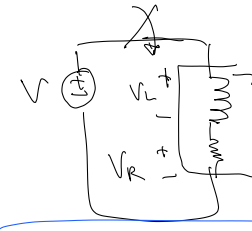
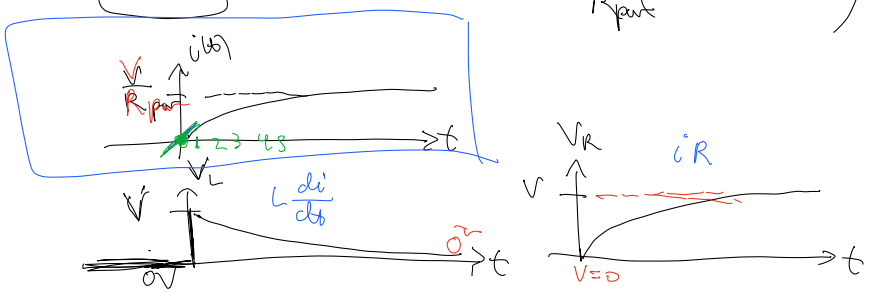
RL circuits

diff. eq.



$$V(t) = L\frac{di}{dt} + iR_{par}$$

$$i(t) = \frac{V}{R_{par}}\left(1 - e^{-Rt/L}\right)$$



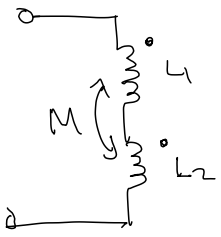
resistance-limited circuit

$t = 0$: $i = 0$

$V_R = 0$ $\quad V = V_L + V_R$

$\rightarrow V_L = V = 10V = L_1\frac{di}{dt}$

$$\frac{di}{dt} = \frac{V}{L_1} = \frac{\Delta i}{\Delta t} = \frac{10V}{L_1}$$



① aiding

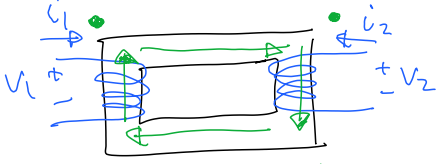
$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

② opposing

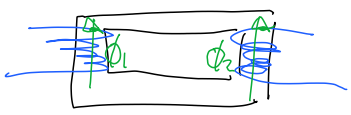
$$V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

① aiding



$$\Phi = \Phi_1 + \Phi_2$$

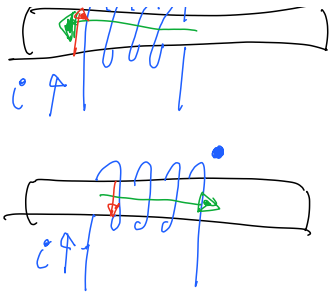
② opposing



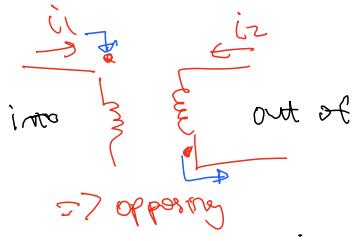
$$\Phi = \Phi_1 - \Phi_2$$

$$V_1 = N_1\frac{d\Phi}{dt}$$

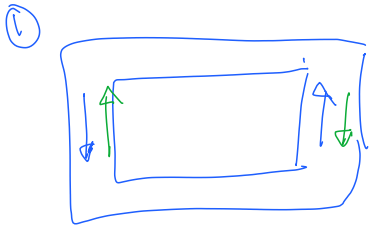
$V_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$

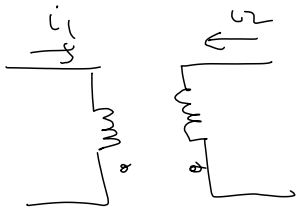


$\Rightarrow$ aiding

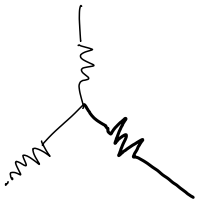


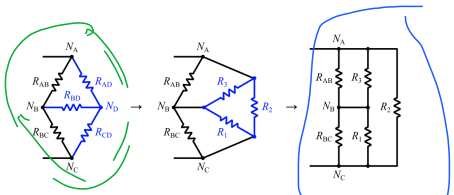
$\Rightarrow$ opposing

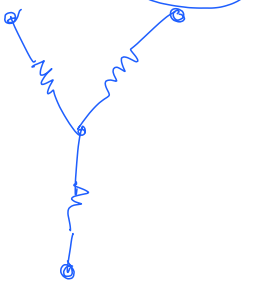


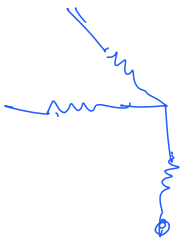


out of the dot (on both sides)

