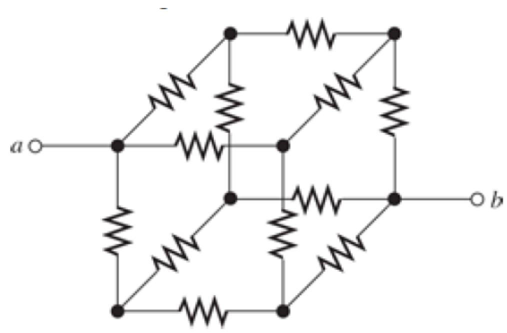
ECE100
Homework-2 solutions

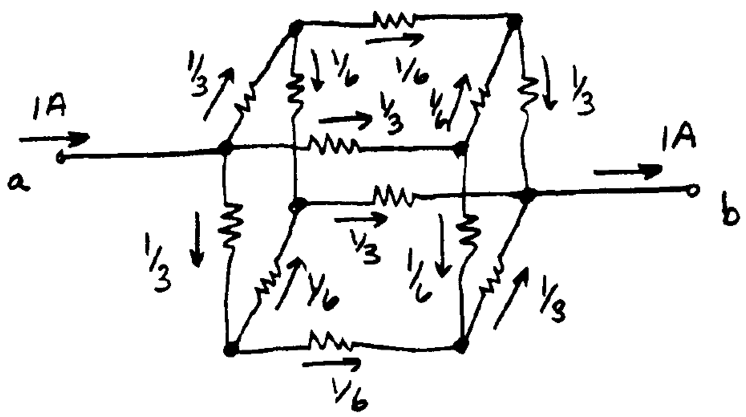
Total Points: 100

Required Readings: Chapter 1, 2 and class notes
Submit your work in a pdf file electronically in the CCLE website before April 11th 11:59 pm. Late homework will not get credit!

1. Twelve 1 Ω resistors are arranged on the edges of a cube, and terminals a and b are connected to diagonally opposite corners of the cube. The problem is to find the resistance between the terminals. Approach the problem this way: Assume that 1 A of current enters terminal a and exits through terminal b. Then, the voltage between terminals a and b is equal to the unknown resistance. By symmetry considerations, we can find the current in each resistor. Then, using KVL, we can find the voltage between a and b.



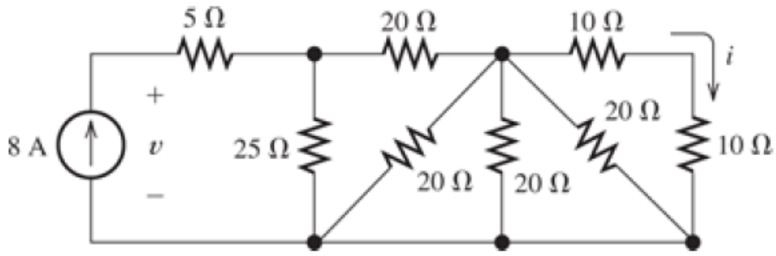
By symmetry, we find the currents in the resistors as shown below:



Then, the voltage between terminals a and b is

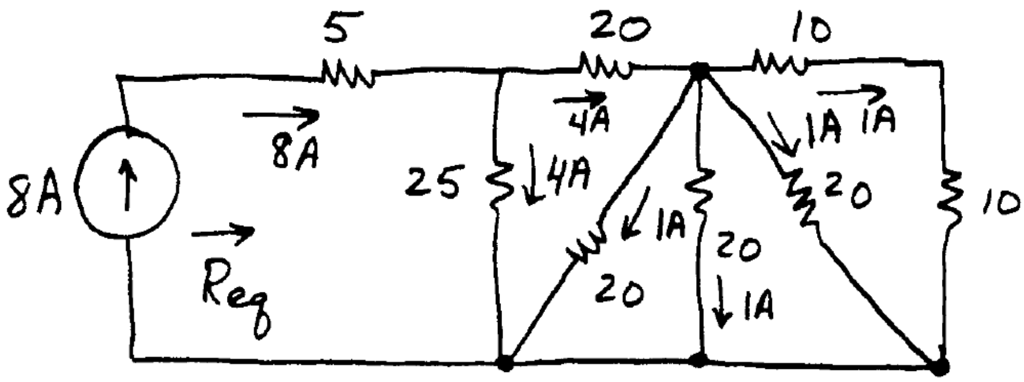
$$v_{ab} = R_{eq} = 1/3 + 1/6 + 1/3 = 5/6$$

