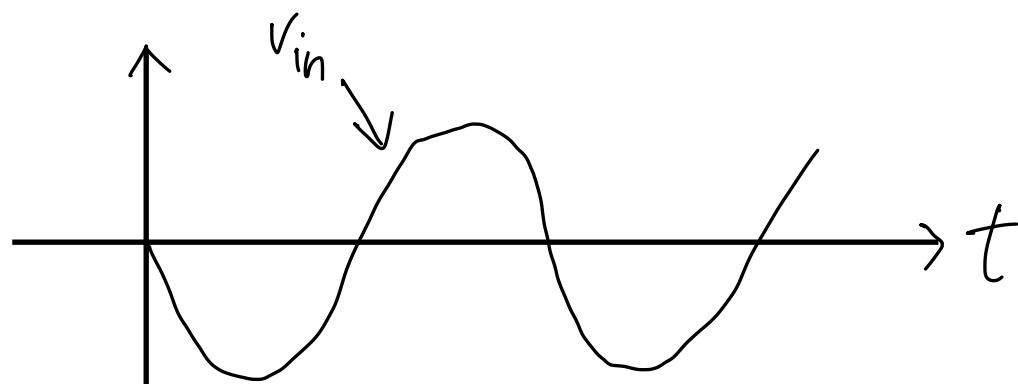
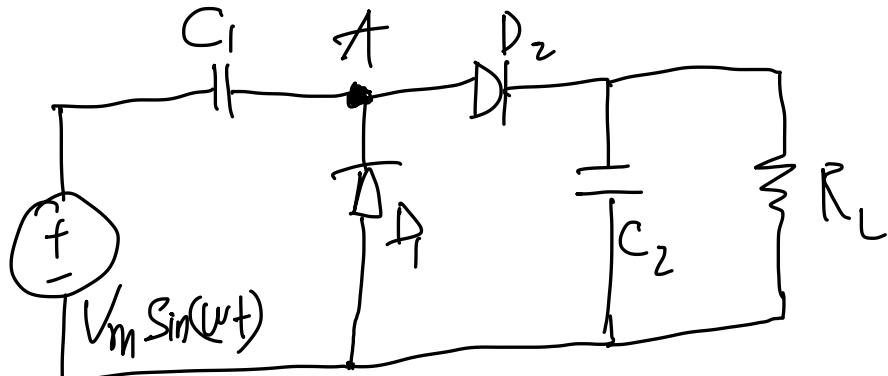


Nhat Hô - HW 8:

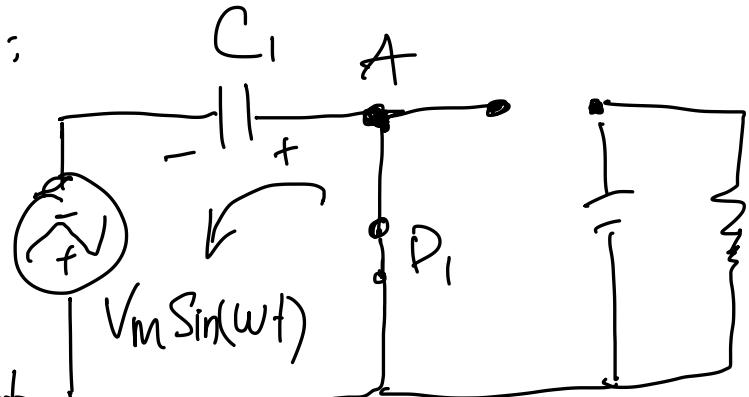
Problem 1: Given the capacitor is very large \Rightarrow discharge only a very small amount per cycle.

- a) Need to sketch the voltage at point A versus time.



⊕ For the negative half-cycle of the input, the D₁ is forward biased and D₂ is reverse biased. Then we have:

\Rightarrow the capacitor C₁ charges to peak voltage of the input. Also, capacitor discharges only a small amount per cycle, it acts as



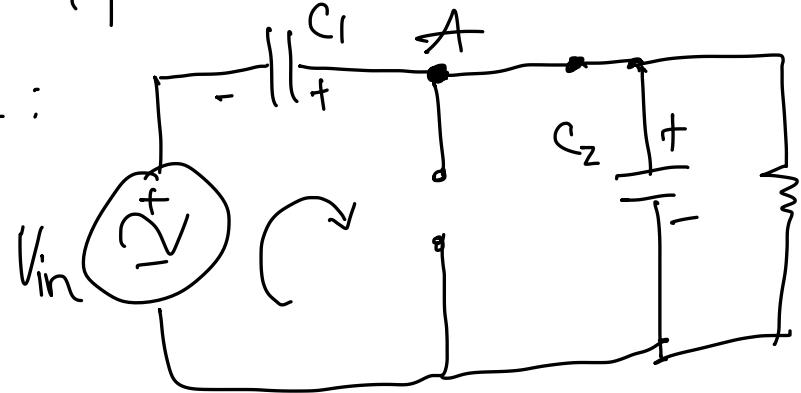
a storage device. Hence, after the first half negative cycle, C_1 is charged to peak value V_m , or $V_{C_1} = V_m$.

