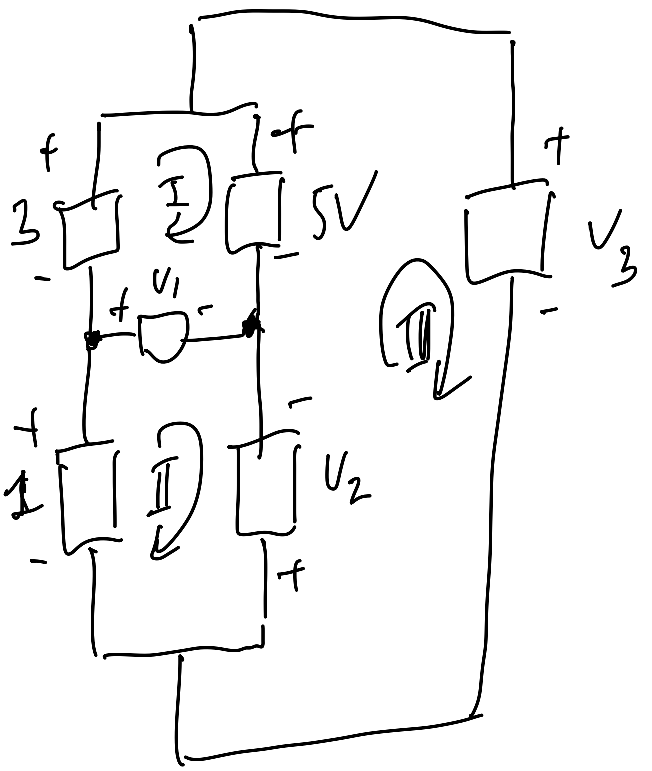
Nhat Ho

Problem1:

a) We have:



Apply KVL in I, we have

$$5 - V_1 - 3 = 0$$

$$\Rightarrow \boxed{V_1 = 2\ (V)}$$

Apply KVL in II, we have

$$V_1 - V_2 - 1 = 0$$

$$\Rightarrow 2 - 1 - V_2 = 0$$

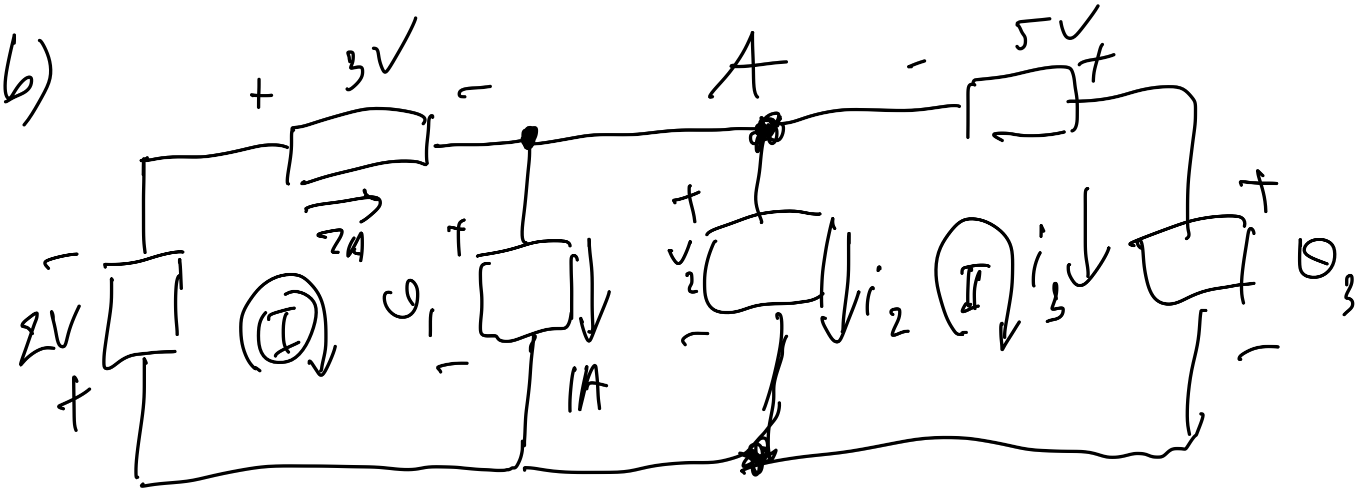
$$\Rightarrow \boxed{V_2 = 1\ (V)}$$

Apply KVL in III, we have

$$-5 + V_3 + V_2 = 0 \Rightarrow V_3 = 5 - V_2$$

$$\Rightarrow \boxed{V_3 = 5 - 1 = 4 (V)}$$

b)



Apply KVL for (I), we have:

$2V + 3V + v_1 = 0 \Rightarrow \boxed{v_1 = -5V}$

Apply KVL for (II), we have:

$-V_2 - 5V + v_3 = 0 \Rightarrow v_3 - V_2 = 5V$

also $V_1 = V_2 \Rightarrow \boxed{V_2 = -5V}$

$\Rightarrow v_3 = v_2 + 5 = -5 + 5 = 0$

$\Rightarrow \boxed{v_3 = 0\ V}$

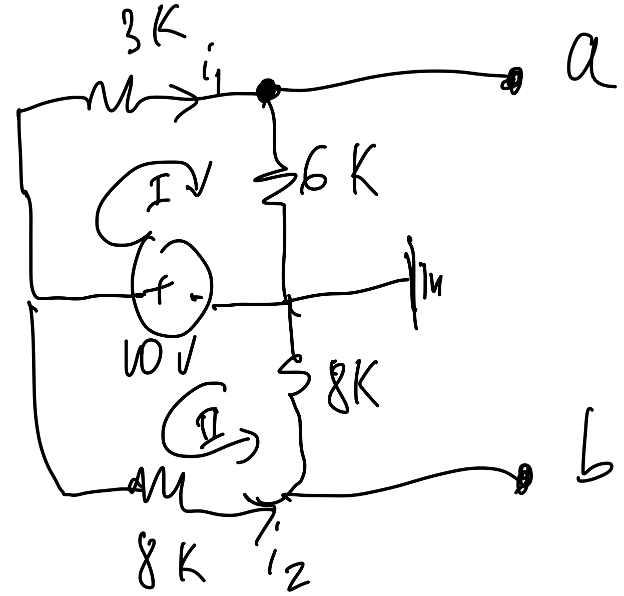
Apply KCL at A, we have:

$2A = 1A + i_2 + i_3 \Rightarrow i_2 + i_3 = 1A$

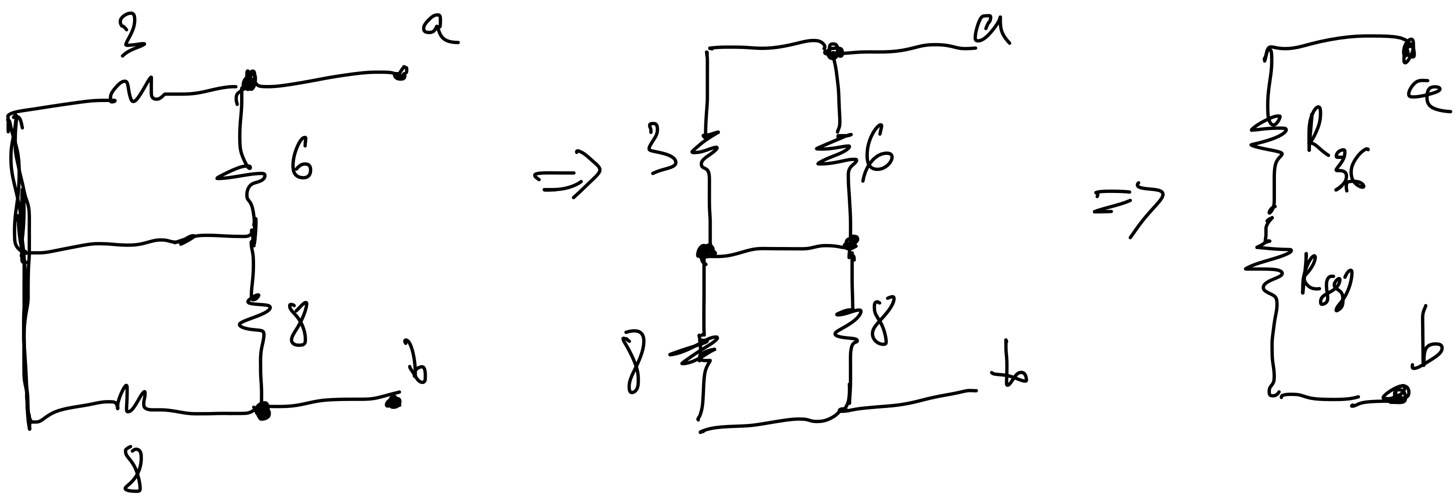
also $\boxed{i_3 = -1A}$ as the diagram

$\Rightarrow \quad i_2 = 1 - i_3 = \boxed{2\ A}$

Problem 2 :



To find R_{th}, we short the circuit for voltage source. We have.



