Home Work 4.

1) Given $\underbrace{\frac{3d^2y(t)}{dt^2} + 19\frac{dy(t)}{dt} + 20y(t)}_{LHS} = \underbrace{2\frac{dx(t)}{dt} - x(t)}_{RHS}, t \geq 0$

$y'(0) = y(0) = 0, \quad x(0) = 0$

a) we have $L_s\{LHS\} = L_s\{\frac{3d^2y(t)}{dt^2}\} + L_s\{19\frac{dy(t)}{dt}\}$

$+ L_s\{20.y(t)\} = 3L_s\{\frac{d^2y(t)}{dt^2}\} + 19L_s\{\frac{dy(t)}{dt}\}$

$+ 20L_s\{y(t)\} = 3[s^2Y(s) - s\underbrace{y(0^-)}_{0} - \underbrace{y'(0^-)}_{0}]$

$+ 19[sY(s) - \underbrace{y(0^-)}_{0}] + 20Y(s)$

$= 3s^2Y(s) + 19sY(s) + 20Y(s) = [3s^2 + 19s + 20]Y(s)$

$\circledast L_s\{RHS\} = L_s\{2\frac{dx(t)}{dt} - x(t)\} \quad (t \geq 0)$

$= 2L_s\{\frac{dx(t)}{dt}\} - L_s\{x(t)\} = 2[sX(s) - \underbrace{x(0^-)}_{0}] - X(s)$

$= 2sX_s - X_s = X(s)(2s - 1)$

$$\Rightarrow L_s\{LHS\} = L_s\{RHS\} \Leftrightarrow (3s^2+19s+20)Y(s) = X(s)(2s-1)$$

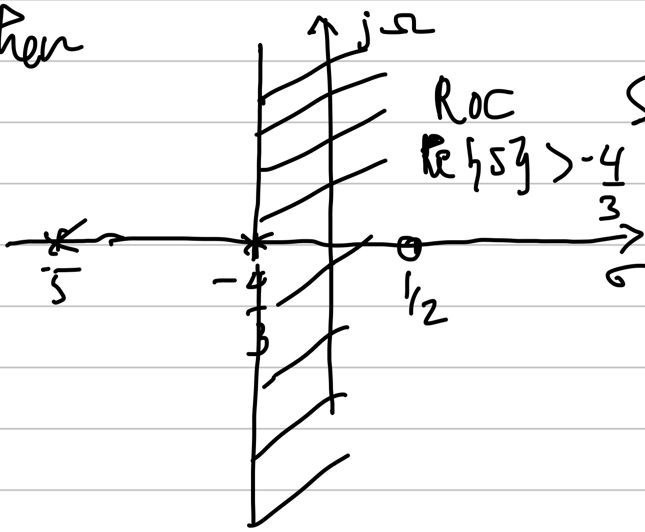
$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{2s-1}{3s^2+19s+20} = \frac{2s-1}{3(s+\frac{4}{3})(s+5)}$$

$$\Rightarrow H(s) = \frac{2s-1}{3(s+\frac{4}{3})(s+5)} = \boxed{\frac{2s-1}{(3s+4)(s+5)}}$$

We have: zero: $2s-1=0 \Leftrightarrow s=\frac{1}{2}$

Poles: $s=-\frac{4}{3}$ & $s=-5$

then



Since all poles are less than 0 & ROC did include the $j\Omega$ axis

$\Rightarrow$ $\boxed{\text{this is stable system}}$

b) Given $x(t) = e^{1/2(t-3)}u(t-3)$

We have $e^{\frac{1}{2}t}u(t) \xrightarrow{L_s} \frac{1}{s-\frac{1}{2}}$, $Re\{s\} > 1$

$$\Rightarrow e^{\frac{1}{2}(t-3)}u(t-3) \xrightarrow{L_s} e^{-s.3}F(s) = e^{-3s} \times \frac{1}{s-\frac{1}{2}}$$

$$\Rightarrow X(s) = \frac{2e^{-3s}}{2s-1}$$

Also $Y(s) = H(s) \cdot X(s)$

$$\Rightarrow Y(s) = \frac{(2s-1)}{(3s+4)(s+5)} \cdot \frac{2e^{-3s}}{(2s-1)} = \frac{2e^{-3s}}{(3s+4)(s+5)}$$

$$= 2e^{-3s}\left[\frac{A}{3s+4} + \frac{B}{s+5}\right] =$$

$$A = \left.\frac{1}{s+5}\right|_{s=-\frac{4}{3}} = \frac{1}{5-\frac{4}{3}} = \frac{3}{11}$$

$$B = \left.\frac{1}{3s+4}\right|_{s=-5} = -\frac{1}{11}$$

$$\Rightarrow Y(s) = 2e^{-3s}\left[\frac{3}{11(3s+4)} - \frac{1}{11(s+5)}\right]$$

$$= \frac{2}{11}e^{-3s}\left[\frac{1}{s+\frac{4}{3}} - \frac{1}{s+5}\right]$$

$$= \frac{2}{11}e^{-3s}\frac{1}{s+\frac{4}{3}} - \frac{2}{11}e^{-3s} \cdot \frac{1}{s+5}$$

