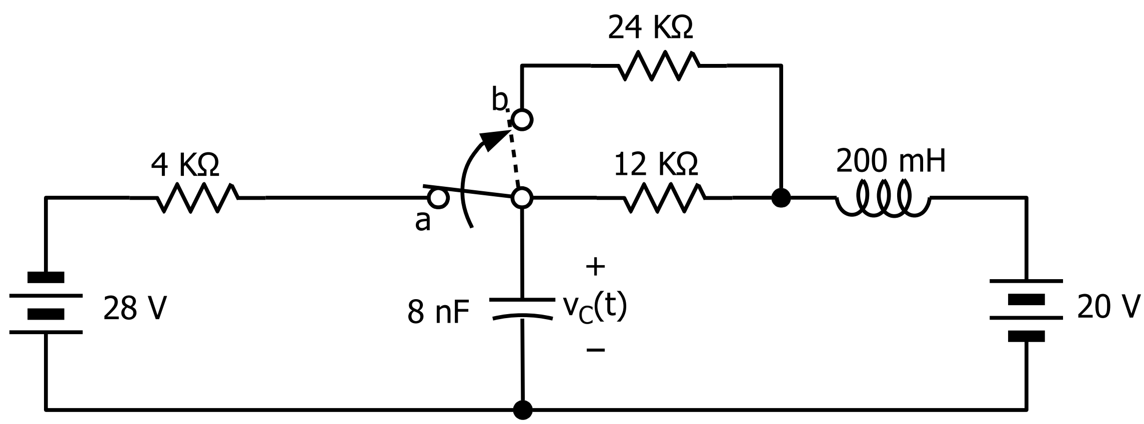
EE3 Fall 2020
Practice Problems 4



1. The switch has been in position a for a <u>long time</u>. All transients have died out. At t = 0, the switch moves instantaneously to position b.

   a. At $t=0^-$ (the last instant that the switch is in position a), what is the current through the capacitor? 0 A

   b. At $t=0^-$ (the last instant that the switch is in position a), what is the voltage across the capacitor? $-28-v_C(0^-)+(4K\Omega)(3mA)=0$; $v_C(0^-)=-16$ V

   c. At $t=0^+$ (the first instant that the switch is in position b), the current through the capacitor is the same as in Part a. True $\boxed{\text{False}}$

   d. At $t=0^+$ (the first instant that the switch is in position b), the voltage across the inductor is the same as at $t=0^-$. True $\boxed{\text{False}}$

   e. At $t=0^+$ (the first instant that the switch is in position b), what is the voltage across the inductor?

   KVL CCW: $-20+v_L(0^+)+(8K\Omega)(3mA)+v_C(0^+)=0$; $v_L(0^+)=12$ V; + at right end

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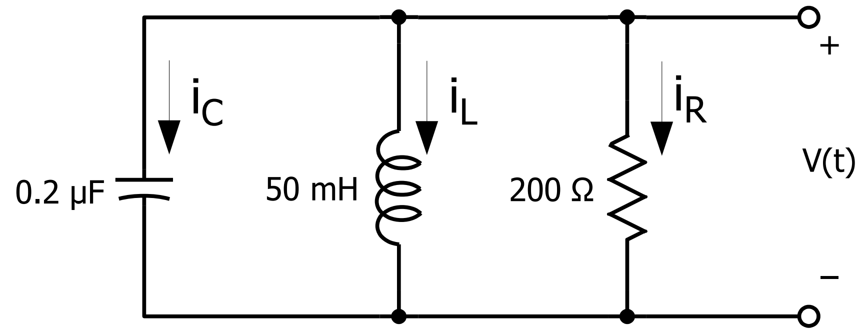

2. This is a second-order circuit. There is an initial voltage on the capacitor $v(0^-)=12$ V, and an initial current in the inductor $i_L(0^-)=30$ mA. In order to solve the differential equation for v(t), the following values must be found:
   a. $i_C(0^+)$
   b. $i_R(0^+)$
   c. $dv(t)/dt|_{t=0^+}$

Using what you know about inductors, capacitors, and KCL, find these values.