④ For the positive half-cycle of the input, D_1 is reverse biased & D_2 is forward biased

In this case, we have:

$$V_A = V_{in} + V_{C_1}$$

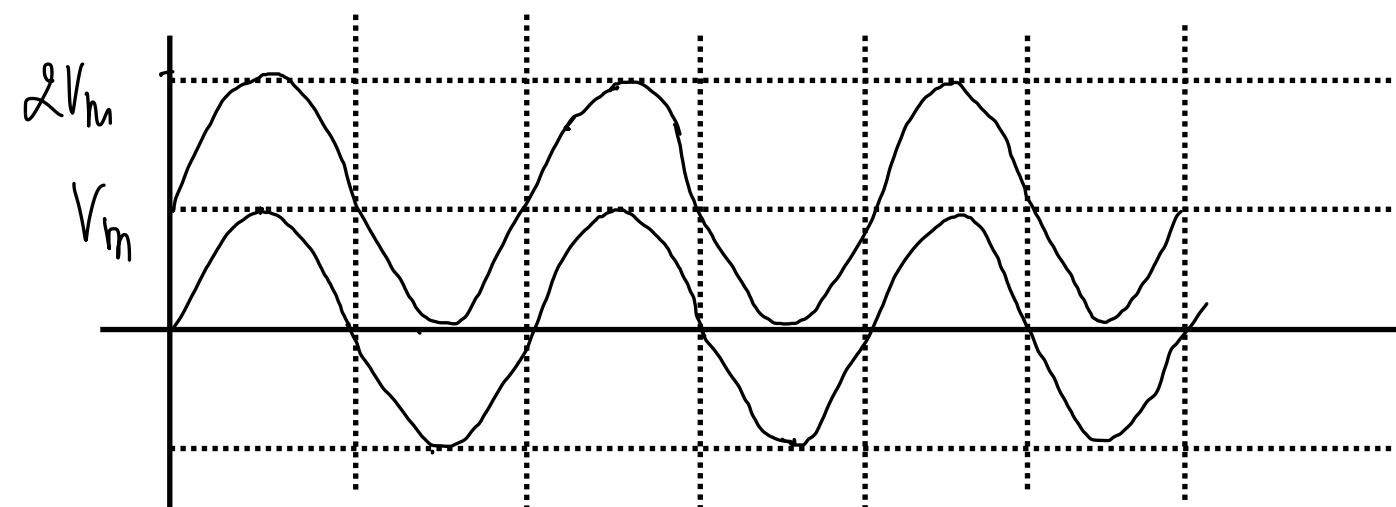
$$= V_m \sin \omega t + V_m$$



$$\Rightarrow V_{A_{max}} = 2V_m \text{ when } V_{in} \text{ max in this half-cycle.}$$

Also, Capacitor C_2 will be charged to twice the peak input voltage $\Rightarrow V_{C_2} = V_L = 2V_m$

④ Sketch the voltage at point A versus time



⊕ Voltage across the load.

From the positive half cycle, we know that the capacitor will be charged upto $2V_m$ voltage

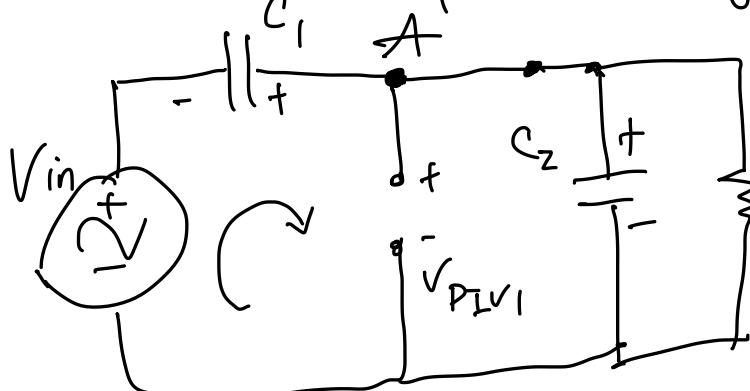
$$\Rightarrow V_L = 2V_m$$

⊕ Because the maximum of the input is V_m while the output voltage will have $2V_m$

\Rightarrow This is called a voltage doubler.

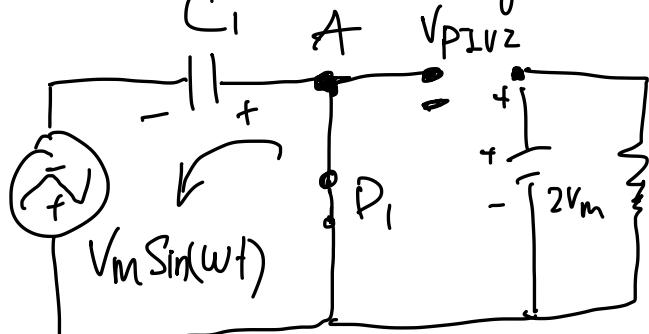
⊕ Peak inverse voltage.

- In the positive cycle, we have:



$$\Rightarrow V_{PIV1} = V_{max} + V_{max} \\ = 2V_m$$

- In the negative cycle, we have:-



$$\Rightarrow V_{PIV2} = 2V_m$$

Problem 2: Given two points in saturation in NMOS

⊕ $V_{GS} = 2V, i_D = 0.2 \text{ mA}$

⊕ $V_{GS} = 3V, i_D = 1.8 \text{ mA}$

In saturation, we have:

$$i_D = K (V_{GS} - V_{th})^2$$

$$\Rightarrow \begin{cases} 0.2 = K (2 - V_{th})^2 \\ 1.8 = K (3 - V_{th})^2 \end{cases} \Rightarrow \frac{1.8}{0.2} = \left(\frac{3 - V_{th}}{2 - V_{th}} \right)^2$$

$$\Rightarrow g = \left(\frac{3 - V_{th}}{2 - V_{th}} \right)^2 \Rightarrow \begin{cases} \frac{3 - V_{th}}{2 - V_{th}} = 3 & \text{(1) Call } V_{th}: V. \\ \frac{3 - V_{th}}{2 - V_{th}} = -3 & \text{(2)} \end{cases}$$

From (1), $3 - V = 3(2 - V) = 6 - 3V$

$$\Rightarrow 2V = 3 \Rightarrow V_{th} = 1.5(V)$$

it is satisfied since $V_{GS} = 2V \geq V_{th} = 1.5V$ (in saturation)

