Math 134 - Homework 4

1. Consider the ODE
$$\dot{x} = \big(\cosh(1+x) - 1\big)\big(x^2 + 2x - r\big).$$
   (a) Find a value r_c and corresponding x_c at which a bifurcation occurs.
   (b) Sketch the bifurcation diagram.
   (c) Taking $y = x - x_c$ and $s = r - r_c$, find a function $f(y, s)$ so that
   $$\dot{y} = f(y, s).$$
   (d) Compute the Taylor series of f at $(y, s) = (0, 0)$ to fourth order.

   Hint: You may wish to use that $\cosh(y) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} y^{2n}$.
   (e) Use both your bifurcation diagram and Taylor series to explain why this is a new type of bifurcation.

2. (Strogatz Exercise 3.5.7) Consider the logistic equation $\dot{N} = rN(1 - \frac{N}{K})$, with initial condition N_0.
   (a) This system has three dimensional parameters r, K, N_0. Find the dimensions of each of these parameters.
   (b) Show that the system can be rewritten in the dimensionless form
   $$\begin{cases} \dfrac{dx}{d\tau} = x(1-x), \\ x(0) = x_0. \end{cases}$$
   for appropriate choices of the dimensionless variables x, x_0, and τ.
   (c) Find a different nondimensionalization in terms of variables u and τ, were u is chosen such that the initial condition is always $u(0) = 1$.

3. Show that $\dot{x} = \ln(1 + x) - rx$ undergoes a transcritical bifurcation at $(x^*, r^*) = (0, 1)$. Use the Transcritical Bifurcation Theorem covered in class.

4. Problem 3.5.6, parts a), b), c) and d), from the textbook.