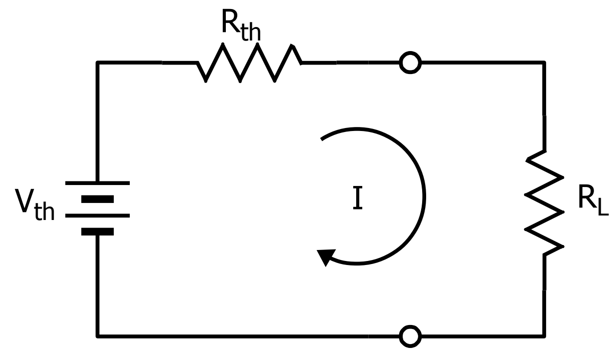
ECE3 FALL QUARTER 2020 LECTURE 5

MAXIMUM POWER TRANSFER

RESLK SENSOR CALIBRATION

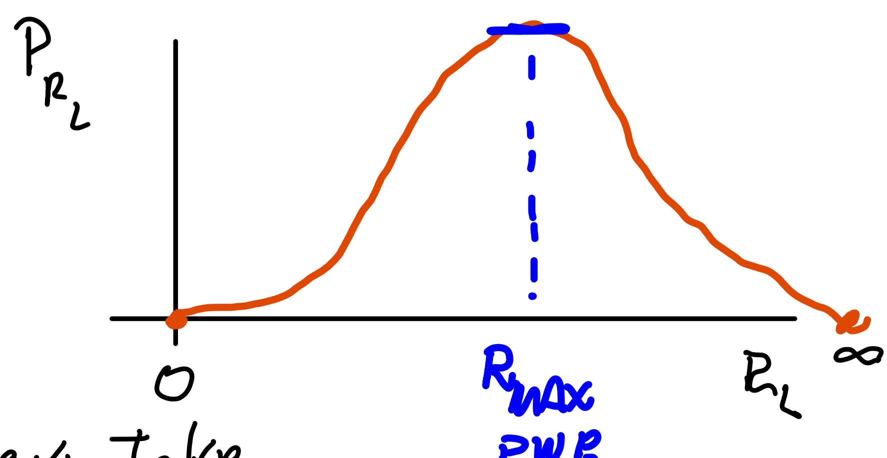
SINUSOIDAL STEADY-STATE (SSS) AC CIRCUITS

MAXIMUM POWER TRANSFER



What value of R_L maximizes power to R_L (P_{RL})?

$$P_{R_L} = I^2 R_L = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$$



To find max, take derivative & set = 0.

$$\left.\frac{dP_{R_L}}{dR_L}\right|_0 = \left.\frac{d}{dR_L}\left[\left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L\right]\right|_0$$

IMPEDANCE MATCHING

$R_L = R_{th}$ for max power to R_L.

RSLK SENSOR CALIBRATION

TAKING PATH SENSOR CALIBRATION MEASUREMENTS

https://youtu.be/swOMZOSCpzM

SENSOR FUSION EXPLANATION

https://youtu.be/KrjccJ-EVjE

SINUSOIDAL STEADY-STATE (SSS) AC CIRCUITS

Why sinusoidal?
Why Steady-State?
Complex Representation
The Complex Plane
AC Voltages and Currents
Complex to Real World
Impedance
Impedance Forms
Impedance vs V/I Phasors
SSS Calculations
Inductive Reactance Derivation
Circuit Analysis

