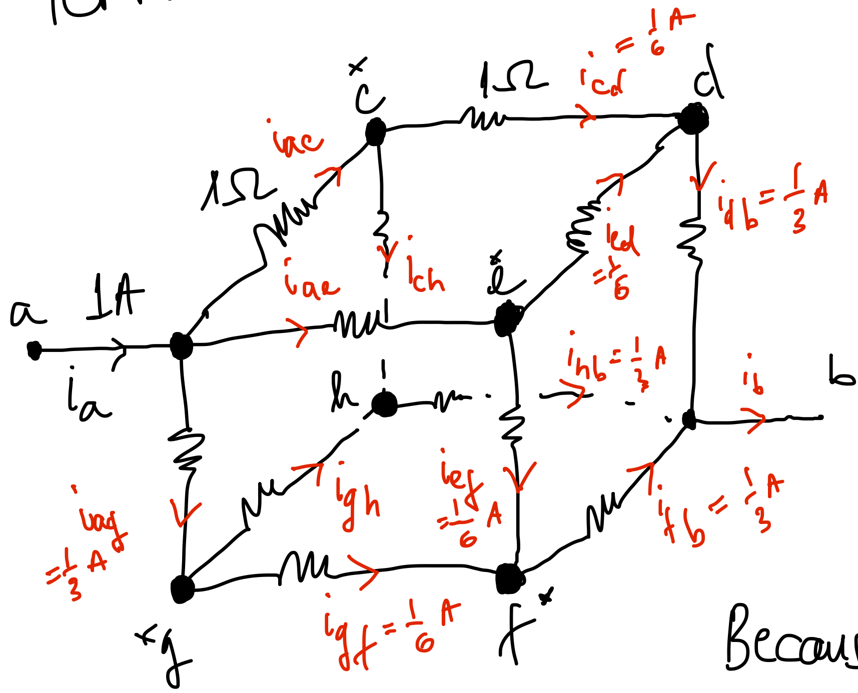
Home Work 2:

Nhat Ho
10535311

1) Find the ΣR between the terminal a & b.



Because we assume there has $i = 1A$ go through $a \to b$, $i_a = 1A$.

Apply KCL at a, we have:

$i_a = i_{ac} + i_{ae} + i_{ag}$

Because all resistor has the same value 1 R and use symmetric consideration,

$i_{ac} = i_{ae} = i_{ag} = \frac{i_a}{3} = \frac{1}{3}(A)$.

Based on the symmetric consideration,

+ apply KCL at c, we have:

$i_{ac} = i_{cd} + i_{ch} = \frac{1}{3}$ & $i_{cd} = i_{ch} \Rightarrow i_{cd} = i_{ch} = \frac{i_{ac}}{2} = \frac{1}{6}A$

+ apply KCL at g, also have $i_{gh} = i_{gf} = \frac{i_{ag}}{2} = \frac{1}{6}(A)$

+ apply KCL at e, also have $i_{ed} = i_{ef} = \frac{i_{ae}}{2} = \frac{1}{6}(A)$

+ apply KCL at f, $i_{fb} = i_{ef} + i_{gf} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}(A)$

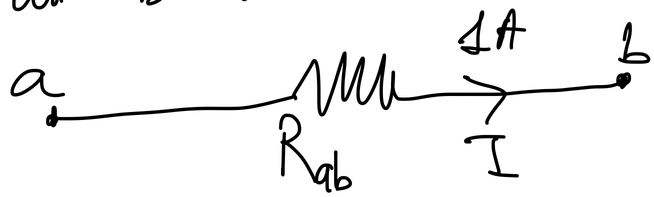
+ Apply KCL at d, $i_{db} = i_{cd} + i_{ed} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}(A)$

+ Apply KCL at h, $i_{hb} = i_{ch} + i_{gh} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}(A)$

+ Apply KCL at b, $i_b = i_{db} + i_{hb} + i_{fb} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\,(A)$

(matching with the assumption)

We can represent the original circuit to the equivalent circuit below:



$\Rightarrow R_{ab} = \frac{V_{ab}}{I} = \frac{V_{ab}}{1}$ (1)

$= V_{ab}\ (R)$

$\Rightarrow$ the voltage between a&b is equal to the unknown resistance. Then apply KVL from $a \to g \to f \to b$, we have:

$V_a - V_b = i_{ag}R + i_{gf}R + i_{fb}R = R(i_{ag} + i_{gf} + i_{fb})$

$\Rightarrow V_{ab} = \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right)A \cdot 1R = \left(\frac{2}{3} + \frac{1}{6}\right)V$

