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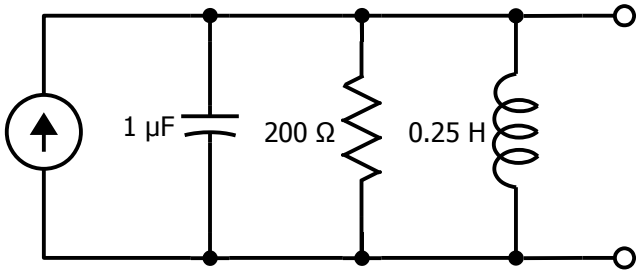
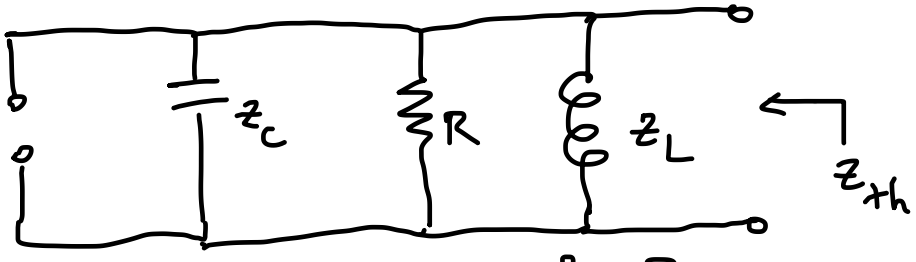
EE3 Fall 2020
Homework Problem 5

If we are trying to find the Thévenin Equivalent circuit, the easy part is V_{th}.
a. If ω = 1000 rad/s, what is Z_{th}?
b. If ω = 2000 rad/s, what is Z_{th}?
c. If ω = 4000 rad/s, what is Z_{th}?



To find Z_{th}, we open circuit for independent current source, we have:



$R = 200\,\Omega$
$L = 0.25\,H$
$C = 1\,\mu F$

$$\Rightarrow Z_{th} = Z_C \,//\, R \,//\, Z_L = \frac{Z_C R}{R + Z_C} \,//\, Z_L$$

$$= \frac{\frac{Z_C R}{R+Z_C} \cdot Z_L}{\frac{Z_C R}{R+Z_C} + Z_L} = \frac{Z_C R Z_L (R+Z_C)}{(R+Z_C)(Z_C R + Z_L(R+Z_C))} = \boxed{\frac{R Z_C Z_L}{R Z_C + R Z_L + Z_L Z_C}}$$

a) $\omega = 1000$ rad/s, $R = 200\,\Omega$, $L = 0.25\,H$, $C = 1\,\mu F = 10^{-6}\,F$

We have: $Z_L = j\omega L = j \times 1000 \text{ rad/s} \times 0.25\,H$

$$\Rightarrow Z_L = 250j\ (\Omega)$$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{1000 \times 1 \times 10^{-6}\,F} = -1000j\ (\Omega)$$

$$Z_{th} = \frac{R Z_C Z_L}{R Z_C + R Z_L + Z_C Z_L} =$$

$$= \frac{200 \times (-1000j) \times 250j}{200 \times (-1000j) + 200 \times 250j + (-1000j) \times 250j}$$

$$= \frac{50 \times 10^6}{250\,000 - 150\,000j} = 147.06 + 88.24j \ (\Omega)$$

$$= 171.5 \angle 30.96^\circ \ (\Omega)$$

$$\Rightarrow \boxed{\begin{aligned} Z_{th} &= 147.06 + 88.24j \ (\Omega) \\ &= 171.5 \angle 30.96^\circ \ (\Omega) \end{aligned}}$$

b) $\omega = 2000 \text{ rad/s}, R = 200\ \Omega, L = 0.25 H, C = 10^{-6} F$

$$\Rightarrow Z_L = j\omega L = j \times 2000 \times 0.25 = 500j \ (\Omega)$$

$$\Rightarrow Z_C = \frac{-j}{\omega C} = \frac{-j}{2000 \times 10^{-6} F} = -500j \ (\Omega)$$

$$\Rightarrow Z_{th} = \frac{R Z_C Z_L}{R Z_C + R Z_L + Z_C Z_L}$$

$$\Rightarrow Z_{th} = \frac{200 \times (-500j) \times (500j)}{200 \times (-500j) + 200 \times 500j + 500j \times (-500j)}$$

$$\Rightarrow Z_{th} = \frac{50 \times 10^6}{250 \times 10^3} = \boxed{200\ (\Omega)}$$

c) $\omega = 4000\ (\text{rad/s})$, $R = 200\ \Omega$, $L = 0.25\ H$,
$C = 1\ \mu F = 10^{-6}\ F$

$$\Rightarrow Z_L = j\omega L = j \times 4000 \times 0.25 = 1000j\ (\Omega)$$

$$\Rightarrow Z_C = \frac{-j}{\omega C} = \frac{-j}{4000 \times 10^{-6}\ F} = -250j\ (\Omega)$$

$$\Rightarrow Z_{th} = \frac{R Z_C Z_L}{R Z_C + R Z_L + Z_C Z_L}$$

$$= \frac{200 \times (-250j) \times 1000j}{200 \times (-250j) + 200 \times 1000j + 1000j \times (-250j)}$$

$$= \frac{50 \times 10^6}{250.10^3 + 150.10^3 j} = 147.06 - 88.24j\ (\Omega)$$

$$= 171.5 \angle -30.96^\circ\ (\Omega)$$

$$\Rightarrow \boxed{Z_{th} = 147.06 - 88.24j\ (\Omega) = 171.5 \angle -30.96^\circ\ (\Omega)}$$