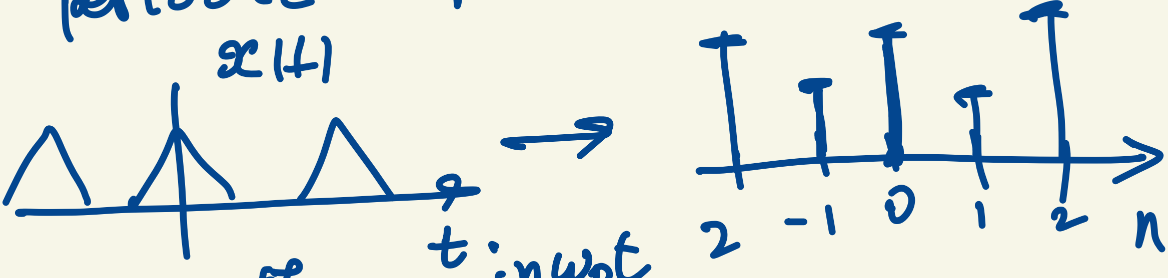
HW deadline is extended
till Friday midnight.

Fourier Transform (Ch. 5)

periodic signal

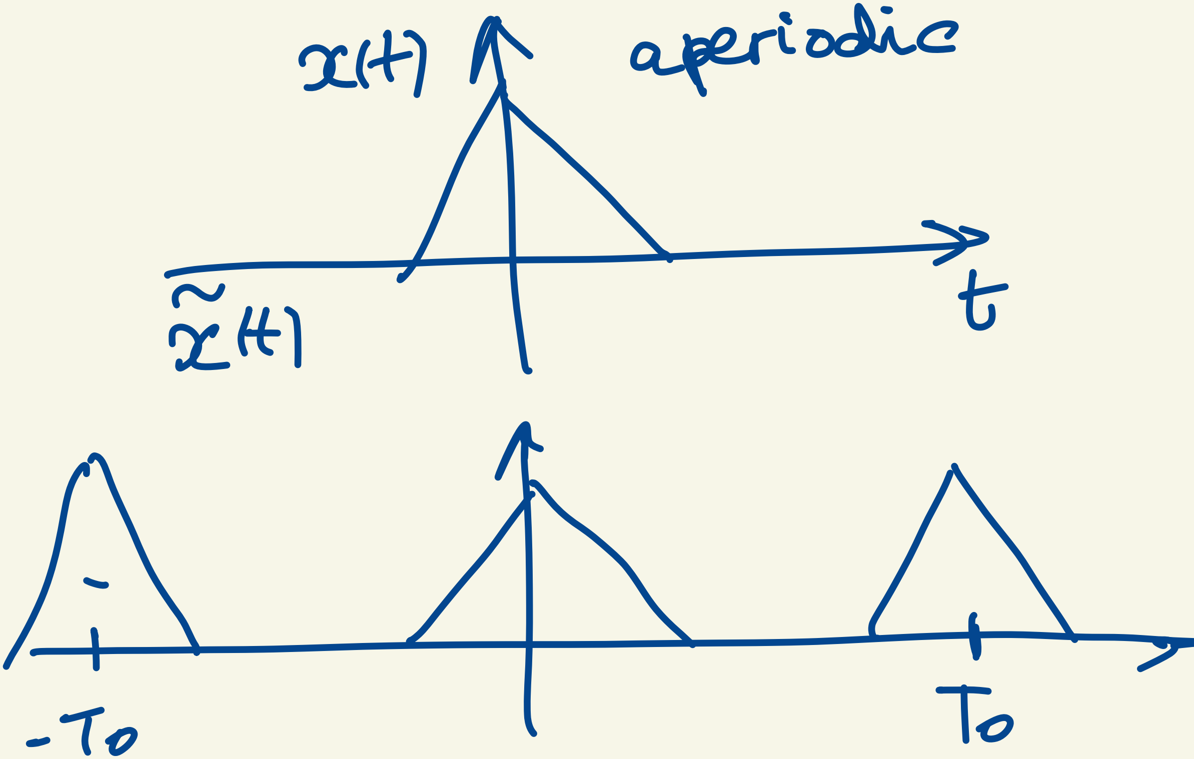


$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$\pm \omega_0, \pm 2\omega_0, \pm 3\omega_0$

How about aperiodic signals?
what can we say about

its frequency content?



$$\lim_{T_0 \to \infty} \underline{\underline{\tilde{x}(t)}} = x(t)$$

↑ apply FS.

$$\rightarrow \tilde{x}(t) = \sum_{n=-\infty}^{+\infty} X_n e^{jn\omega_0 t}, \qquad \omega_0 = \frac{2\pi}{T_0}$$

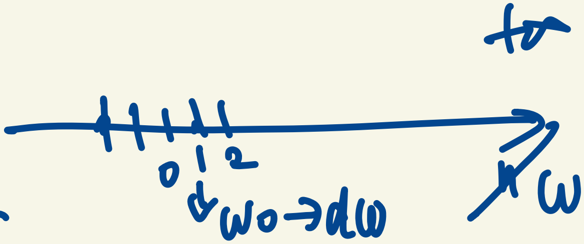
$$X_n = \frac{1}{T_0}\int_{T_0} \tilde{x}(t)\, e^{-jn\omega_0 t}\, dt$$

$$\lim_{T_0\to\infty} \omega_0 = \lim_{T_0\to\infty} \frac{2\pi}{T_0} \rightarrow d\omega$$

$$\lim_{T_0\to\infty} \underline{\underline{X_n \cdot T_0}} = \lim_{T_0\to\infty} \int_{T_0} \tilde{x}(t)\, e^{-j\underline{n\omega_0} t}\, dt$$

$$= \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \underline{X(\omega)}$$