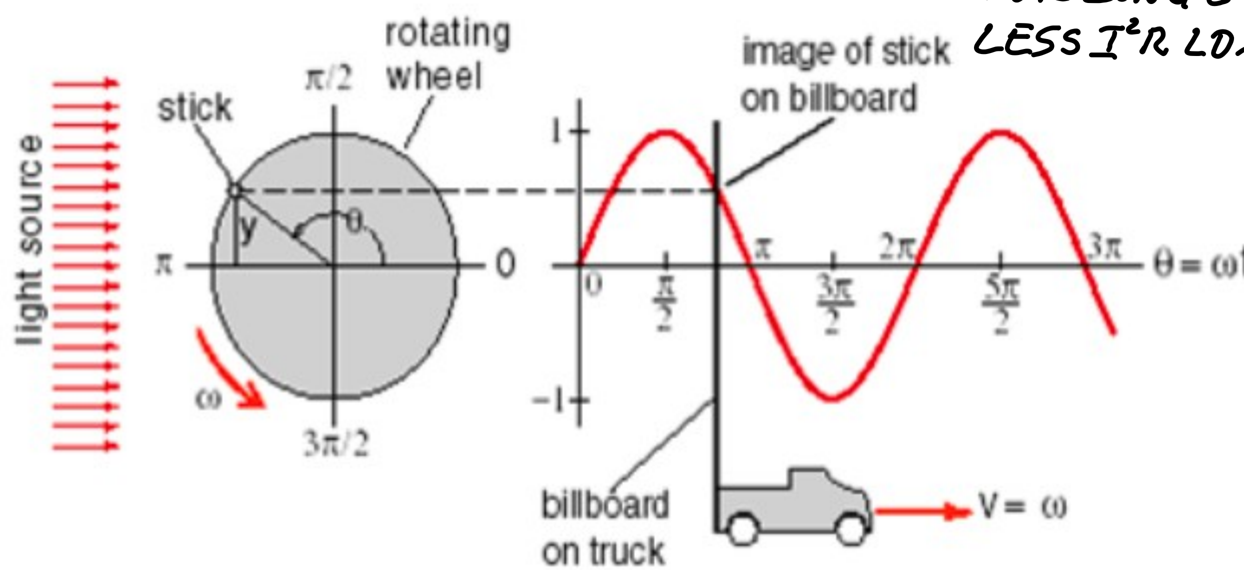
LECTURE 6

Frequency Response
Resonance
Filters
Cutoff Frequency
Filter Types

WHY SINUSOIDAL?

1. Circular motion makes sinusoids

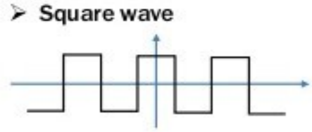
- GENERATORS
- HIGH VOLTAGES FOR LONG LINES: LESS I^2R LOSS.



2. Fourier Series

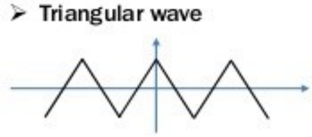
Any repetitive waveform can be represented by a (possibly infinite)series of simusoids of varying amplitude.

- Square wave



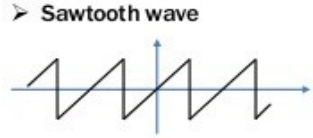
$$\frac{4A}{\pi}\left(\cos\omega_1 t - \frac{1}{3}\cos 3\omega_1 t + \frac{1}{5}\cos 5\omega_1 t - \frac{1}{7}\cos 7\omega_1 t + \cdots\right)$$

- Triangular wave



$$\frac{8A}{\pi^2}\left(\cos\omega_1 t + \frac{1}{9}\cos 3\omega_1 t + \frac{1}{25}\cos 5\omega_1 t + \cdots\right)$$

- Sawtooth wave



$$\frac{2A}{\pi}\left(\sin\omega_1 t - \frac{1}{2}\sin 2\omega_1 t + \frac{1}{3}\sin 3\omega_1 t - \frac{1}{4}\sin 4\omega_1 t + \cdots\right)$$

WHY STEADY-STATE?

Steady-state is main operating mode for many use cases, where efficiency is important.

