Math 134, Spring 2022

Lecture #8: Bifurcations.

Friday April 15th

Learning objectives

Today we will discuss:

- How to draw a bifurcation diagram.
- Transcritical bifurcations.
- Subcritical pitchfork bifurcations.
- Supercritical pitchfork bifurcations.

Bifurcations

Transcritical bifurcations

$$\dot{x} = rx - x^2$$

An example

Consider the equation

$$\dot{x} = r \ln x + x - 1$$

At what value of r does the solution undergo a transcritical bifurcation at $x = 1$?

A) 0

B) 1

C) -1

D) 2

Hint: Recall the Taylor expansion $\ln(1 + y) = y - \frac{1}{2}y^2 + \ldots$.

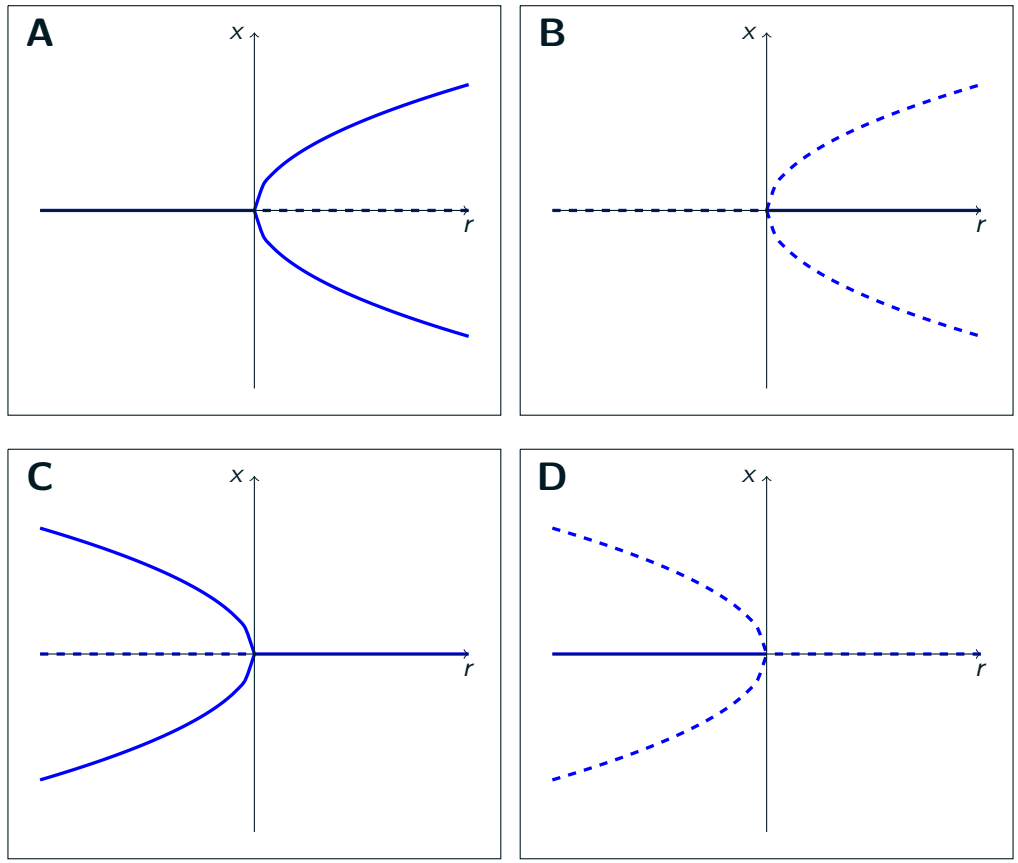
Symmetry

Subcritical pitchfork bifurcation

Consider the ODE

$$\dot{x} = rx + x^3$$

Which of the following is the correct bifurcation diagram?



Supercritical pitchfork bifurcation

$$\dot{x} = rx - x^3$$

See you next time!