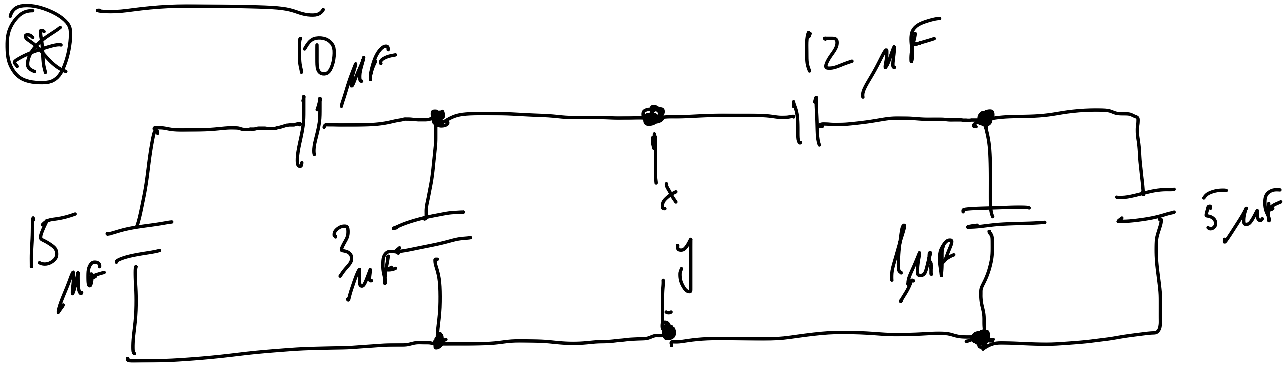
Home Work 3:

Nhat Ho

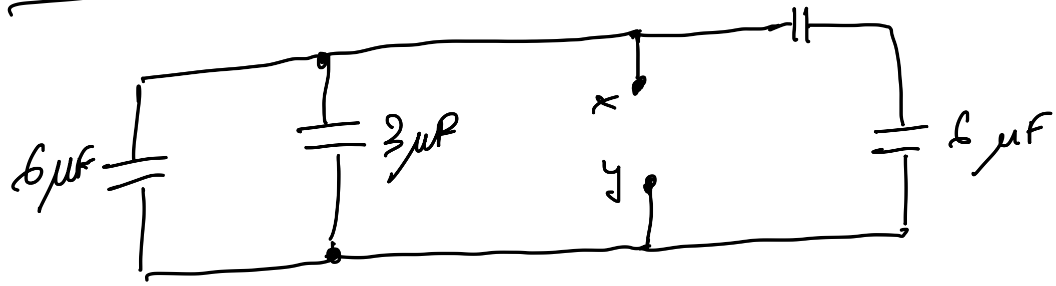
Problem 1:



We have: (1 μF // 5 μF) $\Rightarrow C_{15} = 1\mu F + 5\mu F = 6(\mu F)$

(15 μF series 10 μF) $\Rightarrow C_{1510} = \dfrac{15 \times 10}{15+10} = 6(\mu F)$ $\left(\dfrac{1}{C_{1510}} = \dfrac{1}{15} + \dfrac{1}{10}\right)$

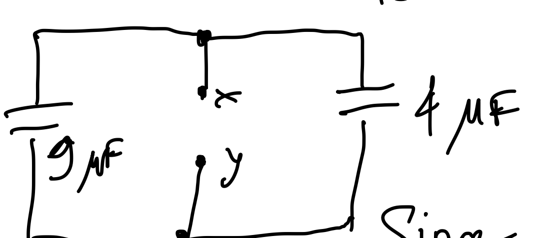
$\Rightarrow$ We have a new circuit:



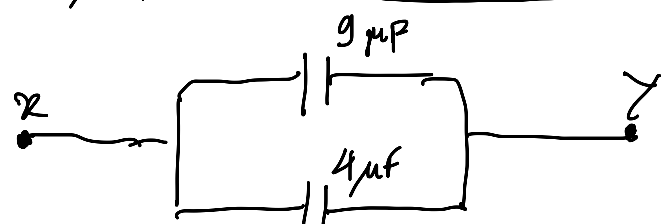
Also, (6 μF // 3 μF) $\Rightarrow C_{63} = 6 + 3 = 9(\mu F)$

For the left most, (12 μF series 6 μF)

$\Rightarrow C_{126} = \dfrac{12 \times 6}{12+6} = 4\ \mu F$ $\Rightarrow$ Continue having:



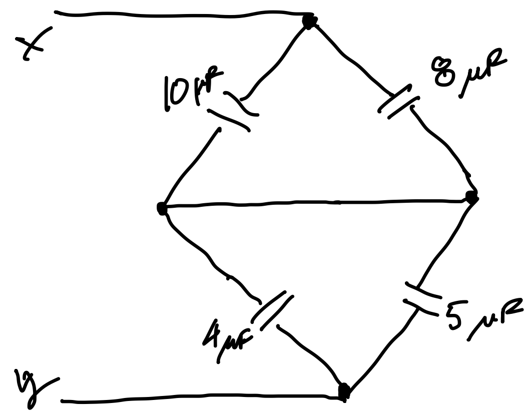
$\Rightarrow$



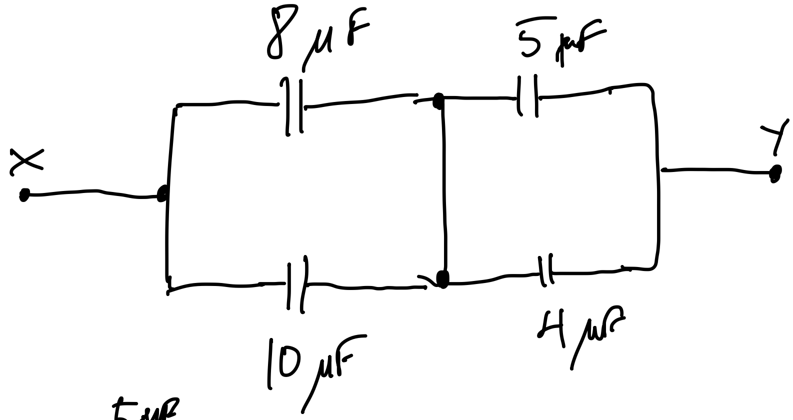
Since 9 μF // 4 μF

$\Rightarrow C_{xy} = 9 + 4 = 13\ \mu F \Rightarrow \boxed{C_{xy} = 13\mu F}$

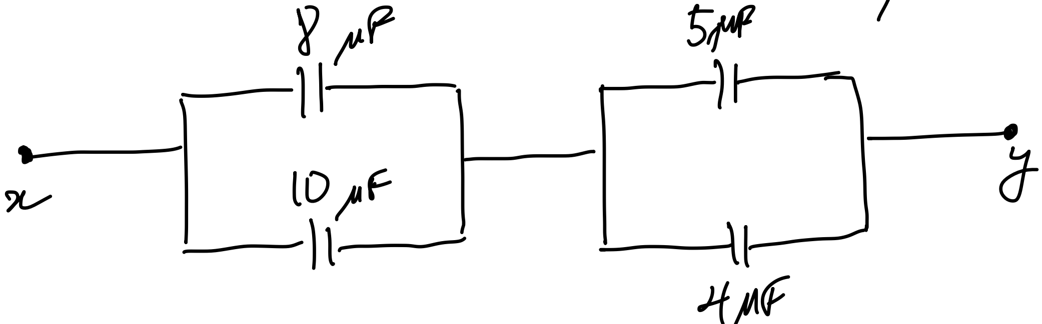
(*)



$\Leftrightarrow$



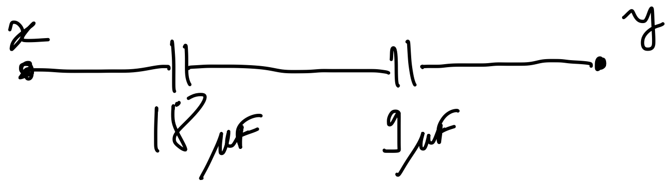
$\Leftrightarrow$



Since ($8\,\mu F$ // $10\,\mu F$) $\Rightarrow C_{810} = 8+10 = 18\,\mu F$

($5\,\mu F$ // $4\,\mu F$) $\Rightarrow C_{54} = 5+4 = 9\,\mu F$.

$\Rightarrow$



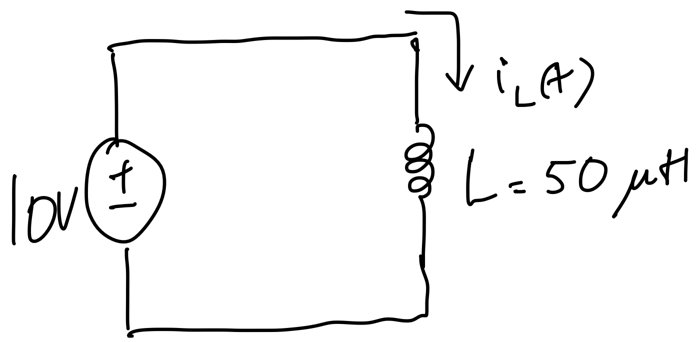
Since $18\,\mu F$ series $9\,\mu F$

$$\Rightarrow C_{xy} = \frac{18\times 9}{18+9} = 6\,(\mu F) \quad \left(\frac{1}{C_{xy}} = \frac{1}{18} + \frac{1}{9}\right)$$

$$\Rightarrow \boxed{C_{xy} = 6\,\mu F}$$

Problem 2: Given $i_L(t=0) = -100mA = -0.1A$

a) What time t_x if
$i = +100\,mA = 0.1A$



We have:

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t_0} v(t)\,dt + i_L(t_0)$$

Since $v(t) = 10V$
$i_L(t_0 = 0) = -0.1A$

$$\Rightarrow i_L(t) = \frac{1}{L}\int_0^t 10\,dt - 0.1\ (A)$$

When $i_{L(t)} = 0.1A$

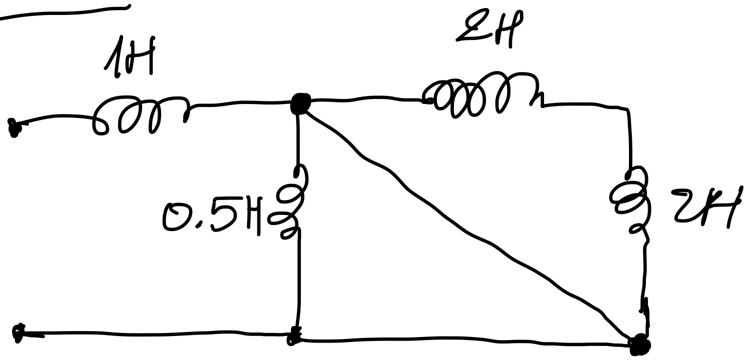
$$\Rightarrow 0.1 + 0.1 = \frac{1}{50.10^{-6}}\int_0^t 10\,dt = 2\times10^4 .\ 10t\Big|_0^t\ (A)$$

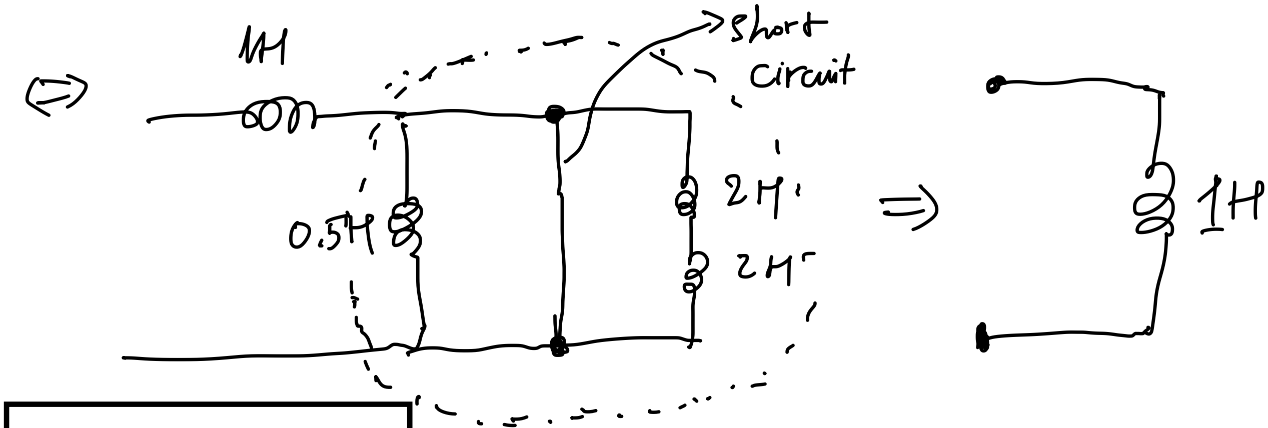
$$\Rightarrow 0.2A = 2\times10^5\,t \Rightarrow t = \frac{0.2\,A}{2\times10^5} = 0.1\times10^{-5}\,(s)$$

