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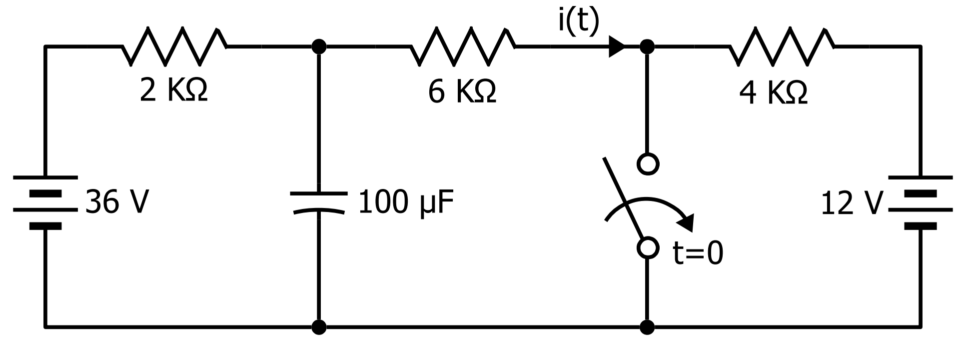
EE3 Fall 2020
Homework Problem ~~6~~ 7

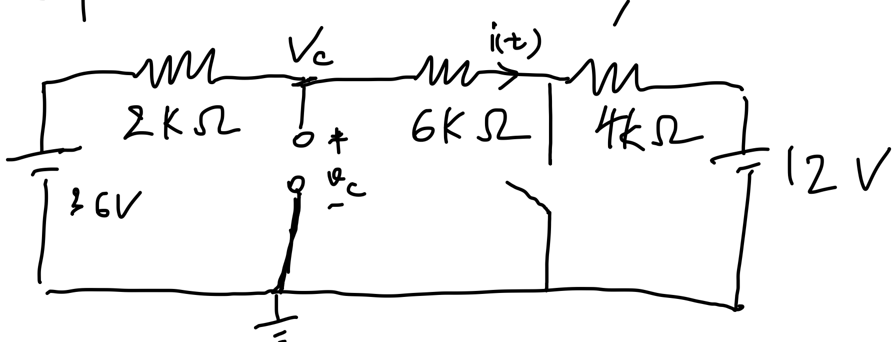
This problem is all about i(t).
a. What is $i(0^-)$?
b. What is $i(0^+)$?
c. What is $i(\infty)$?
d. What is i(t), t>0?



The circuit has been in this conditon for a long time.

This problem can be solved without using a differential equation, though you may do so this way if you wish. Solving it without using a differential equation is done by pulling together a few ideas and bringing them to bear on the problem. That means using the answers to the first 3 parts of this problem, plus the judicious use of a Thévenin equivalent, plus the third slide of Lecture 4 in Week 4 of CCLE.

a) When the switch opens for a long time, the capacitor is opened circuit to DC, We have:



$$\Rightarrow -36 + i(2K\Omega + 6K\Omega + 4K\Omega) + 12 = 0 \text{ (KVL)}$$

$$\Rightarrow 12K\Omega\, i = 36 - 12 = 24 (V)$$

$$\Rightarrow i = i(0^-) = \frac{24\,V}{12000\,\Omega} = \boxed{0.002 (A) = 2 (mA)}$$

