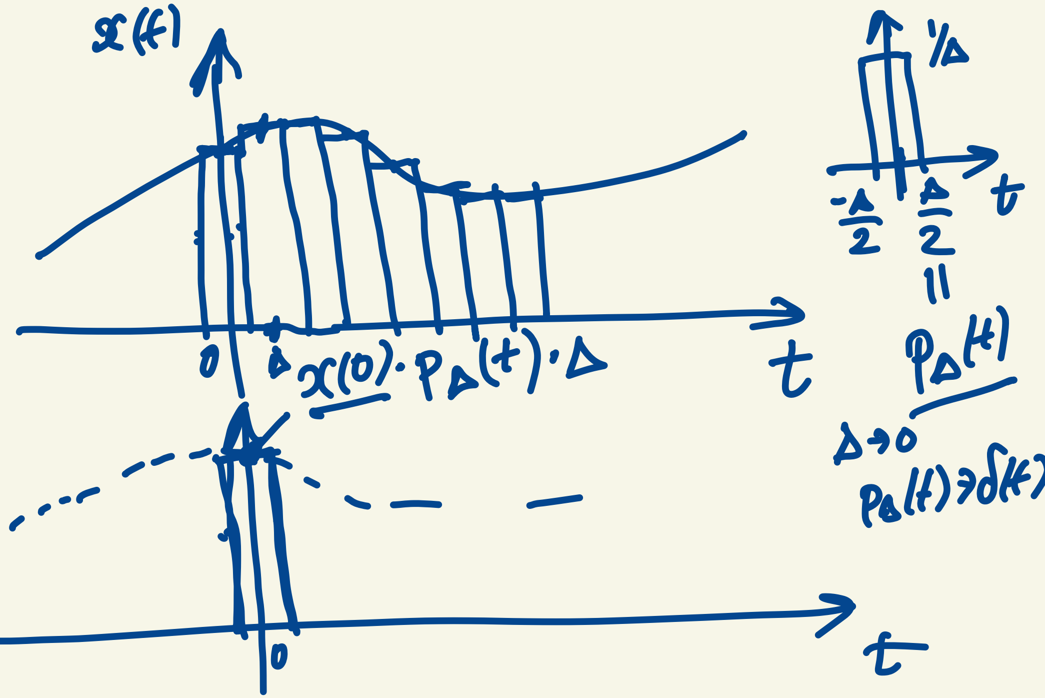
Lecture 3

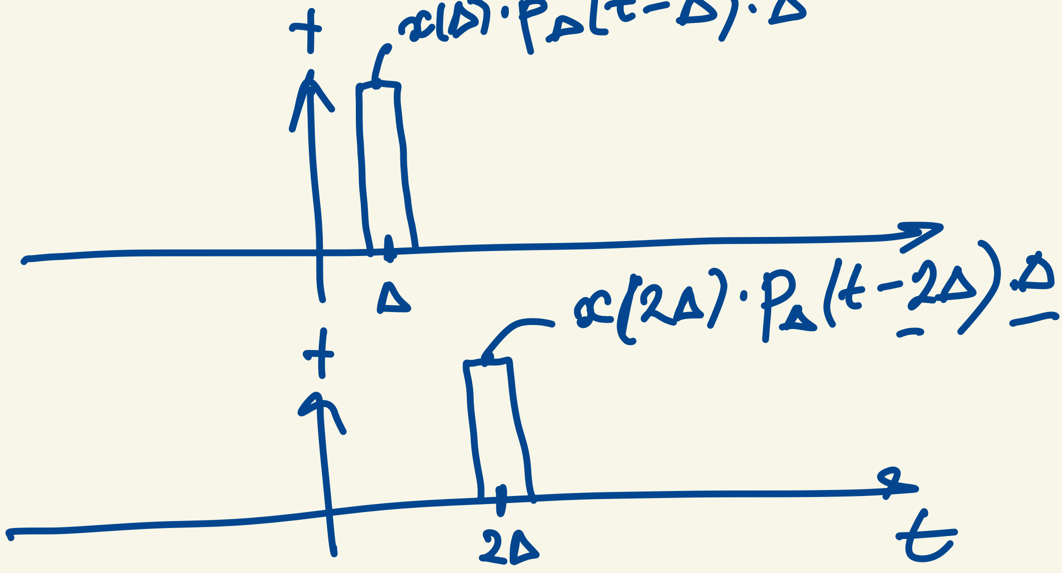
HW#1 was posted on Friday
due Friday 14th @ 11:59pm.