Since $3 \parallel 6 \Rightarrow R_{36} = \frac{3\times 6}{3+6} = \frac{18}{9} = 2K$

$8 \parallel 8 \Rightarrow R_{88} = \frac{8\times 8}{8+8} = 4K$

$\Rightarrow R_{ab} = R_{th} = R_{36} + R_{88} = 2+4 = \boxed{6K}$

apply KVL for I, $\Rightarrow 10 = (3+6)i_1$

$\Rightarrow i_1 = \frac{10}{9}$ (mA) $\Rightarrow v_a = i_1 \times 6K = \frac{10}{9} \times 6$

$\Rightarrow v_a = \frac{60}{9}(V) = \frac{20}{3}(V)$

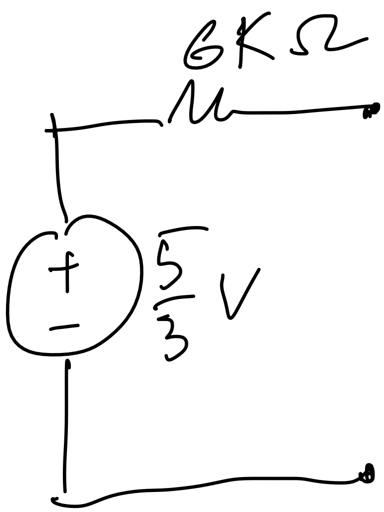
apply KVL for II $\Rightarrow 10 = 16 i_2 \Rightarrow i_2 = \frac{10}{16}$ mA

$\Rightarrow V_b = i_2 \times 8 = \frac{10}{16} \times 8 = 5\ (V)$

$\Rightarrow V_{ab} = V_a - V_b = \frac{20}{3} - 5 = \frac{5}{3}(V)$

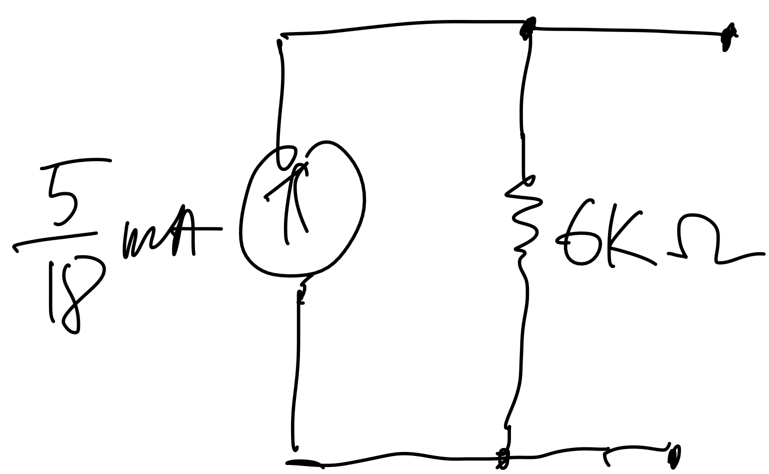
$\Rightarrow V_{th} = \frac{5}{3}(V)$ also $R_{th} = 6K$,