From (2), $3 - V = -3(2 - V) = -6 + 3V$

$$\Rightarrow g = 4V \Rightarrow V = 2.25(V)$$

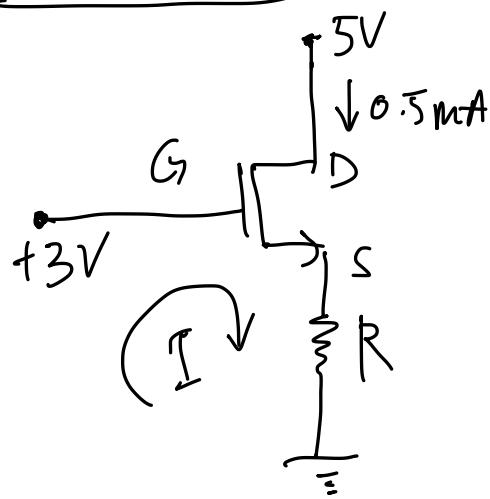
it is not satisfied since $V_{GS} = 2V < V_{th} = 2.25$ (not in saturation)

Also, $1.8 = K (3 - V_{th})^2$

$$\Rightarrow 1.8 = K (3 - 1.5)^2 = 2.25K \Rightarrow K = \frac{1.8}{2.25} = 0.8$$

$$\Rightarrow k = 0.8 (\text{mA/V}^2)$$

Problem 3: Given $V_{th} = 1V$, $K = 0.5 \text{ mA/V}^2$



Find R .

Assume the transistor is in saturation. We have

$$\begin{cases} V_{GS} \geq V_{th} \\ V_{DS} \geq V_{GS} - V_{th} \end{cases}$$

$$i_D = K(V_{GS} - V_{th})^2 \geq 0.5 = 0.5(V_{GS} - 1)^2$$

$$\Rightarrow 1 = (V_{GS} - 1)^2 \Rightarrow V_{GS} - 1 = 1 \Rightarrow V_{GS} = 2$$

$$V_{GS} - 1 = -1 \Rightarrow V_{GS} = 0$$

Since in saturation $V_{GS} \geq V_{th} \Rightarrow V_{GS} = 0$ is rejected

$$\Rightarrow V_{GS} = 2(V) \geq V_{th} = 1V.$$

Apply KVL for \textcircled{I} , we have:

$$3 = V_{GS} + i_D R \Rightarrow R = \frac{3 - V_{GS}}{i_D} = \frac{(3 - 2)V}{0.5 \text{ mA}}$$

$$\Rightarrow R = 2(\text{k}\Omega)$$

Check: We have $V_S = i_D R = 2 \times 0.5 = 1 V$

$$\Rightarrow V_{DS} = V_d - V_s = 5 - 1 = 4(V)$$

$$V_{GS} - V_{th} = 2 - 1 = 1 (V)$$

Since $V_{DS} = 4V > V_{GS} - V_{th} = 1$, it is satisfied the assumption.

$\Rightarrow R = 2(\text{k}\Omega)$ is correct as we assumed.

Problem 4: Given P-MOSFET Pds $V_{th} = -0.5V$ & $k = 0.2 \text{ mA/V}^2$. Assume in saturation, need to find value of V_{gs} such that $i_D = 0.8 \text{ mA}$.

We have, in saturation P-Mosfet:

$$\begin{cases} V_{gs} \leq V_{th} \\ V_{ds} \leq V_{gs} - V_{th} \end{cases} \quad \& i_D = k(V_{gs} - V_{th})^2$$

$$\Rightarrow 0.8 = 0.2(V_{gs} + 0.5)^2 \Rightarrow (V_{gs} + 0.5)^2 = 4$$

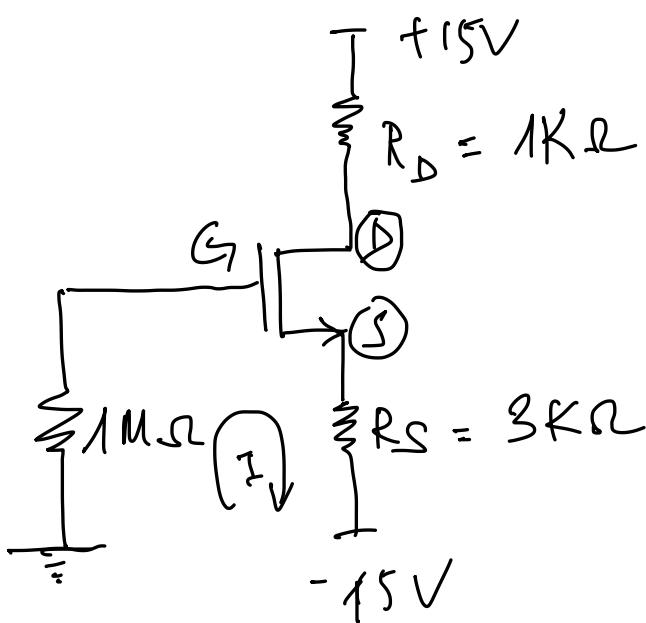
$$\Rightarrow V_{gs} + 0.5 = 2 \Rightarrow V_{gs} = 1.5(V)$$

$$V_{gs} + 0.5 = -2 \Rightarrow V_{gs} = -2.5(V)$$

Since $V_{gs} \leq V_{th} = -0.5V$

$$\Rightarrow V_{gs} = -2.5(V)$$

Prob Pm5: Given $V_{th} = 1V$, $K = 0.25mA/V^2$



Find I_{DQ} & V_{DSQ}

Assume Saturation, we have:

$$\begin{cases} V_{GS} \geq V_{th} = 1V \\ V_{DS} \geq V_{GS} - V_{th} \end{cases}$$

$$i_D = K(V_{GS} - V_{th})^2 = 0.25(V_{GS} - 1)^2 \quad (1)$$

Apply KVL for loop (I), we have:

$$V_{GS} + i_D R_S - 15 = 0 \Rightarrow V_{GS} + i_D 3 = 15$$

$$\Rightarrow V_{GS} = 15 - 3i_D$$

$$\text{From (1)} \Rightarrow i_D = 0.25(15 - 3i_D - 1)^2$$

$$\Rightarrow 0.25(14 - 3i_D)^2 = i_D$$

$$\Rightarrow 49 - 21i_D + 2.25i_D^2 - i_D = 0$$

$$\Rightarrow 2.25i_D^2 - 22i_D + 49 = 0$$

$$\Rightarrow \begin{cases} i_D = 6.346 \text{ mA} \Rightarrow v_{GS} = -4.038(V) \\ i_D = 3.432 \text{ mA} \Rightarrow v_{GS} = 4.704(V) \end{cases}$$

