Problem 2: Given two points in saturation in NMOS

+ $v_{gs} = 2V$, $i_D = 0.2$ mA

+ $v_{gs} = 3V$, $i_D = 1.8$ mA

In saturation, we have:

$$i_D = K(v_{gs} - V_{th})^2$$

$$\Rightarrow \begin{cases} 0.2 = K(2 - V_{th})^2 \\ 1.8 = K(3 - V_{th})^2 \end{cases} \Rightarrow \frac{1.8}{0.2} = \left(\frac{3 - V_{th}}{2 - V_{th}}\right)^2$$

$$\Rightarrow 9 = \left(\frac{3 - V_{th}}{2 - V_{th}}\right)^2 \Rightarrow \left[\begin{array}{l} \frac{3 - V_{th}}{2 - V_{th}} = 3 \quad (1) \\ \frac{3 - V_{th}}{2 - V_{th}} = -3 \quad (2) \end{array}\right.$$ Call V_{th}: V.

From (1), $3 - V = 3(2 - V) = 6 - 3V$

$$\Rightarrow 2V = 3 \Rightarrow \boxed{V_{th} = 1.5 (V)}$$

it is satisfied since $V_{gs} = 2V \geq V_{th} = 1.5V$ (in saturation)

From (2), $3 - V = -3(2 - V) = -6 + 3V$

$$\Rightarrow 9 = 4V \Rightarrow V = 2.25 (V)$$

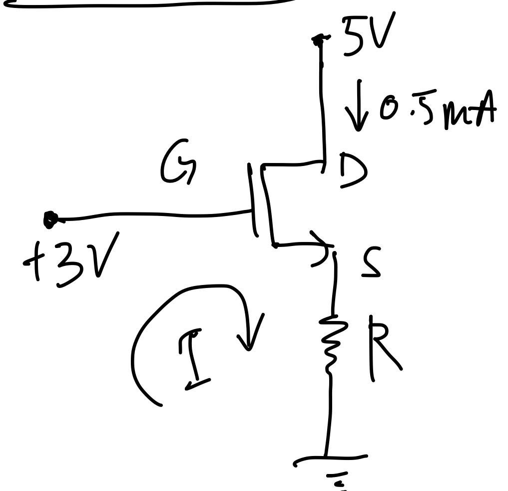
it is not satisfied since $v_{gs} = 2V < V_{th} = 2.25$ (not in saturation)

Also, $1.8 = K(3 - V_{th})^2$

$$\Rightarrow 1.8 = K(3 - 1.5)^2 = 2.25K \Rightarrow K = \frac{1.8}{2.25} = 0.8$$

$$\Rightarrow \boxed{k = 0.8 \ (mA/V^2)}$$

PROBLEM 3: Given $V_{th} = 1V$, $K = 0.5 (mA/V^2)$



Find R.

Assume the transistor is in saturation, we have

$$\begin{cases} v_{gs} \geq V_{th} \\ V_{ds} \geq v_{gs} - v_{th} \end{cases}$$

& $i_D = K(v_{gs} - v_{th})^2 \Rightarrow 0.5 = 0.5 (v_{gs} - 1)^2$

$\Rightarrow 1 = (v_{gs} - 1)^2 \Rightarrow \left[\begin{array}{l} v_{gs} - 1 = 1 \Rightarrow v_{gs} = 2 \\ v_{gs} - 1 = -1 \Rightarrow v_{gs} = 0 \end{array}\right.$

Since in saturation $v_{gs} \geq V_{th} \Rightarrow v_{gs} = 0$ is rejected

$\Rightarrow v_{gs} = 2 (V) \geq V_{th} = 1V.$

Apply KVL for (I), we have:

$$3 = v_{gs} + i_D R \Rightarrow R = \frac{3 - v_{gs}}{i_D} = \frac{(3-2)V}{0.5 mA}$$

$$\Rightarrow \boxed{R = 2 (k\Omega)}$$

Check: we have $V_S = i_D R = 2 \times 0.5 = 1V$

$\Rightarrow V_{ds} = V_d - V_s = 5 - 1 = 4(V)$

$V_{gs} - V_{th} = 2 - 1 = 1\ (V)$

Since $V_{ds} = 4V > V_{gs} - V_{th} = 1$, it is satisfied the assumption.

$\Rightarrow \boxed{R = 2(K\Omega)}$ is corrected as we assump.

Problem 4: Given p-MOSFET has $V_{th} = -0.5V$ & $k = 0.2\ mA/V^2$. Assume in saturation, need to find value of V_{gs} such that $i_D = 0.8mA$.

