ECE 100 (Spring 2021) - Quiz #1

(Format: 3 questions, 50 minutes)

Name: ______________________________

Student ID: __________________________

Score: _______ out of 80

Instructions:

1. Register for the quiz (if you are seeing this, you should have already registered)
2. Once you register for the quiz, you will have 50 minutes to complete the quiz
3. After the quiz, you have 15 minutes to submit and upload your quiz to CCLE (under "Week 3 → Quiz 1").
4. Please fill out this 'End of Quiz' survey to acknowledge that you have completed the quiz and submitted your answer sheet to CCLE: https://forms.gle/n2wxogiiQdKjAT5B6

Rules:

- Quiz is closed book. No computers, cell phones, etc.
- Scientific calculator allowed.
- Box all of your answers & show your work.
- **If you have questions on the exam, please DO NOT post on Piazza. Email instructor(s) directly.**

Quiz Start Time:

Wednesday, April 14th @ 6:00pm PDT

Note: Once you register for the quiz, you will have 1hr 5m to complete & upload your results. (50 minutes to take the exam, 15 minutes to upload).

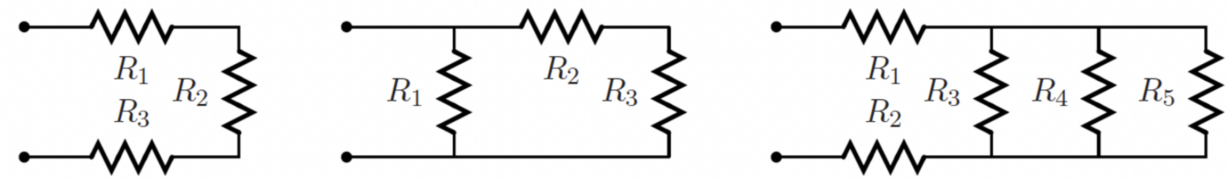
End Time:

Thursday, April 15th @ 11:59am PDT (answer sheet must be submitted by this time)

No late submissions

Problem 1: Circuit Analysis (20 points)

(a) Find the equivalent resistance, as viewed from its port, of each resistor network shown below (3x2 = 6 points)



(i) (ii) (iii)

(i)

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

(ii)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2 + R_3} \quad \Rightarrow \quad \frac{1}{R_{eq}} = \frac{R_1 + R_2 + R_3}{R_1(R_2 + R_3)}$$

$$\boxed{R_{eq} = \frac{R_1 R_2 + R_1 R_3}{R_1 + R_2 + R_3}}$$

(iii)

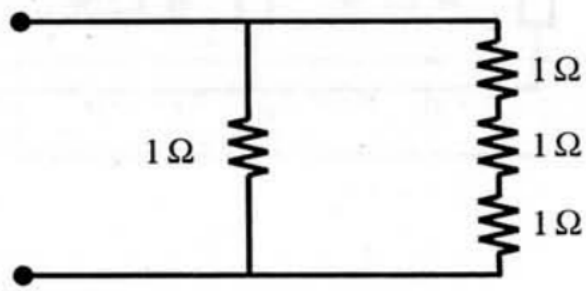
In this circuit we observe that resistors R_3, R_4 and R_5 are in parallel. We also observe that their equivalent resistance R_p is in series with resistors R_1 and R_2. The total equivalent resistance of the network is simply the summation of R_1, R_2 and R_p.

$$R_p = \frac{R_3 R_4 R_5}{R_4 R_5 + R_3 R_5 + R_3 R_4}$$

$$\boxed{R_{eq} = R_1 + R_2 + R_p}$$

(b) Beginning with 1-Ω resistors, synthesize a resistor of (i) 0.75 Ω and (ii) a resistor of 1.5 Ω. Use no more than four 1-Ω resistors in each case. (2x3= 6 points)

(i)

