Math 135 Ordinary Differential Equations

Homework 4

April 22, 2022

Laplace Transforms:

1. Section 50: Problem 5.
2. Section 50: Problem 6.
3. Section 51: Problem 1.
4. Section 52: Problem 2a.
5. Section 52: Problem 5.
6. Prove the following:
   (a) $(f * g)(t) = (g * f)(t)$.
   (b) If f and g are piecewise continuous and of exponential order on $[0, \infty)$, then $(f * g)(t)$ is of exponential order on $[0, \infty)$.
7. Prove the second translation theorem (in time): If $F(s) = L\{f(t)\}(s)$, then
$$L\{u_a(t)f(t-a)\}(s) = e^{-as}F(s) \qquad (a \geq 0).$$
   Here $u_a(t)$ is the unit step function defined as $u_a(t) = 1$, if $t \geq a$, and $= 0$ if $t < a$.
8. Solve the following IVP using the Laplace transform method:
$$y'' - y = t - 2$$
   with $y(2) = 3$ and $y'(2) = 0$.