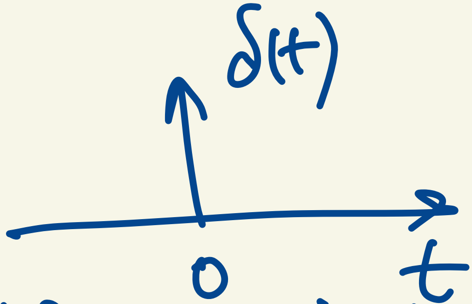
$$\lim_{T_0\to\infty} \underset{n\in\mathbb{Z}}{\underset{\downarrow}{n}}\,\omega_0 = \underset{\in\mathbb{R}}{\omega^{\infty}}$$



$$\boxed{X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \qquad \omega \in \mathbb{R}}$$

Fourier Transform of $x(t)$

ex. $x(t) = \delta(t)$

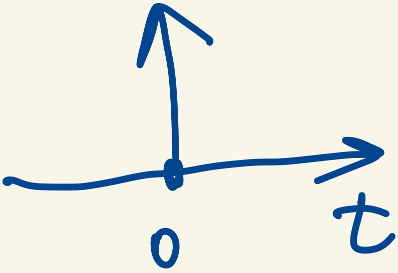


$$\mathcal{F}\{x(t)\} = X(\omega) = \int_{-\infty}^{+\infty} \delta(t) \cdot e^{-j\omega t}\, dt$$

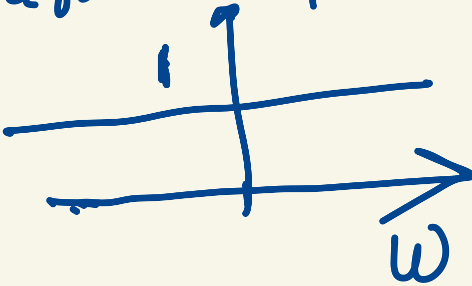
$$= 1$$

Fourier Transform pair.

$$\delta(t) \xrightarrow{\mathcal{F}} 1$$



$|X(\omega)|$ magnitude spectrum



$\measuredangle X(\omega)$ phase spectrum

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ex. $x(t)=e^{-2t}u(t)$

$$X(\omega)=\int_{-\infty}^{+\infty} e^{-2t}\cdot u(t)\cdot e^{-j\omega t}\,dt$$

$$= \underbrace{\int_{0}^{+\infty} e^{-2t} \cdot e^{-\underline{\underline{j\omega t}}} \, dt}_{\mathcal{L}_s\{e^{-2t} \cdot u(t)\} \Big|_{\underline{s = j\omega}}}$$

$\searrow$ ROC

$Re\{s\} > -2$

$$= \frac{1}{s+2} \Big|_{s = j\omega}$$

$$= \frac{1}{2 + j\omega}$$

For causal <u>signals</u>

whose Laplace T.
ROC includes $s=j\omega$
(imaginary axis)