We have: $e^{-\frac{4}{3}t}u(t) \xrightarrow{L_s} \dfrac{1}{s+\frac{4}{3}}$ $\quad Re\{s\} > -\frac{4}{3}$

$$\Rightarrow e^{-\frac{4}{3}(t-3)} u(t-3) \xrightarrow{L_s} e^{-s.3} \frac{1}{s+\frac{4}{3}}$$

$$\Rightarrow \frac{2}{11} e^{-\frac{4}{3}(t-3)} u(t-3) \xrightarrow{L_s} \frac{2}{11} \cdot e^{-3s} \frac{1}{s+\frac{4}{3}}$$

Also: $e^{-5t} u(t) \xrightarrow{L_s} \dfrac{1}{s+5}$ $\quad Re\{s\} > -5$

$$\Rightarrow e^{-5(t-3)} u(t-3) \xrightarrow{L_s} e^{-3s} \frac{1}{s+5}$$

$$\Rightarrow \frac{2}{11} e^{-5(t-3)} u(t-3) \xrightarrow{L_s} \frac{2}{11} e^{-3s} \cdot \frac{1}{s+5}$$

$$\Rightarrow \boxed{y(t) = \frac{2}{11} e^{-\frac{4}{3}(t-3)} u(t-3) - \frac{2}{11} e^{-5(t-3)} u(t-3)}$$

2a) We have $L_s\{LHS\} = L\{\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt}$

$+ 6y(t)\} = [s^3Y(s) - s^2\underbrace{y(0^-)}_{0} - s\underbrace{y'(0^-)}_{0} - \underbrace{y''(0^-)}_{0}]$

$+ 6[s^2Y(s) - s\underbrace{y(0^-)}_{0} - \underbrace{y'(0^-)}_{0}] + 11[sY(s) - \underbrace{y(0^-)}_{0}]$

$+ 6Y(s) = s^3Y(s) + 6s^2Y(s) + 11sY(s) + 6Y(s)$

$= (s^3 + 6s^2 + 11s + 6)Y(s)$

$\circledast$ $L_s\{RHS\} = L_s\{2\frac{d^2x(t)}{dt^2} - 14\frac{dx(t)}{dt} - 16x(t)\}$

$= 2[s^2X(s) - s\underbrace{x(0^-)}_{0} - \underbrace{x'(0^-)}_{0}] - 14[sX(s) - \underbrace{x(0^-)}_{0}]$

$- 16X(s) = 2s^2X(s) - 14sX(s) - 16X(s)$

$= (2s^2 - 14s - 16)X(s)$

$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{2s^2 - 14s - 16}{s^3 + 6s^2 + 11s + 6} = \frac{2(s-8)(s+1)}{(s+3)(s+1)(s+2)}$

$\Rightarrow H(s) = \frac{2(s-8)}{(s+3)(s+2)} \Rightarrow Y(s) = X(s)\frac{2s-16}{s^2+5s+6}$

$$\Rightarrow \quad X(s) \longrightarrow \boxed{\dfrac{1}{s^2+5s+6}} \xrightarrow{u(s)} \boxed{2s-16} \longrightarrow Y(s)$$

