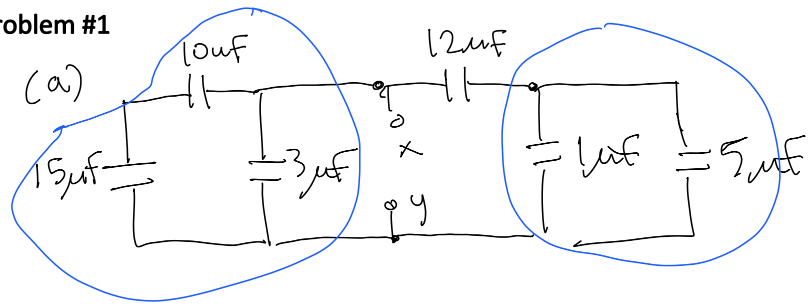
Homework #3 Solutions

Sunday, April 11, 2021   10:11 PM

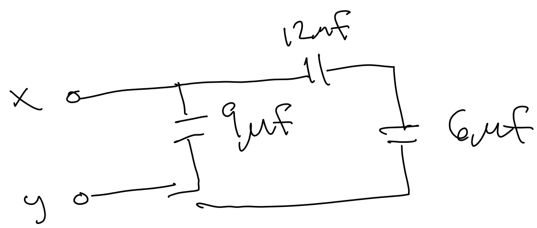
Problem #1

(a)



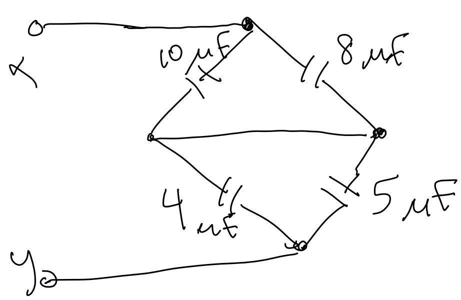
① $\frac{10 \times 15}{10+15} \parallel 3 \equiv \frac{150}{25} + 3 = 9\mu f$

② $1 \parallel 5 \equiv 6\mu f$

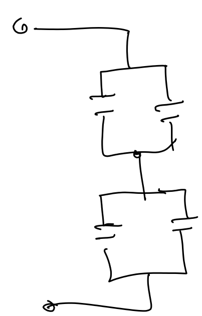


$$C_{eq} \equiv 9\mu f + \frac{6 \times 12}{6+12} = \boxed{13\mu f}$$

(b)

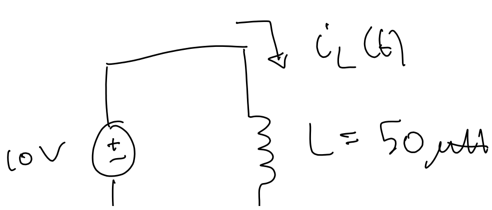


$\equiv$



$$\boxed{C_{eq} \equiv 18 \parallel 9 = 6\mu f}$$

Problem #2



$i_L(t=0) = -100mA$

$V_L(t) = 50\mu H \times \frac{di_L(t)}{dt} = 10V$

$$\rightarrow \frac{di_L}{dt} = 2\times 10^5 \ \frac{A}{s}$$

(a) $i_L(t = t_x) = +100mA$

$$\frac{\Delta i}{\Delta t} = \frac{+100mA - (-100mA)}{\Delta t} = 2\times 10^5 \ \frac{A}{s}$$

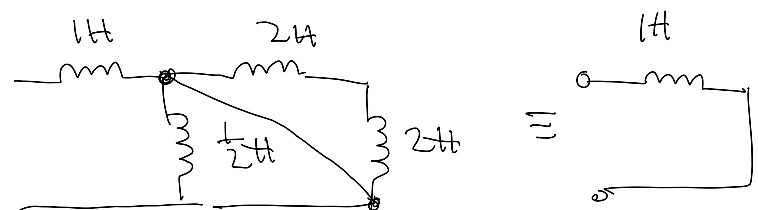
$$\rightarrow \boxed{t_x = \Delta t = 1\mu s}$$

(b) why is this circuit unrealistic?

