inductor acts as a short circuit. With the switch closed, the steady-state current is $(20\,\text{V})/(5\,\Omega) = 4\,A$. With the switch opened, the current eventually approaches $i_L(\infty) = (20\,\text{V})/(20\,\Omega) = 1\,A$. For $t > 0$, the current has the form

$$i_L(t) = K_1 + K_2 \exp(-Rt/L)$$

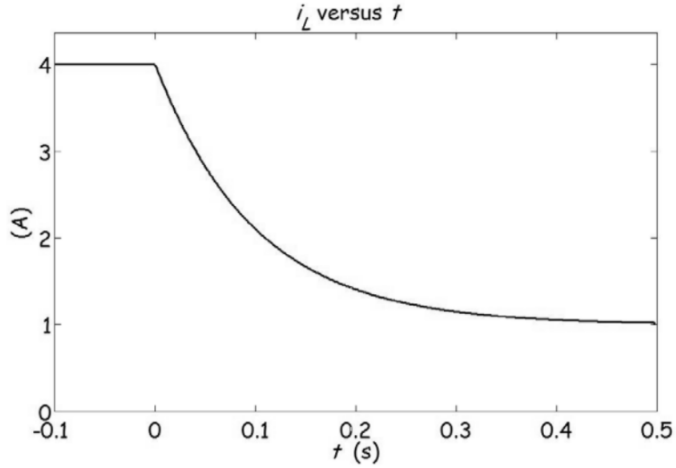
where $R = 20\,\Omega$, because that is the resistance with the switch open. Now, we have

$$i_L(0+) = i_L(0-) = 4 = K_1 + K_2 \qquad i_L(\infty) = 1 = K_1$$

Thus, we have $K_2 = 3$. The current is

$$i_L(t) = 4 \qquad t < 0 \text{ (switch closed)}$$
$$= 1 + 3\exp(-10t) \quad t \geq 0 \text{ (switch open)}$$

i_L versus t



Problem 11 (Note: includes Problems 4.61-4.63)

Hint: See provided notes on 2nd order differential equations

***P4.61.** A dc source is connected to a series *RLC* circuit by a switch that closes at $t = 0$, as shown in **Figure P4.61**. The initial conditions are $i(0+) = 0$ and $v_C(0+) = 0$. Write the differential equation for $v_C(t)$. Solve for $v_C(t)$, if $R = 80\ \Omega$.

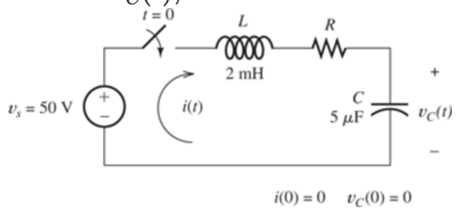


Figure P4.61

***P4.62.** Repeat **Problem P4.61** for $R = 40\ \Omega$.
***P4.63.** Repeat **Problem P4.61** for $R = 20\ \Omega$.

(a) P4.61

Applying KVL to the circuit, we obtain

$$L\frac{di(t)}{dt} + Ri(t) + v_C(t) = v_s = 50 \qquad (1)$$

For the capacitance, we have

$$i(t) = C\frac{dv_C(t)}{dt} \qquad (2)$$

Using Equation (2) to substitute into Equation (1) and rearranging, we have

$$\frac{d^2v_C(t)}{dt^2} + (R/L)\frac{dv_C(t)}{dt} + (1/LC)v_C(t) = 50/LC \qquad (3)$$

$$\frac{d^2v_C(t)}{dt^2} + 4\times10^4\frac{dv_C(t)}{dt} + 10^8 v_C(t) = 50\times10^8$$

We try a particular solution of the form $v_{Cp}(t) = A$, resulting in $A = 50$. Thus, $v_{Cp}(t) = 50$. (An alternative method to find the particular solution is to solve the circuit in dc steady state. Since the capacitance acts as an open circuit, the steady-state voltage across it is 50 V.) Comparing Equation (3) with Equation 4.67 in the text, we find

$$\alpha = \frac{R}{2L} = 2\times10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$$