Firstly, $X(s) = (s^2+5s+6)\,u(s) = \ddot{u} + 5\dot{u} + 6u$

$x_1 = u \Rightarrow \dot{x}_1 = \dot{u} = x_2$

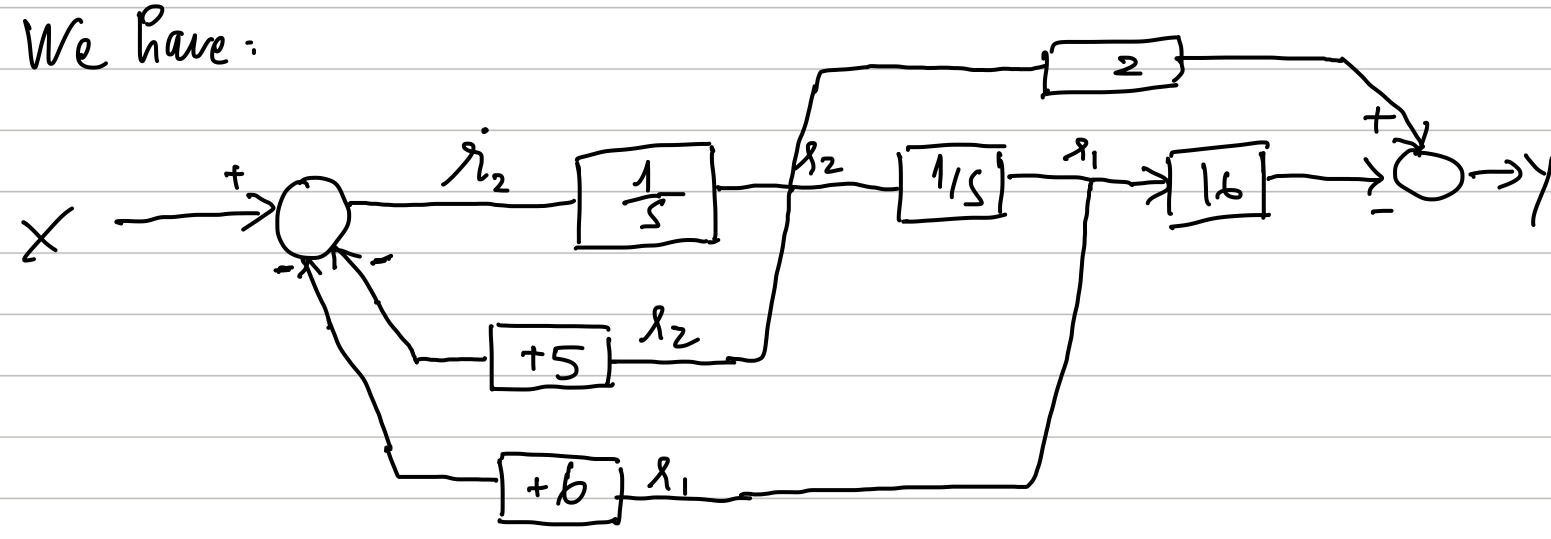
$x_2 = \dot{u} \rightarrow \dot{x}_2 = \ddot{u} = -5\dot{u} - 6u + x = -5x_2 - 6x_1 + x$

Secondly, $Y(s) = (2s-16)\,u(s) = 2\dot{u} - 16u$

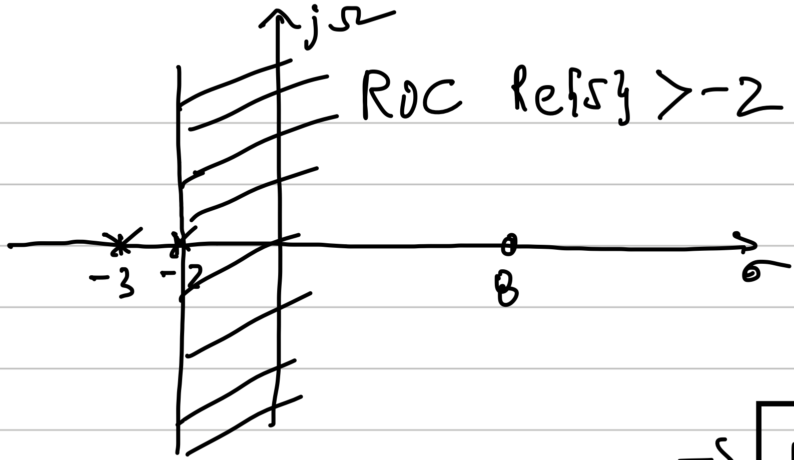
$= 2x_2 - 16x_1$

$$\Rightarrow \begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x \\ y = \begin{bmatrix} -16 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

We have:



b) $H(s) = \dfrac{2(s-8)}{(s+3)(s+2)}$ $\Rightarrow \begin{cases} \text{zero: } s = 8 \\ \text{poles: } s = -3 \ \& \ s = -2 \end{cases}$



Since all real poles < 0

and ROC of H(s) includes

the jΩ axis

$\Rightarrow$ $\boxed{\text{this system is stability}}$

c) $x(t) = u(t+2) - u(t-2)$

We have $u(t) \xrightarrow{L_s} \frac{1}{s}$ $\quad Re\{s\} > 0$

$\rightarrow u(t+2) \xrightarrow{L_s} e^{2s} \frac{1}{s}$ $\quad Re\{s\} > 0$

$\Rightarrow u(t-2) \xrightarrow{L_s} e^{-2s} \frac{1}{s}$ $\quad Re\{s\} > 0$

$\Rightarrow X(s) = e^{2s} \cdot \frac{1}{s} - e^{-2s} \cdot \frac{1}{s} = \frac{1}{s}\left[e^{2s} - e^{-2s}\right]$

