PROJECT 3
MATH 228, WINTER 2023, PROF. KNUT SOLNA

PROBLEM 1 (BINOMIAL MODEL AND OPTION PRICING)

Program a Python function that finds the price of a European call option in the binomial model. The input to the function is the parameters discussed in class $r, u, d, S(0), K, T$ and in addition the numbers of level n of the binomial model.

For $r = 0.05, u = 1.05, d = 0.95, K = 1, T = 2, n = 10$ plot the initial price P_0 as function of the initial value of the underlying $S(0)$ when the payoff maturity T indeed is that of a European call $h(S(T)) = \max(S(T) - K, 0)$.

PROBLEM 2 (VOLATILITY ESTIMATE)

For one of the data sets provided (you choose which), Russel 2000 RUT.csv or Bitcoin BTC-USD.csv, use daily closing prices *Adj Close*, compute an estimate of volatility for a) the whole time series b) when the estimate is based on the observation in a rolling window of one 'trading month' length (21 days) c) one estimate for each 'quarter' (63 trading days). Plot this estimate as a function of the midpoint of the window and quarter respectively. Plot also the returns time series and the price path itself. The volatility estimates should be in annualized units.

PROBLEM 3 (GAINS PROCESSES AND MARTINGALES)

Consider the two (discrete time) gains processes

$$M_n^{(1)} = \sum_{i=0}^{n-1} \zeta_i^{(1)} (B_{t_{i+1}} - B_{t_i}),$$
$$M_n^{(2)} = \sum_{i=0}^{n-1} \zeta_i^{(2)} (W_{t_{i+1}} - W_{t_i}),$$

$n = 1, 2 \cdots$ and $t_{i+1} - t_i = \Delta t; t_0 = 0$. For B, W being two standard Brownian motions with respect to the filtration $\mathcal{F}_{t_i}$ and with correlation coefficient ρ, moreover, with $\zeta_i^{(j)}, j = 1, 2$ being $\mathcal{F}_{t_i}$ measurable.

a) Prove that the following process is a martingale

$$M_n^{(1)} M_n^{(2)} - < M^{(1)}, M^{(2)} >_n$$
$$< M^{(1)}, M^{(2)} >_n = \sum_{i=0}^{n-1} \zeta_i^{(1)} \zeta_i^{(2)} \rho \Delta t.$$

What is the process $< M^{(1)}, M^{(2)} >$ called?

b) Assume that $\zeta_i^{(1)} = 1$, $\zeta_i^{(2)} = 2$ $\rho = 0.5$ and $\Delta t = 0.1$ and simulate realizations of the two processes and plot one example of a realization of the two gains processes.

c) Compute the correlation coefficient

$$\frac{\mathbb{E}[M_n^{(1)} M_n^{(2)}]}{\sqrt{\mathbb{E}[(M_n^{(1)})^2]\mathbb{E}[(M_n^{(2)})^2]}},$$

theoretically and verify your results by estimating the correlation coefficient from simulated gains processes.

Problem 4 (Extra Credit; Price Prediction)

For the data above (Russel 2000 or Bitcoin) prices an estimate of tomorrows price is today's price. Argue why this may or may not be be a good estimate. Compute an estimate of the relative error for this estimate for the last quarter (last 62 days), again you can choose either data set. How does this estimate relate to the volatility estimate in Problem 2? Can do you better (see suggestions in Workout2 or your experience from Project 1)