Math 135-Lec1- Spring 2022

Lecture 1, March 28th

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→ Syllabus

Why do we want to study Differential Equations?

Because they are useful for modeling a big range of phenomena appearing in nature and technological problems

Problem: How does the water temperature of a hot tea evolve?

Experimental data

Model: set a DE

Scientific computing: Solve the DE numerically

Note: we will solve this problem later

Problem

Experimental data

Model

Scientific computing

SIR

- Oncology
- Chemistry
- Biology
- Phisics

Ordinary Differential Equations (ODE)

An ODE is an equation involving an unknown function of a single variable together with one or more of its derivatives.

Example: Find a function $y = y(t)$ such that

$$\frac{dy(t)}{dt} = y(t)$$

unknown function $y(t)$

dependent variable

independent variable

Notation: $\frac{dx(t)}{dt} = \dot{x}(t) = x'(t)$

The order of an ODE is the highest derivative

Examples: Exercise for students

	unknown function	ind. variable	order
$\frac{dy}{dx} = x^2$	y	x	1
$\frac{dy}{dt} = ry$ r const	y	t	1
$\ddot{x} + 5\dot{x} + 2x = 5$	x	t	2
$\dddot{x} + \ddot{x} + x = 1$	x	t	3
$(\dot{x})^2 + x = 1$	x	t	1

* $$\frac{dP}{dt} = kP\left(1-\frac{P}{M}\right)$$ logistic equation

k growth factor

M carrying capacity

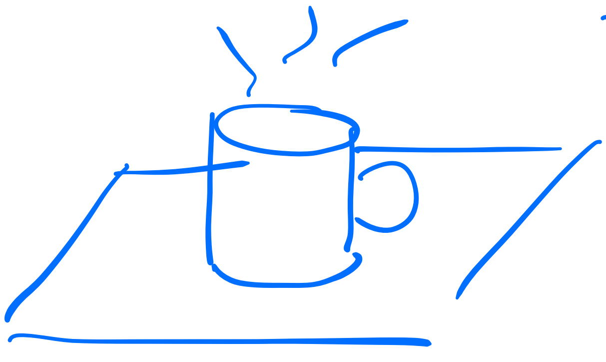
$P(t)$ # of individuals at time t

* $$\frac{dT}{dt} = -k(T-T_s)$$ Newton's law of cooling

k thermal conductivity

T_s room temperature

$T(t)$ temperature at time t



$T_s = 20°C$

$100°C \to 20°C$

The equation

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2}$$

$w = w(x, t)$

$x \in \mathbb{R}$, $t \geq$

is not an ODE since the unknown function w depends upon x and t.

This is an example of a partial DE.

Exercise:

Show that $y(t) := ce^{-t^2}$ is a sol of the first order ODE

$$y' = -2ty$$

where c is any arbitrary real number ($c \in \mathbb{R}$)

Sol:

$$\frac{dy}{dt} = ce^{-t^2}(-2t)$$

$$-2ty = -2tce^{-t}$$

So $y(t) = ce^{-t^2}$ is a solution of $y' = -2ty$.

Question: How many sol. does $y' = -2ty$ have?

∞

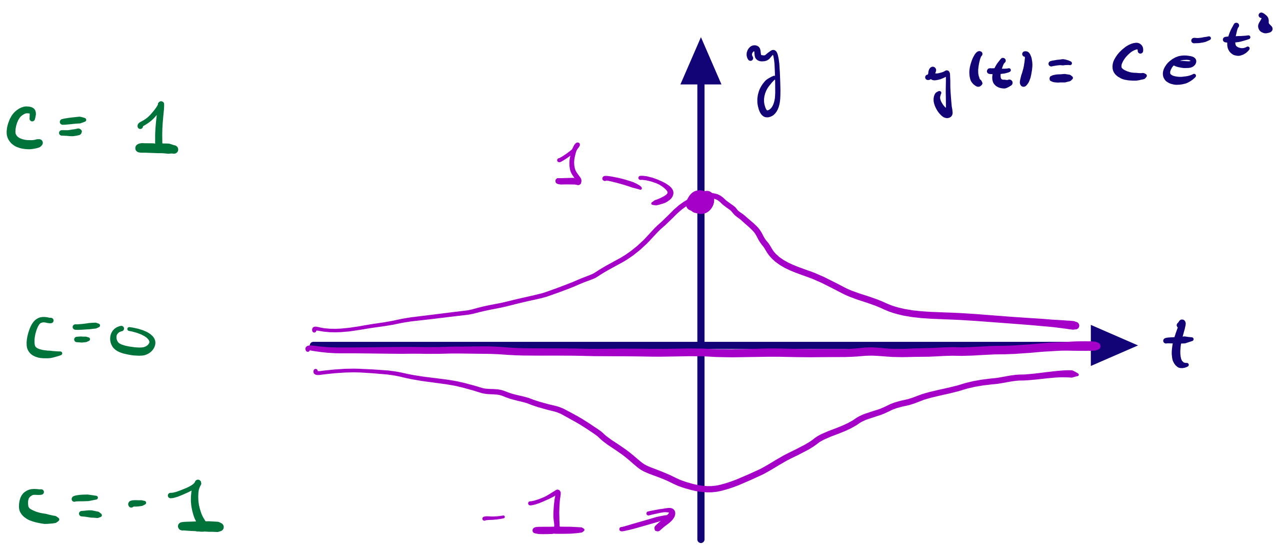
- Every solution to

$$y' = -2ty$$

is of the form $y(t) = Ce^{-t^2}$

- The formula $y(t) = Ce^{-t^2}$ is called general solution to $y' = -2ty$

- The graph of these solutions are called solution curves



Initial Value Problem

Example: Show that the ODE

$$y' = y^2$$

has infinitely many sol.

