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# EE3 Fall 2020
# Homework Problem 3

The switch has been in position a for a long time. At t=0, it moves instantaneously to positon b. Find:

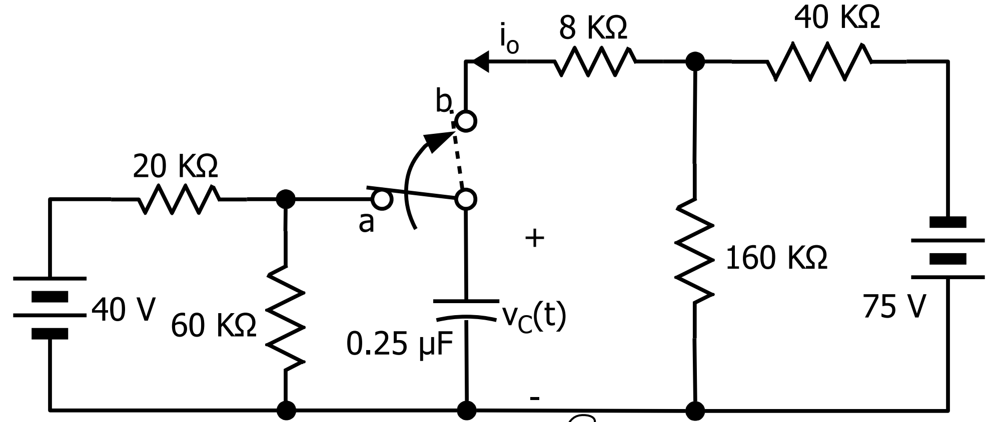
a. $v_c(0^-)$

b. $v_c(0^+)$

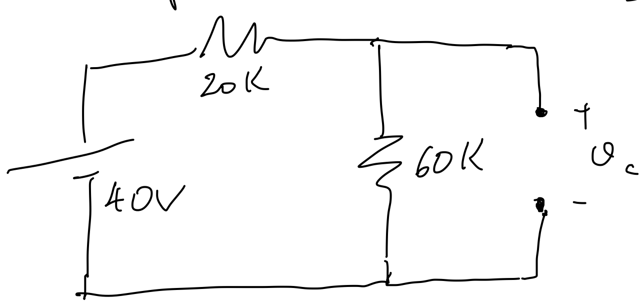
c. $i_o(0^-)$

d. $i_o(0^+)$

e. [EXTRA CREDIT] $v_c(t)$




a) When switch states at a for a long time, the capacitor is open circuit to DC, we have:




$$\Rightarrow v_c = \frac{60K}{60K + 20K} \times 40$$

$$= 30\ (V)$$

$$\Rightarrow \boxed{v_c(0^-) = 30\ (V)}$$

b) At t = 0, the switch move instantaneously to position b

We have: $v_c(0^+) = v_c(0^-) = 30(V)$. Because the capacitors will not allow instantaneous change in voltage.

c) at $t = 0^-$, $i_o = 0$. Because at $t = 0^-$, the circuit is opened, and no current through the 8 KΩ resistor

d) at $t = 0^+$, we have known already $v_c(0^+) = 30V$

And we can have a new circuit:

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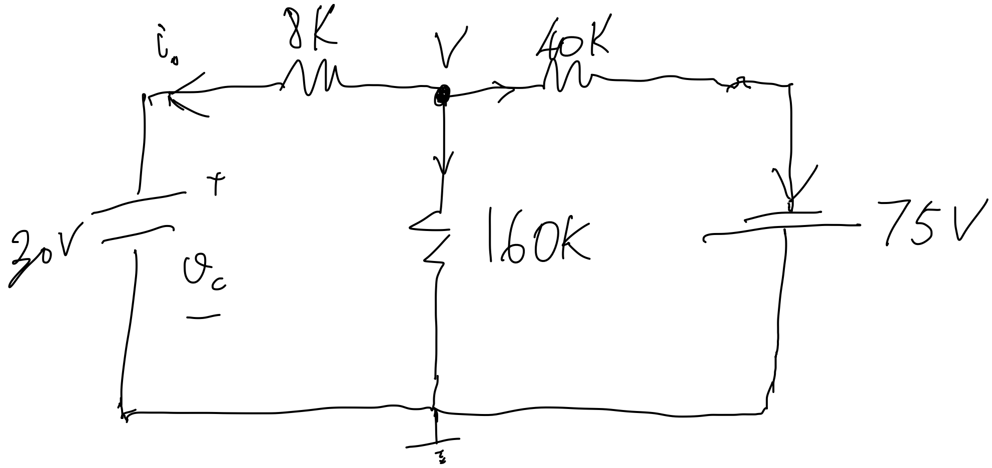