Thus, $Y(s) = X(s) H(s) = \frac{2(s-8)}{(s+3)(s+2)} \cdot \frac{1}{s}\left[e^{2s} - e^{-2s}\right]$

$= 2\left[e^{2s} - e^{-2s}\right] \cdot \frac{s-8}{s(s+3)(s+2)}$

Let $G(s) = \frac{s-8}{s(s+3)(s+2)} = \frac{A_1}{s} + \frac{A_2}{s+3} + \frac{A_3}{s+2}$

$$A_1 = \frac{s-8}{(s+3)(s+2)}\Bigg|_{s=0} = \frac{-8}{6} = \frac{-4}{3}$$

$$A_2 = \frac{s-8}{s(s+2)}\Bigg|_{s=-3} = \frac{-11}{-3\times(-1)} = \frac{-11}{3}$$

$$A_3 = \frac{s-8}{s(s+3)}\Bigg|_{s=-2} = \frac{-10}{-2\times 1} = 5$$

$$\Rightarrow G(s) = -\frac{4}{3}\cdot\frac{1}{s} - \frac{11}{3}\cdot\frac{1}{s+3} + \frac{5}{s+2}$$

$$\Rightarrow Y(s) = 2\left[e^{2s} - e^{-2s}\right]\cdot\left[-\frac{4}{3}\cdot\frac{1}{s} - \frac{11}{3}\frac{1}{s+3} + \frac{5}{s+2}\right]$$

$$= 2\Big[\underbrace{-\frac{4}{3}e^{2s}\cdot\frac{1}{s}} - \underbrace{\frac{11}{3}e^{2s}\frac{1}{s+3}} + \underbrace{5e^{2s}\frac{1}{s+2}} + \underbrace{\frac{4}{3}e^{-2s}\frac{1}{s}}$$

$$+ \underbrace{\frac{11}{3}e^{-2s}\cdot\frac{1}{s+3}} - \underbrace{5e^{-2s}\frac{1}{s+2}}\Big]$$

<u>We have:</u>

$\circledast$ $u(t) \xrightarrow{L_s} \frac{1}{s}$ $\mathrm{Re}\{s\} > 0 \Rightarrow u(t+2) \xrightarrow{L_s} e^{2s}\frac{1}{s}$

$$\Rightarrow -\frac{4}{3}u(t+2) \xrightarrow{L_s} -\frac{4}{3}e^{2s}\cdot\frac{1}{s} \quad \mathrm{Re}\{s\} > 0 \qquad (1)$$

$\circledast \quad e^{-3t} u(t) \xrightarrow{L_s} \frac{1}{s+3} \quad Re\{s\} > -3$

$\Rightarrow e^{-3(t+2)} u(t+2) \xrightarrow{L_s} e^{2s} \frac{1}{s+3}$

$\Rightarrow -\frac{11}{3} e^{-3(t+2)} u(t+2) \xrightarrow{L_s} -\frac{11}{3} e^{2s} \frac{1}{s+3}$ ②

$\circledast \quad e^{-2t} u(t) \xrightarrow{L_s} \frac{1}{s+2} \quad Re\{s\} > -2$

$\Rightarrow e^{-2(t+2)} u(t+2) \xrightarrow{L_s} e^{2s} \frac{1}{s+2}$

$\Rightarrow 5 e^{-2(t+2)} u(t+2) \xrightarrow{L_s} 5 e^{2s} \frac{1}{s+2}$ ③

$\circledast \quad u(t) \xrightarrow{L_s} \frac{1}{s} \quad Re\{s\} > 0 \Rightarrow u(t-2) \xrightarrow{L_s} e^{-2s} \frac{1}{s}$

$\Rightarrow \frac{4}{3} u(t-2) \xrightarrow{L_s} \frac{4}{3} e^{-2s} \frac{1}{s}$. ④

$\circledast \quad e^{-3t} u(t) \xrightarrow{L_s} \frac{1}{s+3} \quad Re\{s\} > -3$

$\Rightarrow e^{-3(t-2)} u(t-2) \xrightarrow{L_s} e^{-2s} \frac{1}{s+3}$

$\Rightarrow \frac{11}{3} e^{-3(t-2)} u(t-2) \xrightarrow{L_s} \frac{11}{3} e^{-2s} \frac{1}{s+3}$ ⑤

$$e^{-2t}u(t) \xrightarrow{L_s} \frac{1}{s+2} \quad Re\{s\} > -2$$

$$\Rightarrow e^{-2(t-2)}u(t-2) \xrightarrow{L_s} e^{-2s}\cdot\frac{1}{s+2}$$

$$\Rightarrow -5e^{-2(t-2)}u(t-2) \xrightarrow{L_s} -5e^{-2s}\frac{1}{s+2} \quad (6)$$