Find one that satisfies $y(0) = 1$

Sol. It is easy to show that

$$y(t) = -\frac{1}{t-c}$$

is a sol. for any $c \in \mathbb{R}$.

Since $y(0) = 1$

$$-\frac{1}{0-c} = 1$$

$\rightarrow \quad c = 1$

$\rightarrow \quad y(t) = -\dfrac{1}{t-1}$ is the unique sol. satisfying $y(0)=1$

Interval of existence

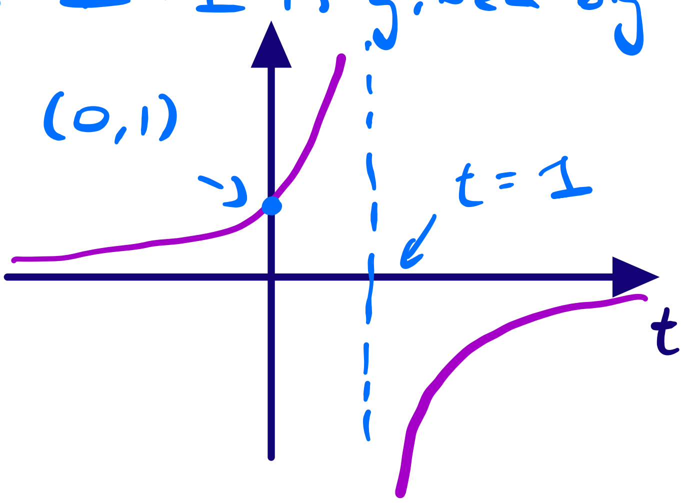
The interval of existence of a sol. to an ODE is defined to be the largest interval over which the sol. can be defined and remain a sol.

Example: Find the interval of existence for the sol. to the IVP

$$\begin{cases} y' = y^2 \\ y(0) = 1 \end{cases}$$

The sol. to this IVP is given by

$$y(t) = -\frac{1}{t-1}$$



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There are three techniques to solve ODEs:

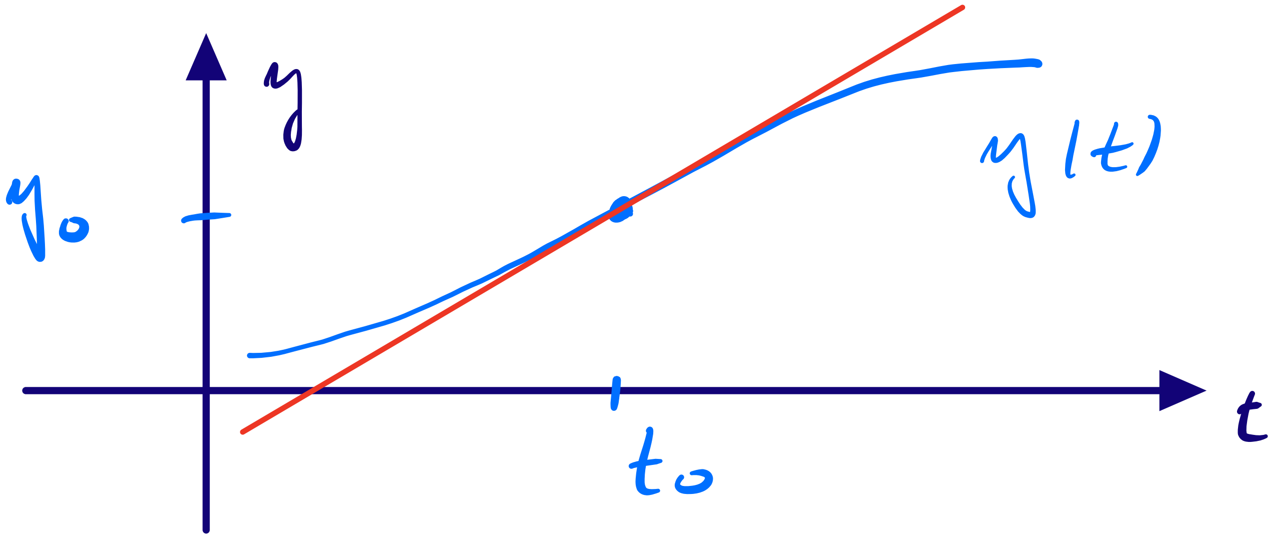
1. Analitic methods (quantitative)
2. Geometric methods (qualitative)
3. Numerical methods (quantita.)

Geometric interpretation

Consider the ODE

$$y' = f(t, y)$$

Let $y = y(t)$ be a sol. and recall that the graph of the function is called a solution curve.



$$y'(t_0) = f(t_0, y_0)$$

Equilibrium point

A point $y_{eq} \in \mathbb{R}$ is called an equilibrium point of the ODE

$$y' = f(y,t)$$

if $f(y_{eq}, t) = 0$ for all t

Example: Find the equilibrium points of

$$y' = y(1-y)$$

Sol.

$$\begin{cases} y = 0 \\ y = 1 \end{cases}$$