We have, in saturation p-mosfet:

$$\begin{cases} v_{gs} \leq v_{th} \\ v_{ds} \leq v_{gs} - V_{th} \end{cases} \quad \& \quad i_D = K(V_{gs} - V_{th})^2$$

$$\Rightarrow 0.8 = 0.2(v_{gs} + 0.5)^2 \Rightarrow (v_{gs} + 0.5)^2 = 4$$

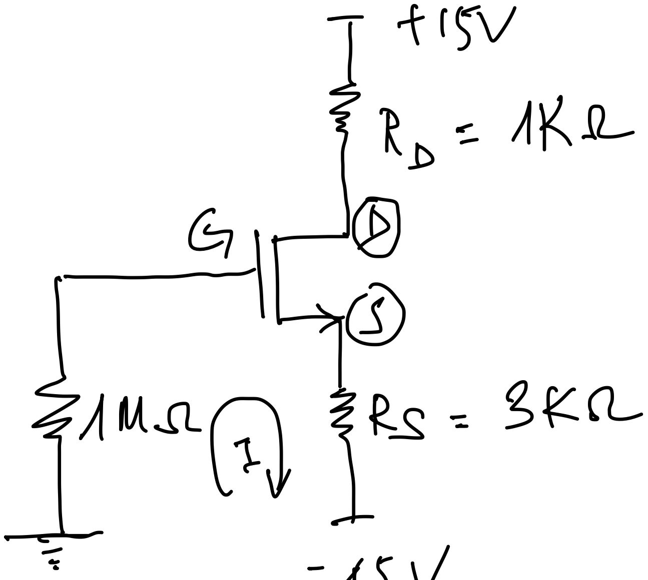
$$\Rightarrow v_{gs} + 0.5 = 2 \Rightarrow v_{gs} = 1.5(V)$$

$$v_{gs} + 0.5 = -2 \Rightarrow v_{gs} = -2.5(V)$$

Since $v_{gs} \leq V_{th} = -0.5V$

$$\Rightarrow \boxed{v_{gs} = -2.5(V)}$$

Problem 5: Given $V_{th} = 1V$, $K = 0.25 mA/V^2$



Find I_{DQ} & V_{DSQ}

Assume saturation, we have:

$$\begin{cases} v_{gs} \geq V_{th} = 1V \\ v_{ds} \geq v_{gs} - v_{th} \end{cases}$$

& $i_D = K(v_{gs} - v_{th})^2 = 0.25(v_{gs} - 1)^2$ (1)

Apply KVL for loop (I), we have:

$$v_{gs} + i_D R_S - 15 = 0 \Rightarrow v_{gs} + i_D 3 = 15$$

$$\Rightarrow v_{gs} = 15 - 3i_D$$

From (1) $\Rightarrow$ $i_D = 0.25(15 - 3i_D - 1)^2$

$$\Rightarrow 0.25(14 - 3i_D)^2 = i_D$$

$$\Rightarrow 49 - 21i_D + 2.25i_D^2 - i_D = 0$$

$$\Rightarrow 2.25i_D^2 - 22i_D + 49 = 0$$

$\Rightarrow$ $i_D = 6.346$ mA $\Rightarrow v_{gs} = -4.038$ (V)

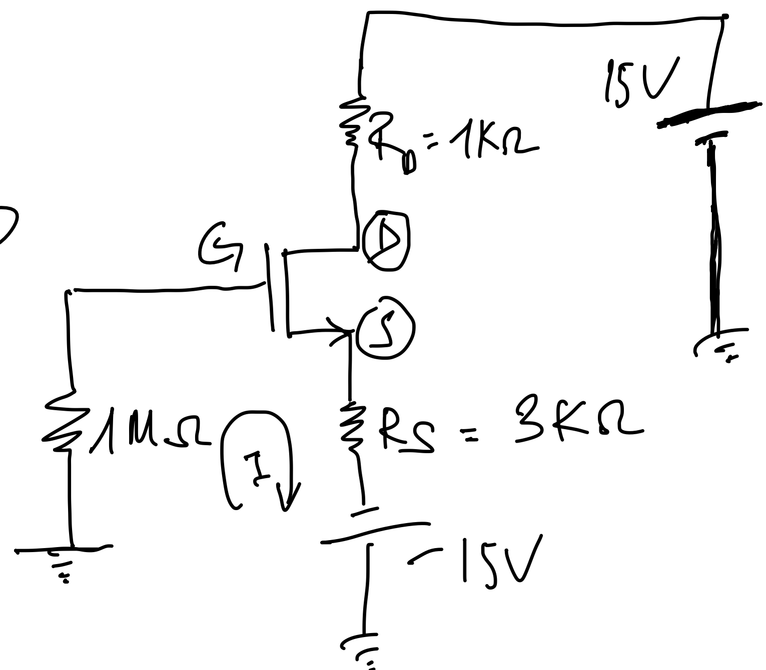
$i_D = 3.432$ mA $\Rightarrow v_{gs} = 4.704$ (V)

