Lecture 2

① odd and even signals

$x(t) = x(-t)$ even

$x(t) = -x(-t)$ odd

Theorem: Any signal can be decomposed into odd and even components.

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$x_e(-t) = \frac{x(-t) + x(-(-t))}{2}$$

$$= \frac{x(-t) + x(t)}{2} = x_e(t)$$

Proof: $x_e(t) + x_o(t) =$

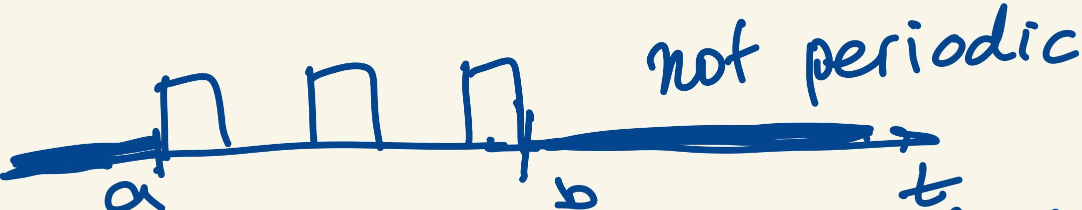
$$= \frac{x(t)+x(-t)}{2} + \frac{x(t)-x(-t)}{2}$$

$$= \frac{x(t) + \cancel{x(-t)} + x(t) - \cancel{x(-t)}}{2}$$

$$= \frac{2x(t)}{2}$$

$$= x(t)$$

Periodic signals



(1) Periodic signals have <u>infinite</u> support.

Finite support signals have property:

$\exists a, b$ s.t. $x(t) = 0$

$t < a$ & $t > b$

② $\exists T_0 : x_p(t + kT_0) = x_p(t)$

$\forall k \in \mathbb{Z}$

T_0 - fundamental period.

Properties of periodic signals

case

a) $x(t)$ is periodic w/ T_0

Then

$y(t) = A + x(t)$

$\Rightarrow y(t)$ is periodic w/ T_0

$$y(t+kT_0) = A + x(t+kT_0)$$

$$= A + x(t)$$

$$= y(t)$$

case

b) $x(t)$ periodic w/ $\underline{T_0}$

$y(t)$ periodic w/ $T_1 = NT_0$

N is integer

$$z(t) = x(t) + y(t)$$

$z(t)$ is periodic w/ $NT_0 = T_1$

Proof:

$$z(t + k \cdot NT_0) = x(t + \underline{kN}\boxed{T_0})$$

$$+ y(t + k\boxed{NT_0})$$

$= x(t) + y(t)$

$= z(t) \Rightarrow z(t)$ is periodic w/ NT_0

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c) $x(t)$ periodic w/ T_0.

$y(t)$ periodic w/ T_1.

s.t. $\boxed{\dfrac{T_1}{T_0} = \dfrac{M}{N}}$ M, N are integers

$MT_0 = NT_1$

$w(t) = x(t) + y(t)$

$\Rightarrow w(t)$ periodic w/ $\underbrace{MT_0 = NT_1}$

Proof:

$$w(t+kNT_1) = x(t+kNT_1)$$

$$+ y(t+\underline{kN}\boxed{T_1})$$

$$= x(t+\underline{kM}\boxed{T_0}) + y(t)$$

$$= x(t) + y(t)$$

$$= w(t)$$

<u>Example</u>

$$x(t) = e^{j2t} = \cos(2t) + j\sin(2t)$$

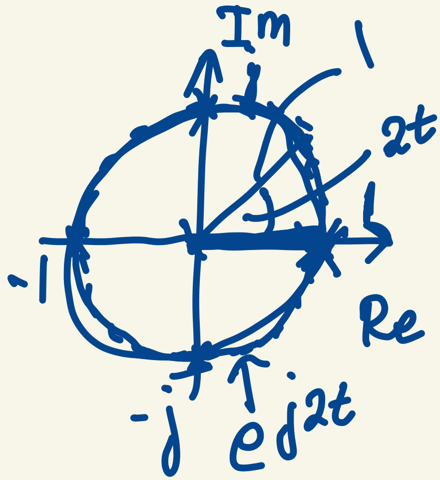
$$y(t) = e^{j\pi t}$$

$x(t)$ is periodic w/ $T_0 =$

$$e^{j2(t+T_0)} = e^{j2t}$$

$$e^{j2t} \cdot e^{j2T_0} = e^{j2t}$$

$$\Leftrightarrow \quad e^{j2T_0} = 1$$



$$e^{j2T_0} = e^{j2\pi}$$

$$\boxed{T_0 = \pi}$$

e^{j2t} w/ $T_0 = \pi$

$e^{j\pi t}$ w/ $T_1 = 2$

$$z(t) = x(t) + y(t) = e^{j2t} + e^{j\pi t}$$

$$\frac{T_1}{T_0} = \frac{2}{\pi} \neq \frac{M}{N}$$

$z(t)$ is not periodic

Ex. $x(t) = e^{j2t}$
$y(t) = e^{j\pi t}$

$w(t) = x(t) \cdot y(t) = e^{j2t} \cdot e^{j\pi t}$
$= e^{j\underbrace{(2+\pi)}_{a} t}$

