Math 135 Ordinary Differential Equations

Homework 1

1. Section 17, Problem 5(b)
Important! We will revisit this type of non-autonomous ODE, called Euler's equidimensional equation, in the PDE section. Also, use the textbook hint to solve this.

2. Write the following linear differential equations with constant coefficients in the form of a system of first order linear differential equations and solve:
(a) $\ddot{x} + \dot{x} - 2x = 0;\ x(0) = x_0,\ \dot{x}(0) = v_0$
(b) $\ddot{x} + x = 0;\ x(0) = x_0,\ \dot{x}(0) = v_0$
Compare the solution of $x(t)$ to the one obtained using the method in Section 17.

3. Show that the initial value problem

$$\dot{x} = |x|^{\frac{1}{2}}, \quad x(0) = 0$$

has four different solutions through the point $(0, 0)$. Sketch these solutions in the (t, x)-plane. Carefully check the domain of the solutions obtained.

4. Section 69, Problem 3. Plot the solutions. Irrespective of the chosen initial approximation, the Picard iterates seem to converge, why?
Hint: For (c), approximate $\cos x$ *by taking appropriate number of terms of its Taylor series.*

5. Section 71, Problem 1

6. (*Following problem will not be graded, solving will help undestand proof of Picard's theorem*)

   a. Give an example of a converging sequence of continuous functions whose limit is not continuous.

   b. State a condition that assures that the limiting function is continuous.

   c. Give a mathematical definition of the key term(s) used in b.