EE3 Fall 2020
Homework Problem 8

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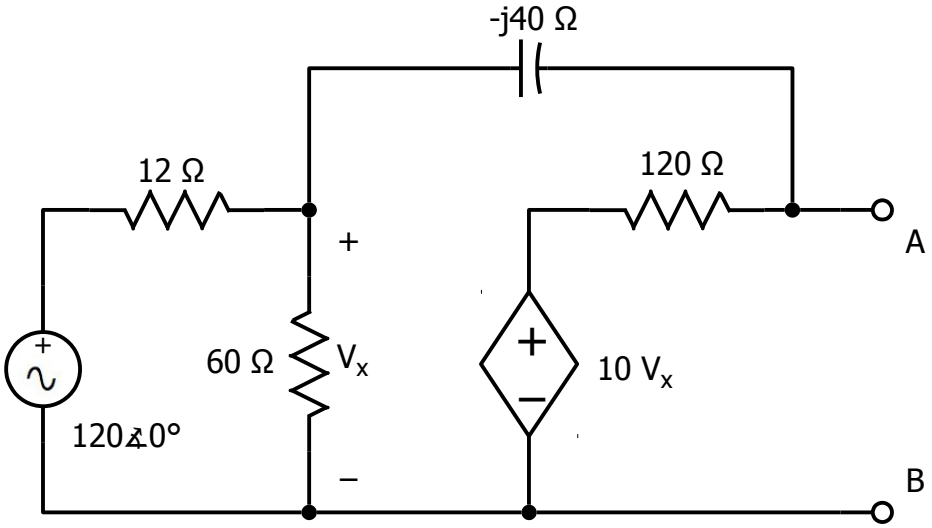
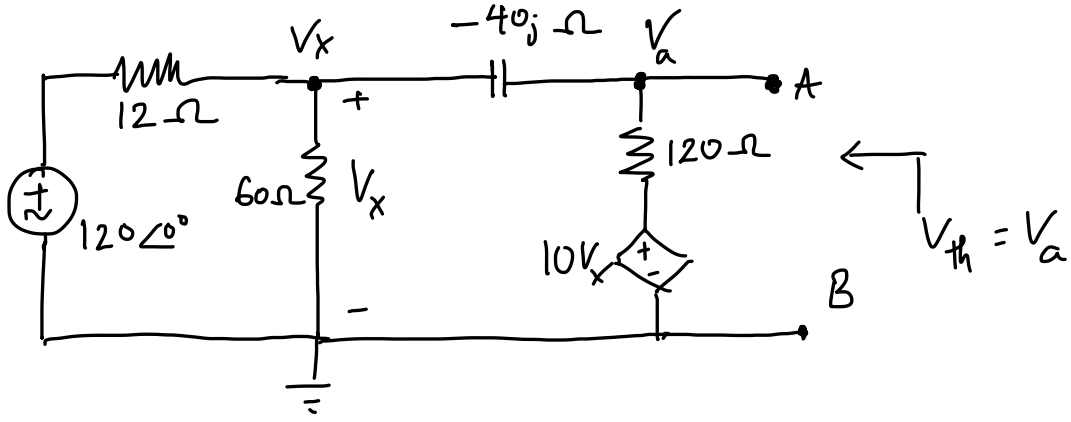
Find the Thévenin Equivalent of this circuit.



We need to find R_{Th} & V_{Th}

① We can have:



$$\begin{cases} \dfrac{V_x - 120\angle 0}{12\,\Omega} + \dfrac{V_x}{60\,\Omega} + \dfrac{V_x - V_a}{-40j} = 0 \ \text{(KCL)} \ \text{(I)} \\ \dfrac{V_a - V_x}{-40j} + \dfrac{V_a - 10V_x}{120} = 0 \quad \text{(KCL)} \ \text{(II)} \end{cases}$$

From (I), $\frac{V_x - 120\angle 0^\circ}{12\Omega} + \frac{V_x}{60\Omega} + \frac{V_x - V_a}{-40j} = 0$

$\Leftrightarrow \frac{5(V_x - 120\angle 0^\circ) + V_x}{60} + \frac{(V_x - V_a)j}{40} = 0$

$\Leftrightarrow 5(V_x - 120\angle 0^\circ) + V_x + 1.5j(V_x - V_a) = 0$

$\Leftrightarrow 5V_x - 600\angle 0^\circ + V_x + 1.5jV_x - 1.5jV_a = 0$

$\Leftrightarrow (6 + 1.5j)V_x - 1.5jV_a = 600\angle 0^\circ = 600$ (III)

From (II), $\frac{V_a - V_x}{-40j} + \frac{V_a - 10V_x}{120} = 0$

$\Leftrightarrow \frac{3j(V_a - V_x)}{120} + \frac{V_a - 10V_x}{120} = 0$

$\Leftrightarrow 3jV_a - 3jV_x + V_a - 10V_x = 0$

$\Leftrightarrow (3j+1)V_a - (3j+10)V_x = 0 \Leftrightarrow (3j+1)V_a = (3j+10)V_x$

$\Leftrightarrow V_x = \frac{3j+1}{3j+10} \cdot V_a$. Plug it into (III), we have:

(III) $\Leftrightarrow (6+1.5j)\cdot\frac{3j+1}{3j+10}V_a - 1.5jV_a = 600$

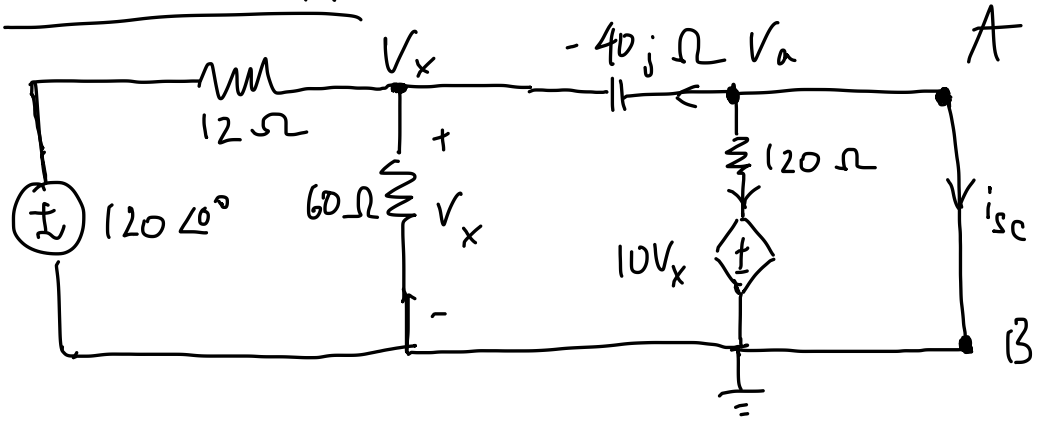
$\Leftrightarrow \left[\frac{(6+1.5j)(3j+1)}{3j+10} - 1.5j\right]V_a = 600$

$\Leftrightarrow \frac{(1.5+19.5j)-1.5j(3j+10)}{3j+10}V_a = 600$

$\Leftrightarrow \frac{6+4.5j}{3j+10}V_a = 600 \Leftrightarrow V_a = \frac{600(3j+10)}{6+4.5j}$

$\Leftrightarrow \boxed{V_a = V_{th} = 784 - 288j\ (V)}$

(+) Find R_{th}: , We have:



$\Rightarrow V_a = V_b = 0V$

$\frac{V_x - 120\angle 0°}{12\Omega} + \frac{V_x}{60\Omega} + \frac{V_x - V_a}{-40j\Omega} = 0$, because $V_a = 0$ (KCL)

$$\Leftrightarrow \frac{V_x - 120\angle 0^\circ}{12\,\Omega} + \frac{V_x}{60\,\Omega} + \frac{V_x}{-40j} = 0$$

$$\Leftrightarrow 5(V_x - 120\angle 0^\circ) + V_x + 1.5j\,V_x = 0$$

$$\Leftrightarrow 5V_x - 600\angle 0^\circ + V_x + 1.5j\,V_x = 0$$

$$\Leftrightarrow (6 + 1.5j)V_x = 600\angle 0^\circ$$

$$\Leftrightarrow V_x = \frac{600\angle 0^\circ}{6 + 1.5j}$$

We also have: $\frac{V_a - V_x}{-40j\,\Omega} + \frac{V_a - 10V_x}{120\,\Omega} + i_{sc} = 0$

$$\Leftrightarrow \frac{-V_x}{-40j} - \frac{10V_x}{120} + i_{sc} = 0$$

$$\Leftrightarrow \frac{-jV_x}{40} - \frac{10V_x}{120} + i_{sc} = 0$$

$$\Leftrightarrow i_{sc} = \frac{jV_x}{40} + \frac{10V_x}{120} = \frac{3jV_x + 10V_x}{120}$$

$$\Leftrightarrow i_{sc} = \frac{(3j+10)V_x}{120} = \frac{3j+10}{120} \cdot \frac{600\angle 0^\circ}{6 + 1.5j}$$

$$\Rightarrow R_{th} = \frac{V_{th}}{i_{sc}} = \frac{(784 - 288j)(6 + 1.5j) \cdot 120}{(3j+10)\,600\angle 0^\circ}$$

$$\Rightarrow \boxed{R_{th} = 91.2 - 38.4j\ (\Omega)}$$

Finally, the Thévenin equivalent of the circuit.