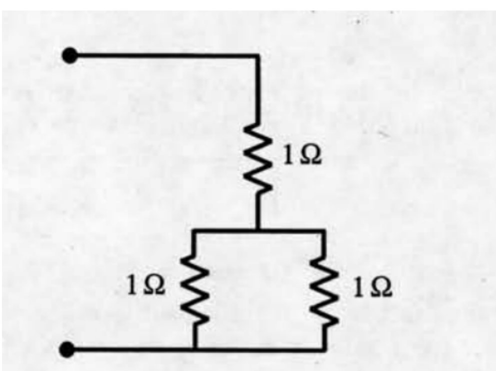


The minimum resistance possible using a series connected circuit is $1\,\Omega$, therefore we must use a parallel combination of resistors to achieve $0.75\,\Omega$ overall. Using equation

$$0.75\Omega = \frac{R_1 R_2}{R_1 + R_2} \quad \Rightarrow \quad 0.75R_1 + 0.75R_2 = R_1 R_2$$

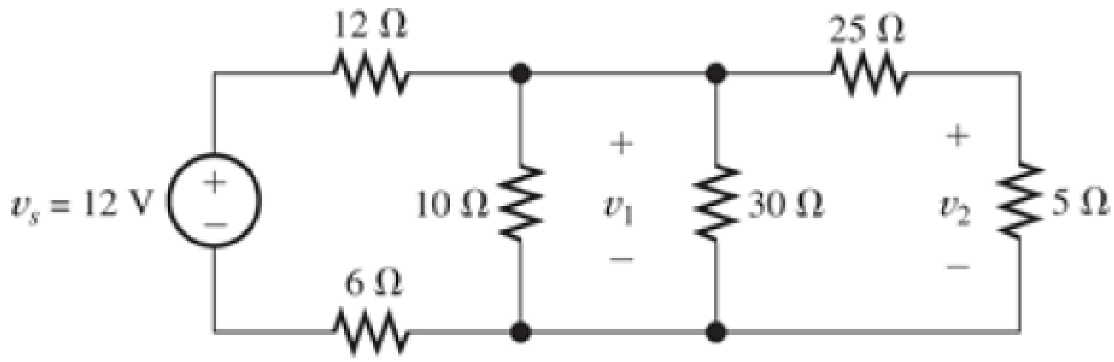
We set $R_1 = 1\,\Omega$ and solve for R_2. The result of this calculation is that $R_2 = 3\,\Omega$. This can be achieved by a series combination of three $1\,\Omega$ resistors as shown in Fig:

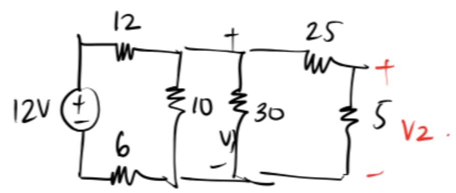
(ii)

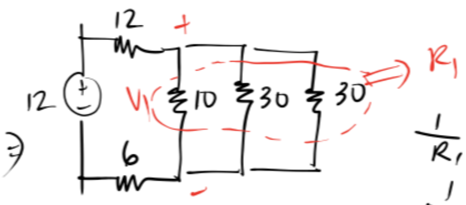


An equivalent resistance of $1.5\,\Omega$ can be achieved by placing a $1\,\Omega$ resistor in series with a parallel combination of two $1\,\Omega$ resistors

(c) Find the voltages V_1 and V_2 and for the circuit shown below. (8 points)

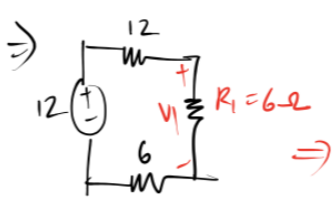






$$\frac{1}{R_1} = \frac{1}{10} + \frac{1}{30} + \frac{1}{30}$$

$$\frac{1}{R_1} = \frac{5}{30} \Rightarrow \boxed{R_1 = 6\,\Omega}$$



$$\Rightarrow V_1 = \frac{6}{(12+6+6)} \cdot 12 = \frac{6 \cdot 12}{24} = 3\,V$$

$$\boxed{V_1 = 3V}$$

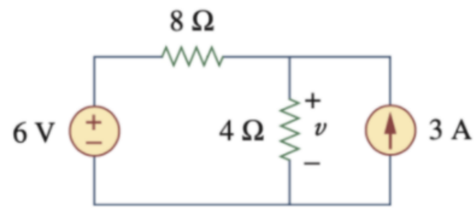
$$V_2 = \frac{(V_1) \cdot 5}{5+25} = \frac{5 \cdot V_1}{30} = \frac{5 \cdot 3}{30} = 0.5\,V$$

$$\boxed{V_2 = 0.5V}$$

Problem 2: Superposition & Source Transformations (20 points)

Hint: The following circuits can be solved using Superposition or Source Transformations.

