Homework 5

ECE 102: Systems and Signals

Winter 2022

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Due Date: 23:59 on 23^{th} February, 2022. Submission via gradescope.
Kindly enroll yourself in the class: ECE 102 on gradescope. Entry code: X3PPGR

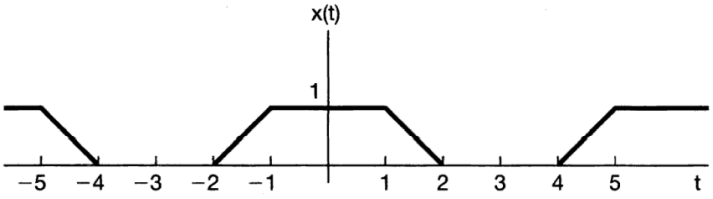
1. Continuous time signal $x(t)$ is periodic with period T_o and fundamental frequency ω_o and has complex Fourier Series coefficients a_k for the k^{th} harmonic. Find the Fourier Series coefficients b_k for the signals $y(t)$ in terms of a_k in each of the following scenarios:

   (a) $y(t) = x(2t-3) + 4\frac{d^2x(t)}{dt^2}$

   (b) $y(t) = \int_{-\infty}^{t+2\alpha} e^{j\omega_o\tau} x(\tau+1)d\tau$

   (c) $y(t) = \frac{dx^3(t)}{dt}$

2. Consider the continuous time signal $x(t)$ shown below



(a)

   (a) Determine the complex Fourier series representation or each of the signal $x(t)$.

   (b) Plot the magnitude and phase spectrum of the Fourier Series coefficients for $x(t)$.

   (c) Derive the Fourier Series coefficients of $x(t)$ from its Laplace transform.

   (d) Express the signal $x(t)$ as a trigonometric fourier series. That is, find a_k and b_k which satisfy

$$x(t) = X_0 + 2\sum_{k=1}^{\infty} a_k \cos(k\Omega_0 t) - 2\sum_{k=1}^{\infty} b_k \sin(k\Omega_0 t)$$

3. Consider a continuous-time LTI system with impulse response $h(t) = e^{-4t}u(t)$ Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

   (a) $x(t) = \sum_{n=-\infty}^{+\infty} \delta(t-n)$

   (b) $x(t) = \sum_{n=-\infty}^{+\infty} (-1)^n \delta(t-n)$

4. (a) Consider the periodic real signal $x(t)$ with the following properties:

      i. The signal period is $T = 6$

      ii. The DC component of the signal is zero

      iii. $x(t) = -x(t-3)$

iv. The Fourier series coefficients $X_k = 0$ for $k > 2$

v. X_1 is a positive real number

vi. $\int_{-3}^{3} |x(t)|^2 = 3$

Find the exact expression of the time-domain signal $x(t)$.

(b) Let $x(t)$ be a periodic signal whose Fourier series coefficients are $a_k = \begin{cases} 2, & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{else} \end{cases}$

(i) Is $x(t)$ real? (ii) Is $x(t)$ even? (iii) Is $dx(t)/dt$ even?

5. The smoothness of signal in time domain determines how its spectrum will look in the frequency domain. Consider two signals with period $T_0 = 2\text{sec}$. The signals are represented as below in the first period: $0 \leq t \leq T_0$:

$$x_1(t) = u(t) - u(t-1)$$
$$x_2(t) = r(t) - 2r(t-1) + r(t-2)$$

Find Fourier series coefficients of $x_1(t)$ and $x_2(t)$ analytically using Fourier series formula. Then, in MATLAB plot magnitude spectra for both signals for $k = -20, -19, \ldots 0, \ldots, 19, 20$ in MATLAB using stem function. Determine which spectra decays faster as k increases and explain how it relates to smoothness of the signal in time domain.
