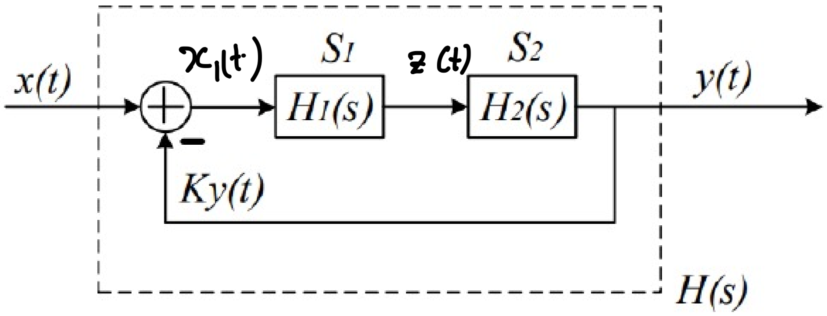
Project

Part 1: Laplace Analysis of a Closed-Loop Control System.

Step 1:

a)



Based on the input-output relationship of S_1 & S_2 system,

* $z(t) = \frac{d}{dt} x_1(t) - x_1(t)$

$z(t) \xrightarrow{L_s} Z(s)$; $x_1(t) \xrightarrow{L_s} X_1(s)$

$\frac{d}{dt} x(t) \xrightarrow{L_s} sX(s) - x(0^-) = sX(s)$

$\Rightarrow Z(s) = sX_1(s) - X_1(s) = (s-1) X_1(s)$

$\Rightarrow \boxed{Z(s) = (s-1) X_1(s)}$ (1)

* $y(t) = \int_{-\infty}^{t} z(\tau) e^{4(\tau - t)} d\tau = \int_{-\infty}^{+\infty} z(\tau) e^{-4(t-\tau)} u(t-\tau) d\tau$

$= \int_{-\infty}^{+\infty} z(\tau) . e^{-4(t-\tau)} u(t-\tau) d\tau = z(t) * h_2(t)$

with $h_2(t) = e^{-4t}u(t)$ & $z(t)$

$\Rightarrow Y(s) = H(s).\ Z(s).$

Also, $e^{-4t}u(t) \xrightarrow{L_s} H(s) = \frac{1}{s+4} \quad Re\{s\} > -4$

$\Rightarrow \boxed{Y(s) = \frac{1}{s+4} Z(s) \quad Re\{s\} > -4}$ (2)

Besides, based on the Figure 1, we have:

$[X(s) - K.Y(s)]\, H_1(s).H_2(s) = Y(s)$

$\Rightarrow X(s).H_1(s)H_2(s) = K\, Y(s) H_1(s) H_2(s) + Y(s)$

$\Rightarrow X(s) H_1(s) H_2(s) = [K H_1(s) H_2(s) + 1]\, Y(s)$

$\Rightarrow \boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s) H_2(s)}{K H_1(s) H_2(s) + 1}}$

b) We have:

$H_1(s) = \frac{Z(s)}{X_1(s)}$

From (1), we have $Z(s) = (s-1)X_1(s)$

$$\Rightarrow \boxed{H_1(s) = s-1, \quad \text{ROC : all } s.}$$

(*) $H_2(s) = \dfrac{Y(s)}{Z(s)}$ From (2) $Y(s) = \dfrac{1}{s+4} Z(s)$

$$\Rightarrow \boxed{H_2(s) = \frac{1}{s+4} \quad \mathrm{Re}\{s\} > -4}$$

c) From part a, $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{H_1(s) H_2(s)}{K H_1(s) H_2(s) + 1}$

$$H_1(s) \cdot H_2(s) = \frac{s-1}{s+4}$$

$$1 + K H_1(s) H_2(s) = 1 + \frac{K(s-1)}{s+4} = \frac{s+4+Ks-K}{s+4}$$

$$= \frac{(K+1)s+4-K}{s+4}$$

$$\Rightarrow H(s) = \frac{(s-1)/(s+4)}{\frac{(K+1)s+4-k}{s+4}} = \frac{s-1}{(k+1)s+4-k}$$

(*) If $k = -1 \Rightarrow H(s) = \frac{1}{5}(s-1)$ ROC: all s

(*) If $k \neq -1$, $(k+1)s + 4 - k = 0 \Leftrightarrow s = \dfrac{k-4}{k+1}$

$$H(s) = \begin{cases} \frac{1}{5}(s-1), & \text{ROC: all } s \text{ with } k=-1 \\ \frac{s-1}{(k+1)s+4-k}, & \text{ROC: } Re\{s\} > \frac{k-4}{k+1} \text{ with } k \neq -1 \end{cases}$$

d) * With $k=-1$, $H(s) = \frac{1}{5}(s-1)$ ROC: all s $\Rightarrow$ ROC is entire s plane $\rightarrow$ $H(s)$ is a stable system.

* With $k \neq -1$, the $H(s)$ system has one pole

$$s = \frac{k-4}{k+1}, \quad \text{ROC: } Re\{s\} > \frac{k-4}{k+1}$$

Then, to make $H(s)$ is stable system, the pole

$s = \frac{k-4}{k+1} < 0$ to make the ROC includes $j\omega$ axis

$\Leftrightarrow -1 < k < 4$

$\Rightarrow$ Combining two cases, the system is stable when

$$\boxed{-1 \leq k < 4}$$

e) Choose $K_1 = 6$ such that the system unstable

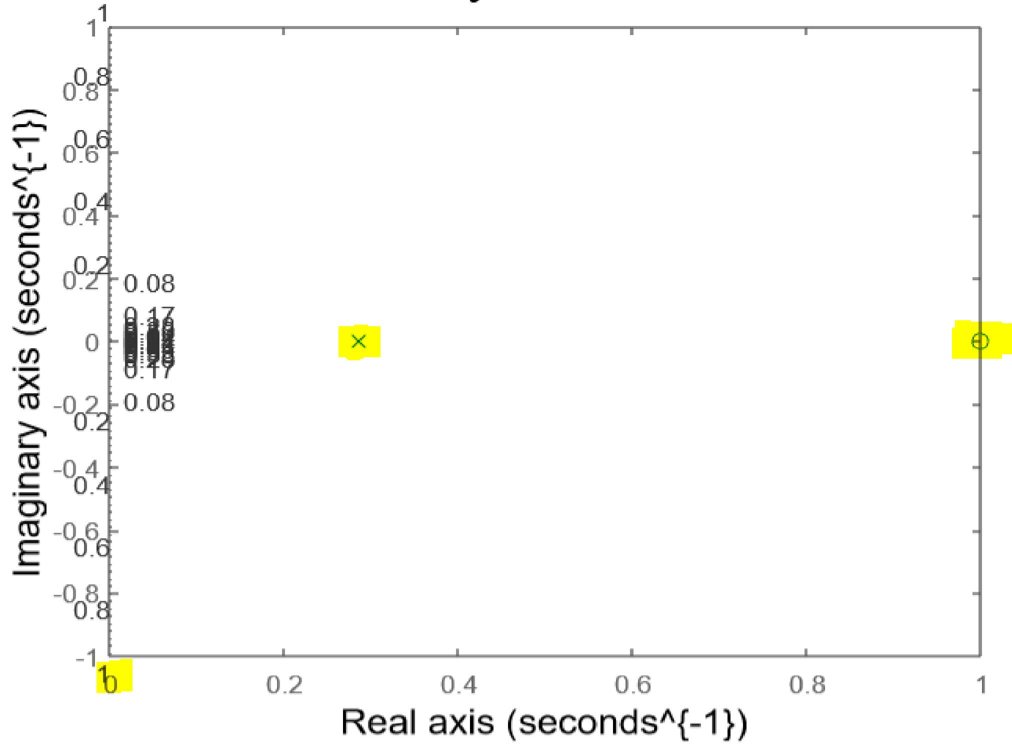
& $K_2 = 3$ such that the system stable

⊛ With $K_1 = 6 \Rightarrow H(s) = \dfrac{s-1}{(k+1)s+4-k} = \dfrac{s-1}{7s+4-6}$

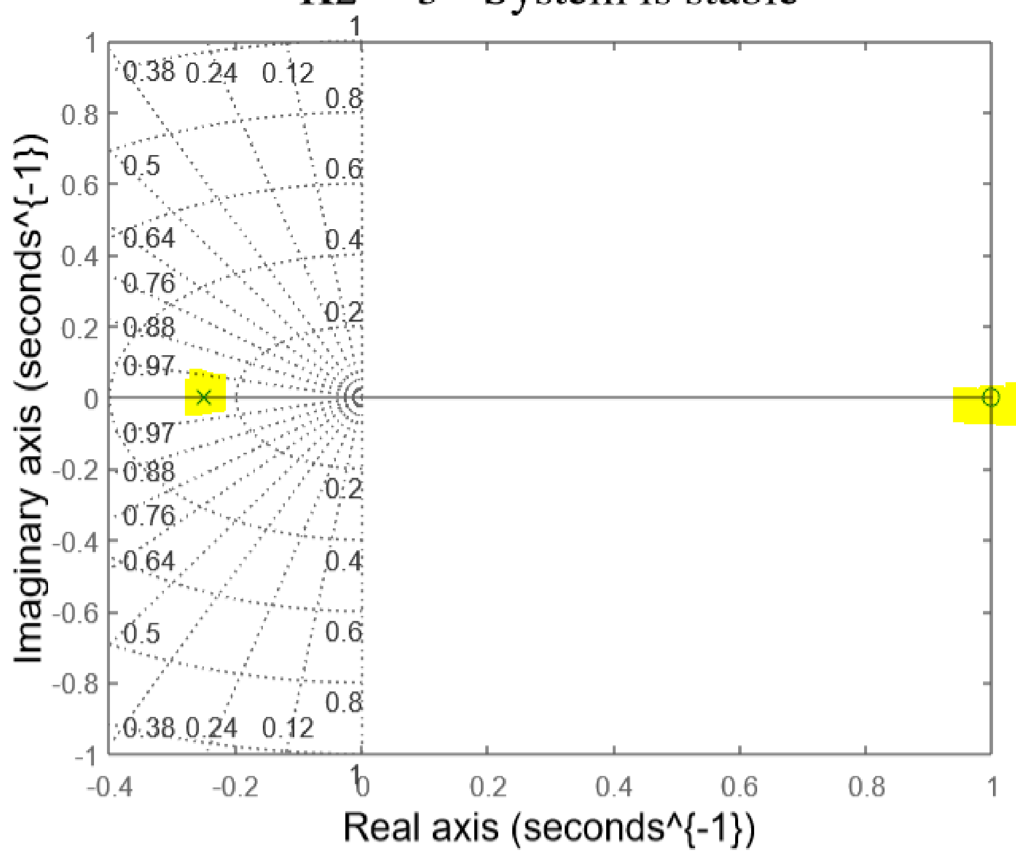
$\Rightarrow H(s) = \dfrac{s-1}{7s-2}$ $Re\{s\} > \frac{2}{7}$ and unstable

⊛ With $K_2 = 3 \Rightarrow H(s) = \dfrac{s-1}{4s+1}$, $Re\{s\} > \frac{-1}{4}$ and stable

K1 = 6 - System is not stable



K2 = 3 - System is stable



Coding part:

```
clear; clc; close all;

sys1 = tf([1 -1],[7 -2]);
figure(1);
h1 = pzplot(sys1);
xlabel('Real axis','Interpreter','latex','fontsize',14);
ylabel('Imaginary axis','Interpreter','latex','fontsize',14);
title(' K1 = 6 - System is not stable','Interpreter','latex','fontsize',18);
grid on;

sys2 = tf([1 -1],[4 1]);
figure(2);
h2 = pzplot(sys2);
xlabel('Real axis','Interpreter','latex','fontsize',14);
ylabel('Imaginary axis','Interpreter','latex','fontsize',14);
title(' K1 = 3 - System is stable','Interpreter','latex','fontsize',18);
grid on;
```

Step 2:

a) $x(t) = \begin{cases} \frac{1}{2}, & 0 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$

* When $K_1 = 6$ & $H(s) = \dfrac{s-1}{7s-2} = \dfrac{1}{7} \cdot \dfrac{s-1}{s-\frac{2}{7}}$ $Re\{s\} > \frac{2}{7}$ (unstable)

$= \frac{1}{7}\left(\dfrac{s - \frac{2}{7} - \frac{5}{7}}{s-\frac{2}{7}}\right) = \frac{1}{7}\left(1 - \frac{5}{7} \cdot \dfrac{1}{s-\frac{2}{7}}\right)$

$= \dfrac{1}{7} - \dfrac{5}{49} \cdot \dfrac{1}{s-\frac{2}{7}}$

$\delta(t) \xrightarrow{L_s} 1 \Rightarrow \frac{1}{7}\delta(t) \longrightarrow \frac{1}{7}$

$e^{\frac{2}{7}t}u(t) \longrightarrow \dfrac{1}{s-\frac{2}{7}} \Rightarrow \dfrac{5}{49} \cdot e^{2/7 t} u(t) \rightarrow \dfrac{5}{49} \cdot \dfrac{1}{s-\frac{2}{7}}$

$$\Rightarrow \boxed{h(t) = \frac{1}{7}\delta(t) - \frac{5}{49} e^{\frac{2}{7}t} u(t)}$$

Applying the convolution for LTI system; we have:

$y(t) = x(t) * h(t)$

$$\circledast \quad K_2 = 3 \Rightarrow H(s) = \frac{s-1}{4s+1} \quad Re\{s\} > \frac{-1}{4} \text{ \& stable}$$

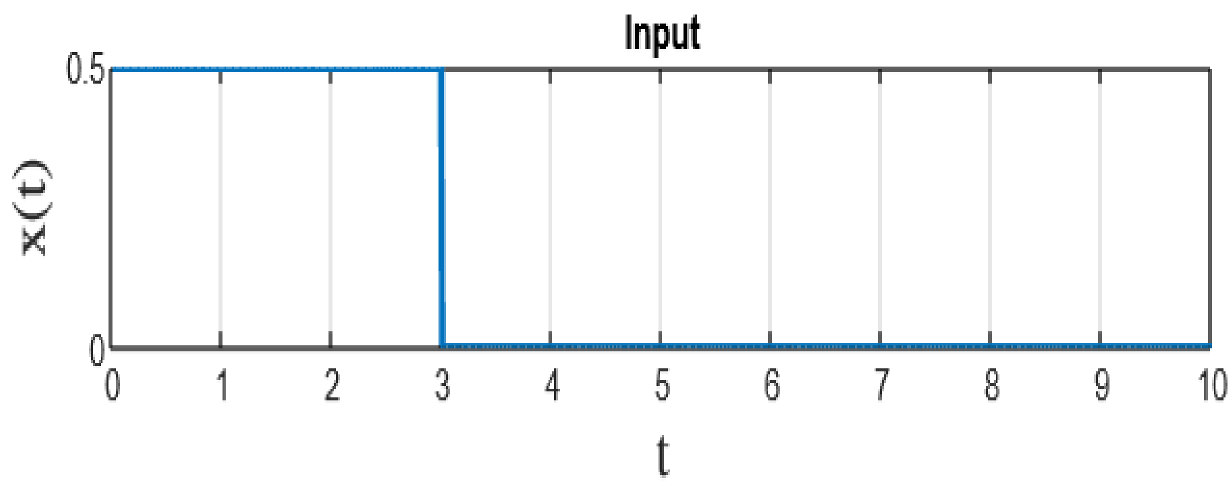
$$\Rightarrow H(s) = \frac{1}{4} \cdot \frac{4(s-1)}{4s+1} = \frac{1}{4}\left[\frac{4s+1-5}{4s+1}\right]$$

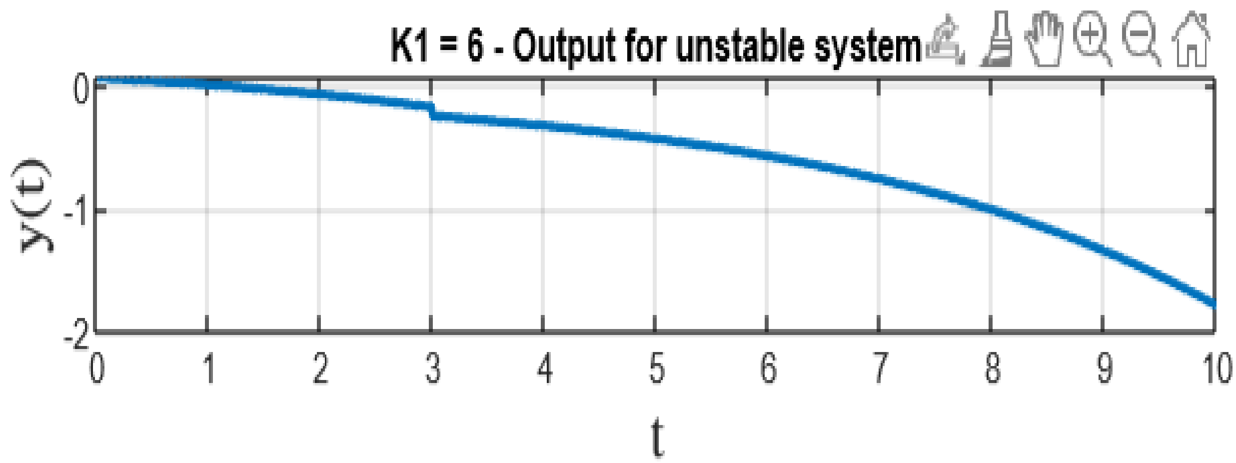
$$= \frac{1}{4}\left[1 - \frac{5}{4(s+1/4)}\right] = \frac{1}{4} - \frac{5}{16} \cdot \frac{1}{s+\frac{1}{4}}$$

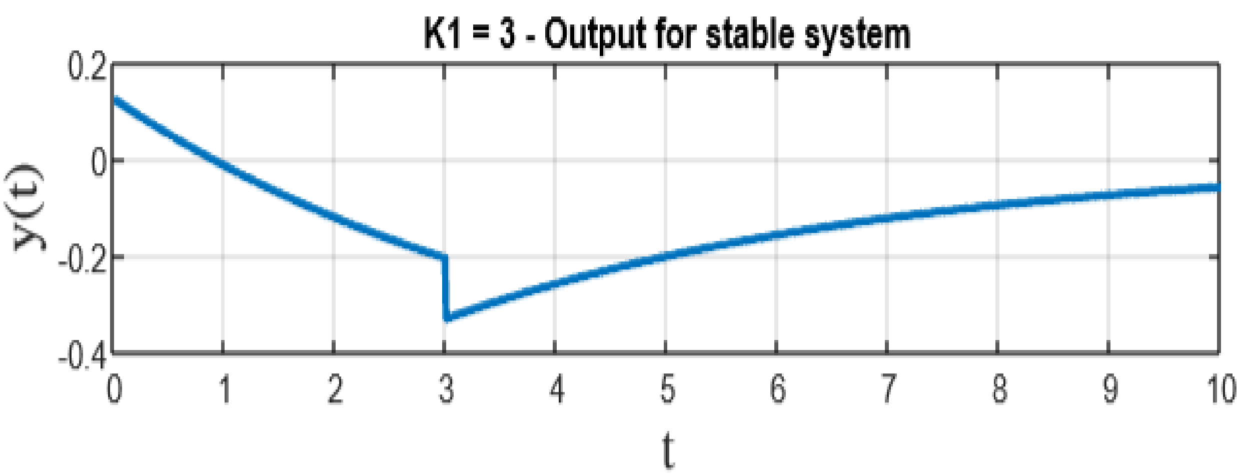
$$\delta(t) \xrightarrow{L_s} 1 \Rightarrow \frac{1}{4}\delta(t) \xrightarrow{L_s} \frac{1}{4}$$

$$\frac{5}{16} e^{-\frac{1}{4}t} u(t) \xrightarrow{L_s} \frac{5}{16} \cdot \frac{1}{s+1/4}$$

$$\Rightarrow \boxed{h(t) = \frac{1}{4}\delta(t) - \frac{5}{16} e^{-1/4 t} u(t).}$$





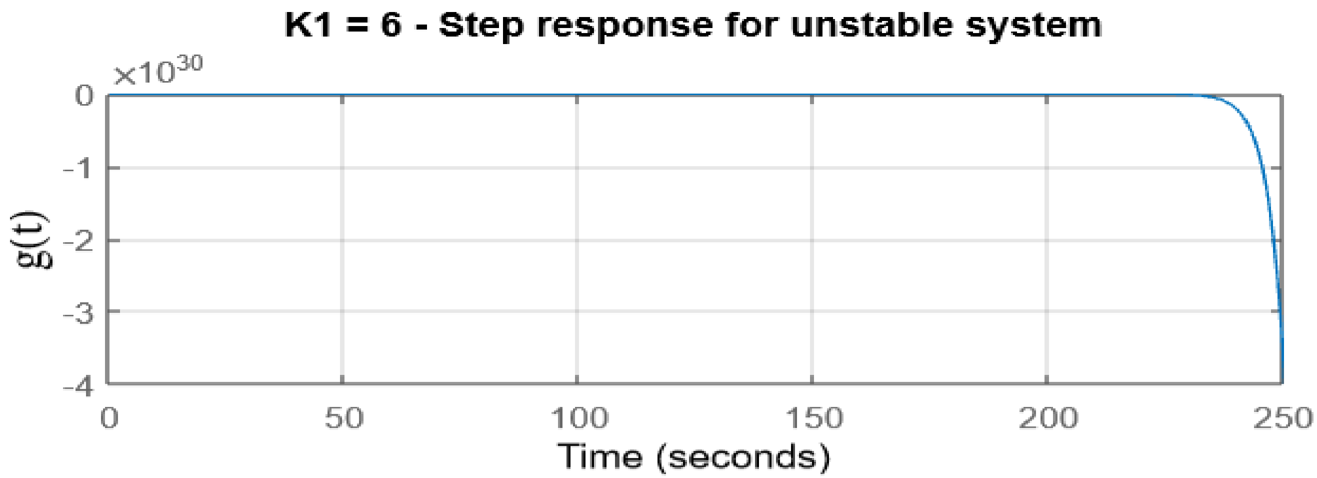


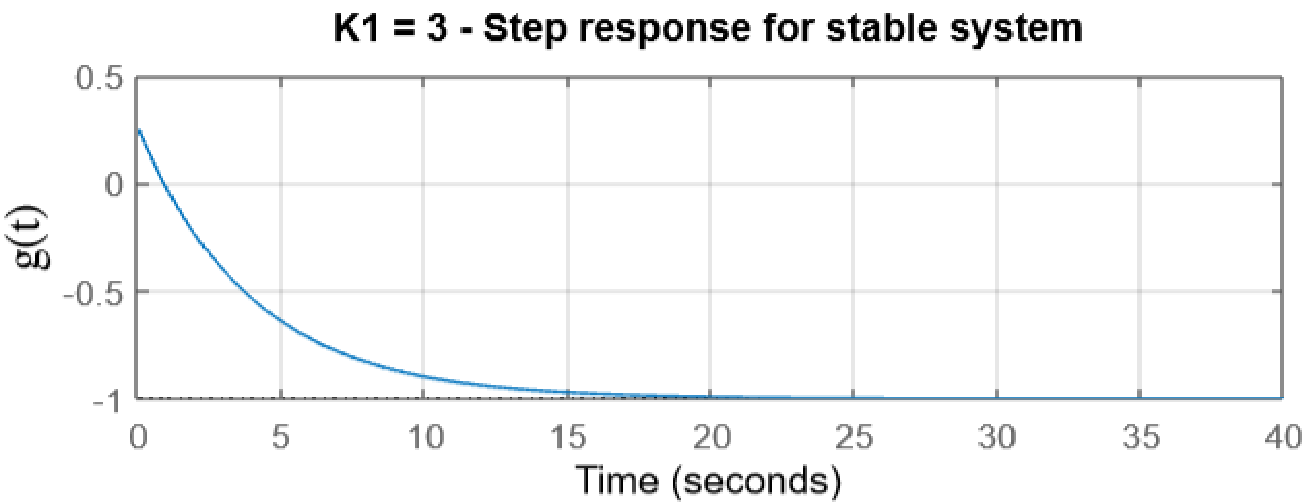
Based on the diagram, we can see the output for both stable or unstable system will be discontinue at t = 3.

Coding part:

```
clear; clc; close all;
sys1 = tf([1 -1],[7 -2]);
sys2 = tf([1 -1],[4 1]);

t = 0:0.01:10;
x = 0.5*double(t<3);
y1 = lsim(sys1,x,t);
y2 = lsim(sys2,x,t);

figure(1);
subplot(3,1,1);
plot(t,x,'LineWidth',2);
grid on;
xlabel('t','Interpreter','latex','fontsize',14);
ylabel('x(t)','Interpreter','latex','fontsize',14);
title('Input');

figure(2);
subplot(3,1,2);
plot(t,y1,'LineWidth',2);
grid on;
xlabel('t','Interpreter','latex','fontsize',14);
ylabel('y(t)','Interpreter','latex','fontsize',14);
title('K1 = 6 - Output for unstable system');

figure(3);
subplot(3,1,3);
plot(t,y2,'LineWidth',2);
grid on;
xlabel('t','Interpreter','latex','fontsize',14);
ylabel('y(t)','Interpreter','latex','fontsize',14);
title('K1 = 3 - Output for stable system');
```

b) Use MATLAB to plot the step response function g(t) for t ∈ [0, 10]





Coding part – continue step 2a.

```
figure(4);
subplot(2,1,1);
step(sys1)
grid on;
ylabel('g(t)','Interpreter','latex','fontsize',14);
title('K1 = 6 - Step response for unstable system');

figure(5);
subplot(2,1,2);
step(sys2);
grid;
ylabel('g(t)','Interpreter','latex','fontsize',14);
title('K1 = 3 - Step response for stable system');
```

Part 2: Control system Frequency Response using the Fourier series and Transform.

Step 3:

From Step 1, we have:

$$H(s) = \begin{cases} \frac{1}{5}(s-1) & \text{ROC: all } s, \quad K=-1 \\ \frac{s-1}{(k+1)s+4-k} & \text{ROC: } Re\{s\} > \frac{K-4}{K+1} \text{ with } K \neq -1 \end{cases}$$

& stable: $-1 \leq K < 4$

a) With $K=2$ ⇒ $H(s)$ is stable system, and

$$H(s) = \frac{s-1}{3s+2} \quad Re\{s\} > \frac{-2}{3}.$$

$$\Rightarrow \boxed{H(\omega) = \frac{j\omega - 1}{3j\omega + 2}}$$

b) Check the range $\omega \in [-10\pi, 10\pi]$

$$H(\omega) = \frac{j\omega - 1}{3j\omega + 2} = \frac{(j\omega-1)(3j\omega-2)}{(3j\omega+2)(3j\omega-2)}$$

$$(j\omega-1)(3j\omega-2) = 3j^2\omega^2 - 2j\omega - 3j\omega + 2$$

$$= -3\omega^2 + 2 - 5j\omega = -(3\omega^2 - 2 + 5j\omega)$$

$$(3j\omega+2)(3j\omega-2) = (3j\omega)^2 - 4$$

$$= -9\omega^2 - 4 = -(9\omega^2+4)$$

$$\Rightarrow H(\omega) = \frac{-3\omega^2 + 2 - 5j\omega}{-(9\omega^2+4)}$$

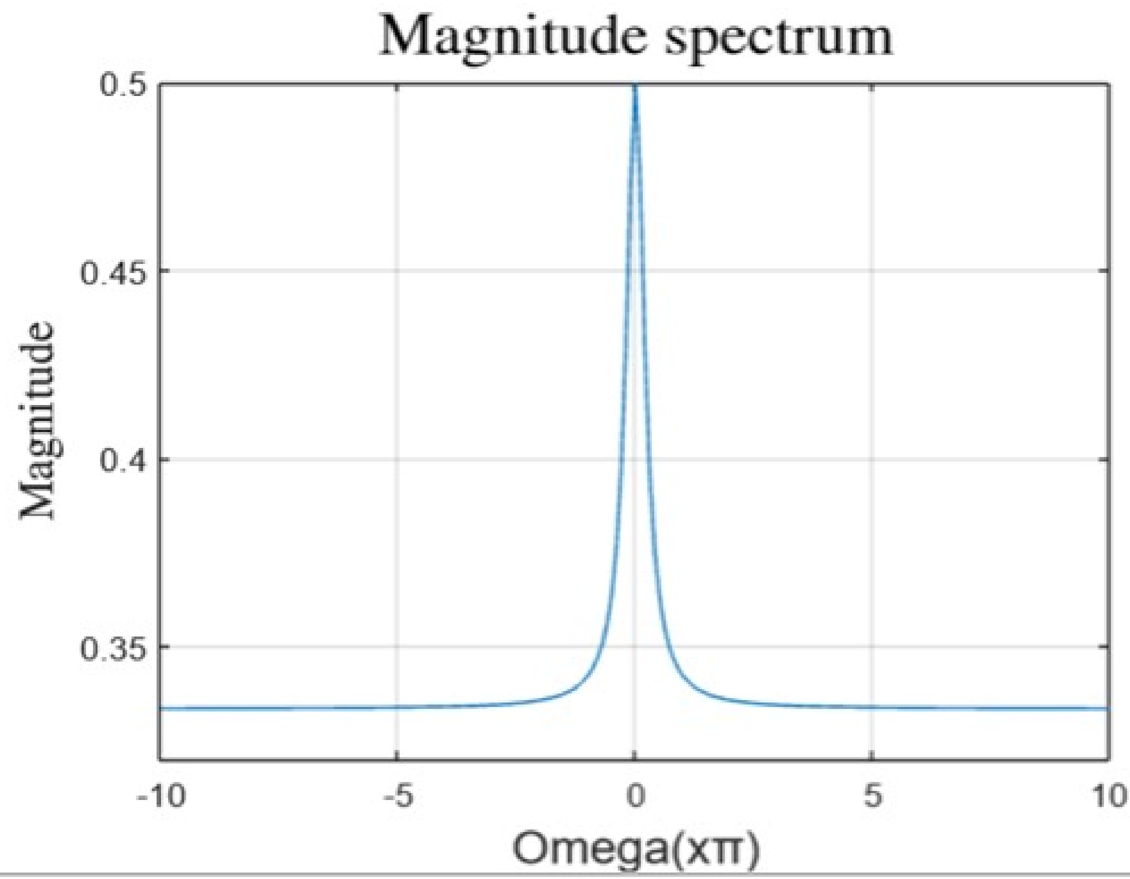
$$\Rightarrow H(\omega) = \frac{3\omega^2 - 2 + 5j\omega}{9\omega^2+4} = \frac{3\omega^2-2}{9\omega^2+4} + j\frac{5\omega}{9\omega^2+4}$$

$$\Rightarrow \boxed{|H(\omega)| = \sqrt{\left(\frac{3\omega^2-2}{9\omega^2+4}\right)^2 + \left(\frac{5\omega}{9\omega^2+4}\right)^2}}$$

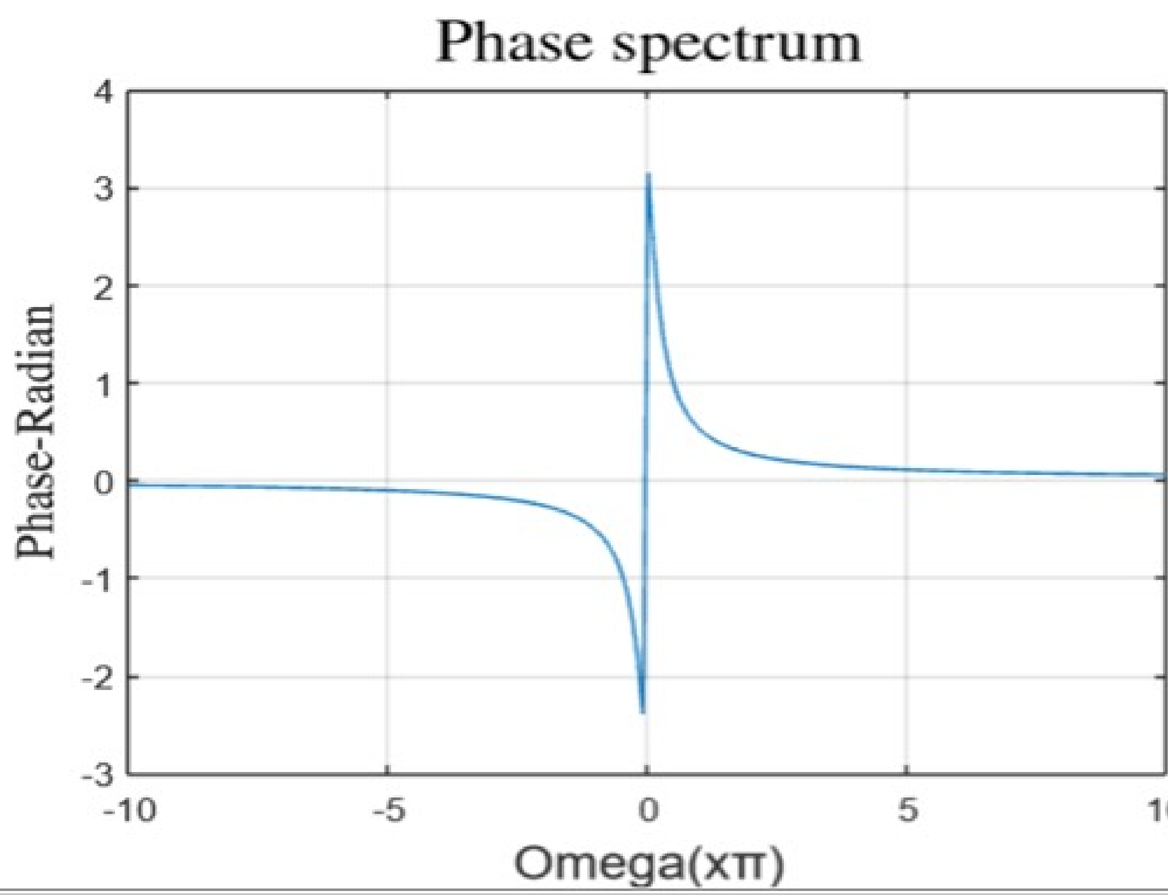
$$\measuredangle H(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\{H(\omega)\}}{\mathrm{Re}\{H(\omega)\}}\right) = \tan^{-1}\left(\frac{5\omega}{9\omega^2+4} \cdot \frac{9\omega^2+4}{3\omega^2-2}\right)$$

$$\Rightarrow \boxed{\measuredangle H(\omega) = \tan^{-1}\left(\frac{5\omega}{3\omega^2-2}\right)}$$

Plot :



Magnitude spectrum
Magnitude
0.5
0.45
0.4
0.35
-10
-5
0
5
10
Omega(xπ)



Phase spectrum
Phase-Radian
4
3
2
1
0
-1
-2
-3
-10
-5
0
5
10
Omega(xπ)

Coding Part

```
clear;
clc;
close all;

w=-10*pi:0.1*pi:10*pi;
H_w=(1i.*w-1)./(3.*1i.*w + 2);

Magnitude = abs(H_w);
Phase = (angle(H_w));

figure(1);
plot(w./pi,Magnitude);
xlabel('Omega(xπ)','Interpreter','latex','fontsize',14);
ylabel('Magnitude','Interpreter','latex','fontsize',14);
title('Magnitude spectrum','Interpreter','latex','fontsize',18);
grid on;

figure(2);
plot(w./pi,Phase);
xlabel('Omega(xπ)','Interpreter','latex','fontsize',14);
ylabel('Phase-Radian','Interpreter','latex','fontsize',14);
title('Phase spectrum','Interpreter','latex','fontsize',18);
grid on;
```

c) Based on the diagram of magnitude spectrum, the system will pass the low frequency and not for the high frequency $\Rightarrow$ this is low pass filter. So, the higher frequency is the one has more heavily attenuated than the lower.

Step 4:

a) $a_1(t) = \begin{cases} t/2, & 0 \le t < 2 \\ 1, & 2 \le t < 4 \end{cases}$

$a_1(t)$ is periodic signal $T_0 = 4(s) \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$

We have: $A_0 = \frac{1}{T_0}\int_{T_0} a_1(t)dt = \frac{1}{4}\left[\int_0^2 \frac{t}{2}dt + \int_2^4 1dt\right]$

$= \frac{1}{4}\left[\frac{t^2}{4}\Big|_0^2 + t\Big|_2^4\right] = \frac{1}{4}\left[(1-0)+(4-2)\right] = \boxed{\frac{3}{4}}$

$A_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt = \frac{1}{4}\left[\int_0^2 \frac{1}{2}te^{-jn\omega_0 t}dt + \int_2^4 e^{-jn\omega_0 t}dt\right]$

$= \frac{1}{8}\int_0^2 te^{-jn\omega_0 t}dt + \frac{1}{4}\int_2^4 e^{-jn\omega_0 t}dt$

⊕ $\int_0^2 te^{-jn\omega_0 t}dt = \int_0^2 te^{at}dt$, $a = -jn\omega_0$, $a^2 = -n^2\omega_0^2$

$= \left(\frac{t}{a} - \frac{1}{a^2}\right)e^{at}\Big|_0^2 = \left(\frac{2}{a} - \frac{1}{a^2}\right)e^{2a} + \frac{1}{a^2}$

⊕ $\int_2^4 e^{-jn\omega_0 t}dt = \int_2^4 e^{at}dt$, $a = -jn\omega_0 t$

$$= \frac{1}{a} e^{at} \Big|_2^4 = \frac{1}{a}\left[e^{4a} - e^{2a}\right]$$

$$\Rightarrow A_n = \frac{1}{8}\left(\frac{2}{a} - \frac{1}{a^2}\right)e^{2a} + \frac{1}{8a^2} + \frac{1}{4a}\left(e^{4a} - e^{2a}\right)$$

$$= \frac{e^{2a}}{4.a} - \frac{e^{2a}}{8a^2} + \frac{1}{8a^2} + \frac{e^{4a}}{4a} - \frac{e^{2a}}{4a}$$

$$= \frac{2a e^{4a}}{8a^2} + \frac{1}{8a^2} - \frac{e^{2a}}{8a^2} = \frac{2ae^{4a} - e^{2a} + 1}{8a^2}$$

$$= \frac{-2jnw_0 e^{-4jnw_0} - e^{-2jnw_0} + 1}{-8n^2w_0^2} = \frac{2jnw_0 e^{-4jnw_0} + e^{-2jnw_0} - 1}{8n^2w_0^2}$$

with $w_0 = \frac{\pi}{2} \Leftrightarrow w_0^2 = \frac{\pi^2}{4}$

$$\Rightarrow A_n = \frac{2jn \cdot \frac{\pi}{2} e^{-4jn\frac{\pi}{2}} + e^{-2jn\pi/2} - 1}{8n^2 \cdot \frac{\pi^2}{4}}$$

$$\Rightarrow A_n = \frac{j\pi n e^{-2nj\pi} + e^{-\pi jn} - 1}{2n^2\pi^2}$$

Also $e^{-2nj\pi} = \cos 2n\pi - j\sin 2n\pi = \cos 2n\pi = 1$

$$e^{-\pi jn} = \cos \pi n - j \sin \pi n = \cos n\pi$$

$$\Rightarrow A_n = \frac{j\pi n \cos 2n\pi + \cos n\pi - 1}{2n^2\pi^2} = \frac{jn\pi + \cos n\pi - 1}{2n^2\pi^2}$$

$$\boxed{\Rightarrow a(t) = \sum_{n=-\infty}^{+\infty} \left[\frac{\cos n\pi - 1}{2n^2\pi^2} + j\frac{n\pi}{2n^2\pi^2} \right] e^{j\frac{\pi}{2}nt} \quad \text{with } X_0 = \frac{3}{4}}$$

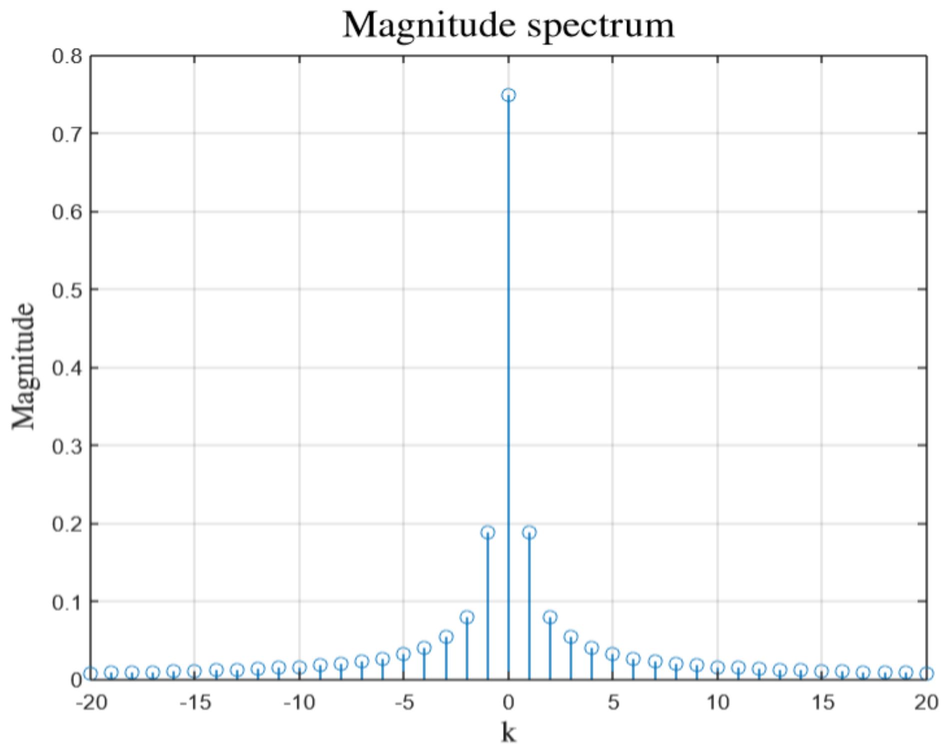
b) $A_n = \frac{\cos n\pi - 1}{2n^2\pi^2} + j\frac{n\pi}{2n^2\pi^2}$

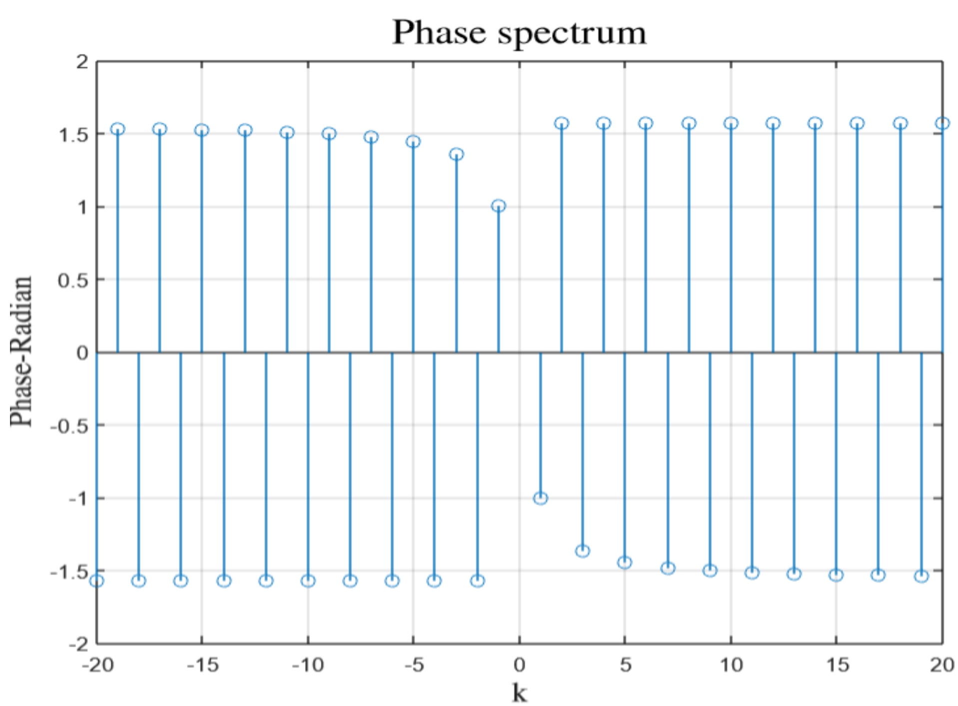
$$\Rightarrow |A_n| = \sqrt{\left(\frac{\cos n\pi - 1}{2n^2\pi^2}\right)^2 + \left(\frac{n\pi}{2n^2\pi^2}\right)^2}, \quad X_0 = \frac{3}{4}$$

$n = \pm 1, \pm 2, \ldots$

$$\measuredangle A_n = \tan^{-1}\left(\frac{\operatorname{Im}\{X_n\}}{\operatorname{Re}\{X_n\}}\right)$$

$$= \tan^{-1}\left(\frac{n\pi}{\cos n\pi - 1}\right)$$





Actually, at k = 0, the phase is going to infinite, but in matlab, it could not show that.

Coding part

```
clc;
close all;
clear all;
phase0 = Inf;
n0 = 3/4;
n1 = linspace(-20,-1,20);
n2 = linspace(1,20,20);

An1 = ((cos(pi.*n1) - 1) ./ (2*(pi.*n1).^2)) + (1i.*n1.*pi)./((2*(pi.*n1).^2));
An2 = ((cos(pi.*n2) - 1) ./ (2*(pi.*n2).^2)) + (1i.*n2.*pi)./((2*(pi.*n2).^2));
Real1 = ((cos(pi.*n1) - 1) ./ (2*(pi.*n1).^2));
Imag1 = (n1.*pi)./((2*(pi.*n1).^2));
Phase1 = atan(Imag1./Real1);
Real2 = ((cos(pi.*n2) - 1) ./ (2*(pi.*n2).^2));
Imag2 = (n2.*pi)./((2*(pi.*n2).^2));
Phase2 = atan(Imag2./Real2);

Mag1 = abs(An1);
Mag2 = abs(An2);
X = [n1, 0, n2];
Mag = [Mag1,n0,Mag2];
Phase = [Phase1, phase0, Phase2];

figure(1)
stem(X,Mag);
xlabel('k','Interpreter','latex','fontsize',14);
ylabel('Magnitude','Interpreter','latex','fontsize',14);
title('Magnitude spectrum','Interpreter','latex','fontsize',18);
grid on;

figure(2)
stem(X,Phase);
xlabel('k','Interpreter','latex','fontsize',14);
ylabel('Phase-Radian','Interpreter','latex','fontsize',14);
title('Phase spectrum','Interpreter','latex','fontsize',18);
grid on;
```

c) With $K=2$, from step 3, we have already:

$$H(\omega) = \frac{j\omega - 1}{3j\omega + 2}$$ is stable system.

Also, from step 4a, we have a(t) has period

$$T_0 = 4(s) \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a(t) = \sum_{n=-\infty}^{+\infty} A_n e^{jn\frac{\pi}{2}t} \Leftrightarrow a(t) = \sum_{k=-\infty}^{+\infty} A_k . e^{jk\frac{\pi}{2}t}$$

Applied for LTI stable system, we have:

$$y(t) = \sum_{k=-\infty}^{+\infty} A_k H(jk\omega_0) e^{jk\omega_0 t}$$

With $$A_k = \frac{\cos k\pi - 1 + j\pi k \cos 2k\pi}{2\pi^2 k^2} = \frac{\cos k\pi - 1 + j\pi k}{2\pi^2 k^2}$$

$$H(jk\omega_0) = \frac{jk\omega_0 - 1}{3jk\omega_0 + 2} = \frac{jk\frac{\pi}{2} - 1}{3jk\frac{\pi}{2} + 2}$$

$$= \frac{jk\pi - 2}{3jk\pi + 4} = \frac{(jk\pi - 2)(3jk\pi - 4)}{(3jk\pi)^2 - 16}$$

$$\text{Numerator} = 3(jk\pi)^2 - 4jk\pi - 6jk\pi + 8$$

$$= -3k^2\pi^2 + 8 - 10jk\pi = -(3k^2\pi^2 - 8 + 10jk\pi)$$

$$\text{Denominator} = -9k^2\pi^2 - 16 = -(9k^2\pi^2 + 16)$$

$$\Rightarrow H(jk\omega_0) = \frac{3k^2\pi^2 - 8 + 10jk\pi}{9k^2\pi^2 + 16}$$

$$\Rightarrow Y_k = A_k H(jk\omega_0) = \frac{\cos k\pi - 1 + j\pi k}{2\pi^2 k^2} \cdot \frac{3k^2\pi^2 - 8 + 10jk\pi}{9k^2\pi^2 + 16}$$

$$\Rightarrow y(t) = \sum_{k=-\infty}^{+\infty} A_k . H(jk\omega_0) e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{+\infty} Y_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} Y_k . e^{jk\frac{\pi}{2}t}$$

$$= \boxed{\sum_{k=-\infty}^{+\infty} \frac{\cos k\pi - 1 + j\pi k}{2\pi^2 k^2} \cdot \frac{3k^2\pi^2 - 8 + 10jk\pi}{9k^2\pi^2 + 16} e^{jk\frac{\pi}{2}t}}$$

✱ Check $(\cos k\pi - 1 + j\pi k)(3k^2\pi^2 - 8 + 10jk\pi)$

$$= (\cos k\pi - 1)(3k^2\pi^2 - 8) + 10jk\pi(\cos k\pi - 1)$$

$$+ j\pi k(3k^2\pi^2 - 8) - 10k^2\pi^2$$

$$= 3k^2\pi^2 \cos k\pi - 8\cos k\pi - 3k^2\pi^2 + 8 - 10k^2\pi^2$$
$$+ j(10k\pi \cos k\pi - 10k\pi + 3k^2\pi^2 \pi k - 8\pi k)$$

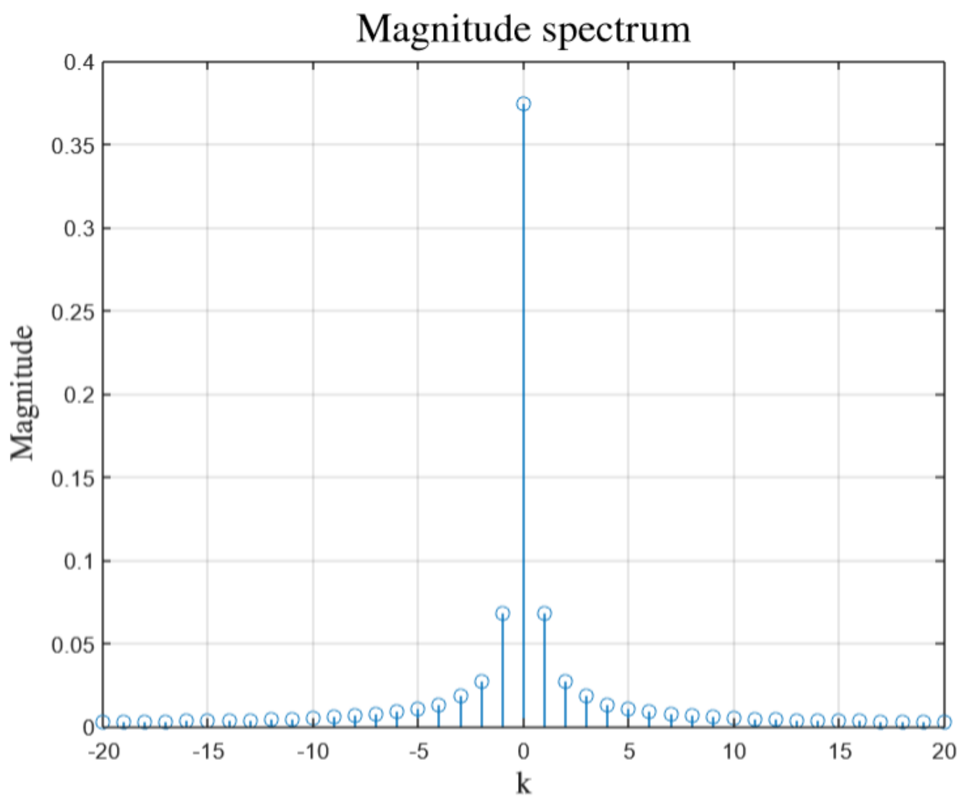
$$= 3k^2\pi^2 \cos k\pi - 8\cos k\pi - 13k^2\pi^2 + 8$$
$$+ j(10k\pi\cos k\pi - 18k\pi + 3k^3\pi^3)$$

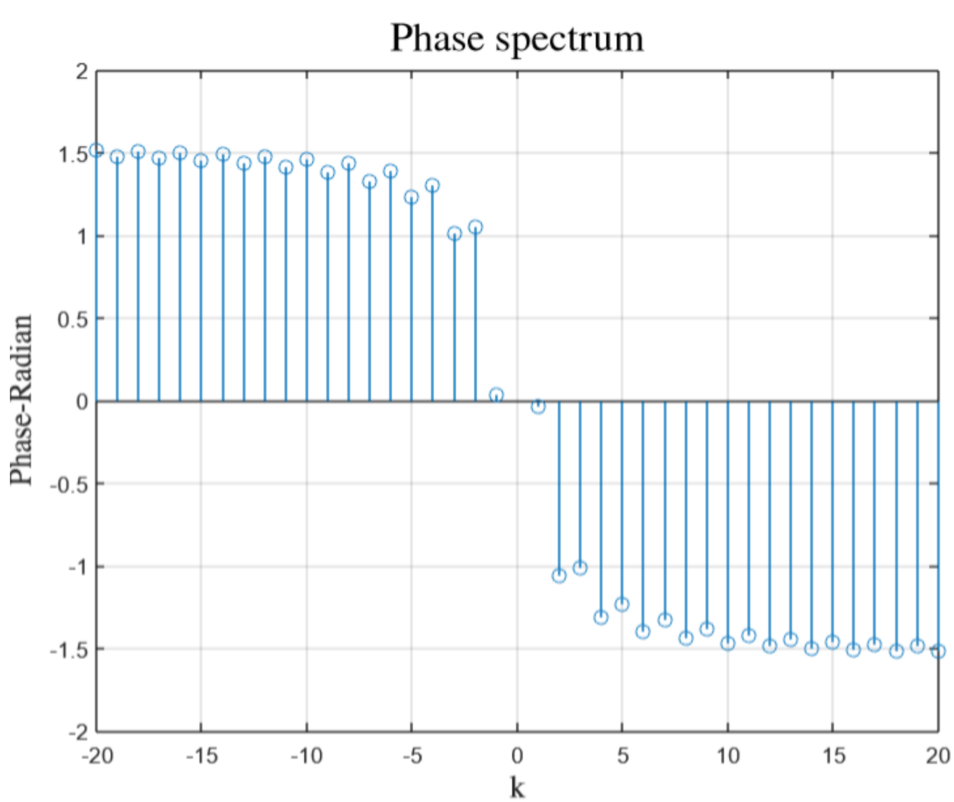
$$\Rightarrow Y_k = \underbrace{\frac{3k^2\pi^2\cos k\pi - 8\cos k\pi - 13k^2\pi^2 + 8}{2\pi^2k^2(9\pi^2k^2+16)}}_{A} + j\underbrace{\frac{10k\pi\cos k\pi - 18k\pi + 3k^3\pi^3}{2\pi^2k^2(9\pi^2k^2+16)}}_{B}$$

$$\boxed{\begin{aligned} &\Rightarrow |Y_k| = \sqrt{A^2+B^2} \\ &\Rightarrow \measuredangle Y_k = \tan^{-1}\left(\frac{10k\pi\cos k\pi - 18k\pi + 3k^3\pi^3}{3k^2\pi^2\cos k\pi - 8\cos k\pi - 13k^2\pi^2 + 8}\right)\end{aligned}}$$

d) We have $Y_0 = X_0 H(0) = \frac{3}{4} \cdot \left.\frac{j\omega - 1}{3j\omega + 2}\right|_{j\omega = 0} = \left|\frac{-3}{8}\right|$

$$\boxed{\Rightarrow Y_0 = \frac{3}{8}} \qquad \boxed{\measuredangle Y_0 = \infty}$$





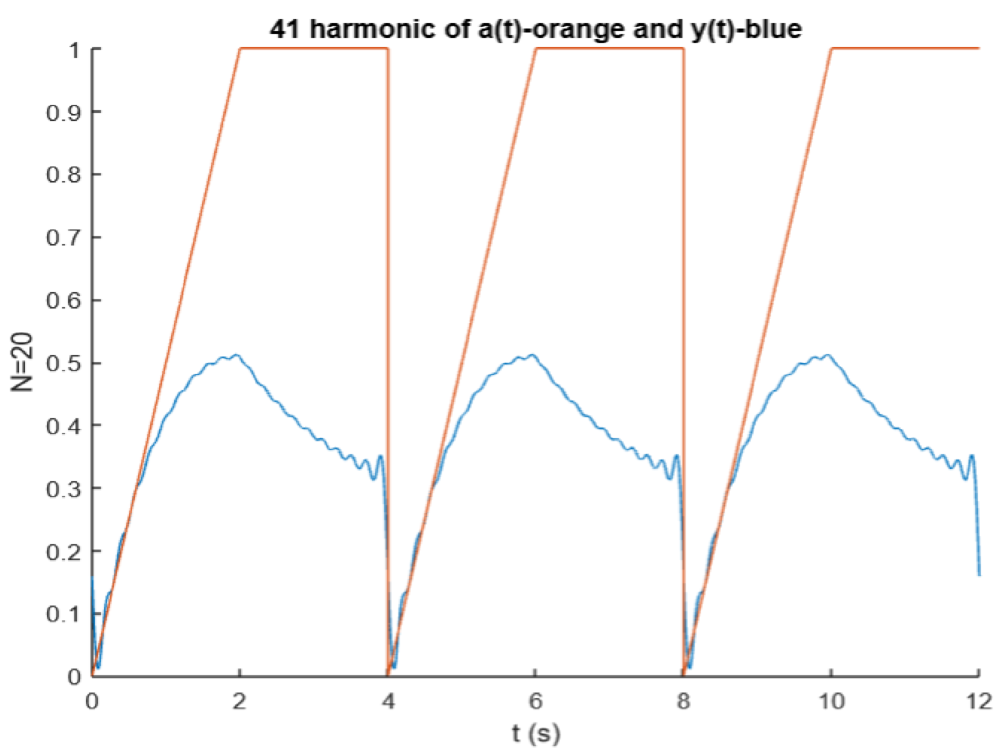
For the phase diagram, there is a missmatch since at t=0, the phase is infinite but matlab could not show that.

Coding part

```
clc;
close all;
clear all;

phase0 = Inf;
n0 = 3/8;
n1 = linspace(-20,-1,20);
n2 = linspace(1,20,20);

An1 = ((cos(pi.*n1) - 1) ./ (2*(pi.*n1).^2)) + (1i.*n1.*pi)./((2*(pi.*n1).^2));
An2 = ((cos(pi.*n2) - 1) ./ (2*(pi.*n2).^2)) + (1i.*n2.*pi)./((2*(pi.*n2).^2));

RealNumerator1 = ((3.* (n1.*pi).^2).*cos(n1.*pi) - 8.*cos(n1.*pi) - 13.*(n1.*pi).^2
+ 8);
RealDenominator1 = 18.*(pi.*n1).^4 + 32.*(pi.*n1).^2;
Real1 = RealNumerator1./RealDenominator1;

ImagNumerator1 = 10.*n1.*pi.*cos(n1.*pi)-18.*n1.*pi+3.*(n1.*pi).^3;
ImagDenominator1 = 18.*(pi.*n1).^4 + 32.*(pi.*n1).^2;
Image1 = ImagNumerator1./ImagDenominator1;

RealNumerator2 = ((3.* (n2.*pi).^2).*cos(n2.*pi) - 8.*cos(n2.*pi) - 13.*(n2.*pi).^2
+ 8);
RealDenominator2 = 18.*(pi.*n2).^4 + 32.*(pi.*n2).^2;
Real2 = RealNumerator2./RealDenominator2;

ImagNumerator2 = 10.*n2.*pi.*cos(n2.*pi)-18.*n2.*pi+3.*(n2.*pi).^3;
ImagDenominator2 = 18.*(pi.*n2).^4 + 32.*(pi.*n2).^2;
Image2 = ImagNumerator2./ImagDenominator2;

Mag1 = sqrt(Real1.^2 + Image1.^2);
Mag2 = sqrt(Real2.^2 + Image2.^2);

Phase1 = atan(Image1./Real1);
Phase2 = atan(Image2./Real2);
```

```
X = [n1, 0, n2];
Mag = [Mag1,n0,Mag2];
Phase = [Phase1, phase0, Phase2];

figure(1)
stem(X,Mag);
xlabel('k','Interpreter','latex','fontsize',14);
ylabel('Magnitude','Interpreter','latex','fontsize',14);
title('Magnitude spectrum','Interpreter','latex','fontsize',18);
grid on;


figure(2)
stem(X,Phase);
xlabel('k','Interpreter','latex','fontsize',14);
ylabel('Phase-Radian','Interpreter','latex','fontsize',14);
title('Phase spectrum','Interpreter','latex','fontsize',18);
grid on;
```

4e)



Coding part

```
clear;
clc;
close all;

N=20;
k1=1:1:N;
k2=-N:1:-1;

omega0=pi./2;
t=0:0.001:12;

a1=t/2;
a2=1;
a3=(t/2)-2;
a4=1;
a5=(t/2)-4;
a6=1;
an=a1.*(t>=0&t<2)+a2.*(t>=2&t<4)+a3.*(t>=4&t<6)+a4.*(t>=6&t<8)+a5.*(t>=8&t
<10)+a6.*(t>=10&t<=12);

A_k1 = ((1i.*pi.*k1.*cos(2.*k1.*pi)) + cos(k1.*pi) - 1) ./ (2.*(k1.*pi).^2);
A_k2 = ((1i.*pi.*k2.*cos(2.*k2.*pi)) + cos(k2.*pi) - 1) ./ (2.*(k2.*pi).^2);

H_k1 = (1i.*k1.*pi - 2) ./ (3i.*k1.*pi + 4);
H_k2 = (1i.*k2.*pi - 2) ./ (3i.*k2.*pi + 4);

Y1= A_k1.*H_k1;
Y2= A_k2.*H_k2;

Y=[Y2 3/8 Y1];%Y2 Y0 Y1

fn = myfs(Y,omega0,t);
hold on;
plot(t,fn);
plot(t,an);
xlabel('t (s)'); ylabel('N=20');
```

```
title('41 harmonic of a(t)-orange and y(t)-blue');

function fn = myfs(Dn,omega0,t)
   N = (length(Dn)-1)/2;
   fn = zeros(size(t));
   for n = -N:N
      D_n = Dn(n+N+1);
      fn = fn + D_n*exp(1i*omega0*n*t);
   end
end
```