1.

(a) $v_1 = 5 - 3 = 2$

$v_2 = -1 + v_1 = 1$

$v_3 = 1 + 3 = 4$

(b) $v_1 = -2 - 3 = -5$

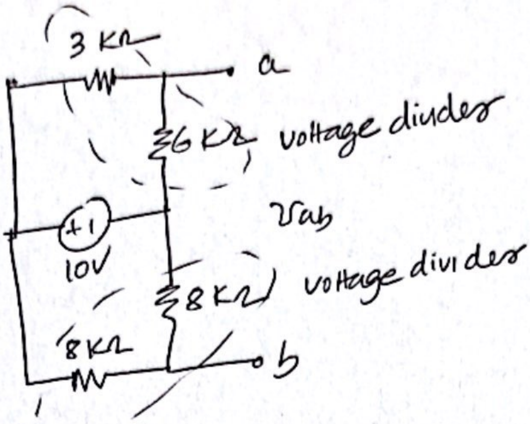
$v_2 = v_1 = -5$

$v_3 = v_2 + 5 = 0$

$i_2 = 2 - 1 - (-1) = 2$

$i_3 = -1$

2.

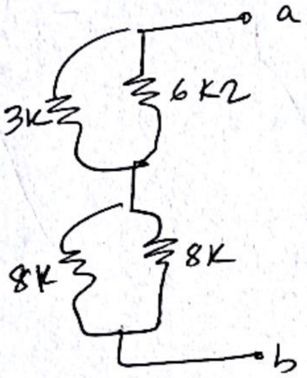


Determining $V_{oc} = V_{ab}$

$$V_{ab} = +10 \times \frac{6}{3+6} - 10 \times \frac{8}{8+8} = \frac{20}{3} - 5$$

$$= +\frac{5}{3} \text{ volts}.$$

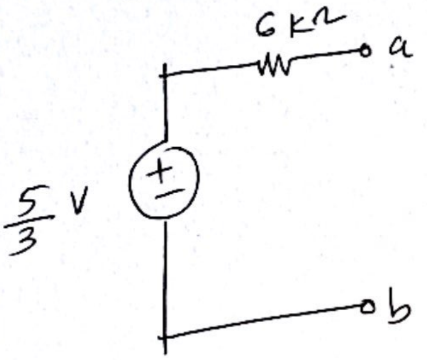
With the 10V source replaced by a short circuit, the circuit reduces to



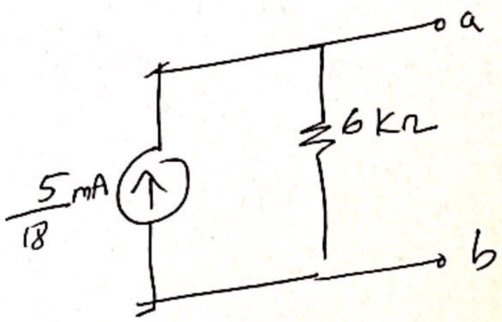
$$R_{th} = 3\|6 + 8\|8$$
$$= 2 + 4 = 6 \text{ K}\Omega$$

$$I_{sc} = \frac{V_{oc}}{R_{th}} = \frac{5/3}{6 \text{ K}} = \frac{5}{18} \text{ mA}$$

Thevenin Equivalent



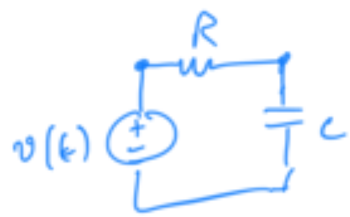
Norton Equivalent



Problem 3

Solution to mid term problem

Subu Mid Term problem



apply KVL

$$\frac{V_c(t) - V(t)}{R} + C\frac{dV_c(t)}{dt} = 0$$

rearranging the eqn. and write $v(t) = t$

$$RC\frac{dV_c(t)}{dt} + V_c(t) = t \quad \text{-----} \; (1)$$

We try a particular solution of the form $V_{cp}(t) = A + Bt$ .... (2)

where A, B are constants to be determined

Substituting (2) in (1) we get

$$RCB + A + Bt = t$$

$$\therefore \; B = 1.$$

$$A = -RC$$

Hence the particular solution is

$$V_{cp}(t) = -RC + t$$

The homogeneous eqn has a solution

$$V_{c_h}(t) = K_1 \exp[-t/RC]$$

[we use the complementary soln. as we know that without the forcing fn the capacitor voltage must decay]

The complete solution is

$$v_c(t) = v_{c_h}(t) + v_{c_p}(t)$$

$$= -RC + t + K_1 \exp(-t/RC)$$

The initial condition at $t=0$ is

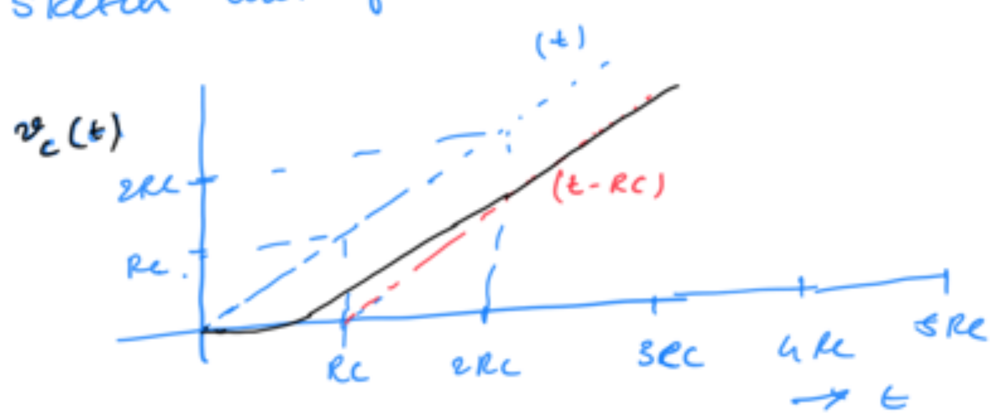
$v_c(0) = 0$ (given)

$\therefore \quad 0 = -RC + K_1 \quad \therefore \quad K_1 = RC$

$$\therefore \quad v_c(t) = v_{c_p}(t) + v_{c_h}(t) = -RC + t + RC\exp[-t/RC]$$

$$= t - RC\left[1 - \exp[-t/RC]\right] \quad \cdots (3)$$

(b) sketch this fn



(c) as can be seen in (b), about 35 time constants is a sufficiently long time (5RC)

at long enough time, the capacitor voltage is

$$v_c(t) \simeq t - RC = 5RC - RC$$

$$= 4RC. = 4 \text{ volts.}$$

(d) the current flowing thru the capacitor must be supplied by the battery

$$i_{c} \stackrel{\text{def}}{=} i_{Battery}(t) = C \frac{dv_c(t)}{dt}$$

$$i_c(t)\Big|_{t=5RC} = C\frac{d}{dt}\left\{ -RC + t + R\exp\left(-{}^{t}/_{RC}\right)\right\} \quad \to 0$$

$$= C\frac{d}{dt}\left[-RC + t\right] = C$$

$$= C$$

$$= 1 \text{ mA.}$$

(e) $P_{batten} = V_{battery} \times i_{batten}$

$$= t \times C\frac{d}{dt}\left[-RC + t + R\exp\left(-{}^{t}/_{RC}\right)\right]$$

$$= \left[C + 1 - t \quad \exp\left(-{}^{t}/_{RC}\right)\right]$$

$= t \times C \left[\ldots \frac{}{C} \ldots \exp(\ldots) \right]$

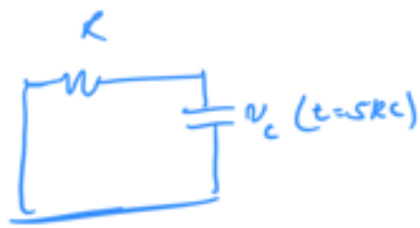
$$= t \times 10^{-3} \left[1 - \frac{t}{10^{-3}} \exp\left(-t \right) \right] \quad \text{--- (4)}$$

(f): the power does two things

(a) heats up the resistor r.
that power is $i_c^2 R$.

(b) Stores energy in the Capacitor
that power is
$P_{batt} - i_c^2 R$.
↳ from em (4)

(g) if the battery runs out the battery is represented by a short cut:



(h) at 10sec
$v_c \simeq t - RC = 10 - 1 = 9$

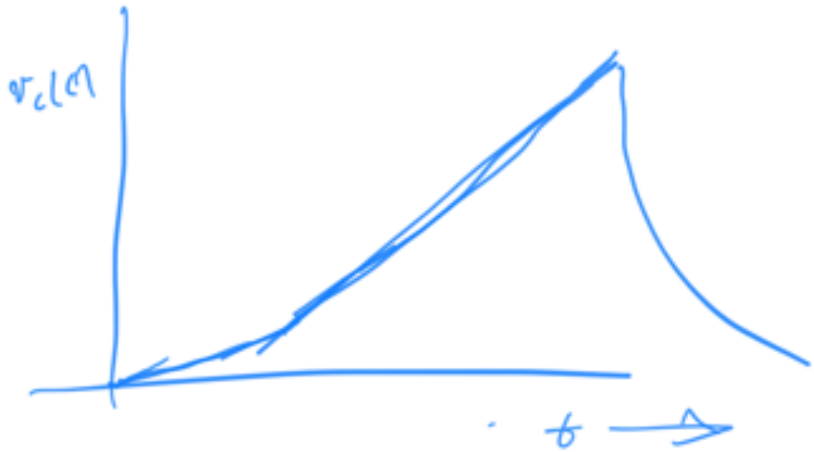
We now have a decay voltage

$$v_c(t) = v_c(10) \cdot e^{-\frac{(t-10)}{RC}}$$

$$\ldots e^{-\frac{(t-10)}{\ldots}}$$

: q e

(c)



Midterm Solutions

Monday, April 26, 2021 10:51 AM

Problem 4

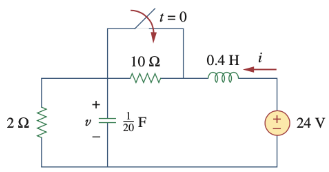
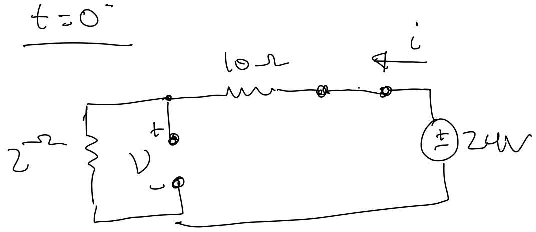


Figure 8.4
For Practice Prob. 8.1.

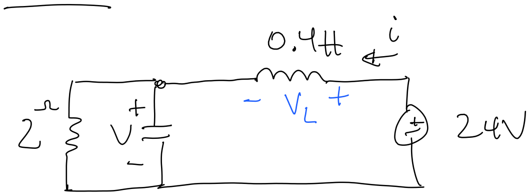
Answer: (a) 2 A, 4 V, (b) 50 A/s, 0 V/s, (c) 12 A, 24 V.

$t = 0^-$



- $i(0^+) = i(0^-) = \dfrac{24V}{(10\Omega + 2\Omega)} = \boxed{2A}$
- $V(0^+) = V(0^-) = \dfrac{2\Omega}{2\Omega + 10\Omega} \times 24V = \boxed{4V}$

$t = 0^+$



$i(0^+) = i(0^-) =$

$V_L = L \dfrac{di}{dt}$

$V(0^+) = V(0^-) = 4V$

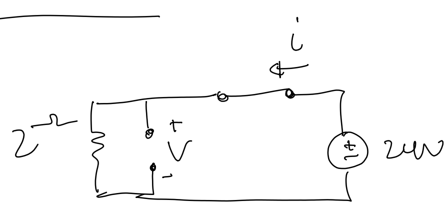
$\Rightarrow V_L(0^+) = 20V \rightarrow \dfrac{di(0^+)}{dt} = \dfrac{1}{L} V_L(0^+)$

$= \left(\dfrac{1}{0.4H}\right) \times 20V$

$\boxed{\dfrac{dV(0^+)}{dt} = 0}$

$\boxed{\dfrac{di(0^+)}{dt} = 50 \dfrac{A}{s}}$

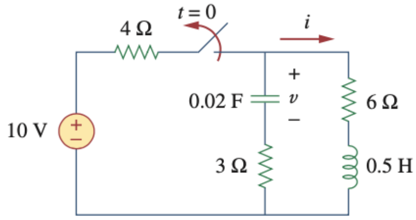
$(t = \infty)$:



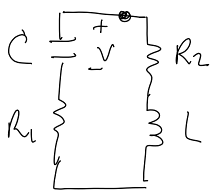
$\boxed{i(\infty) = \dfrac{24V}{2\Omega} = 12A}$

$\boxed{V(\infty) = 24V}$

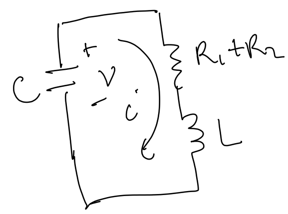
Problem 5



(a) $t > 0$:



OR



$$V = i(R_1+R_2) + L\frac{di(t)}{dt}$$

note: $i = -C\frac{dV}{dt}$

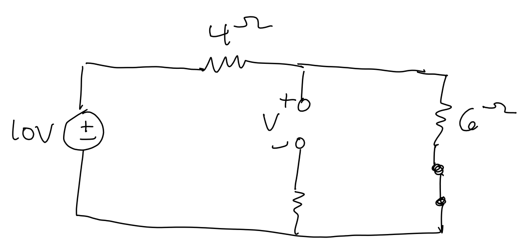
$$\Rightarrow V = -C\frac{dV}{dt}(R_1+R_2) - LC\frac{d^2V(t)}{dt^2}$$

$$\boxed{\frac{d^2V(t)}{dt^2} + \frac{R_1+R_2}{L}\frac{dV(t)}{dt} + \frac{1}{LC}V(t) = 0}$$

$\frac{9}{0.5}$ 100

(b) $V(0^+)$:

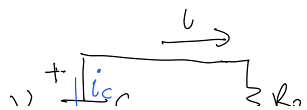
$$V(0^+) = V(0^-) = \frac{6\Omega}{4+6\Omega}10V = \boxed{6V} = \frac{R_2}{R_2+R_3}V_s$$

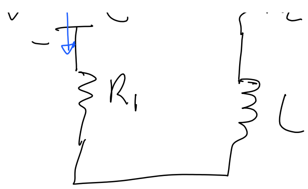


(c) $\frac{dV(0^+)}{dt}$:

note: $i_c = C\frac{dV(0^+)}{dt}$ $\quad i(0^+) = i(0^-) = \frac{10V}{4+6\Omega} = 1A$

note: $i_c = -i$





$$\frac{dV(0^+)}{dt} = -\frac{1}{C}\, i(t=0^+)$$

$$= \frac{-1}{0.02F} \times 1A = \boxed{-50\ \frac{V}{s}}$$

(d) $\underline{t = \infty}$

$$V(t \to \infty) \to 0 \qquad \boxed{V(\infty) \simeq 0V}$$

(e)

$$\zeta = \frac{1}{2} \frac{R_{TOT}}{\sqrt{L/C}} = \frac{1}{2} \frac{(3+6\ \Omega)}{\sqrt{\frac{0.5H}{0.02F}}}$$

$$\boxed{\zeta = 0.9}$$ underdamped

(f)

resonant frequency : $\omega_0 = \frac{1}{\sqrt{LC}} = \boxed{10}\ \frac{rad}{s}$

natural frequency : $\omega_n = \sqrt{1-\zeta^2}\ \omega_0$

$$= 0.436 \times 10 = \boxed{4.36\ \frac{rad}{s}}$$

(g) $\underline{\text{roots:}}$

$$S = -\sigma \pm j\omega_n$$

$$\sigma = \zeta\omega_0 = 0.9 \times 10 = 9\ \frac{rad}{s}$$

$$\boxed{S_1, S_2 = -9 \pm j\,4.36}$$

(h)

$$V(t) = K_1 e^{-\sigma t} \cos(\omega_n t) + K_2 e^{-\sigma t} \sin(\omega_n t) + K_3$$

(i)

(1) $V(\infty) = 0 = K_3 \rightarrow \boxed{K_3 = 0}$

(2) $V(0^+) = 6V$

(3) $\frac{dV(0^+)}{dt} = -50\ \frac{V}{s}$

$\rightarrow (2)\quad V(0^+) = 6V = K_1 + K_2 \cdot 0 = K_1$

$$\Rightarrow \boxed{K_1 = 6V}$$

$$\rightarrow (3)\quad \frac{dV(0^+)}{dt} = -50\frac{V}{s} = \left[-\sigma K_1 e^{-\sigma t}\cos(\omega_n t) - K_1 e^{-\sigma t}\left(\omega_n \sin(\omega_n t)\right)\right]\Big|_{t=0}$$

$$+ \left[-\sigma K_2 e^{-\sigma t}\sin(\omega_n t) + K_2 e^{-\sigma t}\left(\omega_n \cdot \cos(\omega_n t)\right)\right]\Big|_{t=0}$$

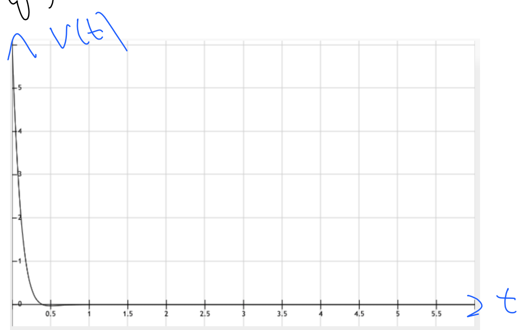
$$-50\frac{V}{s} = -\sigma K_1 + K_2\omega_n$$

$$\rightarrow \quad K_2 = \frac{-50 + \sigma K_1}{\omega_n} = \frac{-50 + (9)(6)}{4.36} = $$

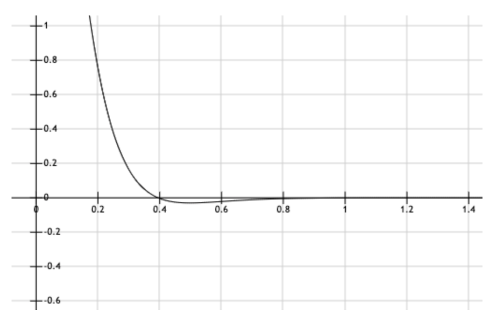
$$= \frac{4}{4.36} \approx \boxed{0.92}$$

$$\Rightarrow \boxed{\begin{array}{l} v(t) \cong 6e^{-\sigma t}\cos(\omega_n t) + 0.92e^{-\sigma t}\sin(\omega_n t) \\ \sigma = 9 \\ \omega_n = 4.36 \end{array}}$$

(j)



Zoomed-in:



minor overshoot