· **Generic representation of signals**

Reminder:
Last lecture we learned new signals
$u(t)$, $\delta(t)$, $r(t)$

$x(t) = \ldots\ldots$





$$x_\Delta(t) = \sum_{k=-\infty}^{+\infty} x(k\Delta)\cdot p_\Delta(t-k\Delta)\cdot\Delta$$

$$x(t) = \lim_{\Delta\to 0} x_\Delta(t)$$

$\Delta \to 0$ $\qquad d\tau \leftarrow$ integration variable

$k\Delta \longrightarrow \tau \qquad -\infty < k < +\infty$

$-\infty < \tau < +\infty$

$p_\Delta(t - k\Delta) \longrightarrow \delta(t-\tau)$

$\Delta \to 0 \; +\infty$

$$\underline{x(t)} = \underline{\int_{-\infty}^{} x(\tau)\delta(t-\tau)d\tau}$$

$$\delta(t) = \frac{du(t)}{dt} \checkmark$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1 \checkmark$$

Big Result

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

Couple of other important properties of $\delta(t)$

$$x(t)\cdot\delta(t) = x(0)\cdot\delta(t)$$

~~a) $x(t)$~~

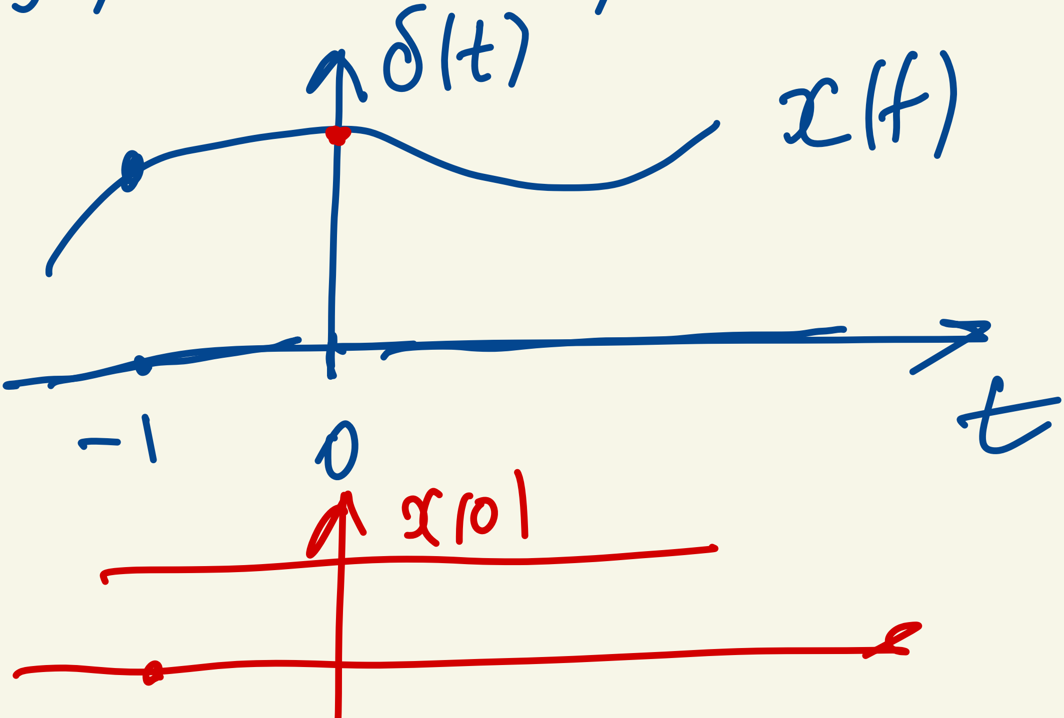
~~b) $x(0)$~~

c) $x(0) \times \delta(t)$