2. Find the values of v and i for the circuit below

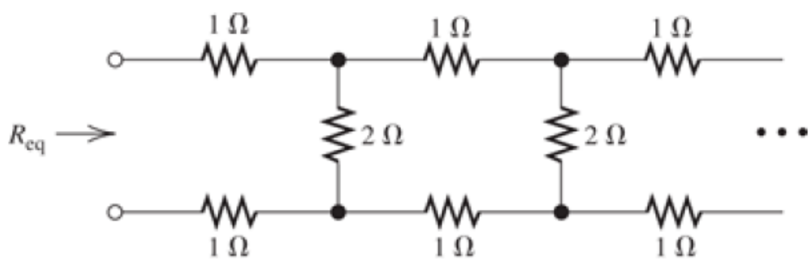


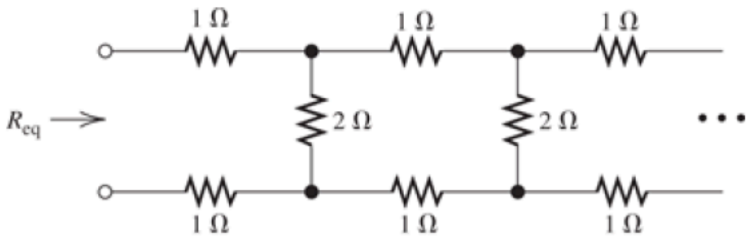
Combining resistors in series and parallel, we find that the equivalent resistance seen by the current source is R_{eq} = 17.5 Ω. Thus,

$v = 8 \times 17.5 = 140$ V. Also, $i = 1$ A.

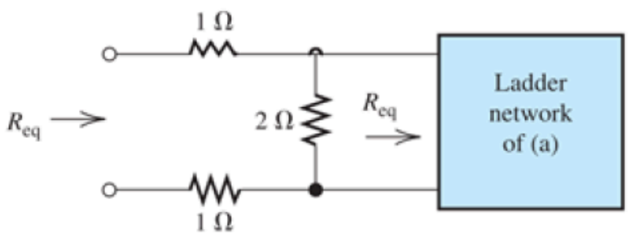


3. Find the equivalent resistance for the infinite network shown in figure below





(a)



(b)

Combining the resistances shown in Figure

$$R_{eq} = 1 + \frac{1}{1/2 + 1/R_{eq}} + 1 = 2 + \frac{2R_{eq}}{2 + R_{eq}}$$

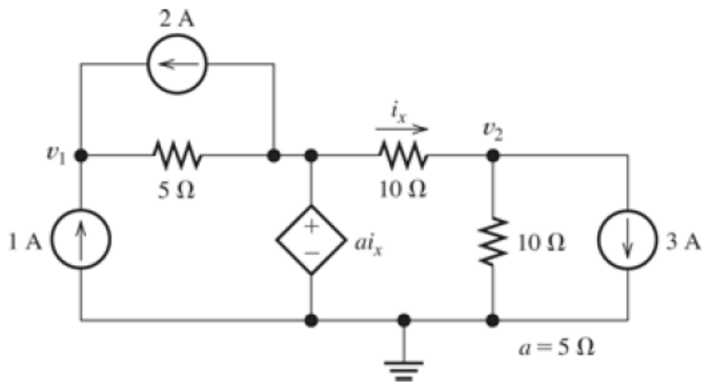
$$R_{eq}(2 + R_{eq}) = 2(2 + R_{eq}) + 2R_{eq}$$

$$(R_{eq})^2 - 2R_{eq} - 4 = 0$$

$$R_{eq} = 3.236\,\Omega$$

($R_{eq} = -1.236\,\Omega$ is another root, but is not physically reasonable.)

4. Solve for the node voltages v_1 and v_2 shown in Figure below



First, we can write:

$$i_x = \frac{5i_x - v_2}{10}$$

Simplifying, we find $i_x = -0.2v_2$.

Then write KCL at nodes 1 and 2:

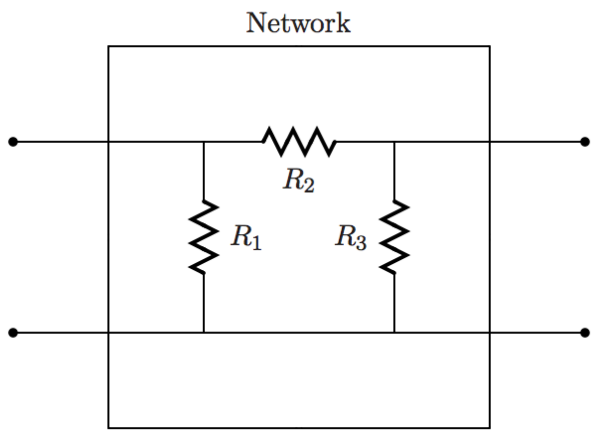
$$\frac{v_1 - 5i_x}{5} = 3 \qquad\qquad \frac{v_2}{10} - i_x = -1$$