Since $v_{GS} \geq V_{th} = 1V$ (Saturation)

$$\Rightarrow v_{GSQ} = 4.704(V) \quad & i_{DQ} = 3.432(\text{mA})$$

* Find v_{DSQ}

Apply KVL for II, we have:

$$-15 + i_D(R_D + R_S) + v_{DS} - 15 = 0$$

$$\Rightarrow v_{DS} + 3.432(1+3) = 30$$

$$\Rightarrow v_{DS} = 30 - 13.728$$

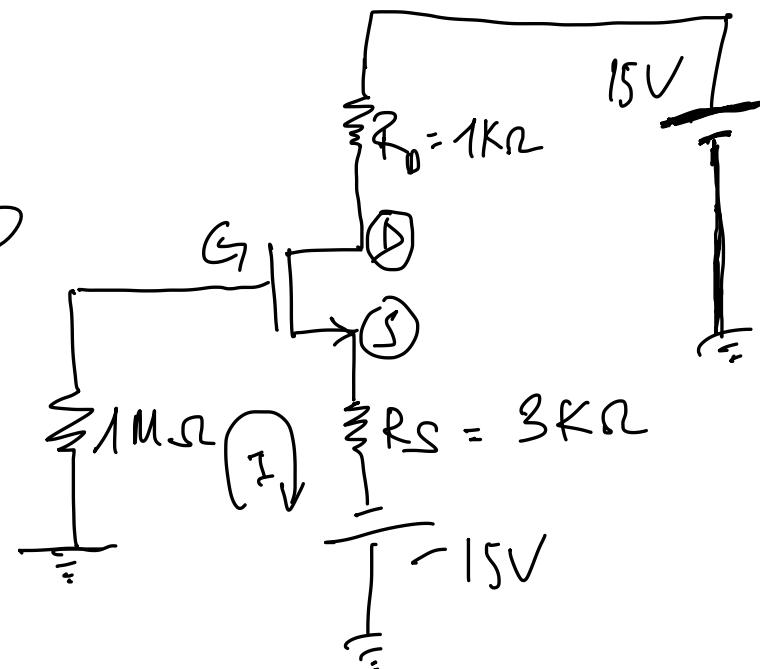
$$\Rightarrow v_{DS} = 16.272(V)$$

$$\text{Check: } v_{GS} - v_{th} = 4.704 - 1 = 3.704(V)$$

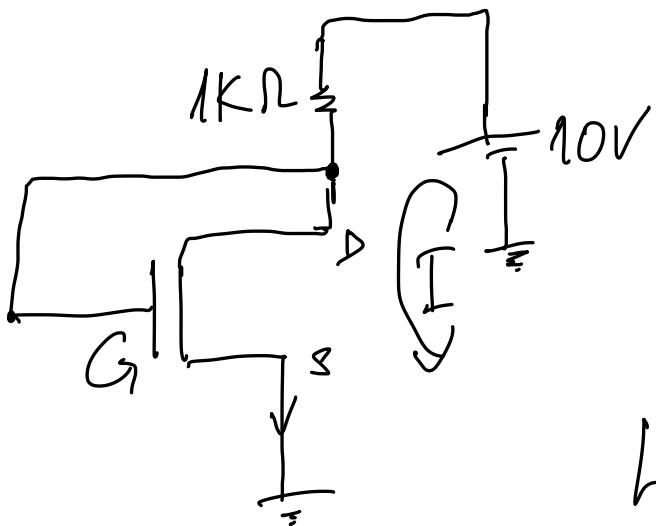
$$\text{Since } v_{DS} = 16.272(V) > v_{GS} - v_{th} = 3.704(V)$$

\Rightarrow Satisfied the assumption

$$\Rightarrow v_{DSQ} = 16.272(V)$$



Problem 6: Given $V_{th} = 1V$, $K = 0.25 \text{ mA/V}^2$



Find I_D & V_{DS}

Assume saturation:

$$V_{GS} \geq V_{th}$$

$$V_{DS} \geq V_{GS} - V_{th}$$

$$I_D = K(V_{GS} - V_{th})^2 = 0.25(V_{GS} - 1)^2 \quad (1)$$

Apply KVL for (1), we have:

$$-10 + I_D R + V_{DS} = 0$$

$$\Rightarrow I_D + V_{DS} = 10. \text{ Since } V_g = V_D$$

$$\Rightarrow I_D + V_{GS} = 10 \Rightarrow I_D = 10 - V_{GS}$$

Plug this into (1), we have:

$$10 - V_{GS} = 0.25(V_{GS} - 1)^2 = 0.25(V_{GS}^2 - 2V_{GS} + 1)$$

$$\Rightarrow 10 - V_{GS} = 0.25V_{GS}^2 - 0.5V_{GS} + 0.25$$

$$\Rightarrow 0.25v_{gs}^2 + 0.5v_{gs} - 9.75 = 0$$

$$\Rightarrow v_{gs} = 5.324 \text{ (satisfied)}$$

$$v_{gs} = -7.324 \text{ (rejected since } v_{gs} < V_{th})$$

$$\Rightarrow i_D = 10 - v_{gs} = 10 - 5.324$$

$$\Rightarrow i_D = 4.676 \text{ mA} \quad \& \quad v_{DSQ} = v_{DS} = 5.324 \text{ V}$$

Check: $v_{gs} - V_{th} = v_{gs} - V_{th} = 5.324 - 1$
 $= 4.324$

$$\Rightarrow v_{ds} = 5.324 > v_{gs} - V_{th} = 4.324$$

\Rightarrow it is consistent with the assumption.