$$\Rightarrow t = 1\times10^{-6}\,(s) \Leftrightarrow \boxed{t = 1\,\mu s}$$

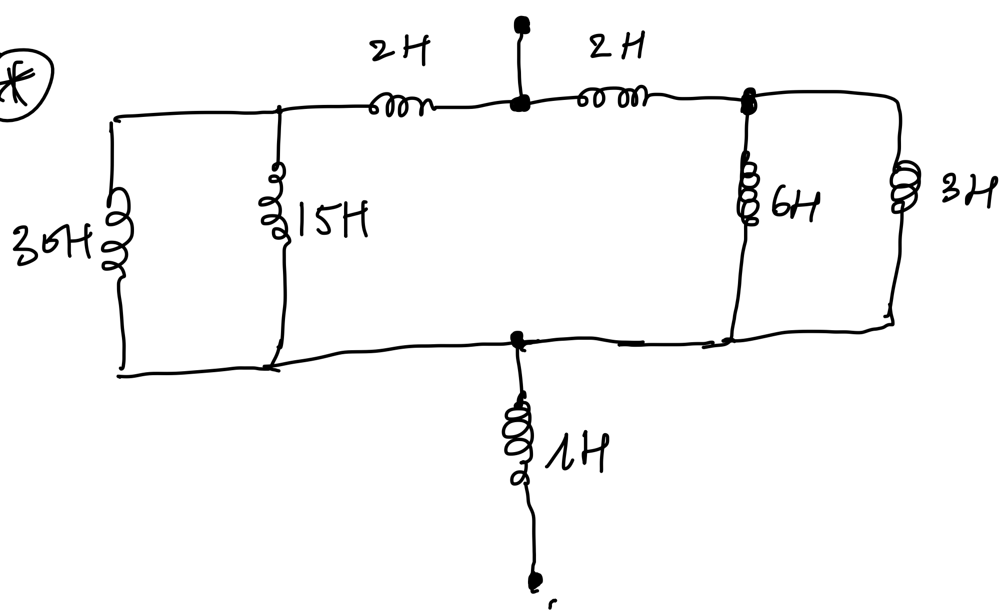
b) Because in this circuit, it only has 1 inductor that does not dissipate the energy. So the process of dissipation will not occur and the oscillation will happen for infinite time. This is not realistic in the real life. Practically, this circuit should have a series of resistance that absorb and dissipate energy to become a realistic model.

Problem 3:





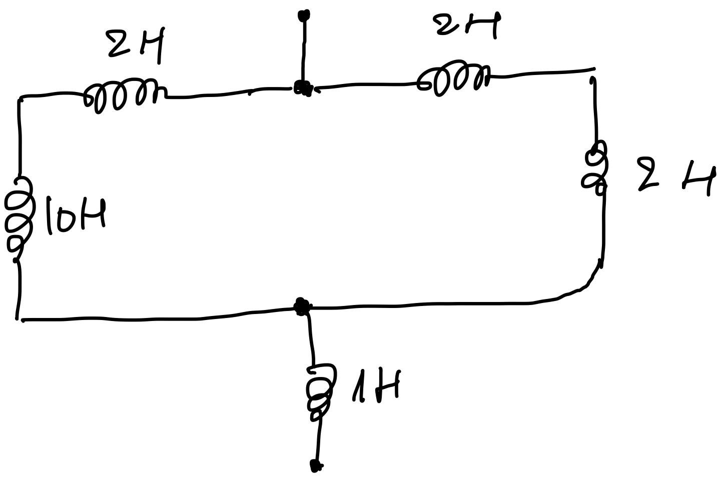
$$\boxed{\Rightarrow L_{eq} = 1\,H}$$



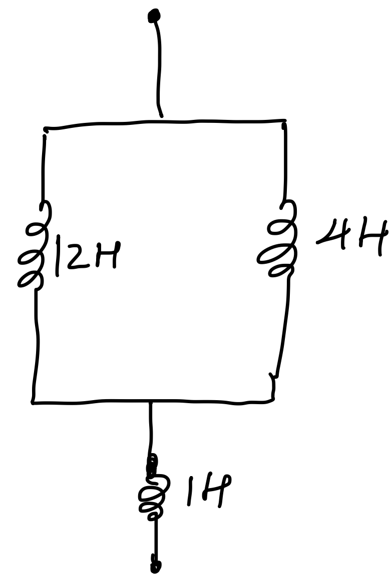
Because $30H \parallel 15H \Rightarrow \dfrac{1}{L_{3015}} = \dfrac{1}{30} + \dfrac{1}{15} \Rightarrow L_{3015} = \dfrac{30 \times 15}{30+15} = 10H$

$(6H \parallel 3H) \Rightarrow \dfrac{1}{L_{63}} = \dfrac{1}{6} + \dfrac{1}{3} \Rightarrow L_{63} = \dfrac{6 \times 3}{6+3} = 2H$

⇒ We have a new circuit:

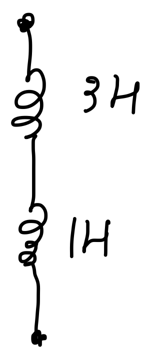


⇔



$$12H \,//\, 4H \Rightarrow L_{124} = \frac{12 \times 4}{12+4} = 3H$$

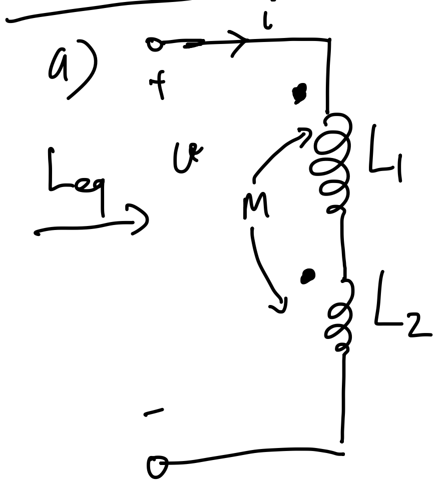
⇒



$$\Rightarrow \boxed{L_{eq} = 3 + 1 = 4\ H}$$

Problem 4 :

a)



