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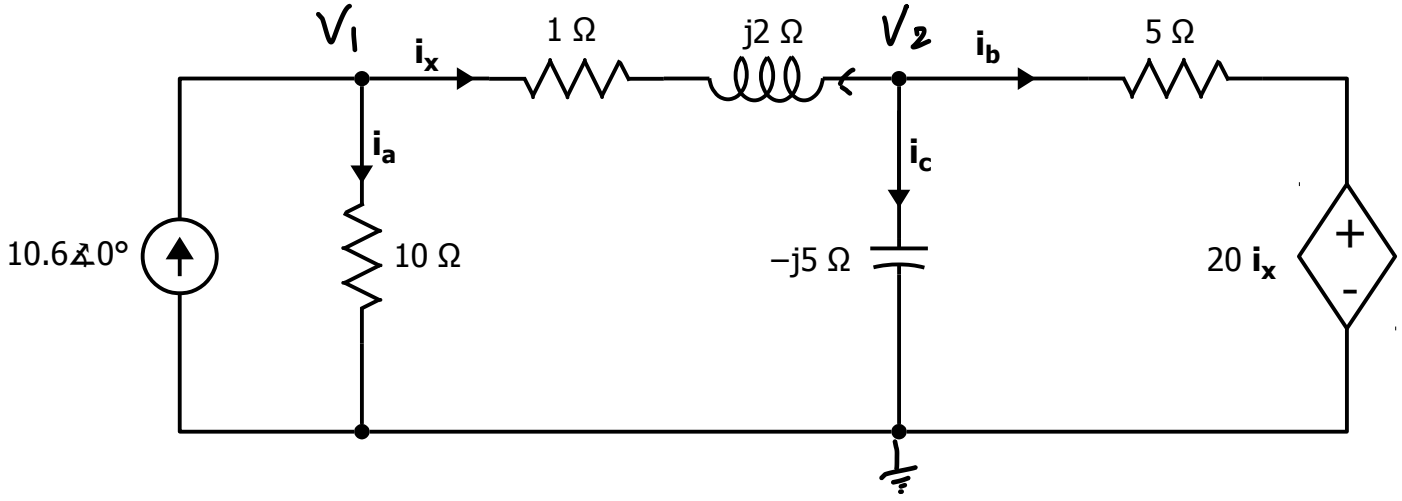
Homework Problem 6



Find the currents $\mathbf{i_a}$, $\mathbf{i_b}$, and $\mathbf{i_c}$ in this sinusoidal steady-state circuit.

Because $\frac{V_1 - V_2}{1+2j} = i_x$

We have:

⊕ $\frac{V_1}{10} + \frac{V_1 - V_2}{1+2j} - 10.6\angle 0^\circ = 0$ (KCL at V_1)

⇒ $V_1(1+2j) + 10(V_1 - V_2) = 10(1+2j) \times 10.6\angle 0^\circ$

⇒ $V_1(1+2j) + 10V_1 - 10V_2 = 106 + 212j$

⇒ $(11+2j)V_1 - 10V_2 = 106 + 212j$ (1)

⊕ Because: $V_2 = i_b \cdot 5\Omega + 20 i_x \Rightarrow i_b = \frac{V_2 - 20 i_x}{5\Omega}$.

⇒ $\frac{V_2 - V_1}{1+2j} + \frac{V_2}{-j5} + \frac{V_2 - 20 i_x}{5} = 0$ (KCL at V_2)

⇒ $-\frac{V_1 - V_2}{1+2j} - \frac{V_2}{5j} + \frac{V_2 - 20 i_x}{5} = 0$

$$\Leftrightarrow -i_x - \frac{V_2}{5j} + \frac{V_2 - 20i_x}{5} = 0$$

$$\Leftrightarrow -5j\, i_x - V_2 + j(V_2 - 20i_x) = 0$$

$$\Leftrightarrow -5j\, i_x - V_2 + jV_2 - 20j\, i_x = 0$$

$$\Leftrightarrow -25j\, i_x + (j-1)V_2 = 0$$

$$\Leftrightarrow 25j\, i_x - (j-1)V_2 = 0$$

$$\Leftrightarrow 25j\, \frac{V_1 - V_2}{1+2j} - (j-1)V_2 = 0$$

$$\Leftrightarrow 25j(V_1 - V_2) - (1+2j)(j-1)V_2 = 0$$

$$\Leftrightarrow 25j\, V_1 - 25jV_2 - (-3-j)V_2 = 0$$

$$\Leftrightarrow 25j\, V_1 - 25jV_2 + (j+3)V_2 = 0$$

$$\Leftrightarrow 25j\, V_1 + (j+3-25j)V_2 = 0$$

$$\Leftrightarrow 25jV_1 + (3-24j)V_2 = 0 \quad (2)$$

From (1) & (2), we have:

$$\begin{cases} (11+2j)V_1 - 10V_2 = 106 + 212j & (1) \\ 25j\, V_1 + (3-24j)V_2 = 0 & (2) \end{cases}$$

$(2) \Leftrightarrow V_2 = \dfrac{-25j\, V_1}{3-24j}$. Plug in to (1), we have:

$$\Rightarrow (11+2j)V_1 + 10\,\frac{25j\, V_1}{3-24j} = 106 + 212j$$

$$\Leftrightarrow (11+2j)(3-24j)V_1 + 250j\, V_1 = (3-24j)(106+212j)$$

$$\Leftrightarrow (81 - 258j)V_1 + 250j\,V_1 = 5406 - 1908j$$

$$\Leftrightarrow (81 - 8j)V_1 = 5406 - 1908j$$

$$\Leftrightarrow V_1 = \frac{5406 - 1908j}{81 - 8j} = 68.4 - 16.8j \ (V)$$

$$\Rightarrow \boxed{V_1 = 68.4 - 16.8j \ (V)}$$

$$\Rightarrow V_2 = \frac{-25j\,V_1}{3 - 24j} = \frac{-25j}{3 - 24j} \times \frac{5406 - 1908j}{81 - 8j}$$

$$\Rightarrow V_2 = \frac{-47,700 - 135,150j}{51 - 1968j}$$

$$\Rightarrow \boxed{V_2 = 68 - 26j \ (V)}$$

So, we have:

$$\textcircled{A}\ i_a = \frac{V_1}{10\,\Omega} = \frac{68.4 - 16.8j}{10\,\Omega} = 6.84 - 1.68j \ (A)$$

$$\textcircled{A}\ i_x = \frac{V_1 - V_2}{1 + 2j} = \frac{68.4 - 16.8j - 68 + 26j}{1 + 2j}$$

$$\Rightarrow i_x = 3.76 + 1.68j \ (A)$$

$$\Rightarrow i_b = \frac{V_2 - 20i_x}{5} = \frac{68 - 26j - 20(3.76 + 1.68j)}{5}$$

$$\Rightarrow i_b = -1.44 - 11.92j \ (A)$$

$$\textcircled{+}\ i_c = \frac{V_2}{-5j} = \frac{68 - 26j}{-5j} = 5.2 + 13.6j \ (A)$$

Finally, We have:

$$i_a = 6.84 - 1.68j \ (A)$$
$$i_b = -1.44 - 11.92j \ (A)$$
$$i_c = 5.2 + 13.6j \ (A)$$