

Discussion 1
ECE 102: Systems and Signals
Winter 2022

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1 Review

1.1 Integration by parts

Find the following definite integrals – (i) $\int x e^{-x} dx$ (ii) $\int x^2 e^{-x} dx$

2 Problems

2.1 Euler's identity and trigonometric identities

Use Euler's identity to obtain an expression for $e^{j(\alpha-\beta)} = e^{j\alpha} e^{-j\beta}$. Obtain its real and imaginary components and show that

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

2.2 Reflection, time shifting, and time scaling

1. Consider a continuous time signal $x(t)$ as described below

$$x(t) = \begin{cases} t + 1, & -1 \leq t \leq 0 \\ -\frac{t}{2} + 1, & 0 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- a) Plot $y(t) = x(t + 1) + x(t - 1)$
- b) Is $y(t)$ an even or odd signal? Express $y(t)$ analytically.
- c) Plot $x(-2t + \frac{3}{2})$

2.3 Even & Odd signal decomposition

Consider the continuous time signal $x(t)$, defined as follows

$$x(t) = \begin{cases} 2, & 0 \leq t \leq 1 \\ t, & 1 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- a) Plot and analytically express the even and odd components, $x_e(t)$ and $x_o(t)$ respectively, of the signal $x(t)$.
- b) Verify that the energy of $x(t)$ is equal to the sum of the energies of $x_e(t)$ and $x_o(t)$.
- c) Find the power of signal $x(t)$.