UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Final Examination

17^{th} March, 2022

Your name : ___________________________________

Instructions

- This exam has 5 questions and 8 pages.
- The exam is closed book. Two double-sided A4 sized cheat sheets are allowed. The use of calculators is permitted.
- All steps and working must be shown. No marks will be awarded for answers without math steps and/or an explanation.
- Write legibly and clearly! Any illegible work will not be graded.
- All plots must be neatly drawn and completely labelled (axes, intercepts, amplitudes) for full credit.

Good Luck!

Table 1: Score Table

Question	Total	Break up	Marks scored	Total score
1	20	5 + 7 + 8		
2	20	10 + 10		
3	20	8 + 5 + 7		
4	20	3 + 3 + 4 + 10		
5	20	12 + 8		
Total	100			

Table 3.1 One-Sided Laplace Transforms

	Function of Time	Function of s, ROC
1.	$\delta(t)$	1, whole s-plane
2.	$u(t)$	$\frac{1}{s}$, $\mathcal{R}e[s] > 0$
3.	$r(t)$	$\frac{1}{s^2}$, $\mathcal{R}e[s] > 0$
4.	$e^{-at}u(t),\ a > 0$	$\frac{1}{s+a}$, $\mathcal{R}e[s] > -a$
5.	$\cos(\Omega_0 t)u(t)$	$\frac{s}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
6.	$\sin(\Omega_0 t)u(t)$	$\frac{\Omega_0}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
7.	$e^{-at}\cos(\Omega_0 t)u(t),\ a > 0$	$\frac{s+a}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
8.	$e^{-at}\sin(\Omega_0 t)u(t),\ a > 0$	$\frac{\Omega_0}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
9.	$2A\, e^{-at}\cos(\Omega_0 t + \theta)u(t),\ a > 0$	$\frac{A\angle\theta}{s+a-j\Omega_0} + \frac{A\angle-\theta}{s+a+j\Omega_0}$, $\mathcal{R}e[s] > -a$
10.	$\frac{1}{(N-1)!}\, t^{N-1}u(t)$	$\frac{1}{s^N}$ N an integer, $\mathcal{R}e[s] > 0$
11.	$\frac{1}{(N-1)!}\, t^{N-1}e^{-at}u(t)$	$\frac{1}{(s+a)^N}$ N an integer, $\mathcal{R}e[s] > -a$
12.	$\frac{2A}{(N-1)!}\, t^{N-1}e^{-at}\cos(\Omega_0 t + \theta)u(t)$	$\frac{A\angle\theta}{(s+a-j\Omega_0)^N} + \frac{A\angle-\theta}{(s+a+j\Omega_0)^N}$, $\mathcal{R}e[s] > -a$

Table 3.2 Basic Properties of One-Sided Laplace Transforms

Causal functions and constants	$\alpha f(t),\ \beta g(t)$	$\alpha F(s),\ \beta G(s)$
Linearity	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
Time shifting	$f(t-\alpha)$	$e^{-\alpha s}F(s)$
Frequency shifting	$e^{\alpha t}f(t)$	$F(s-\alpha)$
Multiplication by t	$t\, f(t)$	$-\frac{dF(s)}{ds}$
Derivative	$\frac{df(t)}{dt}$	$sF(s) - f(0-)$
Second derivative	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0-) - f^{(1)}(0)$
Integral	$\int_{0-}^{t} f(t')dt$	$\frac{F(s)}{s}$
Expansion/contraction	$f(\alpha t)\ \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert}F\left(\frac{s}{\alpha}\right)$
Initial value	$f(0+) = \lim_{s\to\infty} sF(s)$	
Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$	

Simple Real Poles

If $X(s)$ is a proper rational function

$$X(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{\prod_k (s-p_k)} \tag{3.21}$$