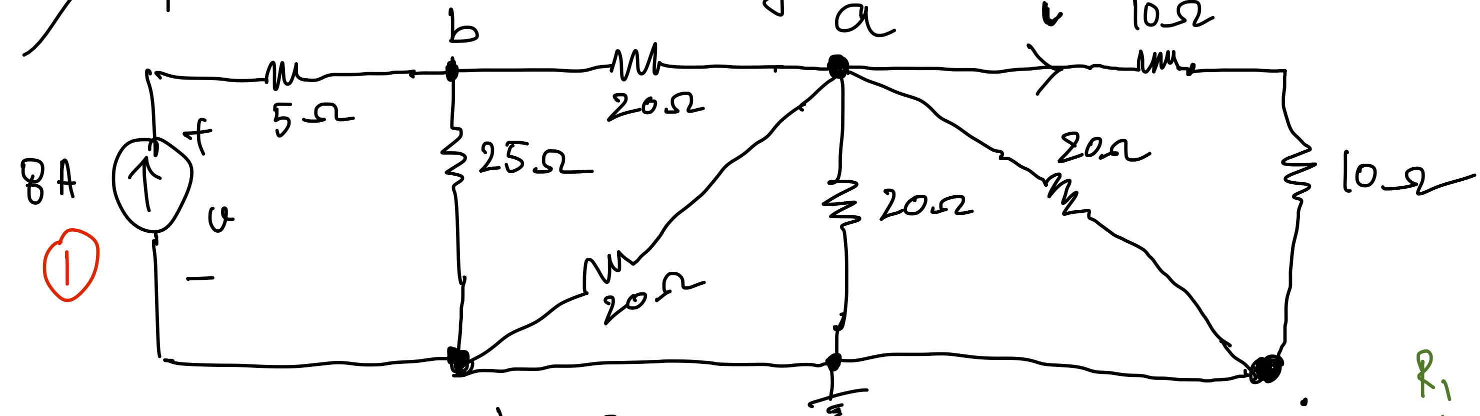
$\Rightarrow \boxed{V_{ab} = \frac{5}{6}V}$. From (1), we have

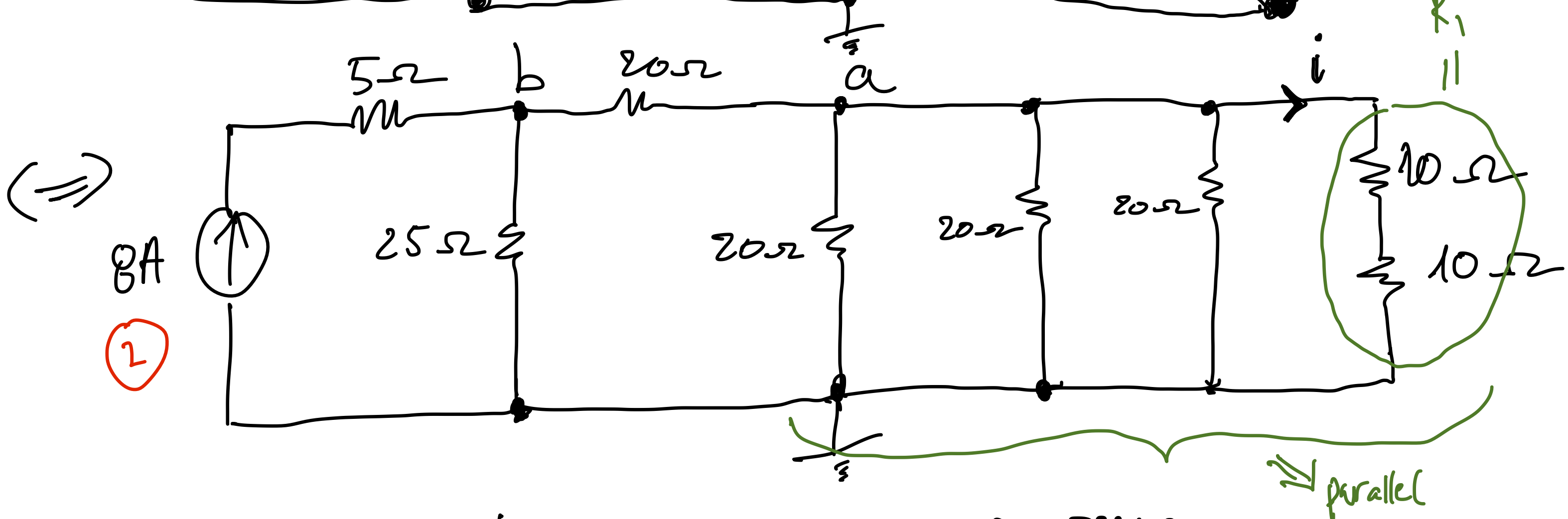
$R_{ab} = \frac{V_{ab}}{1\Omega} = \frac{5}{6}(\Omega)$

