Homework 4

ECE 102: Systems and Signals

Winter 2022

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Due Date: 23:59 on 16^{th} February, 2022. Submission via gradescope.
Kindly enroll yourself in the class: ECE 102 on gradescope. Entry code: X3PPGR

1. **Linear Differential Equations with Laplace Transforms**
   A system S has the following IPOP:

$$3\frac{d^2y(t)}{dt^2} + 19\frac{dy(t)}{dt} + 20y(t) = 2\frac{dx(t)}{dt} - x(t), t \geq 0$$
$$y'(0) = y(0) = 0, x(0) = 0$$

   (a) Find the transfer function of S. Is the system stable?

   (b) Find the output given input: $x(t) = e^{\frac{1}{2}(t-3)}u(t-3)$

2. **Block diagram representation of LTI systems**
   Consider the system S characterized by the differential equation

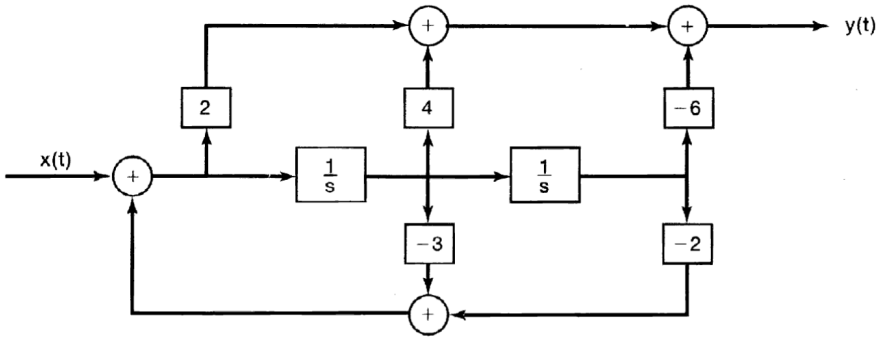
$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = 2\frac{d^2x(t)}{dt^2} - 14\frac{dx(t)}{dt} - 16x(t).$$

   (a) Draw a block diagram realisation of the system using integration and differentiation blocks.

   (b) Draw the pole-zero constellation of system S and comment on its stability.

   (c) Find the output when $x(t) = u(t+2) - u(t-2)$ is applied as input.

3. **Transfer function from system bock diagram**
   Consider the system S whose input output relation are shown by the block diagram below:



   (a) Find the transfer function $H(s)$ for the system.

   (b) Express the system using a differential equation in $x(t)$ and $y(t)$.

   (c) Compute the inverse Laplace transform of $e^{-4s}H(\frac{s}{3} - 4)$.

4. **Fourier series representation**

Fourier proposed to represent a periodic signal as a sum of sinusoids, perhaps an infinite number of them. For instance, consider the representation of a periodic signal $x(t)$ as a sum of cosines of different frequencies

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\Omega_k t + \theta_k)$$

(a) If $x(t)$ is periodic of period T_0, what should the frequencies Ω_k be?

(b) Consider $x(t) = 2 + \cos(2\pi t) - 3\cos(6\pi t + \pi/4)$. Is this signal periodic? If so, what is its period T_0 ? Determine its trigonometric Fourier series as given above by specifying the values of A_k and θ_k for all values of $k = 0, 1, \ldots$

(c) Let the signal $x_1(t) = 2 + \cos(2\pi t) - 3\cos(20t + \pi/4)$ (this signal is almost like $x(t)$ given above, except that the frequency $6\pi \text{rad/sec}$ of the second cosine has been approximated by 20rad/sec). Is this signal periodic? Can you determine its Fourler series as given above by specifying the values of A_k and θ_k, for all values of $k = 0, 1, \ldots$? Explain.

5. **Matlab assignment:**

MATLAB has two functions `laplace` and `ilaplace` to compute the Laplace and Inverse Laplace transform, respectively. For example, the following code computes the Laplace transform of $f(t) = t^5 u(t)$

```
syms t                          %Construct a symbolic variable t
f=t^5*heaviside(t);             %Write the function f(t)
F=laplace(f)                    %Compute Laplace Transform F(s)
```

On the other hand, the following code computes the inverse Laplace transform of $F(s) = \frac{1}{s^2}$

```
syms s                          %Construct a symbolic variable s
F=1/s^2;                        %Write the function F(s)
f=ilaplace(F)                   %Compute Inverse Laplace T. f(t)
```

You can use the following code to evaluate $f(t)$ in the range $t \in [1,4]$ and to plot it

```
fplot(f,[1 4])
```

a) Using MATLAB, compute the Laplace transform of

$$f(t) = (t-5)^4 \exp(-3t)u(t)$$

b) Let

$$F(s) = \frac{1}{s\left(s+2-j\frac{\pi}{3}\right)\left(s+2+j\frac{\pi}{3}\right)}$$

Using MATLAB, find $f(t)$. Evaluate and plot $f(t)$ in the range $t \in [0,2]$.

Note: Include the MATLAB code and the results (MATLAB output, MATLAB figure) in your homework.