We have

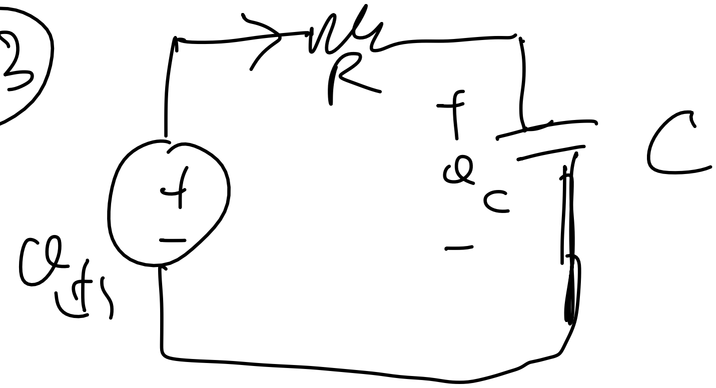


Thevenin

$\Rightarrow$

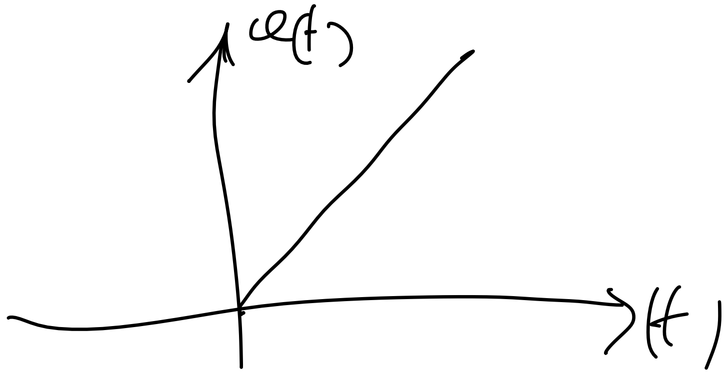


③



$R = 1K$

$C = 1\,mF$



$t < 0 \quad v(t) = 0$

$t \geq 0 \quad v(t) = t$

a) We have

$$-v(t) + iR + v_c = 0$$

$$\Rightarrow -v(t) + RC\frac{dv_c}{dt} + v_c = 0$$

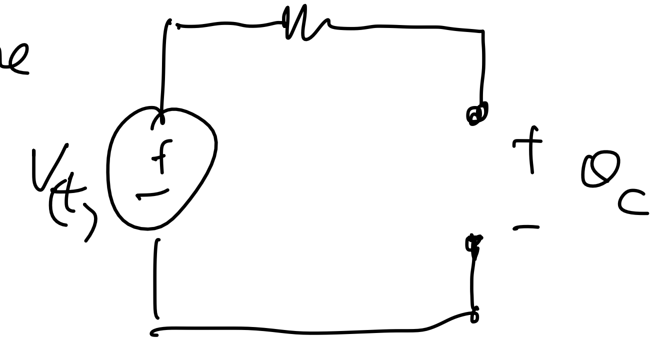
$$\Rightarrow RC\frac{dv_c}{dt} + v_c = v(t)$$

$$\Rightarrow \frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{1}{RC}v(t)$$

Based on this equation we have:

$$v_c(t) = V_c(\infty) - [V_c(\infty) - V_c(0^+)] e^{-t/RC}$$

When $t \to \infty$, we have



$$\Rightarrow v_c(\infty) = V(t) = t$$

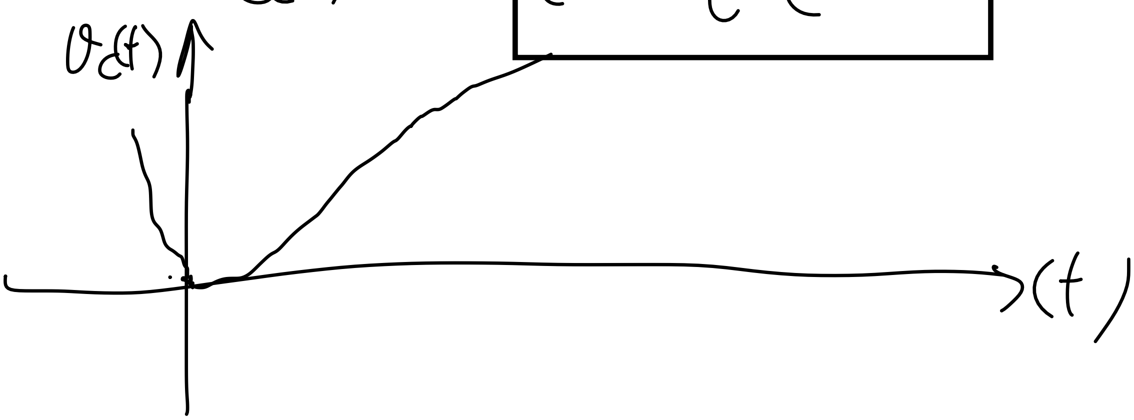
Also at $t = 0$, $v_c(0^+) = 0$

$$\Rightarrow v_c(t) = t - [t - 0] e^{-t/RC}$$

$$\Rightarrow v_c(t) = t - t e^{-t/RC}$$

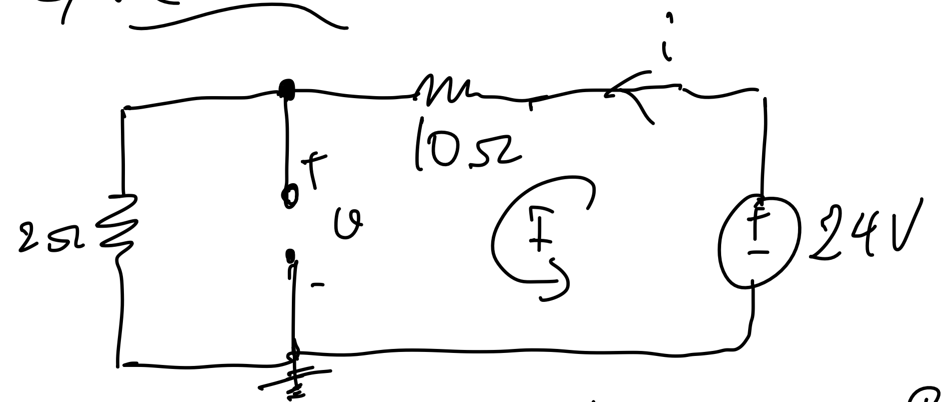
$$RC = 1000 \times 1 \times 10^{-3} = 1 \, (s)$$

$$\Rightarrow v_c(t) = \boxed{t - t e^{-t}}$$



Problem 4: Switch open for a long time

⇒ We have:



apply KVL for loop I, we have:

$$-24 + i(10+2) = 0$$

$$\Rightarrow i = \frac{24}{12} = 2(A)$$

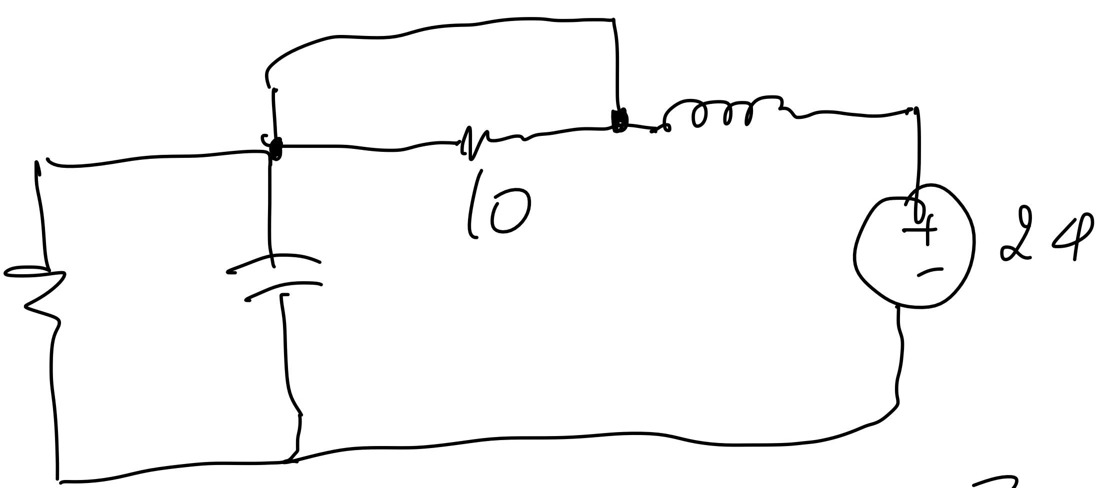
$$\Rightarrow v = i \times 2 = 2 \times 2 = 4(V)$$

a) At $t = 0^-$, there is energy stored in the inductor and capacitors since $v_c = 4V$ and $i_L = 2A$. They are not zero ⇒ stored the energy.

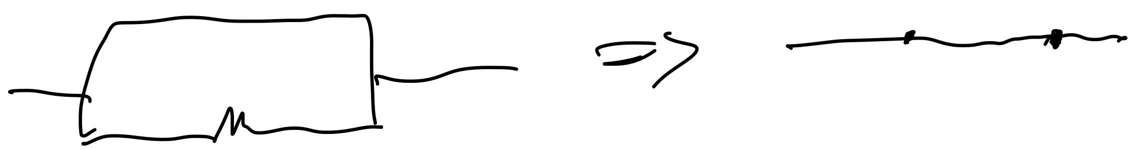
$$\Rightarrow E_L = \frac{1}{2}Li^2 = \frac{1}{2} \times 0.4 \times 2^2 = \boxed{0.8\ J}$$

$$\Rightarrow E_C = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{1}{20} \times 4^2 = \boxed{0.4\ J}$$

b) After switch close, new circuit:



⇒ the $10\,\Omega$ will be shorted and no current go through it

Since 

c) at $t \to 0^-$, we have $v_c(0^-) = 4V$

$i_L(0^-) = 2A$. Since inductor can't change the current instantaneously and capacitor can not change the voltage instantaneously

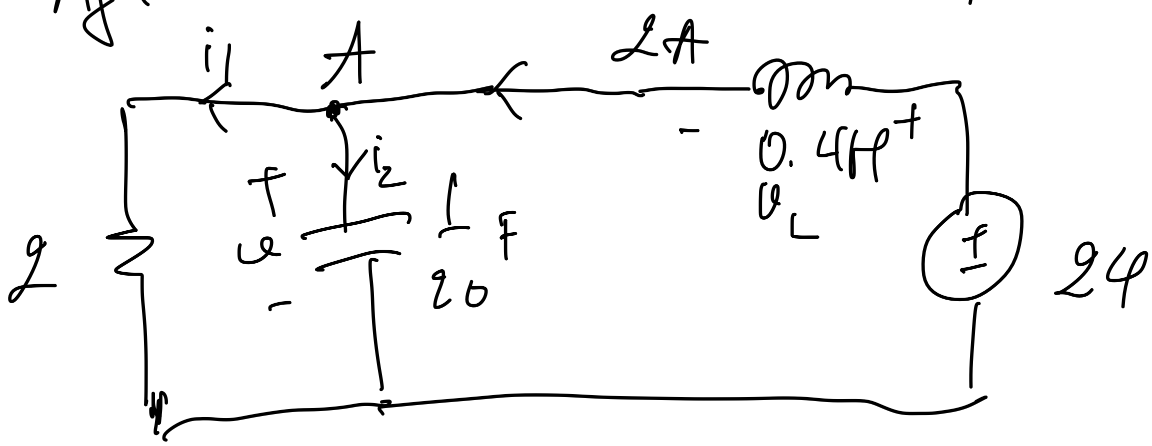
⇒ $\boxed{v_c(0^+) = 4V}$, $\boxed{i_L(0^+) = 2A}$

