Math 134, Spring 2022

Lecture #20: Linear systems

Friday May 13th

Learning objectives

Today we will discuss:

- What it means to say $\boldsymbol{x}^* = 0$ is a stable or unstable node of an uncoupled system.
- What it means to say $\boldsymbol{x}^* = 0$ is a saddle point of an uncoupled system.
- The stable and unstable manifolds associated to a saddle point of an uncoupled system.
- Classification of fixed points for linear systems with distinct real eigenvalues.

Linear systems

Uncoupled linear systems

We say that the linear system

$$\dot{\boldsymbol{x}} = A\boldsymbol{x}$$

is **uncoupled** if

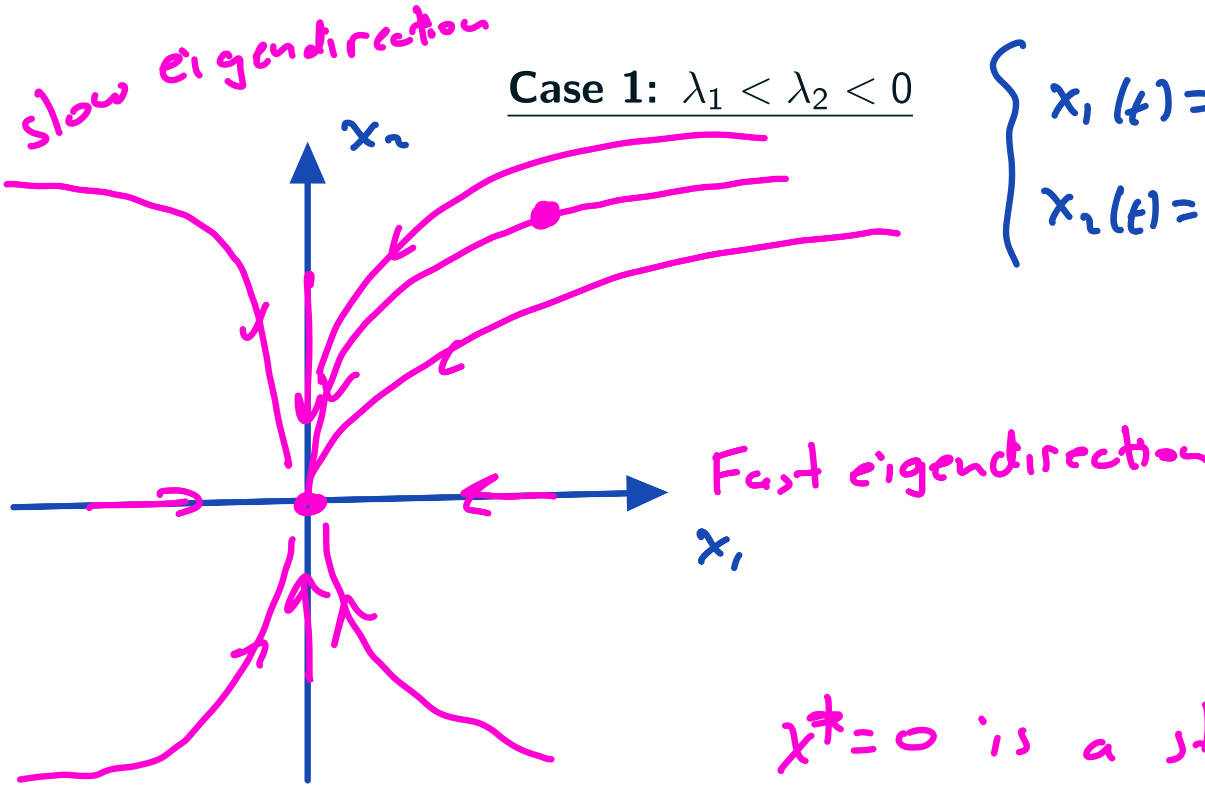
$$A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Assume that $\lambda_1 \neq \lambda_2$

$$\begin{cases} \dot{x}_1 = \lambda_1 x_1 \\ \dot{x}_2 = \lambda_2 x_2 \end{cases} \Rightarrow \begin{cases} x_1(t) = x_1(0) e^{\lambda_1 t} \\ x_2(t) = x_2(0) e^{\lambda_2 t} \end{cases}$$

Case 1: $\lambda_1 < \lambda_2 < 0$

$$\begin{cases} x_1(t) = x_1(0)\, e^{\lambda_1 t} \\ x_2(t) = x_2(0)\, e^{\lambda_2 t} \end{cases}$$

