Project-2

<u>Please submit all work including computer program</u>.

1. (20 points) Compute the pendulum equation:

   dp/dt = - sin q ,   dq/dt = p

   over the time interval [0, 20] with initial data (p,q)(0) = (0,2), by the following multi-step methods (5 points each) at time step h=0.08. Initialize with modified Euler's method. Plot solutions on the (q, p) phase plane, and the energy:

   $E = p^2/2 + (1 - \cos q)$ in time along the solutions.

(A) $$y_{n+4} - 2y_{n+3} + y_{n+2} = h^2\left(\frac{7}{6}f_{n+3} - \frac{5}{12}f_{n+2} + \frac{1}{3}f_{n+1} - \frac{1}{12}f_n\right)$$

(B) $$y_{n+4} - 2y_{n+2} + y_n = h^2\left(\frac{4}{3}f_{n+3} + \frac{4}{3}f_{n+2} + \frac{4}{3}f_{n+1}\right)$$

(C) $$y_{n+4} - 2y_{n+3} + 2y_{n+2} - 2y_{n+1} + y_n = h^2\left(\frac{7}{6}f_{n+3} - \frac{1}{3}f_{n+2} + \frac{7}{6}f_{n+1}\right)$$

d) (5 pts) Comment on the methods above, increase the time interval or reduce the time step to observe more if necessary.

2. (20 points) Compute the ODE model:

$$dy/dt = y^2 - y^3$$
$$y(0) = \delta, \quad t \le 2/\delta.$$

Take δ = 1.e-5, h=0.01, and

a) (5 pts) Implicit midpoint method: $y_{n+1} = y_n + h\, f(y_n/2 + y_{n+1}/2)$

b) (5 pts) Trapezoidal method: $y_{n+1} = y_n + \frac{1}{2} h\,(f(y_n) + f(y_{n+1}))$

c) (5 pts) BDF method: $y_{n+1} = 4/3\, y_n - 1/3\, y_{n-1} + 2h/3\, f(y_{n+1})$

d) (5 pts) Plot and compare the behavior near steady state y =1, adjust h if necessary.