we can say

$$\mathcal{F}\{x(t)\cdot u(t)\} = \mathcal{L}_s\{x(t)\cdot u(t)\}$$

for $s=j\omega$

How about

$$u(t) \xrightarrow{\mathcal{F}} \; ?$$

ROC

$\mathrm{Re}\{s\} > 0$

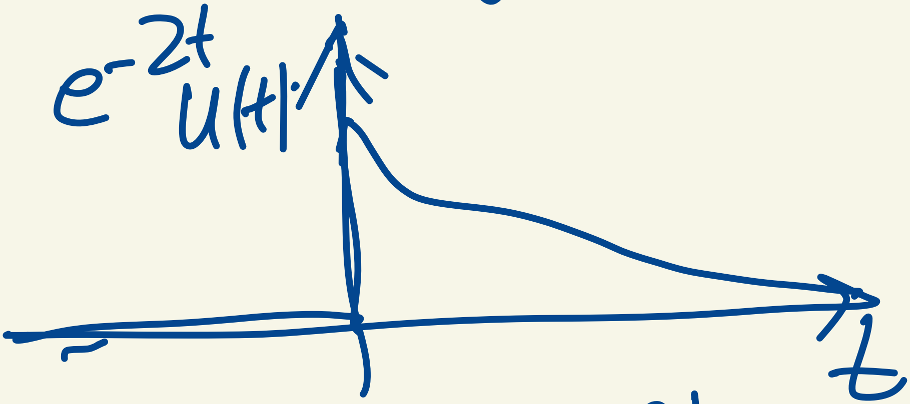
$j\omega$ not included

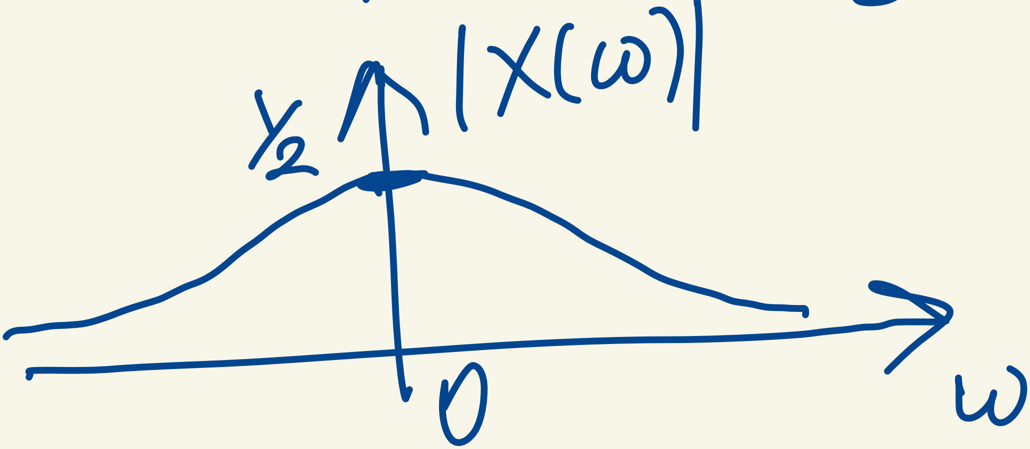
$x(t)$ $X(\omega)$

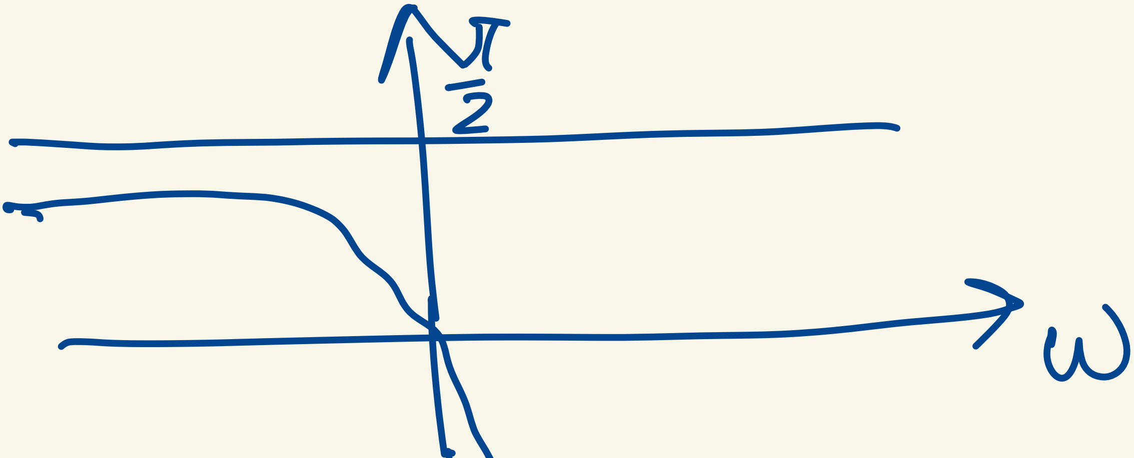
$$e^{-2t}u(t) \longrightarrow \frac{1}{2+j\omega}.$$

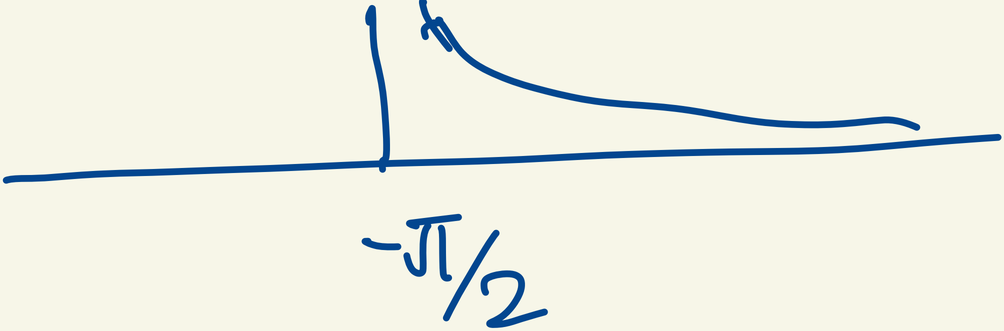
$$|X(\omega)| = \frac{1}{\sqrt{4+\omega^2}}$$

$$\angle X(\omega) = tg^{-1}\left(-\frac{\omega}{2}\right)$$









Inverse Fourier Transf.

$$\underline{\tilde{x}(t)} = \sum_{n=-\infty}^{+\infty} \boxed{X_n}\, e^{jn\omega_0 t} \quad (\lim T_0 \to \infty$$

$$= \sum_{n=-\infty}^{+\infty} \frac{\boxed{2\pi}\, X_n \cdot T_0}{2\pi\, \boxed{T_0}}\, e^{jn\omega_0 t}$$

$$\lim_{T_0 \to \infty} \tilde{x}(t) = \lim_{T_0 \to \infty} \sum_{n=-\infty}^{+\infty} \frac{1}{2\pi} \cdot \underbrace{X_n \cdot T_0} \cdot e^{\widetilde{jn\omega_0 t}} \cdot \underbrace{\frac{2\pi}{T_0}}_{d\omega}$$

$$\boxed{\begin{array}{l} x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega \\ \text{Inverse Fourier transform.} \end{array}}$$

Analysis through Fourier Transform

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

Synthesis through Inverse Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

Existence Of Fourier Transform
" Dirichlet's conditions"

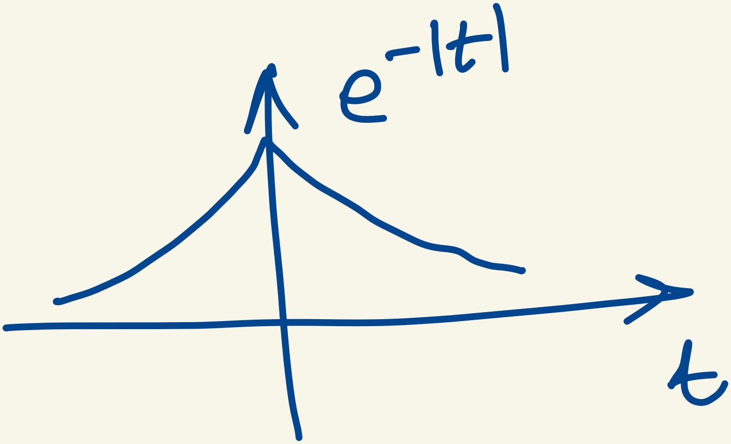
1) $x(t)$ has to be absolutely integrable

$$\int_{-\infty}^{+\infty} |x(t)|\, dt < +\infty$$

2) $x(t)$ has finite number of maxima, minima and discontinuities.