~~d) $\delta(t)$~~

e) ~~$u(t) \cdot x(0)$~~

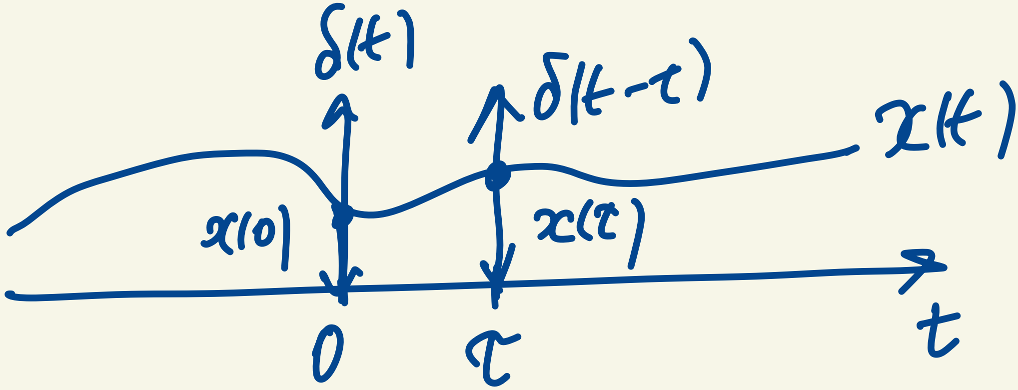
f) ~~$x(0) \cdot \delta(0)$~~



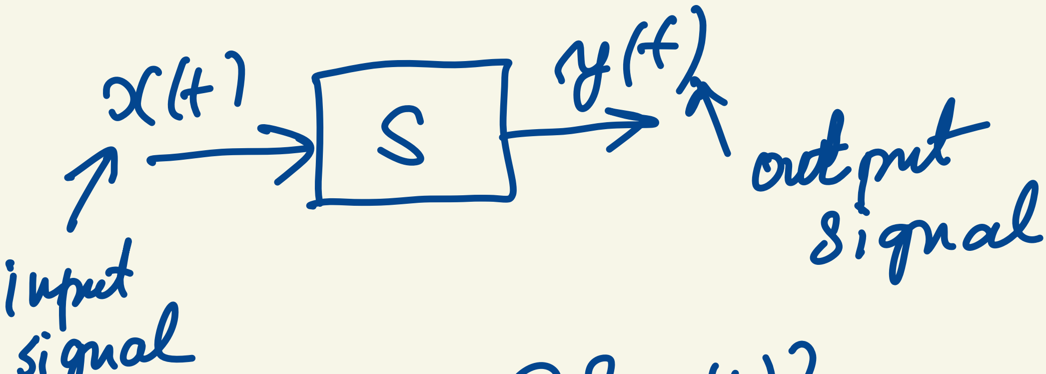
Sampling property of $\delta(t)$

$$x(t)\cdot\delta(t) = x(0)\cdot\delta(t)$$

$$x(t)\cdot\delta(t-\tau) = x(\tau)\cdot\delta(t-\tau)$$



Continuous-Time Systems



$$y(t) = S\{x(t)\}$$

4 properties of Systems

1) Linearity
2) Time-Invariance

} Today.

3) Causality
4) Stability.

Linearity

$$x(t) \rightarrow \boxed{S} \rightarrow y(t)$$

$$y(t) = S\{x(t)\}$$

- Scaling ✓
- Superposition ✓

$$\underline{\alpha \cdot x(t)} \rightarrow \boxed{S} \rightarrow \underline{z(t)} = \underline{S\{\alpha x(t)\}}$$

<u>if S is Linear</u>: $S\{\alpha x(t)\} = \alpha S\{x(t)\}$

$$z(t) = \alpha y(t)$$

$$x_1(t) \rightarrow \boxed{S} \rightarrow y_1(t) = S\{x_1(t)\}$$

$$x_2(t) \rightarrow \boxed{S} \rightarrow y_2(t) = S\{x_2(t)\}$$

$$x_1(t) + x_2(t) \rightarrow \boxed{S} \rightarrow z(t) = S\{x_1(t) + x_2(t)\}$$