$\Rightarrow$ periodic w/ $T_2 = \dfrac{2\pi}{2+\pi}$

product of two periodic signals is not necessarily periodic!

counterexample

$\underline{v(t)} = \underbrace{\left(1 + e^{j2t}\right)} \cdot \underbrace{\left(1 + e^{j\pi t}\right)}$

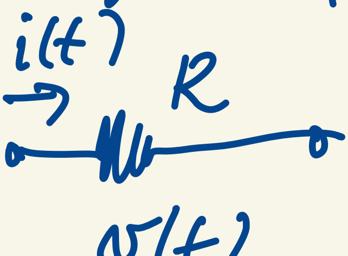
$$= 1 + e^{j2t} + e^{j\pi t} + e^{j2t} \cdot e^{j\pi t}$$

$$= 1 + e^{j2t} + e^{j\pi t} + e^{j(2+\pi)t}$$

$T_0 = \pi$ $\quad$ $T_1 = 2$ $\quad$ $T_2 = \dfrac{2\pi}{2+\pi}$

$$\frac{T_0}{T_1} \neq \frac{M}{N}$$

Signal power



$$p(t) = i(t) \cdot v(t)$$
$$= i^2(t) \cdot R$$

instanteneous power.

$$= \frac{v^2(t)}{R}$$

$$v(t) = i(t) \cdot R$$

Energy of the signal.

$$E = \int_{t_0}^{t_1} p(t)\,dt$$ over $t \in [t_0, t_1]$

$x(t) \rightarrow$ signal

$$E_x \triangleq \int_{-\infty}^{+\infty} x^2(t)\,dt$$

$\rightarrow$ finite energy

$\rightarrow$ infinite energy

In general we have complex signals.

$$\boxed{E_x \triangleq \int_{-\infty}^{+\infty} |x(t)|^2\,dt}$$

$$\boxed{\text{Average power of } x(t) \qquad P_x \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt}$$

e.g. $x(t) = \cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$

what is P_x?

$x(t)$ periodic / w $T_0 = 4$

$\frac{\pi}{2} \cdot T_0 = 2\pi \qquad T_0 = 4$

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \cos^2\left(\frac{\pi}{2}t + \frac{\pi}{4}\right) dt$$

$$= \lim_{\substack{T\to\infty \\ NT_0}} \frac{1}{\cancel{2}\underset{\parallel\ NT_0}{T}} \cdot \cancel{2} \int_0^{T = NT_0} \cos^2\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)dt$$

$$T = N \cdot T_0 \qquad \underset{NT_0}{T} \to \infty \Rightarrow N \to \infty$$

$$= \lim_{N\to\infty} \frac{1}{NT_0} \int_0^{NT_0} \cos^2\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)dt$$

$$= \lim_{N\to\infty} \frac{1}{\cancel{N}T_0} \cdot \cancel{N} \int_0^{T_0} \cos^2\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)dt$$

$$= \frac{1}{T_0} \int_0^{T_0} \cos^2\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)dt$$

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$= \frac{1}{T_0}\int_0^{T_0} \frac{1+\cos\left[2\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)\right]}{2}\,dt$$

$T_0 = 4$

$$= \frac{1}{T_0}\int_0^{T_0} \frac{1}{2}\,dt + \frac{1}{2T_0}\int_0^{T_0} \cos\left(\pi t + \frac{\pi}{2}\right) \cdot dt$$