Substituting for i_x and simplifying, we have

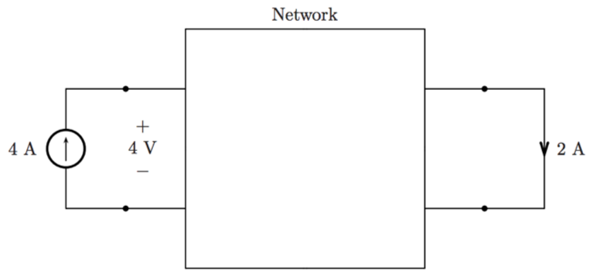
$$v_1 - v_2 = 15 \quad \text{and} \quad 0.3v_2 = -3$$

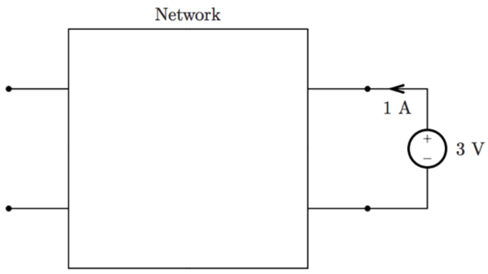
which yield $v_1 = 25\text{ V}$ and $v_2 = -10\text{ V}$.

5. The following network has two ports and three resistors. The resistor values R1, R2 and R3 are unknown.



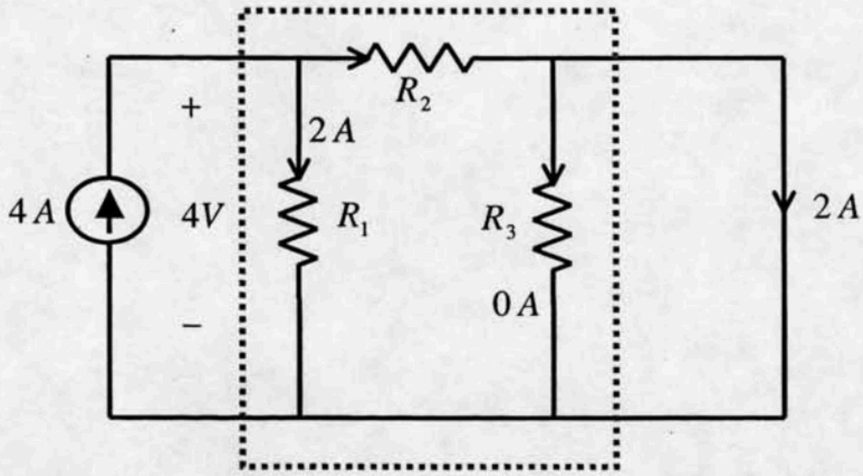
Using the results of the following two experiments performed on the network, find the unknown values of the three resistors.





The first experiment allows us calculate the values of resistors R_1 and R_2. We observe that the output terminal of the network on the right is shorted as shown in Fig: 1.9. This means that no current will flow in R_3. From inspection of the circuit we note that $2A$ of current flows through the short circuit. This current must pass through R_2 and in doing so it causes a voltage drop of $4V$ across the resistor. Therefore from Ohm's law we calculate the value of R_2 to be 2Ω. The remaining $2A$ of current from the current supply must flow through R_1, which also has a voltage drop of $4V$ across it. Therefore the value of R_1 is also 2Ω.

Fig: 1.9



The second experiment allows us calculate the value of R_3 assuming we have already calculated the values of the other resistors. We note that the output terminal of the network is open circuited and so no current can flow through the terminal. Therefore the current entering the network must be split between two parallel paths. The first path is through R_3 and the second path is through the combination of $R_2 + R_1$. We already know the value of $R_2 + R_3$ equals 4Ω. Indirectly we are told the overall resistance of the circuit because we are given the voltage across the circuit is $3V$ and the current flowing into the circuit is $1A$.