We have: $v_L = L\frac{di}{dt}$

$$\Rightarrow \frac{di(0^+)}{dt} = \frac{1}{L} v_L(0^+)$$

$$i_C = C\frac{dv_c}{dt} \Rightarrow \frac{dv(0^+)}{dt} = \frac{1}{C} i_C(0^+)$$

After switch close $t = 0^+$,



$$\Rightarrow i_1(0^+) = \frac{v}{2} = \frac{4}{2} = 2(A)$$

$$\Rightarrow i_2 = 2 - 2 = 0$$

$$\Rightarrow i_c(0^+) = 0 \Rightarrow \boxed{\frac{dv(0^+)}{dt} = \frac{1}{C} i_c(0^+) = 0}$$

Also, at $t = 0^+$, $v = 4V$

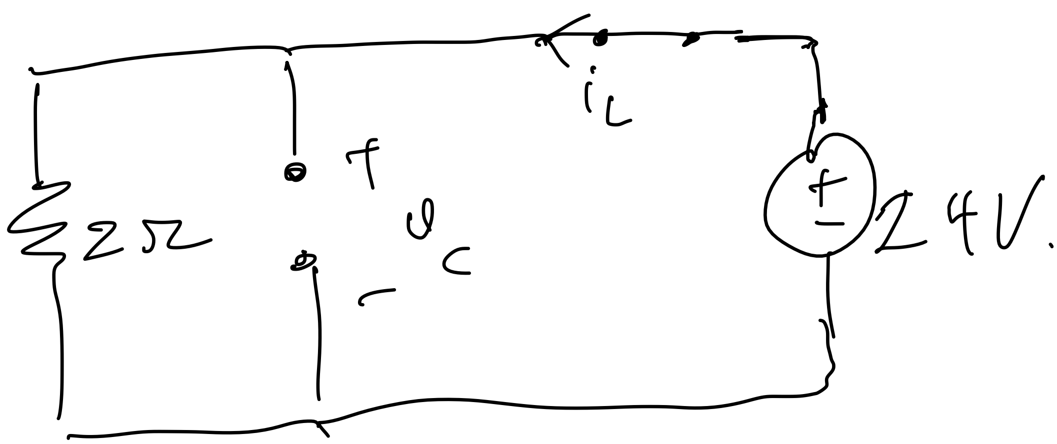
$-24 + v_L(0^+) + 4 = 0$

$\Rightarrow V_L(0^+) = 24 - 4 = 20\,V$

$\Rightarrow \dfrac{di(0^+)}{dt} = \dfrac{1}{L} v_L(0^+) = \dfrac{20}{0.4}$

$\Rightarrow \boxed{\dfrac{di(0^+)}{dt} = 50}$

d) when $t \to \infty$, we have

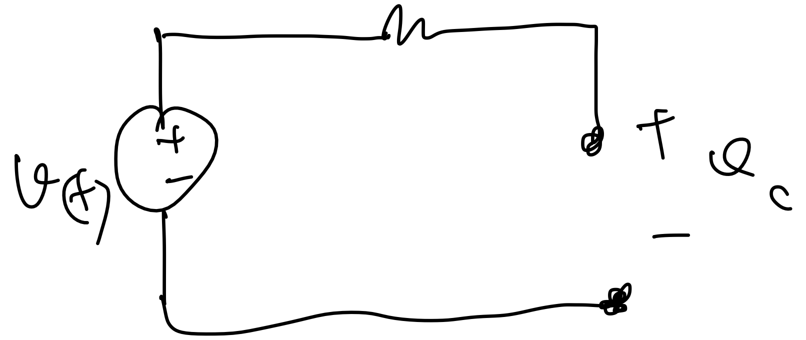


$\Rightarrow i_L(\infty) = \dfrac{24}{2} = \boxed{12\,(A)}$

$v_C(\infty) = 12 \times 2 = \boxed{24\,V}$

Continue 1c.

c) when t is long enough time, we will have:



So $v_c = v(t)$

$\Rightarrow$ the capacitor voltage will be equal to the $v(t)$.

