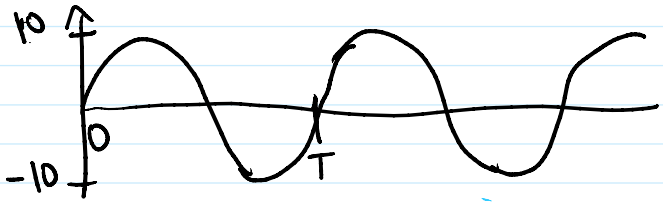
ECE 100 – Homework #1 Solutions

Problem 1

$$i(t) = 10\sin(200\pi t)\ A$$

a)



$$T = \frac{2\pi}{\omega} = \frac{2\pi}{200\pi} = \frac{1}{100} = 10\ \text{ms}$$

b) $Q = \int_0^{5ms} i\,dt$ note: $i = \frac{dq}{dt}$

$$Q = \int_0^{5ms} 10\sin(200\pi t)\,dt$$

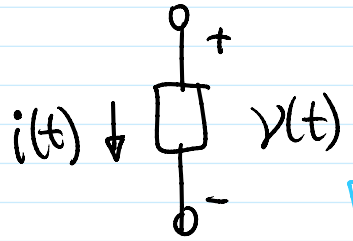
$$= \frac{-10}{200\pi}\cos(200\pi t)\Big|_0^{5ms}$$

$$= \frac{-1}{20\pi}\left[-1 - 1\right] = \boxed{\frac{1}{10\pi}\ [C]}$$ "half cycle"

c) $Q = \int_0^{10ms} 10\sin(200\pi t)\,dt$

$$= \frac{-1}{20\pi}\left[1 - 1\right] = \boxed{0\ [C]}$$ "full cycle"

Problem 2



$v(t) = 10V$, $i(t) = 2e^{-t}\ A$

$$\boxed{p = v(t) \times i(t) = 20e^{-t}\ [W]}$$

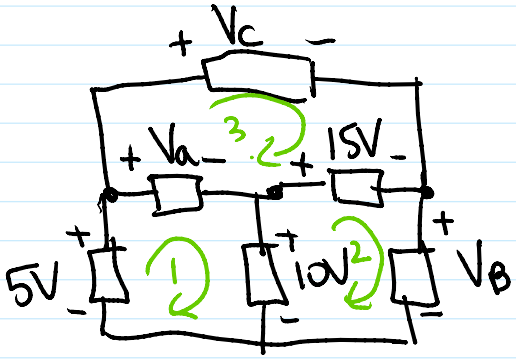
$$\int_0^{\infty} \qquad \int_0^{\infty} \qquad -t \qquad +b_0 \qquad T \qquad T$$

$$E = \int_0^{\infty} p \cdot dt = \int_0^{\infty} 20e^{-t} = -20e^{-t}\Big|_0^{\infty} = -20[0-1]$$

$$\boxed{E = 20\ [J]}$$

∴ absorbed by the element

Problem 3



① $5V - V_A - 10V = 0$

↳ $\boxed{V_A = -5V}$

② $10V - 15V - V_B = 0$

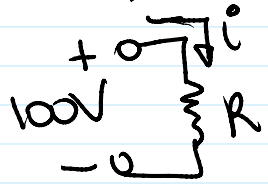
↳ $\boxed{V_B = -5V}$

③ $V_A + 15V - V_C = 0$

$(-5V) + 15V = V_C \rightarrow \boxed{V_C = 10V}$

Problem 4

(a) $P = 100W$ delivered to R



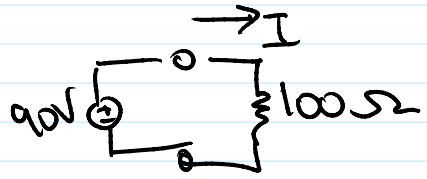
$P = IV = 100\,W$

↳ $I = \frac{100W}{100V} = 1\ Amp$

→ $V = IR \rightarrow R = \frac{V}{I} = \frac{100V}{1A} = \boxed{100\ [\Omega]}$

(b) V reduced to 90V (by 10%)

