Midterm review problems

ECE 102: Systems and Signals

Winter 2022

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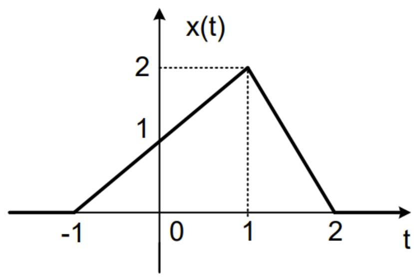
1 Time shifting & scaling, periodicity and energy computation

1. Consider the following real signal $x(t)$



   (a) Sketch the even and odd decomposition of $x(t)$: $x_e(t)$ and $x_o(t)$

   (b) Sketch $x(-2t+3)$, $x(\frac{t}{3}+2)$

   (c) Find the energy of signal $y(t) = \frac{3}{5}x(\frac{t}{3}+2) - jx_e(t-1)x_o(t-2)$, where $j = \sqrt{-1}$.

2. Derive conditions on ω_o and γ_o for the signal $x(t) = \sin\left(\omega_o t + \frac{\pi}{6}\right)\cos\left(\gamma_o t - \frac{\pi}{2}t\right) - \cos^2\left((\omega_o - \gamma_o)t\right)$ to be periodic with period T_o. Compute the power of $x(t)$ in terms of ω_o, γ_o and T_o.

2 Linear Systems: system properties, convolution integral

1. Consider the system S given by the input output relation

$$y(t) = e^{-t}x(t-2) - \int_0^{t/2} x(\tau)e^{2\tau - t}d\tau$$

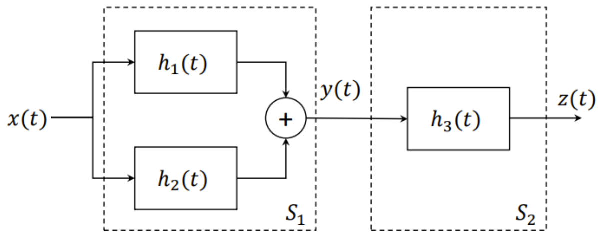
   (a) Compute the impulse response function $h(t, \tau)$ of the system.

   (b) Comment on the time invariance, causality and BIBO stability of the system.

   (c) Determine the output when the input is $x(t) = e^{-t}u(t-2)$

2. Consider a cascaded LTI system S_1S_2 as follows

$$x(t) \to \boxed{S_1} \xrightarrow{y(t)} \boxed{S_2} \to z(t)$$



   where $h_1(t) = \delta(t-1), h_2(t) = \delta(t-2)$, and $h_3(t) = \delta(t-1) - \delta(t-2) + \delta(t-3)$. Let $x(t) = 2(u(t) - u(t-2))$, then

   (a) Find the IPOP between $x(t)$ and $y(t)$. Plot $y(t)$.

   (b) Write the impulse response of the cascade system S_1S_2.

   (c) Comment on the causality of S_1S_2.

   (d) Compute and plot $z(t)$ for the specified input.

3. (a) Prove that for any $x(t)$, $h(t)$ and $g(t)$, the equality $[x(t) * h(t)] * g(t) = x(t) * [h(t) * g(t)]$ holds.

   (b) If $y(t) = x(t) * h(t)$, where $x(t) = u(t+0.5) - u(t-0.5)$ and $h(t) = e^{j\omega_0 t}$, find the value of ω_0 such that $y(0) = 0$.

3 Laplace Transform

1. Compute the Laplace transform of the following signals and specify the ROC.

   (a) $x(t) = \int_0^t \sin(t - \tau)e^{-(2t+3\tau)} \cos(\tau)d\tau$

   (b) $x(t) = (t^2 + te^{-4t})sin(2\omega_o t - \pi) + \int_{\infty}^{t} \tau^2 u(\tau - 2)d\tau$

2. Find the Inverse Laplace transform $f(t)$ for the given functions:

   (a) $F(s) = \frac{s^2+2s+5}{(s^2+a^2)^2(s+3)}$ where a is a real constant

   (b) $F(s) = \frac{2+5se^{-2s}-8e^{-4s}}{s^2+4s+3}$

3. Given below are the transfer functions of the systems S_1, S_2, and S_3

$$H_1(s) = \frac{e^{-2j}}{(s+1)(s-2)} \quad ; \quad H_2(s) = \frac{e^{-4s}s(s-2)}{(s+1)(s+2)} \quad ; \quad H_3(s) = \frac{1}{s(s+1)}$$

   (a) Plot the pole-zero constellations for S_1, S_2, S_3.

   (b) Determine the regions of convergence for which S_1, S_2, and S_3 are BIBO stable. Comment on system causality for the determined ROCs.

   (c) Compute the impulse response functions $h_1(t)$, $h_2(t)$, $h_3(t)$ for the ROCs determined in part (b).

   (d) Consider the cascaded system S_2S_3. Is it BIBO stable? Is it causal? How about system S_1S_2?

   (e) Find the impulse response functions $h_{12}(t)$ and $h_{23}(t)$ by taking inverse laplace transform of $H_{12}(s)$ and $H_{23}(s)$ respectively. Verify your stability result in part (d) by checking whether $h_{12}(t)$ and $h_{23}(t)$ are absolutely integrable.

4. Solve the following differential equation using the Laplace transform:

$$\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} - x(t), \text{t} \geq 0;$$
$$x(0) = 0, y(0) = 0$$