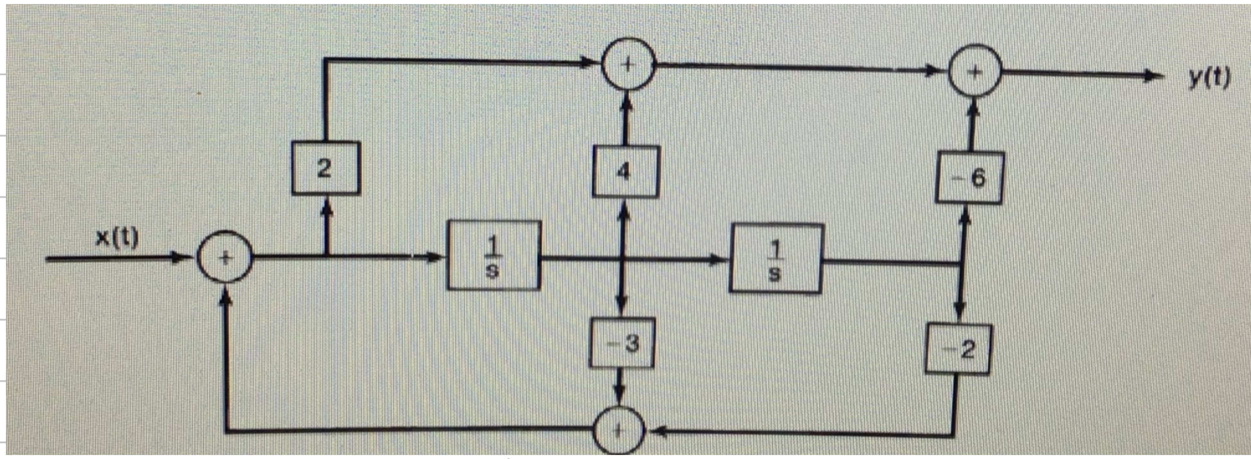
$$\Rightarrow y(t) = 2\left[-\frac{4}{3}u(t+2) - \frac{11}{3}e^{-3(t+2)}\cdot u(t+2) + 5e^{-2(t+2)}u(t+2)\right.$$

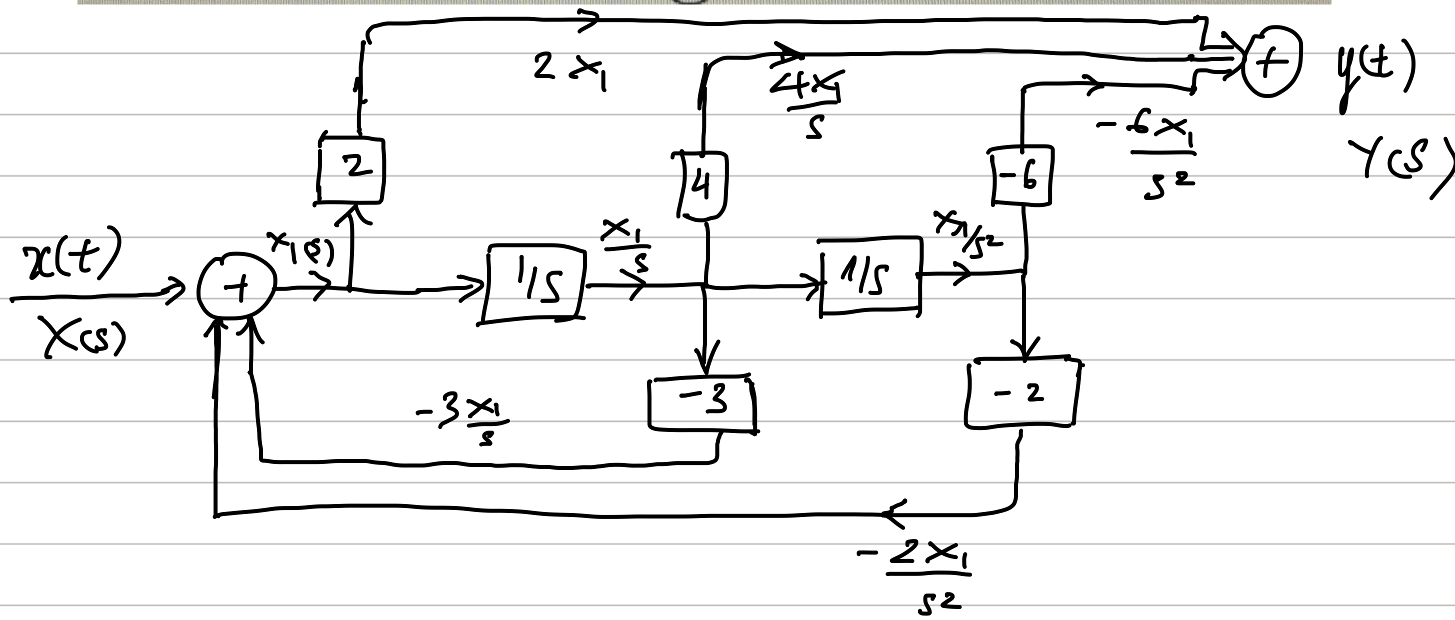
$$+\frac{4}{3}u(t-2) + \frac{11}{3}e^{-3(t-2)}u(t-2)$$