0

$$= \frac{1}{\cancel{T_0}} \cdot \frac{1}{2} \cancel{T_0}$$

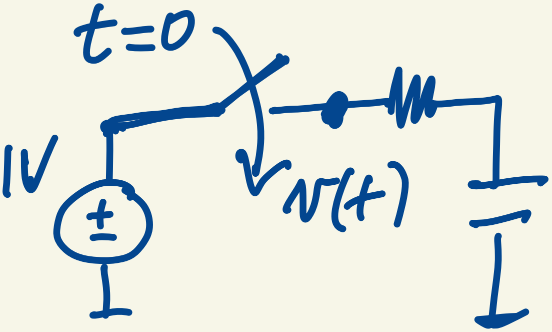
$$= \frac{1}{2}$$

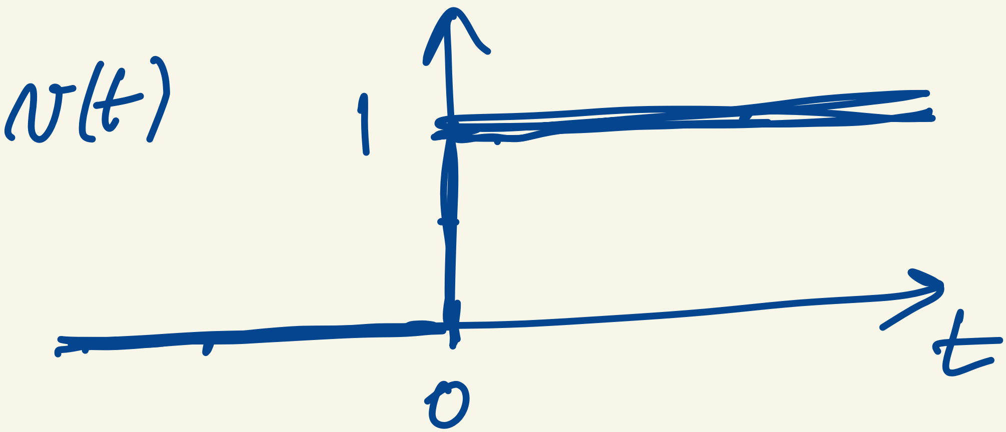
$\pi \cdot T_1 = 2\pi$

$T_1 = 2.$

$$x(t) = \cos(\alpha t + \underline{\beta}) \Rightarrow P_x = \frac{1}{2}$$

Unit Step, Unit Impulse and Unit Ramp Signals



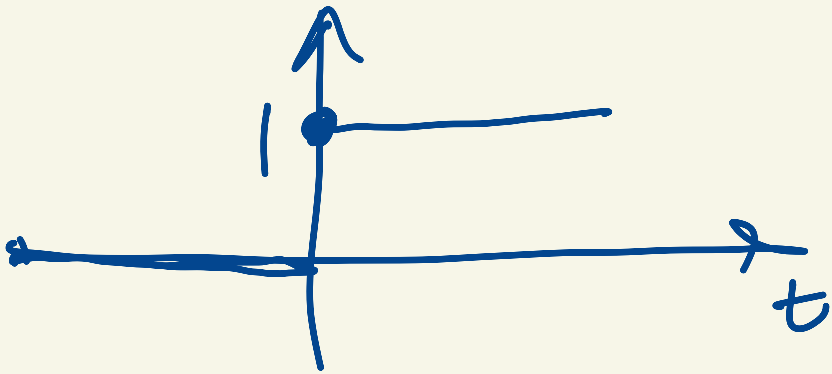


Unit Step $u(t)$

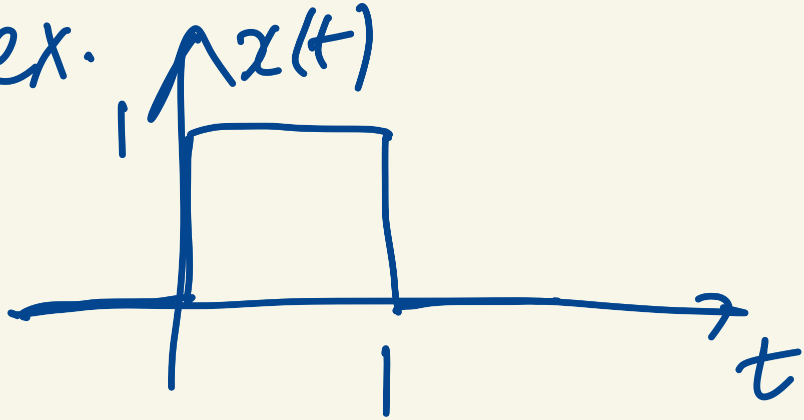
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

in some textbook you may see.

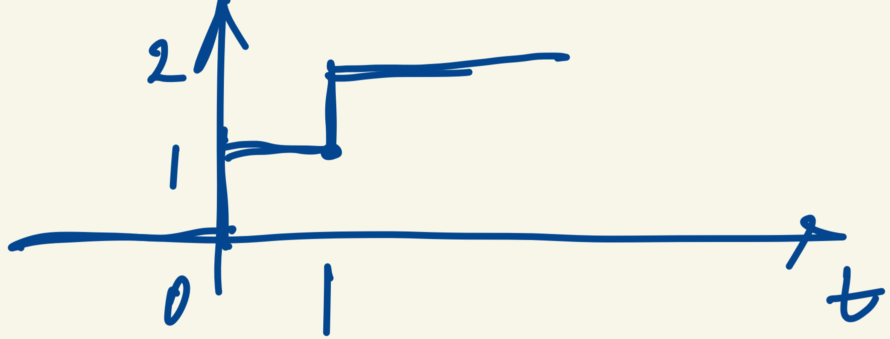
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



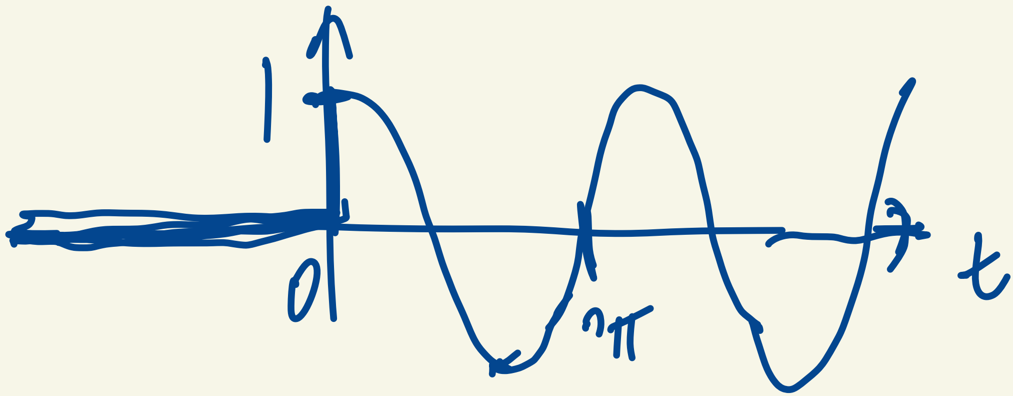
ex.



