Discussion 4

Wednesday, April 20, 2022 7:11 PM

Topics
1. Bifurcations
2. Potential Functions + Bifurcations

3. Hw 3.c / 3.d

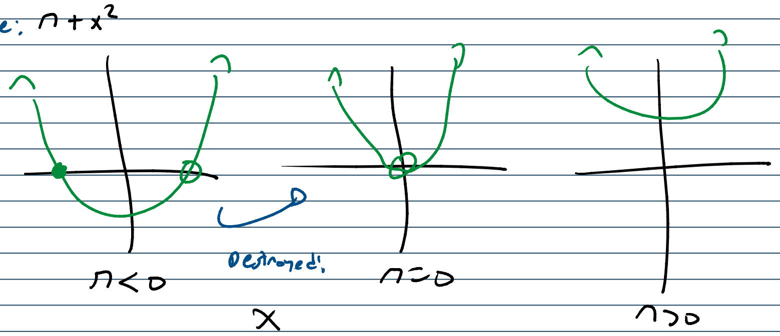
Bifurcations

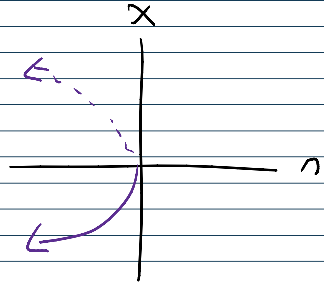
- Saddle Node
- Transcritical
- pitchfork

} - occurs when behavior / location of critical points change as some parameter changes

Saddle Node: Fixed points destroyed and created. Two fixed points collide and destroy each other.

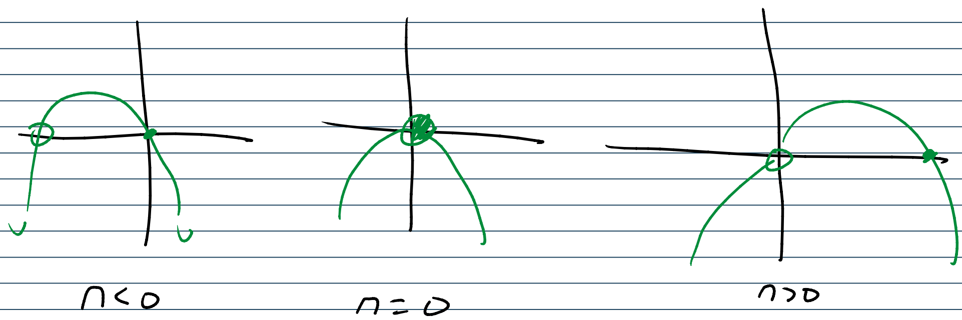
Prototype: $r + x^2$

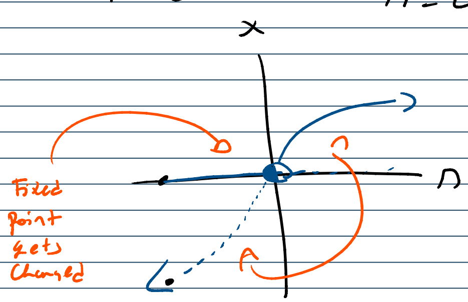




Transcritical: Fixed point isn't destroyed, but instead altered

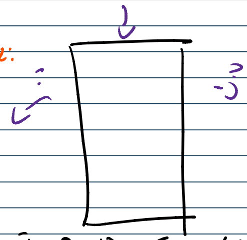
Prototype: $\dot{x} = r x - x^2$





Pitchfork: Looks like a pitchfork
shows a lot in "symmetric" problems

Ex:



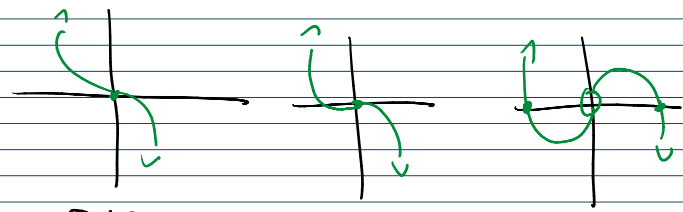
- As load increases on this column, column breaks in L or R w/ no preference

Invariance: $\dot{x} = f(x)$ if $f(x) = -f(-x)$

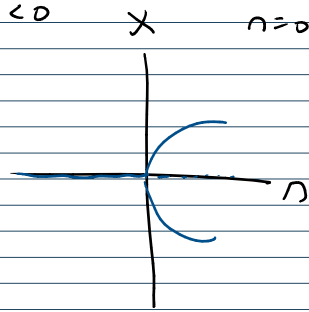
Ex: $\dot{x} = rx - x^3$ $\quad f(-x) = -rx + x^3 = -f(x)$

Implies $(-\dot{x}) = -\dot{x} = -f(x) \to (-\dot{x}) = f(-x)$

- Mathematical description of symmetry



$r<0$ $\quad$ $r=0$ $\quad$ $r>0$



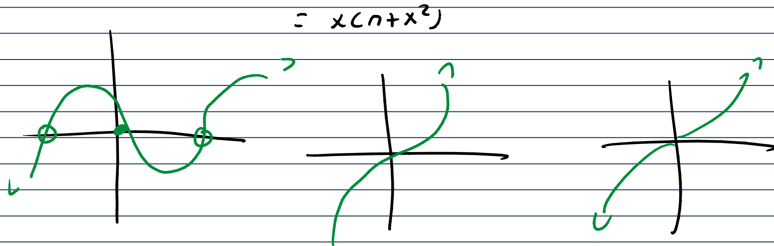
Supercritical: stable branches
Has a "restoring force"

$\dot{x} = rx - x^3$

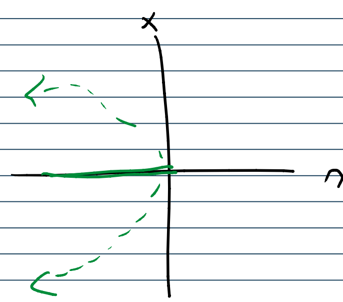
Brings $\dot{x}$ to 0 as x gets big.

Subcritical: Unstable branches, has a "destabilizing force"

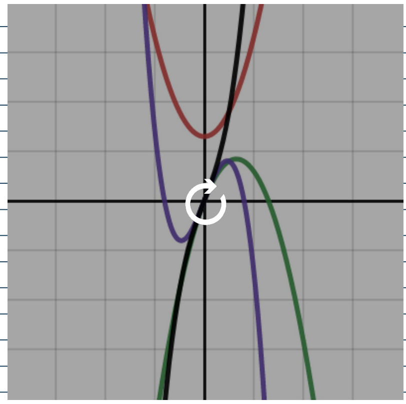
$\dot{x} = rx + x^3$
$= x(r + x^2)$



$r<0$ $\quad$ $r=0$ $\quad$ $r>0$



Desmos | Graphing C



https://www.desmos.com/calculator/zqh74zcihm

Theorem -

If the system

$$\dot{x} = f(x, r)$$

has a bifurcation at $(x, r) = (x^*, r^*)$ then

$$f(x^*, r^*) = 0 \quad \text{and} \quad \frac{\partial f}{\partial x}(x^*, r^*) = 0$$

Note: Can only be used to identify possible bifurcations

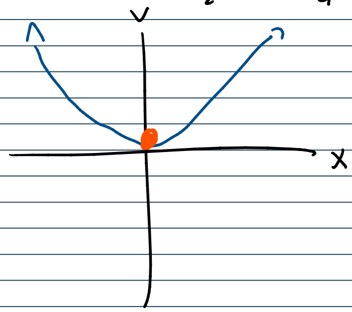
Satisfying this condition is not enough

Need to check if actually bifurcation by phase portrait

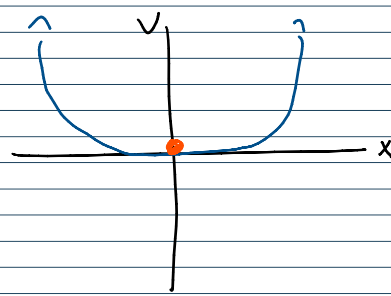
Potential Functions and Bifurcations

Ex: "Plot the potential $V(x)$ for the system $\dot{x} = rx - x^3$, for the cases $r<0$, $r=0$, $r>0$."

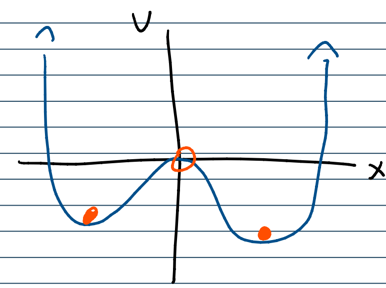
$$V(x) = -\frac{1}{2} r \cdot x^2 + \frac{1}{4} x^4$$



$r<0$



$r=0$

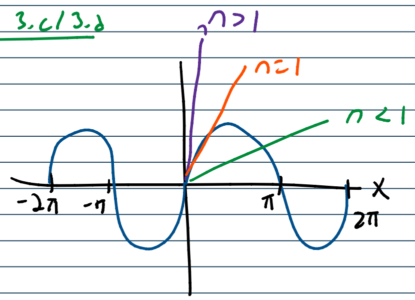


$r>0$

$-\frac{1}{2} r x^2 > 0$, so $V(x)$ is the sum of 2 positive terms

- Can factor into $x^2(-\frac{1}{2}r + x^2)$
- Roots at $x=0, \pm\sqrt{\frac{r}{2}}$

HW 3.c / 3.d



- When $r \geq 1$, $rx = \sin x$ only when $x=0$
- When $r<1$, $rx = \sin x$ at multiple points
  - For the non-zero fixed points, what happens with the stability?