Signals of practical interest and the ones we will use in 102 satisfy these conditions e.g. $x(t) = e^{-|t|} \quad -\infty < t < +\infty$

$$e^{-|t|} = \begin{cases} e^{-t} & t \geq 0 \\ e^{t} & t < 0 \end{cases}$$



$$\mathcal{F}\{x(t)\} = \int_{-\infty}^{+\infty} e^{-|t|} e^{-j\omega t} dt$$

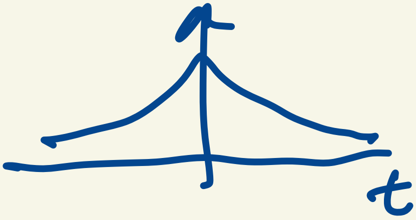
$$= \int_{-\infty}^{0} e^{t} e^{-j\omega t} dt + \int_{0}^{+\infty} e^{-t} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{t(1-j\omega)} dt + \int_{0}^{+\infty} e^{-t(1+j\omega)} dt$$

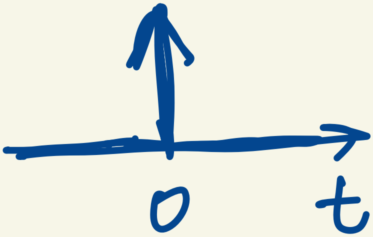
$$= \frac{1}{1-j\omega} e^{t(1-j\omega)} \Big|_{-\infty}^{0} + \left(-\frac{1}{1+j\omega} \cdot e^{-t(1+j\omega)} \right) \Big|_{0}^{+\infty}$$

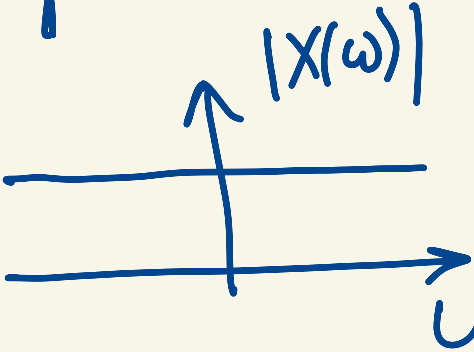
$$= \frac{1}{1-j\omega} + \frac{1}{1+j\omega} = \frac{2}{1+\omega^2}$$

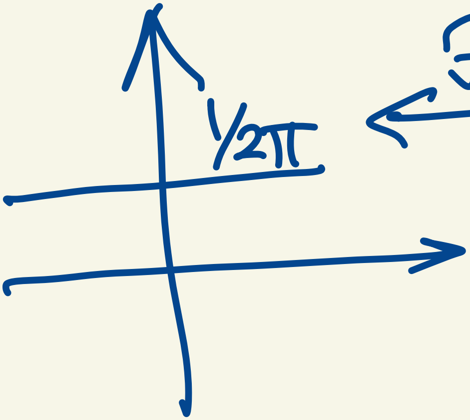
$$x(t) = e^{-|t|} \longrightarrow X(\omega) = \frac{2}{1+\omega^2}$$



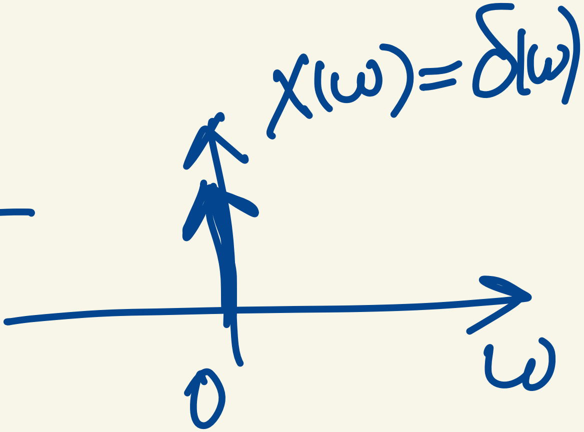
$$\delta(t) \xrightarrow{\mathcal{F}} 1$$