We have : $v = L \dfrac{di}{dt}$

$$v_1 = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$= L_1 \frac{di(t)}{dt} + M \frac{di(t)}{dt}$$

$$v_2 = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$

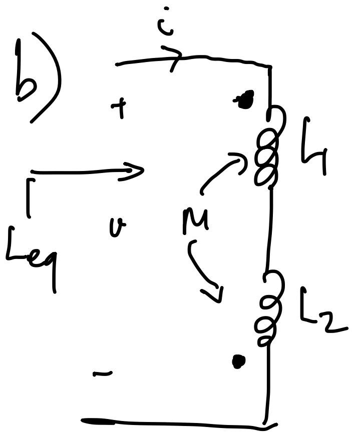
$$= L_2 \frac{di(t)}{dt} + M \frac{di(t)}{dt}$$

$$\Rightarrow v = L_1 \frac{di(t)}{dt} + M \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + M \frac{di(t)}{dt}$$

$$= (L_1 + L_2 + 2M) \frac{di(t)}{dt} = L_{eq} \frac{di(t)}{dt}$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2 + 2M}$$

b)



We have : $v_1 = L_1 \dfrac{di_1(t)}{dt} - M \dfrac{di_2(t)}{dt}$

$$= L_1 \frac{di(t)}{dt} - M \frac{di(t)}{dt}$$

$$v_2 = L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$$

$$= L_2 \frac{di(t)}{dt} - M \frac{di(t)}{dt}.$$

$$\Rightarrow v = v_1 + v_2 = L_1 \frac{di(t)}{dt} - M \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} - M \frac{di(t)}{dt}$$

$$= (L_1 + L_2 - 2M) \frac{di(t)}{dt} = L_{eq} \frac{di}{dt} \Rightarrow \boxed{L_{eq} = L_1 + L_2 - 2M}$$

Problem 5: Ideal transformer with two coils.

$R = 4\,\Omega$, $\phi = \phi_0 \sin \omega t \Rightarrow d\phi/dt = \phi_0 \omega \cos \omega t$

We have $V_1 = N_1 \dfrac{d\phi}{dt} = N_1 \phi_0 \omega \cos \omega t$

$\Rightarrow \boxed{v_1(t) = N_1 \phi_0 \omega \cos(\omega t)}$ (V) ; since $\dfrac{v_1}{v_2} = \dfrac{N_1}{N_2}$

since $v_2(t) = \dfrac{N_2}{N_1} v_1(t) = N_2 \phi_0 \omega \cos(\omega t)$ (V)

Because $v_1(t) = L \dfrac{di_1}{dt} \Rightarrow \dfrac{di_1}{dt} = \dfrac{v_1(t)}{L}$

$$\Rightarrow \frac{di_1}{dt} = \frac{N_1 \phi_0 \omega \cos(\omega t)}{L}$$

$$\Rightarrow i_1(t) = \frac{N_1 \phi_0}{L} \cdot \sin \omega t \text{ (A)} \Rightarrow i_2(t) = \frac{N_1}{N_2} i_1(t)$$

$$\Rightarrow i_2(t) = \frac{N_1^2}{N_2} \cdot \frac{\phi_0}{L} \sin(\omega t) \text{ (A)}$$

We also have $V_2(t) = i_2(t) \cdot R = 4 i_2(t)$

$$\Rightarrow N_2 \phi_0 w \cos wt = 4 \cdot \frac{N_1^2}{N_2} \cdot \frac{\phi_0}{L} \sin(wt)$$

$$\Rightarrow N_2^2 L \phi_0 w \cos wt = 4 N_1^2 \phi_0 \sin wt$$

$$\Rightarrow L = \frac{4 N_1^2 \phi_0 \sin wt}{N_2^2 \phi_0 w \cos wt} = \frac{4 N_1^2}{w \cdot N_2^2} tg(wt)$$

$$\Rightarrow i_1(t) = \frac{N_1 \phi_0}{L} \sin wt = \frac{N_1 \phi_0 \sin wt \; w N_2^2}{4 N_1^2 \, tg(wt)}$$

$$\Rightarrow \boxed{i_1(t) = \frac{\phi_0 w N_2^2 \cos wt}{4 N_1}}$$

Therefore:

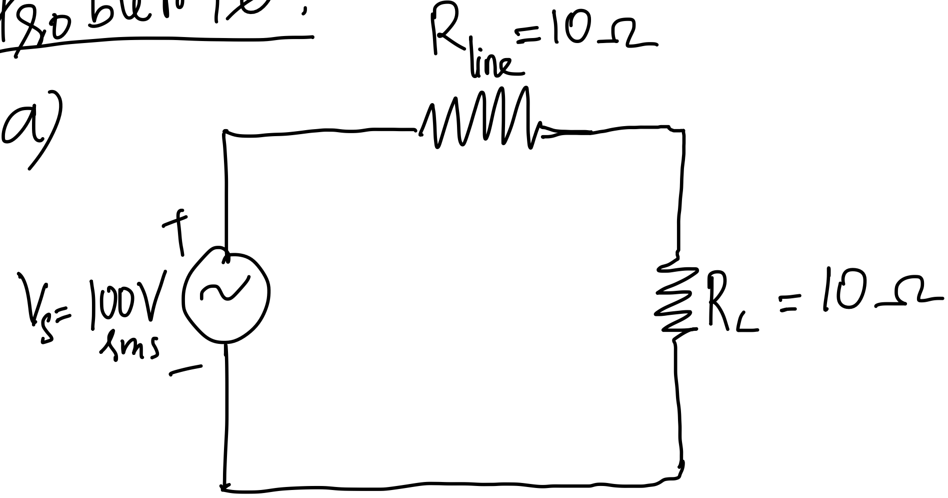
$$\frac{v_1(t)}{i_1(t)} = \frac{N_1 \phi_0 w \cos w(t)}{N_2^2 \phi_0 w \cos wt} \times 4 N_1 = \frac{4 N_1^2}{N_2^2}$$

$$\Rightarrow \boxed{\frac{v_1(t)}{i_1(t)} = \frac{4 N_1^2}{N_2^2}} \quad \text{or} \quad \boxed{R_{eq} = 4 \left(\frac{N_1}{N_2}\right)^2}$$