$$\left.-5e^{-2(t-2)}u(t-2)\right]$$

$$= \boxed{\begin{aligned} &2u(t+2)\left[-\frac{4}{3} - \frac{11}{3}e^{-3(t+2)} + 5e^{-2(t+2)}\right] \\ &+2u(t-2)\left[\frac{4}{3} + \frac{11}{3}e^{-3(t-2)} - 5e^{-2(t-2)}\right] \end{aligned}}$$

3)





Based on the diagram above, we have:

$$\begin{cases} 2X_1(S) + \dfrac{4X_1(S)}{S} + \dfrac{-6X_1(S)}{S^2} = Y(S) & ① \\[2ex] X(S) + \left(-\dfrac{3X_1(S)}{S}\right) + \left(-\dfrac{2X_1(S)}{S^2}\right) = X_1(S) & ② \end{cases}$$

From ②, $X(S) = X_1(S)\left[1 + \dfrac{3}{S} + \dfrac{2}{S^2}\right] = \dfrac{S^2+3S+2}{S^2} \cdot X_1(S)$

$$\Rightarrow X_1(S) = \frac{S^2}{S^2+3S+2} X(S)$$

From ①, $X_1(s)\left[2+\frac{4}{s}-\frac{6}{s^2}\right]=Y(s)$

$$\Rightarrow X_1(s)\left[\frac{2s^2+4s-6}{s^2}\right]=Y(s)$$

$$\Rightarrow \frac{s^2}{s^2+3s+2}\cdot\frac{2[s^2+2s-3]}{s^2}\cdot X(s)=Y(s)$$

$$\Rightarrow \frac{2(s-1)(s+3)}{(s+1)(s+2)}X(s)=Y(s)$$

$$\Rightarrow \boxed{H(s)=\frac{Y(s)}{X(s)}=\frac{2(s-1)(s+3)}{(s+1)(s+2)}}$$

b) We have:

$$Y(s)\left[(s+1)(s+2)\right]=X(s)\left[2(s-1)(s+3)\right]$$

$$\Leftrightarrow Y(s)(s^2+3s+2)=X(s)(2s^2+4s-6)$$

$$\Leftrightarrow s^2Y(s)+3sY(s)+2Y(s)=2s^2X(s)+4sX(s)-6X(s)$$

$$\Leftrightarrow \boxed{\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=2\frac{d^2x(t)}{dt}+4\frac{dx(t)}{dt}-6x(t) \quad (t>0)}$$

c) From part a, $H(s) = \dfrac{2(s-1)(s+3)}{(s+1)(s+2)}$

$$= \frac{2(s^2+2s-3)}{s^2+3s+2}$$

⊛ $s^2+2s-3 = s^2+3s+2-(s+5)$

$$\Rightarrow H(s) = \frac{2(s^2+3s+2)-2(s+5)}{s^2+3s+2} = 2 - \frac{2(s+5)}{s^2+3s+2}$$

Let $G(s) = \dfrac{2(s+5)}{s^2+3s+2} = \dfrac{2(s+5)}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2}$

$$A = \left.\frac{2(s+5)}{s+2}\right|_{s=-1} = \frac{2\times 4}{1} = 8$$

$$B = \left.\frac{2(s+5)}{s+1}\right|_{s=-2} = \frac{2\times 3}{-2+1} = -6$$

$$\Rightarrow G(s) = \frac{8}{s+1} - \frac{6}{s+2}$$

$$\Rightarrow H(s) = 2 - \frac{8}{s+1} + \frac{6}{s+2}$$

We have:

⊛ $\delta(t) \xrightarrow{L_s} 1$ all s.