We have:

$$\frac{V-30}{8K} + \frac{V}{160K} + \frac{V+75}{40K} = 0$$

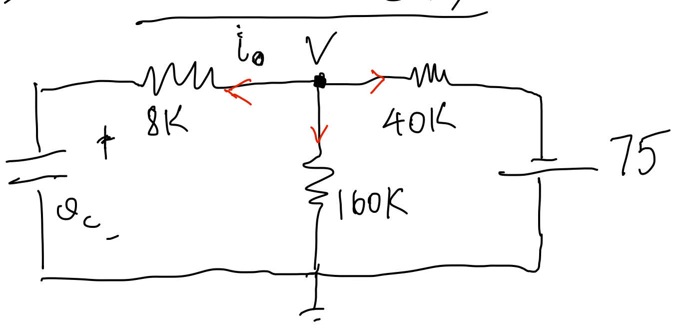
$$\Rightarrow 20(V-30) + V + 4(V+75) = 0$$

$$\Rightarrow 25V = 300 \Rightarrow V = 12\ (V)$$

$$\Rightarrow i_o = \frac{V-30}{8K} = \frac{12-30}{8K} = \frac{-18V}{8K} = -2.25\ (mA)$$

$$\Rightarrow \boxed{i_o(0^+) = -2.25\ (mA)}$$

e) Find $V_c(t)$ We have at $t = 0^+$, $v_c(0^+) = 30V$




$$\begin{cases} \dfrac{V - v_c}{8K} + \dfrac{V}{160K} + \dfrac{V + 75V}{40K} = 0 \\ \dfrac{V - v_c}{8K} = i_o \Rightarrow V = 8K\, i_o + v_c \end{cases}$$

$$\Rightarrow i_o + \frac{8K \cdot i_o + v_c}{160K} + \frac{8K\, i_o + v_c + 75V}{40K} = 0$$

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$$\Rightarrow 160K.i_0 + 8K.i_0 + v_c + 4.(8K i_0 + v_c + 75V)$$

$$= 168K.i_0 + v_c + 32K i_0 + 4v_c + 300V = 0$$

$$= 200K.i_0 + 5v_c + 300V = 0$$

$$= 40K.i_0 + v_c + 60V = 0$$ . Beside $i_0 = i_c = C\frac{dv_c}{dt}$

$$\Rightarrow 40K.C.\frac{dv_c}{dt} + v_c + 60V = 0$$

$$\Rightarrow 40K.C\frac{dv}{dt} = -(v_c + 60V)$$

$$\Rightarrow \frac{dv}{v_c + 60V} = \frac{-dt}{40K.C} \Rightarrow \int_{v_c(0^+)}^{v_c(t)} \frac{dv}{v_c + 60V} = \int_{t=0^+}^{t} \frac{-dt}{40K.C}$$

Also, $v_c(0^+) = 30V$.

$$\Rightarrow \ln(v_c + 60V)\Big|_{30V}^{v_c(t)} = \frac{-t}{40K.C}$$

$$\Rightarrow \frac{v_c(t) + 60V}{90V} = e^{-t/40K.C}$$

$$\Rightarrow v_c(t) = -60V + 90V.e^{-t/40K.C}$$

<u>We have</u>: $\tau = 40K.C = 40\times10^3 \times 0.25\times10^{-6} = 0.01$

$$\Rightarrow v_c(t) = -60V + 90V e^{-t/0.01}$$

$$\Rightarrow \boxed{v_c(t) = -60V + 90V e^{-100t} \quad (V)}$$