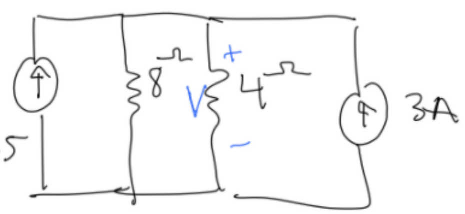
(a) Find voltage, v. (8 points)



two possible approaches:

Approach: ① sourcetransformations

$\frac{6V}{8\Omega} = 0.75$ A



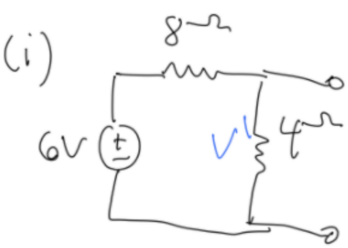
$V = (4\Omega \| 8\Omega) \times (3.75A)$

$= \frac{32}{12}\Omega \times 3.75A = \boxed{10V}$ ☆

Same answer

Approach ② superposition

(i)



$V' = \frac{4}{4+8} \times 6V = \underline{\underline{2V}}$

(ii)

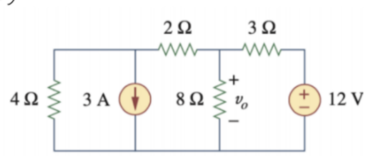
$V'' = (4\Omega \| 8\Omega) \times 3A$

$= \frac{32}{12} \times 3A = \underline{\underline{8V}}$

$\boxed{V = V' + V'' = 10V}$ ☆

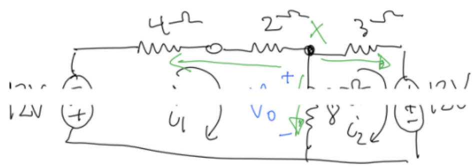
(b) Find voltage, v_o. (12 points)

(b)



can solve using multiple approaches as well

Approach #1:



KCL @ node X:

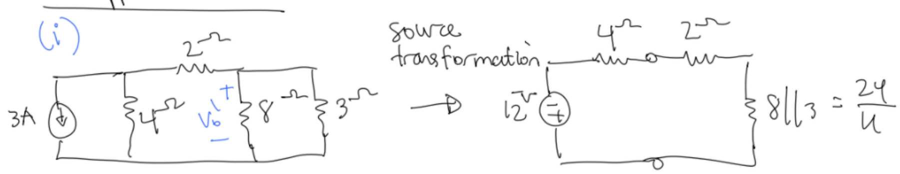
$$\frac{V_o}{8\Omega} + \frac{V_o - (-12V)}{6\Omega} + \frac{V_o - 12V}{3\Omega} = 0$$

$$V_o\left[\frac{1}{8} + \frac{1}{6} + \frac{1}{3}\right] = \frac{12}{3} - \frac{12}{6}$$

$$\boxed{V_o = \frac{2}{\left[\frac{1}{8} + \frac{1}{6} + \frac{1}{3}\right]} = 3.2V}$$

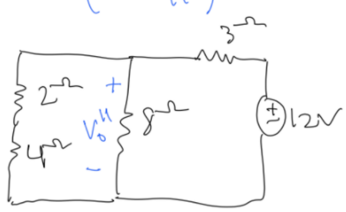
Approach #2:

(i)



$$V_o' = \frac{\frac{24}{11}}{\left(6 + \frac{24}{11}\right)} \cdot -12V = -3.2V$$

(ii)



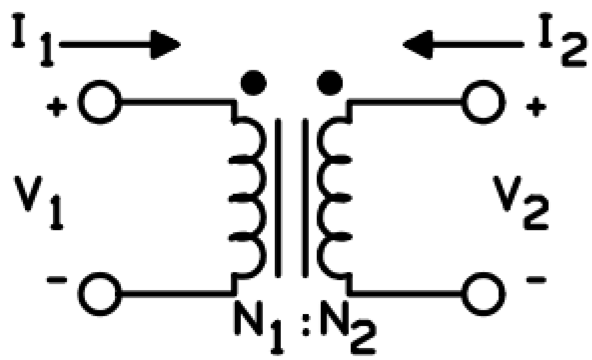
$$V_o'' = \frac{(6\|8)}{(6\|8) + 3\Omega} \times 12V = 6.4V$$

$$\boxed{V_o = V_o' + V_o'' = -3.2V + 6.4V = 3.2V}$$

Problem 3: Transformers (40 points)

We studied the ideal transformer in class.

