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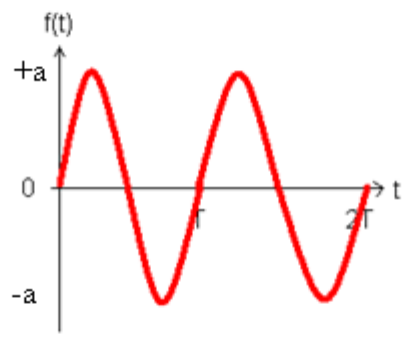
Week 2: Oscilloscopes and Function Generators

RMS of a periodic signal is calculated by first squaring the waveform, then taking its mean over its period, T, then taking the square root. Its definition using the calculus is

$$\text{RMS} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$$

As an example, we will derive the equation for RMS/Vpp for a sine wave. You will be asked to derive the equation for square waves and triangular waves in the pre-lab.



First, $f(t) = a\sin\left(\frac{2\pi t}{T}\right) \rightarrow RMS = \sqrt{\frac{1}{T}\int_0^T \mathrm{a}^2 \sin^2\left(\frac{2\pi t}{T}\right) dt}$

Using the definition of $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$, $RMS = \sqrt{\frac{a^2}{T}\int_0^T \frac{1-\cos\left(\frac{4\pi t}{T}\right)}{2} dt}$

Taking the integral, $RMS = \sqrt{\frac{a^2}{T}\left[\frac{1}{2}t - \frac{\frac{1}{2}\mathrm{T}}{4\pi}\sin\left(\frac{4\pi t}{T}\right)\right]_0^T}$

Evaluating, we get $RMS = \sqrt{\frac{a^2}{T}\left[\frac{1}{2}T\right]}$ (Note that at t = 0 and T, $\sin\left(\frac{4\pi t}{T}\right) = 0$)

Therefore, $RMS = \frac{a}{\sqrt{2}}$, and since $Vpp = 2a$, then $\frac{\boldsymbol{RMS}}{\boldsymbol{Vpp}} = \frac{\mathbf{1}}{\mathbf{2\sqrt{2}}}$

It may for the purposes of your lab helpful to think of RMS in terms of Vpp, like so:

$$RMS = \frac{Vpp}{2\sqrt{2}}$$

Week 2 Prelab

Calculate the ratio RMS/Vpp for the following signals. Show all your work!
Name:

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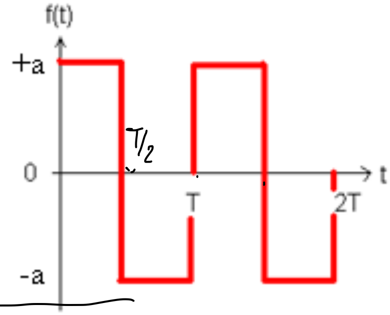
1. Square Wave: RMS / Vpp = ? $\boxed{\frac{1}{2}}$

$$f(t) = \begin{cases} a & \text{for } 0 \le t < T/2 \\ -a & \text{for } \frac{T}{2} \le t \le T \end{cases}$$



$$RMS = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

$$= \sqrt{\frac{1}{T}\left[\int_0^{T/2} a^2\,dt + \int_{T/2}^{T} (-a)^2\,dt\right]}$$

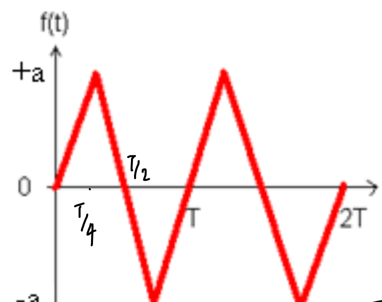
$$= \sqrt{\frac{1}{T}\int_0^T a^2\,dt} = \sqrt{\frac{1}{T}a^2(T-0)} = a$$

Therefore, RMS = a

Since Vpp = 2a

$$\Rightarrow \frac{RMS}{V_{pp}} = \frac{a}{2a} = \boxed{\frac{1}{2}}$$

2. Triangular Wave: RMS / Vpp = ? $\boxed{\frac{1}{2\sqrt{3}}}$



$$f(t) = \begin{cases} \frac{4a}{T}t\,; & 0 \le t < T/4 \\ -\frac{4a}{T}t + 2a\,; & \frac{T}{4} \le t < \frac{3T}{4} \\ \frac{4a}{T}t - 4a\,; & \frac{3T}{4} \le t \le T \end{cases}$$

$$\Rightarrow RMS = \sqrt{\frac{1}{T}\left[\int_0^{T/4}\left(\frac{4a}{T}t\right)^2 dt + \int_{T/4}^{3T/4}\left(-\frac{4a}{T}t + 2a\right)^2 dt + \int_{3T/4}^{T}\left(\frac{4a}{T}t - 4a\right)^2 dt\right]}$$

(next page)

3. If you see a difference by a factor of 10 between the oscilloscope reading and the function generator setting, where is the first place that you should look? Watch the Probe Setting video (https://youtu.be/dtSuTHIviSo) for the answer.

