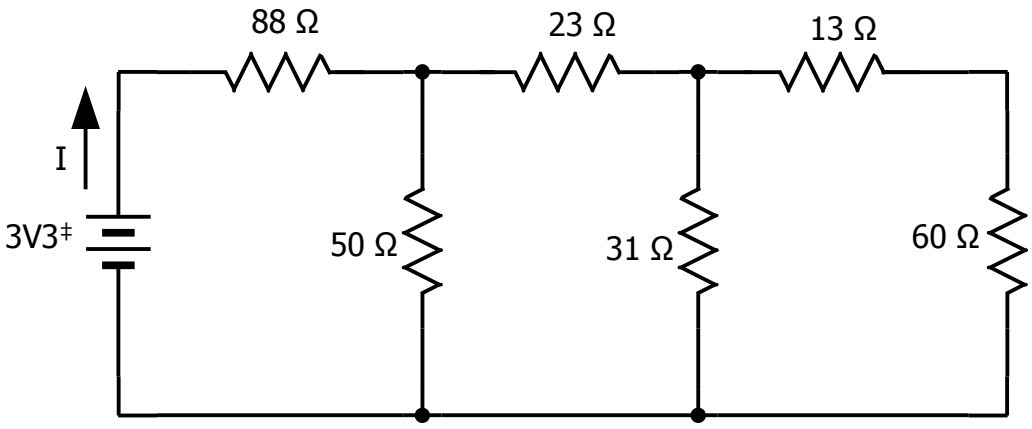
ECE3 Fall 2020
Practice Problems 2

1. This is the problem from Practice Problems 1. Using Node Voltage Analysis, determine the current I.



V_1: top of 50 Ω resistor
V_2: top of 31 Ω resistor

$$\frac{V_1-3.3}{88}+\frac{V_1}{50}+\frac{V_1-V_2}{23}=0$$

$$\frac{V_2-V_1}{23}+\frac{V_2}{31}+\frac{V_2}{73}=o$$

$$1150V_1-1150\cdot3.3+88\cdot23V_1+4400V_1-4400V_2=0$$
$$31\cdot73V_2-31\cdot73V_1+23\cdot73V_2+23\cdot31V_2=0$$

$$\begin{bmatrix}1150+88\cdot23+4400 & -4400\\ -31\cdot73 & 31\cdot73+23\cdot73+23\cdot31\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}1150\cdot3.3\\0\end{bmatrix}$$

$V_1 = 0.70$ V
$V_2 = 0.34$ V

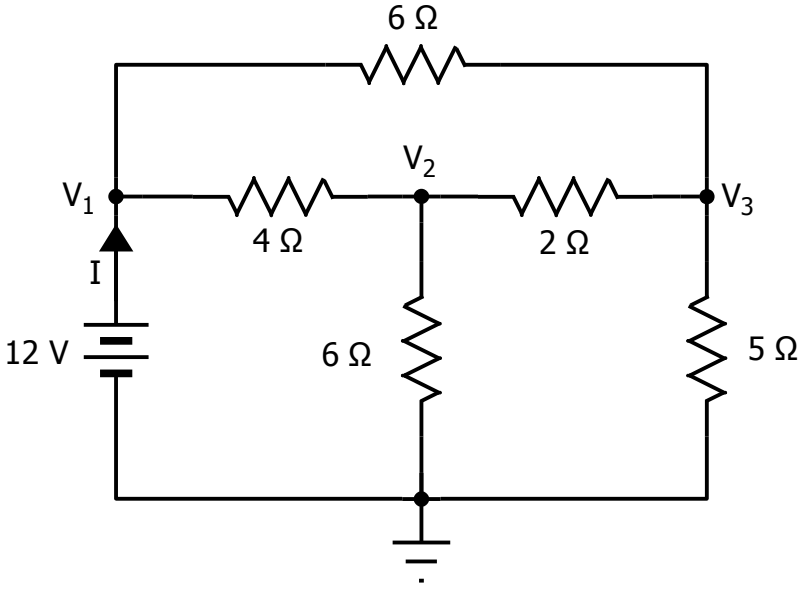
‡ 3V3 is shorthand for 3.3 V. You will see this often on schematics.

ECE3 Fall 2020
Practice Problems 2

2. This is the problem from Lecture 2.

   Using Node Voltage Analysis, V_1, V_2, and V_3.

   Then find the current through the battery.