1. HVAC -- heating, ventilation, & air conditioning
2. Some electric cars: Tesla, Chevy Bolt
3. Wireless communications

ALSO:

4. Calculations are easy (no differential equations!)

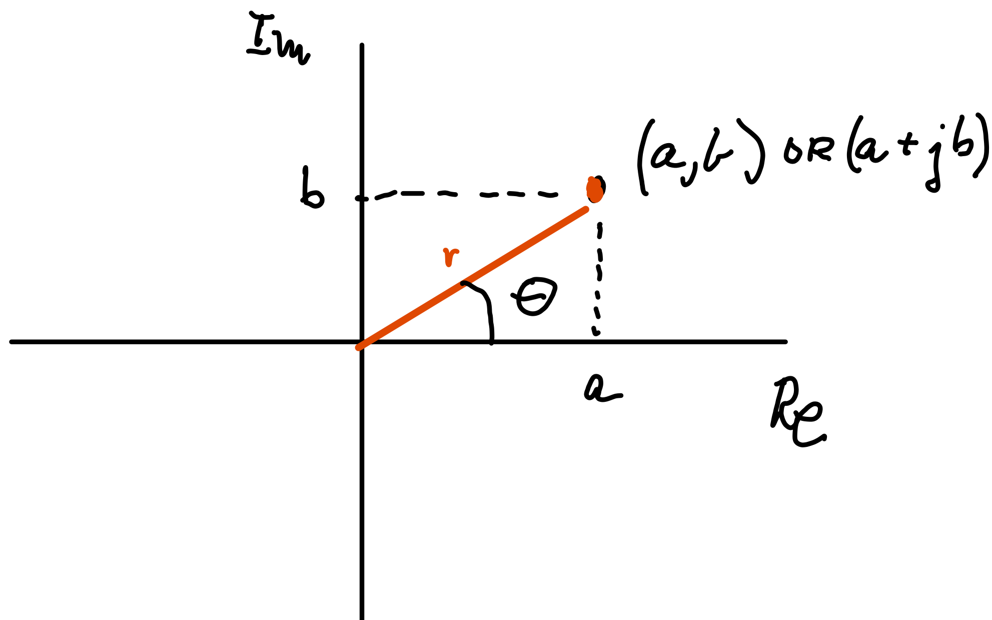
COMPLEX REPRESENTATION

$\sqrt{-1} = j$

complex #s:

- Eulerian: $e^{j\theta}$
- Cartesian / Rectangular: $a + jb$
- Polar: $c\angle\theta$

THE COMPLEX PLANE



$$r^2 = a^2 + b^2$$

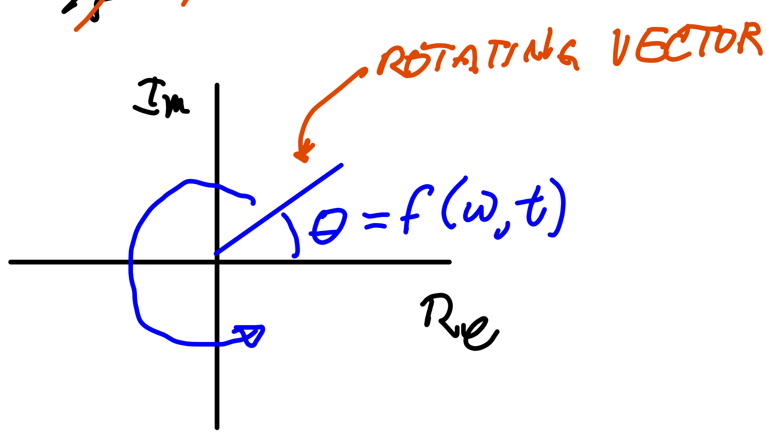
$$\theta = \tan^{-1}\frac{b}{a}$$

COMPLEX TO REAL WORLD

$$v(t) = Ce^{j\theta} = a + jb = C\angle\theta$$

$j\theta = j\omega t$ $\quad \omega$: rad/s $\quad t$: s

$$\frac{\text{rad}}{\cancel{\text{s}}} \times \cancel{\text{s}} = \text{rad}$$



So, for a given frequency ω, $v(t)$ can be written as

$$v(t) = C\cos(\omega t + \theta)$$

ALL REAL #'s!

IMPEDANCE

A superset of resistance.

