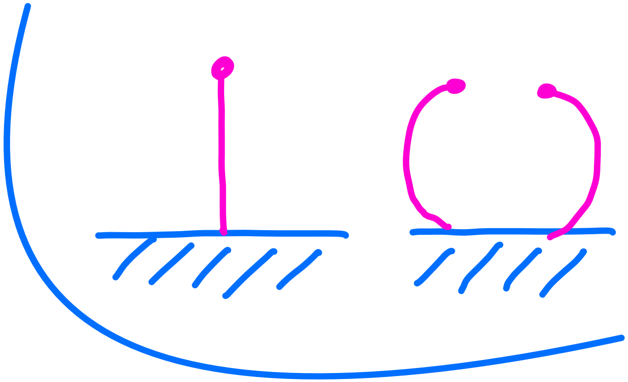
Math 134, Spring 2022

Lecture #9: Bifurcations.

Monday April 18th

Bifurcations

Symmetry



Saddle-node: $\dot{x} = r + x^2$

Transcritical: $\dot{x} = rx - x^2$

Subcritical pitchfork: $\dot{x} = rx + x^3$

$y = -x \rightarrow \dot{y} = -\dot{x} \rightarrow \dot{x} = -\dot{y}$

$\rightarrow x = -y$

SN: $-\dot{y} = r + (-y)^2 \rightarrow \dot{y} = -r - y^2$

T: $\dot{y} = ry + y^2$

SP: $-\dot{y} = r(-y) + (-y)^3$

$\dot{y} = ry + y$

An example: Hysterisis!

$$\dot{x} = rx + x^3 - x^5$$

Taylor's Theorem

Taylor's Theorem

Theorem: Suppose that all partial derivatives of $f(x,r)$ up to order $N+1$ are continuous.

Then,

$$f(x,r) = \sum_{n=0}^{N} \sum_{j=0}^{n} \frac{1}{(n-j)!j!} \frac{\partial^n f}{\partial x^{n-j} \partial r^j}(0,0) x^{n-j} r^j + R_N(x,r),$$

where the remainder term can be written as

$$R_N(x,r) = \sum_{j=0}^{N+1} \frac{1}{(N+1-j)!j!} \frac{\partial^{N+1} f}{\partial x^{N+1-j} \partial r^j}(tx,tr) x^{N+1-j} r^j,$$

for some $0 < t < 1$.

A special case

An example

Consider the function

$$f(x, r) = (r^2 + x)e^x$$

Which of the following is the correct Taylor series expansion to quadratic order at $x = 0,\ r = 0$?

A) $x + \frac{1}{2}x^2 + \frac{1}{2}r^2 + \dots$

B) $x - x^2 + r^2 + \dots$

C) $x + r - xr + \dots$

D) $x + x^2 + r^2 + \dots$

See you next time!