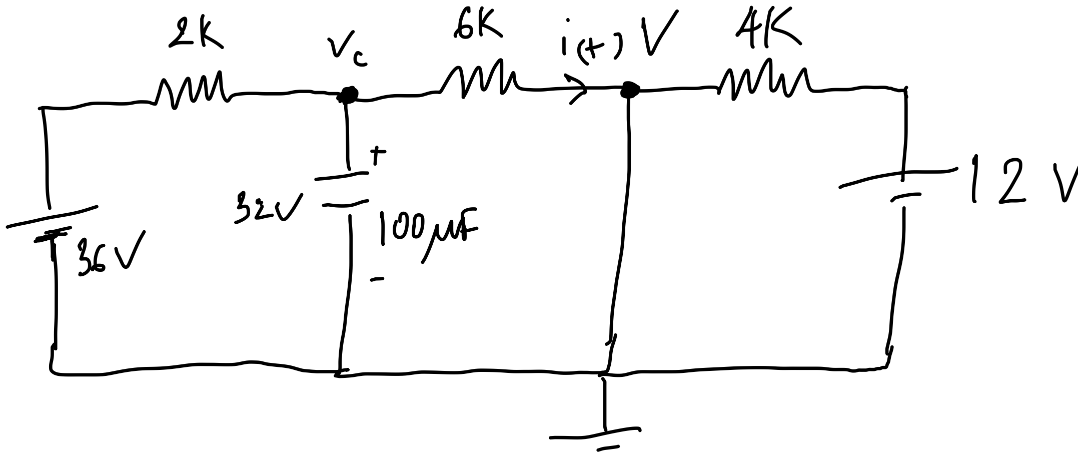
We also have: $-36 + i.2K\Omega + v_c = 0$

$$\Rightarrow v_c = v_c(0^-) = 36 - 0.002A \times 2000\,\Omega = 32(V)$$

b) At $t = 0^+$, the switch closes

$\Rightarrow v_c(0^-) = v_c(0^+) = 32\,(V)$ because the capacitor will not allow instantaneous change in voltage.

We also have:

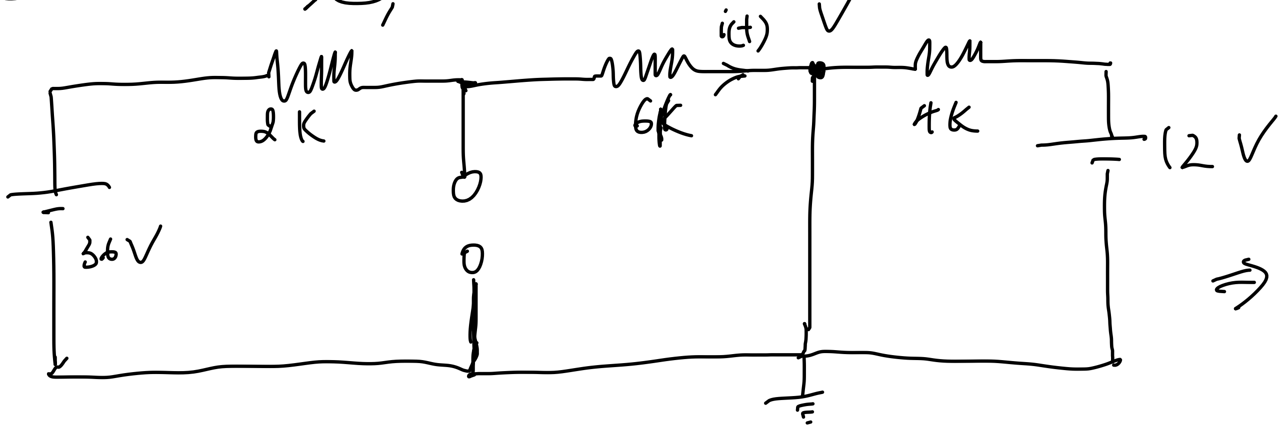


KCL:

$\Rightarrow V = 0(V) \Rightarrow i = i(0^+) = \dfrac{V_c - V}{6k\Omega} = \dfrac{32V - 0}{6K}$

$\Rightarrow i = i(0^+) = \boxed{\dfrac{16}{3}\,(mA) \simeq 5.33\,(mA)}$

c) After closing the switch, the capacitor will be discharged, so when $t \to \infty$, the capacitor is opened circuit to DC, we have:

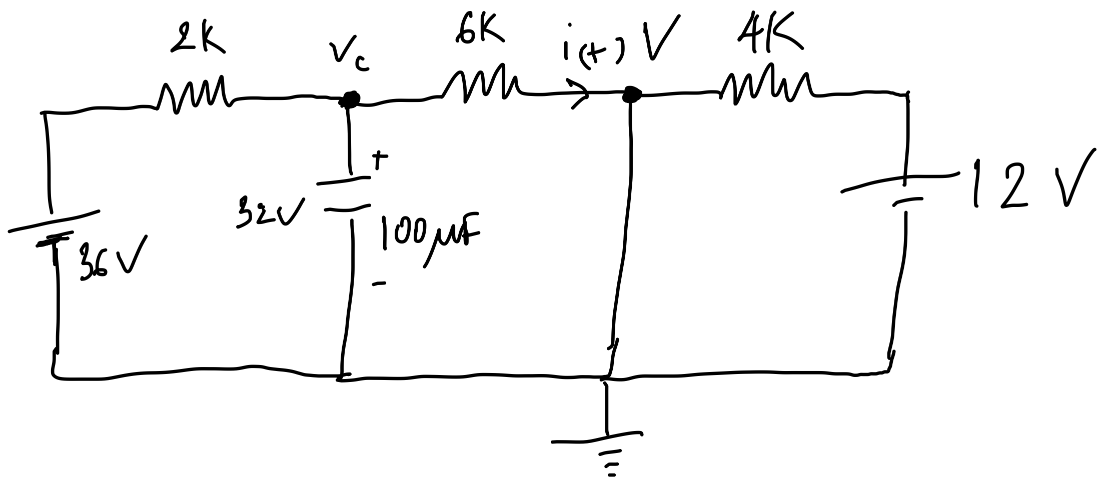


$\Rightarrow V = 0$

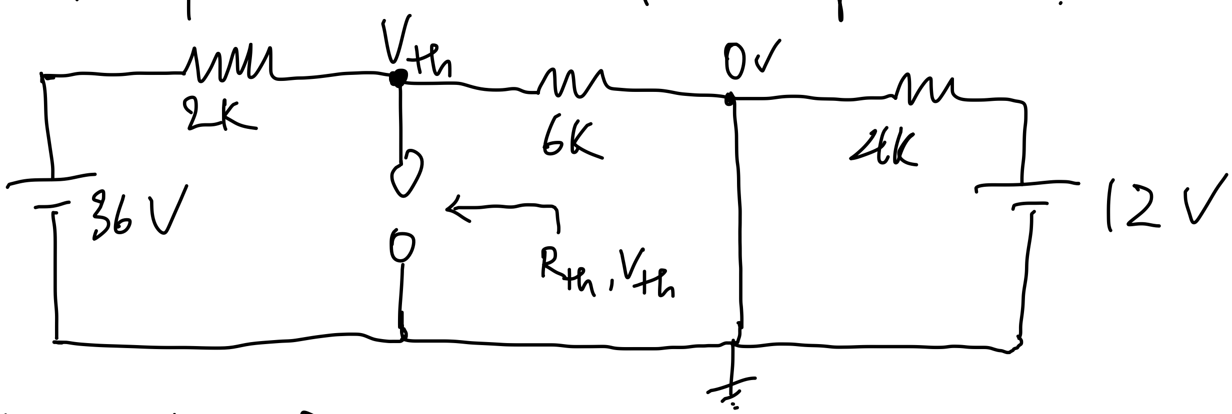
We have $i = \dfrac{36 - V}{2K\Omega + 6K\Omega} = \dfrac{36 - 0}{8K\Omega} = \dfrac{36V}{8K\Omega}$

$\rightarrow \boxed{i = i(\infty) = 4.5(mA)}$

d) When $t > 0$, we have the circuit:



⊕ Firstly, we can apply the Thévenin equivalent; and the ports are at the capacitor..



* We find the V_{th}.

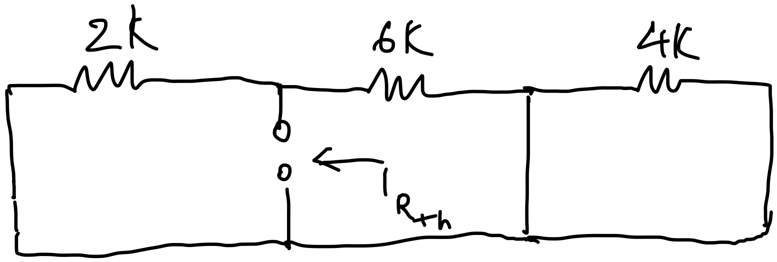
$$\frac{V_{th} - 36}{2K} + \frac{V_{th} - 0}{6K} = 0$$

$$\Leftrightarrow 3(V_{th} - 36) + V_{th} = 0 \Leftrightarrow 3V_{th} + V_{th} = 3 \times 36$$

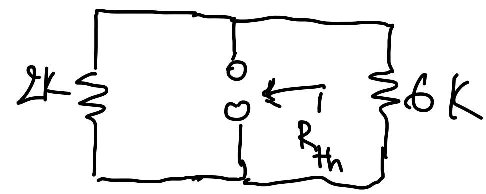
$\Leftrightarrow \; 4V_{th} = 108\,(V) \Rightarrow V_{th} = 27\,(V)$

* Find R_{th}.

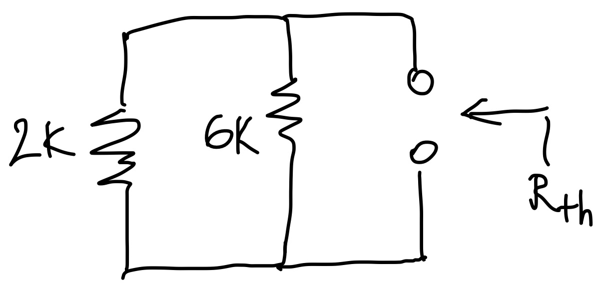
Because we only have independent source, so we can short the voltage sources, so:



$\Leftrightarrow$

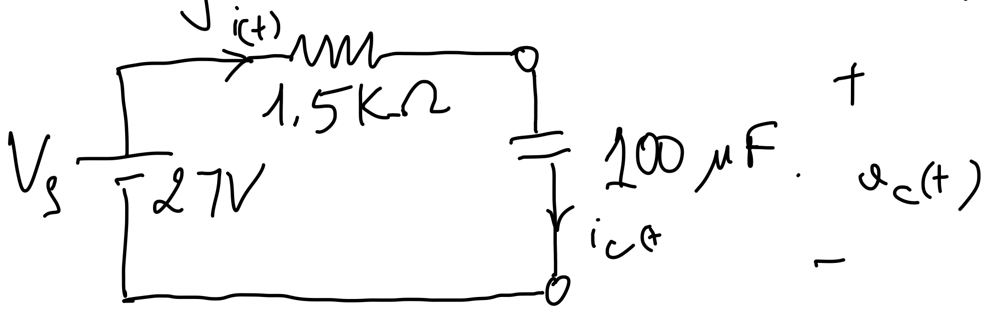


$\Leftrightarrow$



$$\Rightarrow R_{th} = 2K \,//\, 6K = \frac{2 \times 6}{2+6} = 1.5\,(K\Omega)$$

Finally we can have the Thévenin equivalent circuit:



From there, we apply the equation for RC circuit in third slide of Lecture 4 in Week 4, we have

$$v_c(t) = V_s - \left[V_s - V_{co}(0^+)\right] e^{-t/RC}$$

Also, at $t \to 0^+$, $V_{co} = 32V$, $RC = 1.5 \times 1000 \times 100 \times 10^{-6} = 0.15\,(s)$

$$\Rightarrow v_c(t) = 27 - (27 - 32)e^{-t/0.15}$$

$$\Rightarrow v_c(t) = 27 + 5e^{-\frac{20}{3}t} \ (V)$$

(*) We can prove at below:

$$-27 + 1.5K\, i(t) + v_c(t) = 0 \quad , \quad v_c(0^+) = 32(V)$$

$$\Leftrightarrow 1.5K.C.\frac{dv_c}{dt} + v_c(t) = 27 \ (V)$$

$$\Leftrightarrow 1.5KC . \frac{dv_c}{dt} = -v_c + 27$$

$$\Leftrightarrow \int \frac{dv}{27 - v_c} = \int \frac{dt}{1.5KC} \Leftrightarrow \int \frac{dv}{v_c - 27} = -\int \frac{dt}{1.5KC}$$

$$\Leftrightarrow \ln(v_c - 27)\Big|_{v_0 = 32V,\, t \to 0^+}^{v_t} = \frac{-t}{1.5KC} \Leftrightarrow \ln\frac{v_t - 27}{32 - 27} = \frac{-t}{1.5KC}\Big|_{0^+}^{t}$$

$$\Leftrightarrow \ln\frac{v_t - 27}{5} = \frac{-t}{1.5K.C} = \frac{-t}{1.5 \times 1000 \times 100 \times 10^{-6}} = \frac{-t}{0.15}$$

$$\Rightarrow v_t - 27 = 5e^{-\frac{20}{3}t}$$

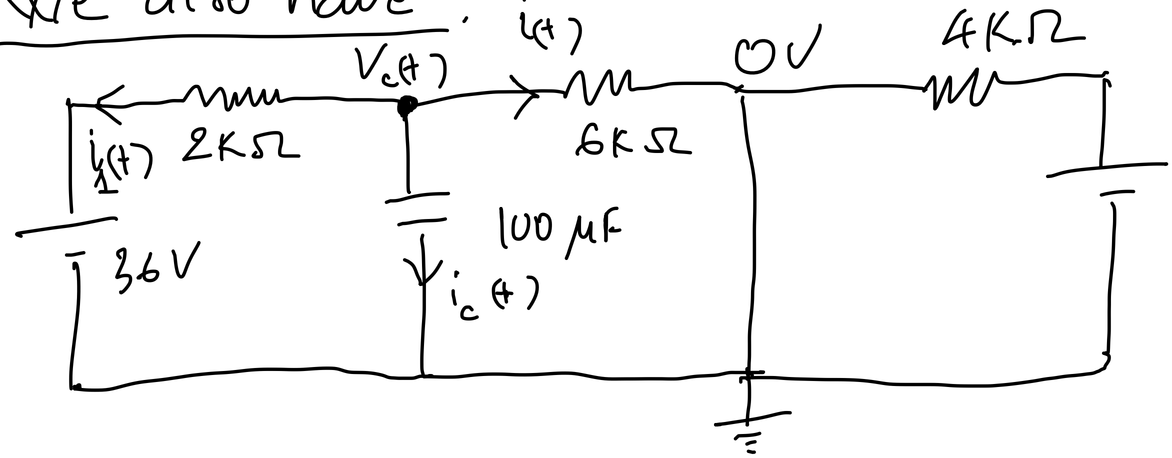
$$\Rightarrow v_c(t) = 27 + 5e^{-\frac{20}{3}t} \ (V)$$

$$\Rightarrow \frac{dv_c(t)}{dt} = 5.\left(-\frac{20}{3}\right)e^{-\frac{20}{3}t} = -\frac{100}{3}.e^{-\frac{20}{3}t} \ (V)$$

$$\Rightarrow i_c(t) = C\frac{dv_c(t)}{dt} = 100\times 10^{-6} \times \frac{-100}{3} e^{-\frac{20}{3}t}$$

$$= \frac{-1}{300} e^{-\frac{20}{3}t} (A) = -\frac{10}{3} e^{-\frac{20}{3}t} (mA)$$

We also have:



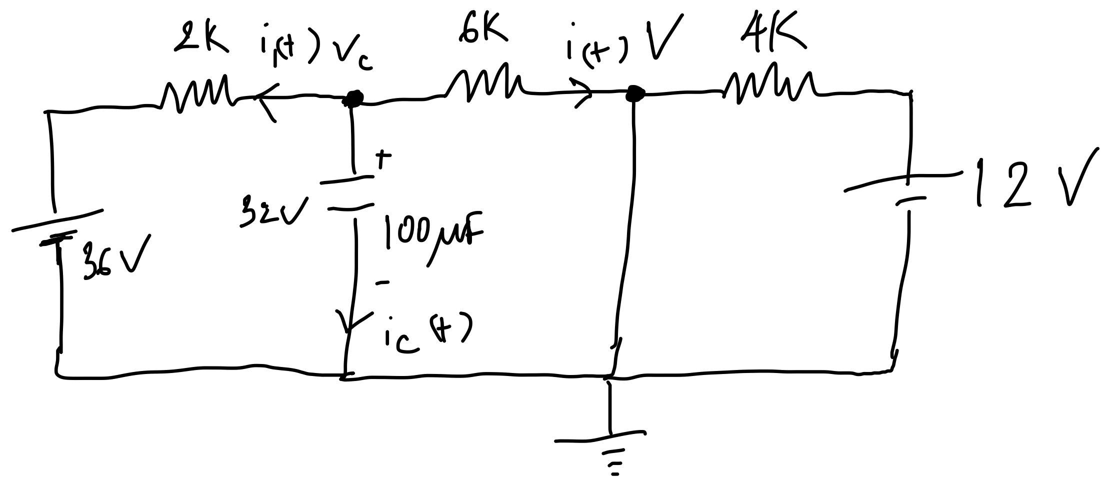
$$\Rightarrow i(t) = \frac{V_c(t) - 0V}{6K\Omega} (mA)$$

$$= \frac{27 + 5e^{-\frac{20}{3}t} (V)}{6\,K\Omega}$$

$$\boxed{\Rightarrow i(t) = \frac{27 + 5e^{-\frac{20}{3}t}}{6} (mA) (t>0)}$$

+ Seconly, instead of using Thévenin, we can solve this problem by using a differential equation, then compare the result.

We have: At $t = 0^+$, $v_c(t) = V_0 = 32\,(V)$



$$\frac{V_c - 36}{2K} + i_c + \frac{V_c - 0}{6K} = 0$$

$\Leftrightarrow$ $3(V_c - 36) + 6K.i_c + V_c = 0$

$\Leftrightarrow$ $3V_c + V_c + 6i_c = 108\,(V)$

$\Leftrightarrow$ $4V_c + 6i_c = 108 \Leftrightarrow 4V_c + 6C\frac{dv_c}{dt} = 108$

$\Leftrightarrow$ $6K.C\frac{dv_c}{dt} = 108 - 4V_c$

$\Leftrightarrow$ $1.5K.C\frac{dv_c}{dt} = 27 - V_c$

$$\Leftrightarrow \frac{dv_c}{27-v_c} = \frac{dt}{1.5KC} \Leftrightarrow \int \frac{dv_c}{v_c-27} = -\int \frac{dt}{1.5K.C}$$

$$\Leftrightarrow \ln(v_c-27)\Big|_{v_0}^{v_c(t)} = \frac{-t}{1.5K.C}\Big|_0^t$$

$$\Leftrightarrow \ln \frac{v_c(t)-27}{32-27} = \frac{-t}{1.5K.C}$$

$$\Leftrightarrow v_c(t) = 27 + 5e^{-t/1.5K.C}$$

Also, $1.5K.C = 1.5 \times 1000 \times 100 \times 10^{-6} = 0.15(s)$

$$\Leftrightarrow v_c(t) = 27 + 5e^{-\frac{20}{3}t} \quad (V)$$

$$\Rightarrow i(t) = \frac{v_c(t) - 0}{6K} = \frac{27+5e^{-\frac{20}{3}t}}{6K} \quad (mA)$$

$$\Rightarrow \boxed{i(t) = \frac{27+5e^{-\frac{20}{3}t}}{6} \quad (mA)}$$

As we can see, with 2 ways of solution we have:

$$\boxed{t>0,\quad i(t) = \frac{27+5e^{-\frac{20}{3}t}}{6} \ (mA) = \frac{27+5e^{-\frac{20}{3}t}}{6000} \ (A)}$$