Since we have $\alpha > \omega_0$, this is the overdamped case. The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -0.2679\times10^4$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -3.732\times10^4$$

The complementary solution is

$$v_{Cc}(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

and the complete solution is

$$v_C(t) = 50 + K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C \frac{dv_C(t)}{dt}\Big|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 50 + K_1 + K_2$$

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = s_1 K_1 + s_2 K_2$$

Solving, we find $K_1 = -53.87$ and $K_2 = 3.867$. Finally, the solution is

$$v_C(t) = 50 - 53.87 \exp(s_1 t) + 3.867 \exp(s_2 t)$$

(b) 4.62

As in the solution to P4.61, we have

$$v_{Cp}(t) = 50$$

$$\alpha = \frac{R}{2L} = 10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$$

Since we have $\alpha = \omega_0$, this is the critically damped case. The roots of the characteristic equation are equal:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -10^4$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10^4$$

The complementary solution is given in Equation 4.75 in the text:

$$v_{Cc}(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

and the complete solution is

$$v_C(t) = 50 + K_1 \exp(s_1 t) + K_2 t \exp(s_1 t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C \frac{dv_C(t)}{dt}\Big|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 50 + K_1$$

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = s_1 K_1 + K_2$$

Solving, we find $K_1 = -50$ and $K_2 = -50 \times 10^4$. Finally, the solution is

$$v_C(t) = 50 - 50 \exp(s_1 t) - (50 \times 10^4) t \exp(s_1 t)$$

(c) 4.63

As in the solution to P4.61, we have

$$v_{Cp}(t) = 50$$

$$\alpha = \frac{R}{2L} = 0.5 \times 10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$$

Since we have $\alpha < \omega_0$, this is the under-damped case. The natural frequency is given by Equation 4.76 in the text:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = 8.660 \times 10^3$$

The complementary solution is given in Equation 4.77 in the text:

$$v_{Cc}(t) = K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

and the complete solution is

$$v_C(t) = 50 + K_1 \exp(-\alpha t)\cos(\omega_n t) + K_2 \exp(-\alpha t)\sin(\omega_n t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C\frac{dv_C(t)}{dt}\Big|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 50 + K_1$$

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = -\alpha K_1 + \omega_n K_2$$

Solving, we find $K_1 = -50$ and $K_2 = -28.86$. Finally, the solution is

$$v_C(t) = 50 - 50\exp(-\alpha t)\cos(\omega_n t) - (28.86)\exp(-\alpha t)\sin(\omega_n t)$$

Problem 12

P4.64. Consider the circuit shown in **Figure P4.64** in which the switch has been open for a long time prior to $t = 0$ and we are given $R = 25\ \Omega$.

a. Compute the undamped resonant frequency, the damping coefficient, and the damping ratio of the circuit after the switch closes.
b. Assume that the capacitor is initially charged by a 25-V dc source not shown in the figure, so we have $v(0+) = 25\ \mathrm{V}$. Determine the values of $i_L(0+)$ and $v\prime(0+)$.
c. Find the particular solution for *v*(*t*).
d. Find the general solution for *v*(*t*), including the numerical values of all parameters.

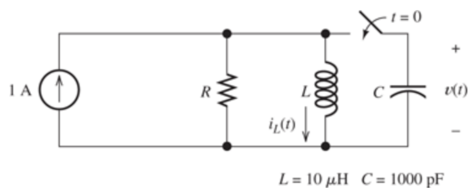


Figure P4.64

(a) Using Equation 4.103 from the text, the damping coefficient is

$$\alpha = \frac{1}{2RC} = 20 \times 10^6$$

Equation 4.104 gives the undamped resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \times 10^6$$

Equation 4.71 gives the damping ratio

$$\zeta = \alpha / \omega_0 = 2$$

Thus, we have an overdamped circuit.

