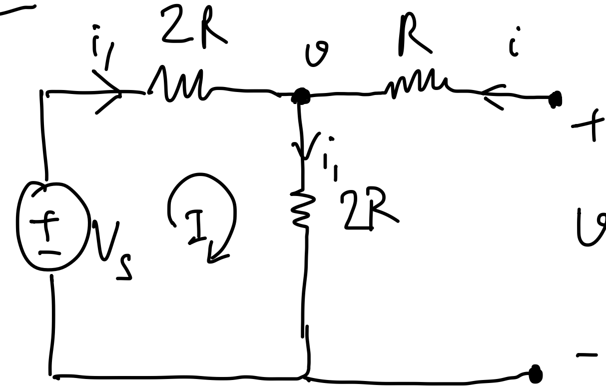
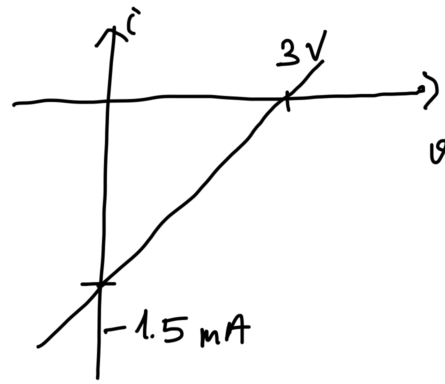
Home Work 4:

Problem 1:





a) From the graph, we have:

$v_0 = 3V$ when $i = 0 \Leftrightarrow$ open circuit $\rightarrow \boxed{v_{th} = 3V}$

$i_{sc} = -1.5mV$ when $v = 0 \Leftrightarrow$ Short circuit

$$\Rightarrow Z_{th} = \left|\frac{V_0}{i_{sc}}\right| = \left|\frac{3V}{1.5\times10^{-3}A}\right| = 2\times10^{3}(\Omega)$$

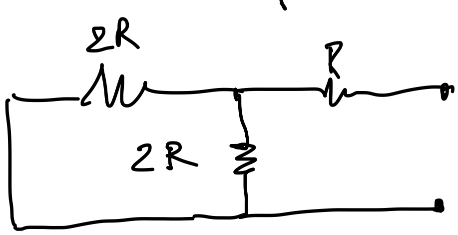
$$= \boxed{2\ k\Omega}$$

b) ⊛ When $\begin{cases} i = 0 \\ v = 3V \end{cases} \Rightarrow$ apply KVL for loop (I), we have.

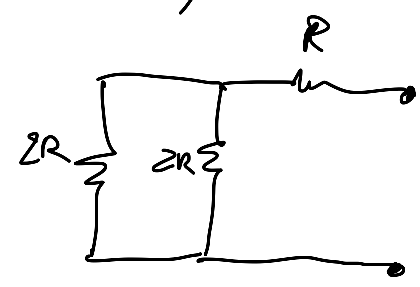
$V_s = 4Ri_1$, also $v = 2Ri_1 = 3V \Rightarrow i_1 = \frac{3}{2R}(A)$

$\rightarrow V_s = 4R \times \frac{3}{2R}$ $\boxed{6\ (V)}$

⊛ We have the new circuit when finding the Z_{th}:
(Short circuit for voltage source V_s)



$\Leftrightarrow$

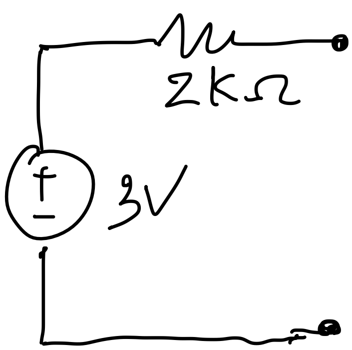


$\Rightarrow Z_{th} = (2R \parallel 2R)$ series $R = \dfrac{2R \times 2R}{2R + 2R} + R = 2R$

From a, we know $Z_{th} = 2\,k\Omega \Rightarrow 2R = 2\,k\Omega$

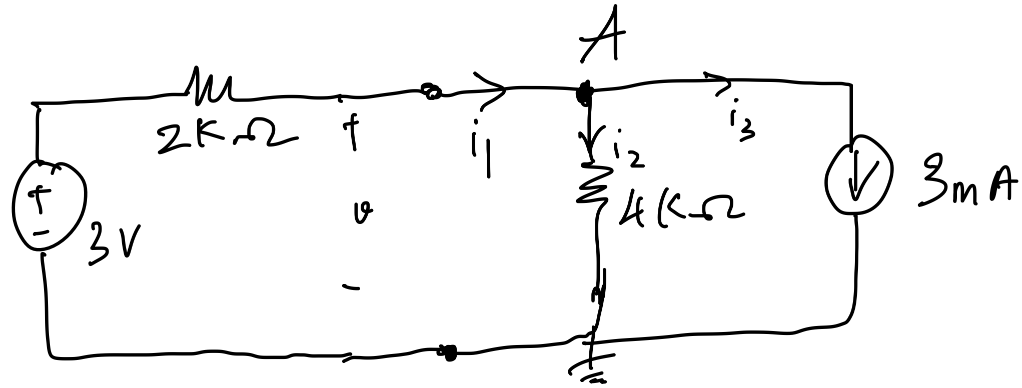
$\Rightarrow \boxed{R = 1\,(k\Omega)}$

c) From a & b, we have the equivalent Thevenin:



$\Rightarrow$

The new circuit:



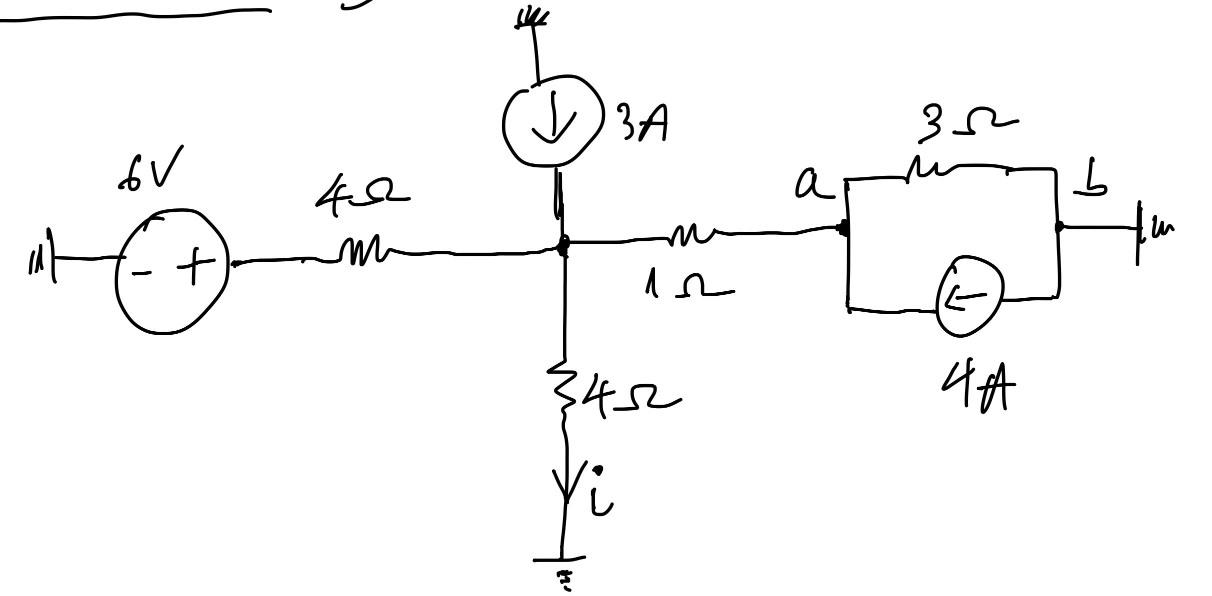
Apply KCL at A $\Rightarrow i_1 = i_2 + i_3$

$$\Rightarrow \frac{3-v}{2k\Omega} = \frac{v}{4k\Omega} + 3\,mA \Rightarrow \frac{3-v}{2k} - \frac{v}{4k} = 3\,mA$$

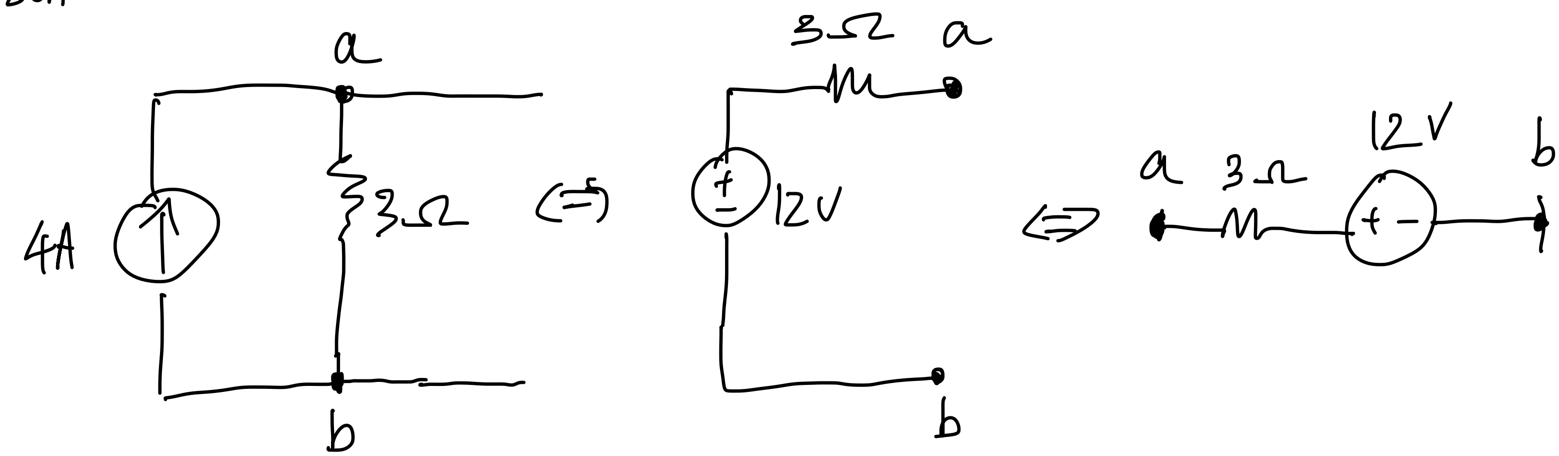
$$\Rightarrow 2(3-v) - v = 3 \times 4 = 12$$

$$\Rightarrow 6 - 2v - v = 12 \Rightarrow -6 = 3v \Rightarrow \boxed{v = -2V}$$

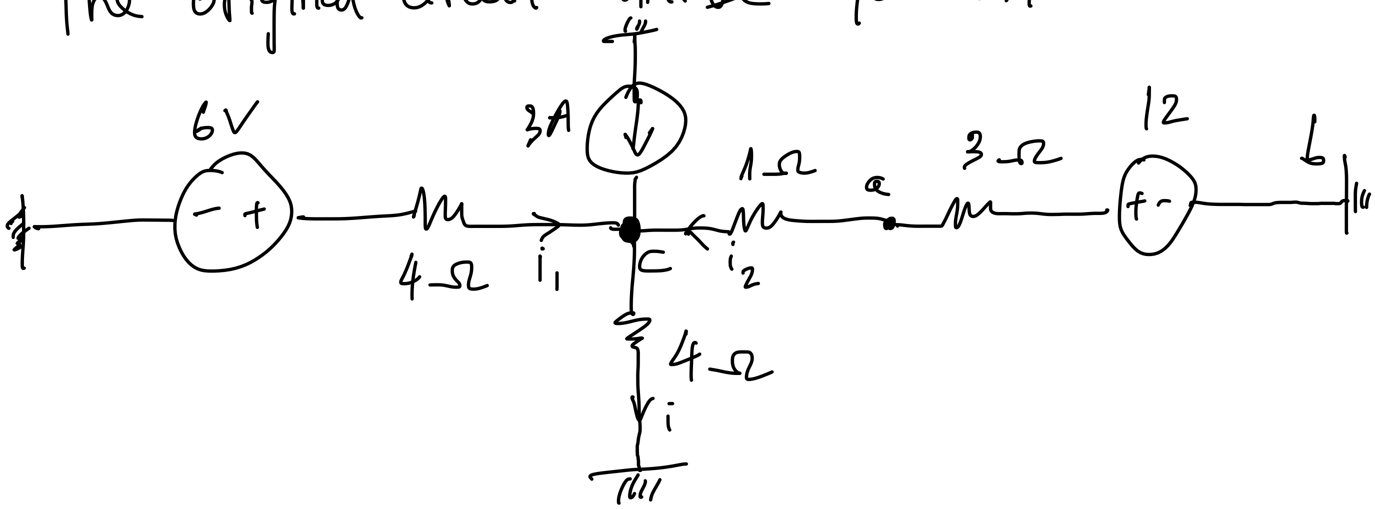
Problem 2: Determine the current i:



Since we have:



The original circuit will be equivalent:



Apply KCL at C, we have: $i_1 + i_2 + 3 = i$

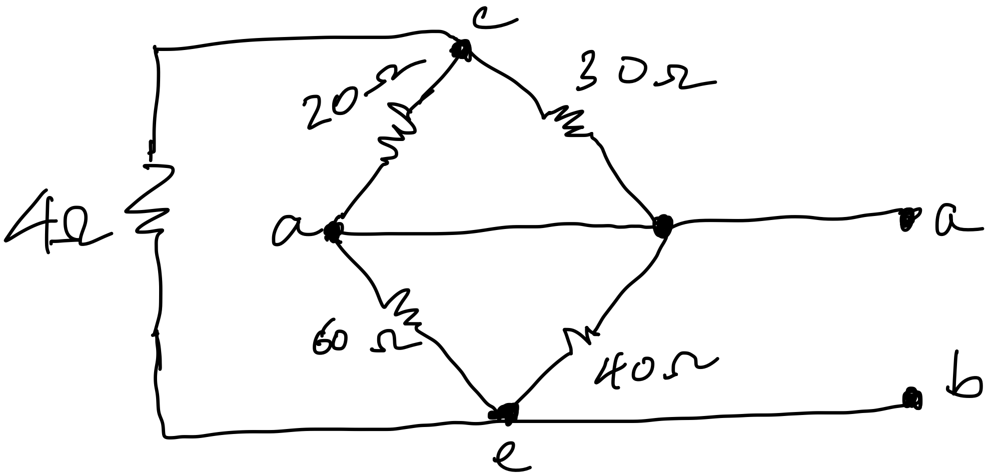
$$\Rightarrow \frac{6 - v_c}{4} + 3 + \frac{12 - v_c}{4} = \frac{v_c}{4}$$

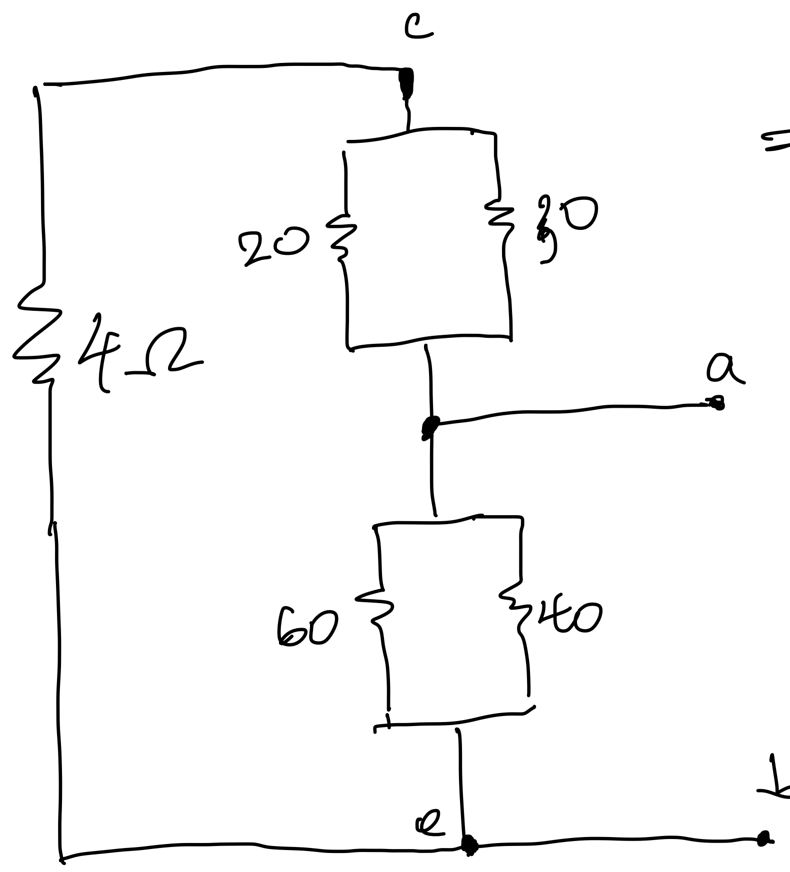
$$\Rightarrow 6 - v_c + 12 + 12 - v_c = v_c$$

$$\Rightarrow 30 = 3v_c \Rightarrow v_c = 10 \ (V)$$

$$\Rightarrow i = \frac{10}{4} = \boxed{2.5 \ A}$$

Problem 3:





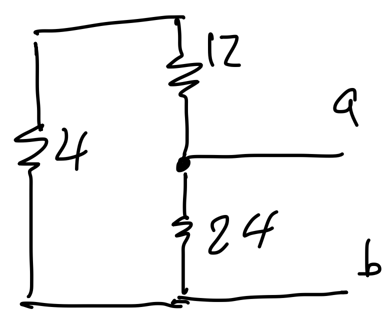
$20 \parallel 30$

$$\Rightarrow R_{20-30} = \frac{30 \times 20}{30+20} = 12\,\Omega$$

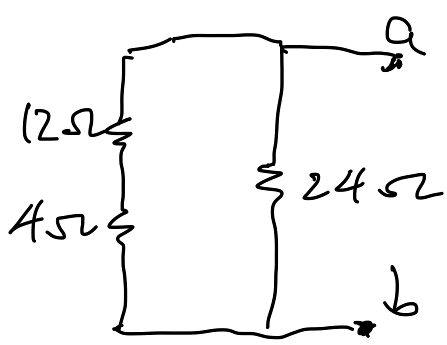
$60 \parallel 40$

$$\Rightarrow R_{60-40} = \frac{60 \times 40}{60+40} = 24\,\Omega$$

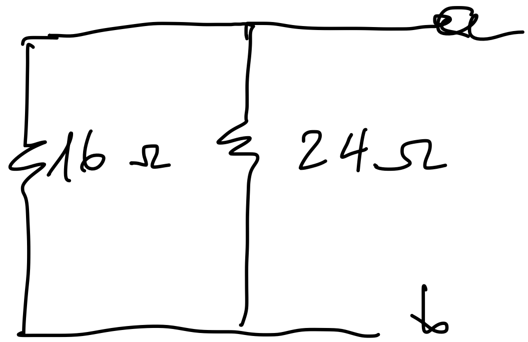
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$

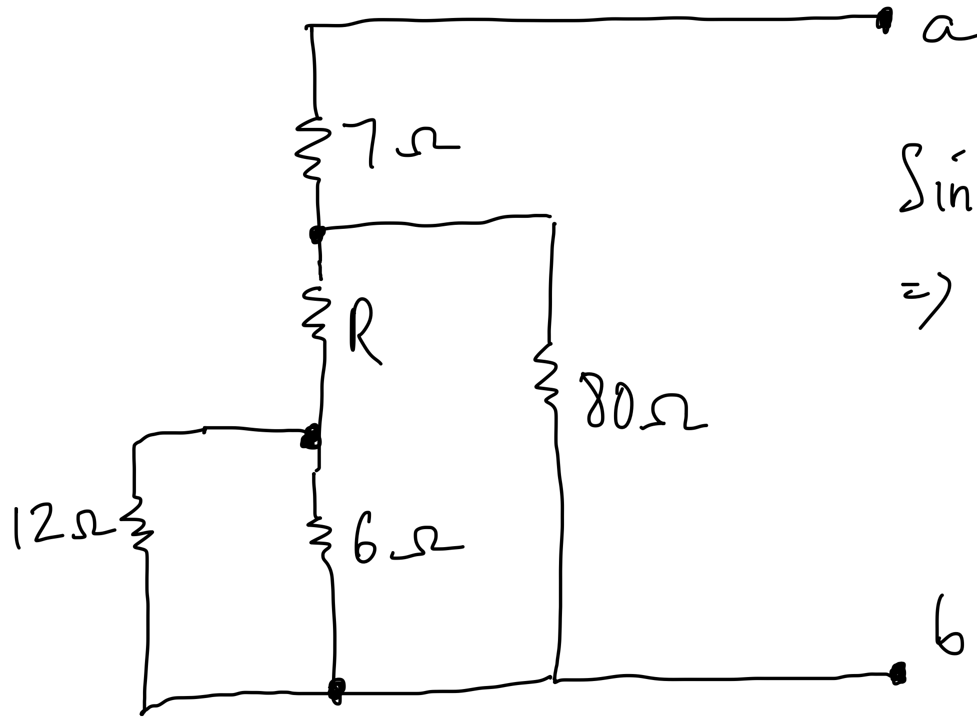


Since $16\,\Omega \parallel 24\,\Omega$

$$\Rightarrow R_{ab} = \frac{16 \times 24}{16 + 24}$$

$$\Rightarrow \boxed{R_{ab} = 9.6\,(\Omega)}$$

Problem 4: Given $R_{ab} = 23\,\Omega$



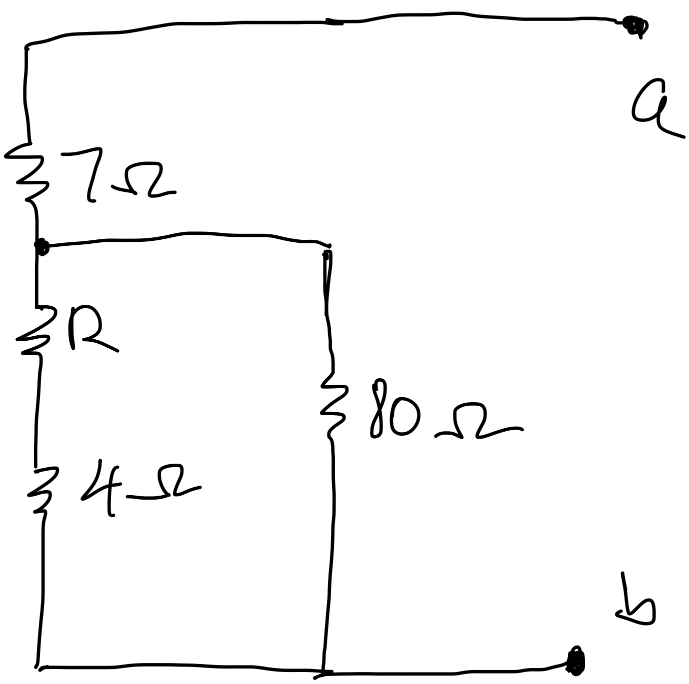
Since $12\,\Omega \parallel 6\,\Omega$

$\Rightarrow R_{12\_6} = \frac{12 \times 6}{12+6} = 4\,\Omega$

$(R \text{ series } 4\,\Omega) \parallel 80\,\Omega$

$\Rightarrow (R+4) \parallel 80\,\Omega$

$\Rightarrow R_{4\_80} = \frac{(R+4)\,80}{R+84}$



Also R_{4_80} series $7\,\Omega \Rightarrow$

$R_{ab} = 7 + R_{4\_80} = 23\,(\Omega)$

$$\Rightarrow R_{4-80} = 16\ \Omega$$

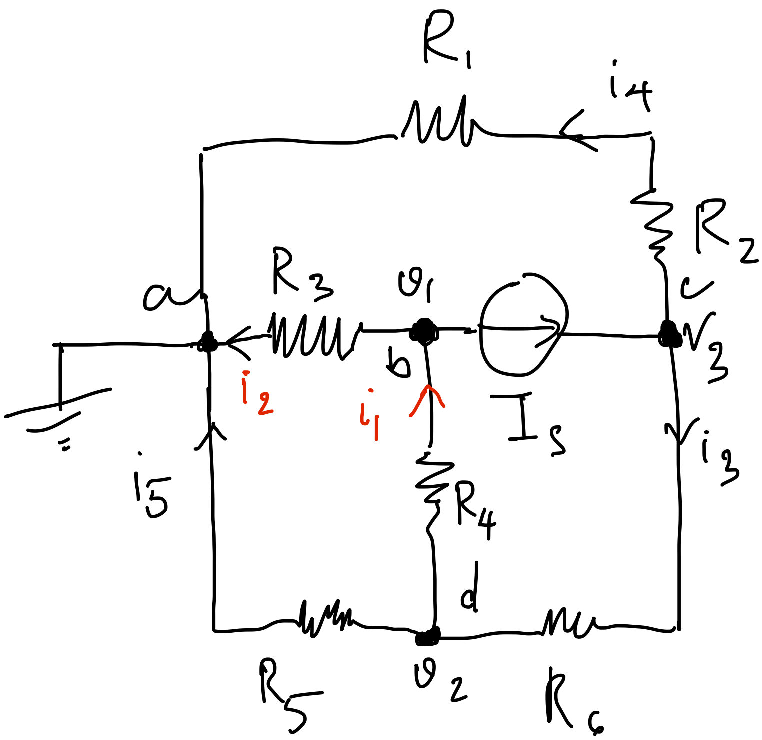
$$\Rightarrow \frac{(R+4)\times 80}{R+84} = 16(\Omega)$$

$$\Rightarrow 80R + 320 = 16R + 1344$$

$$\Rightarrow 64R = 1024$$

$$\Rightarrow \boxed{R = 16(\Omega)}$$

Problem 5: Given:



$R_1 = 15\,\Omega$

$R_2 = 5\,\Omega$

$R_3 = 20\,\Omega$

$R_4 = 10\,\Omega$

$R_5 = 8\,\Omega$

$R_6 = 4\,\Omega$

$I_s = 5A$

Apply KCL at b, we have:

$$i_1 = i_2 + I_s = i_2 + 5$$

$$\Rightarrow \frac{v_2 - v_1}{R_4} = \frac{v_1}{R_3} + 5$$

$$\Rightarrow \frac{v_2 - v_1}{10} = \frac{v_1}{20} + 5$$

$$\Rightarrow 2v_2 - 2v_1 = v_1 + 100$$

$$\Rightarrow 2v_2 - 3v_1 = 100 \quad (1)$$

Apply KCL at c, we have:

$$I_s = i_3 + i_4 \Rightarrow 5 = \frac{v_3 - v_2}{R_6} + \frac{V_3 - 0}{R_2 + R_1}$$

$$\Rightarrow 5 = \frac{v_3 - v_2}{4} + \frac{v_3}{20}$$

$$\Rightarrow 100 = 5v_3 - 5v_2 + v_3$$

$$\Rightarrow 6v_3 - 5v_2 = 100 \quad (2)$$

Apply KCL at a, we have:

$$i_2 + i_4 + i_5 = 0 \Rightarrow \frac{v_2}{R_5} + \frac{v_3}{R_1 + R_2} + \frac{v_1}{R_3} = 0$$

$$\Rightarrow \frac{v_2}{8} + \frac{v_3}{20} + \frac{v_1}{20} = 0$$

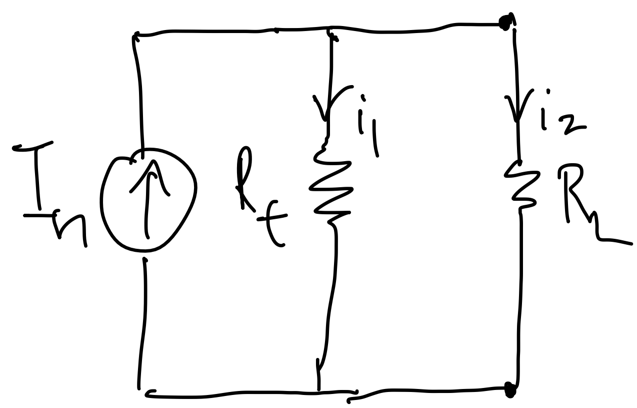
$$\Rightarrow 5v_2 + 2v_3 + 2v_1 = 0$$

$$\Rightarrow 2v_1 + 5v_2 + 2v_3 = 0 \quad (3)$$

From (1), (2) & (3), we have:

$$\begin{cases} -3v_1 + 2v_2 = 100 \\ -5v_2 + 6v_3 = 100 \\ 2v_1 + 5v_2 + 2v_3 = 0 \end{cases} \Rightarrow \boxed{\begin{cases} v_1 = -30.56\,(V) \\ v_2 = 4.167\,(V) \\ v_3 = 20.14\,(V) \end{cases}}$$

Problem 6 :



* Find expression for the power delivered to the load in term of I_n, R_t & R_L

$$i_2 = \frac{R_t}{R_t + R_L} I_n$$

(divde current)

$$\Rightarrow P_{R_L} = i_2^2 R_L = \left(\frac{R_t}{R_t + R_L} I_n \right)^2 \times R_L$$

$$\Rightarrow \boxed{P_{R_L} = \frac{R_t^2 R_L}{(R_t + R_L)^2} I_n^2}$$

* Assuming I_n, R_t are fixed value, R_L is variable, show that maximum power delivered for $R_L = R_t$

let $x = R_L$, $R_t = a$ (constant), I_n constant

$$P = \frac{a^2 x}{(a+x)^2} I_n \quad (R_t = a > 0 \ \& \ R_L = x > 0)$$

since they are value of resistor

We have $\frac{dP}{dx} = a^2 I_n \left[\frac{x}{(a+x)^2}\right]'$

Also: $\left[\frac{x}{(a+x)^2}\right]' = \frac{(a+x)^2 - 2x(a+x)}{(a+x)^2}$

$$\Rightarrow \frac{dP}{dx} = 0 \Leftrightarrow (a+x)^2 - 2x(a+x) = 0$$

$$\Leftrightarrow (a+x)(a+x-2x) = 0$$

$$\Leftrightarrow (a+x)(a-x) = 0 \Leftrightarrow \left[\begin{array}{l} a = -x \text{ (reject)} \\ a = x \end{array}\right.$$

$$\Rightarrow \frac{dP}{dx} = 0 \Leftrightarrow a = x \Leftrightarrow R_t = R_L$$

Since $P = \frac{a^2 x}{(a+x)^2}$ will get maximum

when $\frac{dP}{dx} = 0 \Rightarrow$ the power which is delivered for R_L will get maximum value when $R_L = R_t$.

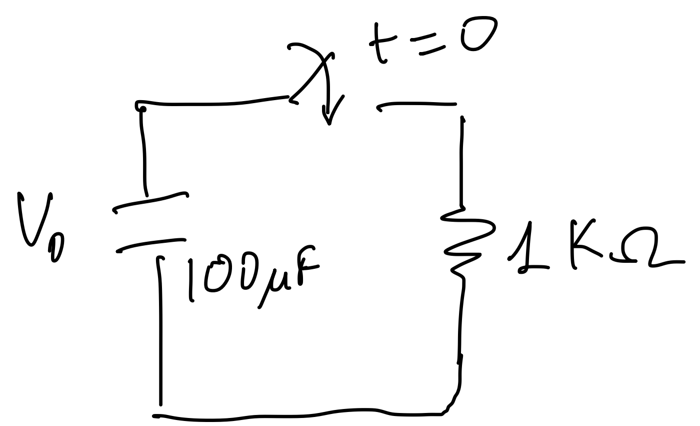
* We have already $P = \frac{R_t^2 R_L}{(R_t + R_L)^2} \times I_n^2$

With $R_L = R_t$

$\Rightarrow P = \frac{R_t^2 \times R_t}{4R_t^2} \times I_n^2 = \frac{R_t}{4} \times I_n^2$

$$\Rightarrow \boxed{P_{max} = \frac{I_n^2 R_t}{4}}$$

Problem 7:



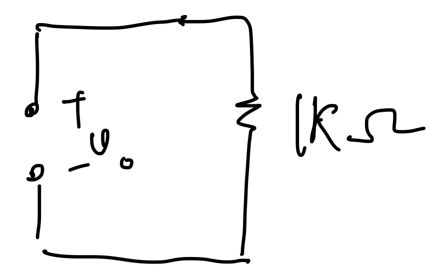
a capacitance is initially charge to 1000V

$\Rightarrow V_0 = 1000V$ at $t=0^+$

(Since capacitor does not allow the voltage change instantaneously)

$\Rightarrow V(0^+) = 1000V$

Also when $t \to \infty$ $\Rightarrow$



$\Rightarrow v(\infty) = 0\,V$

Also: $v_c(t) = V(\infty) - \left(v(\infty) - v(0^+)\right)e^{-t/RC}$

$\Rightarrow v_c(t) = V_0\, e^{-t/RC} = 1000\, e^{-t/RC}\ (V)$

$R = 1000\,\Omega$, $C = 100\mu F = 100\times10^{-6}F$

$\Rightarrow RC = 1000 \times 100 \times 10^{-6} = 0.1$

$\Rightarrow v_c(t) = 1000\, e^{-10t}\ (V)$

$\Rightarrow [v_c(t)]^2 = (1000)^2 e^{-20t}$

$\Rightarrow$ Energy of capacitor:

$$E(t) = \frac{1}{2} C [v_c(t)]^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 1000^2 e^{-20t}$$

$$= 50 e^{-20t} \quad (J)$$

Since at $t = 0^+$, we have the initial energy of capacitance $\Rightarrow E_0 = 50 e^{-0} = 50 (J)$

Since 50% of the initial energy of capacitance has been dissipated in the resistance $\Rightarrow$ the capacitance remains 50%.

$\Rightarrow$ at t_2, we have:

$$E(t_2) = 50 e^{-20t_2} = 50\% \text{ of } 50 J$$

$$= 0.5 \times 50 = 25 (J)$$

$$\Rightarrow e^{-20t_2} = \frac{25}{50} = 0.5$$

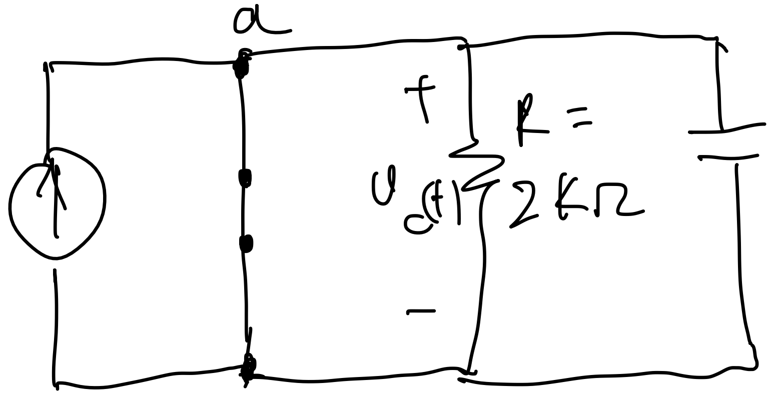
$$\Rightarrow t_2 = \frac{\ln 0.5}{-20} \Rightarrow t_2 \simeq 0.03466 (s)$$

$$\Rightarrow \boxed{t_2 = 34.66 \text{ ms}}$$

Problem 8: $v_c(t)$

* Switch was closed before $t=0$
* open after $t=0$

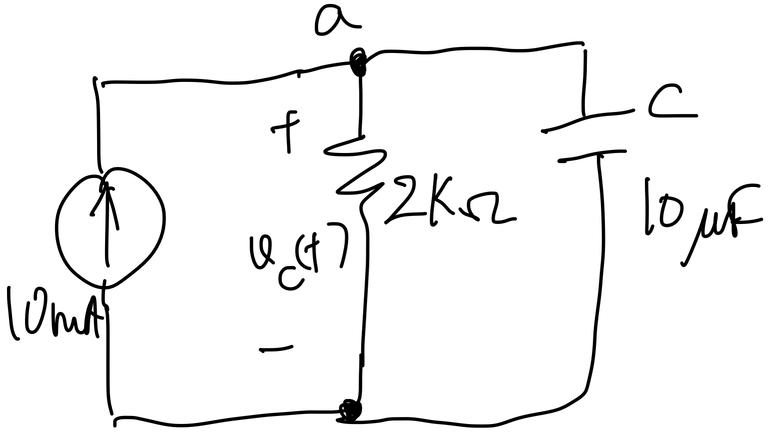
At $t=0^-$ ⇒ switch was closed, we have



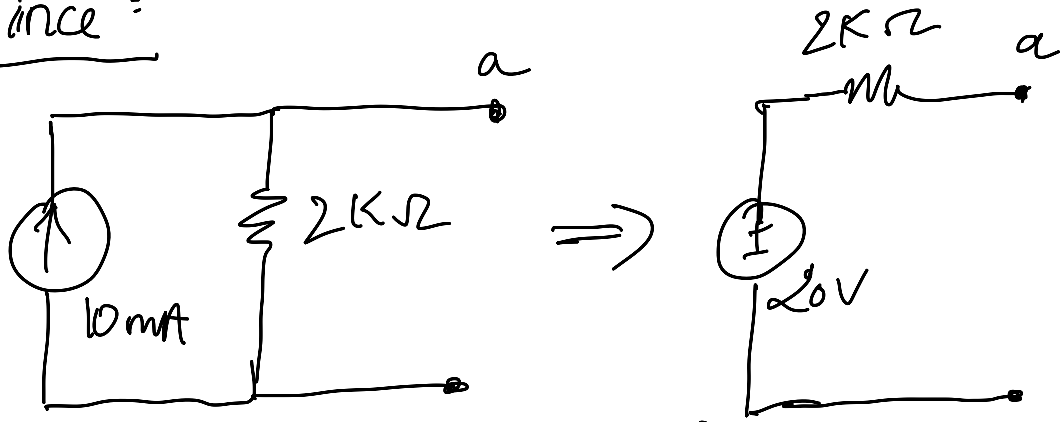
$\Rightarrow V_c(0^-) = 0(V)$ (short circuit at a)

$\Rightarrow V_c(0^+) = 0$ since the capacitor doesn't allow the voltage change instantaneously.

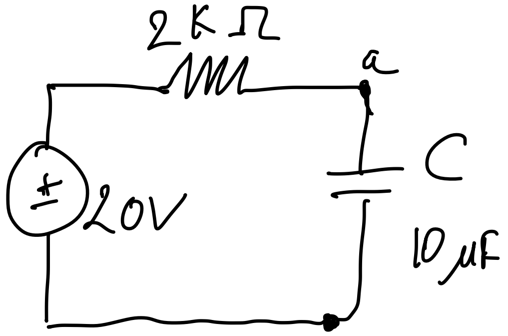
Then $t=0^+$, We have:



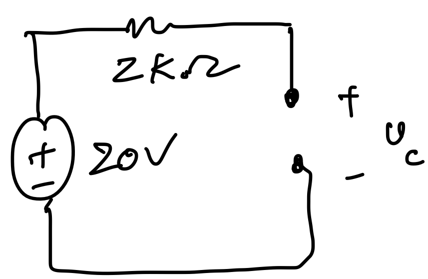
Since:



Thevenin equivalent.

⇒ 

When $t \to \infty$, we have



:

$$\Rightarrow v_c(\infty) = 20V.$$

$$\text{Also, } \tau = RC = 2000\,\Omega \times 10 \times 10^{-6} = 0.02\,(s)$$

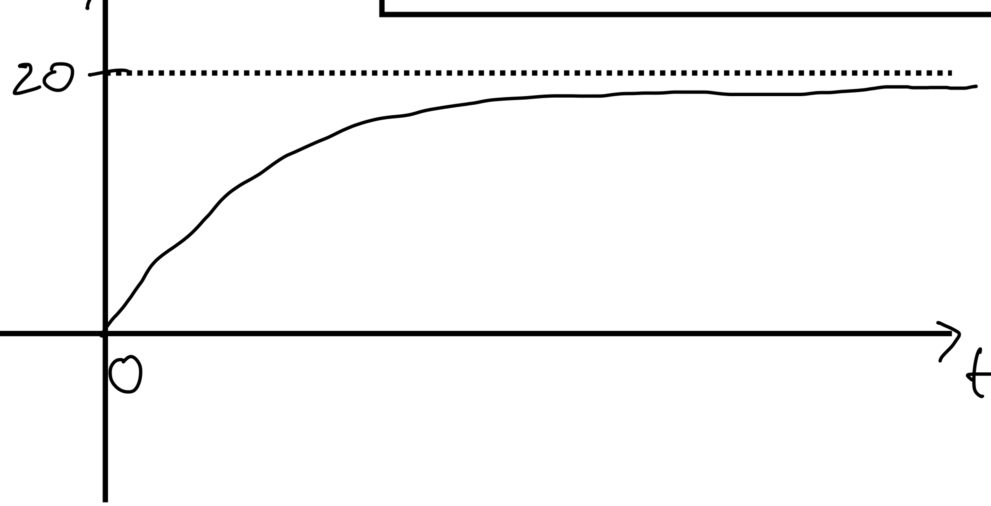
$$\Rightarrow v_c(t) = v(\infty) - \left(v(\infty) - v(0^+)\right) e^{-t/RC}$$

$$\Rightarrow v_c(t) = 20 - (20 - 0)\,e^{-50t} \quad (V)$$

$$v_c(t) = \boxed{20 - 20e^{-50t}\ (V)}$$

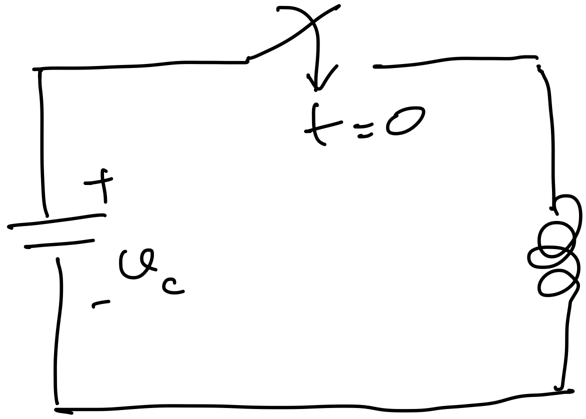
$t = 0 \Rightarrow v_c = 0$

$t = \infty \Rightarrow v_c = 20$



Problem 9: Determine the max of I_L

close at $t = 0$

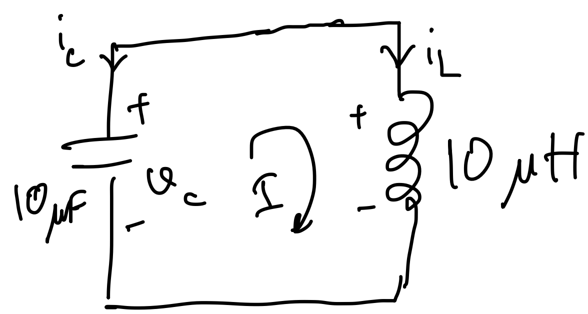


$V_c(0) = 10V$

$i_L(0) = 0\,(A)$

Since capacitor doesn't allow the voltage change instantaneously & inductor doesn't allow the current change instantaneously $\Rightarrow v_C(0^+) = 10V$

$i_L(0^+) = 0\,(A)$.



Using the energy conservation, we can have the maximum energy of L will be the same as the maximum energy of C that is initial energy of capacitance.

Also $E_C = \frac{1}{2} C [V_C(t)]^2$

In the initial time $t = 0^+$, We have

$v_c(0^+) = 10V$

$\Rightarrow$ the maximum energy of capacitance:

$$E_{C_{max}} = \frac{1}{2} C \times 10^2 = \frac{1}{2} \times 10 \times 10^{-6} F \times 10^2$$

$$= 0.0005 \text{ (J)}$$

Beside, the energy of inductance:

$$E_L = \frac{1}{2} L [i_L(t)]^2$$

Since L is constant $\Rightarrow$ the maximum of inductance $\Leftrightarrow$ the maximum of current I_L

$$\Rightarrow E_{L_{max}} = \frac{1}{2} L I_{L_{max}}^2$$

Using the energy conservation we mentioned before, we have:

$$E_{C_{max}} = E_{L_{max}}$$

$$\Rightarrow \frac{1}{2} L I_{L_{max}}^2 = 0.0005\ (J)$$

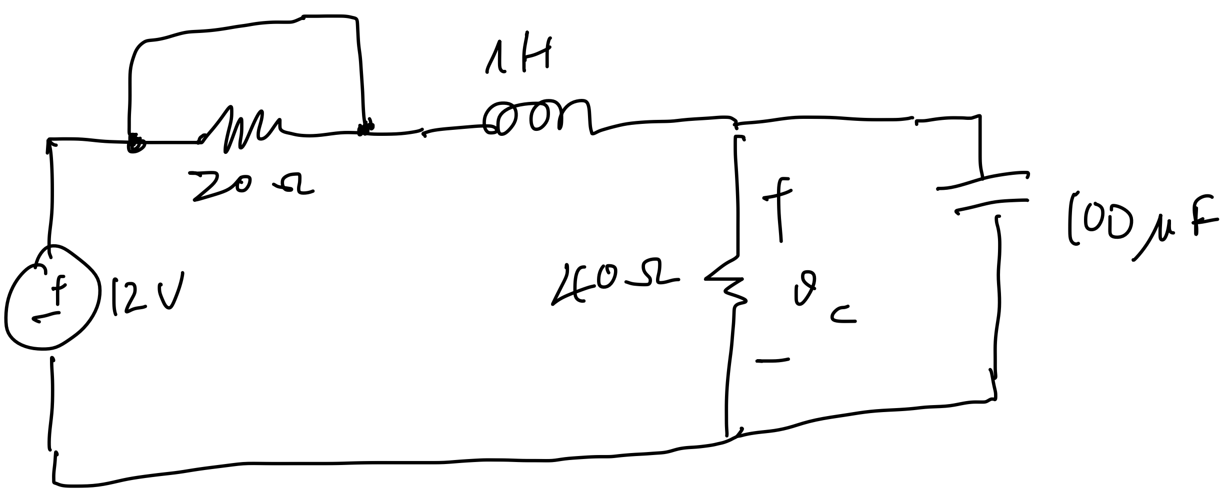
$$\Rightarrow \frac{1}{2} \times 10 \times 10^{-6} I_{L_{max}}^2 = 0.0005$$

$$\Rightarrow I_{L_{max}}^2 = 100$$

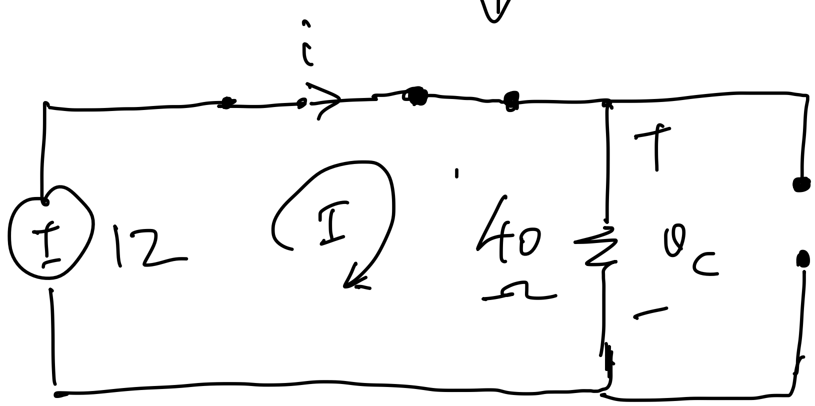
$$\Rightarrow \boxed{I_{L_{max}} = 10\ (A)}$$

Problem 10: For a long time prior to $t=0$, the switch closed. Find v_c prior $t=0 \Rightarrow t \to 0^-$

Prior to $t=0 \Leftrightarrow$ We have:



$\Downarrow$



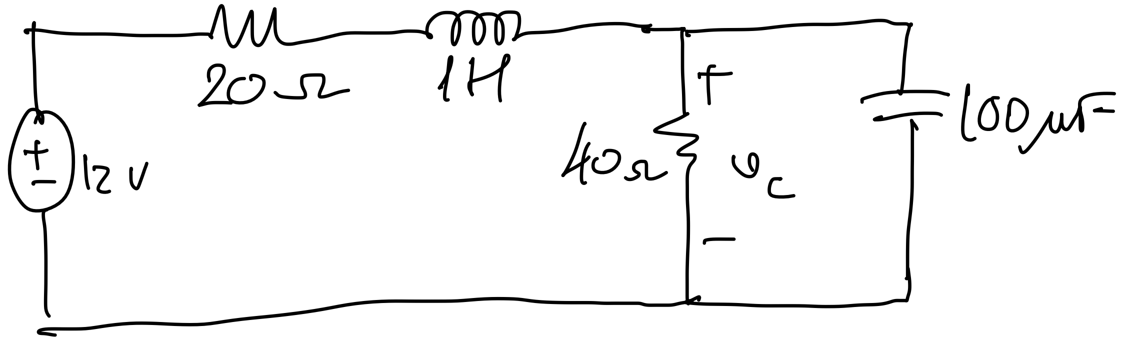
Apply KVL for I $\Rightarrow 12 = 40i \Rightarrow i = \frac{12}{40}(A)$

$\Rightarrow i = 0.3(A)$

$\Rightarrow v_c = 40\Omega \times \frac{12}{40} = 12(V)$

$\Rightarrow \boxed{v_c(0^-) = 12V}$ & $i_L(0^-) = 0.3(A)$

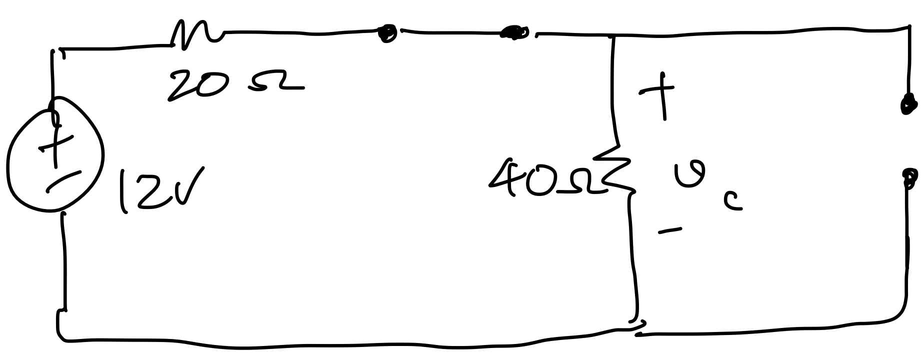
After opening the switch, We have:



Since capacitor doesn't allow the voltage change instantaneously & inductor doesn't allow the current change instantaneously $\Rightarrow v_c(0^+) = 12\ V$

$i_L(0^+) = 0.3\ (A)$

After open switch for a long time, We have:



$$\Rightarrow v_c = \frac{40}{20+40} \times 12 = 8(V) \quad \text{(Voltage divider)}$$

$$\Rightarrow \boxed{v_c(\infty) = 8V}$$