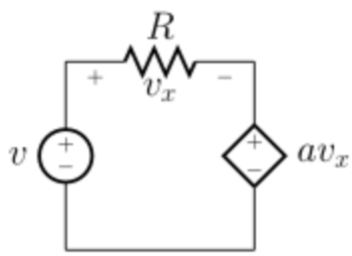
Discussion 1A (Week 2)

Wednesday, April 7, 2021 10:57 AM

Today:

- **Review Quiz #3 answers**
- **Ideal transformers**
- **Resistive networks**
- Example Questions
- Magnetics review (https://www.ieee.li/pdf/introduction_to_power_electronics/chapter_12.pdf)

Quiz 3 Answers:



solve by KVL:

① $V = IR + aV_x$

② $V_x = IR$

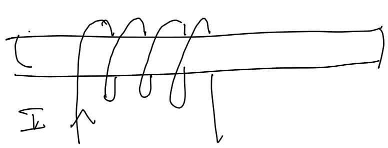
$\rightarrow V = IR + a(IR) = I(1+a)R$

$V = 100V, \; R = 20, \; I = 1A$

$100\,V = (1+a)20 \rightarrow 1+a = 5$

$\rightarrow \boxed{a = 4}$

Inductance (Sections 3.4 - 3.7)

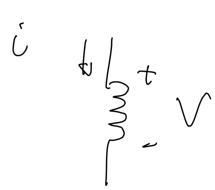


$$\boxed{V(t) = L\frac{di}{dt}}$$

$$\boxed{i(t) = \frac{1}{L}\int V(t)\,dt}$$

* when current changes in value the resulting magnetic flux changes, Φ

* time-varying magnetic flux in a coil induces a voltage across a coil



by convention

Energy stored in a inductor: $E_{cap} = \frac{1}{2}CV^2$ (this from lecture)

$$E_{inductor} = \int P_{inductor}\, dt$$

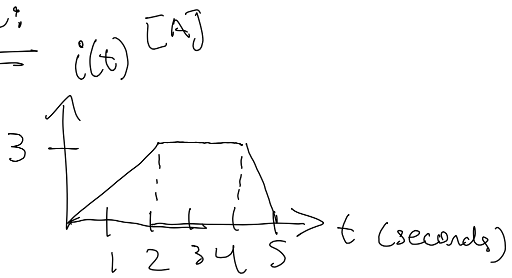
$$p(t) = I(t)V(t) = i(t) \times \left(L\frac{di(t)}{dt}\right)$$

$$E = \int p(t)\,dt = \int_0^t \left(L\frac{di}{\cancel{dt}} \times i(t)\, \cancel{dt}\right)$$

* assumption: $i(t=0^-)=0$ (initial condition)

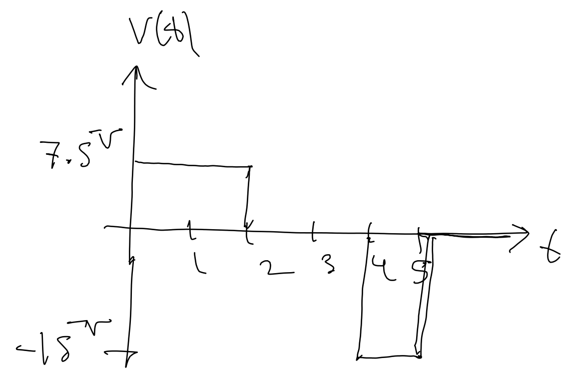
$$\rightarrow E_{ind} = \int_0^t L \times i(t)\, di = \boxed{\frac{1}{2}Li^2}$$

Example:



$L = 5H$

$$v = L\frac{di}{dt}$$



$t=0 \rightarrow t=2$: $\frac{di}{dt} = \frac{3}{2} = 1.5$

$V(t) = (5H) \times 1.5 = 7.5V$

$t=2 \rightarrow t=4$:

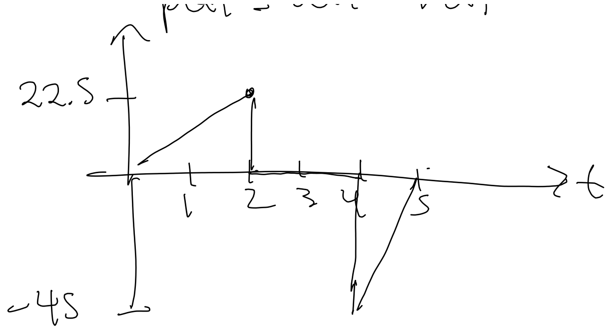
$v = 0V$ b/c $\frac{di}{dt} = 0$

$t=4 \rightarrow t=5$:

$\frac{di}{dt} = -3 \rightarrow v(t) = (5H)(-3) = -15V$

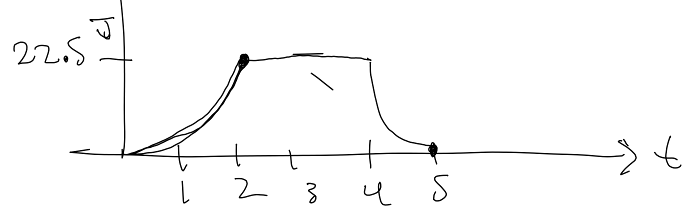
• what is $p(t)$?

$p(t) = i(t) \times V(t)$



what is Energy stored across the inductor over time?

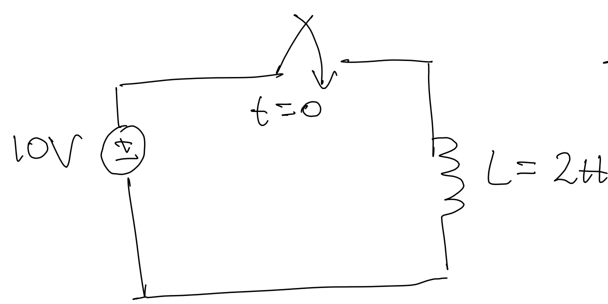
$$E = \int_0^t p(t)\, dt$$

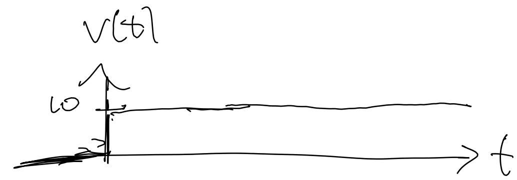


$$E(t=2) = \int_0^2 \left(\frac{22.5}{2} t \right) dt = \frac{22.5}{2} \times \frac{1}{2} t^2 \Big|_0^2 = \boxed{22.}$$

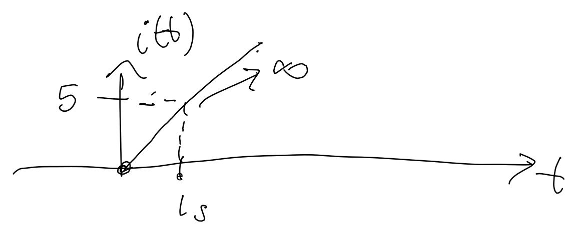
$$E = \int_4^5 (45t - 225)\, dt = \left(\frac{45t^2}{2} - 225t \right) \Big|_4^5 = -22$$

Example #2 :





$$V = L \frac{\Delta i}{\Delta t}$$

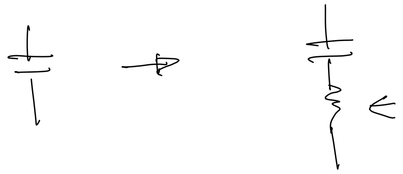


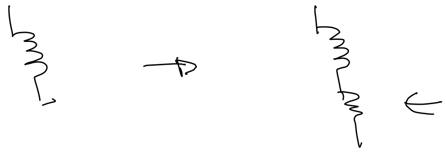
$$10V = 2H \times \frac{di}{dt}$$

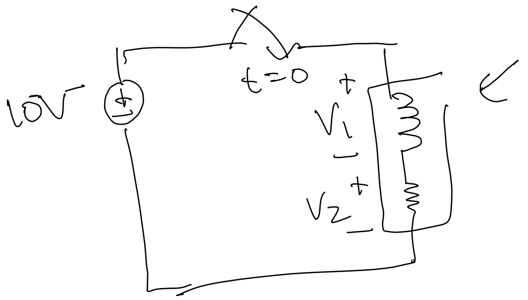
$$\frac{di}{dt} = \frac{10V}{2H} = 5$$

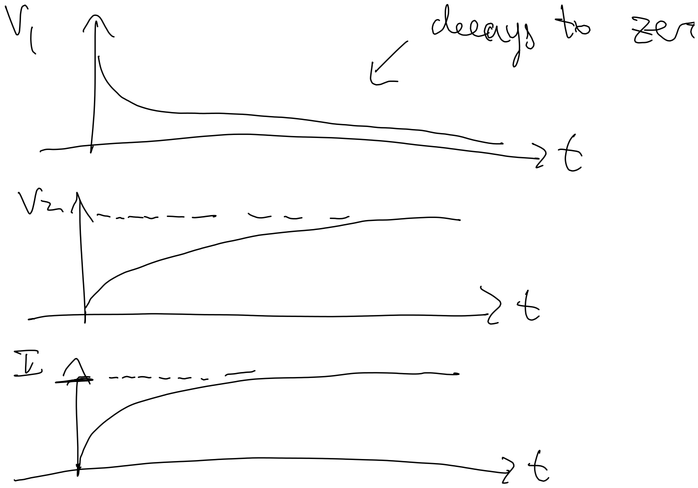
what happens as $t \to \infty$:

$$i \longrightarrow \infty$$









$$V(t) = IR + L\frac{di}{dt}$$

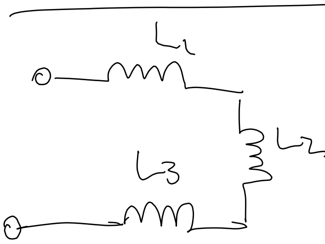
↳ differential equation

$$I(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow I(\infty) = \frac{V}{R}$$

* resistance-limited circuit

Series Inductance

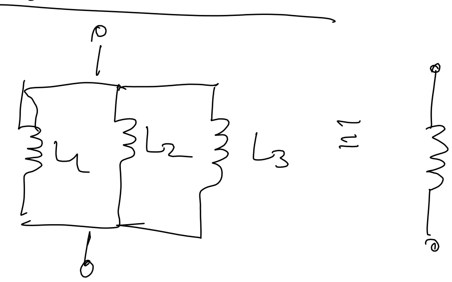


$$V(t) = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$$

$$= L_{TOTAL}\frac{di}{dt}$$

$$\boxed{\therefore L_{TOTAL} = L_1 + L_2 + L_3}$$

Parallel Inductance



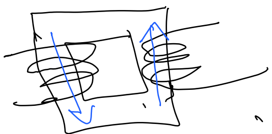
$L_1 \| L_2 \| L_3$

$$\boxed{L_{TOTAL} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}}$$

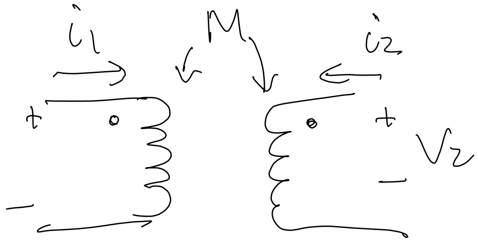
Transformers: Section 14.5 (ideal transformers)

Transformers: Section 14.5 (...)

↳ Section 14.1 - 14.4 cover useful background regarding electromagnetics



aiding:

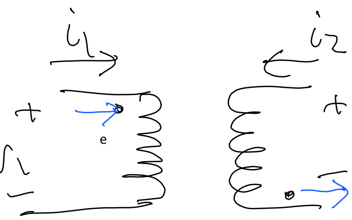


$$V_1 = L_1 \frac{di_1}{dt} + M_{21}\frac{di_2}{dt}$$
$$V_2 = L_2 \frac{di_2}{dt} + M_{12}\frac{di_1}{dt}$$

note: $M_{12} = M_{21} = \underline{M}$

* magnetic flux produced by one coil can either aid or oppose the flux produced by the other coil

opposing

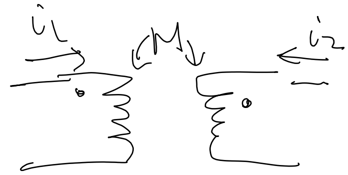


$$V_1 = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$$
$$V_2 = L_2 \frac{di_2}{dt} - M\frac{di_1}{dt}$$

Example:

$L_1 = 1H$ $L_2 = 2H$ $M = 1H$

$i_1(t) = \sin(10t)$
$i_2(t) = \frac{1}{2}\sin(10t)$



Solve on your own!

Find $V_1(t)$ & $V_2(t)$

answers:

$$\boxed{\begin{aligned} V_1(t) &= 15\cos(10t) \; [V] \\ V_2(t) &= 20\cos(10t) \; [V] \end{aligned}}$$

sanity check:

$I_1 > I_2$

↳ $V_1 < V_2$

$P = IV$

Ideal Transformers:

coupling coefficient:

$$k = \frac{\quad}{\sqrt{L_1 L_2}}$$

M

$$V_1(t) = N_1 \frac{d\Phi}{dt}$$

$$V_2 = N_2 \frac{d\Phi}{dt}$$

$k=1$: implies perfect coupling

$$\Phi = \Phi_1 = \Phi_2$$

$$\rightarrow \boxed{V_2 = \frac{N_2}{N_1} V_1}$$

$$\boxed{n = \frac{N_2}{N_1}}$$

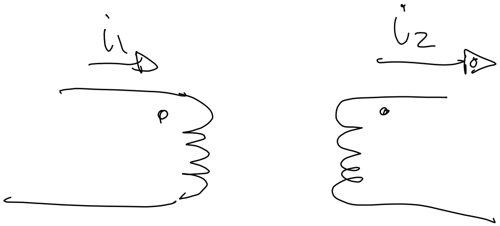
$\hookrightarrow n$: turns ratio

$$\boxed{i_2(t) = \frac{-i_1(t)}{\left(\frac{N_2}{N_1}\right)} = \frac{-N_1}{N_2} i_1(t) = \frac{-1}{n} i_1(t)}$$

normal convention

book

the book uses a slightly different convention:



$$\frac{i_2(t)}{i_1(t)} = \frac{N_1}{N_2} = \frac{1}{n}$$