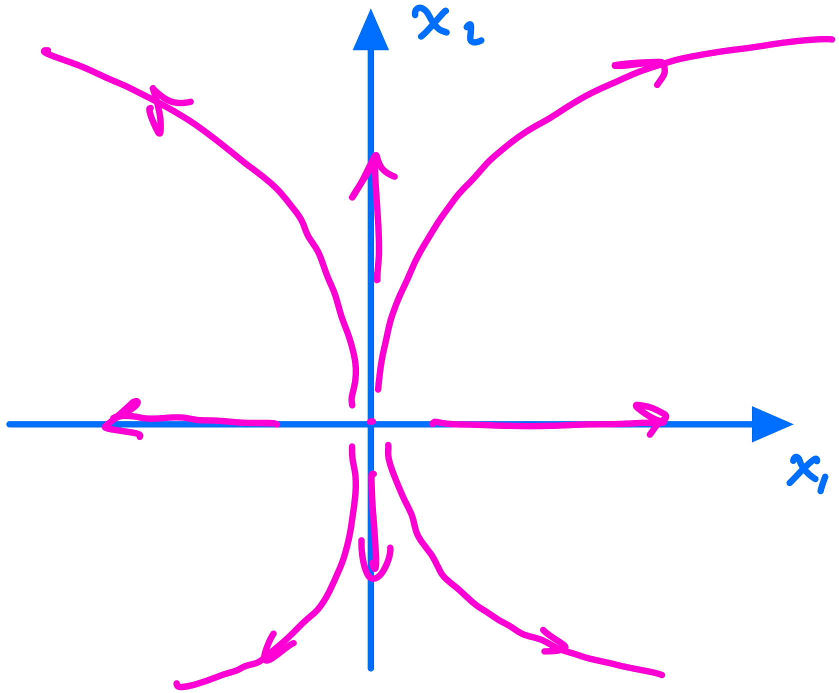


$x^* = 0$ is a stable node

Case 2: $\lambda_1 > \lambda_2 > 0$

$$\begin{cases} x_1(t) = x_1(0)\, e^{\lambda_1 t} \\ x_2(t) = x_2(0)\, e^{\lambda_2 t} \end{cases}$$

slow eigendirection



$\dot{\underline{x}} = A\, \underline{x}$

Fast eigendirection

$x^* = 0$ is an unstable node

Case 3: $\lambda_1 < 0 < \lambda_2$

Case 4: $\lambda_1 = 0$ and $\lambda_2 \neq 0$

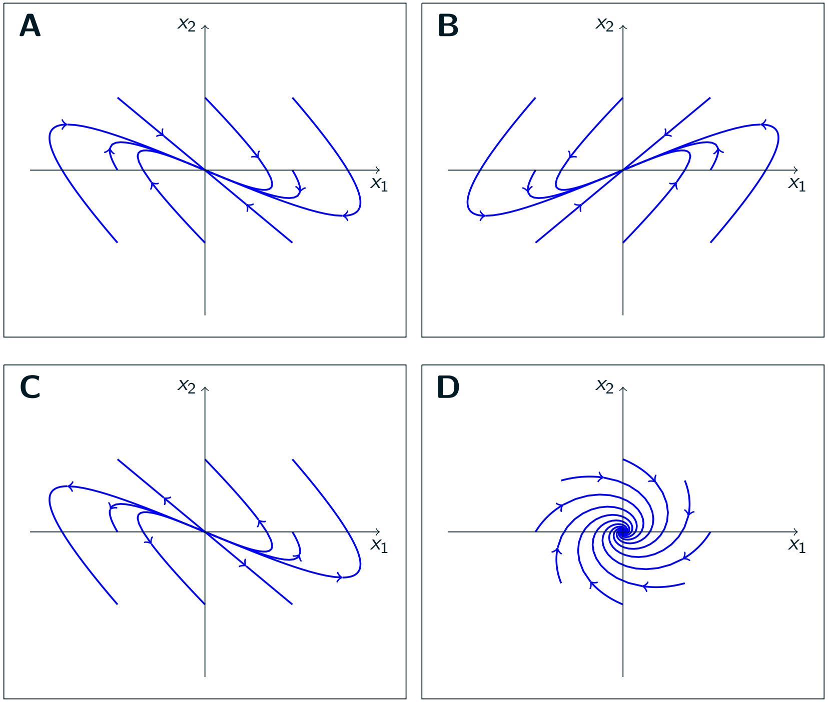
An example

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \boldsymbol{x}$$

An example

Which of the following phase portraits corresponds to the system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix} \boldsymbol{x}$$



See you next time!