Problem 7: In transistor small signal analysis

Definition of g_m & s_d

* The relationship between V_{GS} and I_D is called Trans conductance, and denoted by g_m . This relationship is the partial derivative of drain current i_D with the gate source voltage V_{GS} at Q point, then

$$g_m = \left. \frac{\partial i_D}{\partial V_{GS}} \right|_{Q\text{-point}}$$

* The drain resistance is the partial derivative of the drain current i_D with the V_{DS} at Q point

$$\frac{1}{s_d} = \left. \frac{\partial i_D}{\partial V_{DS}} \right|_{Q\text{-point}}$$

Problem 8: Given an unusual type of FET with

$$i_D = 3v_{gs}^3 + 0.1v_{ds} \quad , \quad v_{gSQ} = 1V, v_{dSQ} = 10V$$

Need to find value of g_m & λ_d , $\left. \begin{array}{l} i_D (\text{mA}) \\ v_{gSQ} (\text{V}) \\ v_{dSQ} (\text{V}) \end{array} \right\}$

We Have : $g_m = \frac{\partial i_D}{\partial v_{gs}} \Big| Q \text{ point}$

$$\frac{\partial i_D}{\partial v_{gs}} = g v_{gs}^2$$

$$\Rightarrow \boxed{g_m = g v_{gSQ}^2 = g (\text{mS})}$$

$$\frac{1}{\lambda_d} = \frac{\partial i_D}{\partial v_{ds}} \Big| Q \text{ point}$$

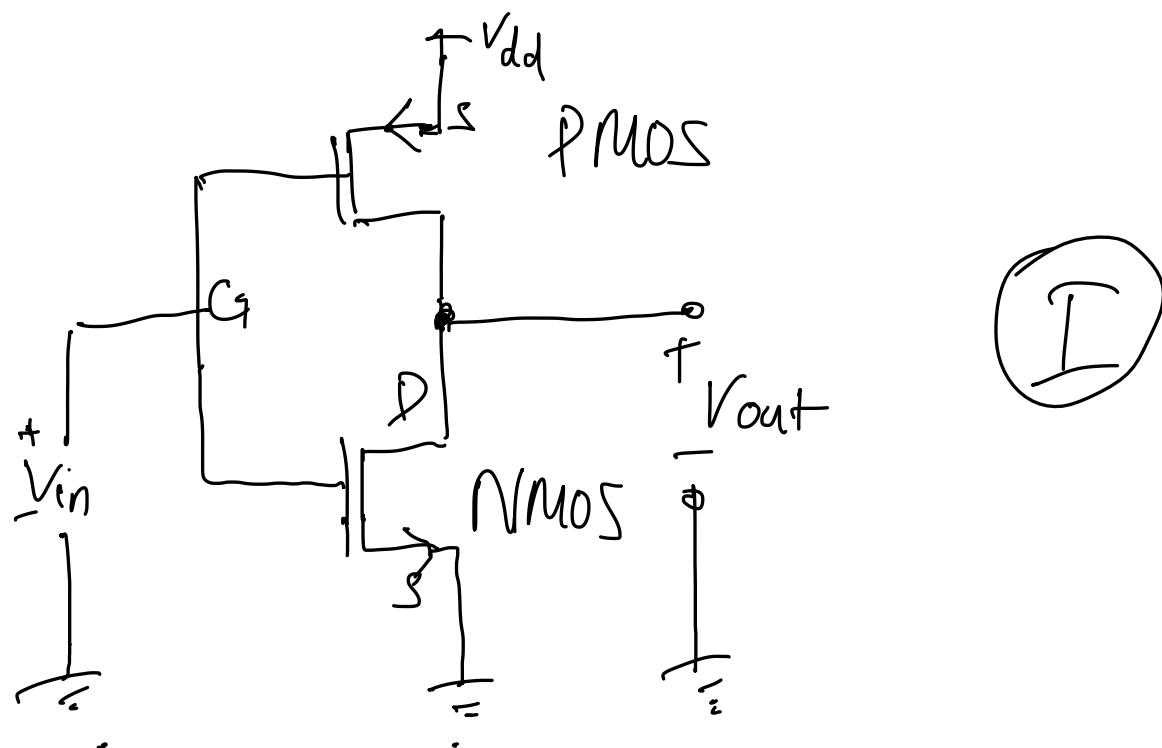
Also, $\frac{\partial i_D}{\partial v_{ds}} = 0.1 \Rightarrow \frac{1}{\lambda_d} = 0.1$