$$i_L(0^+) = i_L(0^-) = 30 \text{ mA}$$
$$v(0^+) = v(0^-) = 12 \text{ V}$$

a. $i_C(0^+)+i_L(0^+)+i_R(0^+) = 0$

$$i_C(0^+) = (-30-60) = -90 \text{ mA}$$

b. $i_R(0^+)=\dfrac{v(0^+)}{R} = \dfrac{12}{200} = 60 \text{ mA}$

c. Because $i_C(0^+) = C \left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = \frac{-90\text{e-}3}{0.2\text{e-}6} = -450 \text{ KV/s}$$

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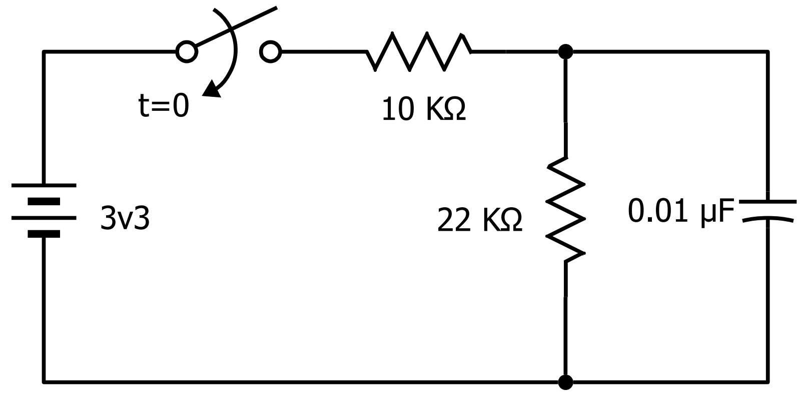
3a. Find the time constant τ of this circuit. This will require solving the differential equation for the circuit.

3b. Then, find only the Thévenin resistance R_{th} of the circuit to the left of the capacitor (consider the capacitor to be the load).

3c. Then, compute R_{th}*C and compare to the τ from 3a.



$$\frac{v_C-3.3}{10e3}+\frac{v_C}{22e3}+1e\text{-}8\frac{dv_C}{dt}=0$$

$$\frac{dv_C}{dt}+(1e8)\left(\frac{1}{10e3}+\frac{1}{22e3}\right)v_C=(1e8)\left(\frac{3.3}{10e3}\right)$$

$$P=1.45e4;\ Q=3.3e4;\ e^{\int_0^t PdX}=e^{1.45e4\,t}$$

$$e^{1.45e4\,t}\frac{dv_C}{dt}+1.45e4\,e^{1.45e4\,t}v_C=3.3e4\,e^{1.45e4\,t}$$

$$\frac{d}{dt}\left(e^{1.45e4t}v_C\right)=3.3e4\,e^{1.45e4\,t}$$

$$e^{1.45e4\,t}v_C=\int 3.3e4\,e^{1.45e4\,t}dt=\frac{3.3e4}{1.45e4}e^{1.45e4\,t}+C=2.27\,e^{1.45e4\,t}+C$$

$$v_C=2.27+C\,e^{-1.45e4\,t}$$

$$v_C(0^+)=2.27+C \ \rightarrow\ C=-2.27$$

$$v_C(t)=2.27\left(1-e^{-1.45e4\,t}\right)$$

$$\text{Time constant }\ \tau=\frac{1}{1.45e4}=6.875e\text{-}5\ \text{s}$$

R_{th} of network assuming C is the load:

$$R_{th}=10e3\|22e3=6.875e3\ \Omega$$

$$R_{th}\cdot C=(6.875e3)(1e\text{-}8)=6.875e\text{-}5\ \text{s}$$

CONCLUSION: for a first-order capacitive circuit with step input, $\tau=\dfrac{1}{P}$!

Also, $\tau=C\cdot R_{th}$, where C is the load! NOTE: not a mathematical proof.

