Homework 3

ECE 102: Systems and Signals

Winter 2022

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Due Date: 23:59 on 28^{th} January, 2022. Submission via gradescope.
Kindly enroll yourself in the class: ECE 102 on gradescope. Entry code: X3PPGR

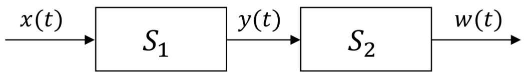
1. Compute the impulse response functions $h(t)$ for each of the LTI systems given below. Comment on the causality and BIBO stability of the systems.

   (a) $S_1: \quad y(t) = \int_{-\infty}^{t-1} e^t \cos(2\tau + 2 - 2t)x(\tau)e^{-\tau+2}d\tau$

   (b) $S_2: \quad y(t) = e^{-t}\int_{-\infty}^{t} e^{\tau}[\cos(t)\cos(\tau) - \sin(t)\sin(\tau)]x(\tau)d\tau$

   (c) $S_3: \quad y(t) = \int_{-\infty}^{t-1} e^{-(t-\tau)}x(\tau - 2)d\tau$

2. Given below are two cascaded LTV systems S_1 and S_2. The input-output relation for system S_1 is given by eqn. (1), where $x(t)$ and $y(t)$ are inputs and output of system S_1, respectively. The input-output relation for system S_2 is given by eqn. (2), where $\mathrm{y}(t)$ and $\mathrm{w}(t)$ are inputs and output of system S_2 respectively.



$$y(t) = x(t)u(t) - \int_{-\infty}^{t-2} e^{-(t-\tau)}x(\tau)u(\tau)d\tau, t \in (-\infty, \infty) \qquad (1)$$

$$w(t) = \int_{-\infty}^{t} y(\sigma)u(\sigma)d\sigma, t \in (-\infty, \infty) \qquad (2)$$

   (a) Compute IRF of S_1 and S_2 : $h_1(t,\tau)$ and $h_2(t,\tau)$ respectively.

   (b) Compute IRF $h_{12}(t,\tau)$ of cascaded system S_1S_2.

   (c) Is the cascaded system (with input $x(t)$ and output $w(t)$) a stable system? Explain.

   (d) Obtain the IRF $h_{21}(t,\tau)$

3. **Matlab assignment 1:** Consider signal $y(t) = \mathrm{sinc}(t)$, where $\mathrm{sinc}(t) = \frac{\sin \pi t}{\pi t}\ \forall -\infty < t < \infty$. Using Matlab, implement and plot the following signals over the range $t \in [-5, 5]$. Your final submission for this question should consist of six plots and the Matlab code.

   (a) $y(2t - 1)$

   (b) $y^2(t)$

   (c) $z(t) = y(t) * y(t)$

   (d) Verify by means of Matlab plots whether $z(2t - 1)$ is equal to $\{y(t) * y(2t - 1)\}$ or $\{y(2t - 1) * y(2t - 1)\}$ or neither.

4. **Matlab assignment 2:** For the following problems, include screenshots/images of your MATLAB figures. You should have 5 total figures for this problem. Also, copy and paste your MATLAB code into your homework.

   a) Using MATLAB, plot $x(t)$ and $h(t)$ individually over the range $t \in [-3, 3]$.

   $$x(t) = \sin(2\pi t)u(t+1)u(-t+1) \quad ; \quad h(t) = u(t) - u(t-1)$$

   b) Compute $y_1(t) = x(t) * h(t)$ ($*$: convolution operator) analytically as a piecewise function.

   (c) Solve for $y_2(t) = x(t) \cdot h(t)$ (where $\cdot$ is the multiplication operator) analytically.

   (d) Using MATLAB, plot $y_1(t)$ and $y_2(t)$ individually over the range $t \in [-3, 3]$. Are $y_1(t)$ and $y_2(t)$ the same?

   (e) Using MATLAB, numerically convolve $x(t)$ and $h(t)$ and plot the result over the same range. Confirm that the result is the same as $y_1(t)$.

Hint for Matlab assignments: Here is a sample code snippet to help you get started. The following code plots the signal $z(t) = \sin(2\pi t) - \frac{1}{2}\cos\left(3\frac{\pi}{5} - \frac{\pi}{3}t\right)$ in the time interval $[-3, 3]$.

```
clear; clc; close all;
t=linspace(-3,3);    % Define time axis : 100 equally spaced points in [-3,3]
z = sin(2*pi*t) - 0.5*cos(3*pi/5-pi/3*t);    % Define signal z(t) as a function of t

figure(1);   % Create figure with label (1)
plot(t,z);   % Plot signal z(t) against t
xlabel('t','Interpreter','latex','fontsize',16); ylabel('z(t)','Interpreter','latex','fontsize',16); % Label the x and y axes
title('z(t) vs t','Interpreter','latex','fontsize',18); % Set the graph title
grid on; % Apply a mesh grid to the graph
```

Some useful Matlab functions:

(a) *heaviside(x)* : generates unit step function. *eg.* u=*heaviside(t);* generates function $u(t)$

(b) *conv(x,y)*: returns convolution of input signals/vectors x and y.

(c) *Elementwise multiplication of vectors in matlab*: x.*y

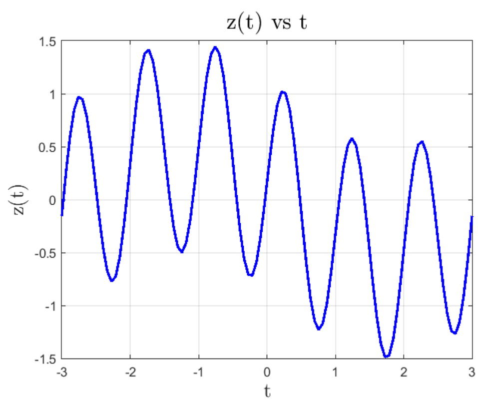


Figure 1: Output of sample code snippet