Table 5.1 Basic Properties of the Fourier Transform

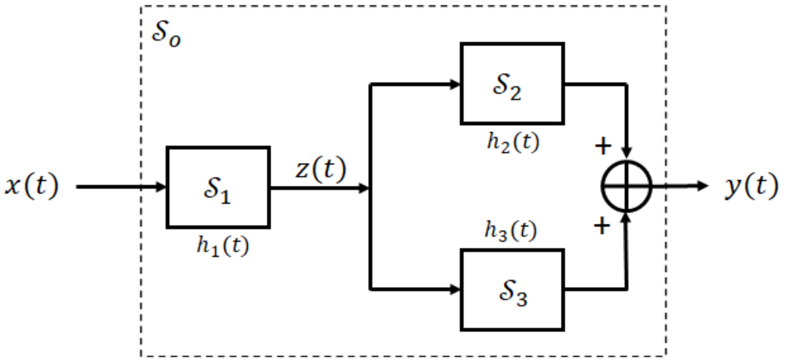
	Time Domain	Frequency Domain
Signals and constants	$x(t), y(t), z(t), \alpha, \beta$	$X(\Omega), Y(\Omega), Z(\Omega)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\Omega) + \beta Y(\Omega)$
Expansion/contraction in time	$x(\alpha t), \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert} X\left(\frac{\Omega}{\alpha}\right)$
Reflection	$x(-t)$	$X(-\Omega)$
Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(\Omega)\rvert^2 d\Omega$
Duality	$X(t)$	$2\pi x(-\Omega)$
Time differentiation	$\frac{d^n x(t)}{dt^n}$, $n \geq 1$, integer	$(j\Omega)^n X(\Omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\Omega)}{d\Omega}$
Integration	$\int_{-\infty}^{t} x(t')dt'$	$\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$
Time shifting	$x(t-\alpha)$	$e^{-j\alpha\Omega} X(\Omega)$
Frequency shifting	$e^{j\Omega_0 t} x(t)$	$X(\Omega - \Omega_0)$
Modulation	$x(t)\cos(\Omega_c t)$	$0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$
Periodic signals	$x(t) = \sum_k X_k e^{jk\Omega_0 t}$	$X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$
Symmetry	$x(t)$ real	$\lvert X(\Omega)\rvert = \lvert X(-\Omega)\rvert$
		$\angle X(\Omega) = -\angle X(-\Omega)$
Convolution in time	$z(t) = [x * y](t)$	$Z(\Omega) = X(\Omega)Y(\Omega)$
Windowing/multiplication	$x(t)y(t)$	$\frac{1}{2\pi}[X * Y](\Omega)$
Cosine transform	$x(t)$ even	$X(\Omega) = \int_{-\infty}^{\infty} x(t)\cos(\Omega t)dt$, real
Sine transform	$x(t)$ odd	$X(\Omega) = -j\int_{-\infty}^{\infty} x(t)\sin(\Omega t)dt$, imaginary

Table 5.2 Fourier Transform Pairs

	Function of Time	Function of Ω
1	$\delta(t)$	1
2	$\delta(t-\tau)$	$e^{-j\Omega\tau}$
3	$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$
4	$u(-t)$	$\frac{-1}{j\Omega} + \pi\delta(\Omega)$
5	$\text{sgn}(t) = 2[u(t) - 0.5]$	$\frac{2}{j\Omega}$
6	$A, \quad -\infty < t < \infty$	$2\pi A\delta(\Omega)$
7	$Ae^{-at}u(t), \; a > 0$	$\frac{A}{j\Omega + a}$
8	$Ate^{-at}u(t), \; a > 0$	$\frac{A}{(j\Omega + a)^2}$
9	$e^{-a\lvert t\rvert}, \; a > 0$	$\frac{2a}{a^2 + \Omega^2}$
10	$\cos(\Omega_0 t), \quad -\infty < t < \infty$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
11	$\sin(\Omega_0 t), \quad -\infty < t < \infty$	$-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$
12	$A[u(t+\tau) - u(t-\tau)], \; \tau > 0$	$2A\tau\frac{\sin(\Omega\tau)}{\Omega\tau}$
13	$\frac{\sin(\Omega_0 t)}{\pi t}$	$u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$
14	$x(t)\cos(\Omega_0 t)$	$0.5[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$

Question 1 (20 marks)

Given below is the block diagram of a cascaded LTI causal system $\mathcal{S}_o$, comprising of three system blocks: $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$, with impulse response functions $h_1(t)$, $h_2(t)$ and $h_3(t)$ respectively.



- System $\mathcal{S}_o$ is given by the input output relation shown below, where $x(t)$ is the input and $y(t)$ is the output.

$$y(t) = x(t) - 9\int_{-\infty}^{t} x(\sigma)(t-\sigma)e^{-2(t-\sigma)}d\sigma$$

- When an input $x(t) = e^{-5(t-3)}u(t-3)$ is applied to the block $\mathcal{S}_1$, we get the output as $z(t) = \delta(t-3) + 2e^{-2(t-3)}u(t-3)$.

- Further, $\mathcal{S}_2$ and $\mathcal{S}_3$ are single pole systems with no zeros. $\mathcal{S}_2$ has the higher magnitude pole.

(a) Find the transfer function $H_o(s)$ and indicate its ROC. (5 marks)

(b) Find the transfer functions $H_1(s)$, $H_2(s)$ and $H_3(s)$. (7 marks)

(c) Find the transfer function $\tilde{H}(s)$ of a system whose impulse response function is given by

$$\tilde{h}(t) = \int_{-\infty}^{\infty} e^{-(4t+\tau)}h_2(\tau)h_3(t-\tau)d\tau$$

Indicate its ROC. (8 marks)

Question 2 (20 marks)