Since $v_{gs} \geq V_{th} = 1V$ (Saturation)

$\Rightarrow v_{gsQ} = 4.704$ (V) & $\boxed{i_{DQ} = 3.432 \text{ (mA)}}$

(*) Find V_{DSQ}

Apply KVL for I, we have:



$-15 + i_D(R_D + R_S) + v_{ds} - 15 = 0$

$\Rightarrow v_{ds} + 3.432(1+3) = 30$

$\Rightarrow v_{ds} = 30 - 13.728$

$\Rightarrow v_{ds} = 16.272$ (V)

Check: $v_{gs} - v_{th} = 4.704 - 1 = 3.704$ (V)

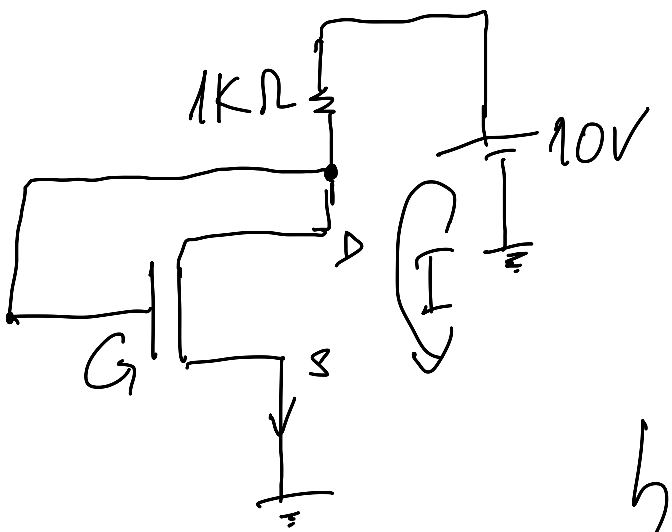
Since $v_{ds} = 16.272$ (V) $> v_{gs} - v_{th} = 3.704$ (V)

$\Rightarrow$ Satisfied the assumption

$\Rightarrow$ $\boxed{v_{dsQ} = 16.272 \text{ (V)}}$

Problem 6: Given $V_{th} = 1V$, $K = 0.25\, mA/V^2$

find I_D & V_{ds}



Assump saturation:

$$\begin{cases} v_{gs} \geq v_{th} \\ v_{ds} \geq v_{gs} - v_{th} \end{cases}$$

& $i_D = K(v_{gs} - v_{th})^2 = 0.25(v_{gs} - 1)^2$ (1)

Apply KVL for (I), we have:

$-10 + i_D R + v_{DS} = 0$

$\Rightarrow i_D + v_{DS} = 10$. Since $v_g = v_D$

$\Rightarrow i_D + v_{gs} = 10 \Rightarrow i_D = 10 - v_{gs}$

Plug this into (1), we have:

$10 - v_{gs} = 0.25(v_{gs} - 1)^2 = 0.25(v_{gs}^2 - 2v_{gs} + 1)$

$\Rightarrow 10 - v_{gs} = 0.25 v_{gs}^2 - 0.5 v_{gs} + 0.25$

$\Rightarrow 0.25v_{gs}^2 + 0.5v_{gs} - 9.75 = 0$

$\Rightarrow \begin{cases} v_{gs} = 5.324 \text{ (satisfied)} \\ v_{gs} = -7.324 \text{ (rejected since } v_{gs} < V_{th}) \end{cases}$

$\Rightarrow i_D = 10 - v_{gs} = 10 - 5.324$

$$\Rightarrow \boxed{i_{DQ} = 4.676 \text{ (mA)} \ \& \ v_{gsQ} = v_{DSQ} = 5.324 \text{(V)}}$$

<u>Check</u>; $v_{gs} - V_{th} = v_{gs} - V_{th} = 5.324 - 1$
$= 4.324$

$\Rightarrow v_{ds} = 5.324 > v_{gs} - V_{th} = 4.324$

$\Rightarrow$ it is corrected with the assumption.