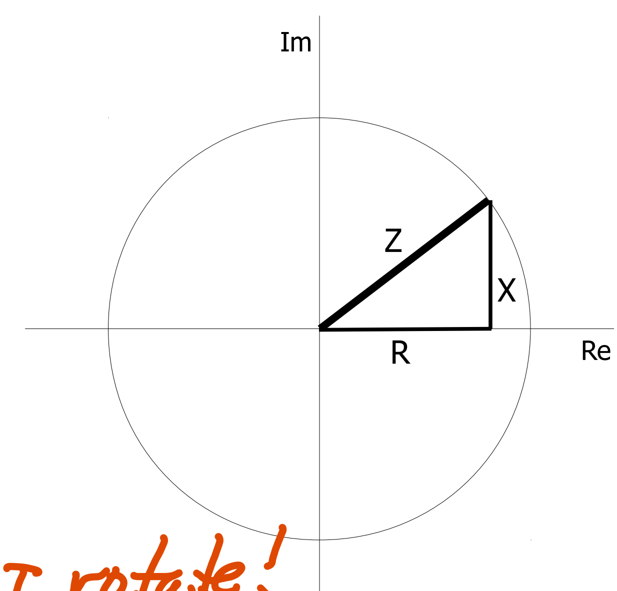
IMPEDANCE (Z)

- Resistance (R)
- Reactance (X, X_L, X_C)

  inductive → X_L X_C ← capacitive
- Reactive Impedance (Z_L, Z_C)

It is convenient to represent impedance as a complex-number quantity.

$$Z = R + jX$$



Z is a constant.
Vector does NOT rotate!

REACTANCE, REACTIVE IMPEDANCE, AND IMPEDANCE

Reactance: REAL #

Ex, $X_C = \frac{1}{\omega C}$

$X_L = \omega L$

Reactive Impedance IMAGINARY #

Ex; $Z_C = \frac{1}{j\omega C} = -j\left(\frac{1}{\omega C}\right)$

$Z_L = j\omega L$

Impedance COMPLEX #

Ex; $Z = R + jX$

$Z = R - j\left(\frac{1}{\omega C}\right)$

$Z = R + j\omega L$

IMPEDANCE vs PHASORS

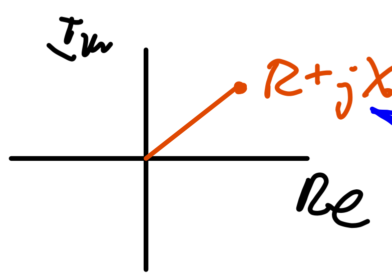
Both are complex.

Impedances represent **CONSTANT** quantities,

Phasors represent **TIME-VARYING** quantities,

Impedance

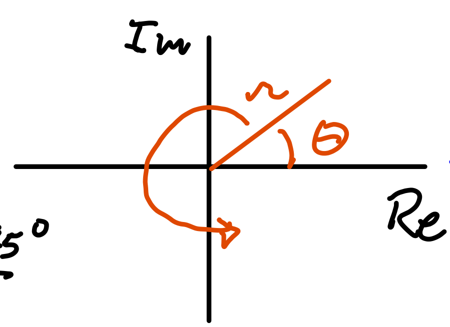
Ex: $Z_{LOAD} = 5 + j6$



DOES NOT ROTATE!

Phasor

Ex: $v(t) = 12 \angle 45^\circ$

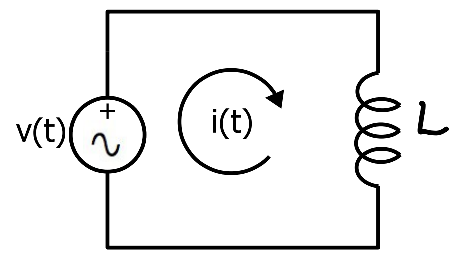


ROTATES, EVEN THOUGH THE "ωt" FACTOR IS ONLY UNDERSTOOD TO BE THERE ("ωt" IS NOT INCLUDED IN PHASOR NOTATION).

SSS CALCULATIONS

All ckt calculations in SSS are the same as for DC <u>except</u> the calculations use complex #s. Only complex <u>arithmetic</u>.

REACTIVE IMPEDANCE DERIVATION



Let $i(0) = i_0$

Let $v(t) = e^{j\omega t}$

We want $X_L = \frac{v(t)}{i(t)}$

$-e^{j\omega t} + L\frac{di}{dt} = 0$

Guess: $i(t) = Ae^{st}$ $\frac{di}{dt} = Ase^{st}$

$i(0) = i_0 = Ae^{0} = A$

$e^{j\omega t} = L i_0 s e^{st} \Rightarrow s = j\omega$ [by similarity of exponents]

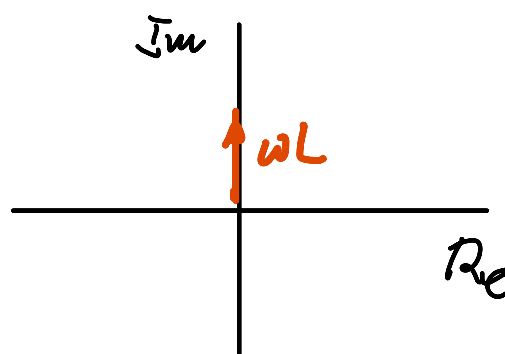
$e^{j\omega t} = L i_0 j\omega e^{j\omega t} \Rightarrow 1 = L i_0 j\omega \Rightarrow i_0 = \frac{1}{j\omega L}$

So $A = \frac{1}{j\omega L}$!

Apply to guess: $i(t) = Ae^{j\omega t} = \frac{1}{j\omega L}e^{j\omega t}$

$X_L = \frac{v(t)}{i(t)} = \frac{e^{j\omega t}}{\frac{e^{j\omega t}}{j\omega L}} = j\omega L$

VOILA!

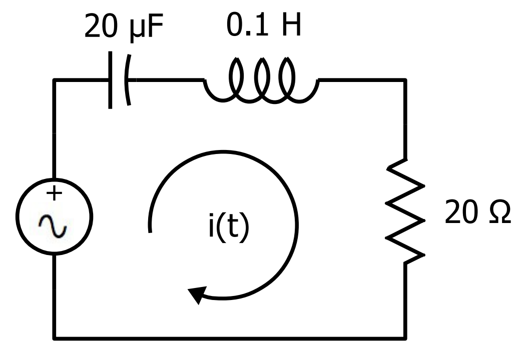


SIMPLE CIRCUIT ANALYSIS

$$Z_C = -j\left(\frac{1}{\omega C}\right) = -j\left(\frac{1}{(1e3)(20e{-}6)}\right)$$

$$Z_C = -j50$$

$$Z_L = j\omega L = j100$$



v(t) = 10∠ 0°

$\omega = 1000$ rad/s

$$\text{KVL: } -10\angle 0^\circ + iZ_C + iZ_L + iR = 0$$

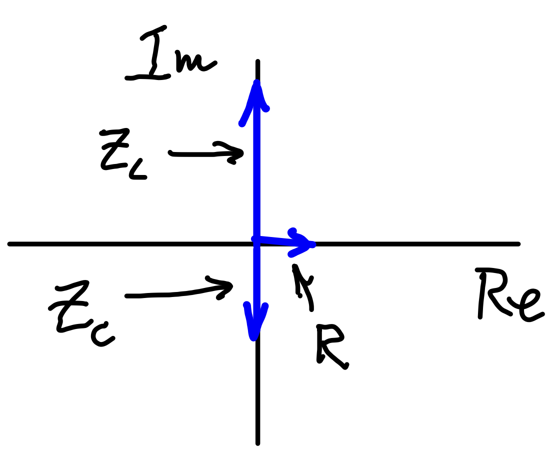
$$Z_{TOT} = -j50 + j100 + 20 = 20 + j50$$

$$i\,Z_{TOT} = i(20 + j50) = 10\angle 0^\circ$$

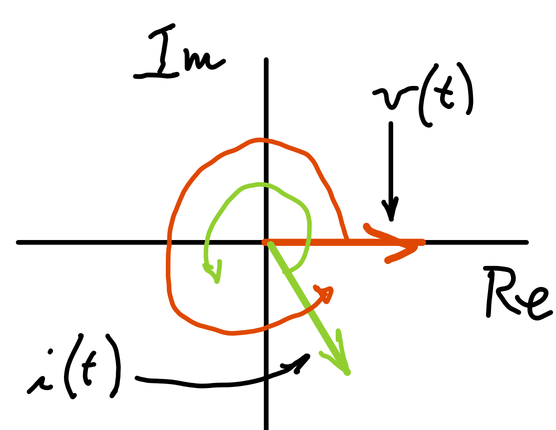
$$i = \boxed{\frac{10\angle 0}{20 + j50}} \approx 0.18\angle -68^\circ$$

LEARN TO DO THIS ON YOUR CALCULATOR IN YOUR SLEEP!

IMPEDANCES



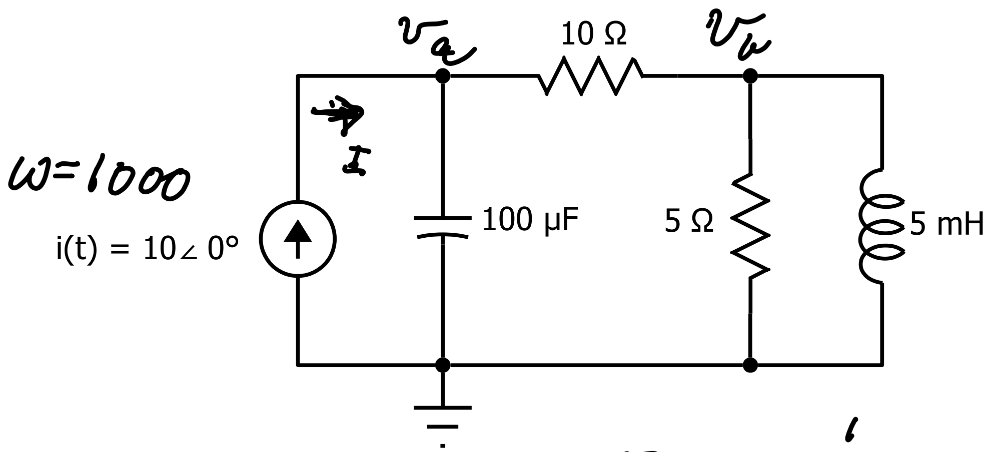
PHASORS



BOTH VOLTAGE & CURRENT PHASORS ROTATE AT 1000 RAD/S.

CURRENT LAGS BEHIND VOLTAGE.

AC NODE-VOLTAGE CIRCUIT ANALYSIS



$$Z_L = j(1e3)(5e{-3}) = +j5 \qquad Z_C = \frac{1}{j(1e3)(100e{-6})} = -j10$$

$$-I + \frac{v_a}{-j10} + \frac{v_a - v_b}{10} = 0$$

$$\frac{v_b - v_a}{10} + \frac{v_b}{5} + \frac{v_b}{j5} = 0$$

$$\left(\frac{-j10}{-j10}\right)(-10) + \frac{v_a}{(-j10)} + \left(\frac{-j}{-j}\right)\frac{v_a - v_b}{10} = 0$$

$$\left(\frac{j}{j}\right)\frac{v_b - v_a}{10} + \left(\frac{2j}{2j}\right)\frac{v_b}{5} + \left(\frac{2}{2}\right)\frac{v_b}{j5} = 0$$

$$-j100 - v_a + j(v_a - v_b) = 0$$

$$j(v_b - v_a) + j2v_b + 2v_b = 0$$

$$(-1+j)v_a - jv_b = j100$$

$$-jv_a + (2+j3)v_b = 0$$

$$v_a = 58.8 - j64.7 = 87.5\angle{-47.7^\circ}$$

$$v_b = 23.5 - j6.9 = 24.3\angle 166^\circ$$