An LTI causal system $\mathcal{S}$ has impulse response $h(t)$ given by

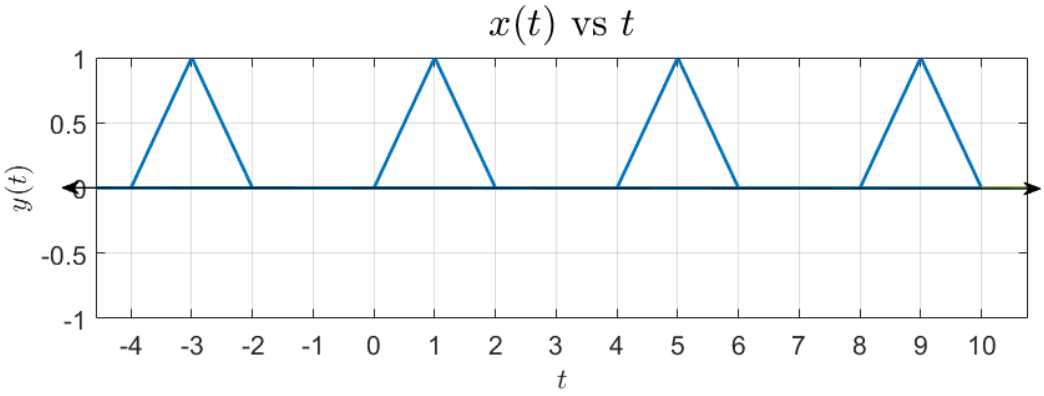
$$h(t) = \int_0^t \sin(3\tau)e^{-3(t-\tau)}d\tau$$

(a) Find the Frequency response function $H(\omega)$ of the system. (10 marks)

(b) An input $x(t) = 1 + 3\cos(3t)$ applied to the system results in output $y(t)$. Sketch the magnitude response $|Y(\omega)|$. (10 marks)

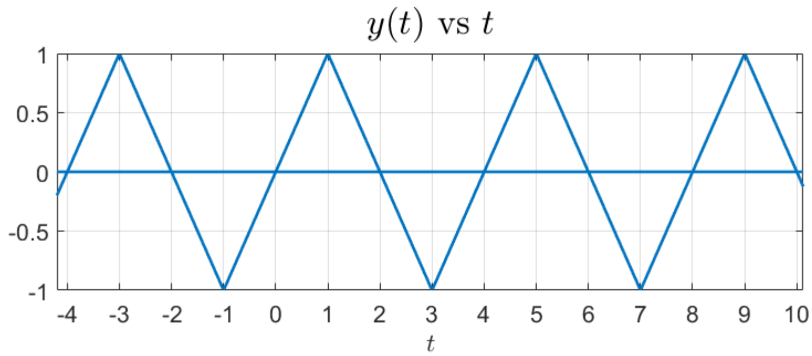
Question 3 (20 marks)

Consider a periodic signal $x(t)$ with period $T_o = 4$.



(a) Find the complex Fourier series coefficients (X_k) of $x(t)$. (8 marks)

(b) Consider the periodic signal $y(t)$ with period $T_o = 4$. (5 marks)



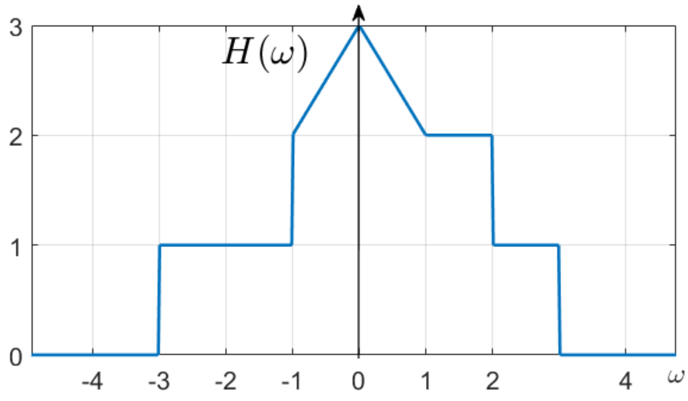
Use properties of Fourier Series to find the complex Fourier Series coefficients (Y_k) of $y(t)$ from X_k computed in part (a). Verify that the even harmonics of signal $y(t)$ are zero.

(c) Sketch the magnitude response $|Z(\omega)|$ of $z(t)$, where (7 marks)

$$z(t) = \left\{ y(t) \; * \; 7sinc\left(\frac{7\pi t}{4}\right) \right\}$$

Question 4 (20 marks)

Consider a system $\mathcal{S}$ with impulse response $h(t)$ and frequency response $H(\omega)$ as shown below.



(a) Without finding $h(t)$ explicitly, find $\int_{-\infty}^{\infty} h(2t)dt$ (3 marks)

(b) Without finding $h(t)$ explicitly, find $h(0)$ (3 marks)

(c) Without finding $h(t)$ explicitly, compute $\int_{-\infty}^{\infty} |h(t)|^2 dt$ (4 marks)

(d) Find the Inverse Fourier transform $h(t)$ without performing any integration. (10 marks)
Hint: Use linearity property of Fourier transform to decompose $H(\omega)$. Thereafter, use sinc-rec Fourier transform pairs and properties of Fourier transforms.

Question 5 (20 marks)

Consider a real system with impulse response function $h(t) = 2sinc^2(\pi t)\,[1 + 2\cos(2\pi t)]$.

(a) Find the frequency response $H(\omega)$ and sketch its magnitude response. (12 marks)

Hint: Use multiplication property of Fourier transform.

$$x(t)y(t) \stackrel{\mathcal{F}}{\rightarrow} \frac{1}{2\pi} X(\omega) * Y(\omega)$$

(b) Find the energy contained in output $y(t)$ when an input $\delta(t)$ is applied to the above system, using Parseval's theorem. (8 marks)