Problem 7: In transitor small signal analysis

Denifition of g_m & r_d

* The relationship between V_{GS} and I_D is called Trans conductance, and denoted by g_m. This relationship is the partial derivative of drain current i_D with the gate source voltage v_{GS} at Q point, then

$$\boxed{g_m = \left.\frac{\partial i_D}{\partial V_{GS}}\right|_{Q\text{-point}}}$$

* The drain resistance is the partial derivative of the drain current i_D with the v_{DS} at Q point

$$\boxed{\frac{1}{r_d} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Q\text{-point}}}$$

Problem 8: Given an unusual type of FET with

$i_D = 3v_{gs}^3 + 0.1 v_{ds}$. $v_{gsQ} = 1V$, $v_{dsQ} = 10V$

Need to find value of g_m & r_d, $\left\{ \begin{array}{l} i_D\ (mA) \\ v_{gsQ}\ (V) \\ v_{DSQ}\ (V) \end{array} \right.$

We have:

$$g_m = \left. \frac{\partial i_D}{\partial v_{gs}} \right|_{Q\ point}$$

$$\frac{\partial i_D}{\partial v_{gs}} = 9 v_{gs}^2$$

$$\Rightarrow \boxed{g_m = 9 v_{gsQ}^2 = 9\ (mS)}$$

$$\frac{1}{r_d} = \left. \frac{\partial i_D}{\partial v_{DS}} \right|_{Q\ point}$$

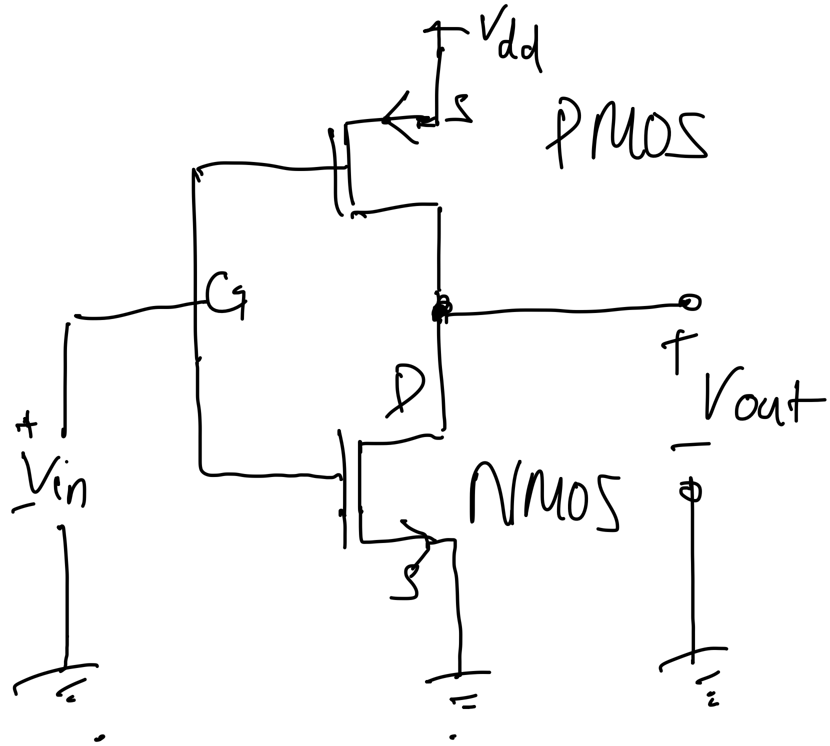
Also, $\frac{\partial i_D}{\partial v_{DS}} = 0.1 \Rightarrow \frac{1}{r_d} = 0.1$

$$\Rightarrow \boxed{r_d = 10\ (K\Omega)}$$

Problem 9: Need to draw the circuit diagram of a two-input CMOS AND gate

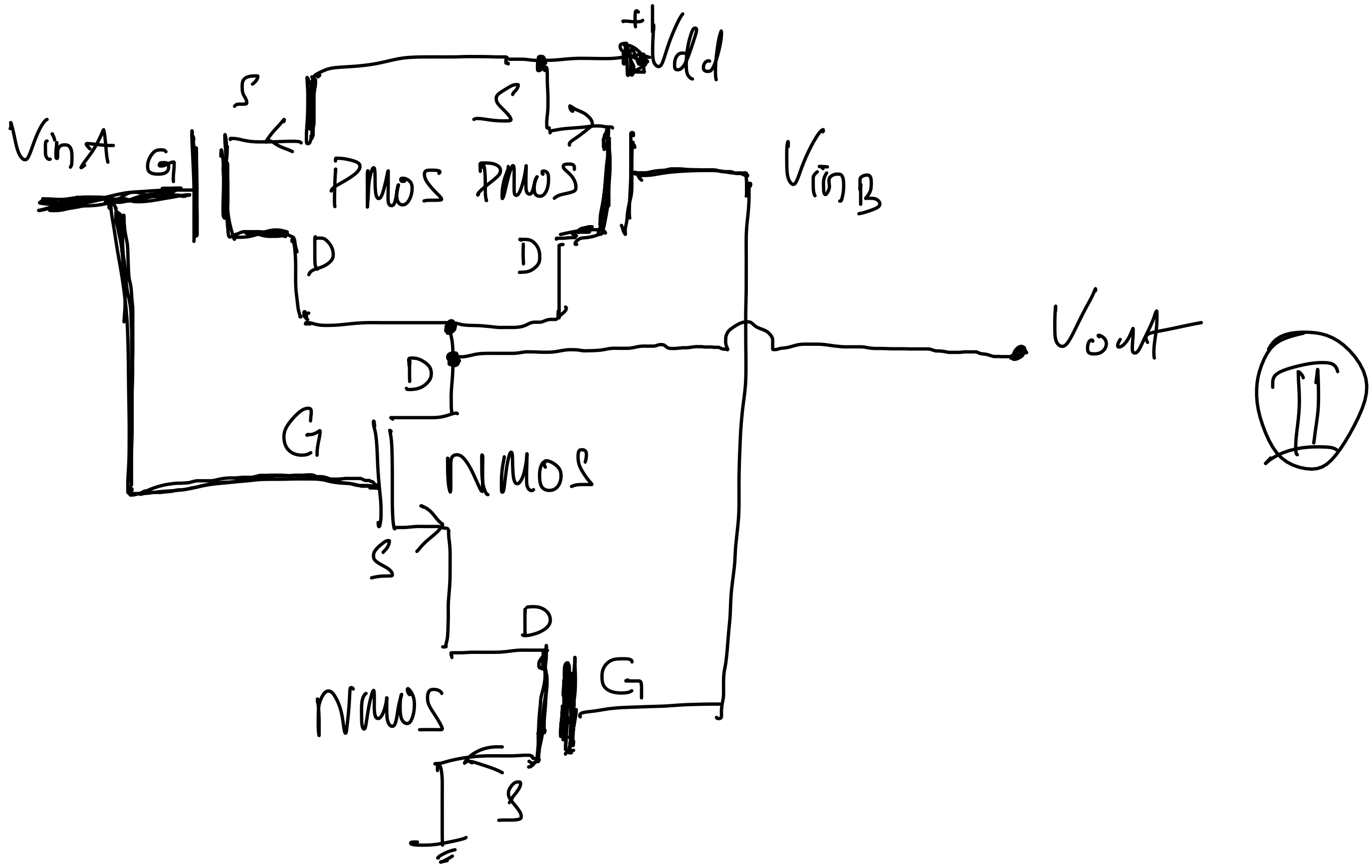
As the hint, we use an inverter connect to the output of the two input CMOS NAND gate

As we learn in the class, we have a diagram for the CMOS inverter below:

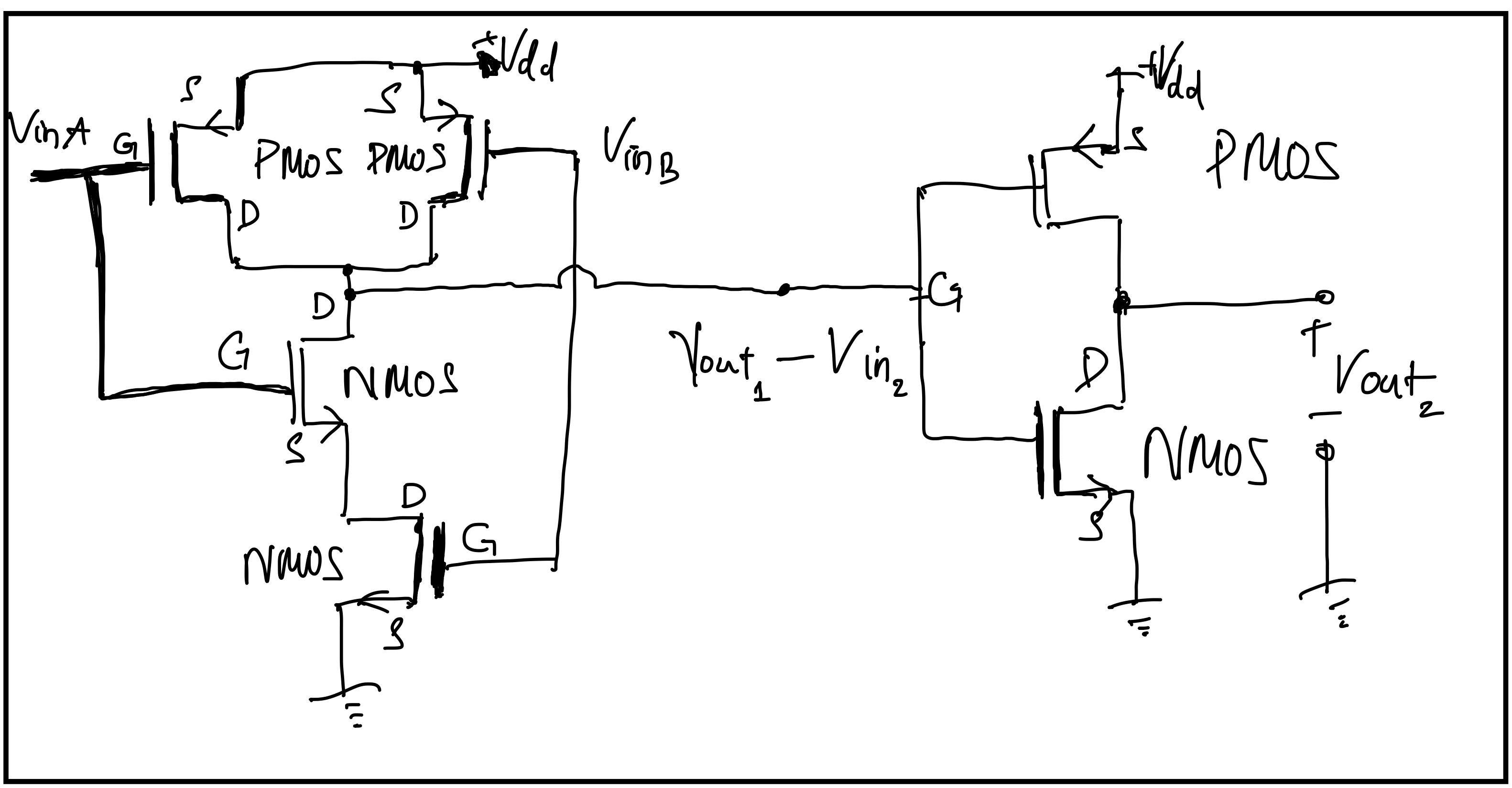


I

We also learn about the two-input CMOS NAND gate.



From I & II, we have the circuit diagram of two-input CMOS AND gate.

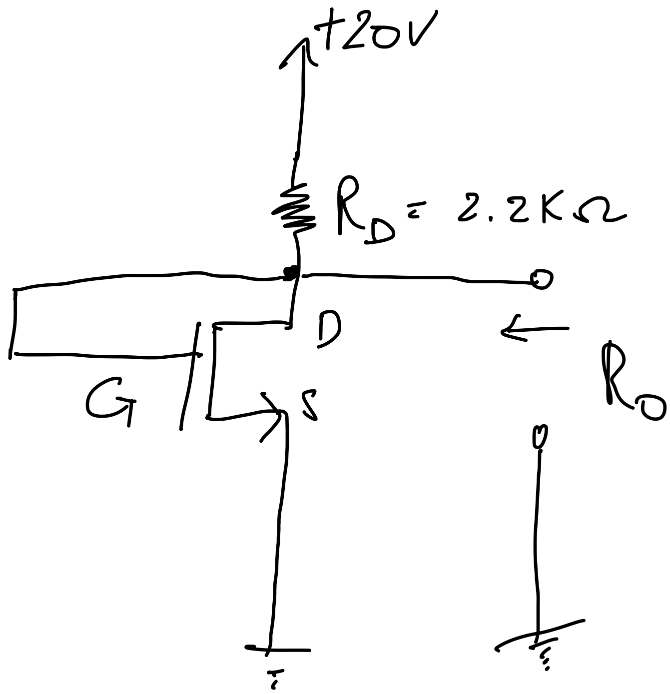


Problem 10: Given $V_{th} = 3V$, $K = 0.5 mA/V^2$

* Find V_{DSQ} & I_{DQ}

We have $V_G = V_D$

$\Rightarrow v_{DS} = V_{GS}$



Assump saturation:

$$\begin{cases} v_{gs} \geq V_{th} = 3V \\ V_{DS} \geq v_{gs} - v_{th} \end{cases}$$

& $i_D = K(v_{gs} - v_{th})^2 = 0.5(v_{gs} - 3)^2$ (1)

Also, apply KVL, we have:

$20 = i_D R_D + v_{DS}$ sin $v_{DS} = v_{gs}$

$\Rightarrow 20 = i_D \times 2.2 + v_{gs} \Rightarrow i_D = \dfrac{20 - v_{gs}}{2.2}$ (mA)

plug into (1), we have:

$$\frac{20 - v_{gs}}{2.2} = 0.5(v_{gs} - 3)^2$$

$$\Rightarrow 20 - v_{gs} = 1.1(v_{gs}^2 - 6v_{gs} + 9)$$

$$\Rightarrow 20 - v_{gs} = 1.1 v_{gs}^2 - 6.6 v_{gs} + 9.9$$

$$\Rightarrow 1.1 v_{gs}^2 - 5.6 v_{gs} - 10.1 = 0$$

$$\Rightarrow \begin{cases} v_{gs} = 6.5\,(V) & \text{satisfied assumption.} \\ v_{gs} = -1.412\,(V) & \text{rejected since } -1.412 < V_{th} = 3V \end{cases}$$

$$\Rightarrow \boxed{v_{gSQ} = 6.5(V)}$$

$$\boxed{I_{DQ} = \frac{20 - v_{gSQ}}{2.2} = 6.1364(mA)}$$

Check: $V_{DS} = V_{GS}$ & $V_{th} = 3 > 0$

$\Rightarrow v_{DS} = V_{GS} \geq v_{gs} - V_{th} \Rightarrow$ satisfied the assumption.

* Find value of g_m.

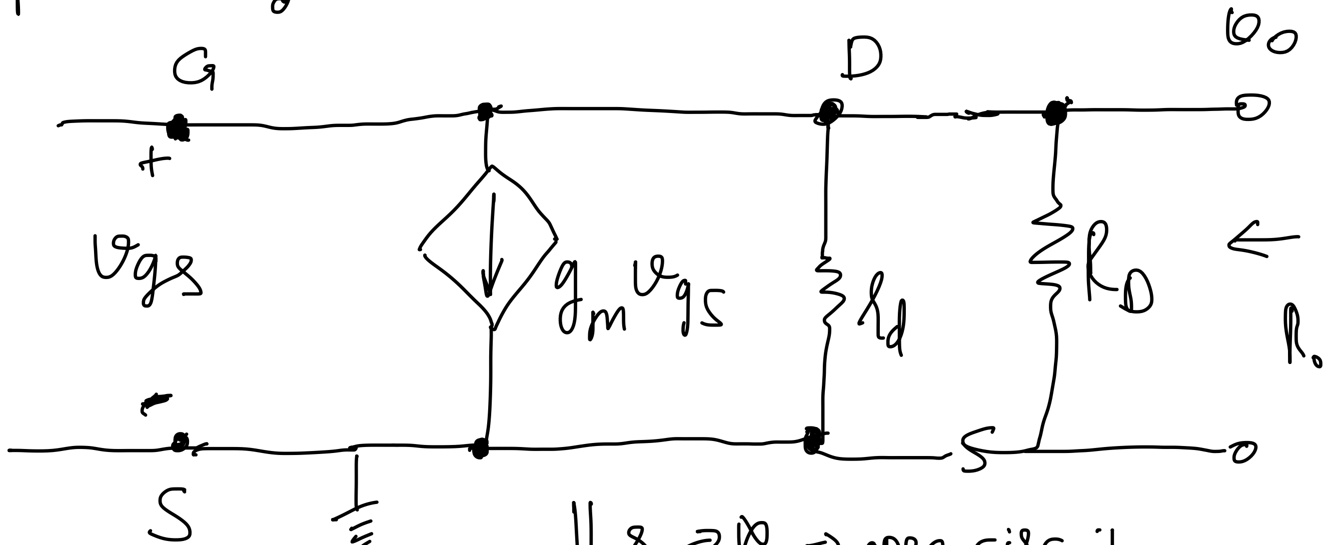
$$I_{DQ} = K(V_{gSQ} - V_{th})^2$$

$$g_m = \left.\frac{\partial I_{DQ}}{\partial V_{GS}}\right|_{Q\,point} = 2K(V_{GSQ} - V_{th})$$

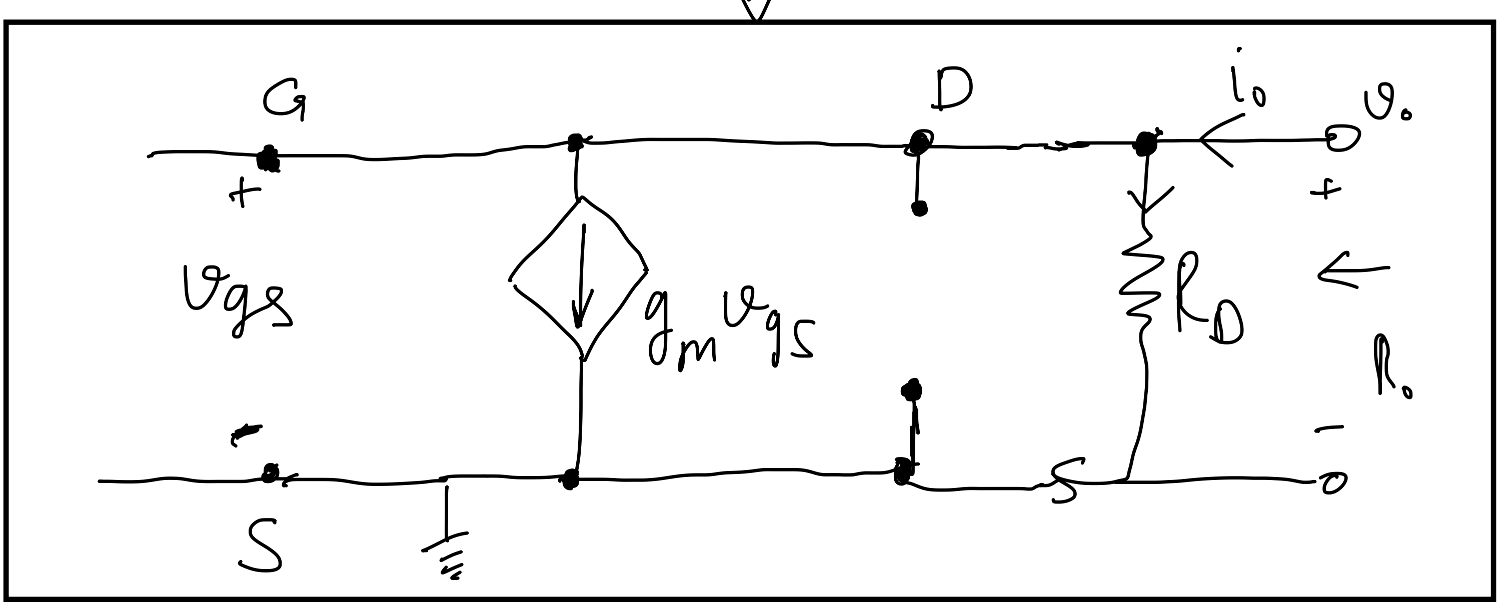
$$\Rightarrow g_m = 2 \times 0.5 \times (6.5 - 3) = \boxed{\begin{array}{l} 3.5\,(mS) \\ = 3.5\,(mA/V) \end{array}}$$

* Draw the small-signal equivalent:

With assump the $r_d \to \infty$



$r \to \infty \Rightarrow$ open circuit



* Apply KCL at D, we have:

$$i_o = \frac{v_o}{R_D} + g_m v_{gs} \text{ , also } v_o = v_{gs}$$

$$\Rightarrow i_o = \frac{v_o}{R_D} + g_m v_o \text{ . Also, } i_o = \frac{v_o}{R_o}$$

$$\Rightarrow \frac{v_o}{R_o} = \frac{v_o}{R_D} + g_m v_o \Rightarrow \frac{1}{R_o} = g_m + \frac{1}{R_D}$$

$$\Rightarrow \boxed{R_o = \frac{R_D}{1+g_m R_D} = \frac{1}{g_m + \frac{1}{R_D}}}$$

$$\Rightarrow R_o = \frac{1}{3.5(mA/V) + \frac{1}{2.2K\Omega}} = 0.2529(K\Omega)$$

$$\Rightarrow \boxed{R_o = 252.874\ \Omega}$$