$$x(t) = u(t) - u(t-1)$$



$x(t) = u(t) + u(t-1)$

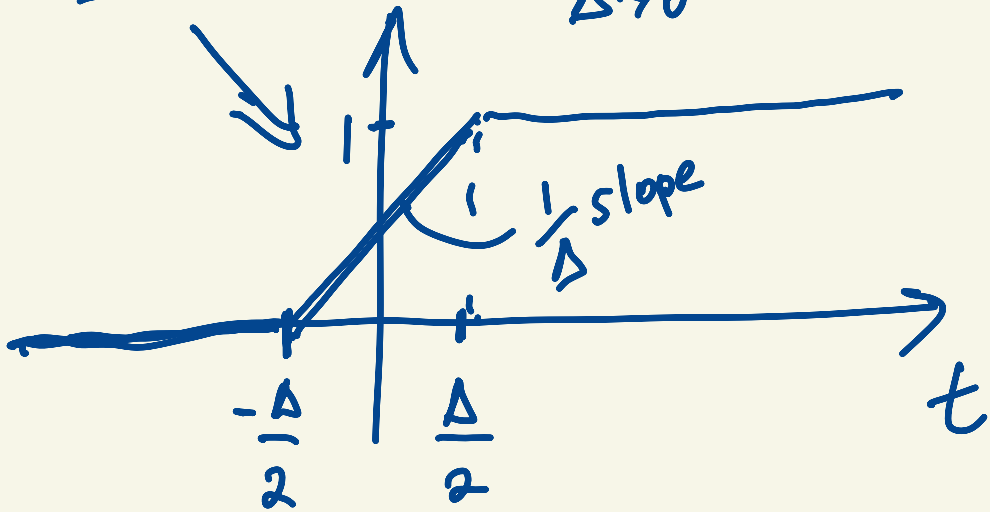


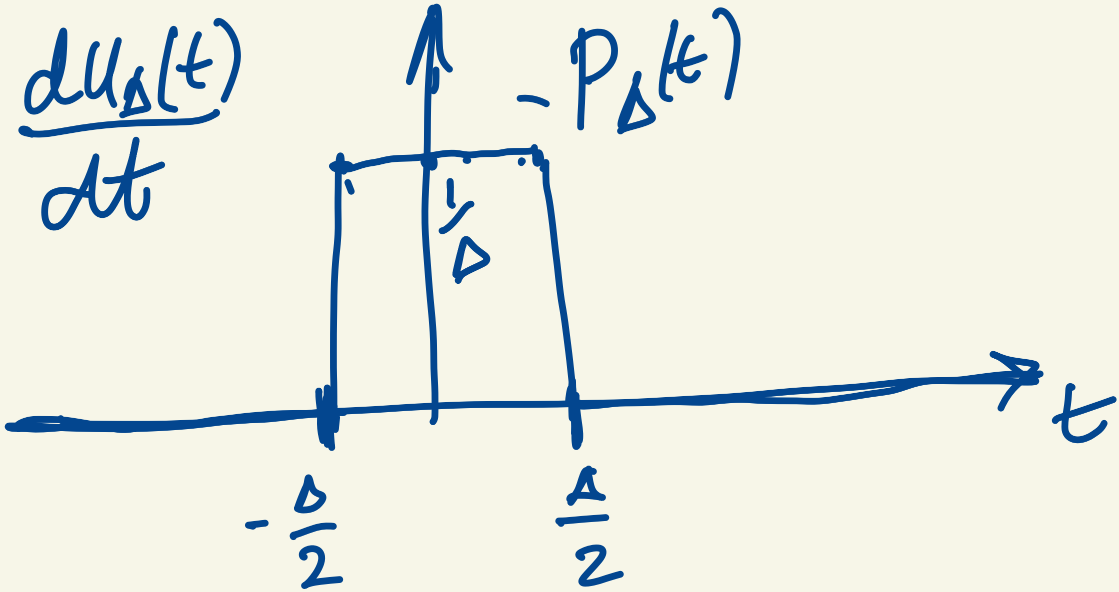
$x(t) = \cos(t) \cdot u(t)$

$\frac{du(t)}{dt} = ?$

$u_\Delta(t)$

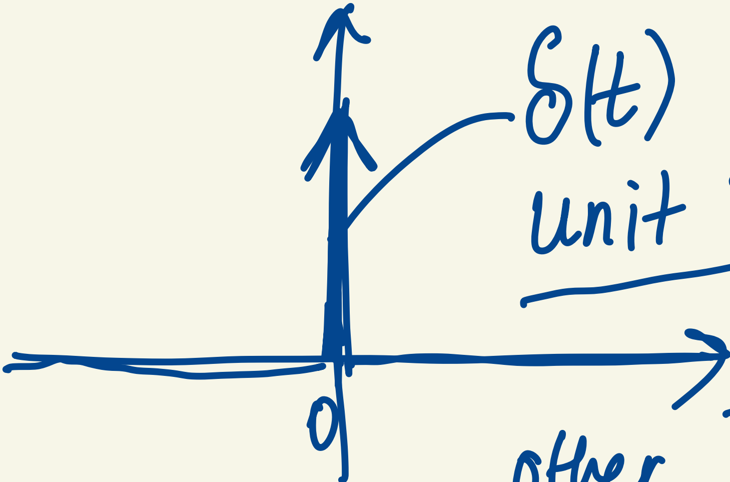
$$\lim_{\Delta \to 0} u_\Delta(t) = u(t)$$





