# Math 135 Ordinary Differential Equations

Homework 2

*Note: When the textbook says 'f satifies a Lipschitz condition', it implies that f is Lipschitz continuous in the dependent variable y, not the independent variable x.*

1. Section 70, Problem 3

2. Section 70, Problem 5

3. Determine if this function is Lipschitz continuous (specify the domain)

$$f(x) = \begin{bmatrix} -x_1 + x_1 x_2 \\ x_2 - x_1 x_2 \end{bmatrix}$$

4. Prove that, if $f, h : \mathbb{R} \to \mathbb{R}$ are locally Lipschitz over some bounded domain $D$, then $f + h$, $fh$, and $f \circ h$ are locally Lipschitz.

**Application of Picard's theorem:**

5. Section 70, Problem 6. (Note: the DE is $y' = |y|$.)

6. Section 70, Problem 7.

7. Does Picard's theorem apply to the following IVP, if yes, what does it imply? What is maximal interval of existence of the solution; you may find this by evaluating the explicit solution. Observe how the interval of existence changes with $y_0$.

$$\frac{dy}{dt} = ty^3, \; y(0) = y_0$$