$\Rightarrow$ the resistance between a&b:

$\boxed{R = \frac{5}{6}(\Omega)}$

2) Find the values of v & i





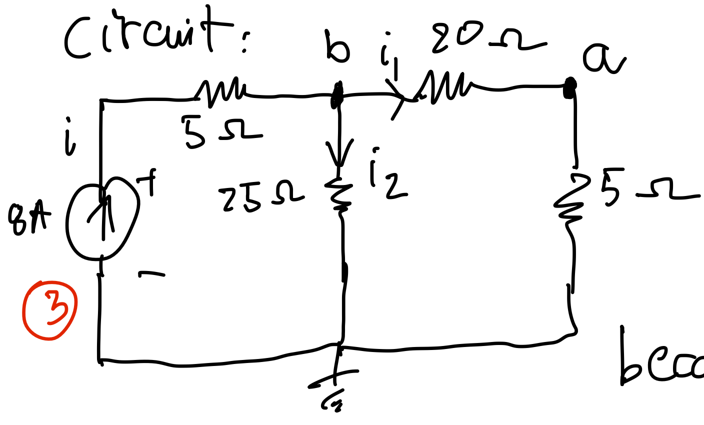
As we can see, two resistor 10Ω are series

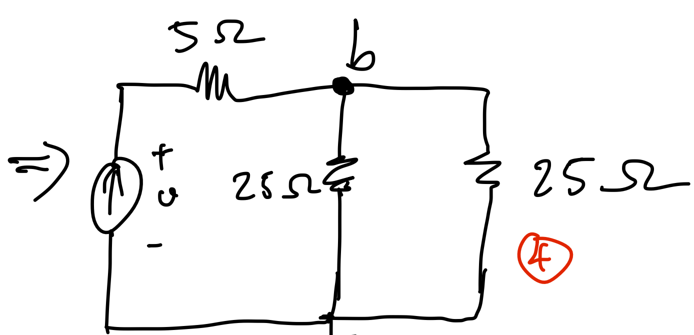
$\Rightarrow R_1 = 10 + 10 = 20 (\Omega)$

Then the last 3 resistor 20Ω are parallel and also parallel with $R_1 = 20\Omega$

$\Rightarrow$ <u>We have</u>: $\frac{1}{R_2} = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{4}{20}$

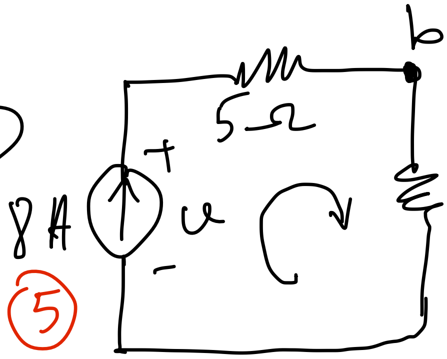
$\Rightarrow R_2 = \frac{20}{4} = 5 (\Omega)$, we can have the new reduced circuit:





because 20Ω series with 5Ω

$\Rightarrow$



$R = \frac{25.25}{25+25} = 12.5\,\Omega$

besause 25Ω parallel with 25Ω

Based on circuit (5), apply KVL, We have:

$-v + iR = 0 \Rightarrow v = iR = 8A \times (5 + 12.5)\Omega$

$\Rightarrow v = 8 \times 17.5 = \boxed{140V}$

Also, $-v + i.5\Omega + v_b \Rightarrow v_b = v - 5.i$

$\Rightarrow v_b = 140V - 5 \times 8 = 100\,(V)$

Apply KCL at b for circuit (3), we have:

$i = i_1 + i_2 \Rightarrow i = \frac{V_b - V_a}{20\Omega} + \frac{V_b - 0}{25\Omega}$