→ I $\qquad I = \frac{V}{\quad} = \frac{90V}{\quad} = 0.9A$

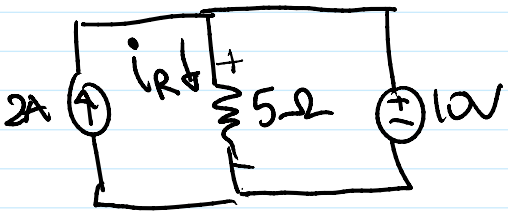


$$I = \frac{V}{R} = \frac{90V}{100\Omega} = 0.9A$$

$$P = IV = 0.9A \times 90V = \boxed{81\,[W]}$$

∴ power reduced by >10%
(reduced by 19%)

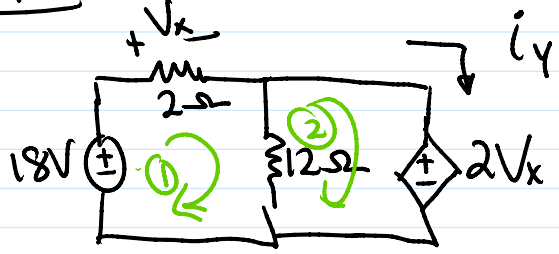
Problem 5



$$V = i_R \times 5\Omega = 10V$$

$$\therefore \boxed{i_R = 2\,[A]}$$

Problem 6



$$V_x = i_1(2\Omega)$$

V_A

① $18V - i_1(2\Omega) - (i_1 - i_2)(12\Omega) = 0$

② $2V_x + (i_2 - i_1)(12\Omega) = 0$

rewriting:

① $18V - i_1(2\Omega) = V_A$

② $2V_x = +V_A \rightarrow V_A = +2V_x$

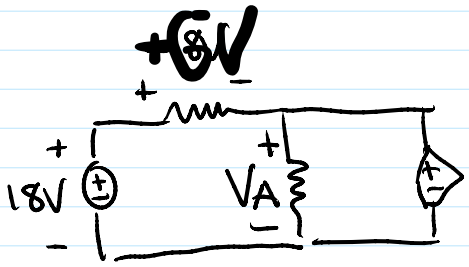
$$\Rightarrow \quad 18V - 2i_1 = +2V_x = +2(2i_1) = +4i$$

$$18V = 2i_1 + 4i_1 = +6i_1$$

(6*i1)

$$i_1 = \frac{18V}{6\Omega} = 3A$$

$$V_x = i_1 \times 2\Omega = +6V$$



$$18V - (+6V) - V_A = 0$$

$$\rightarrow V_A = 12V$$

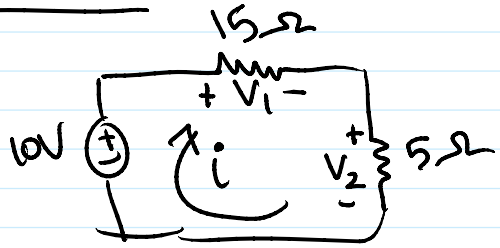
$$V_A = 12V = (i_1 - i_2) \times 12\Omega$$

note : $i_2 = i_y$

$$\frac{12V}{12\Omega} = i_1 - i_y \rightarrow i_y = i_1 - 1A$$

$$\therefore i_y = +3A - 1A = +2A$$

Problem 7



a.) KVL

$$10V = V_1 + V_2$$

b.) Ohm's Law

$$V_1 = i \times 15\Omega \qquad V_2 = i \times 5\Omega$$

c.) $$10V = i \times 15\Omega + i \times 5\Omega = i \times 20\Omega$$

$$\therefore i = 0.5[A]$$

d.)

voltage source : $P_{source} = 10V \times 0.5A = 5W$ (delivered)

voltage source: P_{source}

R_1: $P_1 = V_1 i = 15 i^2 = 3.75\,W$ (absorbed by R_1)

R_2: $P_2 = V_2 i = 5 i^2 = 1.25\,W$ (absorbed by R_2)

$P_{source} = P_1 + P_2$ $\surd$