Homework 6

ECE 102: Systems and Signals

Winter 2022

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Due Date: 23:59 on 7^{th} March, 2022. Submission via gradescope.

Kindly enroll yourself in the class: ECE 102 on gradescope. Entry code: X3PPGR

1. An LTI causal system is described by the equation:

$$\frac{dy(t)}{dt} + \int_0^t y(t-\tau)\tau^2 e^{3-\tau} d\tau = \frac{dx(t)}{dt} - x(t), t > 0$$
$$y(0) = 0, x(t) = y(t) = 0 \text{ for } t < 0$$

   (a) Find $H(s)$

   (b) The periodic signal $|\cos^2(t)| + 1$ is applied to the system. Find the Fourier coefficients of the output $\tilde{y}(t)$.

   (c) Find the Fourier series coefficients of $z(t) = \tilde{y}(t-3) * \tilde{y}(2t)$

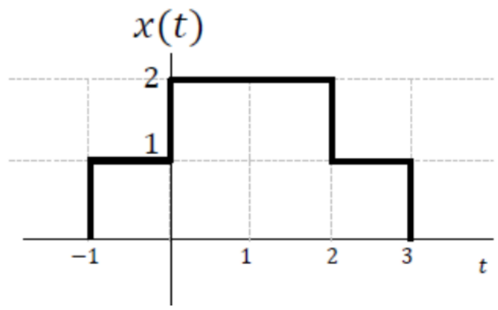
2. Find the Fourier transform of the following signals:

   (a) $x_1(t) = \begin{cases} t^2, 0 \le t < 1 \\ 0, \text{ otherwise} \end{cases}$

   (b) $x_2(t) = [u(t+2) - u(t-2)]\cos(100t) + 1$

   (c) $x_3(t) = \int_{-\infty}^{t} \cos(5(t-\sigma))\delta(\sigma - 2)d\sigma$

3. Consider the time-domain real signal $x(t)$ with a Fourier Transform $X(\omega)$, where $x(t)$ is shown below.



   (a) Find $X(0)$ and $\int_{-\infty}^{\infty} X(\omega)d\omega$

   (b) Compute $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

   (c) Compute $\int_{-\infty}^{\infty} X(\omega)Y(\omega)d\omega$, where $Y(\omega) = \frac{2\sin(\omega)}{\omega}e^{j2\omega}$
   Hint: For any real signals $f(t)$ and $g(t)$, we have:

$$\int_{-\infty}^{\infty} f(t)g(-t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)G(\omega)d\omega$$

(d) Sketch the inverse Fourier transform of $\mathrm{Re}\{X(\omega)\}$.
Note: You can answer all parts without explicitly evaluating $X(\omega)$

4. Consider a signal $x(t)$ with Fourier transform $X(j\omega)$. Suppose you are given the following facts:

   i. $x(t)$ is real and non-negative.
   ii. $\mathcal{F}^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$, where A is a constant.
   iii. $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$

   Determine a closed form expression for $x(t)$ and find the constant A.

5. Consider the following signal:

$$x(t) = e^{-t}\sin(2\pi t)u(t) + \delta(t-2)$$

   (a) Use the Laplace transform to find the Fourier transform $X(\Omega)$ of signal $x(t)$
   (b) Compute amplitude and phase spectrum.
   (c) Use MATLAB to plot amplitude and phase spectrum for $\Omega \in [-10, 10]$.