Because if $\frac{di_L}{dt}$ is always > 0, then it implies that $i_L(t) \rightarrow \infty$ (as $t \rightarrow \infty$). The circuit would stop working well before that point as it would either (1) burn up or (2) parasitic resistances would limit the current that can flow through the inductor ("resistance-limited circuit")

Problem #3

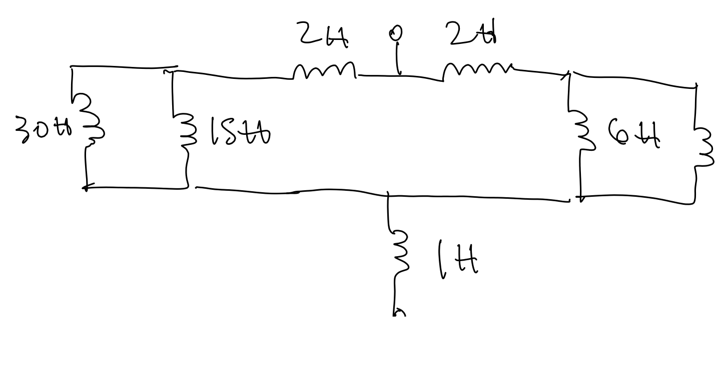
(a)



$\boxed{L_{eq} = 1H}$

* $\frac{1}{2}$H and 4H (2+2) inductors have no effect as they are in parallel w/ a short circuit

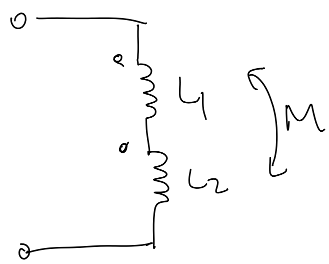
(b)



$$L_{eq} = 1H + [(30 \| 15) + 2] \| [(6 \| 3) + 2]$$
$$= 1H + 12H \| 4H = 1H + 3H = \boxed{4H}$$

Problem #4

(a)

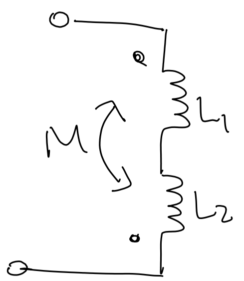


$$v(t) = v_1(t) + v_2(t)$$
$$v_1(t) = L_1 \frac{di}{dt} + M \frac{di}{dt}$$
$$v_2(t) = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$\boxed{v(t) = [L_1 + L_2 + 2M] \frac{di}{dt}}$$

$$\boxed{\therefore L_{eq} = L_1 + L_2 + 2M}$$

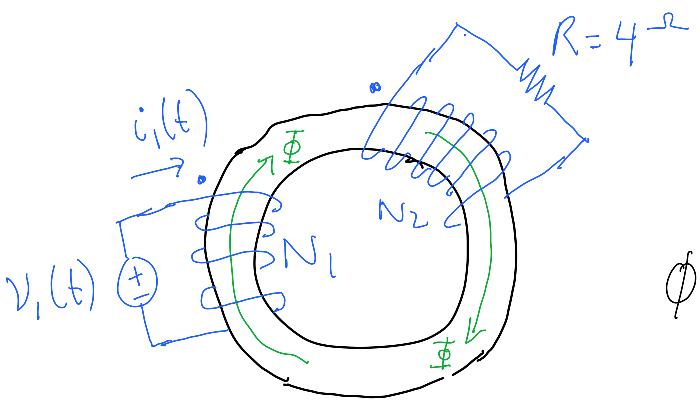
(b)



$$v_1(t) = L_1 \frac{di}{dt} - M \frac{di}{dt}$$
$$v_2(t) = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$\rightarrow \boxed{L_{eq} = L_1 + L_2 - 2M}$$

Problem #5



$$\phi = \phi_0 \sin(\omega t)$$

(a) $\underline{v_1(t)}$

$$V_1(t) = N_1 \frac{d\phi}{dt} = \boxed{N_1 \phi_0 \omega \cos(\omega t)}$$

(b) $\underline{i_1(t)}$

$$i_2(t) = \frac{V_2(t)}{4\Omega} = \frac{1}{4\Omega} \times N_2 \phi_0 \omega \cos(\omega t)$$

$$i_2 = \frac{1}{n} i_1 = \frac{N_1}{N_2} i_1 \rightarrow i_1(t) = \frac{N_2}{N_1} i_2(t)$$

$$i_1(t) = \frac{N_2}{N_1} \times \frac{1}{4\Omega} \times N_2 \phi_0 \omega \cos(\omega t)$$

$$\boxed{i_1(t) = \frac{N_2^2}{N_1} \times \frac{1}{4\Omega} \times \phi_0 \omega \cos(\omega t)}$$

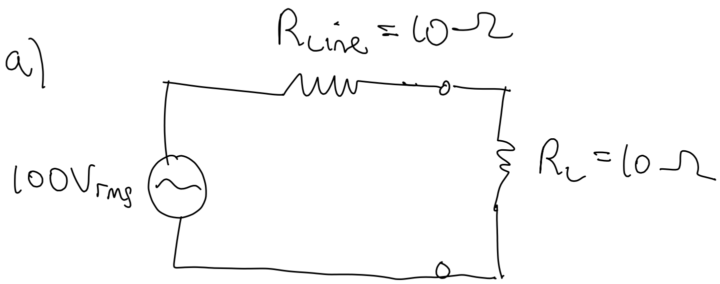
(c) $\underline{V_1(t) / i_1(t):}$

$$\frac{V_1(t)}{i_1(t)} = \frac{N_1 \cancel{\phi_0 \omega} \cancel{\cos(\omega t)}}{\left(\frac{N_2^2}{N_1} \times \frac{1}{4\Omega} \times \cancel{\phi_0} \omega \cancel{\cos(\omega t)}\right)}$$

$$\boxed{\frac{V_1(t)}{i_1(t)} = \frac{N_1^2}{N_2^2} \times 4\Omega}$$

Problem #6

(a)

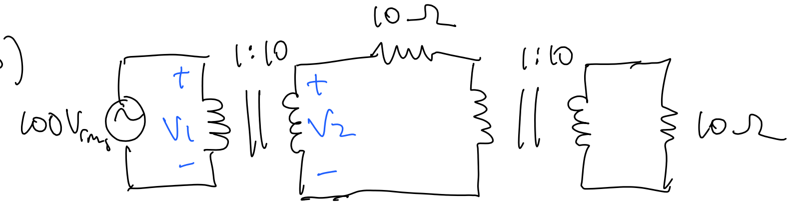


$$I = 5 A_{rms}$$

- $P_{delivered} = 100 V_{rms} \times 5 A_{rms} = \underline{500 W}$ (average)
- $P_{dissipated\text{-}line} = I^2 R = (5A)^2 \times 10\Omega = \underline{250 W}$
- $P_{dissipated\text{-}load} = I^2 R = (5A)^2 \times 10\Omega = \underline{\underline{250 W}}$

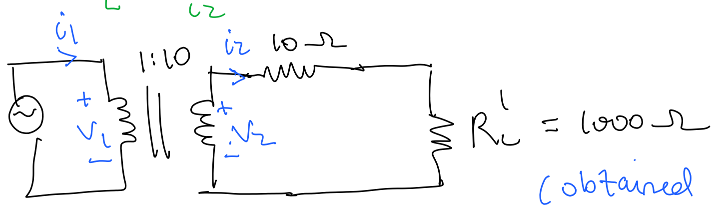
- Efficiency $= \dfrac{P_{Load}}{P_{delivered}} = \dfrac{250\,W}{500\,W} = \boxed{50\%}$

(b)



$$\left.\begin{array}{ll} (1) & V_1 = 10 V_2 \\ (2) & i_1 = \frac{1}{10} i_2 \\ (3) & R_L = \frac{V_2}{i_2} = 10\,\Omega \end{array}\right\} \quad \frac{V_1}{i_1} = \frac{10 V_2}{\left(\frac{1}{10} i_2\right)} = 100 \times \frac{V_2}{i_2} = 100 \times R_L$$

$$= \underline{\underline{1000\,\Omega}}$$

→ 

(obtained by "reflecting")

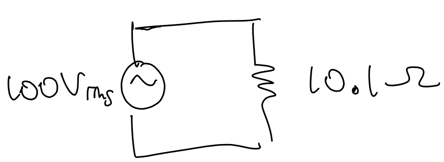
- reflect again:

(1) $V_1 = \frac{1}{10} V_2$

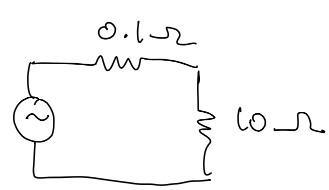
(2) $i_1 = 10\, i_2$

(3) $\frac{V_2}{i_2} = (10 + 1000)\,\Omega = \underline{\underline{1010\,\Omega}}$

$$\Rightarrow \quad \frac{V_1}{i_1} = \frac{1}{100} \times 1010\,\Omega = \underline{\underline{10.1\,\Omega}}$$



OR



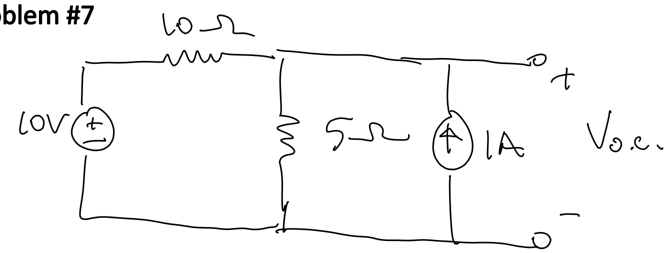
- $P_{delivered} = \dfrac{(100\,V_{rms})^2}{10.1\,\Omega} \simeq \underline{\underline{990.1\,W}}$

- $P_{dissipated\text{-}line} = I^2 R = \left(\dfrac{100\,V_{rms}}{10.1\,\Omega}\right)^2 \times 0.1\,\Omega \approx \underline{\underline{9.8\,W}}$

- $P_{dissipated\text{-}load} = I^2 R \approx \underline{980.3\,W}$

- Efficiency $= \dfrac{P_{load}}{P_{delivered}} = \dfrac{980.3\,W}{990.1\,W} \simeq \boxed{99\%}$

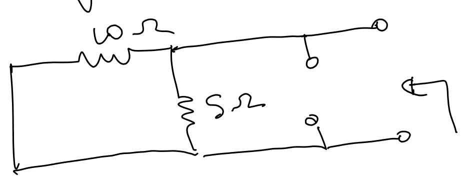
Problem #7



- by KCL: $1A = \dfrac{V_{o.c.}}{5\Omega} + \dfrac{V_{o.c.} - 10V}{10\Omega}$

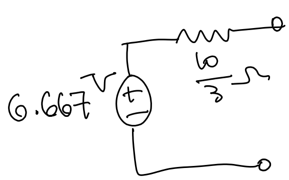
$V_{o.c} = 6.667\,V$

- zeroing sources:

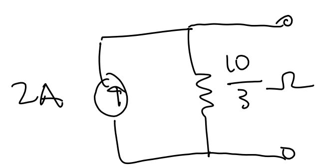


$R_{TH} = 10 \| 5 = \dfrac{10}{3}\,\Omega$

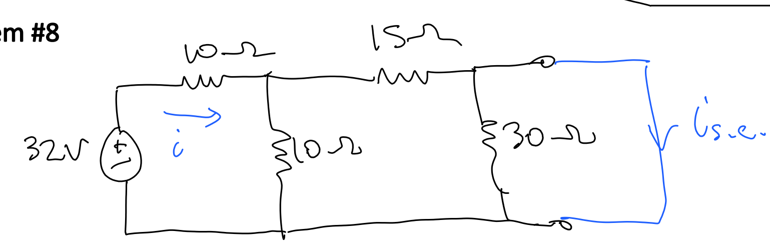
① Thevenin

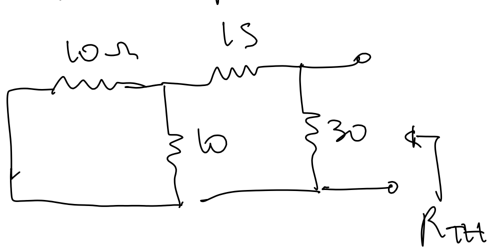


② Norton



Problem #8





$R_{TH} = 30 \| (15 + 10\|10)$

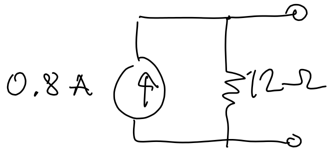
$= 30 \| 20 = 12\,\Omega$

- $i_{s.c.}$: $i = \dfrac{32V}{[10 + 10\|15]} = \dfrac{32V}{16\Omega} = 2A$

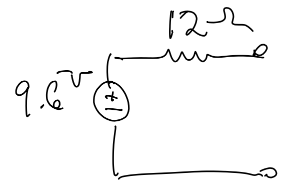
$[10 + w \| 10]$

$$i_{s.c.} = \frac{10}{10+15} 2A = \boxed{\frac{4}{5} A}$$

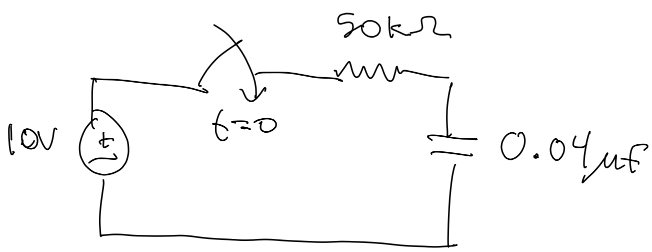
norton



thevenin



Problem #9



$* \; V_c(0^+) = -10V$

$V_c[\infty] = 10V$

$$V_c(t) = V[\infty] - (V[\infty] - V[0])\, e^{-t/RC}$$

$$= 10V - (10 - (-10V))\, e^{-t/RC}$$

$$\boxed{V_c(t) = 10V - (20V) e^{-t/RC}}$$

$$V_c(t_0) = 10 - 20\, e^{-t_0/RC} = 0V$$

$$e^{-t_0/RC} = \frac{1}{2}$$

$$\longrightarrow \quad t_0 = -RC \times \ln\left(\frac{1}{2}\right) =$$

$$= -(50k\Omega)(0.04\mu F) \ln\left(\frac{1}{2}\right)$$

$$\approx \boxed{1.386ms}$$