$$\Rightarrow 2\delta(t) \xrightarrow{L_s} 2 \quad \text{all } s$$

$$e^{-t}u(t) \xrightarrow{L_s} \frac{1}{s+1} \qquad Re\{s\} > -1$$

$$\Rightarrow 8e^{-t}u(t) \xrightarrow{L_s} \frac{8}{s+1}$$

$$e^{-2t}u(t) \xrightarrow{L_s} \frac{1}{s+2} \qquad Re\{s\} > -2$$

$$\Rightarrow 6e^{-2t}u(t) \xrightarrow{L_s} \frac{6}{s+2}$$

$$\Rightarrow h(t) = 2\delta(t) - 8e^{-t}u(t) + 6e^{-2t}u(t)$$

$$\xrightarrow{L_s} H(s).$$

Since we have $H(s) = 2 - \frac{8}{s+1} + \frac{6}{s+2}$

$$\Rightarrow H\left(\frac{s}{3}-4\right) = 2 - \frac{8}{\frac{s}{3}-4+1} + \frac{6}{\frac{s}{3}-4+2}$$

$$\left.\begin{aligned} \frac{s}{3}-4+1 &= \frac{s}{3}-3 = \frac{s-9}{3} \\ \frac{s}{3}-4+2 &= \frac{s}{3}-2 = \frac{s-6}{3} \end{aligned}\right\} \Rightarrow H\left(\frac{s}{3}-4\right) = 2 - \frac{24}{s-9} + \frac{18}{s-6}$$

$$\Rightarrow e^{-4s} H\left(\frac{s}{3}-4\right) = e^{-4s}\left[2 - \frac{24}{s-9} + \frac{18}{s-6}\right]$$

<u>We have:</u>

* $\delta(t) \xrightarrow{L_s} 1 \quad \text{all } s \quad \Rightarrow \delta(t-4) \xrightarrow{L_s} e^{-4s}$

$$\Rightarrow 2\delta(t-4) \xrightarrow{L_s} 2e^{-4s}$$

* $e^{9t} u(t) \xrightarrow{L_s} \frac{1}{s-9} \quad Re\{s\} > 9$

$$\Rightarrow e^{9(t-4)} u(t-4) \xrightarrow{L_s} e^{-4s} \cdot \frac{1}{s-9}$$

$$\Rightarrow 24e^{9(t-4)} u(t-4) \longrightarrow 24e^{-4s} \cdot \frac{1}{s-9}$$

* $e^{6t} u(t) \xrightarrow{L_s} \frac{1}{s-6} \quad Re\{s\} > 6$

$$\Rightarrow e^{6(t-4)} u(t-4) \xrightarrow{L_s} e^{-4s} \cdot \frac{1}{s-6}$$

$$\Rightarrow 18e^{6(t-4)} \cdot u(t-4) \xrightarrow{L_s} 18e^{-4s} \cdot \frac{1}{s-6}$$

$$\Rightarrow R(t) = 2\,\delta(t-4) - 24e^{9(t-4)}u(t-4) + 18e^{6(t-4)}u(t-4)$$

$$\xrightarrow{L_s} e^{-4s}H\left(\frac{s}{3}-4\right)$$

So, inverse of Laplace transform of $e^{-4s}H\left(\frac{s}{3}-4\right)$

is $\boxed{2\,\delta(t-4) - 24e^{9(t-4)}u(t-4) + 18e^{6(t-4)}u(t-4)}$

4) $$x(t) = \sum_{k=0}^{\infty} A_k \cos(\Omega_k t + \theta_k)$$

a) If $x(t)$ is periodic of period T_0, then the frequencies

$$\Omega_0 = \frac{2\pi}{T_0} \quad \& \quad \Omega_k = k \cdot \left(\frac{2\pi}{T_0}\right) \quad k = 0, 1, 2, \dots$$

b) $$x(t) = 2 + \underbrace{\cos(2\pi t)}_{x_1(t)} - \underbrace{3\cos(6\pi t + \pi/4)}_{x_2(t)}$$

We have $x_1(t)$ periodic with $T_{01} = \frac{2\pi}{2\pi} = 1(s)$

$x_2(t)$ periodic with $T_{02} = \frac{2\pi}{6\pi} = \frac{1}{3}$

Since $\frac{T_{01}}{T_{02}} = \frac{1}{1/3} = 3 = \frac{M}{N} = \frac{3}{1} \Rightarrow M = 3, N = 1$

$\Rightarrow$ $\boxed{x(t) \text{ is periodic with } T_0 = 3T_{02} = T_{01} = 1(s)}$

(*) We have trigonometric Fourier is given:

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\Omega_k t + \theta_k) = \sum_{k=0}^{\infty} A_k \cos(2\pi k t + \theta_k)$$

$$= A_0 \cos(\theta_k) + A_1 \cos(2\pi t + \theta_1) + A_2 \cos(4\pi t + \theta_2)$$

$$+ A_3 \cos(6\pi t + \theta_3) + \dots + \dots$$

$$= 2 + \cos(2\pi t) + (-3)\cos(6\pi t + \pi/4)$$

$$\Rightarrow \boxed{\begin{cases} A_0 = 2, \ \theta_0 = 0 \\ A_1 = 1, \ \theta_1 = 0 \\ A_2 = 0, \ \theta_2 = 0 \\ A_3 = -3, \ \theta_3 = \pi/4 \\ A_k = 0, \ \theta_k = 0 \text{ with } k \geq 4 \end{cases}}$$

c) $x_1(t) = 2 + \underbrace{\cos(2\pi t)}_{y_1(t)} - \underbrace{3\cos(20t + \pi/4)}_{y_2(t)}$