$$\Rightarrow \boxed{\lambda_d = 10 (\text{K} \cdot \Omega)}$$

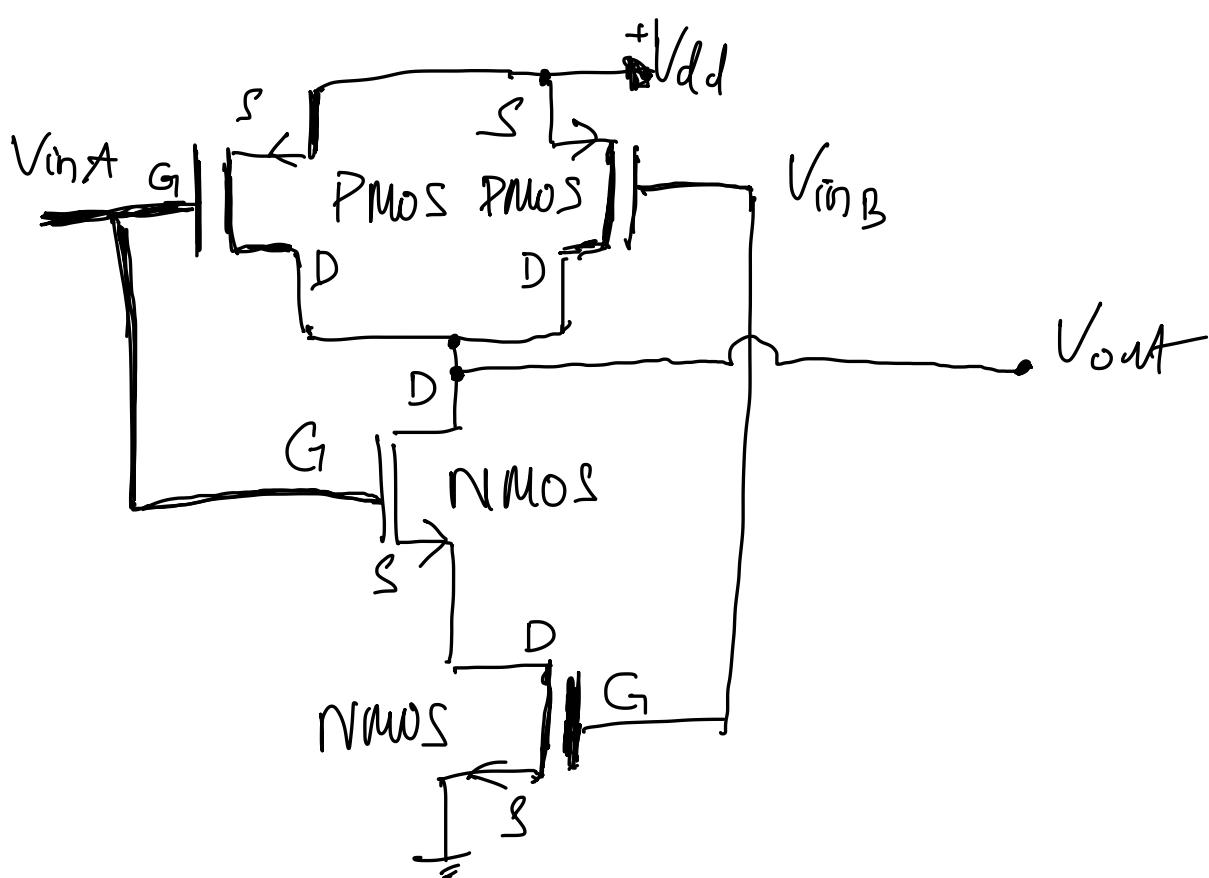
Problem 1: Need to draw the circuit diagram of a two-input CMOS AND gate

As the hint, we use an inverter connect to the output of the two input CMOS NAND gate

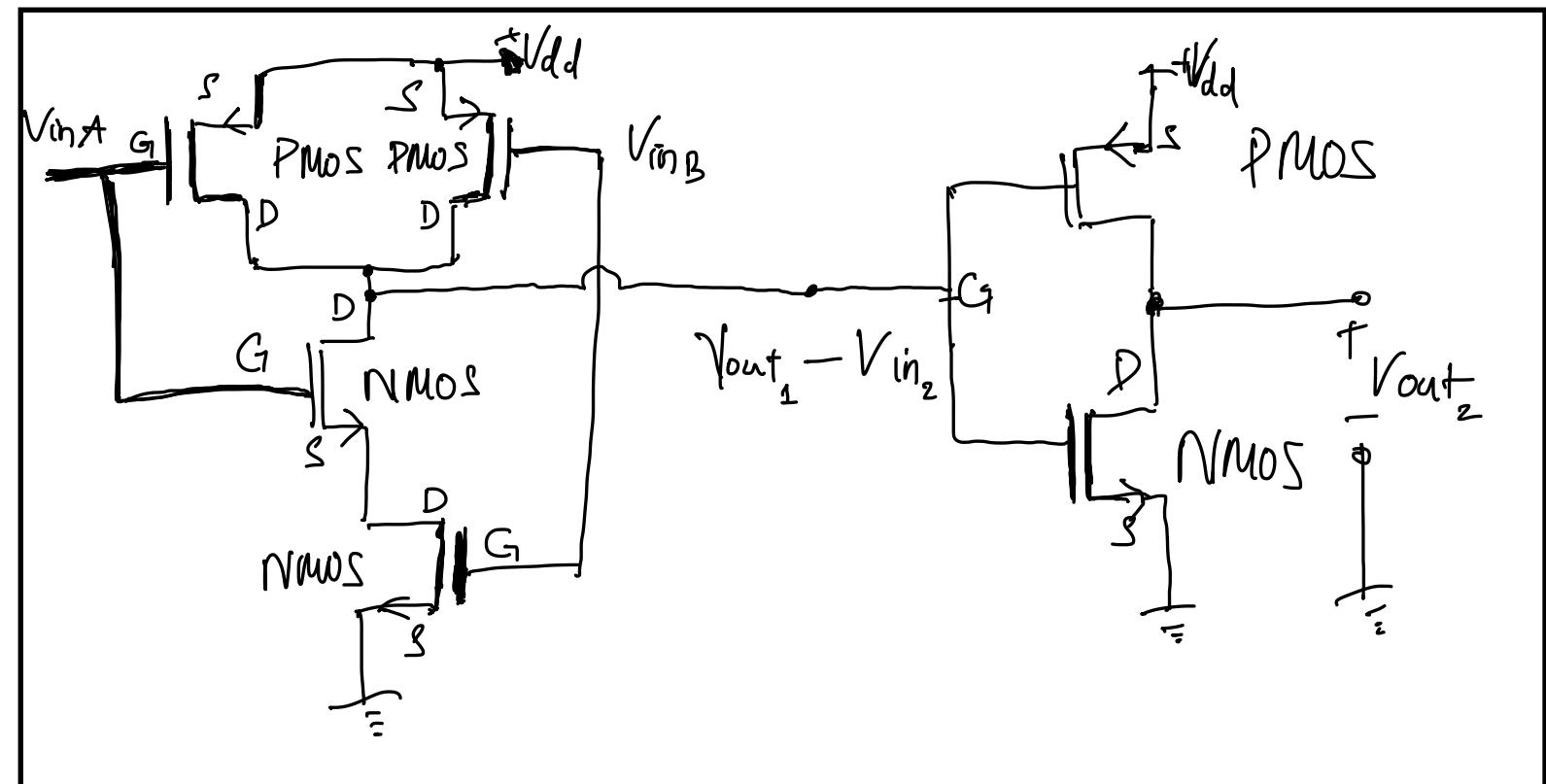
As we learn in the class, we have a diagram for the CMOS inverter below:



We also learn about the two-input CMOS NAND gate.



From (I) & (II), we have the circuit diagram of two-input CMOS AND gate.



Problem 10: Given $V_{th} = 3V$, $K = 0.5 \text{ mA/V}^2$

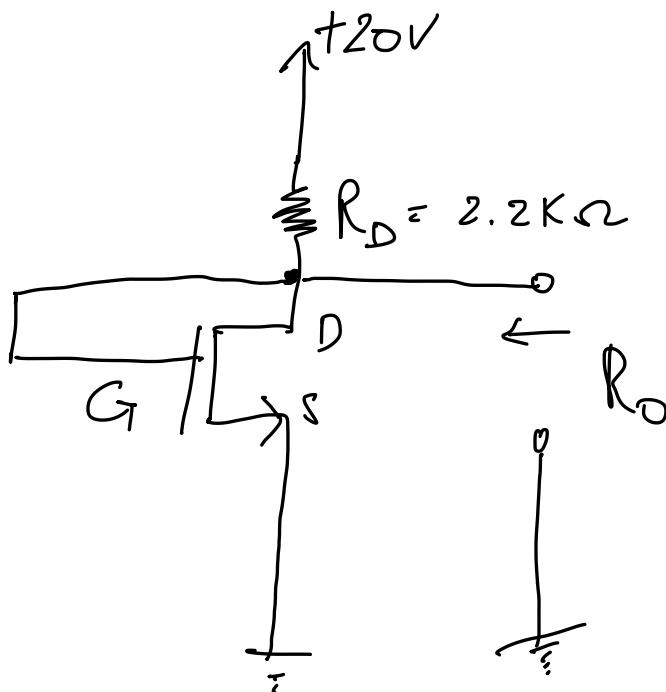
* Find V_{DSQ} & I_{DQ}

We have $V_G = V_D$

$$\Rightarrow V_{DS} = V_{G,S}$$

Assume Saturation:

$$\begin{cases} V_{GS} \geq V_{th} = 3V \\ V_{DS} \geq V_{GS} - V_{th} \end{cases}$$



$$\& i_D = K(V_{GS} - V_{th})^2 = 0.5(V_{GS} - 3)^2 \quad (1)$$

Also, apply KVL, we have:

$$20 = i_D R_D + V_{DS} \quad \text{Since } V_{DS} = V_{GS}$$

$$\Rightarrow 20 = i_D \times 2.2 + V_{GS} \Rightarrow i_D = \frac{20 - V_{GS}}{2.2} \text{ (mA)}$$

Plug into (1), we have:

$$\frac{20 - V_{GS}}{2.2} = 0.5(V_{GS} - 3)^2$$

$$\Rightarrow 20 - \upsilon_{gs} = 1.1 (\upsilon_{gs}^2 - 6\upsilon_{gs} + 9)$$

$$\Rightarrow 20 - \upsilon_{gs} = 1.1 \upsilon_{gs}^2 - 6.6 \upsilon_{gs} + 9.9$$

$$\Rightarrow 1.1 \upsilon_{gs}^2 - 5.6 \upsilon_{gs} - 0.1 = 0$$

$\Rightarrow \boxed{\upsilon_{gs} = 6.5 \text{ (V)}}$ Satisfied assumption.

$\boxed{\upsilon_{gs} = -1.412 \text{ (V)}}$ Rejected since $-1.412 < V_{th} = 3$

$$\Rightarrow \boxed{\upsilon_{g_{sQ}} = 6.5 \text{ (V)}}$$

$$I_{DQ} = \frac{20 - \upsilon_{g_{sQ}}}{2.2} = 6.1364 \text{ (mA)}$$

Check: $V_{DS} = V_{GS} \& V_{th} = 3 > 0$

$\Rightarrow \upsilon_{DS} = V_{GS} \geq \upsilon_{gs} - V_{th} \Rightarrow$ Satisfied the assumption.

* Find value of g_m .

