Discussion 1B (Week 4)

Thursday, April 22, 2021 8:59 AM

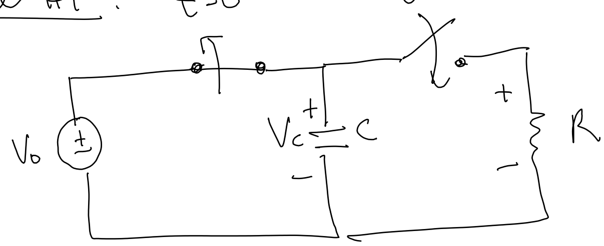
Today:

RC circuits

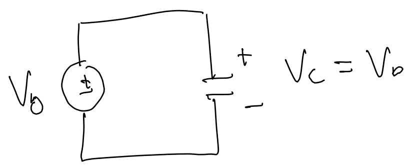
RL circuits

steady-state analysis

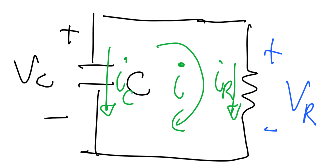
Example #1:



$t < 0$:



$t \geq 0$:



$V_c = V_R$

$V_c(t = 0^-) = V_0$

$V_c(t = 0^+) = V_0$

$$V_c(t) = V_0 e^{-t/RC}$$

KCL:

$$i_c = -i_R \quad \leftarrow \quad \begin{cases} i_c = -i \\ i_R = i \end{cases}$$

$$i_R + i_c = 0$$

$$\frac{dV_c(t)}{dt} + \frac{V_c(t)}{RC} = 0$$

(1) FODE:

$$V_c(t) = K_1 + K_2 e^{st}$$

st |

$$\left(K s e^{st}\right) + \frac{1}{RC}\left(K_1 + K_2 e\right) = 0$$

$$\left(s + \frac{1}{RC}\right) K_2 e^{st} + \frac{K_1}{RC} = 0$$

$$(1 + sRC) K_2 e^{st} + K_1 = 0$$

(1) $1 + sRC = 0 \rightarrow s = \frac{-1}{RC}$

(2) $\rightarrow K_1 = 0$

$$V_c(t) = K_2 e^{-t/RC}$$

$V_c(0^-) = V_0$

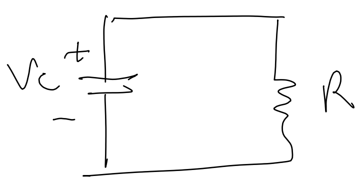
$V_c(0^+) = V_0$

$$V_c(t=0) = K_2 e^0 = V_0$$

$$\rightarrow K_2 = V_0$$

$$\boxed{V_c(t) = V_0 e^{-t/RC}}$$

KVL:



$V_c(t) = V_R(t)$

$V_c(t) - V_R(t) = 0$

$$\frac{1}{C}\int i_c\,dt - i_R R = 0 \qquad \therefore\ i_c = -i_R$$

$$\frac{1}{C}\int i_c\,dt + i_c R = 0 \qquad i_c = C\frac{dV_c}{dt}$$

$$\frac{1}{\cancel{C}}\cancel{\int} \cancel{C}\left(\frac{dV_c}{\cancel{dt}}\right)\cancel{dt} + \left(C\frac{dV_c}{dt}\right)R = 0$$

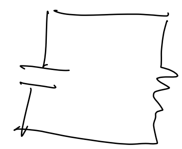
$$\int dV_c + RC\frac{dV_c}{dt} = 0$$

$$V_c + RC\frac{dV_c}{dt} = 0$$

Little trick (only true for $\underbrace{\text{FODE}}_{\text{1st order diff.}}$)

$$\boxed{V(t) = V[\infty] - \left(V[\infty] - V[0]\right) e^{-t/\tau}}$$

previous example:

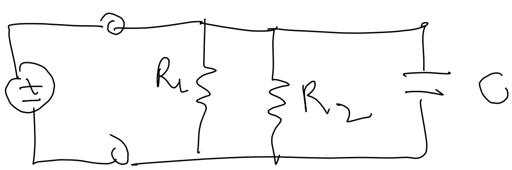


initial: $V_c[0] = V_0$

final: $V_c[\infty] = 0$

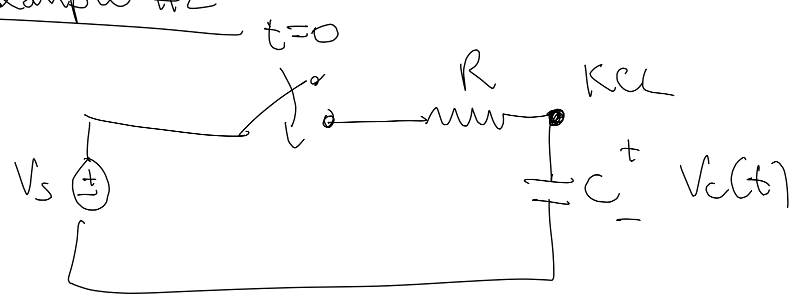
$$V_c(t) = 0 - (0 - V_0)\, e^{-t/\tau}$$