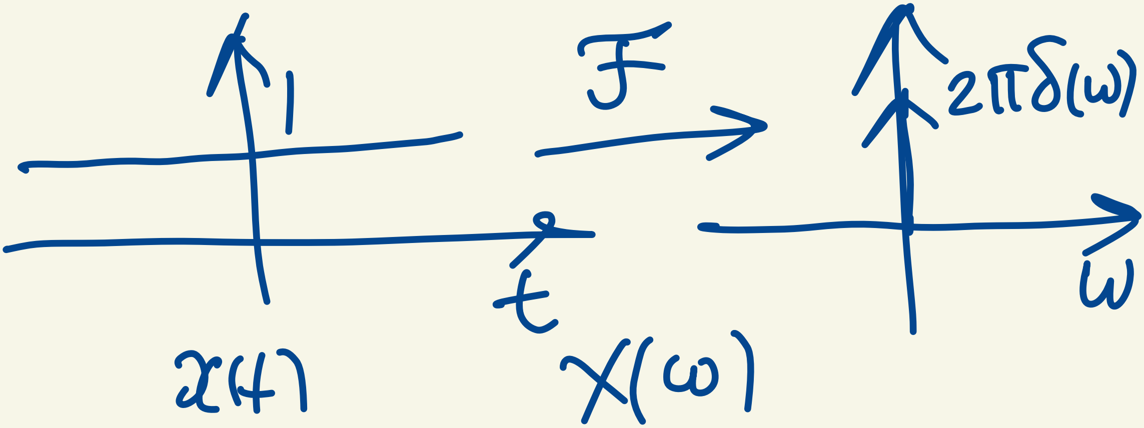


$$\xleftarrow{\mathcal{F}^{-1}}$$



$$\mathcal{F}^{-1}\{\delta(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\delta(\omega)\cdot e^{j\omega t}\,d\omega$$

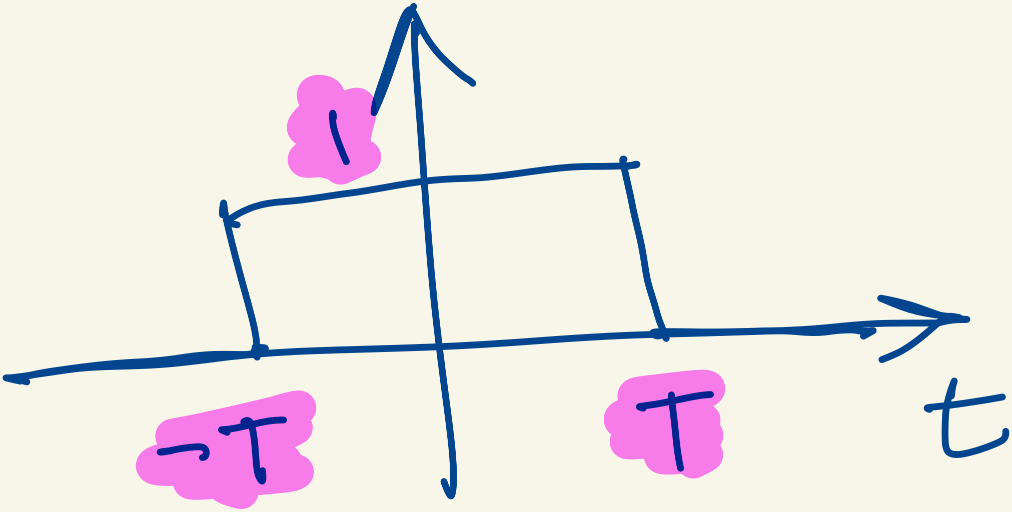
$$= \frac{1}{2\pi}$$



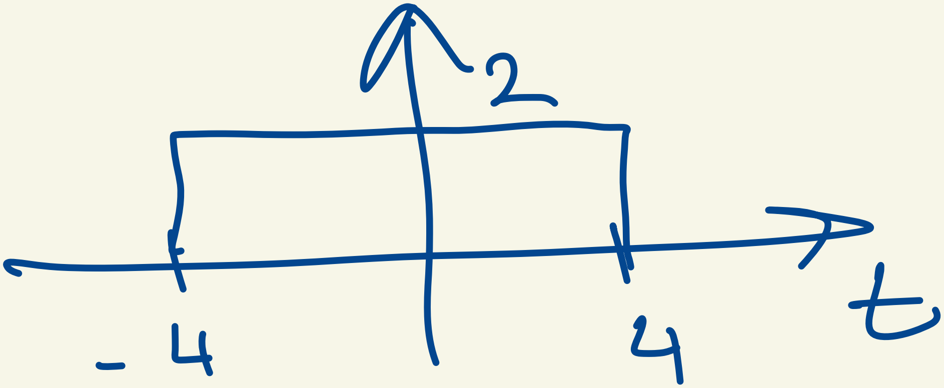
$$\boxed{\begin{aligned} \delta(t) &\rightarrow 1 \\ 1 &\rightarrow 2\pi\delta(\omega) \end{aligned}}$$

pairs that are related.

Another important signal



$$u(t+T) - u(t-T) \triangleq \mathrm{rec}(t, T)$$



$$2\,\mathrm{rec}(t, 4)$$

$$\mathcal{F}\{\mathrm{rec}(t, T)\} =$$

$$= \int_{-T}^{T} 1 \cdot e^{-j\omega t}\, dt$$

$$= -\frac{1}{j\omega} e^{-j\omega t} \Big|_{-T}^{T}$$

$$= -\frac{2}{2j\omega} \left[e^{-j\omega T} - e^{j\omega T} \right]$$

$$= \frac{2}{\omega} \left[\frac{e^{j\omega T} - e^{-j\omega T}}{2j} \right]$$

$$= \frac{2T}{\omega T} \sin(\omega T)$$

$$\frac{\sin x}{x} \triangleq \operatorname{sinc}(x)$$

$$= 2T \operatorname{sinc}(\omega T)$$

$$\operatorname{sinc}(0) = 1$$

$$\underline{\mathrm{rec}(t,T)} \xrightarrow{\mathcal{F}} \underbrace{2T}_{X(0)}\mathrm{sinc}(\omega T)$$

$$X(0) = 2T$$

$$X(\omega)\Big|_{\omega=0} = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

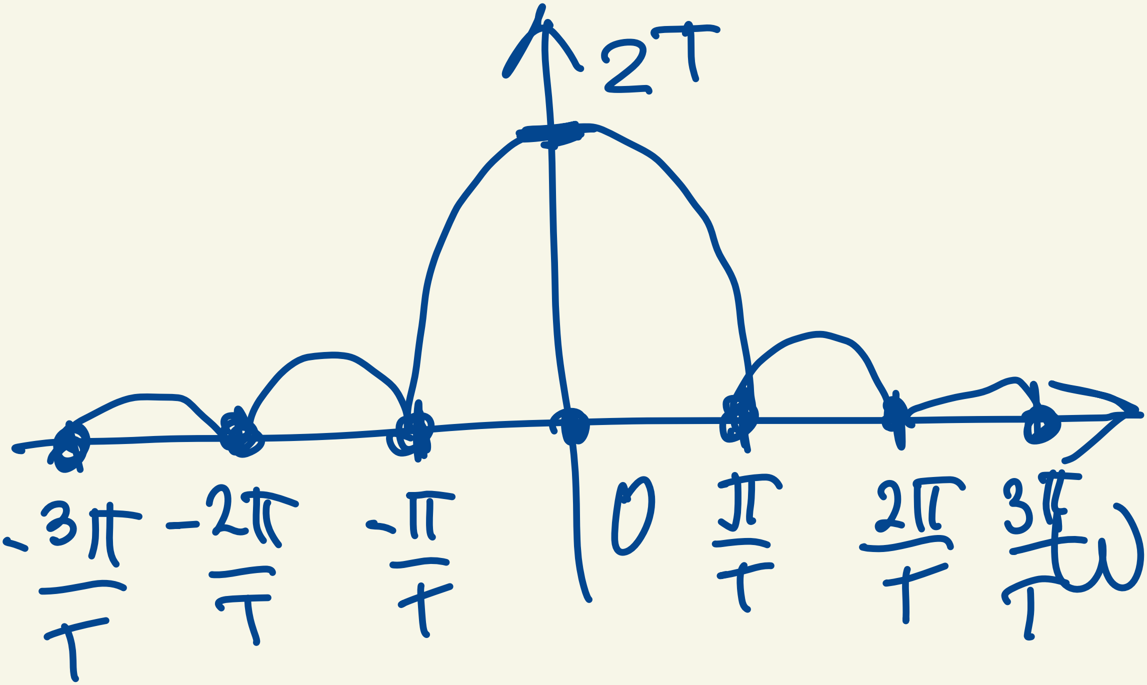
$$\boxed{X(0) = \int_{-\infty}^{+\infty} x(t)\, dt \quad \text{area under } x(t)}$$

$$x(t) = \mathrm{rec}(t,T) \quad X(0) = 2T$$

$$\mathrm{rec}(t,T) \xrightarrow{F} 2T\,\mathrm{sinc}(\omega T)$$

$$|X(\omega)| = \left|2T\,\mathrm{sinc}(\omega T)\right|$$

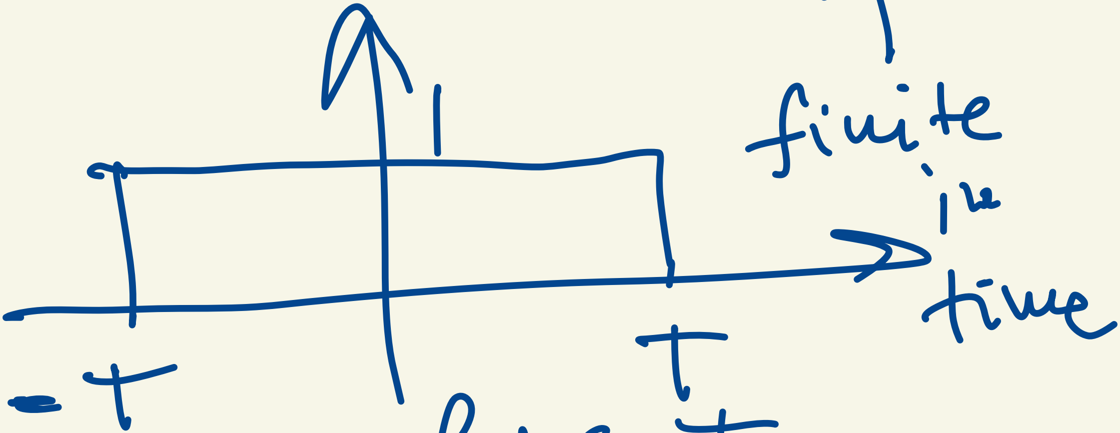
$$= \left|2T\,\frac{\sin(\omega T)}{\omega T}\right|$$



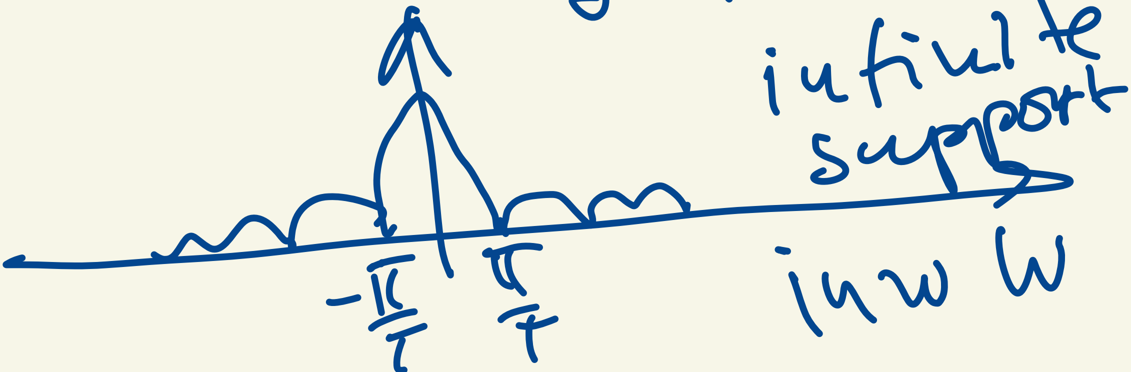
$|X(\omega)| = 0$ $\quad \sin(\omega T) = 0$