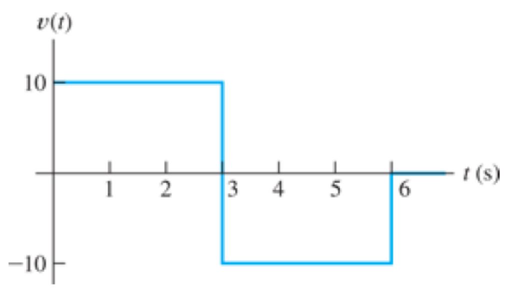
Equivalent resistance of circuit: $R_{eq} = \dfrac{3V}{1A} = 3\Omega \quad \Rightarrow \quad 3\Omega = \dfrac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$

$$R_1 + R_2 = 4\Omega \quad \Rightarrow \quad 12 + 3R_3 = 4R_3 \quad \Rightarrow \quad R_3 = 12\Omega$$

The final values of the resistors are as follows:

$$\boxed{R_1 = 2\Omega \qquad R_2 = 2\Omega \qquad R_3 = 12\Omega}$$

6. The voltage across a 2-H inductance is shown in Figure below. The initial current in the inductance is $i_L(0)=0$. Sketch the current, power, and stored energy to scale versus time.



Given,

Inductance, $L = 2\text{H}$

The voltages from the waveform are

$v(t) = 10\ \text{V} \qquad \text{for } 0 < t \le 3\ \text{s}$

$v(t) = -10\ \text{V} \qquad \text{for } 3\text{s} < t \le 6\ \text{s}$

Now,

Current through the inductor, $i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0)$

For $0 < t \le 3\ \text{s}$

$$i(t) = \frac{1}{2}\int_0^t 10dt + i(0)$$

$$i(t) = \frac{1}{2}[10t]_0^t + 0$$

$$i(t) = \frac{1}{2}[10t]$$

$$\therefore i(t) = 5t\ \text{A}$$

For $3 < t \le 6\text{s}$

$$i(t) = \frac{1}{2}\int_3^t -10\ dt + i(3\text{s})$$

$$i(t) = \frac{1}{2}[-10t]_3^t + 15$$

$$i(t) = -\frac{1}{2}[10t - 30] + 15$$

$$i(t) = -5t + 15 + 15$$

$$\therefore i(t) = -5t + 30\ \text{A}$$

Power, $p(t) = v(t) \times i(t)$

For $0 < t \le 3\ \text{s}$

$$p(t) = 10 \times 5t$$

$$\therefore p(t) = 50t\ \text{W}$$

For $3\text{s} < \text{t} \le 6\text{s}$

$$p(t) = -10 \times (-5t + 30)$$

$$\therefore p(t) = 50t - 300\ \text{W}$$

Energy stored

$$w(t) = \frac{1}{2} L i^2(t)$$

For $0 < t \leq 3$ s

$$w(t) = \frac{1}{2} \times 2 \times (5t)^2$$

$$\therefore w(t) = 25t^2 \text{ J}$$

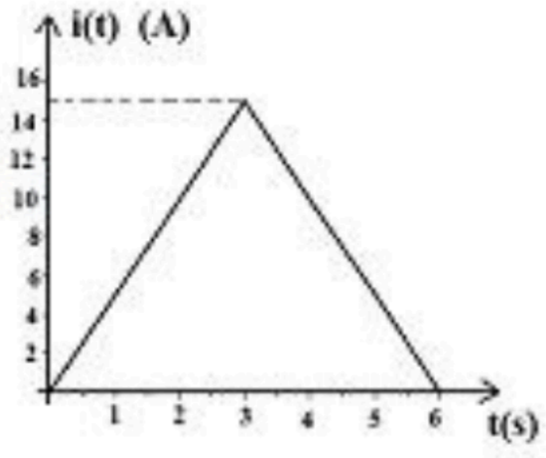
For $3 \text{ s} < t \leq 6$ s

$$w(t) = \frac{1}{2} \times 2 \times (-5t + 30)^2$$

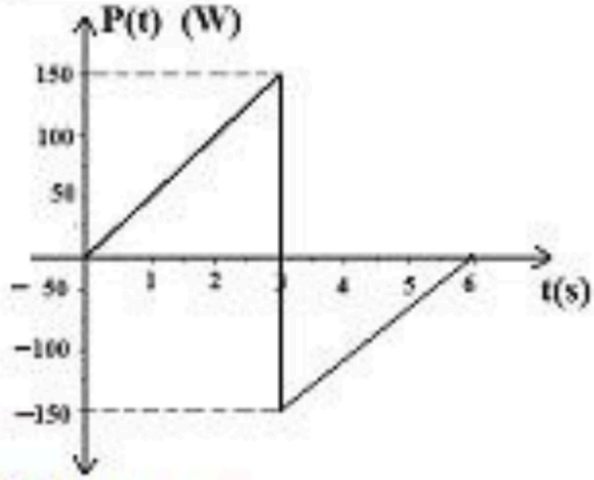
$$\therefore w(t) = 25t^2 + 900 - 300t$$

<u>Waveforms</u> :–

<u>Current:</u>



Power:



Energy: