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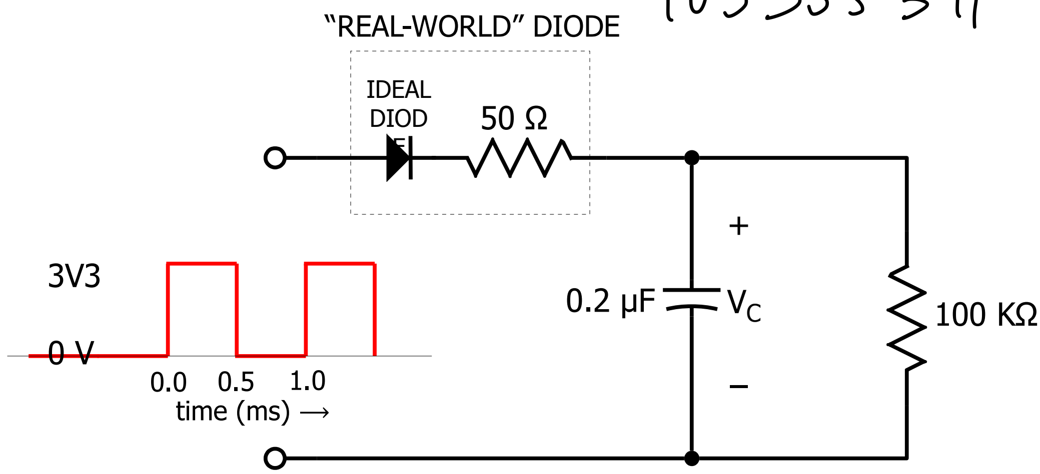
EE3 Fall 2020
Homework Problem 4



The input voltage has been at 0 V for a long time. All transients have died out. At time t = 0.0, a 3V3 pulse train begins. NOTE: the device inside the dashed-line box in an approximation of a real-world diode: when the left end is positive with respect to the right end, it is **forward-biased** and its resistance is 50 Ω. When the left end is negative, it is **reverse-biased** and its resistance is ∞.

a. When the input voltage is 3V3, the diode is forward-biased; 3V3 > v_C. What is the charging time constant? Neglect the effect of the 100 KΩ resistor.

b. When the input voltage is 0 V, the diode is reverse-biased, v_C > 0 V, and the capacitor can discharge only through the 100 KΩ resistor. What is the discharging time constant?

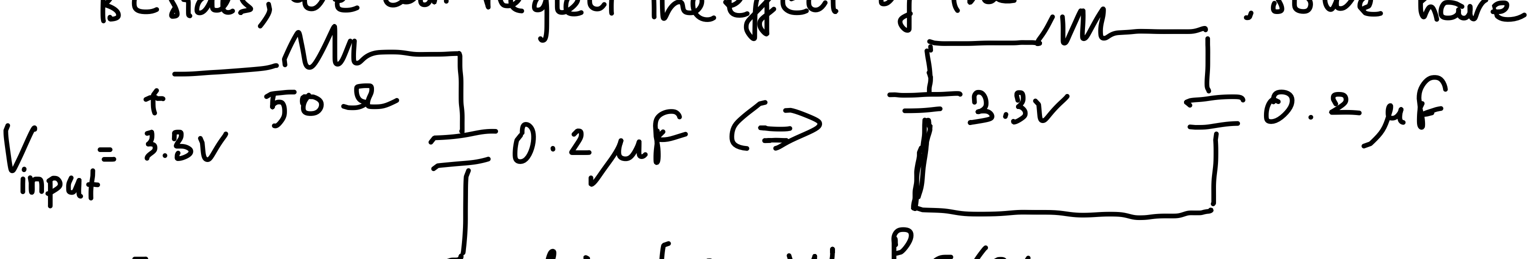
c. What is v_C at time 0.0+?

d. What is v_C at time 0.5+? You will need to evaluate the equation describing the behavior a shown on Slide 3 of Lecture 4 on CCLE Week 3.

e. What is v_C at time 1.0+? Remember that v_C cannot change in the transition from 0– to 0+.

f. Based on what you see in (d.) and (e.), predict the value of v_C at time 1.5+.

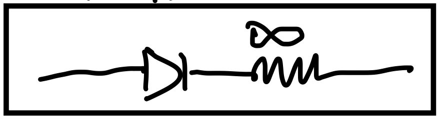
a) Because when $V_{input} = 3.3V$, the forward-biased happens and at this time, the capacitor will be charged by the V_{input}. Besides, we can neglect the effect of the 100 kΩ, so we have:



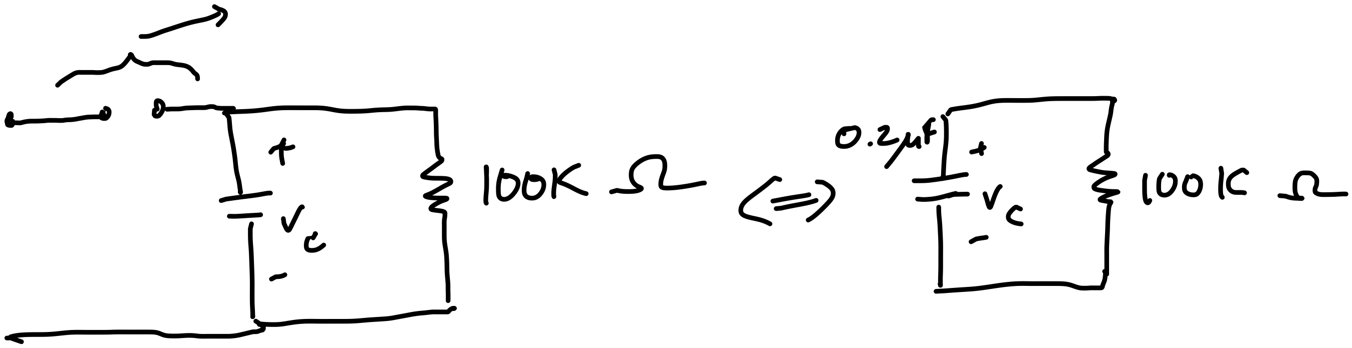
As we proved in class, we have:

$\Rightarrow \tau = RC = 50\,\Omega \times 0.2 \times 10^{-6} = 10^{-5}$ (s)

$\Rightarrow$ charging time constant: $\boxed{\tau = 10^{-5} (s)}$

b) When the input $V = 0V$, the diode is reverse biased, at that time, the resistance of diode is ∞  . So, the capacitor can discharge only through the $100K\Omega$ resistor $\Rightarrow$ we have:

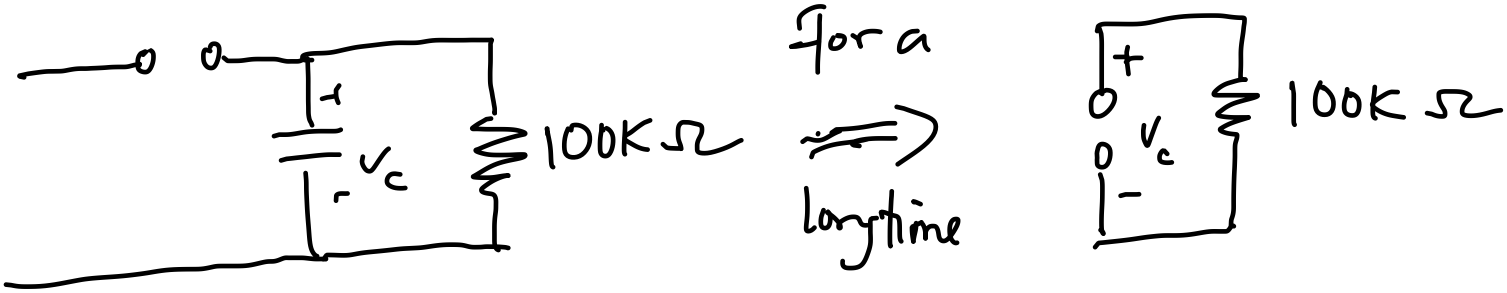
reverse-biased $\Rightarrow R = \infty \Leftrightarrow$ no current $\Leftrightarrow$ open circuit



$$\Rightarrow \tau = RC = 100 \times 1000 \times 0.2 \times 10^{-6}$$

$$\Rightarrow \boxed{\tau = 0.02 \ (s)}$$

c) Because the input voltage has been at 0V for a long time, so at $t = 0^-$ we have:

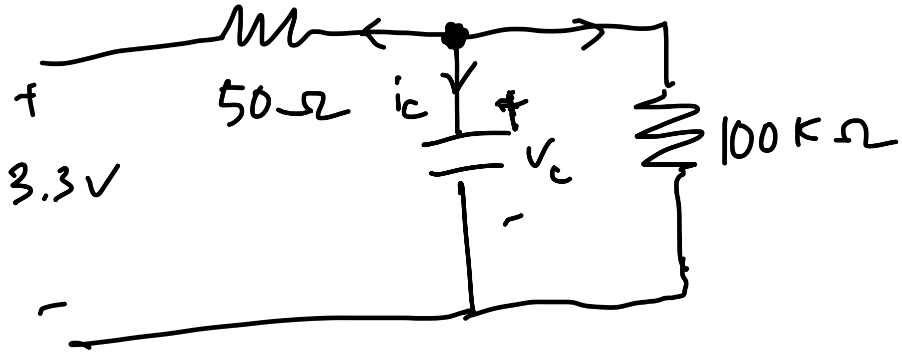


$\Rightarrow v_c(0^-) = 0V$. Since the voltage of capacitor can not change instantaneously

$$\Rightarrow \boxed{v_c(0^+) = 0\,V}$$

d) Find v_c at $t = 0.5^+$

At $t = 0^+ \to 0.5^-$, $V_{input} = 3.3V > V_c$, we have:



We have known already $v_c(0^+) = 0V$

We have: $$\frac{V_c - 3.3V}{50\,\Omega} + i_c + \frac{V_c}{100000\,\Omega} = 0 \text{ (KCL)}$$

$$\Rightarrow \frac{2000\,(V_c - 3.3V) + V_c}{100\,000\,\Omega} + C\frac{dv_c}{dt} = 0$$

$$\Leftrightarrow \frac{2001\,v_c - 6600V}{100\,000\,\Omega} + \frac{0.2\times10^{-6}\times 100\,000\,\frac{dv_c}{dt}}{100\,000\,\Omega} = 0$$

$$\Leftrightarrow 2001\,v_c - 6600 = -0.02\frac{dv_c}{dt}$$

$$\Leftrightarrow \frac{dv_c}{2001v_c - 6600} = -50\,dt$$

$$\Leftrightarrow \frac{dv_c}{v_c - \frac{6600}{2001}} = -50\times 2001\ dt$$

$$\Leftrightarrow \int \frac{dv_c}{v_c - \frac{6600}{2001}} = \int (-100050)\,dt$$

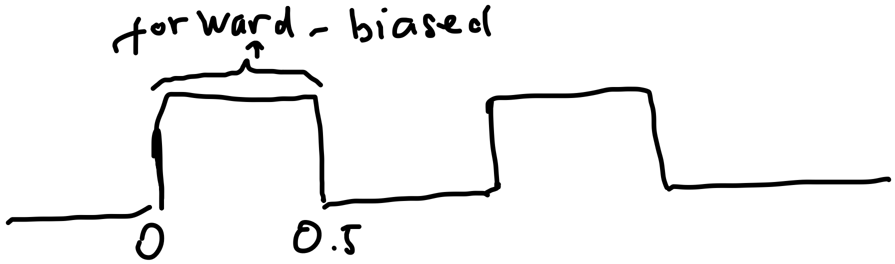
$$\Leftrightarrow \ln \frac{v_c - \frac{6600}{2001}}{0 - \frac{6600}{2001}} = -100050\,t$$

$$\Rightarrow v_c - \frac{6600}{2001} = -\frac{6600}{2001} e^{-100050t}$$

$$\Rightarrow v_c = \frac{6600}{2001} - \frac{6600}{2001} e^{-100050t}$$

$$\Rightarrow v_c(t) = \frac{6600}{2001}\left(1 - e^{-100050t}\right) (V)$$

Also :



$\Rightarrow$ From $t = 0^+ \longrightarrow t = 0.5^-$

we will have the voltage of capacitor will follow

$$v_c(t) = \frac{6600}{2001}\left(1 - e^{-100050t}\right) \text{ (V)}$$

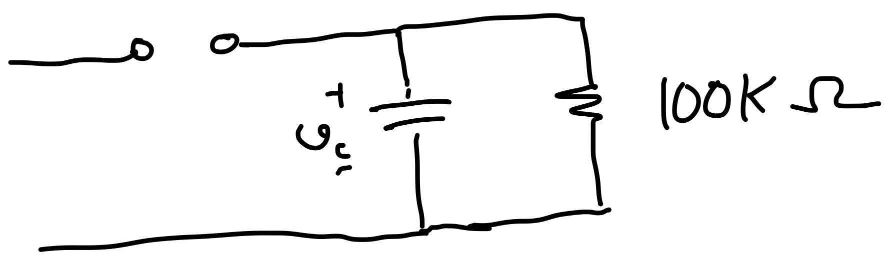
$$\Leftrightarrow v_c(t) = \frac{2200}{667}\left(1 - e^{-100050t}\right) \text{ (V)}$$

$\Rightarrow$ at $t = 0.5^-$,

$$v_c = \frac{6600}{2001}\left(1 - e^{-100050 \times 0.5}\right)$$

$$\Rightarrow v_c(0.5^-) \approx \frac{6600}{2001} \approx 3.298 \text{ (V)}$$

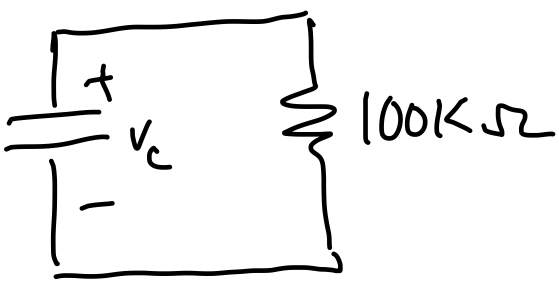
At $t = 0.5^+$, the voltage input begins equal 0 $\Rightarrow$ reverse-biased happens again. We have:



Because the voltage of capacitor cannot change instantaneously $\rightarrow$ $\boxed{v_c(0.5^-) = v_c(0.5^+) = 3.298V}$

e) As we can see, start from $t = 0.5^+$, the reverse-biased happen again that makes the capacitor discharge, and we have:

$t = 0.5^+ \longrightarrow 1^- \iff$ capacitor discharge:



$V_c(0.5^+) = 3.298(V)$

In this basic RC circuit

We have: $\tau = RC = 100\times10^3 \times 0.2\times10^{-6}$

$\Rightarrow \tau = 0.02$

$$v_c + i_c R = 0 \iff v_c + C\frac{dv_c}{dt}R = 0$$

$$\Rightarrow v_c = -RC\frac{dv_c}{dt} \iff \frac{dv_c}{v_c} = -\frac{1}{RC}dt$$

$$\iff \int_{0.5^+}^{t}\frac{dv_c}{v_c} = \int_{0.5^+}^{t} -\frac{1}{RC}dt \iff \ln v_c\Big|_{0.5^+}^{t} = \frac{-t}{RC}\Big|_{0.5^+}^{t}$$

$$\iff \ln\frac{v_c(t)}{v_c(0.5^+)} = -\frac{1}{RC}(t - 0.5^+)$$

$$\iff \ln\frac{v_c(t)}{3.298V} = -50(t - 0.5^+)$$

$$\Rightarrow v_c(t) = 3.298\times e^{-50(t-0.5^+)} \ (V)$$

at $t = 1^- \Rightarrow v_c(1^-) = 3.298 \times e^{-50 \times 0.5}$

$\Rightarrow v_c(1^-) = 4.58 \times 10^{-11} (V) \approx 0 (V)$

$\Rightarrow$ at $t = 1^-$, $v_c = 0 (V)$.

$\Rightarrow$ at $t = 1^+$, $\boxed{v_c(1^+) = v_c(1^-) = 0\,V}$

because the voltage of capacitor can not change instantaneously.

f)

From part d & e, we realize the capacitor will be charged and discharged continuously with the same period 0.5 s.

Particularly, in 0.5 s from $0^+ \rightarrow 0.5^-$, the capacitor will be charged to have $V_c \approx 3.298 (V)$, and then it will be discharge from $0.5^+ \rightarrow 1^-$ to have $V_c \approx 0 (V)$.

Similarly, from $t = 1^+ \to 1.5^-$, the capacitor will be continued charged to have $V_c(1.5^-) \approx 3.298(V)$. Then, we can predict the voltage of capacitor at $t = 1.5^+$ equal v_c at $t = 1.5^-$

$$\Rightarrow \quad v_c(1.5^+) = v_c(1.5^-) = \boxed{3.298(V)}$$