d) we have $i_c = C \frac{dv_c}{dt}$

also $v_c(t) = t - te^{-t}$

$\Rightarrow \frac{dv_c(t)}{dt} = 1 - [e^{-t} - te^{-t}]$

$= 1 - e^{-t} + te^{-t}$

$$\Rightarrow \boxed{i_C(t) = 10^{-3}\left(1 - e^{-t} + te^{-t}\right) \ (A)}$$

e) We have:

$$p(t) = v(t)\, i(t)$$

$$= \boxed{10^{-3}\cdot\left(t - te^{-t}\right)\left(1 - e^{-t} + te^{-t}\right)}$$

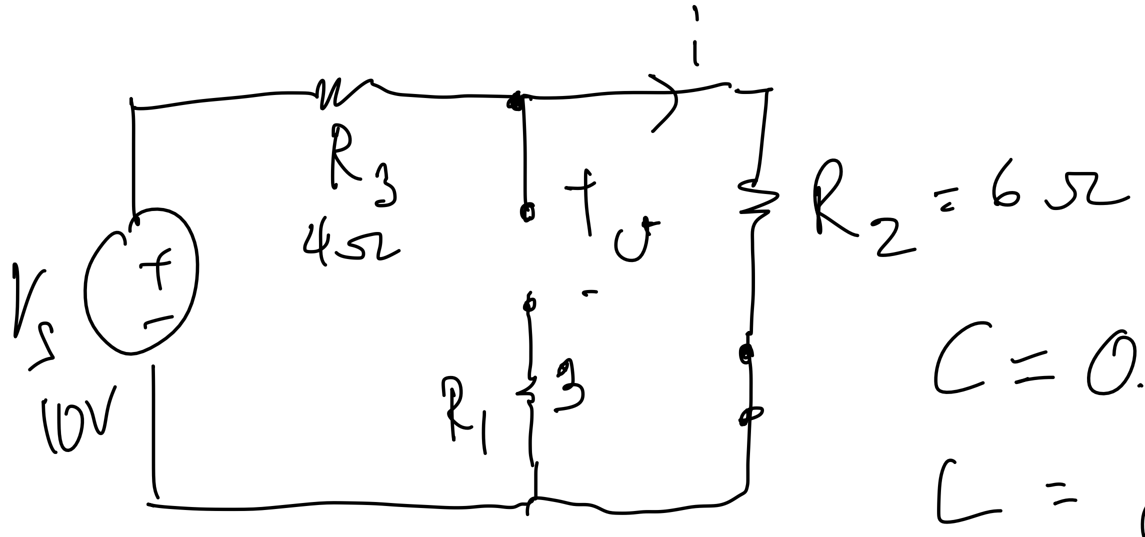
$$= 10^{-3}\left(t - te^{-t} + t^2e^{-t} - te^{-t} + te^{-2t} - t^2e^{-2t}\right)$$

$$= \boxed{10^{-3}\left(t - 2te^{-t} + t^2e^{-t} + te^{-2t} - t^2e^{-2t}\right)}$$

f) The power delivered by the battery will be dissicipated on the resistor

Problem 5: [I do b first]

b) at $t \to 0^-$, switch close for a long time, we have:



$C = 0.02F$

$L = 0.5H$

Apply KVL for the loop, we have:

$$10 = i(R_3 + R_2) = 10i$$

$\Rightarrow \quad i = 1A \Rightarrow i_L(0^-) = 1A.$

$\Rightarrow v = iR_2 = 1A \times 6\Omega = 6(V)$

$\Rightarrow v_c(0^-) = 6V$

After open switch at $t \to 0^+$, the capacitor could not change voltage instantaneously

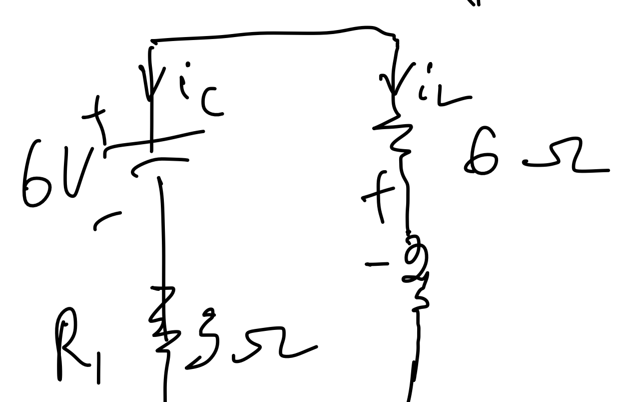
$$\Rightarrow \boxed{V_c(0^-) = V_c(0^+) = 6V}$$

c) Find $\dfrac{dv(0^+)}{dt}$

We have : $i_c = C \dfrac{dv_c}{dt}$

$$\Rightarrow \left.\frac{dv_c}{dt}\right|_{t=0^+} = \frac{1}{C} \cdot i_c(0^+)$$