$$\boxed{V_c(t) = V_0\, e^{-t/\tau} \qquad \tau = RC}$$



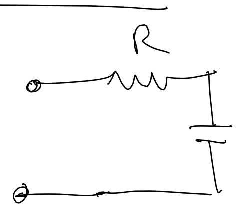
$\tau = (R_1 \| R_2)\, C$

Example #2

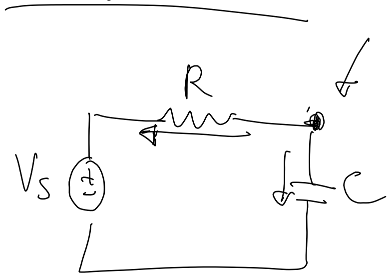


KCL

$t < 0$:



$t \geq 0$:



$V_c(t=0^-) = 0V$

$V_c(t=0^+) = 0V$

$$\frac{V_c - V_s}{R} + C\frac{dV_c}{dt} = 0$$

$$RC\frac{dV_c}{dt} + V_c(t) = V_s$$

FODE:

$$V_c(t) = K_1 + K_2 e^{st}$$

$$(1 + RCs)K_2 e^{st} + K_1 = V_s$$

(1) $1 + RCs = 0 \rightarrow s = \frac{-1}{RC}$

(2) $K_1 = V_s$

$$V_c(t) = V_s + K_2 e^{-t/RC}$$

$$V_c(0) = 0V = V_s + K_2 e^0$$

$$\rightarrow K_2 = -V_s$$

$$\boxed{V_c(t) = V_s - V_s e^{-t/RC}}$$

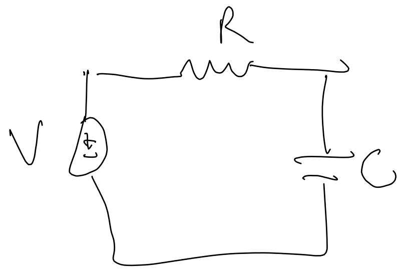
trick:

$V_c[0] = 0V \qquad V_c[\infty] = V_s$

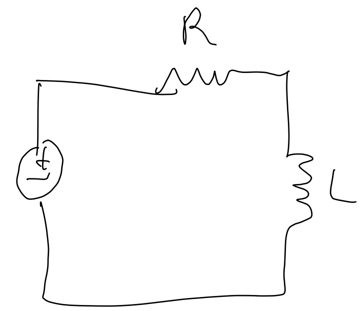
$$V_c(t) = V_s - (V_s - 0)e^{-t/\tau}$$

$$\boxed{V_c(t) = V_s - V_s e^{-t/RC}}$$

FODE:



$\tau = RC$



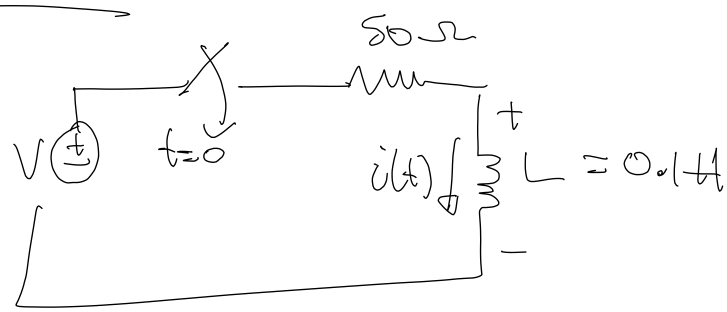
$\tau = \frac{L}{R}$

Second order:

$$V = I\left(R + sL + \frac{1}{Cs}\right)$$

$$\frac{V}{I} = \underline{LCs^2 + RCs + 1}$$

Example #3:



$V_L(\infty) = 0V$

$i(\infty) = \frac{V_s}{R}$

$i(t = 0^-) = 0A$

$i(t = 0^+) = 0A$

KVL:

$$V_s = i(t)R + L\frac{di(t)}{dt}$$

FODE:

$$i(t) = K_1 + K_2 e^{st}$$

$$V_s = RK_1 + (R + sL)K_2 e^{st}$$

(1) $R + sL = 0 \rightarrow \boxed{s = -\frac{R}{L}}$

(2) $V_s = RK_1 \rightarrow \boxed{K_1 = \frac{V_s}{R}}$

$$i(t) = \frac{V_s}{R} + K_2 e^{-\frac{R}{L}t} \leftarrow e^{-t/\tau}$$

$$\boxed{\tau = \frac{L}{R}}$$

$$i(0) = 0A = \frac{V_s}{R} + K_2 e^0$$

$$\rightarrow K_2 = \frac{-V_s}{R}$$

$$\boxed{i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-t/\tau}}$$

trick:

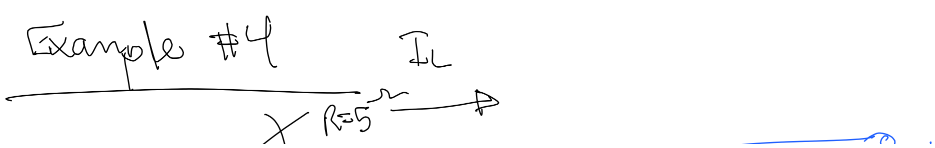
$$i[0] = 0A$$

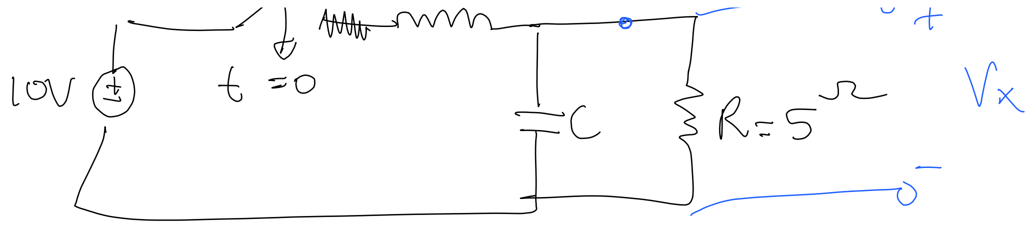
$$i[\infty] = \frac{V_s}{R}$$

$$i(t) = \frac{V_s}{R} - \left(\frac{V_s}{R} - 0\right) e^{-t/\tau}$$

$$\star \quad i[t] = i[\infty] - \left(i[\infty] - i[0]\right) e^{-t/\tau}$$

Example #4

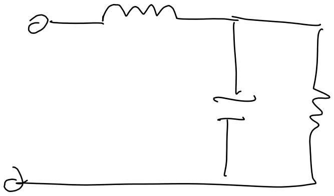




$V_x = V_c = V_R$

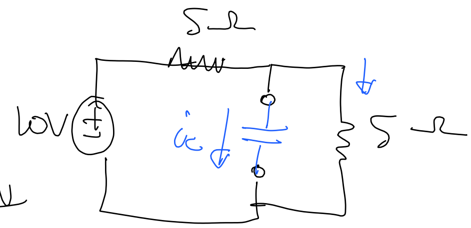
$V_x(t=0) = 0V$

$i_L(t=0) = 0A$



$V_x(t\to\infty) = 5V$

$i_L(t\to\infty) = \frac{10V}{10\Omega} = 1A$



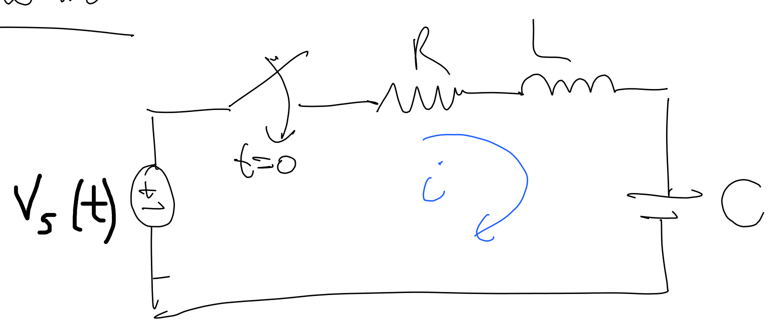
$i_c(\infty) = 0$

$V_c(\infty) = iR$

inductor: short ckt

cap: open-ckt.

Example #5



by KVL:

$V = L\frac{di}{dt}$

$$V_s(t) = i(t)R + L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt$$

$$V_s(t) = \frac{di(t)}{dt}R + L\frac{d^2 i}{dt^2} + \frac{1}{C}i(t)$$

$$LC\frac{d^2 i}{dt^2} + RC\frac{di}{dt} + i(t) = \frac{dV_s(t)}{dt}$$