Homework 1

ECE 102: Systems and Signals

Winter 2022

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Due Date: 23:59 on 14th January, 2022. Submission via gradescope.

Kindly enroll yourself in the class: ECE 102 on gradescope. Entry code: X3PPGR

1 Problems

1. A continuous-time signal $x(t)$ is shown in Figure 1. Sketch and label carefully each of the following signals:

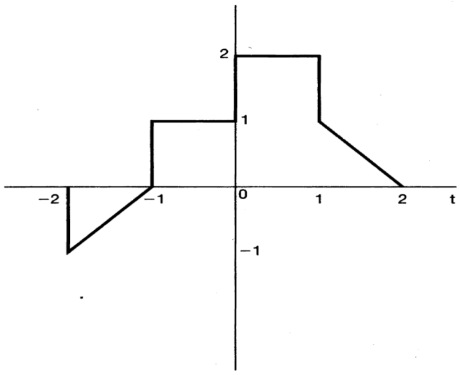


Figure 1: $x(t)$

   (a) $x(t-1)$

   (b) $x(4-t)$

   (c) $[x(t-2)-2x(-t+1)]\,u(t-\frac{1}{2})$

2. (a) Using complex exponentials, prove that:

$$a(\mathrm{t}) = \cos(\theta t)\sin(\psi t) = \frac{1}{2}(\sin((\theta+\psi)t) - \sin((\theta-\psi)t))$$

   (b) Can $a(t)$ be periodic? If so, use $\theta = 2\pi$ to find the value of ψ where $a(t)$ has a period of 3.

   (c) Determine the fundamental period of the signal $x(t) = 2\cos(10t+1) - \sin(4t-1)$

3. Consider the periodic signal $x(t) = \cos(3\Omega_o t) + 5\cos(\Omega_o t)\,, -\infty < t < \infty$, and $\Omega_o = \pi$. The frequencies of the two sinusoids are harmonically related (that is, one is a multiple of the other).

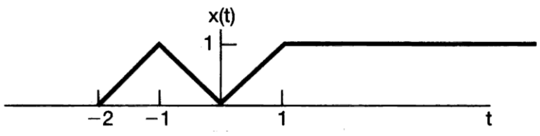
   (a) Determine the period T_o of $x(t)$.

   (b) Compute the power P_x of $x(t)$.

   (c) Verify that the power P_x is the sum of the powers P_1 of $x_1(t) = \cos(3\Omega_o t)$ and P_2 of $x_2(t) = 5\cos(\Omega_o t)$, for $\Omega_o = \pi$.

(d) In the above case, we see that there is superposition of the powers because the frequencies are harmonically related. Suppose that $\gamma(t) = \cos(t) + \cos(\frac{\pi}{2}t)$ where the frequencies are not harmonically related. Find out whether $\gamma(t)$ is periodic or not. Indicate how you would find the power P_γ of $\gamma(t)$. Would $P_\gamma = P_1 + P_2$ where P_1 is the power of $\cos(t)$ and P_2 is the power of $\cos(\frac{\pi}{2}t)$? Explain what is the difference with respect to the case of harmonic frequencies.

4. (a) Determine and sketch the even and odd parts of the signal depicted in the figure below. Label your sketches carefully.



(b) Show that the energy of a general continuous time signal $x(t)$ can be expressed as the sum of the energies of its even and odd components. That is,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x_e(t)|^2 dt + \int_{-\infty}^{\infty} |x_o(t)|^2 dt$$