$\Delta \to 0$ $\quad u_\Delta(t) \to u(t)$

$$p_\Delta(t) = \frac{du_\Delta(t)}{dt} \longrightarrow \delta(t)$$



$\delta(t)$

Unit Impulse

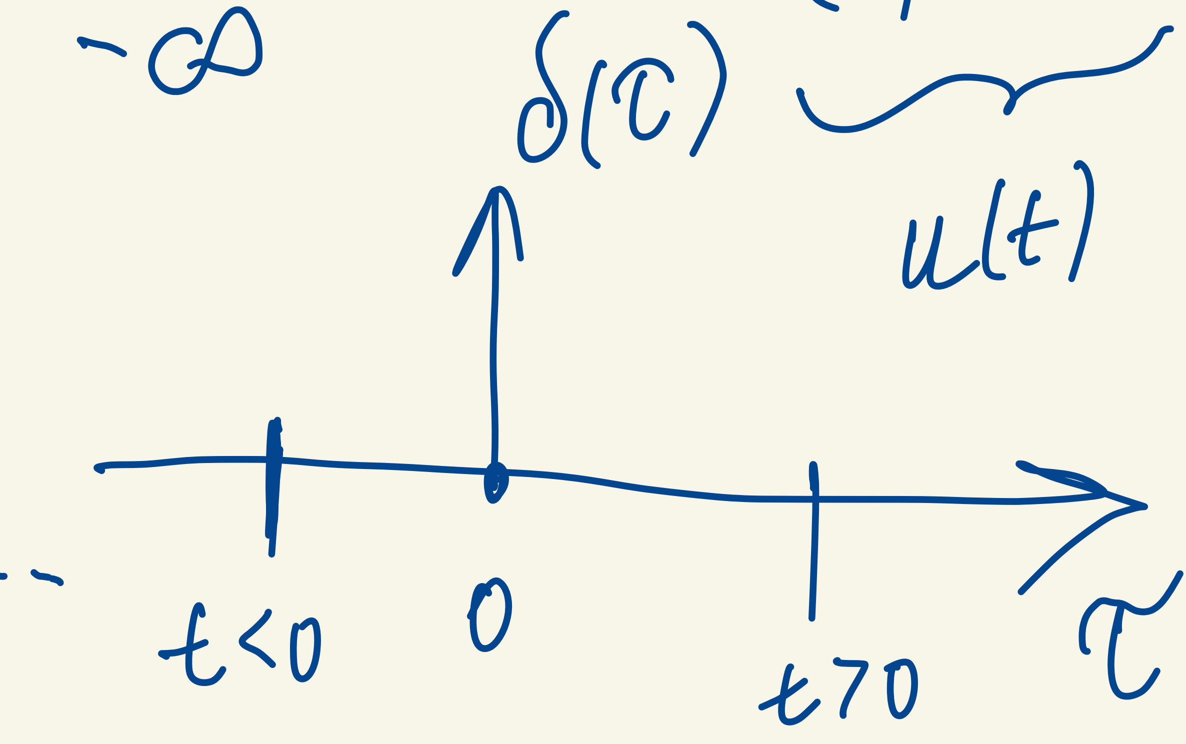
other

Dirac Delta function

Delta function

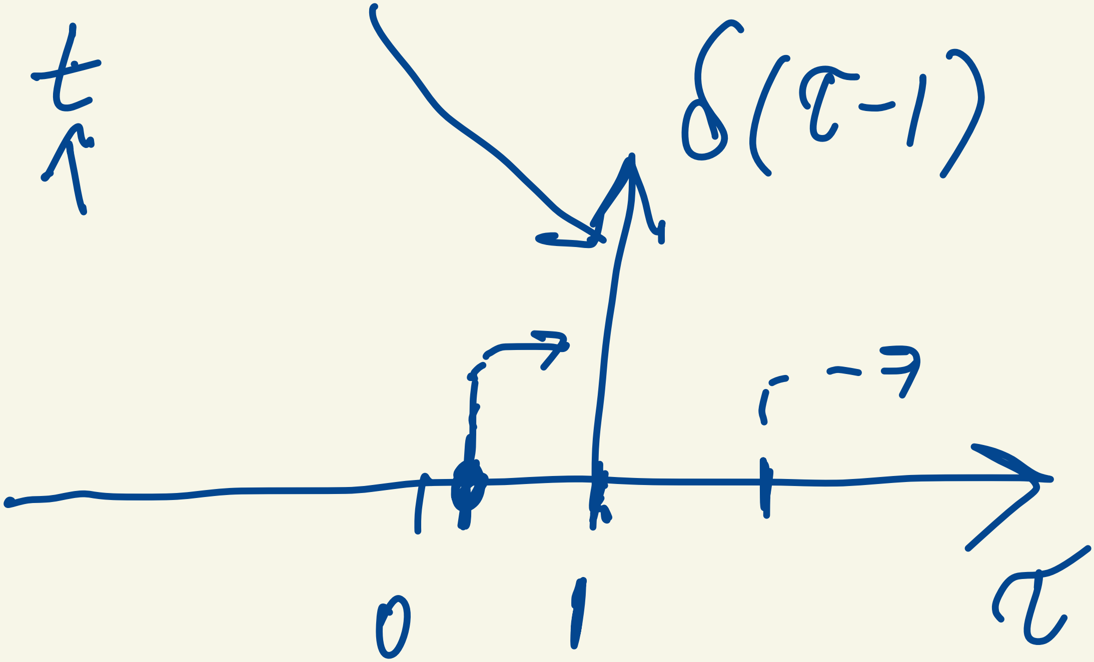
$$\boxed{\begin{aligned} \delta(t) &\triangleq \frac{du(t)}{dt} \\ \int_{-\infty}^{+\infty} \delta(t)\,dt &= 1 \end{aligned}}$$