$\Leftrightarrow$ technique "reflection"

Problem 6:

a)



We have: $I_L = \frac{V_s}{R_{line} + R_L} = \frac{100V}{(10+10)\,\Omega} = 5\,(A)$

* Power delivered by the source:

$P_S = V_S I_L = 100 \times 5 = \boxed{500\ W}$

* Power dissipated in the line resistance:

$P_{line} = I_L^2 R_{line} = 5^2 \times 10 = \boxed{250\ W}$

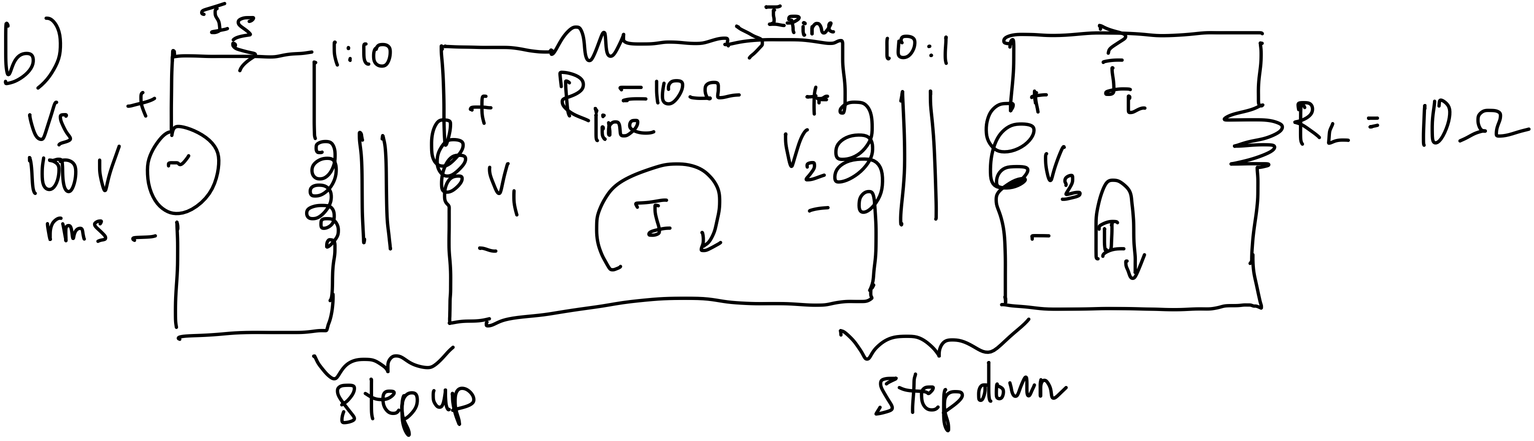
* Power delivered to the load:

$P_L = I_L^2 R_L = 5^2 . 10 = \boxed{250\ W}$

$\Rightarrow$ the efficiency:

$\eta = \frac{P_L}{P_S} \times 100 = \frac{250}{500} \times 100 = \boxed{50\%}$

b)



We have:

$$\frac{V_s}{V_1} = \frac{1}{10} \Rightarrow V_1 = 10V_s \quad (1)$$

also, $\dfrac{I_s}{I_{line}} = \dfrac{10}{1} \Rightarrow I_s = 10\, I_{line} \Leftrightarrow I_{line} = \dfrac{I_s}{10} \quad (2)$

Apply KVL for I, $V_1 = I_{line} \cdot R_{line} + V_2$

From (1) & (2), $\Rightarrow 10V_s = \dfrac{I_s}{10} \cdot R_{line} + V_2$

$\Rightarrow V_2 = 10V_s - \dfrac{I_s}{10} R_{line} \quad (3)$

Besides, $\dfrac{V_2}{V_3} = \dfrac{10}{1} \Rightarrow V_2 = 10V_3$; $\dfrac{I_{line}}{I_L} = \dfrac{1}{10} \quad (4)$

$\Rightarrow I_L = 10\, I_{line}$. From (2) $I_{line} = \dfrac{I_s}{10} \Rightarrow I_s = 10\, I_{line}$

$\Rightarrow I_s = I_L \quad (5)$

Apply KVL for II, We have: $V_3 = I_L R_L$

$\Rightarrow 10V_3 = 10\, I_L R_L$. Sin $V_2 = 10V_3$ (from (4))

$I_s = I_L$ (from (5)) (6)

$\Rightarrow V_2 = 10 I_s R_L$. Since $V_2 = 10V_s - \frac{I_s}{10} R_{line}$ from (2)

$\Rightarrow 10V_s - \frac{I_s}{10} R_{line} = 10 I_s R_L$

Plug $V_s = 100V$, $R_{line} = 10\,\Omega = R_L$

$\Rightarrow 10 \times 100 - \frac{I_s}{10} \cdot 10 = 10 \cdot I_s \cdot 10$

$\Rightarrow 1000 - I_s = 100 I_s \Rightarrow I_s = \frac{1000}{101} = 9.901A$

$\Rightarrow$ * Power delivered by the source:

$P_s = V_s \cdot I_s = 100V \times 9.901A = \boxed{990.1(W)}$

Also, $I_s = I_L$ (from (6))

$\Rightarrow$ * Power delivered to the load:

$P_L = I_L^2 R_L = 9.901^2 A \times 10 = \boxed{980.3(W)}$

From (4), $I_{line} = \frac{I_L}{10} = \frac{9.901}{10} = 0.9901(A)$

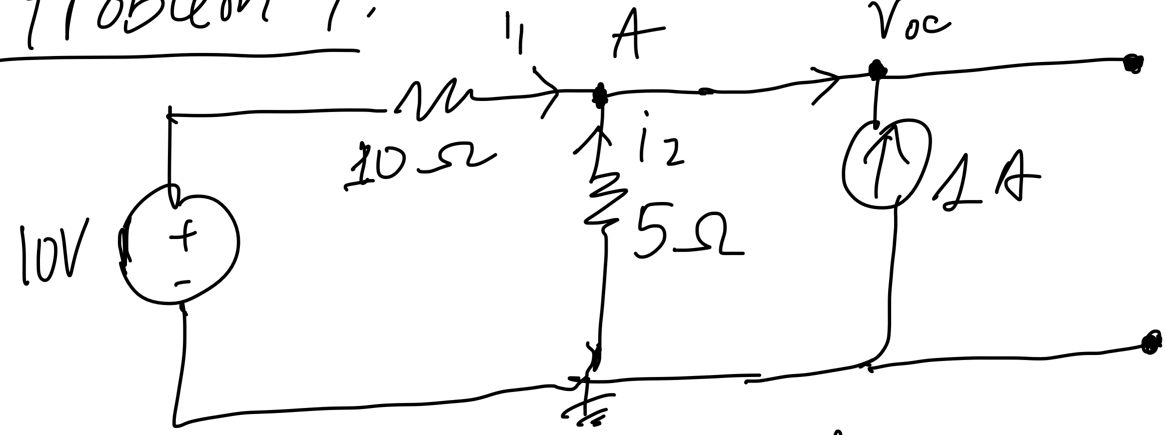
$\Rightarrow$ power dissipated in the line resistance:

$P = I_{line}^2 R_{line} = (0.9901)^2 \times 10\,\Omega = \boxed{9.803(W)}$

$\Rightarrow$ The efficiency: $\eta = \frac{P_L \times 100}{P_s} = \frac{980.3}{990.1} \times 100 = 99.01\%$

$\Rightarrow \boxed{\eta = 99.01\%}$ $\Rightarrow$ the efficiency of delivering power to the load is increasing by using transformer.

Problem 7:



Apply KCL at A, we have:

$$i_1 + i_2 + 1 = 0$$

$$\frac{10 - V_{oc}}{10} + \frac{0 - V_{oc}}{5} + 1 = 0$$

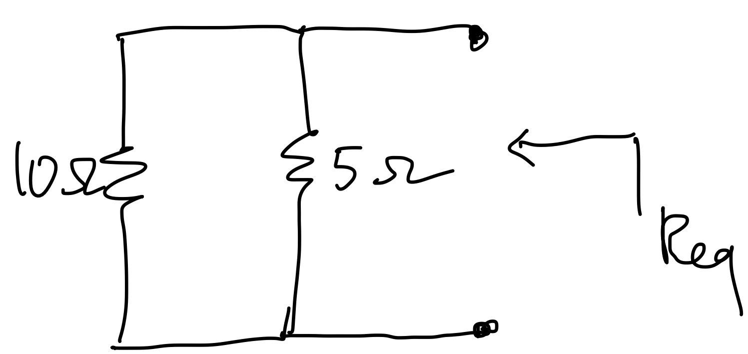
$$\Rightarrow 10 - V_{oc} + 2(0 - V_{oc}) + 10 = 0$$

$$\Rightarrow 10 - V_{oc} - 2V_{oc} + 10 = 0$$

$$\Rightarrow 20 = 3V_{oc} \Rightarrow V_{oc} = \frac{20}{3}(V) = V_{th}$$

* Finding the equivalent resistance:

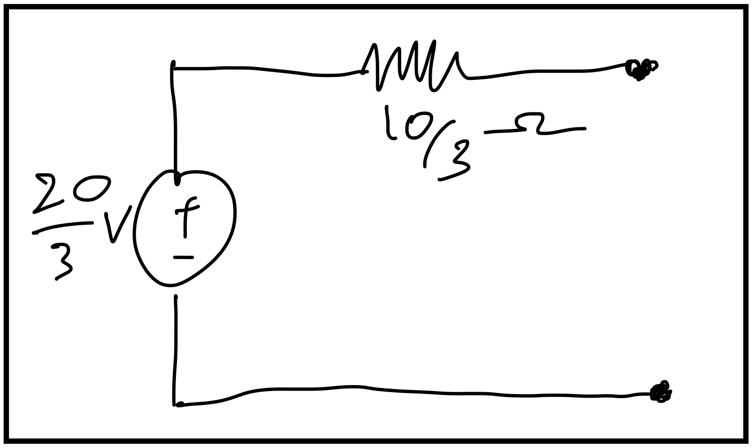
We have the new circuit:



$10\Omega \,//\, 5\Omega$

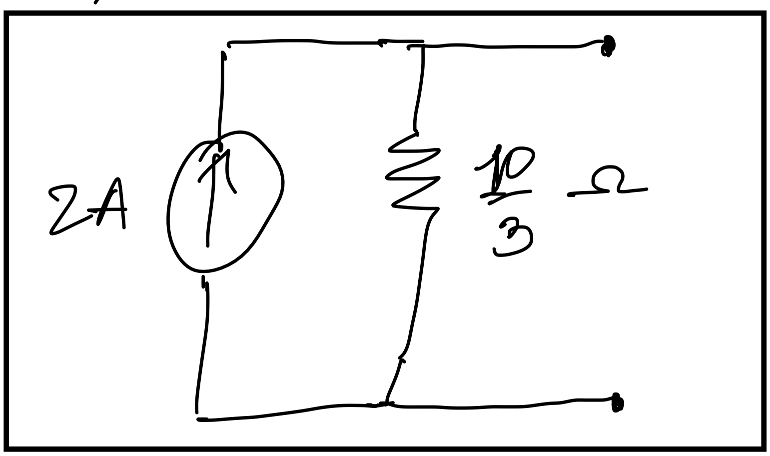
$$\Rightarrow R_{eq} = \frac{5 \times 10}{5 + 10} = \frac{10}{3}(\Omega)$$

$\Rightarrow$ We have the Thevenins equivalent circuit:

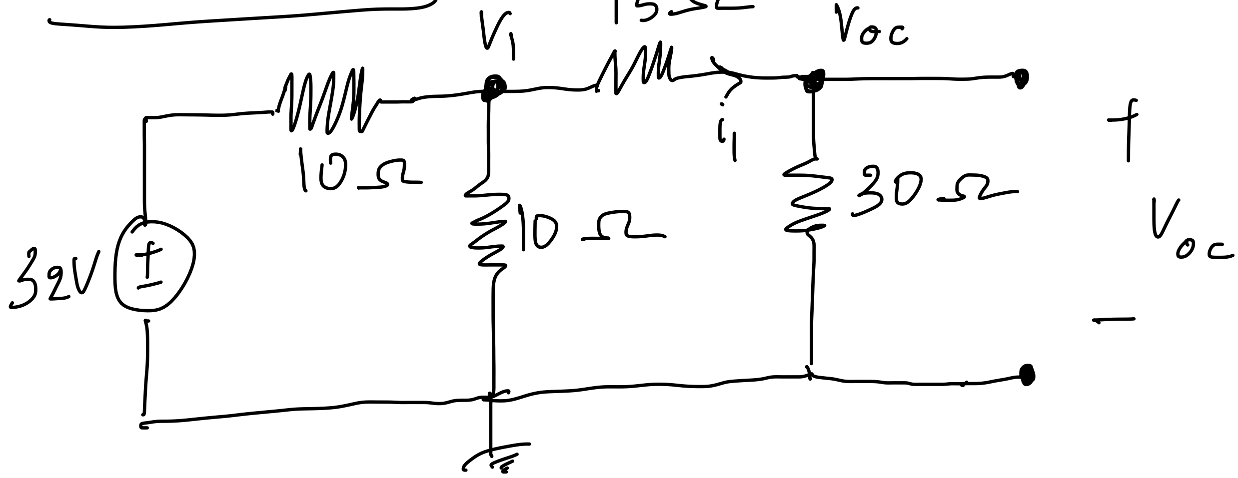


$$\Rightarrow I_N = \frac{20}{3} \cdot \frac{3}{10} = 2A$$

$\Rightarrow$ The Norton's equivalent circuit is:



Problem 8 :



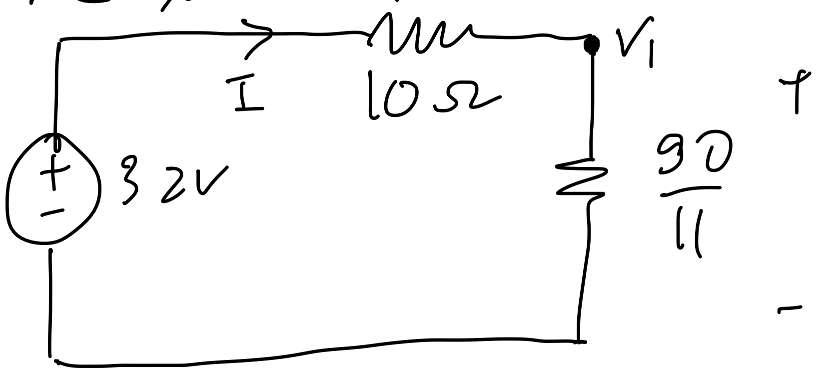
(1)

We have $(15\Omega$ series $30\Omega) \parallel 10\Omega$

$\Rightarrow R_{15-30} = 15 + 30 = 45\,\Omega$

$\Rightarrow R_{15-30-10} = \dfrac{45 \times 10}{45 + 10} = \dfrac{90}{11}(\Omega)$

We have the new circuit :



$\Rightarrow I = \dfrac{32V}{\left(10 + \frac{90}{11}\right)\Omega}$

(Ohm's Law)

$\Rightarrow V_1 = I \cdot \dfrac{90}{11} = \dfrac{32V}{\left(10 + \frac{90}{11}\right)} \cdot \dfrac{90}{11} = 14.4\,(V)$

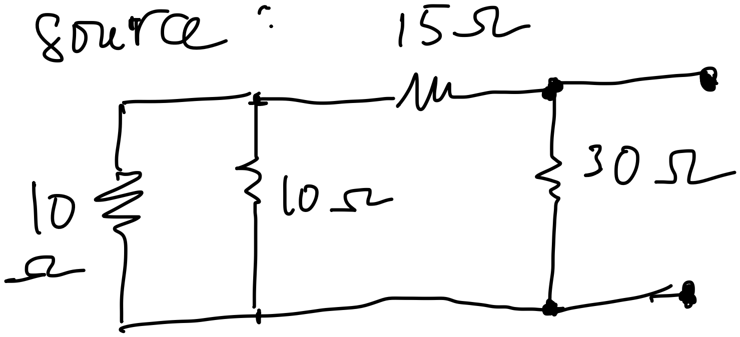
Go back to circuit (1), We have $V_1 = (15 + 30)\, i_1$

$\Rightarrow i_1 = \dfrac{V_1}{45}(A)$. Also $V_{oc} = i_1 \times 30\,\Omega$

$$\Rightarrow V_{oc} = \frac{V_1}{45} \times 30 = \frac{2}{3} \times 14.4 = 9.6(V)$$

(+) Find the equivalent resistance:

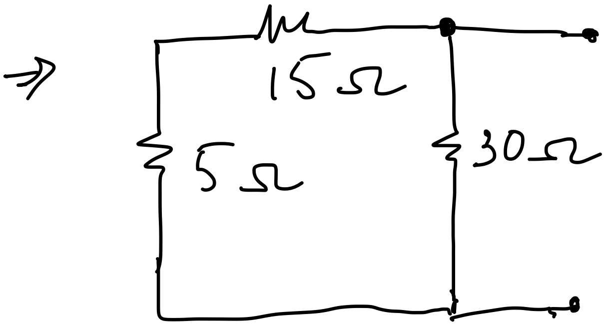
We have a new circuit by shorting the voltage source:



Since $10\Omega \parallel 10\Omega$

$$\Rightarrow R_{1010} = \frac{10 \times 10}{10+10} = 5\Omega$$

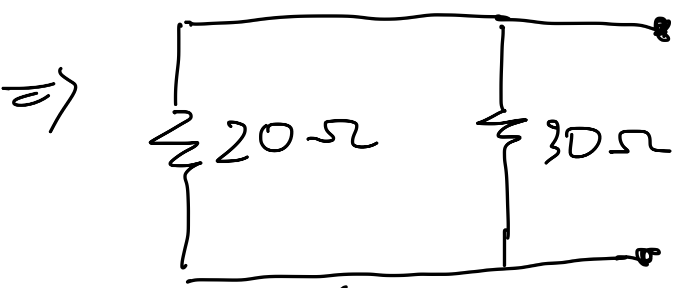
$\Rightarrow$



Since 15Ω series to 5Ω

$$\Rightarrow R_{15\text{-}5} = 15 + 5 = 20\Omega$$

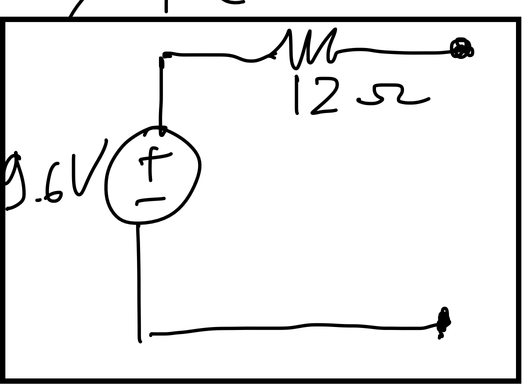
$\Rightarrow$



$20\Omega \parallel 30\Omega$

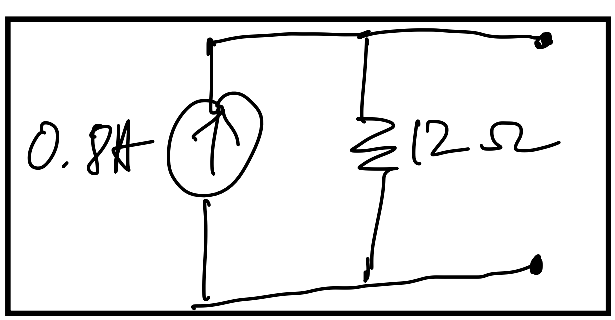
$$\Rightarrow R_{20\text{-}30} = \frac{20 \times 30}{20+30} = 12\Omega$$

$\Rightarrow$ The Thevenin equivalent circuit:



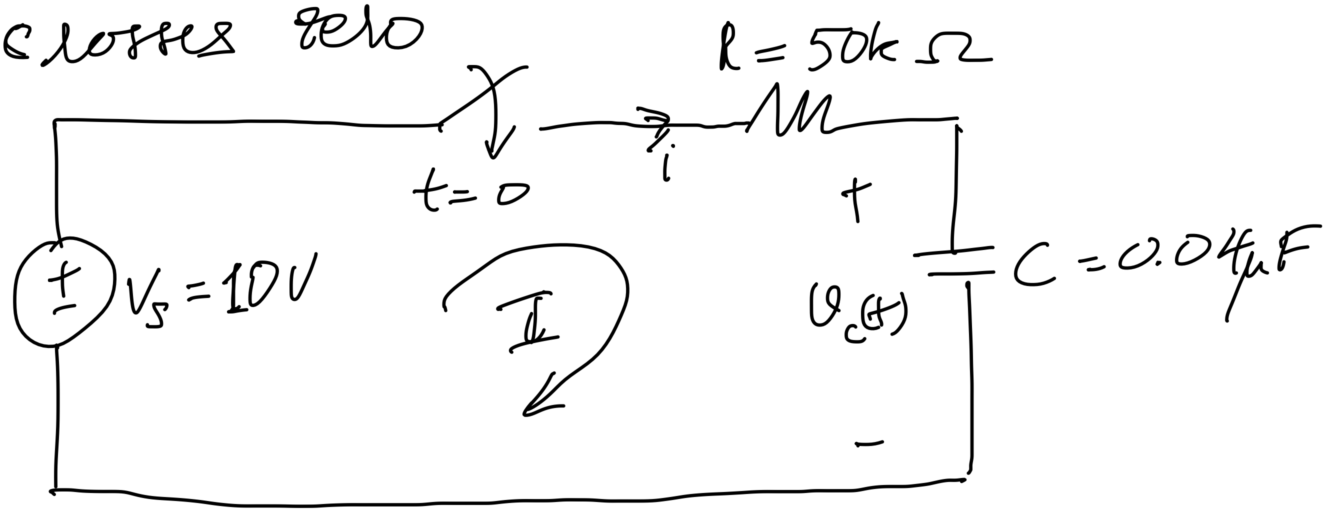
$$\Rightarrow I_N = \frac{9.6V}{12\Omega} = 0.8A$$

The Norton equivalent circuit:



Problem 9: $v_c(0^+) = -10V$. Find $v_c(t)$.

Determine the time t_0 at which the voltage crosses zero



Apply KVC for loop (I), we have: (at $t = 0^+$)

$-V_s + Ri_c + v_c(t) = 0$. Also $i_c = C\dfrac{dv_c(t)}{dt}$

$$\Rightarrow -V_s + RC\frac{dv_c(t)}{dt} + v_c(t) = 0$$

$$\Rightarrow RC\frac{dv_c(t)}{dt} + v_c(t) = V_s \quad (1)$$

We can use the solution form of this linear first order differential equation, we have the form of $v_c(t) := K_1 + K_2e^{st}$ with K_1 & K_2 are constant (4.13 in text book)

$\Rightarrow \dfrac{dv_c(t)}{dt} = K_2 s e^{st}$, then plug it into (1), we have:

$$RC\,K_2 s e^{st} + K_1 + K_2 e^{st} = V_s$$

$$\Leftrightarrow K_2 e^{st}(RCs+1) + K_1 = V_s$$

For equality, the coefficient of e^{st} must be 0

$$\Rightarrow \begin{cases} K_1 = V_s = 10 \\ s = \dfrac{-1}{RC} = \dfrac{-1}{50\times10^3 \times 0.04\times10^{-6}} = -500 \end{cases}$$

$$\Rightarrow v_c(t) = K_1 + K_2 e^{st} = 10 + K_2 e^{-500t}$$

as a given, $v_c(0^+) = -10V$

$$\Rightarrow -10 = 10 + K_2 \Rightarrow K_2 = -20$$

$\Rightarrow$ the expression for the voltage across the capacitor by the time:

$$\boxed{V_c(t) = 10 - 20e^{-500t} \quad (V)}$$

When $V_c(t) = 0 \Rightarrow 20e^{-500t} = 10$

$$\Rightarrow e^{-500t} = 0.5 \Rightarrow -500t = \ln 0.5$$

$$\Rightarrow t = \frac{-\ln 0.5}{500} = 0.0013863 \text{ (s)}$$

$$\Leftrightarrow \boxed{t = 1.3863 \text{ (ms)}}$$