Firstly, we should check the probe attenuation by looking at the channel 1 (yellow) ratio on the oscilloscope. If the ratio is 10:1, it means we are connecting a 10-to-1 attenuation probe to the oscilloscope that causes the wrong measurement. So, we need to fix it by 41 adjusting the knob to have a channel 1's ration is 1:1

$$+\ \int_0^{T/4}\left(\frac{4a}{T}t\right)^2 dt = \int_0^{T/4}\left(\frac{4a}{T}\right)^2 t^2\, dt = \frac{16a^2}{T^2}\,\frac{t^3}{3}\Big|_0^{T/4}$$

(prove for number 2)

$$= \frac{16a^2}{T^2}\cdot\frac{1}{3}\left(\frac{T}{4}\right)^3 = \frac{16a^2\times T^3}{T^2\times 3\times 4^3} = \boxed{\frac{a^2 T}{12}}$$

$$+\ \int_{T/4}^{3T/4}\left(-\frac{4a}{T}t+2a\right)^2 dt = \int_{T/4}^{3T/4}\left(\frac{4a}{T}t-2a\right)^2 dt$$

$$= (2a)^2\int_{T/4}^{3T/4}\left(\frac{2t}{T}-1\right)^2 dt$$

Let $x = \frac{2t}{T} - 1$

$t = T/4 \Rightarrow x = -1/2$

$t = \frac{3T}{4} \Rightarrow x = \frac{1}{2}$

$dx = \frac{2}{T}dt \Rightarrow dt = \frac{T}{2}dx$

$$\Rightarrow 4a^2\int_{-\frac{1}{2}}^{1/2} x^2\,\frac{T}{2}\,dx$$

$$= 4a^2\cdot\frac{T}{2}\,\frac{x^3}{3}\Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{2a^2T}{3}\left(\frac{1}{8}+\frac{1}{8}\right) = \frac{2a^2T}{3\cdot 4}$$

$$= \boxed{\frac{a^2T}{6}}$$

$$+\ \int_{3T/4}^{T}\left(\frac{4a}{T}t-4a\right)^2 dt = \int_{3T/4}^{T}(4a)^2\left(\frac{t}{T}-1\right)^2 dt$$

$$= 16a^2 \int_{3T/4}^{T} \left(\frac{t}{T} - 1\right)^2 dt$$

Let $x = \frac{t}{T} - 1$

$\Rightarrow t = 3T/4 \Rightarrow x = -1/4$

$t = T \Rightarrow x = 0$

$dx = \frac{dt}{T} \Rightarrow T\,dx = dt$

$$\Rightarrow 16a^2 \int_{-1/4}^{0} x^2 T\,dx$$

$$= 16a^2 T \left. \frac{x^3}{3} \right|_{-1/4}^{0} = 16a^2 T \left(0 - \frac{1}{3}\left(-\frac{1}{4}\right)^3\right)$$

$$= 16a^2 T \cdot \frac{1}{192} = \boxed{\frac{1}{12} a^2 T}$$

$$\Rightarrow RMS = \sqrt{\frac{1}{T}\left(\frac{a^2T}{12} + \frac{a^2T}{6} + \frac{a^2T}{12}\right)} = \sqrt{\frac{4a^2}{12}}$$

$\Rightarrow RMS = \frac{a}{\sqrt{3}}$, Since $V_{pp} = 2a$

$$\Rightarrow \frac{RMS}{V_{pp}} = \frac{a}{\sqrt{3} \times 2a} = \boxed{\frac{1}{2\sqrt{3}}}$$

4. If you see a difference by a factor of 2 between the oscilloscope reading and the function generator setting, where is the first place that you should look? Watch the Function Generator Output Impedance video (https://youtu.be/-8Dv1oOjD9w) for the answer. The first place we should look is the resistance load. Because the function generator (FG) has a default 50Ω load, in real case, the load is oscilloscope having 1 MΩ in input impedance. To have a correct measurement, we need to adjust the load of FG to have the high Z load (it means 1 MG load)

5. Why would you ever want to use AC coupling on an oscilloscope? Watch the AC Coupling video (https://www.youtube.com/watch?v=dtSuTHIviSo&t=6s) for the answer.

Because the AC coupling can filter out the DC signal component from a AC-DC signal. This will increase the solution of the signal measurement on the oscilloscope.

Week 2 Prelab End