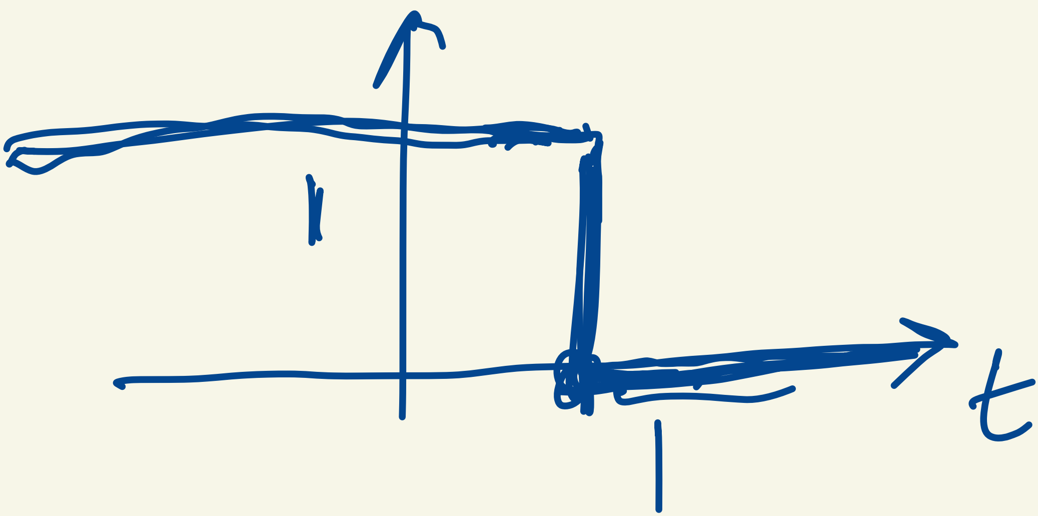
$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = \underbrace{\begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}}_{u(t)}$$



$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

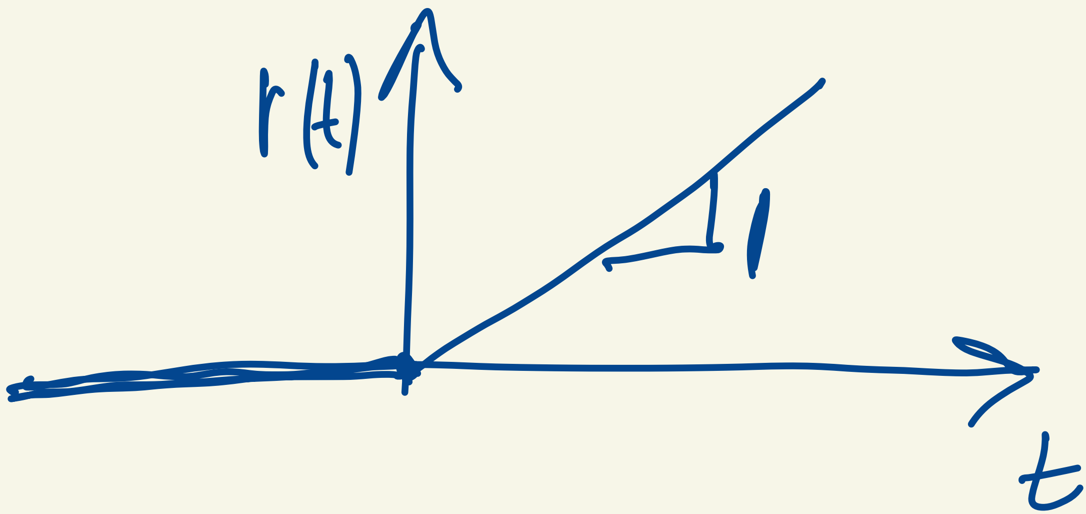
$$\underbrace{\int_{t}^{+\infty} \delta(\tau-1)\,d\tau}= \begin{cases} 1 & t<1 \\ 0 & t>1 \end{cases}$$





$u(-t+1)$

Unit ramp $r(t)$



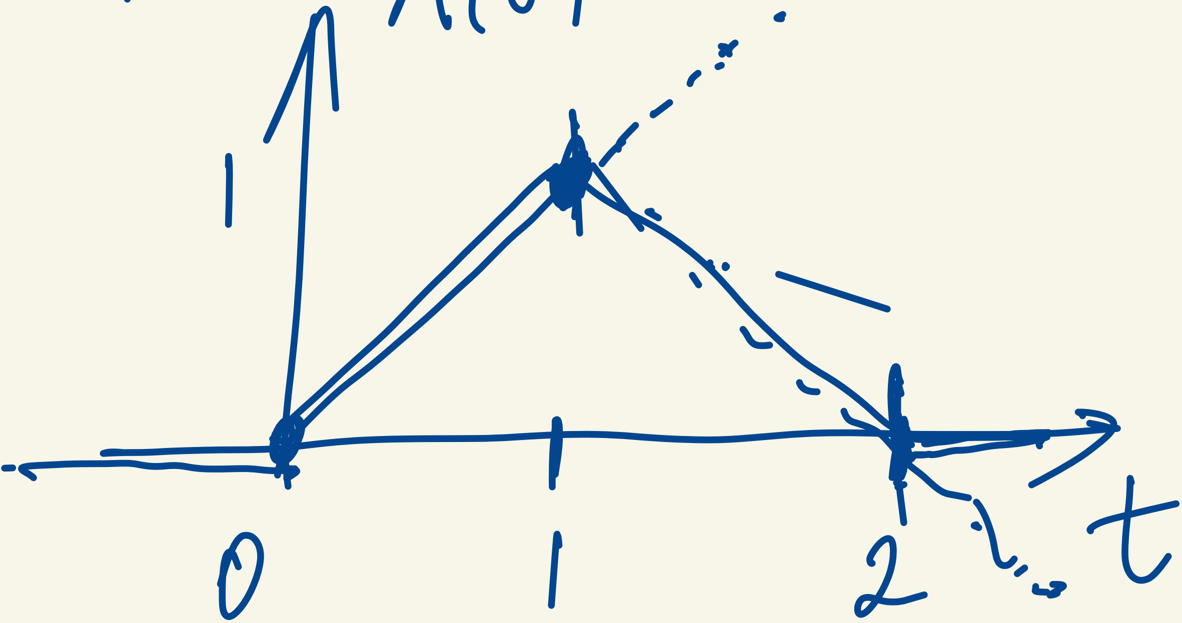
$$r(t) = \begin{cases} t & t > 0 \\ 0 & t < 0 \end{cases}$$

$$r(t) = t \cdot u(t)$$

$$\frac{dr(t)}{dt} = u(t)$$

$$\frac{d^2 r(t)}{dt^2} = \delta(t)$$

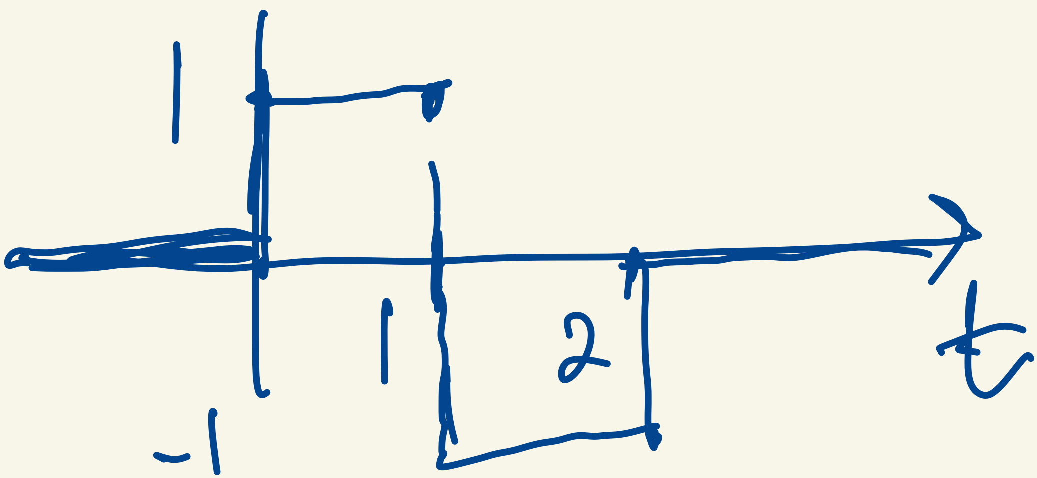
Ex. $\Lambda(t)$



$$\Lambda(t) = \begin{cases} 0 & t < 0 \\ t & 0 \le t \le 1 \\ -t+2 & 1 \le t < 2 \\ 0 & \text{o.w.} \end{cases}$$

