Project-3

<u>Please submit all work including computer program</u>.

1. (40 points) Compute the stochastic Taylor-Green system below to model particle motion in the ocean and atmosphere:

$$d\,p = -\sin(q)\,dt + \sigma\,d\,W_1\,,\quad d\,q = \sin(p)\,dt + \sigma\,d\,W_2$$

over the time interval [0, T], T=5000, with initial data (p, q)(0) = (0,0), by the following methods at time step h=0.05. Calculate effective diffusivity:

$$D_E := \lim_{t\to\infty} E[\,p^2(t)\,]/(2\,t) \approx E[\,p^2(T)]/(2T)$$

where E[●] is approximated by empirical mean over 5000 independent realizations, at σ = 1.e-1, 1.e-2, 1.e-3, 1.e-4, 1.e-5, 1.e-6.

a) (17 pts) Euler-Maruyama (EM) method:

$$p_{n+1} = p_n - h\sin(q_n) + \sigma\,\Delta W_1$$
$$q_{n+1} = q_n + h\sin(p_n) + \sigma\,\Delta W_2$$

b) (17 pts) Symplectic Splitting (SS) method:

$$p_{n+1} = p_n - h\sin(q_n) + \sigma\,\Delta W_1$$
$$q_{n+1} = q_n + h\sin(p_n - h\sin(q_n)) + \sigma\,\Delta W_2$$

where $\Delta W_1, \Delta W_2$ are independent random variables of the form $h^{1/2}\,N(0,1)$.

c) (6 pts) Plot D_E vs. σ on log-log scale, and compare with theoretical result $D_E = O(\sigma)$, or $\log D_E = \log\sigma$ + const. Comment on the methods above. Note that without the drift velocity (or currents in the ocean or atmosphere), $p = \sigma W_1$, and $D_E = \sigma^2/2$.