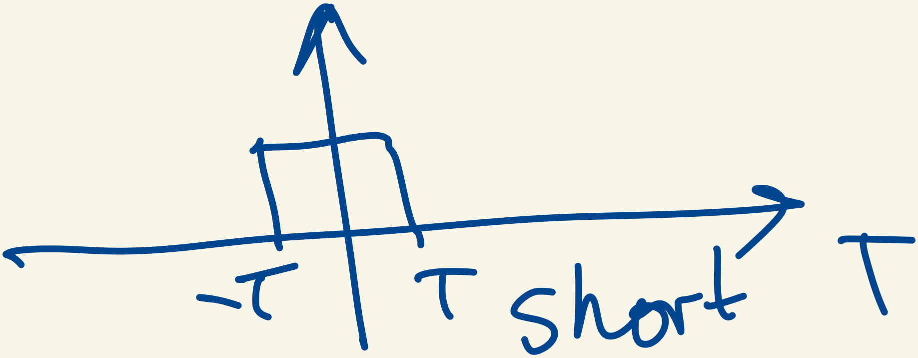
$$\omega T = k\pi$$

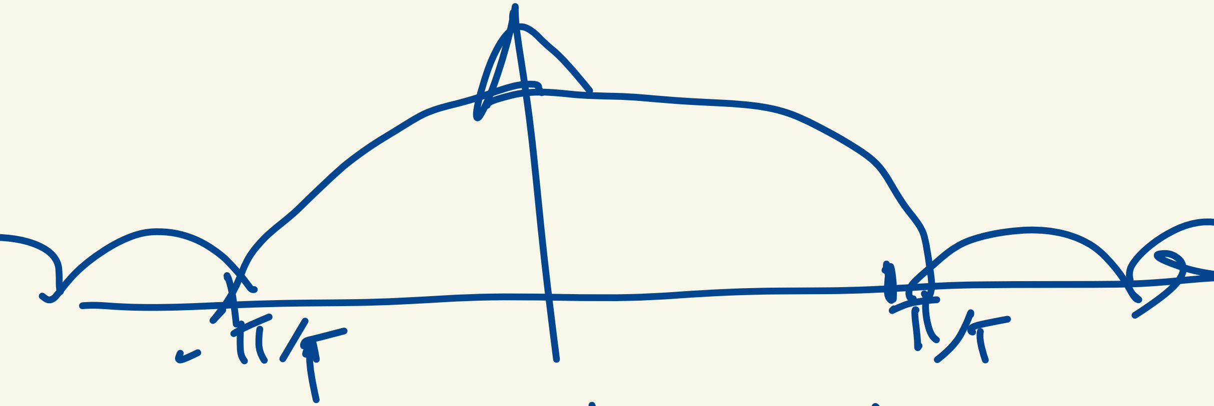
$$\omega = \frac{k\pi}{T}$$

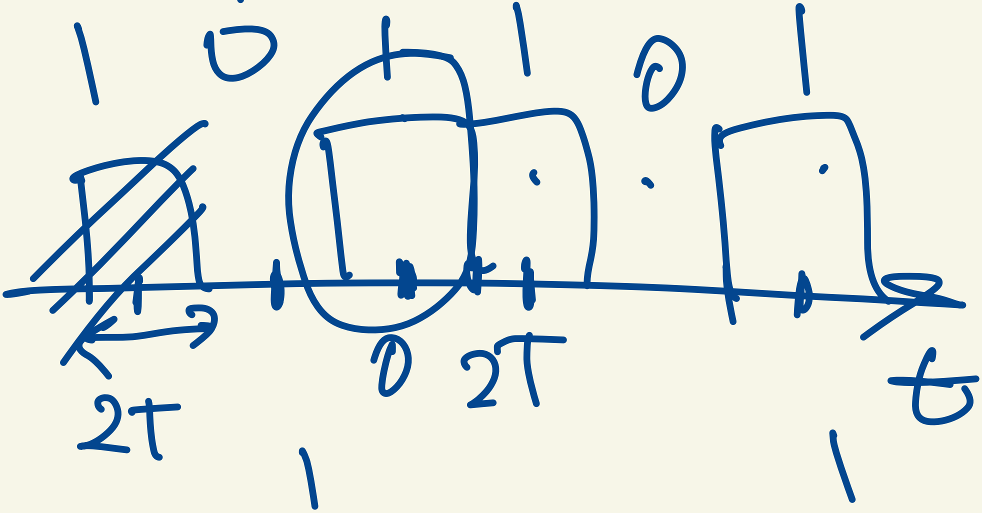


long T









$$1\cdot \mathrm{rec}(t,\tau)+1\cdot \mathrm{rec}(t-2\tau)$$

$$\overset{0}{+}\ 1\ \overset{1}{\text{rec}}(t-6T,T).$$

$$s(t)=\sum_{i=0}^{N} b_i\,\text{rec}(t-i2T,T)$$

$N\cdot 2T \rightarrow$ total time

I send N bits,

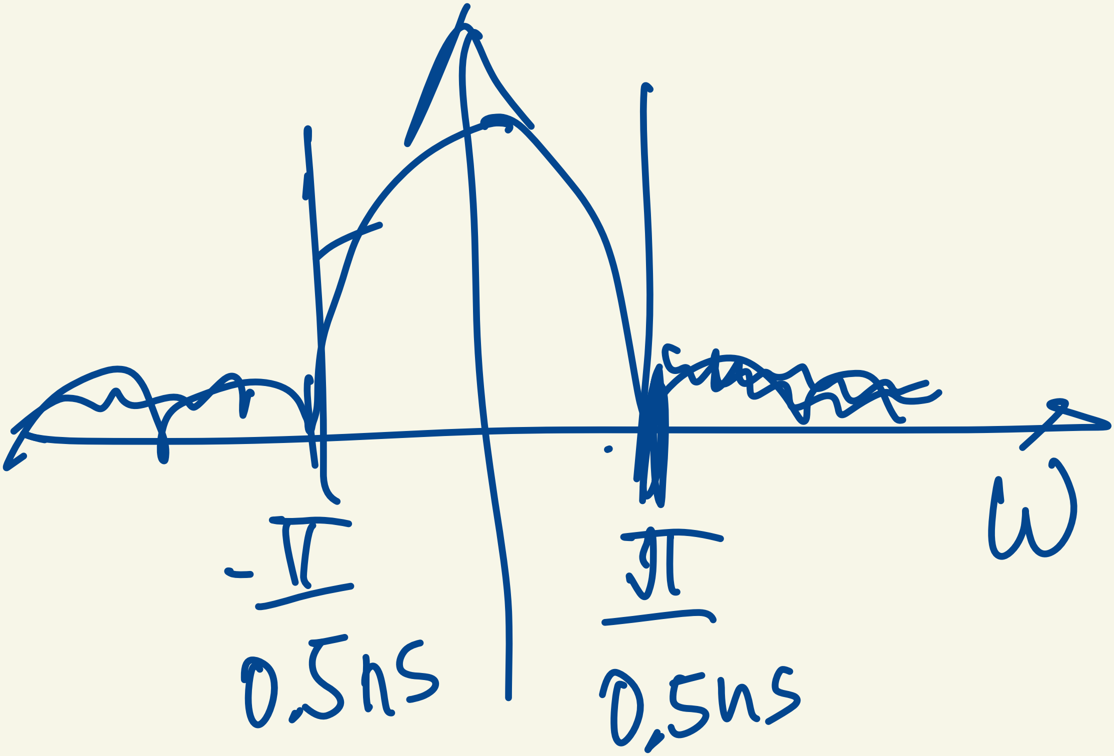
