UCLA MATH 135, WINTER 2022, MIDTERM EXAM 1

Students MUST COPY AND SIGN the following honor pledge AT THE TOP of the paper submitted for their exam solutions or else receive a failing grade by department policy:

I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this exam.

Print name: Sign name:

This is an open-book and open-note examination. Please show all your work. Partial credit will be given to partial answers. There are 1+4 = 5 problems for a total of 40 points. Test is designed to be completed in 1 hour. Posted: January 28, 8:00am. To be completed and uploaded by January 29, 7:59am (Pacific standard times). Between these times, as much time as desired is allowed to work on the exam.

Please note that there is a table of common Laplace transforms that is posted on Canvas along with this exam. These may be of use for the problems below.

[0] ∞ *points*:

Did you sign the honor pledge? If not, turn back right now to the cover page and sign it.

[1] *Short answer questions, 2 points each for 10 points total*:

(a) *True or false?* If $L[f(x)] = F(p)$, then

$$L[f(\alpha x)] = \alpha F\left(\frac{p}{\alpha}\right)$$

for $\alpha > 0$. You do not need to justify your answer.

(b) *True or false?* For all $x \geq 1$, the function $f(x) = \exp(\sqrt{x}/2)$ is of exponential order. You do not need to justify your answer.

(c) Compute the Laplace transform $L[f]$ for $f(x) = x\,\sin(3x)$.

(d) Suppose $h \in C^3$. What is the Laplace transform of

$$\frac{d^3h}{dx^3}\,?$$

(e) Suppose that $r = \pm 2i$ are the roots of the characteristic equation. What is the differential equation this corresponds to, and what is the general solution?

[2] *10 points*: Suppose that $\exists M, c > 0$ such that

$$|f(x)| \leq M e^{cx}, \qquad \forall x$$

i.e. that f is of exponential order on the real line. Show that

$$\lim_{p \to \infty} \left| p \, F(p) \right| < \infty$$

must be finite, where $F(p)$ denotes the Laplace transform of f.

[3] *10 points*: Compute the Laplace transform of the square-wave function $s(x)$ given by

$$s(x) = \begin{cases} 1 & x \in [0, \pi) \cup [2\pi, 3\pi) \cup [4\pi, 5\pi) \cup \dots \\ 0 & x \in [\pi, 2\pi) \cup [3\pi, 4\pi) \cup [5\pi, 6\pi) \cup \dots \end{cases}$$

[4] *10 points*: In class we loosely defined the Dirac delta distribution $\delta(x)$ to be the 'function' that was infinite at $x = 0$ and zero everywhere else, and we took as a more formal definition $\delta(x)$ to be the function such that

$$L[\delta(x)] = 1.$$

Compute the solution to the IVP given by

$$\begin{cases} y''(x) + 25y(x) = f(x) \\ y(0) = 1, y'(0) = 0 \end{cases}$$

where $f(x) = \delta(x - 3)$.