(a) Draw the symbol for an ideal transformer. (4 points)



N_1=N_P and N_2=N_S

(b) If the transformer has N_P primary turns and has a turns ratio of n, what are the number of turns in the secondary coil N_S? (4 points)

$n = N_S/N_P \rightarrow N_S = n \times P$

(c) For this ideal transformer, if the primary voltage is V_P, what is the secondary voltage V_S? Does V_S depend on the current being drawn out of the secondary turns, I_S? (4 points)

$V_S = n \times V_P$

No, in an ideal transformer V_S is independent of the current being drawn by the secondary load.

(d) For a secondary current I_S, what is the primary current, I_P?
(4 points)

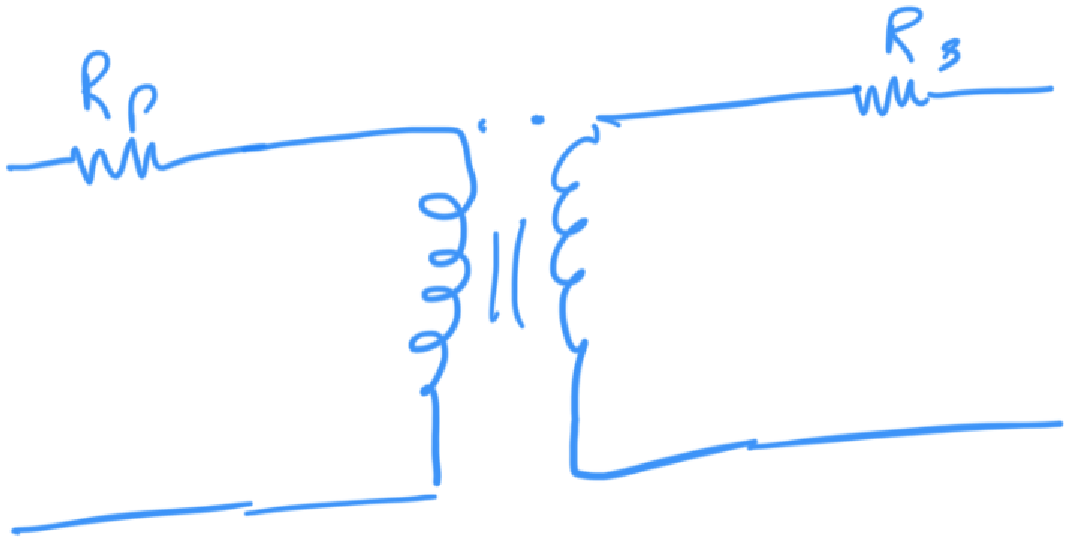
Input power=out power
$V_P \times I_P = V_S \times I_S$, hence $I_S = V_P \times I_P / V_S = I_P / n$

(e) What is the output power P_S? What is the input power P_P? What is the transformer efficiency, η_1? (4 points)