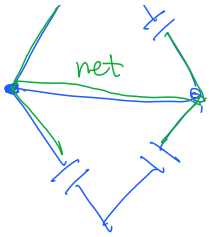


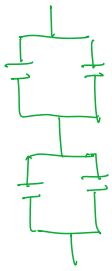


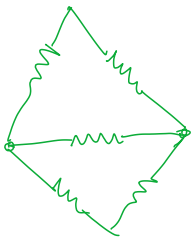




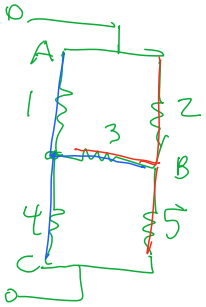
net = node



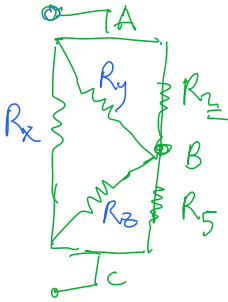




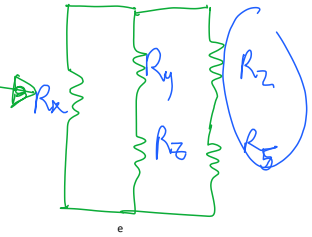
Y ⟷ Δ



① redraw



unchanged

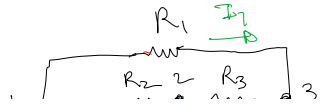


R_x, R_y, R_z be re-computed using Δ ⟷ Y transform

Tools:

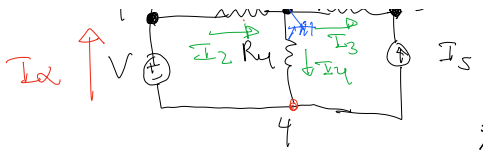
1. KVL
2. KCL
3. Ohm's Law
4. source transformations
5. superposition
6. series/parallel circuits

★ KCL is more useful



node 2: ground

$V_2 = 0$



* always defined I flowing into node = 0

node 1:

$$I_\alpha - I_1 - I_2 = 0$$

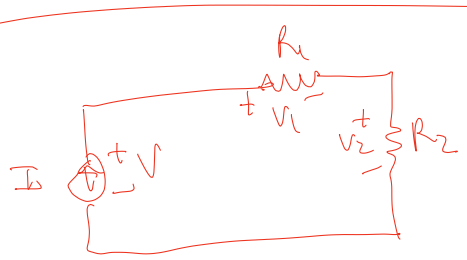
$$I_1 = \frac{V_1 - V_3}{R_1} \qquad I_2 = \frac{V_1 - V_2}{R_2}$$

what is I_α?

* voltage source

don't use KCL

· current can be any value



$$V - IR_1 - IR_2 = 0 \quad \text{(KVL)}$$

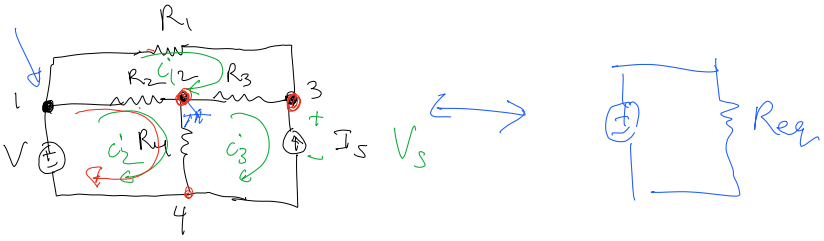
how would solve for V?

$$V = I(R_1 + R_2)$$

N-1 loops, M-1 nodes } 3 variables

(0V)

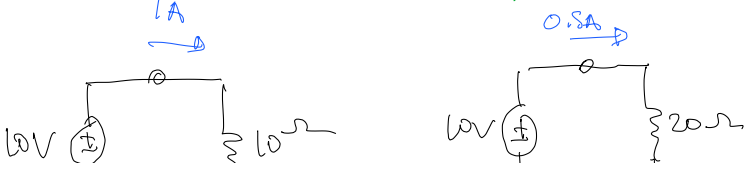
cannot solve KCL @ node 1 easily



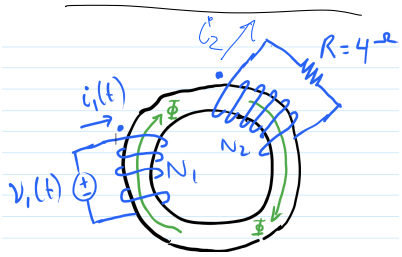
loop #3: $-V_S - (i_3 - i_2)R_4 - (i_3 - i_1)R_3 = 0$

What is V_S?

→ V_S can be any value



ideal transformer:



$$V_1 = L_1 \frac{di}{dt}$$

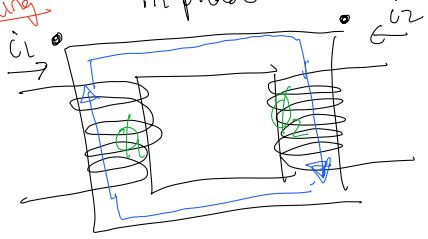
$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \rightarrow V_2 = \frac{N_2}{N_1} V_1$$

$$i_2 = \frac{N_1}{N_2} i_1 =$$

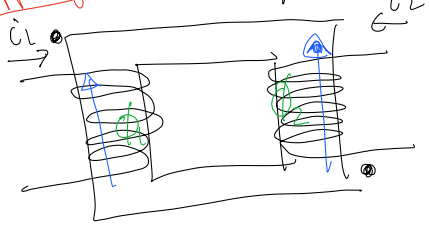
$$V_1 = N_1 \frac{d\Phi}{dt} \qquad V_2 = N_2 \frac{d\Phi}{dt}$$

---

① aiding "in phase"



② opposing "out of phase"



$$\phi = \phi_1 + \phi_2$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

@ side 1:

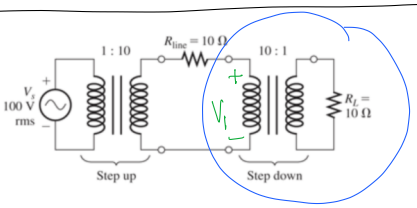
$$\phi = \phi_1 - \phi_2$$

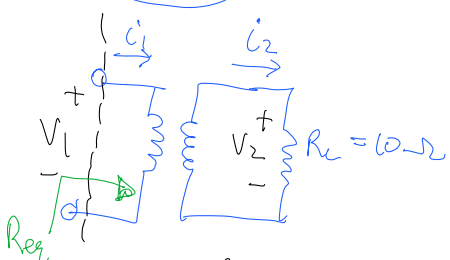
@ side 2:

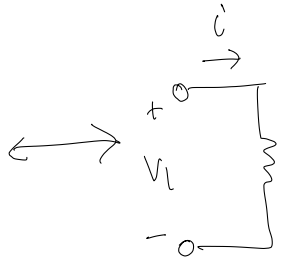
$$\phi = \phi_2 - \phi_1$$

$$V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$







given:

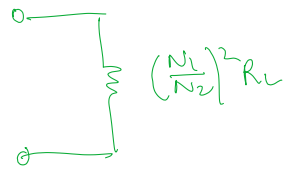
$$V_2 = i_2 R_L \qquad V_1 = \frac{N_1}{N_2} V_2 \qquad V_2 = \frac{N_2}{N_1} V_1$$

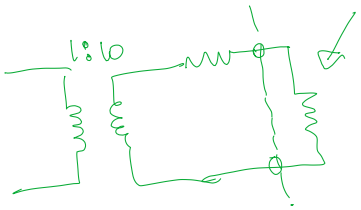
$$V_1 = \frac{N_1}{N_2} i_2 R_L \qquad i_2 = \frac{N_1}{N_2} i_1 \qquad i_2 = \frac{N_1}{N_2} i_1$$

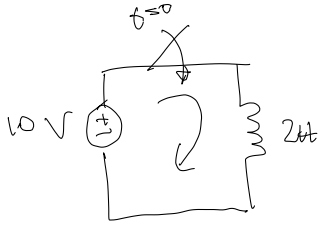
$$V_1 = \frac{N_1}{N_2}\left(\frac{N_1}{N_2}\right) i_1 R_L$$

$$\frac{V_1}{i_1} = \left(\frac{N_1}{N_2}\right)^2 R_L \quad \leftarrow \quad R_{eq} = \frac{V_1}{i_1}$$

technique: "reflection"

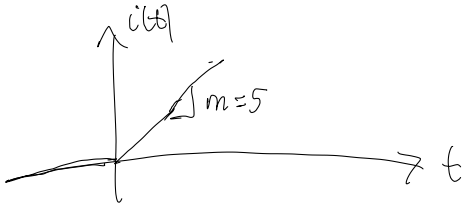






$$10V - (2\Omega)\frac{di}{dt} = 0$$

$$\frac{di}{dt} = \frac{10V}{2\Omega} = 5\,\frac{A}{s}$$



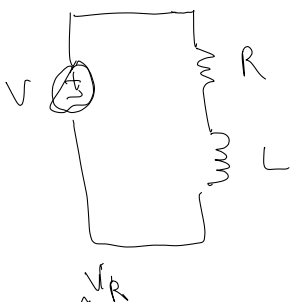
$$i(t = 1s) = 5A$$

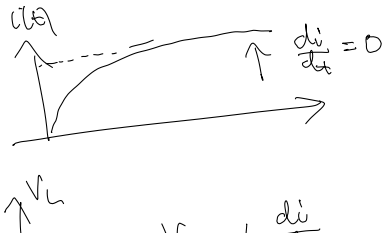
$$i(t = 1000000\,s) = 5 \times 10^6 A$$

$t \to \infty$: $i(t) \to \infty$

differential

$$V = iR + L\frac{di}{dt}$$





"resistance - limited circuit"