We have $y_1(t)$ is periodic with $T_{01} = \frac{2\pi}{2\pi} = 1 \ (s)$

$y_2(t)$ is periodic with $T_{02} = \frac{2\pi}{20} = \frac{\pi}{10} \ (s)$

Then $\frac{T_{01}}{T_{02}} = \frac{1}{\pi/10} = \frac{10}{\pi} = \frac{M}{N} \Rightarrow M$ & N are not both integer number

$\Rightarrow$ $x_1(t)$ is not periodic $\Rightarrow$ Fourier could not be applied $\Rightarrow$ We could not determine its Fourier series.

5.

a)

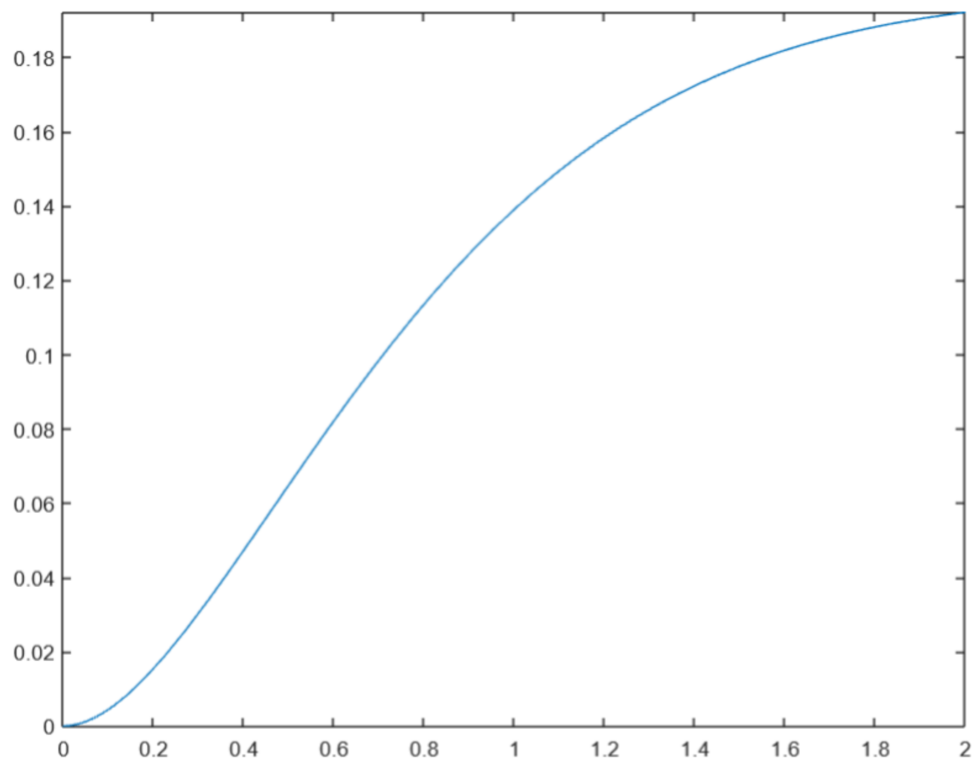
```
%% Clear Cache
close all;
clc;

%% Problem 5
% Part a
syms t
f=(t-5)^4*exp(-3*t)*heaviside(t);
F=laplace(f)
```

```
F =

300/(s + 3)^3 - 120/(s + 3)^4 + 24/(s + 3)^5 + (500*((5*s)/4 + 11/4))/(s + 3)^2
```

b)

```
% Part b
syms s;
F=1/(s*(s+2-1i*pi/3)*(s+2+1i*pi/3));
f=ilaplace(F)
fplot(f,[0 2]);
```

```
f =

- 9/((- 6 + pi*1i)*(6 + pi*1i)) - (exp((t*(- 6 + pi*1i))/3)*9i)/(2*pi*(- 6 + pi*1i)) - (exp(-(t*(6 + pi*1i))/3)*9i)/(2*pi*(6 + pi*1i))
```



Full code:

```
%% Clear Cache
clc;
clear all;
close all;

%% Problem 5
% Part a
syms t
f=(t-5)^4*exp(-3*t)*heaviside(t);
F=laplace(f)

% Part b
syms s;
F=1/(s*(s+2-1i*pi/3)*(s+2+1i*pi/3));
f=ilaplace(F)
fplot(f,[0 2]);
```