Node V_1: $V_1 = 12$ V

Node V_2: $\frac{V_2-12}{4}+\frac{V_2-0}{6}+\frac{V_2-V_3}{2}=0$

Node V_3: $\frac{V_3-12}{6}+\frac{V_3-V_2}{2}+\frac{V_3}{5}=0$

$3V_2+2V_2+6V_2-6V_3=36$

$5V_3+15V_3+6V_3-15V_2=60$

$$\begin{bmatrix} 11 & -6 \\ -15 & 26 \end{bmatrix}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 36 \\ 60 \end{bmatrix}$$

$V_2 = 6.61$ V

$V_3 = 6.12$ V

KCL @ V_1:

$\frac{V_1-V_2}{4}+\frac{V_1-V_3}{6}-I=0$

$I = 2.33$ A

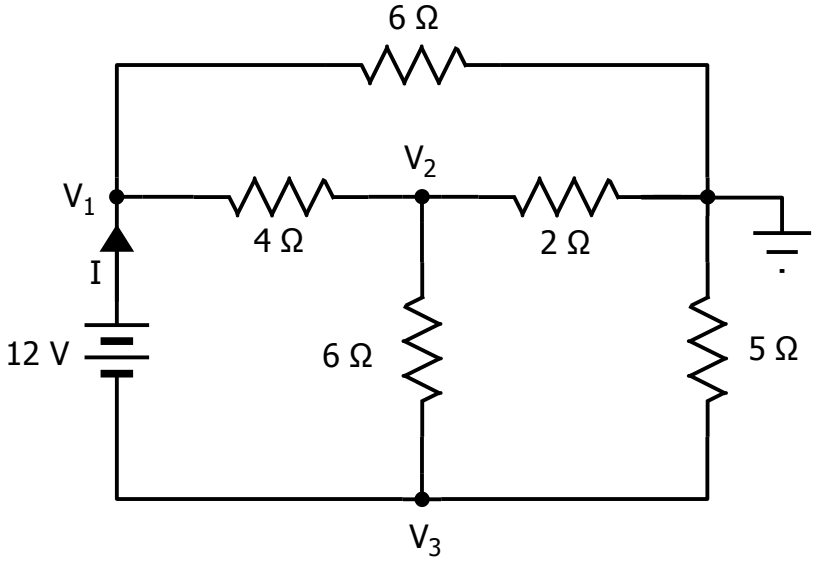
ECE3 Fall 2020
Practice Problems 2

3. This is the same circuit as Problem 2. The reference node has moved.

   Using Node Voltage Analysis, find V_1, V_2, and V_3.

   Then find the current through the battery.

   (Note: with the ground at a different node, you now have 3 unknown nodes. But you also know the voltage relationship between 2 of them.)



$$V_1 - V_3 = 12$$
$$\frac{V_2 - V_1}{4} + \frac{V_2 - V_3}{6} + \frac{V_2}{2} = 0$$
$$\frac{V_1 - V_2}{4} + \frac{V_1}{6} - I = 0$$
$$\frac{V_3 - V_2}{6} + \frac{V_3}{5} + I = 0$$

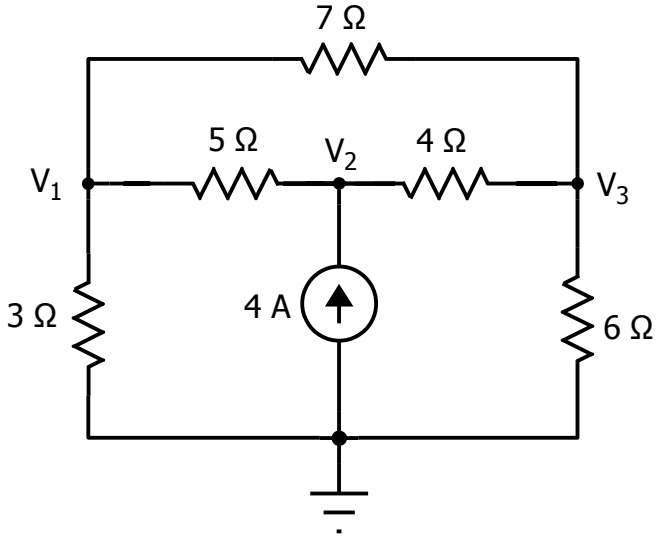
$$3V_2 - 3V_1 + 2V_2 - 2V_3 + 6V_2 = 0$$
$$3V_1 - 3V_2 + 2V_1 - 12I = 0$$
$$5V_3 - 5V_2 + 6V_3 + 30I = 0$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ -3 & 11 & -2 & 0 \\ 5 & -3 & 0 & -12 \\ 0 & -5 & 11 & 30 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 = 5.88 \text{ V}$$
$$V_2 - 0.49 \text{ V}$$
$$V_3 = -6.12 \text{ V}$$
$$I = 2.33 \text{ A}$$

ECE3 Fall 2020
Practice Problems 2

4. Find the three voltages. You may need to dig a little to work this problem.



$$\frac{V_1-V_2}{5}+\frac{V_1}{3}+\frac{V_1-V_3}{7}=0$$
$$\frac{V_2-V_1}{5}-4+\frac{V_2-V_3}{4}=0$$
$$\frac{V_3-V_1}{7}+\frac{V_3-V_2}{4}+\frac{V_3}{6}=0$$

$$21V_1-21V_2+35V_1+15V_1-15V_3=0$$
$$4V_2-4V_1-80+5V_2-5V_3=0$$
$$24V_3-24V_1+42V_3-42V_2+28V_3=0$$

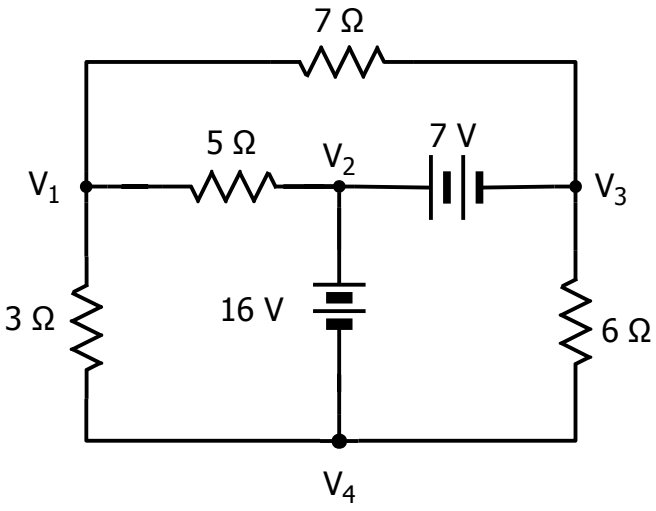
$$71V_1-21V_2-15V_3=0$$
$$-4V_1+9V_2-5V_3=80$$
$$-24V_1-42V_2+94V_3=0$$

$$V_1=7.2\ \text{V}$$
$$V_2=17.4\ \text{V}$$
$$V_3=9.6\ \text{V}$$

ECE3 Fall 2020
Practice Problems 2

5. Choose (judiciously) a reference node. Then find the current through the 7 V battery.

   You may need to dig a little to work this problem. Note: if you choose to solve this problem using Mesh Current Analysis, you are on your own.



CHOOSING V_4 TO BE THE REFERENCE NODE,
$V_2 = 16$ V, and $V_3 = 9$ V

$$\frac{V_1-16}{5}+\frac{V_1}{3}+\frac{V_1-9}{7}=0$$

$$21V_1-(16)(21)+35V_1+15V_1-135=0$$

$$71V_1=471$$

$$V_1=6.6 \text{ V}$$

KCL at the right end of the 7 V battery (currents assumed to be leaving unless known otherwise):

$$\frac{V_3-0}{6}+\frac{V_3-V_1}{7}+I_7=0$$

$$1.50+0.34+I_7=0$$

$$I_7=-1.84 \text{ A (left to right, into the node)}$$

CHOOSING V_2 TO BE THE REFERENCE NODE,
$V_4 = -16$ V, and $V_3 = -7$ V

$$\frac{V_1+16}{3}+\frac{V_1}{5}+\frac{V_1+7}{7}=0$$

$$35V_1+(16)(35)+21V_1+15V_1+105=0$$

$$71V_1=-665$$

$$V_1=-9.4 \text{ V}$$

KCL at the right end of the 7 V battery (currents assumed to be leaving unless known otherwise):

$$\frac{V_3-V_4}{6}+\frac{V_3-V_1}{7}+I_7=0$$

$$1.50+0.34+I_7=0$$

$$\frac{-7-(-16)}{6}+\frac{-7-(-9.4)}{7}+I_7=0$$

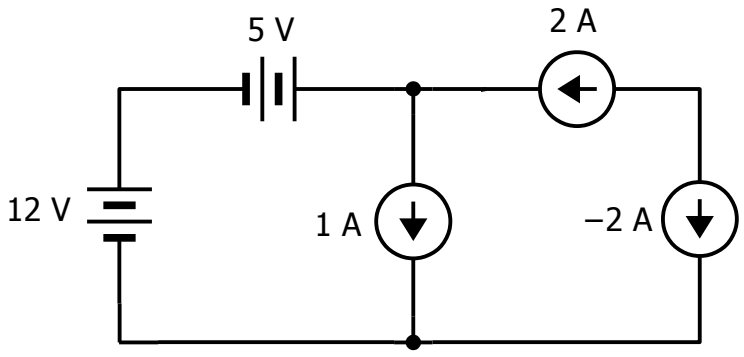
$$1.50+0.34+I_7=0$$

$$I_7=-1.84 \text{ A (left to right, into the node)}$$

ECE3 Fall 2020
Practice Problems 2

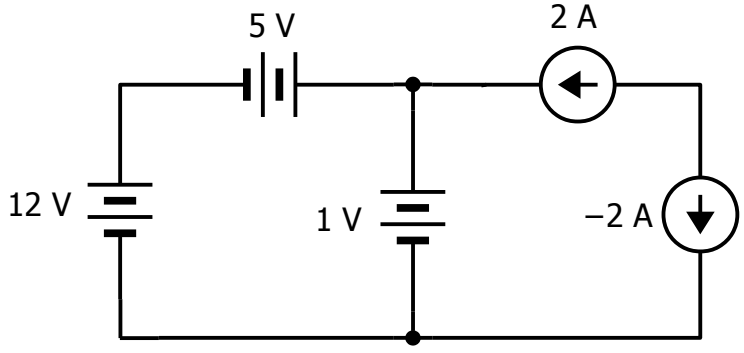
These problems will exercise your knowledge of KVL and KCL. Be sure to watch the video. "Legal" means that KCL and KVL are satisfied.

4. Is this a "legal" circuit? If not, why not?



Circuit is legal.

5. Is this a "legal" circuit? If not, why not?



Circuit is illegal.

ECE3 Fall 2020
Practice Problems 2