$$R=\frac{\#\text{bits}}{\text{total time}}=\frac{N}{N\cdot 2T}$$

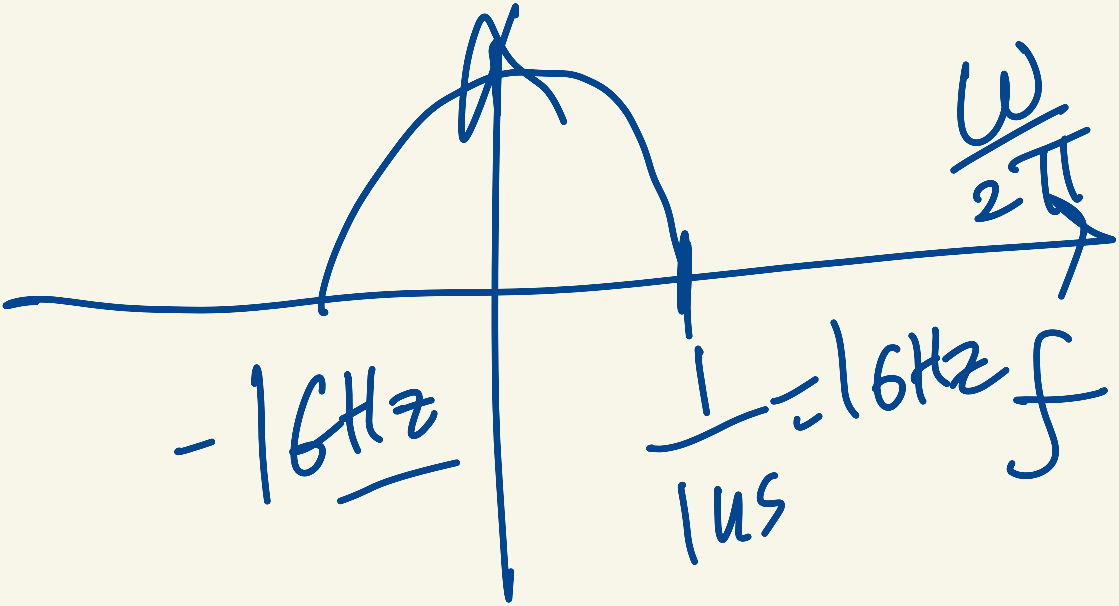
$$R = \frac{1}{2T} \left[\frac{\text{bits}}{s}\right]$$

$$R = 1\,\frac{\text{Gbits}}{s} = 10^9\,\frac{\text{bits}}{s}$$

$$\boxed{2T} = \frac{1}{R} = 1\text{ns}.$$

$$\text{rec}(t, 0.5\text{ns})$$


$-\frac{\pi}{0.5ns}$
$\frac{\pi}{0.5ns}$
ω


$-1GHz$
$\frac{1}{1us} = 1GHz$
$\frac{\omega}{2\pi}$
f

Radio Spectrum