$$I_{DQ} = k (V_{g_{sQ}} - V_{th})^2$$

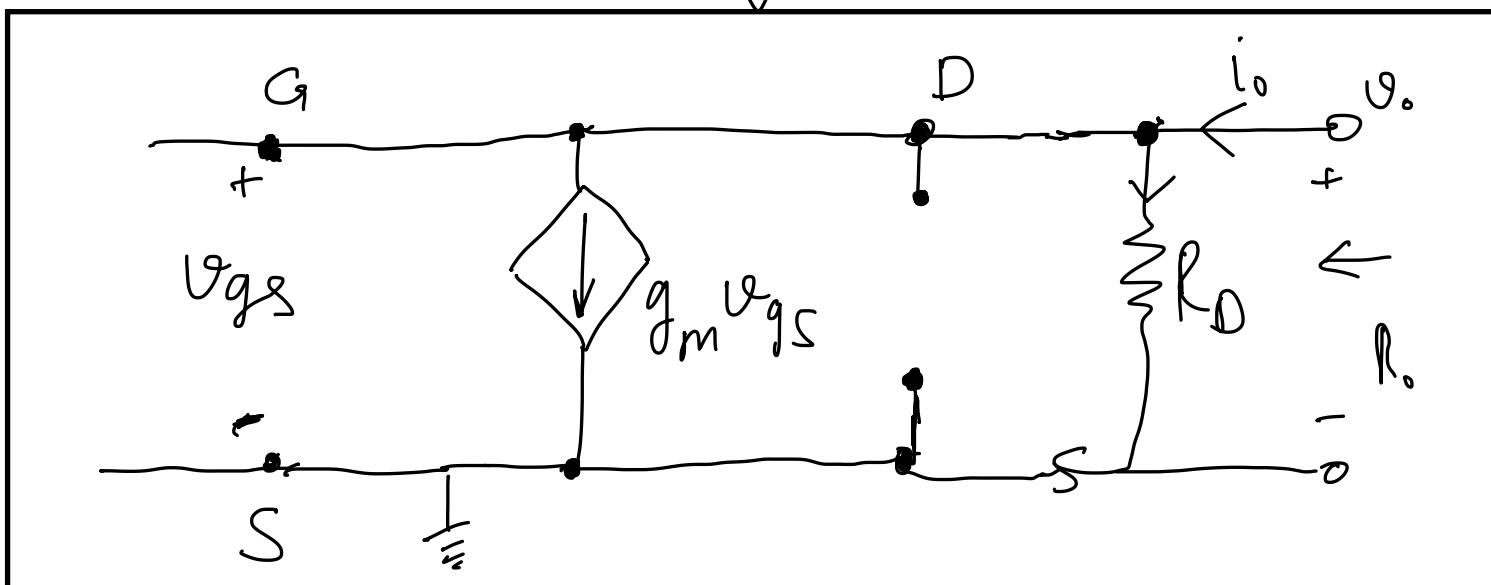
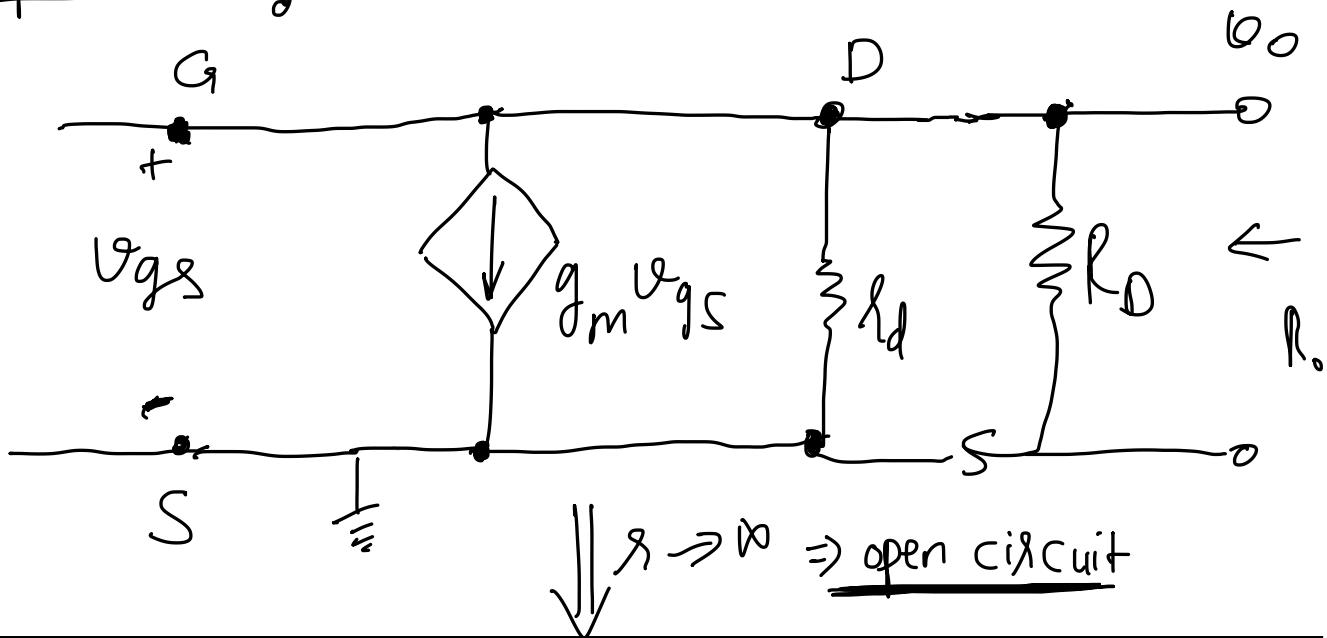
$$g_m = \left. \frac{\partial I_{DQ}}{\partial V_{GS}} \right|_{Q\text{point}} = 2K(V_{G,50} - V_{th})$$

$$\Rightarrow g_m = 2 \times 0.5 \times (6.5 - 3) = 3.5 (\mu S)$$

$$= 3.5 (mA/V)$$

* Draw the small-signal equivalent:

With assumption the $I_d \rightarrow 0$



* Apply KCL at D, we have:

$$i_o = \frac{v_o}{R_D} + g_m v_{gs}, \text{ also } v_o = v_{gs}$$

$$\Rightarrow i_o = \frac{v_o}{R_D} + g_m v_o. \text{ Also, } i_o = \frac{v_o}{R_D}$$

$$\Rightarrow \frac{v_o}{R_D} = \frac{v_o}{R_D} + g_m v_o \Rightarrow \frac{1}{R_D} = g_m + \frac{1}{R_D}$$

$$\Rightarrow R_D = \frac{R_D}{1 + g_m R_D} = \frac{1}{g_m + \frac{1}{R_D}}$$

$$\Rightarrow R_D = \frac{1}{3.5(\text{mA/V}) + \frac{1}{2.2\text{k}\Omega}} = 0.2529(\text{k}\Omega)$$

$$\Rightarrow R_D = 252.874 \Omega$$