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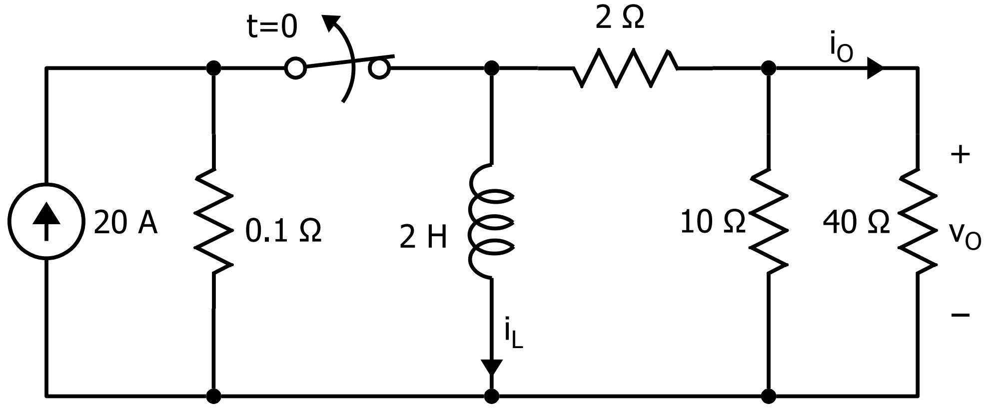
4. The switch has been in the position shown for a long time. Find:
   a. $i_L(0^+)$
   b. $i_O(0^+)$
   c. $v_O(0^+)$
   d. τ for $t=0^+$
   e. $i_L(0^+)$ for all $t>0$
   (HINT: refer to the Lecture 4 video at 22 minutes.)



a. $i_L(0^+) = i_L(0^-) = 20\text{ A}$

b. $i_O(0^+) = -20\left(\dfrac{10}{10+40}\right) = -4\text{ A}$

c. $v_O(0^+) = (-4)\cdot(40) = -160\text{ V}$

d. $\tau = \dfrac{L}{R_{th}}$

$R_{th} = 2 + 10\|40 = 2 + 8 = 10\,\Omega$

$\tau = \dfrac{2}{10} = 0.2\text{ s}$

e. Refer to the Lecture 4 video at 22 minutes.

$i_L(t) = i_O(0^+)\,e^{\frac{-t}{\tau}} = 20\,e^{-5t}$

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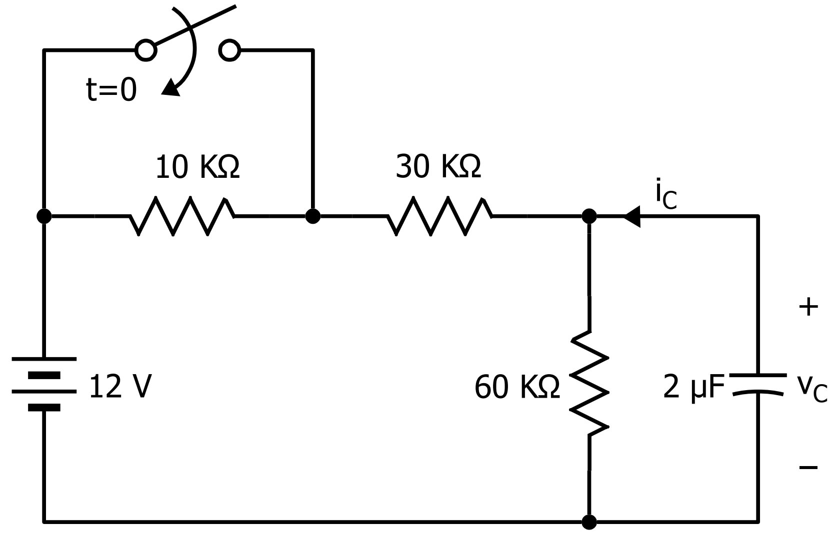
5. The switch has been open for a long time. Find:
   a. $v_C(0^-)$
   b. $v_C(0^+)$
   c. $v_C(\infty)$
   d. $i_C(0^-)$
   e. $i_C(0^+)$



a. $v_C(0^-) = 12\left(\dfrac{60}{60+40}\right) = 7.2\ \text{V}$

b. $v_C(0^+) = v_C(0^-) = 7.2\ \text{V}$

c. $v_C(\infty) = 12\left(\dfrac{60}{60+30}\right) = 8\ \text{V}$

d. $i_C(0^-) = 0$

e. $\dfrac{7.2-12}{30\text{e}3} + \dfrac{7.2}{60\text{e}3} - i_C(0^+) = 0$

$$i_C(0^+) = \frac{21.6-24}{60\text{e}3} = 0.04\ \text{mA}$$