Nhat Ho - Quiz 2.

Problem 1:

a) $i_1(t) = 4\cos(\omega t + 30) = 4\angle 30^\circ$

$i_2(t) = 5\sin(\omega t - 20) \Rightarrow i_2(t) = 5\cos(\frac{\pi}{2} - \omega t + 20)$

$\Rightarrow i_2(t) = 5\cos(-\omega t + 110^\circ) = 5\cos(\omega t - 110^\circ)$

$\Rightarrow i_2 = 5\angle -110^\circ$

$\Rightarrow i_T = i_1(t) + i_2(t) = 4\angle 30^\circ + 5\angle -110^\circ$

$= 3.22\angle -56.98^\circ$

$\Rightarrow \boxed{i_T(t) = 3.22\cos(\omega t - 56.98^\circ)}$ (A)

b) $4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75)$

$= 50\angle 75^\circ$

Also $\omega = 2$ (rad/s)

We have: in phasor domain, we change the previous equation to:

$$4i(t) + 8i(t) \times \frac{1}{2j} - 3i(t) \times 2j$$

$$= 50 \angle 75^\circ$$

Since $\int_{-\infty}^{t} dt \Leftrightarrow \frac{1}{jw} = \frac{1}{w} \angle -90^\circ$

$$= \frac{1}{2j}$$

& $\frac{d}{dt} = jw = 2j$

$\Rightarrow$ $4i(t) + 8i(t) \times \frac{1}{2j} - 3i(t) \times 2j$

$$= 50 \angle 75^\circ$$

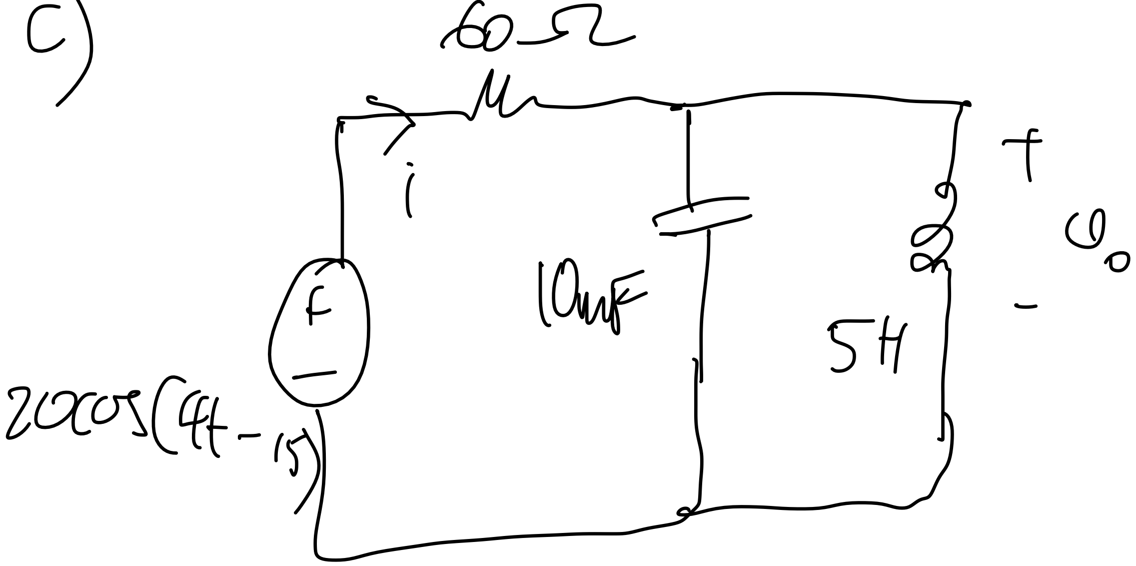
$\Rightarrow i(t) \left[2 - 5j\right] = \frac{50}{2} \angle 75^\circ$

$\Rightarrow$ $= 25 \angle 75^\circ$

$$\Rightarrow i(t) = 4.64 \angle 143.198^{\circ} \ (A)$$

$$\Rightarrow \boxed{i(t) = 4.64 \cos(2t + 143.198^{\circ}) \ (A)}$$

c)




We have: $\omega = 4$ (rad/s)

$$\Rightarrow Z_C = \frac{-j}{\omega C} = \frac{-j}{4 \times 10 \times 10^{-3}}$$

$$= -25j\ (\Omega)$$

$$Z_L = j\omega L = j \times 4 \times 5 = 20j\ (\Omega)$$

$$\text{Since } Z_C \parallel Z_L \Rightarrow Z_{CL} = \frac{Z_C \times Z_L}{Z_C + Z_L}$$

$$= \frac{-25 \times 20 j^2}{-25j + 20j} = \frac{500}{-5j} = -\frac{100}{j} = \frac{-100j}{j^2}$$

$$\Rightarrow Z_{CL} = 100j\ (\Omega)$$

$$\Rightarrow Z_{eq} = R + Z_{CL} = 60 + 100j\ (\Omega)$$

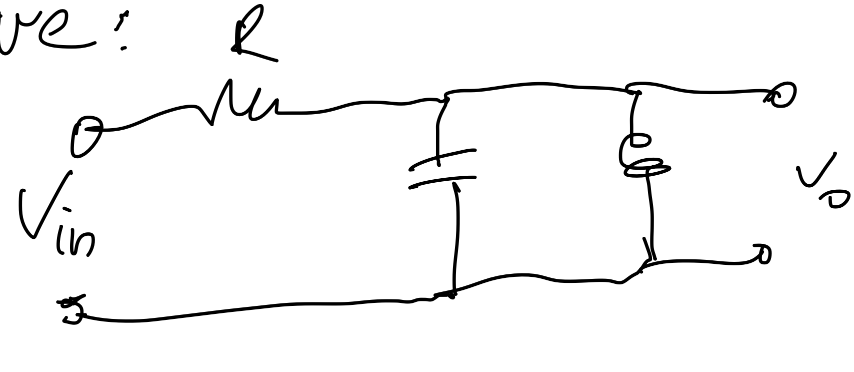
$$\Rightarrow i = \frac{V}{Z_{eq}} = \frac{20\angle -15^\circ}{60 + 100j}$$

$$\Rightarrow v_o = i \times Z_{CL} = \frac{20\angle -15}{60+100j} \times 100j$$

$$= 17.15\ \angle 15.964^\circ$$

$$\Rightarrow \boxed{v_o(t) = 17.15 \cos(4t + 15.964^\circ)}$$

d) We have:



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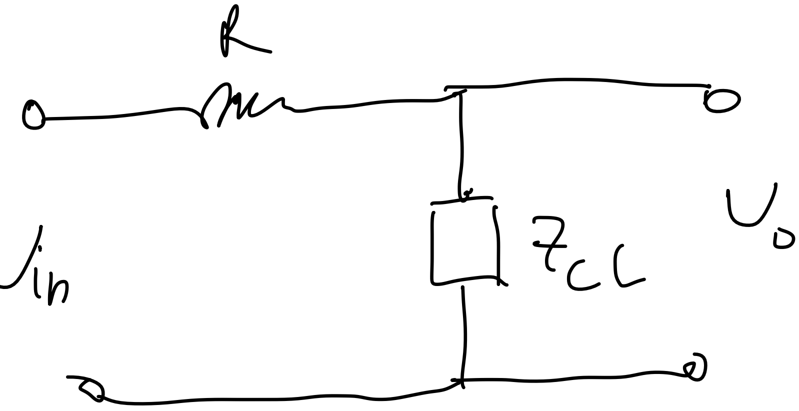



$$\frac{V_o}{V_{in}} = \frac{Z_{CL}}{R + Z_{CL}} \quad \text{(voltage divider)}$$

$$= \frac{1}{1 + \frac{R}{Z_{CL}}}$$

$$Z_{CL} = Z_C \parallel Z_L = \frac{Z_C \times Z_L}{Z_C + Z_L}$$

$$Z_C = \frac{1}{\omega C j} \quad \Rightarrow Z_C \times Z_L = \frac{j\omega L}{\omega C j} = \frac{L}{C}$$

$$Z_L = j\omega L$$

$$\Rightarrow Z_C + Z_L = \frac{1}{\omega C j} + j\omega L = \frac{1 + j\omega L j\omega C}{\omega C j}$$

$$= \frac{1 - \omega^2 LC}{\omega C j} \Rightarrow Z_{CL} = \frac{L}{\underline{C}} \cdot \frac{j\omega \underline{C}}{1 - \omega^2 LC}$$

$$\Rightarrow Z_{CL} = \frac{j\omega L}{1-\omega^2 LC}$$

$$\Rightarrow \frac{V_o}{V_{in}} = \frac{1}{1+\frac{R(1-\omega^2 LC)}{j\omega L}}$$

$$= \frac{1}{1+\frac{R}{j}\left(\frac{1}{\omega L} - \omega C\right)}$$

when $\omega \to 0 \Rightarrow \frac{V_o}{V_{in}} \to 0$

When $\omega \to \infty \Rightarrow \frac{V_o}{V_{in}} \to 0$

$\Rightarrow V_{out} \to 0$

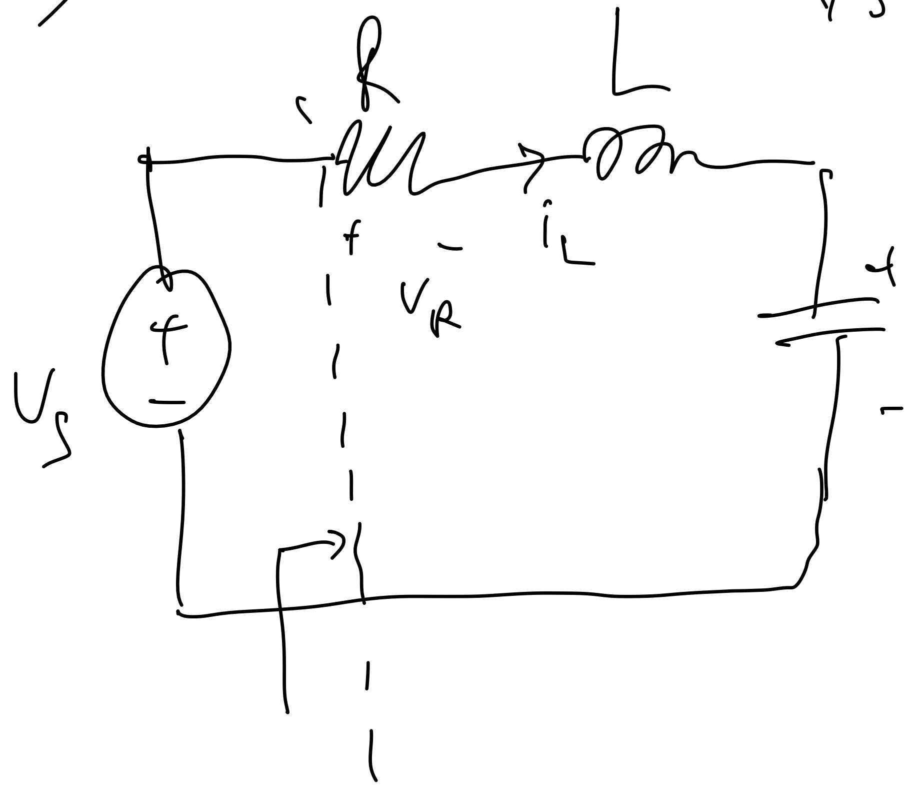
$\Rightarrow$ the type of filter can be construct is

$\boxed{\text{band - pass.}}$

2)

$$V_s(t) = V_m \sin(\omega t)$$

$$\Rightarrow V_s = V_m \angle 0^\circ$$




$$i_C = C\frac{dv_c}{dt}$$

$$i_C = i_L = C\frac{dv_c}{dt}$$

a) $$-v_s + i_L R + v_L + v_c = 0$$

$$\Rightarrow -v_s + i_L R + L\frac{di}{dt} + v_c = 0$$

$$\Rightarrow 0 + R\frac{di_L}{dt} + L\frac{d^2 i}{dt} + \frac{dv_c}{dt} = 0$$

$$\Rightarrow R\frac{di_L}{dt} + L\frac{d^2 i}{dt} + \frac{1}{C} i_L = 0$$

$$\Rightarrow \frac{d^2 i}{dt} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{CL} i_L = 0$$

$$\Rightarrow s^2 + \frac{R}{L} s + \frac{1}{CL} = 0$$

$$\Rightarrow s^2 + 2\xi\omega_0 s + \omega_0^2 = 0$$

$$\Rightarrow s_{1,2} = \left[-\xi \pm \sqrt{\xi^2 - 1}\right]\omega_0$$

$$\Rightarrow i(t) = K_1 e^{s_1(t)} + K_2 e^{s_2 t}$$

$$\Rightarrow \boxed{V_R(t) = \left[K_1 e^{s_1(t)} + K_2 e^{s_2(t)}\right] \times R}$$

b) We have:

$$\begin{cases} Z_L = j\omega L \\ Z_C = -\dfrac{j}{\omega C} \\ R \end{cases} \Rightarrow Z_{eq} = j\omega L - \frac{j}{\omega C} + R$$

$$\Rightarrow i = \frac{V_S}{Z_{eq}} = \frac{V_S}{j\omega L - \frac{j}{\omega C} + R}$$

$$\rightarrow V_R(t) = i \times R$$

$$\Rightarrow V_R(t) = \frac{V_S}{j\omega L - \frac{j}{\omega C} + R} \times R$$

$$V_R(t) = \boxed{\frac{V_m \angle 0^\circ}{j\omega L - \frac{j}{\omega C} + R} \times R} \ (V)$$

c) $\boxed{Z_3 = j\omega L - \frac{j}{\omega C} + R}$ $(\Omega)$

d) $H(\omega j) = \frac{V_R}{V_s(j\omega)}$

We have: $V_R = \frac{V_S \times R}{j\omega L - \frac{j}{\omega C} + R}$

$\Rightarrow \frac{V_R}{V_S} = \frac{R}{R + j\omega L - \frac{j}{\omega C}}$

$\Rightarrow \boxed{H(\omega j) = \frac{R}{R + j\omega L - \frac{j}{\omega C}}}$

e)

$$H(\omega j) = \frac{R}{R + j\omega L - \frac{j}{\omega C}} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{1}{1 + \frac{j}{R}\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{1 + j\,\frac{\omega^2 LC - 1}{R\omega C}}$$

$$\boxed{\Rightarrow |H(\omega j)| = \frac{1}{\sqrt{1 + \left(\frac{\omega^2 LC - 1}{R\omega C}\right)^2}}}$$

$$\boxed{\Rightarrow \angle H(\omega j) = -\arctan\left(\frac{\omega^2 LC - 1}{R\omega C}\right)}$$

f)

g) We have $f_0$ at resonance

$\Leftrightarrow Z_L = Z_C$

$\Rightarrow f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$\Rightarrow \boxed{\omega_0 = 2\pi f_0 = \dfrac{1}{\sqrt{LC}}}$

h) $\omega = \omega_0 \Rightarrow$ the output phase will be equal to 0.

i) $Q_S = \dfrac{2\pi f_0 L}{R} = \dfrac{1}{\sqrt{LC}} \cdot \dfrac{L}{R}$

$= \boxed{\dfrac{1}{R}\sqrt{\dfrac{L}{C}}}$

i) 3dB: $\Rightarrow \Delta\omega = \omega_H - \omega_L = \frac{R}{L} = \frac{\omega_0}{Q_S}$

$\Rightarrow \omega_H$ & $\omega_L$ is the cut-off frequency of half power with 3DB frequency.