

Math 135 - Lecture 2  
Spring 2022  
Midterm  
5/6/2022

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Name:

Student ID:

Please sign the following honor statement. **If you do not sign this, you will receive 0 points.**

*I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.*

Signed:

This exam contains 6 pages (including this cover page) and 7 problems. There are a total of 100 points available.

- Attempt all questions.
  - You may use additional blank pages as required.
  - Please write legible.
  - You can use a cheat sheet (both pages).
  - You can use a simple calculator (i.e. with no plotting capabilities).
  - For those of you taking your test through Zoom: **Posting problems to online forums or “tutoring” websites counts as interaction with another person so it is strictly forbidden.**
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1. (15 points) Given the piecewise continuous function

$$f(x) = \begin{cases} 2 & \text{if } 0 \leq x < 1, \\ 1 & \text{if } x \geq 1, \end{cases}$$

compute its Laplace Transform.

2. (15 points) Compute the Laplace Transform of  $f(x) = x \sin(3x)$ .

3. (15 points) Let us consider a function  $f \in C^3(\mathbb{R})$  such that  $f'''$  is of exponential order  $\alpha$  and  $f(0) = 1$ ,  $f'(0) = 0$ ,  $f''(0) = 1$ . Compute  $\mathcal{L}\{f'''(t)\}(s)$ .

4. (5 points) Show that the function  $h(x) = |x|^\alpha$  where  $x \in [-1, 1]$  is not locally Lipschitz continuous at  $x = 0$ , for any  $\alpha \in (0, 1)$ .

5. (15 points) Solve the initial value problem

$$\frac{dx}{dt} + 50x = 5,$$

with  $x(0) = 0$ , using the Laplace Transform method.

6. (20 points) Consider the equation IVP

$$\begin{cases} \frac{dx}{dt} = \sin(x) \\ x(0) = 1. \end{cases}$$

- (a) Show that there exists a unique solution to this problem. Justify your answer.
- (b) Write down the integral equation equivalent to this problem.
- (c) Compute the first two Picard iterations.

7. (15 points) Consider the following Integral equation

$$y(x) = e^x + \int_0^x e^{x-t} y(t) dt .$$

Solve it using the Laplace Transform.