<u>if S is Linear</u> $S\{x_1(t) + x_2(t)\}$

$$= S\{x_1(t)\} + S\{x_2(t)\}$$

$$z(t) = y_1(t) + y_2(t)$$

Linear Systems must satisfy both scaling and superposition

$$\alpha_1 x_1(t) + \alpha_2 x_2(t) \longrightarrow \boxed{S} \longrightarrow$$

$$S\{\alpha_1 x_1(t) + \alpha_2 x_2(t)\} \overset{\text{superpos.}}{=}$$

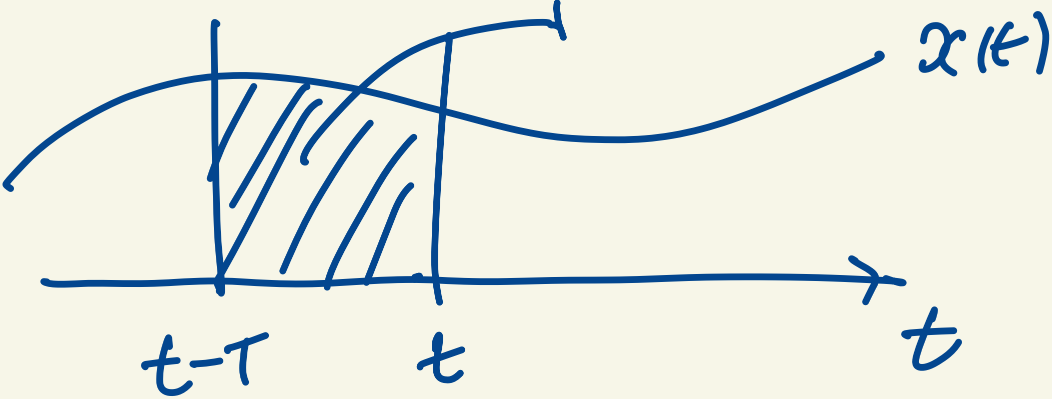
$$S\{\alpha_1 x_1(t)\} + S\{\alpha_2 x_2(t)\} \overset{\text{scaling}}{=}$$

$$\alpha_1 S\{x_1(t)\} + \alpha_2 S\{x_2(t)\}$$

Example: $\xrightarrow{x(t)} \boxed{S} \longrightarrow y(t)$

$$y(t) = \frac{1}{T} \int_{t-T}^{t} \boxed{x(\tau)}\, d\tau + B$$

B, T are constants.



Biased averaging.

$$\alpha x(t) \longrightarrow \boxed{S} \longrightarrow z(t) \overset{?}{=} \alpha y(t)$$

$$z(t) = S\{\alpha x(t)\} = \frac{1}{T}\int_{t-T}^{t} \boxed{\alpha x(\tau)}\, d\tau + B$$

$$\alpha y(t) = \frac{\alpha}{T}\int_{t-T}^{t} x(\tau)\, d\tau + \alpha B$$

$Z(t) \neq \alpha y(t)$

except when $B = 0$

S is in general
non-Linear. (NL)

iff $B = 0$ then S is **L**

L means Linear

Example:

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) = |x(t)|$$

$$\alpha x(t) \longrightarrow \boxed{S} \longrightarrow z(t) = S\{\alpha x(t)\}$$

$$= |\alpha x(t)|$$

$$= |\alpha| \cdot |x(t)|$$

$$z(t) \stackrel{?}{=} \alpha y(t)$$

$$|\alpha| |x(t)| \stackrel{?}{=} \alpha |x(t)|$$

$$|\alpha| \neq \alpha$$

$$x_1(t) \longrightarrow \boxed{S} \longrightarrow y_1(t) = |x_1(t)|$$

$$x_2(t) \longrightarrow \boxed{S} \longrightarrow y_2(t) = |x_2(t)|$$

$$x_1(t) + x_2(t) \longrightarrow \boxed{S} \longrightarrow z(t)$$

$$z(t) = |x_1(t) + x_2(t)|$$

$$\| \, ?$$

$$y_1(t) + y_2(t)$$

no $|x_1(t)| + |x_2(t)| \neq |x_1(t) + x_2(t)|$

Time-Invariance (TI)

independent from Linearity

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) = S\{x(t)\}$$

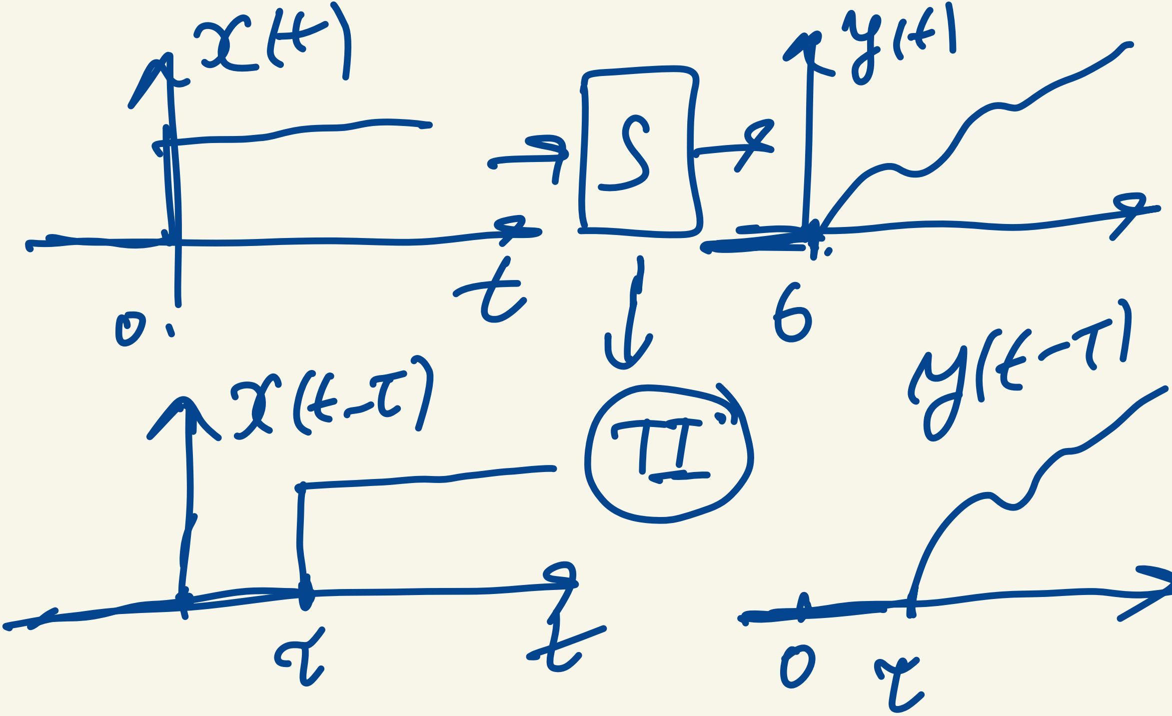
$$x(t-\tau) \longrightarrow \boxed{S} \longrightarrow z(t) = S\{x(t-\tau)\}$$

if S is TI

$$z(t) = y(t-\tau)$$

if S is TI

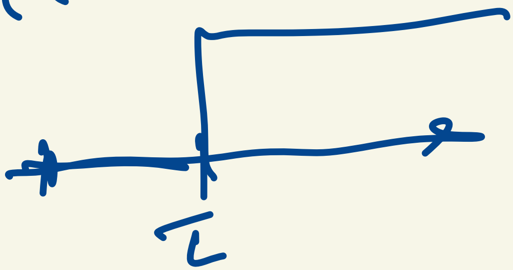
$$x(t-\tau) \longrightarrow \boxed{S} \longrightarrow y(t-\tau)$$



if S is Time-Varying
(TV)

$$x(t-\tau) \rightarrow \boxed{S} \rightarrow z(t) \neq y(t-\tau)$$

$x(t-\tau)$



$\rightarrow$

$z(t)$

So for 4 different system properties

1) L + TI = LTI

2) L + TV = LTV

3) NL + TI

4) NL + TV

Example:

$$m(t) \rightarrow \boxed{S} \rightarrow y(t) = m(t)\cos(\Omega_0 t)$$

input   input.

$$\boxed{m(t-\tau)} \rightarrow \boxed{S} \rightarrow z(t) = \boxed{m(t-\tau)} \cdot \cos(\Omega_0 t)$$

we need to check if

$$\underbrace{z(t)}_{①} \overset{③}{\overset{?}{=}} \underbrace{y(t-\tau)}_{②}$$

① $z(t) = m(t-\tau) \cdot \cos(\Omega_0 t)$

② $y(t-\tau) = m(t-\tau)\cos(\Omega_0(t-\tau))$

replace t w/ $t-\tau$

③ compare. $z(t) \neq y(t-\tau)$

S is TV!

$$n(t) = m(t-\tau) \longrightarrow \boxed{S} \longrightarrow z(t) = n(t) \cdot \cos(\Omega_0 t)$$

$$= m(t-\tau)\cos(\Omega_0 t)$$

Example

$$\underline{\underline{x(t)}} \longrightarrow \boxed{S} \longrightarrow y(t) = \int_{-\infty}^{+\infty} (t-\sigma)\,\underline{\underline{x(\sigma)}}\,d\sigma$$

$$-\infty < t < +\infty$$

$$\underline{x(t-\tau)} \longrightarrow \boxed{S} \longrightarrow \underline{z(t)}$$

$$\overset{1}{z}(t) \overset{3}{\overset{?}{=}} y(t-\overset{2}{\tau})$$

$$z(t) = S[x(t-\tau)]$$

$$= \int_{-\infty}^{+\infty} (t-\sigma)\cdot x(\sigma-\tau)\, d\sigma$$

$\sigma \to \lambda+\tau$, $\sigma \to \lambda+\tau$, $d\sigma \to d\lambda$

$$y(t-\tau) = \int_{-\infty}^{+\infty} (t-\tau-\sigma)\, x(\sigma)\, d\sigma$$

change of variable in $z(t)$

$$\sigma - \tau = \lambda$$

$$\sigma = \lambda + \tau \qquad \sigma \to -\infty \quad \lambda \to -\infty$$

$$d\sigma = d\lambda + 0 \qquad \sigma \to +\infty \quad \lambda \to +\infty$$

$d\tau = 0$ τ is constant.

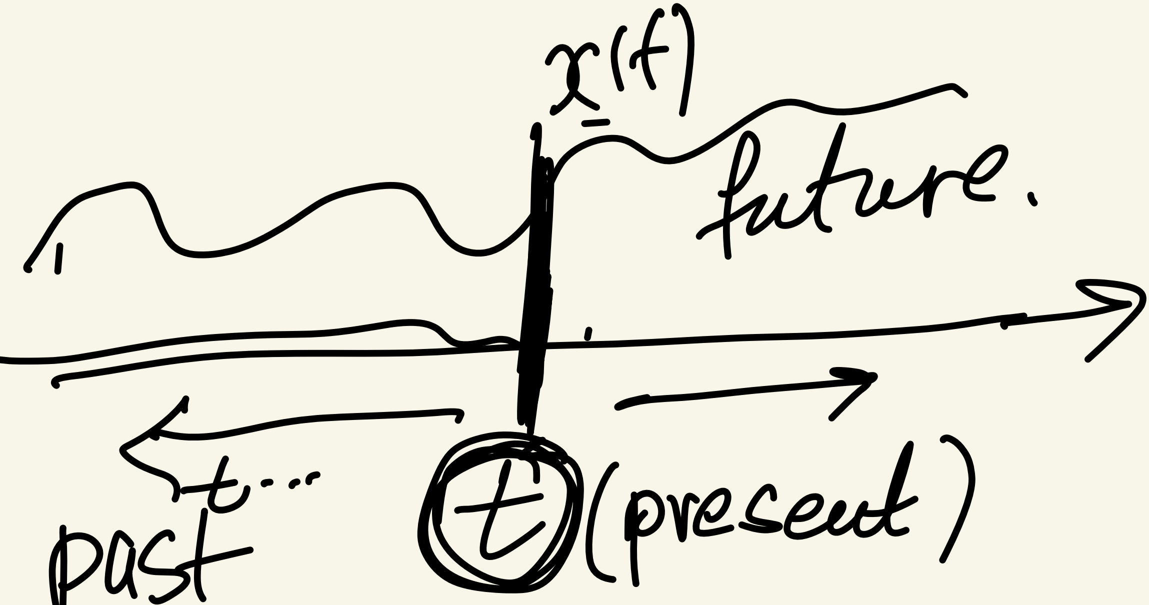
$$z(t) = \int_{-\infty}^{+\infty} h(t-\tau-\lambda)\,x(\lambda)\,d\lambda$$

$$z(t) = y(t-\tau) \quad S \text{ is } TI$$

Causality (C)

Def. A system is said to be <u>causal</u> if the output at any time t depends only on the value of the input $x()$ up to time t:

$\{x(\sigma) : \sigma \leq t\}$.



Example:

$$y(t) = \int_{-\infty}^{t} x(\sigma)\,d\sigma$$

S is C.

Example

$$y(t) = \int_{0}^{+\infty} e^{-(t-\sigma)}\,x(\sigma)\,d\sigma$$

S is NC.

Special case:

$\underline{y(t)} = \underline{2x(t)} + 3$

S is C. depends only on present.

S has no memory

$\Rightarrow$ memoryless

$y(t) = 2x(t+3)$

NC.

$y(t) = 2x(t-2)$

C.

C, C memoryless

or NC