Homework 2

ECE 102: Systems and Signals

Winter 2022

Instructor: Prof. Danijela Cabric

Due Date: 23:59 on 21st January, 2022. Submission via gradescope.

Kindly enroll yourself in the class: ECE 102 on gradescope. Entry code: X3PPGR

---

1. Given below are the input/output relations of four systems, where $x(t)$ is the input and $y(t)$ is the system output. Classify each system as (i) Linear/ Non-linear (ii) Time variant/ Time invariant (iii) Causal/ Non-causal with proper justification.
   Also find the output $y(t)$ for each system, when input $x(t) = u(t-2) - u(t-4)$ is applied.

   (a) $y(t) = \int_{-\infty}^{2t} x(\tau + 3)d\tau$

   (b) $y(t) = x(t)\sin(\pi t)$

   (c) $y(t) = \frac{dx(t)}{dt}$

   (d) $y(t) = x(2-t) + x(2+t)$

2. Consider the following input/output relationship for a system S :

$$y(t) = x(t) - \int_{t-1}^{t+1} e^{|t-\tau|}x(\tau)d\tau$$

   (a) Rewrite $y(t)$ in the form $y(t) = \int_{-\infty}^{\infty}[?]d\tau$ where [?] is a function to be determined.

   (b) Classify S as Linear/ Non-linear, Time varying/ Time invariant, Causal/ Non-causal. Justify your answer.

   (c) Find the output $y(t)$ given: $x(t) = e^{-t}u(t+2)$

3. Consider an LTI system whose response to the signal $x_1(t)$ in Figure 1(a) is the signal $y_1(t)$ illustrated in Figure 1(b).

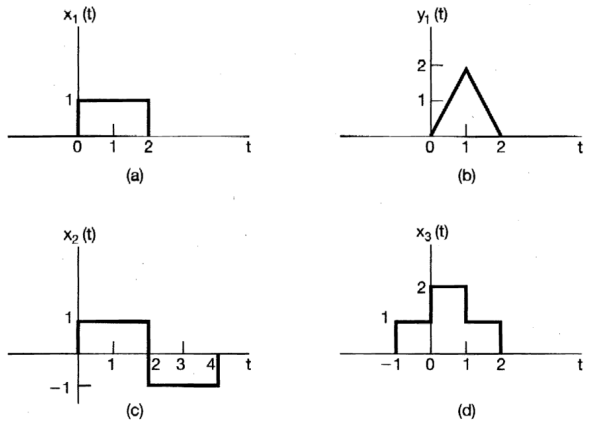

Figure 1

(a) Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure 1(c).

(b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure 1(d).

(c) Consider the LTI system with input/output relation $y(t) = \int_{t-2}^{t} x(\tau)d\tau$. Find the impulse response $h(t)$. Sketch the output when an input of $x(t) = u(t+2)+u(t)-2u(t-1)+\delta(t-1)$ is applied to the system.

4. Given the following input-output relation (IPOP) of a system:

$$y(t) = \int_{-\infty}^{\infty} e^{-t}(t-\tau)^2 u(\tau+t)x(\tau-2)d\tau, t \in (-\infty, \infty).$$

a) Find impulse response of the system $h(t, \tau)$. Is the system time variant (TV) or time invariant (TI)? Is it causal (C) or non-causal (NC)?

b) Find the corresponding output, $y(t)$, given an input of:

$$x(t) = \delta(t-2) - e^{-t}u(t+1), \quad t \in (-\infty, \infty)$$