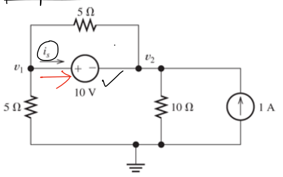
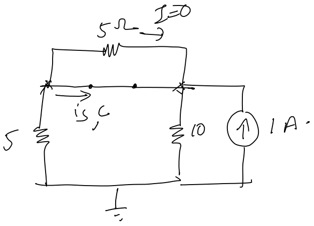
April 23

Thursday, 22 April 2021 10:07 PM

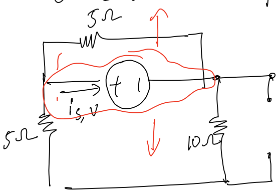
Superposition

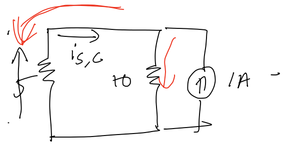




Voltage source = Short

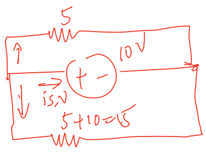
Current sources = open





$$I_{5\Omega} = \frac{1A \times 10}{5+10} = \frac{2}{3} = 0.667$$

$$\boxed{i_{S_C} = -0.667}$$



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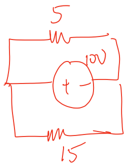
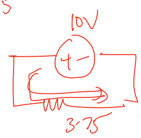


$R_{eq} = 5 \| 15 = 3.75\Omega$.

=



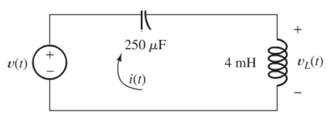
$$I = \frac{10}{3.75} = 2.66A$$

$$\boxed{i_{s,v} = -2.66A}$$

$i_s = i_{s,v} + i_{s,c}$

$= -2.66 + -0.667 = -3.33\ A$.

Find $i(t)$, $v_L(t)$, $v(t)$, the energy stored in the capacitance, the energy stored in the inductance, and the total stored energy for the circuit of Figure P3.70, given that $v_C(t) = 10\cos(1000t)$ V. (The argument of the cosine function is in radians.) Show that the total stored energy is constant with time. Comment on the results.



First order

The general form of the solution is $\boxed{x(t) = A + B\exp(-t/\tau)}$. A is the steady-state solution for $t >> 0$. We determine the value of the desired current or voltage immediately after $t = 0$, denoted by $x(0+)$. Then, we solve $x(0+) = A + B$ for the value of B. Finally, we determine the Thévenin resistance R_t from the perspective of the energy storage element (i.e., the resistance seen looking back into the circuit with the energy storage element removed) and compute the time constant: $\boxed{\tau = R_t C}$ for a capacitance or $\boxed{\tau = L/R_t}$ for an inductance.

A : steady state.

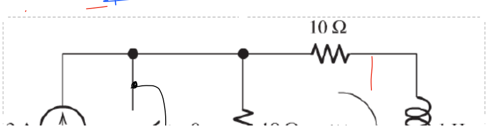
$t >> 0$.

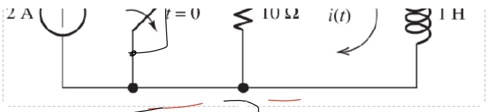
$t = 0$

$x(0^+)$

$x(0^+) = A + B\exp(-\frac{0}{\tau})$

$x(0^+) = A + B$.



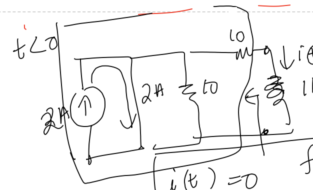


$\tau = R_t C$; $\frac{L}{R_t}$

$\tau = R_t C$

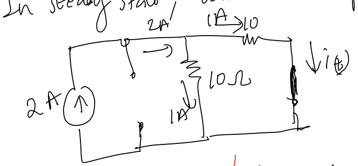
$\tau = \frac{L}{R}$

$t<0$

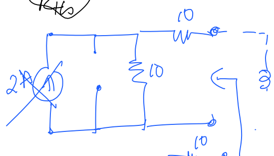


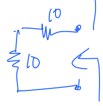
$\boxed{i(t) = 0 \quad \text{for } t<0}$

In steady state, with switch open $(t>>0)$



$\frac{L}{R_{th}}$





$i(\infty) = 1\ A$

$\boxed{i(t) = A + B\exp\left(\frac{-t}{\tau}\right)}$ for $t>0$

$i(0^-) = i(0^+) = 0 = A + B\exp\left(\frac{-0}{\tau}\right)$

$\boxed{0 = A + B}$ ——— ①

$R_{th} = 10+10 = 20$

$i(\infty) = 1$ $\quad \boxed{1 = A + B\exp\left(\frac{-\infty}{\tau}\right)} = A + 0$ ——— ②

from eq ① & ② $\boxed{B=-1}$ $\boxed{A=1}$ $\tau = ?$

$\tau = \frac{L}{R} = \frac{L}{20}$

$i(t) = 1 + -\exp\left(\frac{-t \cdot 20}{1}\right)$

$\boxed{i(t) = 1 - \exp(-20t)}$ for $t>0$

Second order circuits

Solution of the Second-Order Equation

We will see that the circuit equations for currents and voltages in circuits having two energy-storage elements can always be put into the form of Equation 4.63. Thus, let us consider the solution of

L, C

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t) \qquad (4.64)$$

where we have used *x*(*t*) for the variable, which could represent either a current or a voltage.

$x(t) =$ current / voltage

Here again, the general solution *x*(*t*) to this equation consists of two parts: a particular solution $x_p(t)$

$x(t) = x_p(t) + x_c(t)$

Particular Solution.

The particular solution is any expression $x_p(t)$ that satisfies the differential equation

$$\frac{d^2x_p(t)}{dt^2} + 2\alpha\frac{dx_p(t)}{dt} + \omega_0^2 x_p(t) = f(t)$$

One way to determine the particular solution is to assume that it is a constant $[x_p(t) = K]$, ~~substitute into~~ the differential equation, and solve for *K*.

$x_p(t)$

A second method is to replace the inductors by short circuits the capacitances by open circuits and solve for the steady-state dc response.

$\boxed{x_p(t) = K}$

Complementary Solution.

The complementary solution $x_c(t)$ is found by solving the homogeneous equation, which is obtained by substituting 0 for the forcing function *f*(*t*). Thus, the homogeneous equation is

$$\frac{d^2x_c(t)}{dt^2} + 2\alpha\frac{dx_c(t)}{dt} + \omega_0^2 x_c(t) = 0 \qquad (4.67)$$

$\boxed{x = Ke^{st}}$

In finding the solution to the homogeneous equation, we start by substituting the trial solution $x_c(t) = Ke^{st}$. This yields

$\frac{dx}{dt}$

$$s^2Ke^{st} + 2\alpha sKe^{st} + \omega_0^2 Ke^{st} = 0 \qquad (4.68)$$

$\frac{d^2x}{dt^2}$

Factoring, we obtain

$$\left(s^2 + 2\alpha s + \omega_0^2\right)Ke^{st} = 0 \qquad (4.69)$$

Since we want to find a solution Ke^{st} that is nonzero, we must have

$$s^2 + 2\alpha s + \omega_0^2 = 0 \qquad (4.70)$$

This is called the **characteristic equation**.

The **damping ratio** is defined as

$$\zeta = \frac{\alpha}{\omega_0} \qquad (4.71)$$

The form of the complementary solution depends on the value of the damping ratio. The roots of the characteristic equation are given by

$\zeta > 1$

$\zeta = 0$

$\zeta < 1$

S

> The form of the complementary solution depends on the value of the damping ratio.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad (4.72)$$

and

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \qquad (4.73)$$

We have three cases depending on the value of the damping ratio ζ compared with unity.

1. *Overdamped case* $(\zeta > 1)$. If $\zeta > 1$ (or equivalently, if $\alpha > \omega_0$), the roots of the characteristic equation are real and distinct. Then the complementary solution is

$$x_c(t) = K_1e^{s_1t} + K_2e^{s_2t} \qquad (4.74)$$

In this case, we say that the circuit is **overdamped**.

If the damping ratio is greater than unity, we say that the circuit is overdamped, the roots of the characteristic equation are real, and the complementary solution has the form given in **Equation 4.74**.

$\frac{\alpha}{\omega_0} = 1$

2. *Critically damped case* $(\zeta = 1)$. If $\zeta = 1$ (or equivalently, if $\alpha = \omega_0$), the roots are real and equal. Then, the complementary solution is

$$\boxed{x_c(t) = K_1 e^{s_1 t} + K_2 t e^{s_1 t}} \quad (4.75)$$

$K_1 e^{s_1 t} + K_2 t\, e^{s_1 t}$

If the damping ratio equals unity, the circuit is critically damped, the roots of the characteristic equation are real and equal, and the complementary solution has the form given in **Equation 4.75**.

In this case, we say that the circuit is **critically damped**.

3. *Underdamped case* $(\zeta < 1)$. Finally, if $\zeta < 1$ (or equivalently, if $\alpha < \omega_0$), the roots are complex. (By the term *complex*, we mean that the roots involve the imaginary number $\sqrt{-1}$.) In other words, the roots are of the form

$$\boxed{s_1 = -\alpha + j\omega_n} \quad \text{and} \quad \boxed{s_2 = -\alpha - j\omega_n}$$

$\xi < 1$

in which $j = \sqrt{-1}$ and the **natural frequency** is given by

$$\boxed{\omega_n = \sqrt{\omega_0^2 - \alpha^2}} \quad (4.76)$$

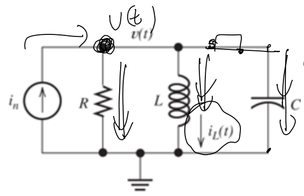
$\omega_n = \sqrt{\omega_0^2 - \alpha^2}$

For complex roots, the complementary solution is of the form

$$\boxed{x_c(t) = K_1 e^{-\alpha t}\cos(\omega_n t) + K_2 e^{-\alpha t}\sin(\omega_n t)}$$

In this case, we say that the circuit is **underdamped**.

$v(t)$



$C\frac{dv_t}{dt}$

α, ω_0, ξ, ζ

ω_n

$\xi = \frac{\alpha}{\omega_0}$

$i_L(t) = x(t)$

ζ

s_1, s_2

ω_n

We can analyze this circuit by writing a KCL equation at the top node of **Figure 4.31(b)** which results in

$$C\frac{dv(t)}{dt} + \frac{1}{R}v(t) + \frac{1}{L}\int_0^t v(t)\,dt + i_L(0) = i_n(t) \quad (4.100)$$

$V = L\frac{di}{dt}$

This can be converted into a pure differential equation by taking the derivative with respect to time:

$$C\frac{d^2v(t)}{dt^2} + \frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) = \frac{di_n(t)}{dt} \quad (4.101)$$

$i = \frac{1}{L}\int_0^t v\,dt + i_L(0)$

Dividing through by the capacitance, we have

$f(t)$

$$\boxed{\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = \frac{1}{C}\frac{di_n(t)}{dt}} \quad (4.102)$$

Now, if we define the damping coefficient

$$\boxed{\alpha = \frac{1}{2RC}} \quad (4.103)$$

$2\alpha = \frac{1}{RC}$

the undamped resonant frequency

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad (4.104)$$

$2\alpha\frac{dv}{dt} = \frac{1}{RC}\frac{dv}{dt}$

and the forcing function

$$\boxed{f(t) = \frac{1}{C}\frac{di_n(t)}{dt}} \quad (4.105)$$

$\omega_0^2\, x(t)$

the differential equation can be written as

$$\frac{d^2v(t)}{dt^2} + 2\alpha\frac{dv(t)}{dt} + \omega_0^2 v(t) = f(t) \quad (4.106)$$

$\omega_0^2\, v(t) = \frac{1}{LC}v(t)$

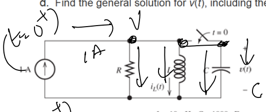
$\omega_0 = \frac{1}{\sqrt{LC}}$

ω_0 = undamped

P4.64. Consider the circuit shown in **Figure P4.64** in which the switch has been open for a long time prior to t=0 and we are given R=25 Ω.

a. Compute the undamped resonant frequency, the damping coefficient, and the damping ratio of the circuit after the switch closes.
b. Assume that the capacitor is initially charged by a 25-V dc source not shown in the figure, so we have v(0+)=25 V. Determine the values of i L (0+) and v′(0+).
c. Find the particular solution for $v(t)$.
d. Find the general solution for $v(t)$, including the numerical values of all parameters.

$(t = 0^+) \rightarrow V$



$L = 10\ \mu H \quad C = 1000\ pF$

$\alpha = \frac{1}{2RC} = 20 \times 10^{6}$

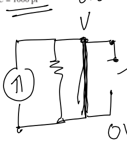
$-C\frac{dv(0^+)}{dt}$

$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \times 10^{6}$

$V(t = 0^+)$

V

$i_L(0^+)$



$\xi = \frac{\alpha}{\omega} = \frac{20 \times 10^6}{10 \times 10^6} = 2$

$V(0^+) = 25$

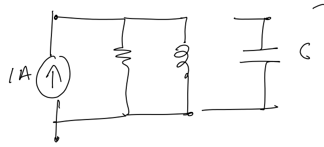
overdamped $\Rightarrow$ $\xi > 1$ $\quad 2 > 1$

$V'(0^+) = \frac{dV_c}{dt}\Big|_{at\ t = 0^+}$

$t = 0^-$

$i_L(0^+) = i_L(0^-)$

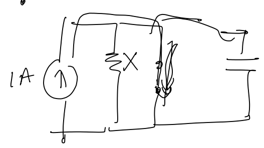
$V_c(0^+) = V_c(0^-)$



$\boxed{i_L(0^-) = 1\ A}$

$t = -\infty \quad -10\,s$

$t = 0^-$



$t = 0$

$t = 0^+$

$\boxed{i_L(0^-) = i_L(0^+) = 1\ A}$

$$\frac{v(0^+)}{R} + i_L(0^+) + C\frac{dv}{dt} = 1$$

$v(0^+) = 25$

$R = 25$, $i_L(0^+) = 1$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = v'(0^+) = -10^9 \text{ V/s}$$

(c) particular steady state

$V_p(t) = 0$

(d) $s_1 = -2.679\times10^6$

$s_2 = -37.32\times10^6$

$$V_c(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

$Vp(t) = 0$

$K_1, K_2]$ $\quad V(t) = V_c(t) + V_p(t)$

$v(0^+) = 0$, $v'(0^+) = -10^9$

$$\boxed{V(t) = -1.934\exp(-2.679\times10^6 t) + 26.93\exp(-37.3\times10^6 t)}$$