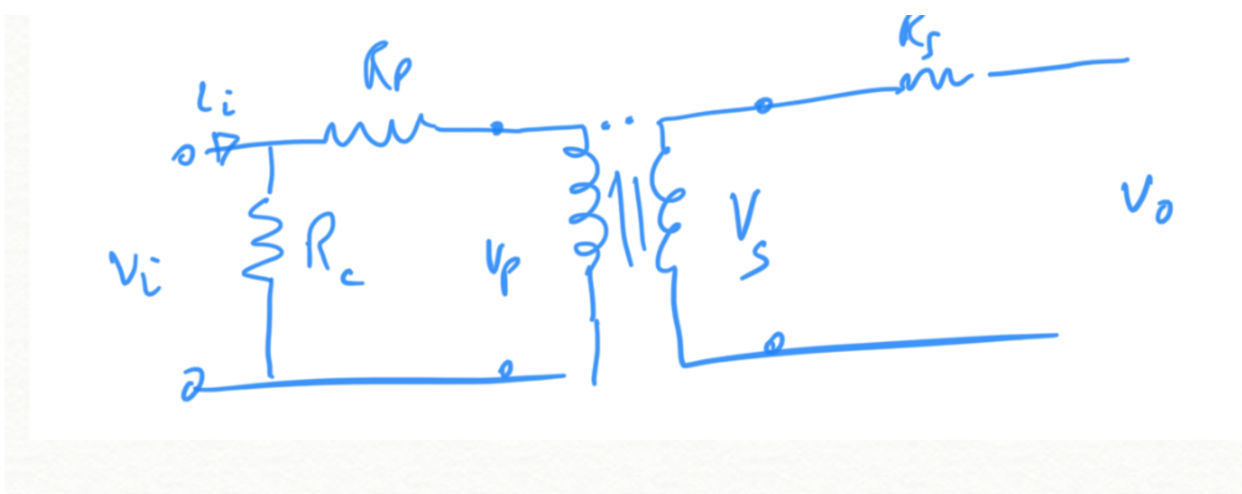
For an ideal Transformer: $P_P = V_P \times I_P = P_S$
Efficiency, η_1=100%

(f) In practice, the transformer is not ideal. If the coils have a resistance, ρ_τ / turn, how would you represent this non-ideality in the symbol for the transformer? Draw it. (4 points)

On the primary side you would add a series resistance $R_P = N_P \times \rho_\tau$
On the secondary side add a series resistance $R_S = N_S \times \rho_\tau$



(g) Now let's assume the transformer also has core losses. Assume that the core losses only depend on the input voltage, V_i, to the transformer. How would you represent this loss on the model for the transformer? (4 points)
Hint: some of the primary current will be diverted to heating up the transformer core (the iron part of the transformer).



The assumption here is that all the core losses can be approximated by a shunt resistor on the input side. This shunt resistor could be either in parallel with the input voltage as we have shown here or it could be in parallel with the transformer windings. The R_C represents two types of losses: the core losses defined by the hysteresis properties of the core material as well as eddy currents that can be induced in the core iron. Remember these are all models and there may not be a unique answer. The way we create these models is so as to approximate as best as possible the measurement data.

(h) For an input current, I_i, and an input voltage, V_i, what is the output voltage, V_o, and output current, I_o? (4 points)

Since we have not yet studied the AC impedance of inductors we will approximate with a DC analysis

The input current I, will divide at the input node: the current through R_C is V_i/R_C, so by KCL the current in R_P and the coil is

$$I_P = (I_i - V_i/R_C)$$

And the primary voltage at the primary coil is:

$$\begin{aligned} V_P &= V_i - I_P R_P \\ &= V_i - (I_i - V_i/R_C) \times R_P \end{aligned}$$

Now we will invoke what we did in part (d) and say that the current in the secondary coil is:

$$I_S = (I_i - V_i/R_C)/n \quad (n = N_S/N_P)$$

And the voltage at the secondary coil is:

$V_S = n \times V_P = n \times [V_i - (I_i - V_i/R_C) \times R_P]$

This current flows through R_S and produces a voltage drop of $I_S \times R_S$

Hence the output voltage available at the load is:

$V_o = V_S - I_S \times R_S$

$V_o = (n \times [V_i - (I_i - V_i/R_C) \times R_P]) - ((I_i - V_i/R_C)/n) \times R_S$

(i) What is the efficiency of this non-ideal transformer, $\eta_{\text{non-ideal}}$?
(4 points)

The input power is: $P_i = V_i \times I_i$

The output power is $P_o = V_o \times I_o$ (Note: $I_o = I_S$)

$P_o = [(n \times [V_i - (I_i - V_i/R_C) \times R_P]) - I_S \times R_S] \times I_S$

where: $I_S = ((I_i - V_i/R_C)/n)$

Non-ideal Efficiency is:

$\eta_{non-ideal} = P_o/P_i = \frac{[(n \times [V_i - (I_i - V_i/R_C) \times R_P]) - I_S \times R_S] \times I_S}{V_i \times I_i} \quad (I_S = ((I_i - V_i/R_C)/n))$

(j) Which do you think is a bigger factor: the series resistance of the coils or the shunt conductance of the core losses? (4 points)

Usually it’s the series resistance especially when you have a large number of turns.