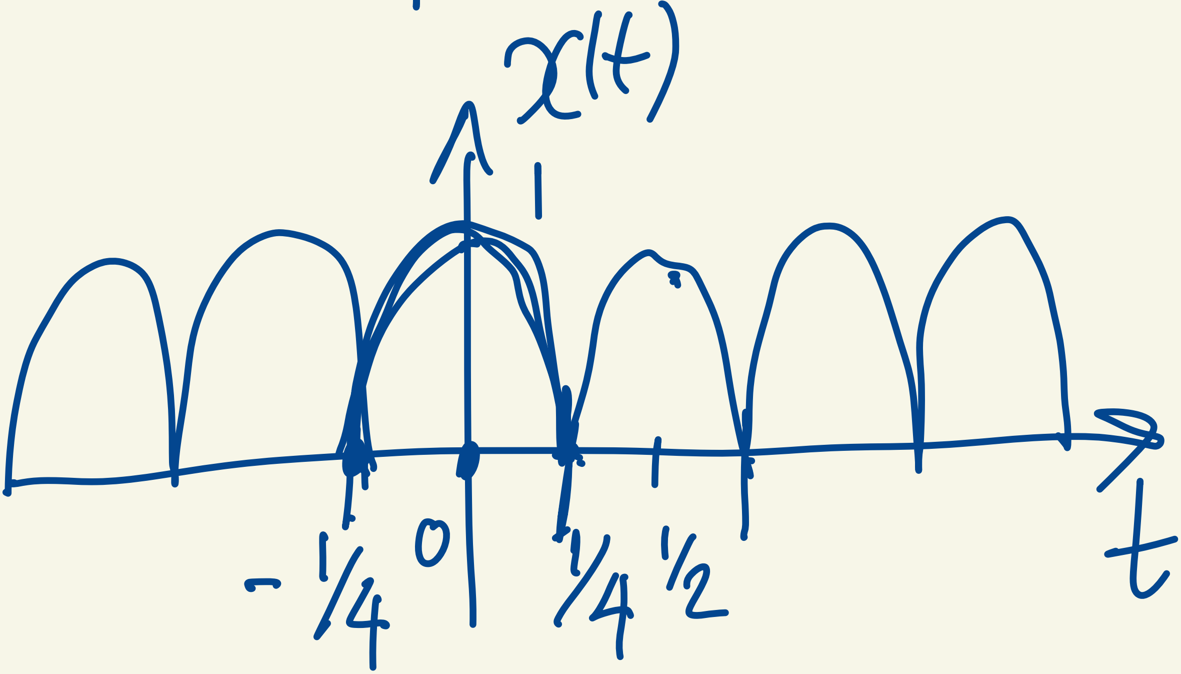
$$\Lambda(t) = r(t) - 2r(t-1) + r(t-2)$$

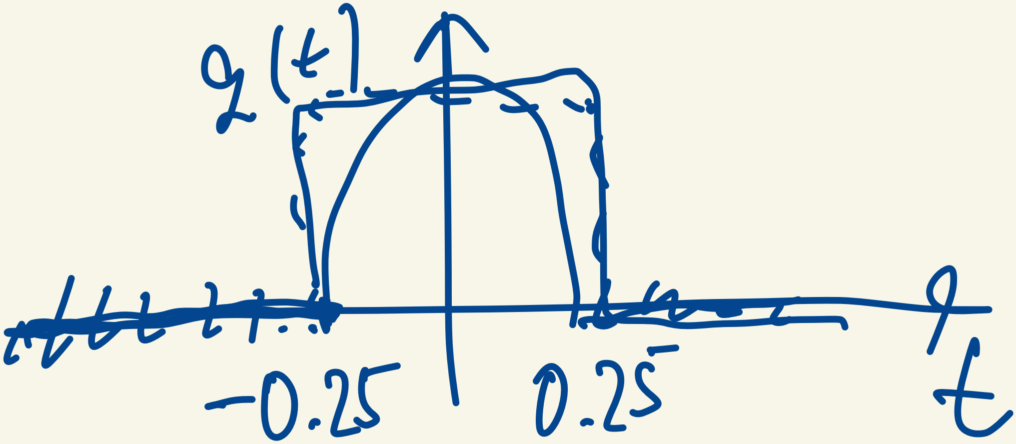
$$\frac{d\Lambda(t)}{dt} = u(t) - 2u(t-1) + u(t-2)$$



Ex. rectified signal

$$x(t) = \left|\cos(2\pi t)\right|$$





$$\underline{g(t)} = \cos(2\pi t) \cdot \left[u(t+0.25) - u(t-0.25)\right]$$

$$x(t) = \sum_{K=-\infty}^{+\infty} g(t + K \cdot \underline{0.5})$$