(b) Because the switch has been open for a long time the inductor acts as a short circuit and we have $i_L(0-) = 1$ A. Because the current in the inductance cannot change instantaneously in this circuit, we have $i_L(0+) = 1$ A. Writing a current equation at $t = 0+$, we have

$$\frac{v(0+)}{R} + i_L(0+) + Cv'(0+) = 1$$

Substituting $R = 25\ \Omega$, $v(0+) = 25$ V and $i_L(0+) = 1$ A, yields

$$v'(0+) = -10^9\ \text{V/s}$$

(c) Under steady-state conditions, the inductance acts as a short circuit. Therefore, the particular solution for $v(t)$ is:

$$v_p(t) = 0$$

(d) The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

Problem 13

Refer to Figure 1 below. The inductor has no initial energy (Hint: $i_L(0^+) = 0A$)

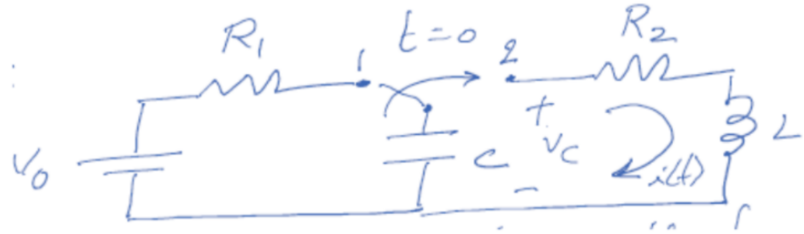


Figure 1.

a. Derive the differential equation that governs the current i(t) for time $t \geq 0$.

b. What is the characteristic equation that governs the behavior of this circuit?

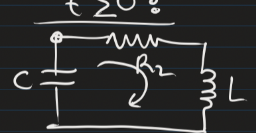
For the following assume $R_1 = 5\ k\Omega$, $R_2 = 16\ \Omega$, $L = 400\ nH$, $C = 4\ nF$, and $V_0 = 10$ Volts.

c. Derive an expression for the voltage across the capacitor, $V_C(t)$ for $t \geq 0$. Assume that the circuit achieved steady state before $t = 0$ i.e. assume that the circuit came into being at $t = -\infty$.

d. Draw a rough sketch of the waveform.

(a)

$t \geq 0$:



KVL:

- $V_c(t) - iR_2 - L\frac{di}{dt} = 0$

note: $i_L(t) = i(t) = -C\frac{dV_c(t)}{dt}$

$$\Rightarrow V_c(t) - \left(-C\frac{dV_c(t)}{dt}\right)R_2 - L\left(\frac{d}{dt}\left(-C\frac{dV_c}{dt}\right)\right) = 0$$

$$\Rightarrow LC\frac{d^2V_c(t)}{dt^2} + R_2C\frac{dV_c(t)}{dt} + V_c(t) = 0$$

$$\Rightarrow \boxed{\frac{d^2V_c(t)}{dt^2} + \frac{R_2}{L}\frac{dV_c(t)}{dt} + \frac{1}{LC}V_c(t) = 0}$$

(b)

$$s^2 + \frac{R_2}{L}s + \frac{1}{LC} = 0$$

OR...

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

where $\underline{\underline{\omega_0 = \frac{1}{\sqrt{LC}}}}$ and $\underline{\underline{\zeta = \frac{1}{2}\frac{R_2}{\sqrt{\frac{L}{C}}}}}$

$R_1 = 5k\Omega$.
$R_2 = 16\Omega$
$L = 400nH$
$C = 4nF$
$V_0 = 10V$

$$s^2 + 40\times10^6 s + 6.25\times10^{14} = 0$$

OR...

$$\omega_0 = 25\times10^6$$
$$\zeta = 0.8$$

(c) initial conditions:

$V_c(0^+) = V_0 = 10V$

$i_L(0^+) = i(0^+) = -C\frac{dV_c}{dt}\Big|_{t=0} = 0A$

final condition:

$V_c(\infty) = 0V$

roots:
$$s = \left[-\zeta \pm \sqrt{1-\zeta^2}\right]\omega_0$$

$$s_1, s_2 = -2\times10^{-7} \pm j\,1.5\times10^{-7}$$

$$V_c(t) = K_1 e^{-2\times10^{-7}t}\cos(1.5\times10^{-7}t) + K_2 e^{-2\times10^{-7}t}\sin(1.5\times10^{-7}t) + \cancel{K_3}^{\,0}$$

(1) $V_c(\infty) = \underline{K_3 = 0}$

(2) $V_c(0) = \underline{K_1 = V_0 = 10V}$

(3) $$\frac{dV_c(t)}{dt} = \left[-2\times10^{-7}K_1 e^{-\sigma t}\cos(\omega_n t) + K_1 e^{-\sigma t}(-\omega_n \sin(\omega_n t))\right] + \left[-2\times10^{-7}K_2 e^{-\sigma t}\sin(\omega_n t) + K_2 e^{-\sigma t}(+\omega_n\cos(\omega_n t))\right]$$

$$\frac{dV_c(t)}{dt}\Big|_{t=0} = \left[-2\times10^{-7}K_1\right] + \left[K_2\omega_n\right] = 0$$

$$\Rightarrow K_2 = \frac{2\times10^{-7}K_1}{\omega_n} = \frac{(2\times10^{-7})(10V)}{(1.5\times10^{-7})}$$

$$K_2 = \frac{40}{3}$$

$$\boxed{V_c(t) = 10e^{-\sigma t}\cos(\omega_n t) + \frac{40}{3}e^{-\sigma t}\sin(\omega_n t)}$$

Problem 14

Refer to Figure 2 below. Both switches changed from their position 1→2, respectively, at time t=0, after the circuits having already achieved steady state.

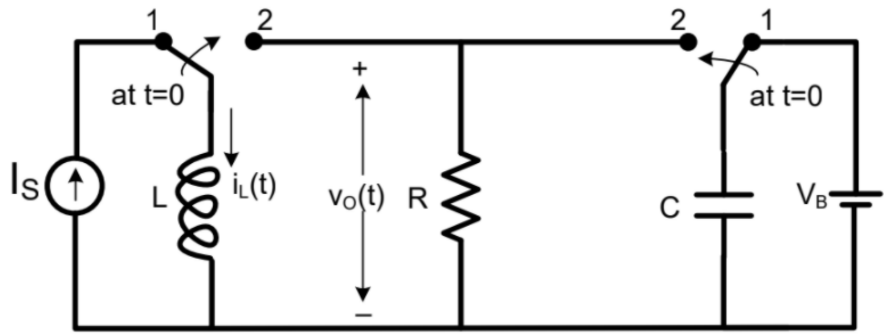


Figure 2.

For parts (a) through (d), express your answers in terms of V_B, I_S, R, L, and C.

a. Derive a differential equation in terms of $v_O(t)$ that characterizes circuit behavior for $t \geq 0$.

b. What is the characteristic equation of this circuit for time $t \geq 0$?

c. Determine the value of $v_O(t)$ just after $t = 0$.

d. Determine the value of $\left.\dfrac{dv_O(t)}{dt}\right|_{t=0+}$.

For the rest, use $V_B = 2V$, $I_S = 0.6mA$, $C = 1nF$, $L = 64nH$, and $R = 5$ Ohms.

e. Calculate the damping factor of the circuit. What kind of damping is this?

f. Derive an expression for the complete solution of $v_O(t)$ in this circuit.

g. Derive an expression for the current, $i_L(t)$ in this circuit.

h. Draw a rough sketch of the solution, $v_O(t)$. Do not forget to show the initial and final values of $v_O(t)$. If $v_O(t)$ shows some ringing, mark the period and the time constant of the envelope as well.

i. What will happen if R were infinitely large? Derive an expression for $v_O(t)$ and sketch it.

For your own practice. Please attend office hours if you would like to discuss.