We know, after close switch, we have:



Since

$i_c(0^+) = -i_L(0^+)$

$\Rightarrow i_c(0^+) = -i_L(0^+)$

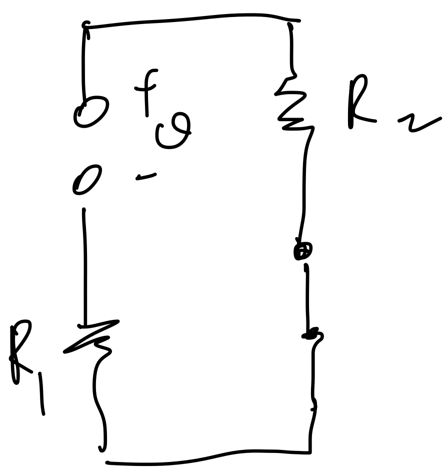
From above, we have already $i_L(0^-) = 1A$
Also, the inductor could not change current instantaneously $\Rightarrow i_L(0^+) = i_L(0^-) = 1A$

$\Rightarrow i_C(0^+) = -1A$.

$$\Rightarrow \left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{1}{0.02} \times (-1)$$

$$= \boxed{-50}$$

f) When $t \to \infty$, we have:

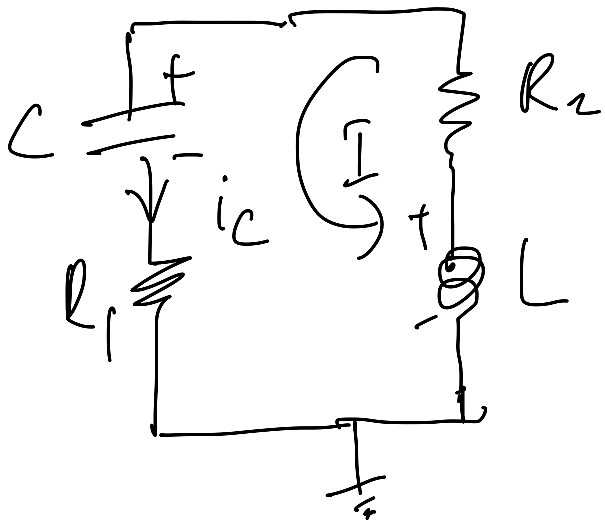


$\Rightarrow v(t) = 0$ when $t \to \infty$

$\boxed{\text{or } v(\infty) = 0}$

back to 5a:

at $t = 0^+$:



We have already:

$$\begin{cases} v_C(0^+) = 6V \\ i_L(0^+) = 1A \end{cases}$$

Apply KVL for I, we have:

also $i_C = -i_L$

$$\Rightarrow i_L(t) = -C\frac{dv_C}{dt}$$

$$v_C + i_C(R_1+R_2) - v_L = 0$$

$$\Rightarrow v_C + C\frac{dv_C}{dt}(R_1+R_2) - L\frac{di_L}{dt} = 0$$

$$\Rightarrow v_C + C\frac{dv_C}{dt}(R_1+R_2) - L\frac{d}{dt}\left(-C\frac{dv_C}{dt}\right) = 0$$

Call $R = R_1+R_2$

$$\Rightarrow v_c(t) + RC\frac{dv_c}{dt} + LC\frac{d^2 v_c(t)}{dt^2} = 0$$

$$\Rightarrow \frac{d^2 v_c(t)}{dt^2} + \frac{R}{L}\frac{dv_c(t)}{dt} + \frac{1}{LC}v_c(t) = 0$$

$$\Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\Leftrightarrow s^2 + 2\xi\omega_0 s + \omega_0^2 = 0$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$; $\xi = \frac{1}{2}\cdot\frac{R_2}{\sqrt{\frac{L}{C}}}$

$L = 0.5H$

$C = 0.02F$ $\Rightarrow LC = 0.01$

$$\Rightarrow \boxed{\omega_0 = 10} \; ; \; \boxed{\xi = \frac{6}{2\times 5} = 0.6}$$

e) $\varepsilon = 0.6 < 1 \Rightarrow$ underdamp.

f) We have already $\omega_0 = 10$

$\omega_n = \sqrt{1-\varepsilon^2}\,\omega_0 = \sqrt{1-0.6^2} \cdot 10$

$= \boxed{8}$

g) roots are :

$s_1, s_2 = -\varepsilon\omega_0 \pm j\omega_0\sqrt{1-\varepsilon^2}$

$= -0.6 \times 10 \pm j \times 10 \times 0.8$

$= \boxed{-0.6 \pm 8j}$

$$\Rightarrow \boxed{v_c(t) = K_1 e^{-0.6t}\cos(8t) + K_2 e^{-0.6t}\sin(8t)}$$