Math 134 - Homework 7

1. (From Strogatz §6.3) For each of the following systems, find the fixed points, classify them, sketch the neighboring trajectories, and try to fill in the rest of the phase portrait.

(a)
$$\begin{cases} \dot{x} = x - y \\ \dot{y} = x^2 - 4 \end{cases}$$

(b)
$$\begin{cases} \dot{y} = x^2 - 4 \\ \dot{x} = y + x - x^3 \\ \dot{y} = -y \end{cases}$$

(c)
$$\begin{cases} \dot{x} = xy - 1 \\ \dot{y} = x - y^3 \end{cases}$$

2. (Strogatz Exercise 6.3.9) Consider the system

$$\begin{cases} \dot{x} = y^3 - 4x \\ \dot{y} = y^3 - y - 3x \end{cases}$$

- (a) Find all the fixed points and classify them.
- (b) Show that the line x = y is invariant, i.e., any trajectory that starts on it stays on it.
- (c) Show that $|x(t) y(t)| \to 0$ as $t \to \infty$ for all other trajectories.
 - (<u>Hint:</u> Form a differential equation for x y.)
- (d) Sketch the phase portrait.
- 3. Question 6.3.12
- 4. Question 6.3.13
- 5. Question 6.3.14
- 6. Question 6.3.15