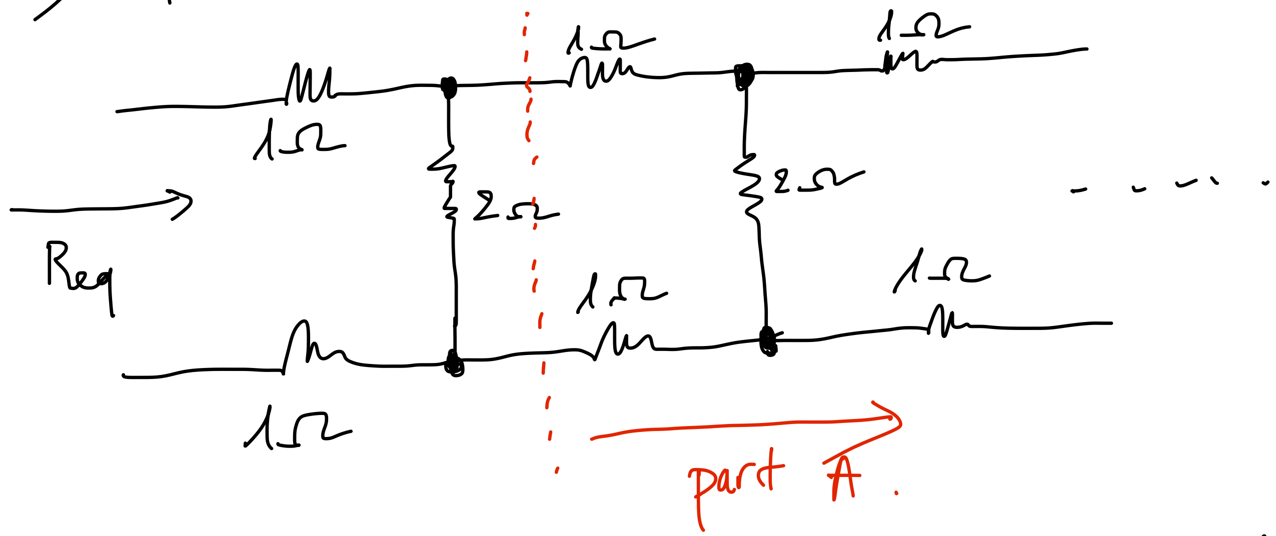
$\Rightarrow 8 = \frac{100 - V_a}{20} + \frac{100V}{25\Omega} = \frac{100 - V_a}{20} + 4$

$\Rightarrow \frac{100 - V_a}{20} = 8 - 4 = 4 \Rightarrow V_a = 100 - 80 = 20(V)$

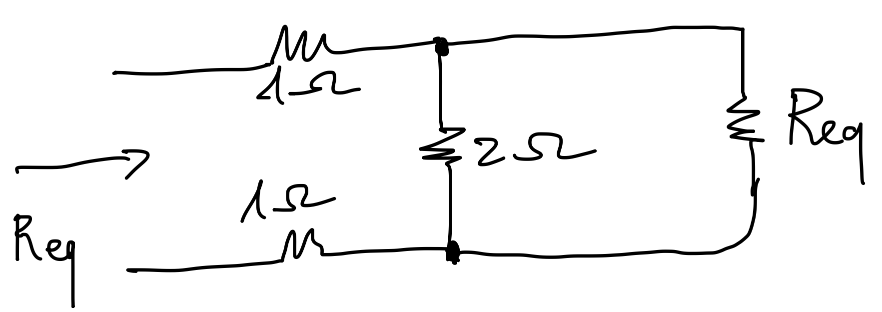
Go back to the circuit (2), we have:

$\frac{V_a}{10\Omega + 10\Omega} = i \Rightarrow i = \frac{20V}{20\Omega} = \boxed{1A}$

3) Find the equivalent resistance:



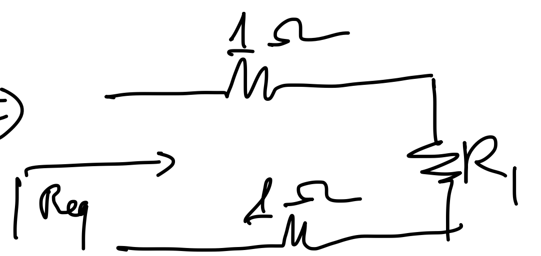
We divide the original circuit as the above picture. Called part A is the circuit has n resistors which has value 2 Ω. ⇒ the orignial circuit has (n+1) resistors 2 Ω. Because the original circuit is the infinite network ⇒ $n \to \infty$ means $n \approx n+1$. In the other words, the total resistance of part A ≈ Req. Based on this idea we can redraw the circuit:



Called $R_1 = (2\Omega \,//\, Req)$

$$\Rightarrow R_1 = \frac{2\,Req}{2+Req} \quad (1)$$

⇔



$$\Rightarrow Req = 1\Omega + 1\Omega + R_1$$

$$\Rightarrow Req = 2 + \frac{2\,Req}{2+Req} = \frac{2(2+Req)+2\,Req}{2+Req} = \frac{4Req+4}{2+Req}$$

For easier calculation, called $R_{eq} = x$ $(x > 0)$

$$\Rightarrow x = \frac{4x+4}{2+x} \Rightarrow x^2 + 2x = 4x + 4$$

$$\Rightarrow x^2 - 2x - 4 = 0 \Leftrightarrow \left[\begin{array}{l} x = \dfrac{2+\sqrt{4+16}}{2} \\ x = \dfrac{2-\sqrt{4+16}}{2} \end{array}\right.$$

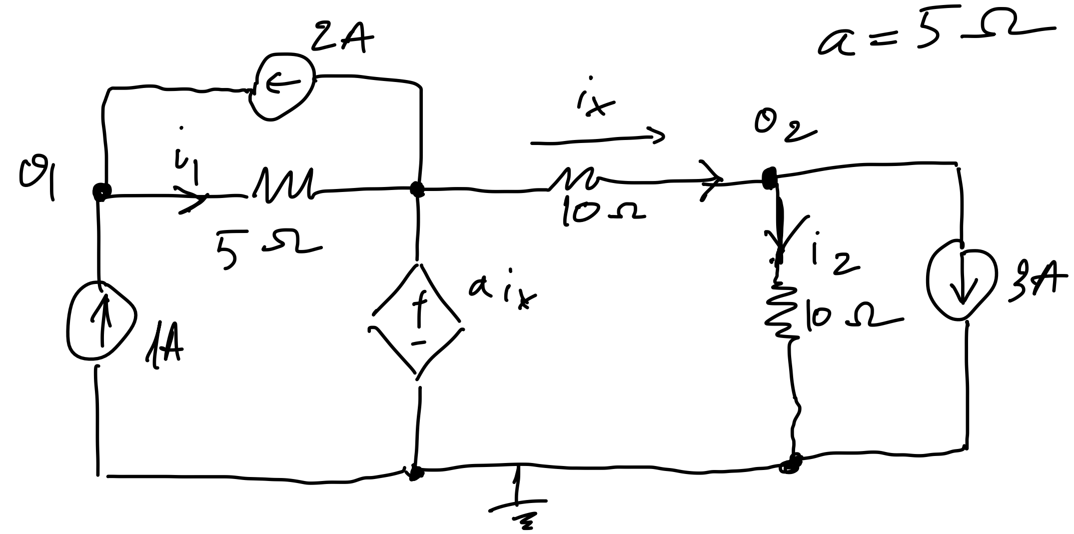
$$\Leftrightarrow \left[\begin{array}{l} x = \dfrac{2+\sqrt{20}}{2} = \dfrac{2+2\sqrt{5}}{2} = 1+\sqrt{5} \\ x = \dfrac{2-\sqrt{20}}{2} = \dfrac{2-2\sqrt{5}}{2} = 1-\sqrt{5} < 0 \Rightarrow \text{rejection} \end{array}\right.$$

(value of resistor can not be negative)

$$\Rightarrow R_{eq} = x = \boxed{1+\sqrt{5} \simeq 3.236\,(\Omega)}$$

4) Solve for the node voltage v_1 & v_2.

$a = 5\,\Omega$



Apply KCL at v_1, we have: $i_1 = 2 + 1 = 3A$.

$$\Leftrightarrow \frac{v_1 - a i_x}{5\,\Omega} = 3A \Rightarrow v_1 - a i_x = 15\ (V)$$

$$\Rightarrow v_1 - 5 i_x = 15 \quad (1)$$

Apply KCL at v_2, we have: $i_x = i_2 + 3A$.

$$\Leftrightarrow \frac{a i_x - v_2}{10\,\Omega} = \frac{v_2 - 0}{10\,\Omega} + 3A$$

$$\Rightarrow 5 i_x - v_2 = v_2 + 30V \Rightarrow 5 i_x - 2 v_2 = 30\ (V) \quad (2)$$

Also: $\frac{a i_x - v_2}{10} = i_x \Rightarrow 5 i_x - v_2 = 10 i_x$ (Ohm's Law)

$$\Rightarrow v_2 = -5 i_x \quad (3)$$

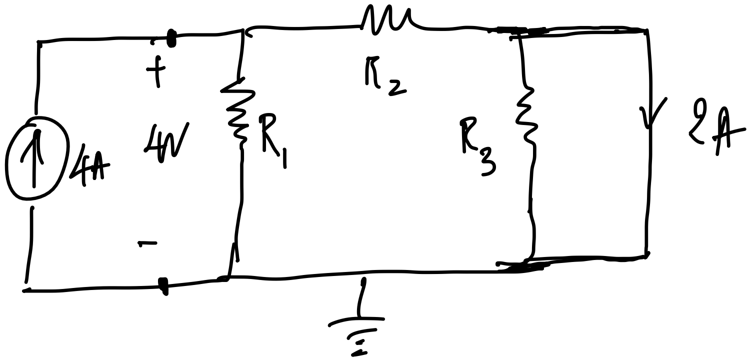
Plug (3) into (2), we have: $-v_2 - 2 v_2 = 30V$

$$\Rightarrow \boxed{v_2 = -10V} \Rightarrow i_x = \frac{v_2}{-5} = \frac{-10}{-5} = 2(A)$$

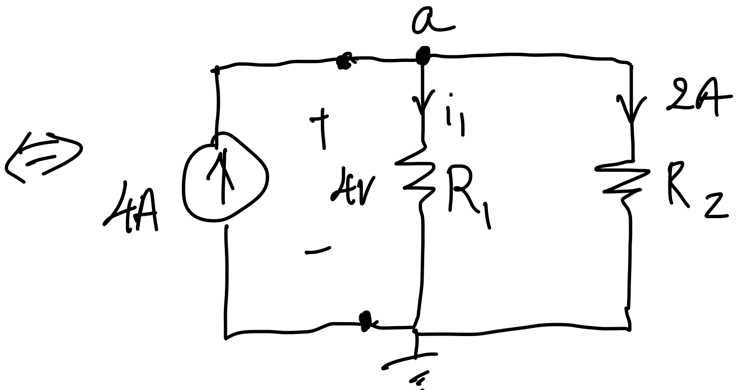
Plug i_x into (1) $\Rightarrow v_1 = 15 + 5 i_x = 15 + 10 = \boxed{25\ (V)}$

5) Find R_1, R_2, R_3.

* First experiment:



Because R_3 is shorted, so there is no current go throught it, we can have a new circuit.



Apply KCL at a, we have:

$4 = i_1 + 2A$

$\Rightarrow i_1 = 2A$.

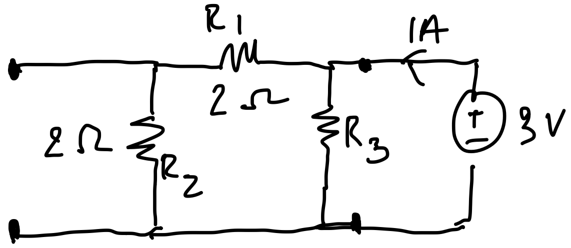
Also, $i_1 R_1 = v_a - 0 = 4V$ (Ohm's Law)

$\Rightarrow 2R_1 = 4V \Rightarrow R_1 = \frac{4}{2} = 2(\Omega)$

Beside: $V_a - 0 = 2A \cdot R_2 \Rightarrow 2R_2 = 4 \Rightarrow R_2 = 2(\Omega)$ (Ohm's Law)

$$\Rightarrow \boxed{\begin{array}{l} R_1 = 2(\Omega) \\ R_2 = 2(\Omega) \end{array}}$$

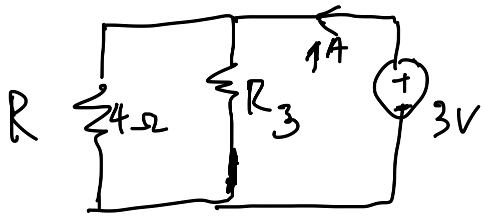
* Second Experiment:



In this case, we have R_1 series to R_2, called $R = R_1 + R_2$

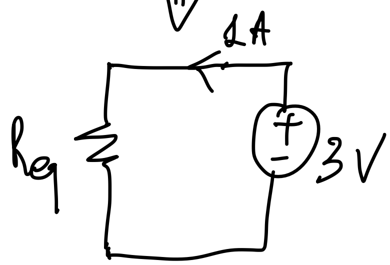
$\Rightarrow R = 4\Omega$.

So we can have the new circuit.: Let $R_{eq} = (R \parallel R_3)$



$$\Rightarrow (R \parallel R_3) \Rightarrow R_{eq} = \frac{R \cdot R_3}{R + R_3} = \frac{4R_3}{4 + R_3} \quad (*)$$

<u>Apply Ohm's Law:</u>



$$\Rightarrow R_{eq} \cdot I = V$$

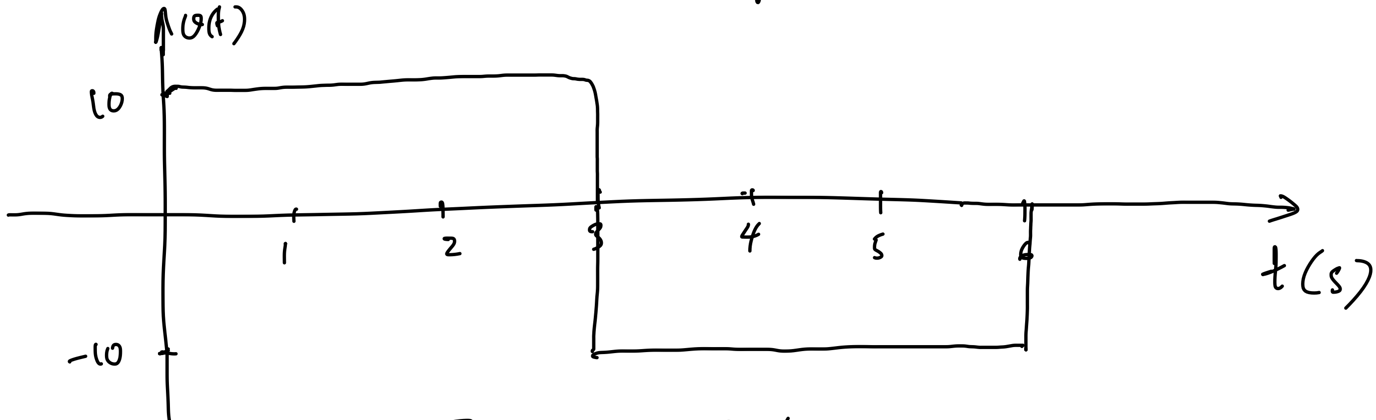
$$\Rightarrow R_{eq} \cdot 1A = 3V \Rightarrow R_{eq} = 3\,\Omega$$

$\Rightarrow$ Plug $R_{eq} = 3\,\Omega$ into $(*)$, we have:

$$\frac{4R_3}{4 + R_3} = 3\,\Omega \Rightarrow 4R_3 = 3(4 + R_3)$$

$$\Rightarrow 4R_3 = 12 + 3R_3 \Rightarrow \boxed{R_3 = 12\,(\Omega)}$$

⑥ Given $L = 2H$, $i_L(0) = 0$. Need to sketch the current, power, & stored energy to scale versus time.



Based on the Figure, we have:

$v(t) = 10V$, $t = 0 \to 3s$

$v(t) = -10V$, $t = 3 \to 6s$

Also, $i_L(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i_L(t_0) = \frac{1}{2}\int_{t_0}^{t} v(t)\,dt + i_L(t_0)$

* For $t = 0 \to 3s$, $v(t) = 10V$, and $i_L(0) = 0$

$\Rightarrow i_L(t) = \frac{1}{2}\int_0^t 10\,dt = 5\int_0^t dt = 5t$ (A)

* For $t = 3 \to 6s$, $v(t) = -10V$, and at $t = 3s$, $i_L(3s) = 5 \times 3 = 15A$

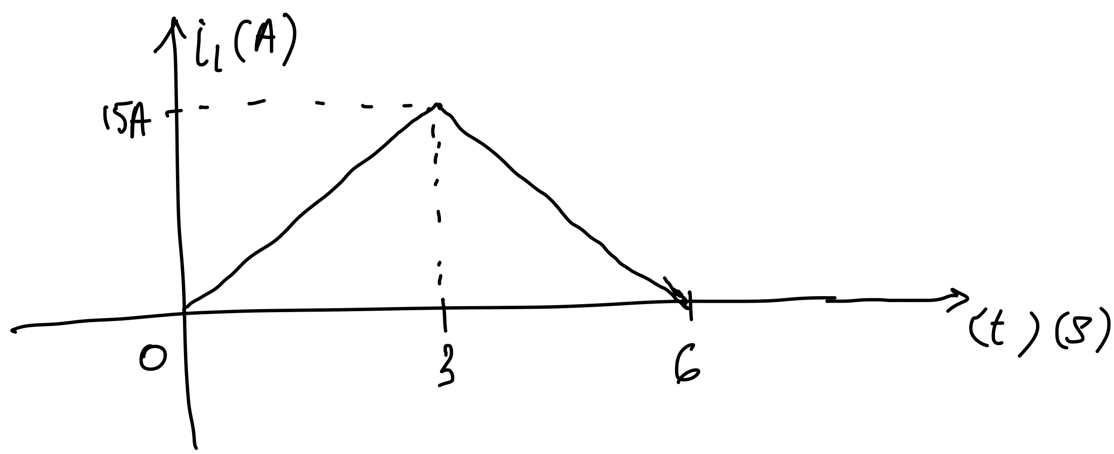
$\Rightarrow i_L(t) = \frac{1}{2}\int_3^t (-10)\,dt + 15$ (A)

$\Rightarrow i_L(t) = -5\int_3^t dt + 15 = -5(t-3) + 15 = -5t + 30$ (A)

$\Rightarrow$ We have:

$$i_L(t) = \begin{cases} 5t\ (A), & t = 0 \to 3s \\ -5t + 30\ (A), & t = 3s \to 6s \end{cases}$$

$\Rightarrow$ <u>the figure of current:</u>



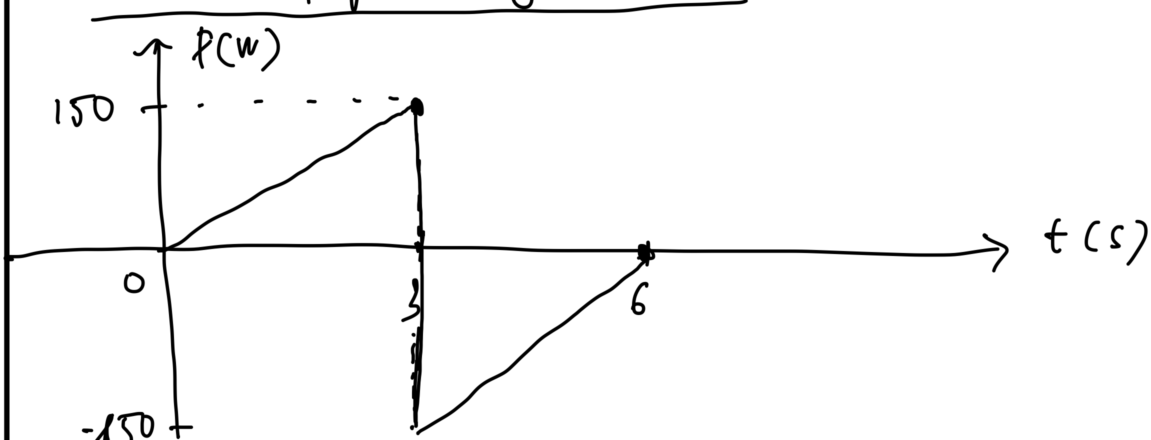
Also, $p(t) = v(t)\, i(t)$

* For $t = 0 \to 3s$, $\begin{cases} v(t) = 10\ (V) \\ i(t) = 5t\ (A) \end{cases} \Rightarrow P(t) = 50t\ (W)$

* For $t = 3s \to 6s$, $\begin{cases} v(t) = -10V \\ i(t) = -5t + 30\ (A) \end{cases}$

$\Rightarrow P(t) = -10(-5t + 30) = 50t - 300\ (W)$

$\Rightarrow$ <u>the figure of power:</u>



We have: $W_{(t)} = \frac{1}{2} L i_L^2(t) = \frac{1}{2} \times 2\, i(t)^2 = i_L^2(t)$

Since:

$$i_L(t) = \begin{cases} 5t\ (A), & t = 0 \to 3s \\ -5t + 30\ (A), & t = 3s \to 6s \end{cases}$$

$\Rightarrow$ • For $t = 0 \to t = 3s$:

$$W(t) = i_L^2(t) = (5t)^2 = 25t^2\ (J)$$

• For $t = 3s \to 6s$,

$$W(t) = i_L^2(t) = (-5t + 30)^2 = 25t^2 - 300t + 900\ (J)$$

Then the figure of stored energy: