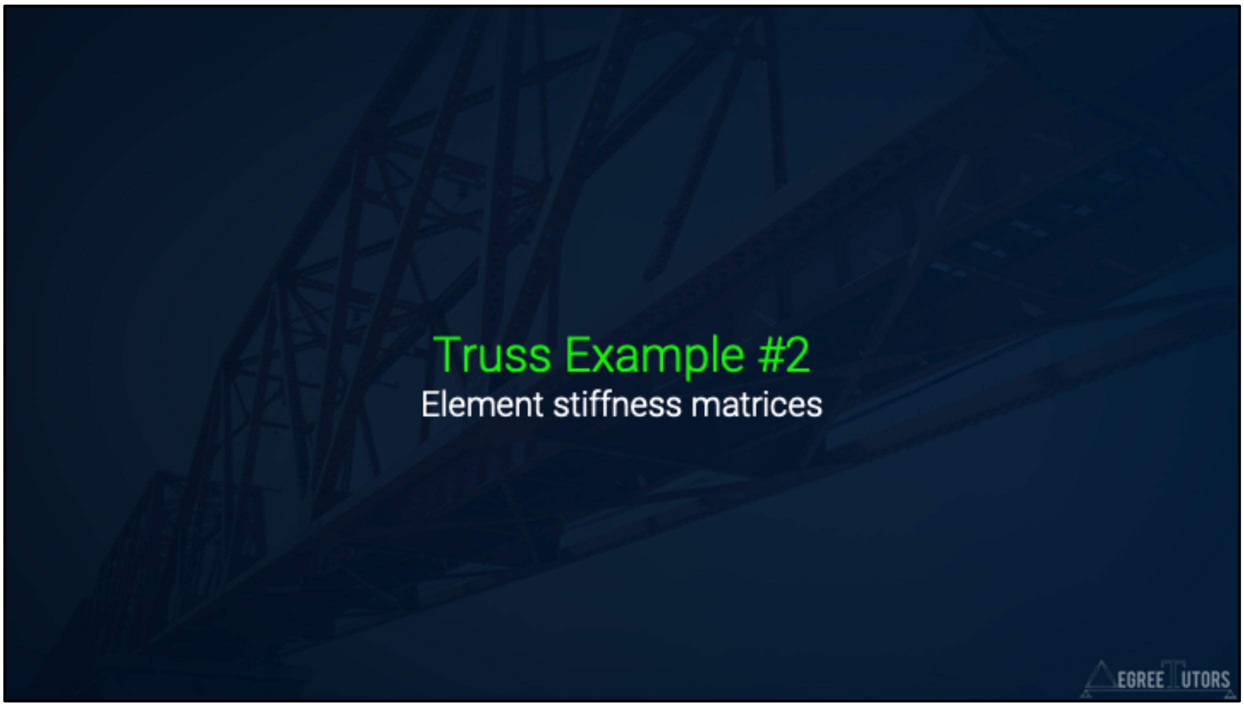
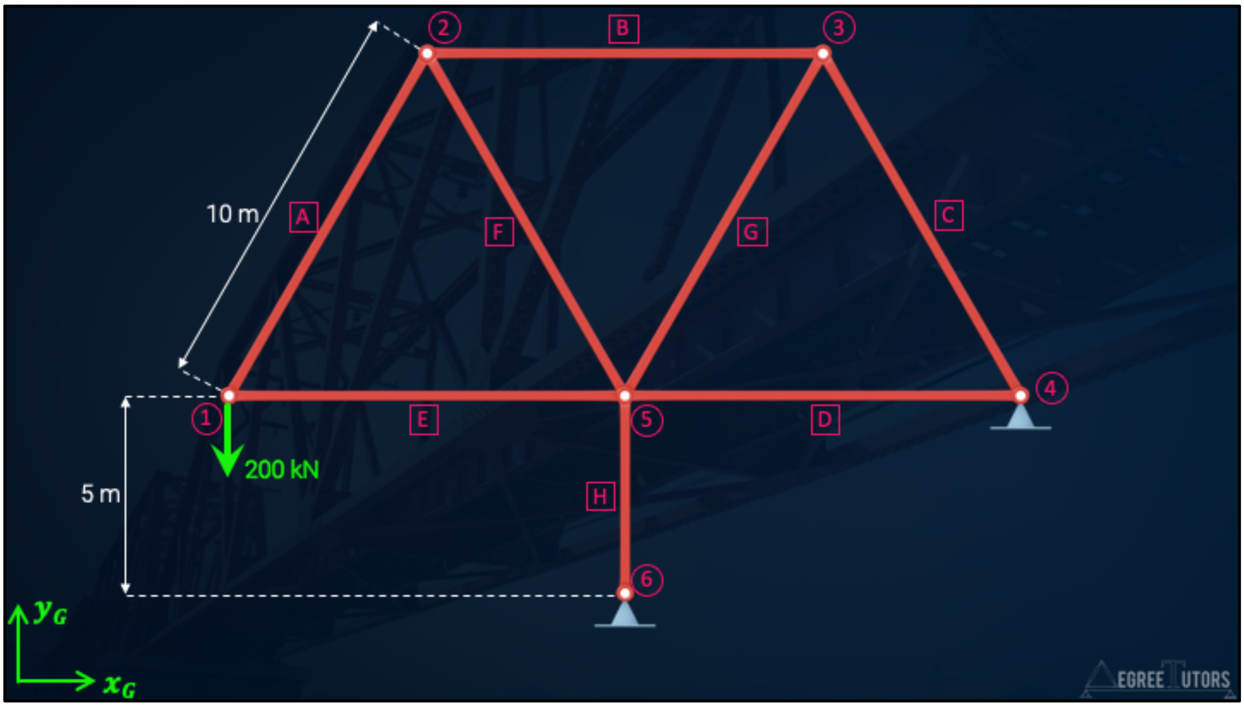

Truss Example #2
Element stiffness matrices



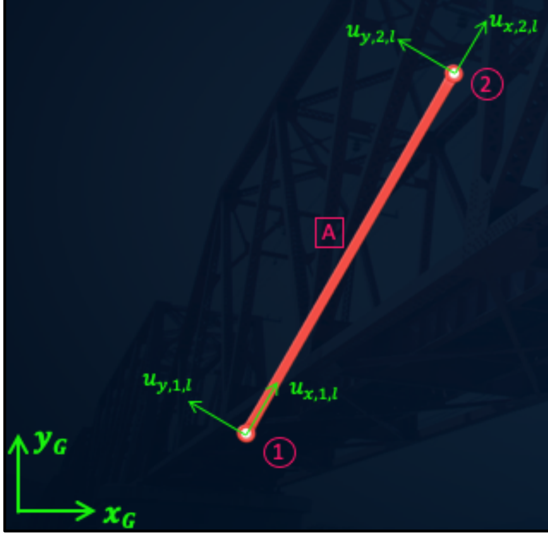
10 m
5 m
200 kN
1
2
3
4
5
6
A
B
C
D
E
F
G
H
y_G
x_G
DEGREE TUTORS

Member A

Measure all angles **anticlockwise** from the **global positive x-axis** with **node i at the origin** where $i < j$

$L_A = 10\ m,$

$\theta = 1.047\ rads\ (60°)$



$$K_{A,G} = \frac{EA}{L_A}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

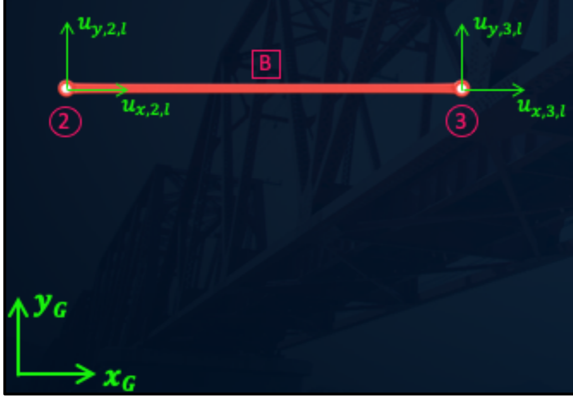
$$K_{A,G} = EA\begin{bmatrix} 0.025 & 0.043 & -0.025 & -0.043 \\ 0.043 & 0.075 & -0.043 & -0.075 \\ -0.025 & -0.043 & 0.025 & 0.043 \\ -0.043 & -0.075 & 0.043 & 0.075 \end{bmatrix}$$

Member B

Measure all angles anticlockwise from the global positive x-axis with node i at the origin where $i < j$

$L_B = 10\ m,$

$\theta = 0\ rads\ (0°)$



$$K_{B,G} = \frac{EA}{L_B}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

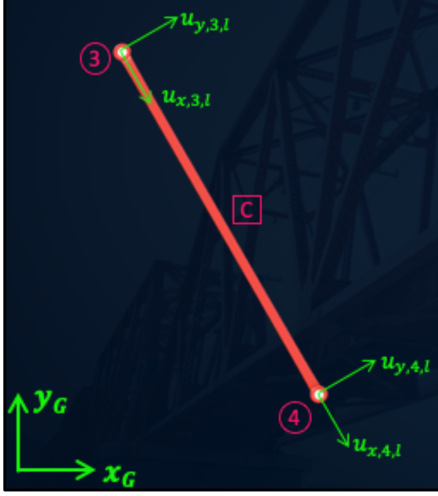
$$K_{B,G} = EA\begin{bmatrix} 0.1 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Member C

Measure all angles **anticlockwise** from the **global positive x-axis** with **node i at the origin** where $i < j$

$$L_C = 10\ m,$$

$$\theta = 5.236\ rads\ (300°)$$



$$K_{C,G} = \frac{EA}{L_C}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$K_{C,G} = EA\begin{bmatrix} 0.025 & -0.043 & -0.025 & 0.043 \\ -0.043 & 0.075 & 0.043 & -0.075 \\ -0.025 & 0.043 & 0.025 & -0.043 \\ 0.043 & -0.075 & -0.043 & 0.075 \end{bmatrix}$$

Member D

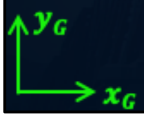
Measure all angles **anticlockwise** from the **global positive x-axis** with **node i at the origin** where $i < j$

$L_D = 10\ m,$

$\theta = 3.142\ rads\ (180°)$

$$K_{D,G} = \frac{EA}{L_D}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$K_{D,G} = EA\begin{bmatrix} 0.1 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



Member E

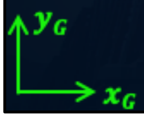
Measure all angles anticlockwise from the global positive x-axis with node i at the origin where $i < j$

$L_E = 10\ m,$

$\theta = 0\ rads\ (0°)$

$$K_{E,G} = \frac{EA}{L_E}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

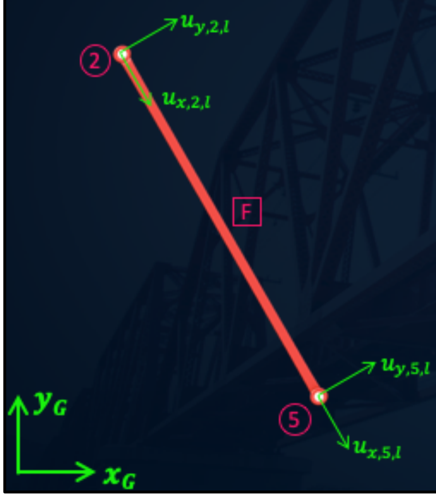
$$K_{E,G} = EA\begin{bmatrix} 0.1 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



Member F

Measure all angles anticlockwise from the global positive x-axis with node i at the origin where $i < j$

$$L_F = 10\,m,$$
$$\theta = 5.236\,rads\ (300°)$$

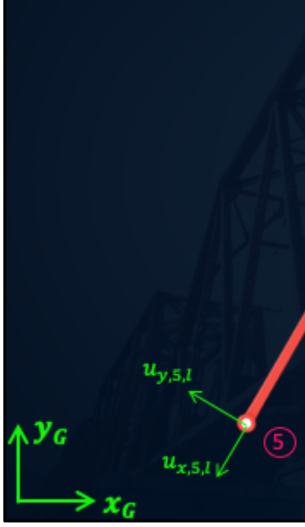


$$K_{F,G} = \frac{EA}{L_F}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$K_{F,G} = EA\begin{bmatrix} 0.025 & -0.043 & -0.025 & 0.043 \\ -0.043 & 0.075 & 0.043 & -0.075 \\ -0.025 & 0.043 & 0.025 & -0.043 \\ 0.043 & -0.075 & -0.043 & 0.075 \end{bmatrix}$$

Member G

Measure all angles anticlockwise from the global positive x-axis with node i at the origin where $i < j$

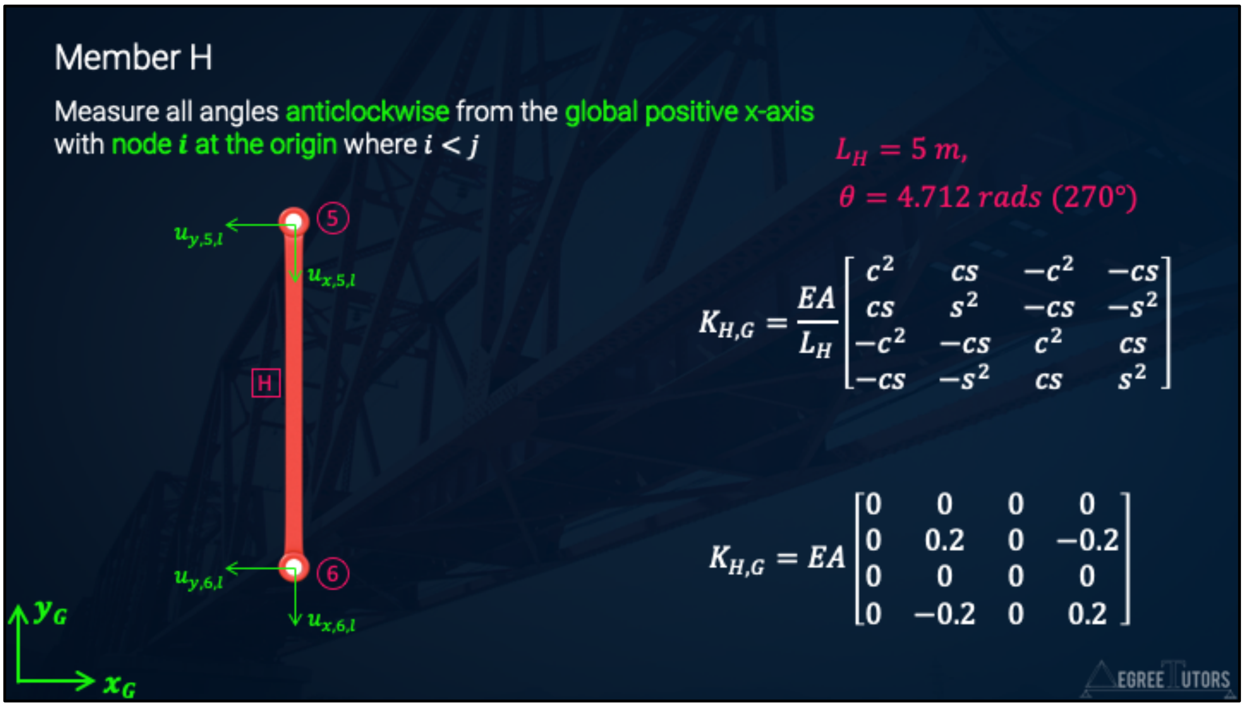


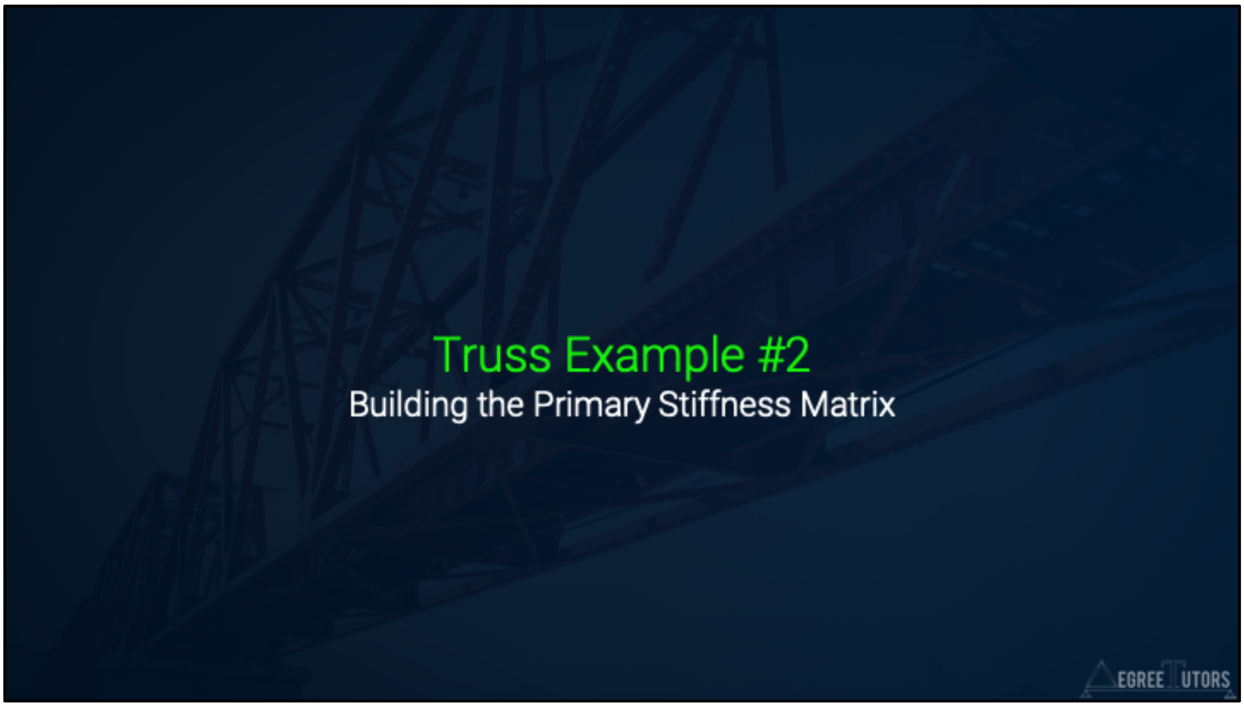
$L_G = 10\ m,$

$\theta = 4.189\ rads\ (240°)$

$$K_{G,G} = \frac{EA}{L_G}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$K_{G,G} = EA\begin{bmatrix} 0.025 & 0.043 & -0.025 & -0.043 \\ 0.043 & 0.075 & -0.043 & -0.075 \\ -0.025 & -0.043 & 0.025 & 0.043 \\ -0.043 & -0.075 & 0.043 & 0.075 \end{bmatrix}$$


Member H
Measure all angles anticlockwise from the global positive x-axis with node i at the origin where $i < j$
$L_H = 5\ m,$
$\theta = 4.712\ rads\ (270°)$
5
6
H
$u_{y,5,l}$
$u_{x,5,l}$
$u_{y,6,l}$
$u_{x,6,l}$
y_G
x_G
$K_{H,G} = \frac{EA}{L_H}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$
$K_{H,G} = EA\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & -0.2 \\ 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & 0.2 \end{bmatrix}$
EGREE UTORS


Truss Example #2
Building the Primary Stiffness Matrix
DEGREE TUTORS

Combine element stiffness matrices into a primary stiffness matrix

Remember, each element stiffness matrix is divided into 4 quadrants...

$$K_{A,G} = EA\begin{bmatrix} \boxed{\begin{matrix} 0.025 & 0.043 \\ 0.043 & 0.075 \end{matrix}} & \boxed{\begin{matrix} -0.025 & -0.043 \\ -0.043 & -0.075 \end{matrix}} \\ \boxed{\begin{matrix} -0.025 & -0.043 \\ -0.043 & -0.075 \end{matrix}} & \boxed{\begin{matrix} 0.025 & 0.043 \\ 0.043 & 0.075 \end{matrix}} \end{bmatrix}$$

$$K_{A,G} = EA\begin{bmatrix} \boxed{K_A11} & \boxed{K_A12} \\ \boxed{K_A21} & \boxed{K_A22} \end{bmatrix}$$

$$K_{B,G} = EA\begin{bmatrix} K_B11 & K_B12 \\ K_B21 & K_B22 \end{bmatrix}$$

$$K_{C,G} = EA\begin{bmatrix} K_C11 & K_C12 \\ K_C21 & K_C22 \end{bmatrix}$$

$$K_{D,G} = EA\begin{bmatrix} K_D11 & K_D12 \\ K_D21 & K_D22 \end{bmatrix}$$

$$K_{E,G} = EA\begin{bmatrix} K_E11 & K_E12 \\ K_E21 & K_E22 \end{bmatrix}$$

$$K_{F,G} = EA\begin{bmatrix} K_F11 & K_F12 \\ K_F21 & K_F22 \end{bmatrix}$$

$$K_{G,G} = EA\begin{bmatrix} K_G11 & K_G12 \\ K_G21 & K_G22 \end{bmatrix}$$

$$K_{H,G} = EA\begin{bmatrix} K_H11 & K_H12 \\ K_H21 & K_H22 \end{bmatrix}$$

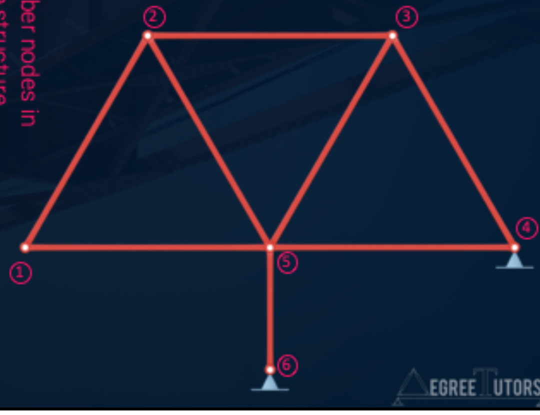
Set up the primary stiffness matrix template

Number nodes in the structure

$$
K_P = \begin{array}{c c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} & \\
\begin{bmatrix}
k11 & k12 & k13 & k14 & k15 & k16 \\
k21 & k22 & k23 & k24 & k25 & k26 \\
k31 & k32 & k33 & k34 & k35 & k36 \\
k41 & k42 & k43 & k44 & k45 & k46 \\
k51 & k52 & k53 & k54 & k55 & k56 \\
k61 & k62 & k63 & k64 & k65 & k66
\end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\end{array}
$$

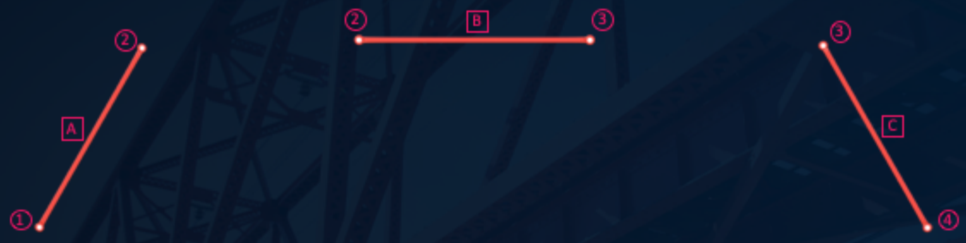
Number nodes in the structure

Each matrix element k represents a 2x2 matrix – so the template expands to become 12x12 when filled



DEGREE TUTORS

Fill in the template with the quadrants of the element stiffness matrices



$$K_P = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & & & & \end{array} \\ \begin{bmatrix} K_A11 & K_A12 & 0 & 0 & 0 & 0 \\ K_A21 & K_A22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{0} & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ \\ \\ \\ \\ \end{array}$$

$$\boxed{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$K_P = \begin{array}{c} \begin{array}{cccccc} & 2 & 3 & & & \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_B11 & K_B12 & 0 & 0 & 0 \\ 0 & K_B21 & K_B22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ 2 \\ 3 \\ \\ \\ \\ \end{array}$$

$$K_P = \begin{array}{c} \begin{array}{cccccc} & & 3 & 4 & & \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_C11 & K_C12 & 0 & 0 \\ 0 & 0 & K_C21 & K_C22 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \\ 3 \\ 4 \\ \\ \\ \end{array}$$

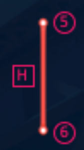
Fill in the template with the quadrants of the element stiffness matrices

$$K_P = \begin{array}{cc} \begin{matrix} & & & 4 & 5 & \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_D11 & K_D12 & 0 \\ 0 & 0 & 0 & K_D21 & K_D22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} \\ \\ \\ 4 \\ 5 \\ \\ \end{matrix} \end{array}$$

$$K_P = \begin{array}{cc} \begin{matrix} 1 & & & & 5 & \end{matrix} & \\ \begin{bmatrix} K_E11 & 0 & 0 & 0 & K_E12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ K_E21 & 0 & 0 & 0 & K_E22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 1 \\ \\ \\ \\ 5 \\ \\ \end{matrix} \end{array}$$

$$K_P = \begin{array}{cc} \begin{matrix} & 2 & & & 5 & \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_F11 & 0 & 0 & K_F12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_F21 & 0 & 0 & K_F22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} \\ 2 \\ \\ \\ 5 \\ \\ \end{matrix} \end{array}$$

Fill in the template with the quadrants of the element stiffness matrices

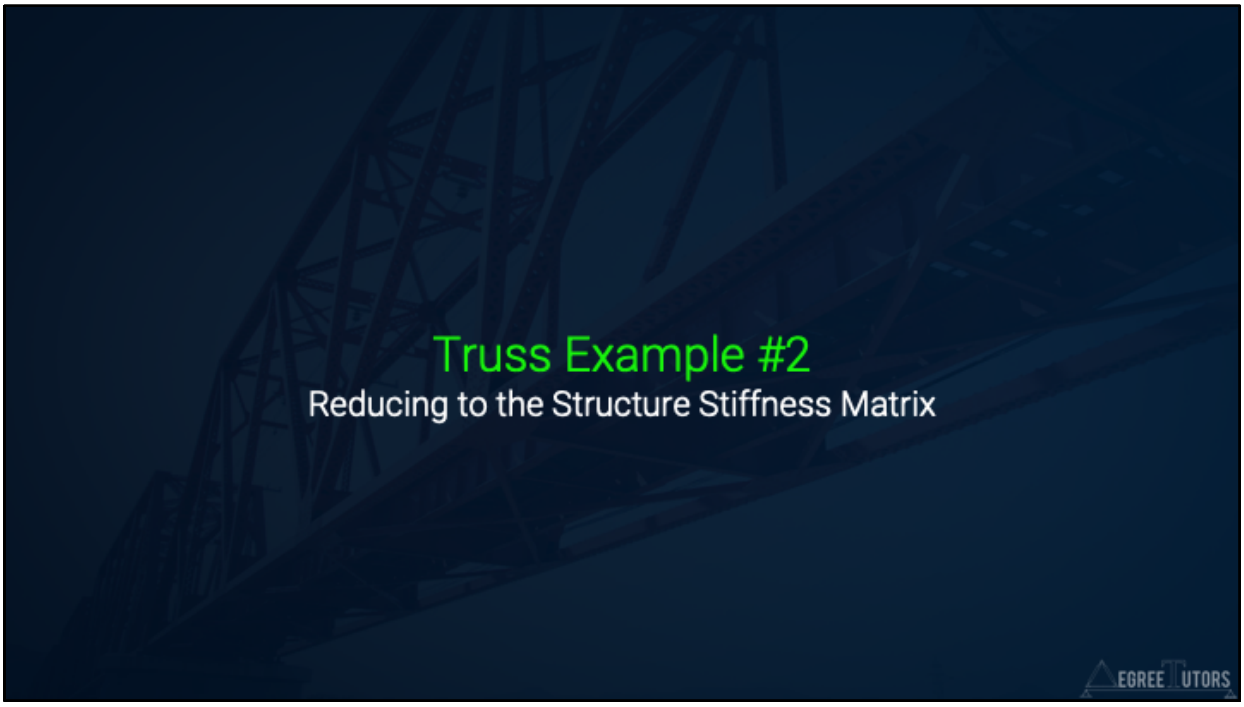


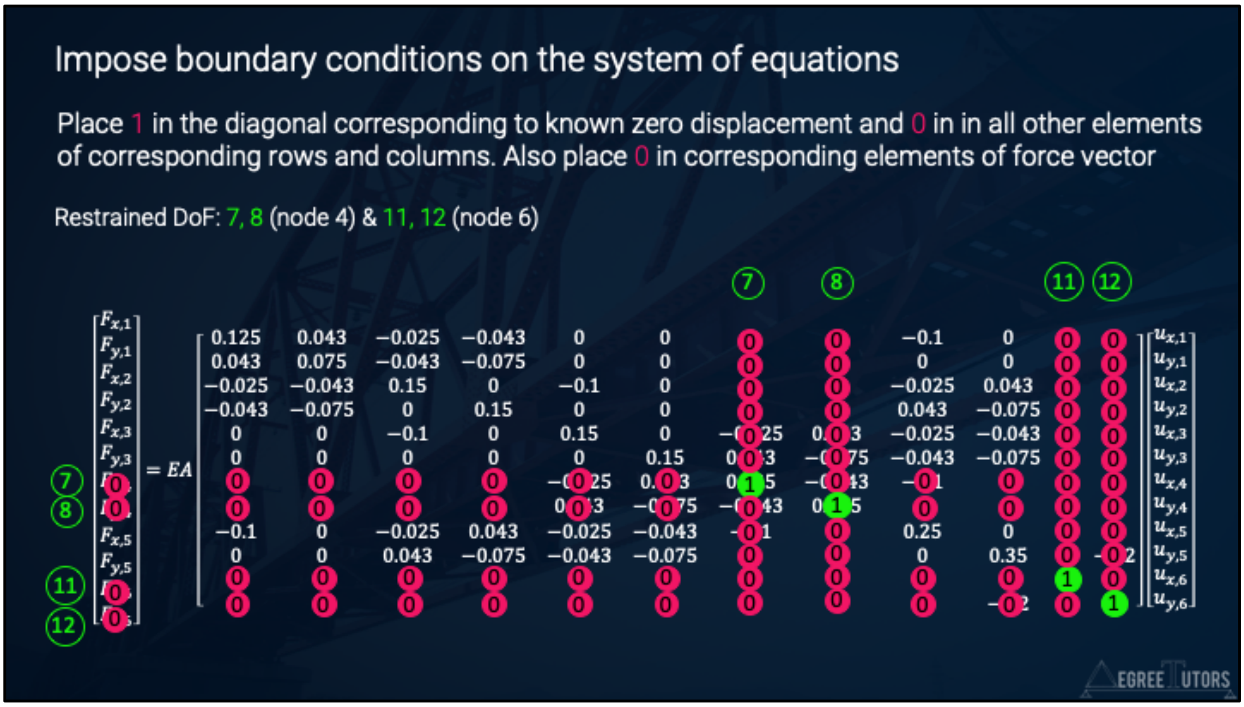
$$K_P = \begin{array}{cc} \begin{array}{cccccc} & & 3 & & 5 & \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_G 11 & 0 & K_G 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_G 21 & 0 & K_G 22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} \\ \\ 3 \\ \\ 5 \\ \\ \end{array} \end{array}$$

$$K_P = \begin{array}{cc} \begin{array}{cccccc} & & & & 5 & 6 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_H 11 & K_H 12 \\ 0 & 0 & 0 & 0 & K_H 21 & K_H 22 \end{bmatrix} & \begin{array}{c} \\ \\ \\ \\ 5 \\ 6 \end{array} \end{array}$$

Fill in the template with the quadrants of the element stiffness matrices

$$K_P = \begin{bmatrix} 0.125 & 0.043 & -0.025 & -0.043 & 0 & 0 & 0 & 0 & -0.1 & 0 & 0 & 0 \\ 0.043 & 0.075 & -0.043 & -0.075 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.025 & -0.043 & 0.15 & 0 & -0.1 & 0 & 0 & 0 & -0.025 & 0.043 & 0 & 0 \\ -0.043 & -0.075 & 0 & 0.15 & 0 & 0 & 0 & 0 & 0.043 & -0.075 & 0 & 0 \\ 0 & 0 & -0.1 & 0 & 0.15 & 0 & -0.025 & 0.043 & -0.025 & -0.043 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.15 & 0.043 & -0.075 & -0.043 & -0.075 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.025 & 0.043 & 0.125 & -0.043 & -0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.043 & -0.075 & -0.043 & 0.075 & 0 & 0 & 0 & 0 \\ -0.1 & 0 & -0.025 & 0.043 & -0.025 & -0.043 & -0.1 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0.043 & -0.075 & -0.043 & -0.075 & 0 & 0 & 0 & 0.35 & 0 & -0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 00 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0.2 \end{bmatrix}$$

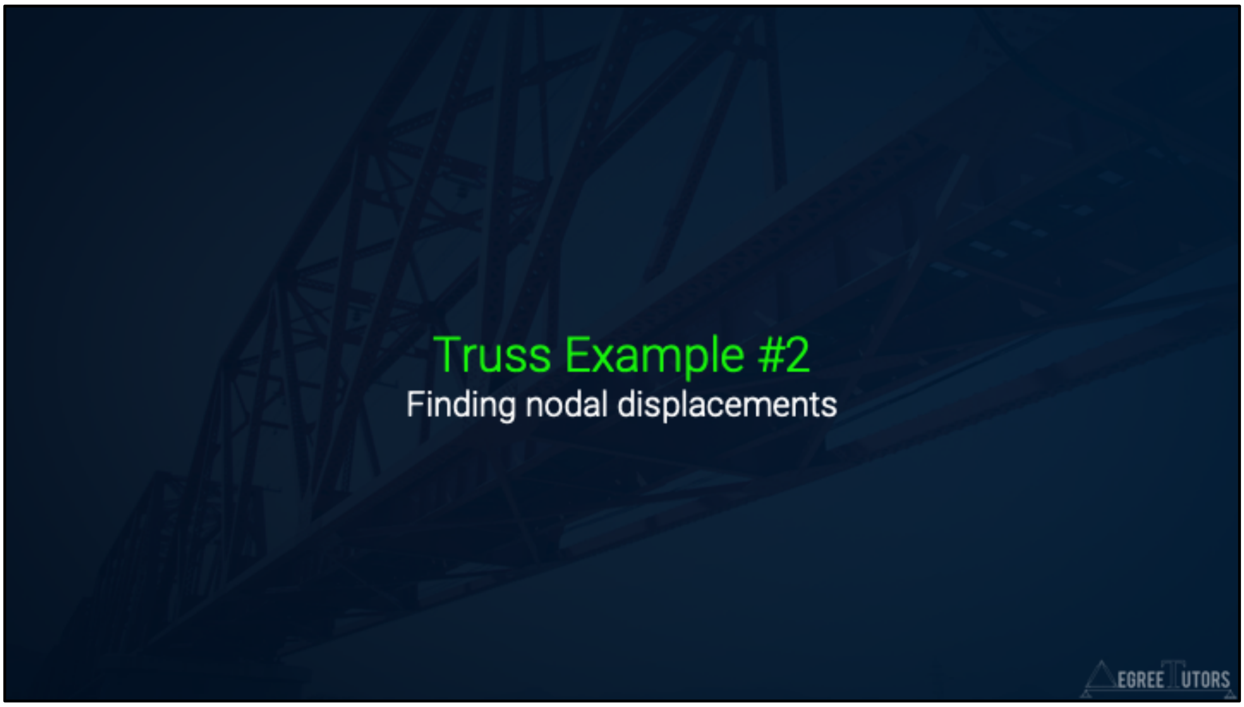

Truss Example #2
Reducing to the Structure Stiffness Matrix


Impose boundary conditions on the system of equations
Place 1 in the diagonal corresponding to known zero displacement and 0 in in all other elements of corresponding rows and columns. Also place 0 in corresponding elements of force vector
Restrained DoF: 7, 8 (node 4) & 11, 12 (node 6)
DEGREE TUTORS

Impose boundary conditions on the system of equations

8 x 8 structure stiffness matrix

$$EA\begin{bmatrix} 0.125 & 0.043 & -0.025 & -0.043 & 0 & 0 & -0.1 & 0 \\ 0.043 & 0.075 & -0.043 & -0.075 & 0 & 0 & 0 & 0 \\ -0.025 & -0.043 & 0.15 & 0 & -0.1 & 0 & -0.025 & 0.043 \\ -0.043 & -0.075 & 0 & 0.15 & 0 & 0 & 0.043 & -0.075 \\ 0 & 0 & -0.1 & 0 & 0.15 & 0 & -0.025 & -0.043 \\ 0 & 0 & 0 & 0 & 0 & 0.15 & -0.043 & -0.075 \\ -0.1 & 0 & -0.025 & 0.043 & -0.025 & -0.043 & 0.25 & 0 \\ 0 & 0 & 0.043 & -0.075 & -0.043 & -0.075 & 0 & 0.35 \end{bmatrix}$$


Truss Example #2
Finding nodal displacements

8 unknown displacements…8 linear simultaneous equations

$$\begin{bmatrix} F_{x,1} \\ F_{y,1} \\ F_{x,2} \\ F_{y,2} \\ F_{x,3} \\ F_{y,3} \\ F_{x,5} \\ F_{y,5} \end{bmatrix} = EA \begin{bmatrix} 0.125 & 0.043 & -0.025 & -0.043 & 0 & 0 & -0.1 & 0 \\ 0.043 & 0.075 & -0.043 & -0.075 & 0 & 0 & 0 & 0 \\ -0.025 & -0.043 & 0.15 & 0 & -0.1 & 0 & -0.025 & 0.043 \\ -0.043 & -0.075 & 0 & 0.15 & 0 & 0 & 0.043 & -0.075 \\ 0 & 0 & -0.1 & 0 & 0.15 & 0 & -0.025 & -0.043 \\ 0 & 0 & 0 & 0 & 0 & 0.15 & -0.043 & -0.075 \\ -0.1 & 0 & -0.025 & 0.043 & -0.025 & -0.043 & 0.25 & 0 \\ 0 & 0 & 0.043 & -0.075 & -0.043 & -0.075 & 0 & 0.35 \end{bmatrix} \begin{bmatrix} u_{x,1} \\ u_{y,1} \\ u_{x,2} \\ u_{y,2} \\ u_{x,3} \\ u_{y,3} \\ u_{x,5} \\ u_{y,5} \end{bmatrix}$$

$$\{F\} = [K_S]\{U\}$$

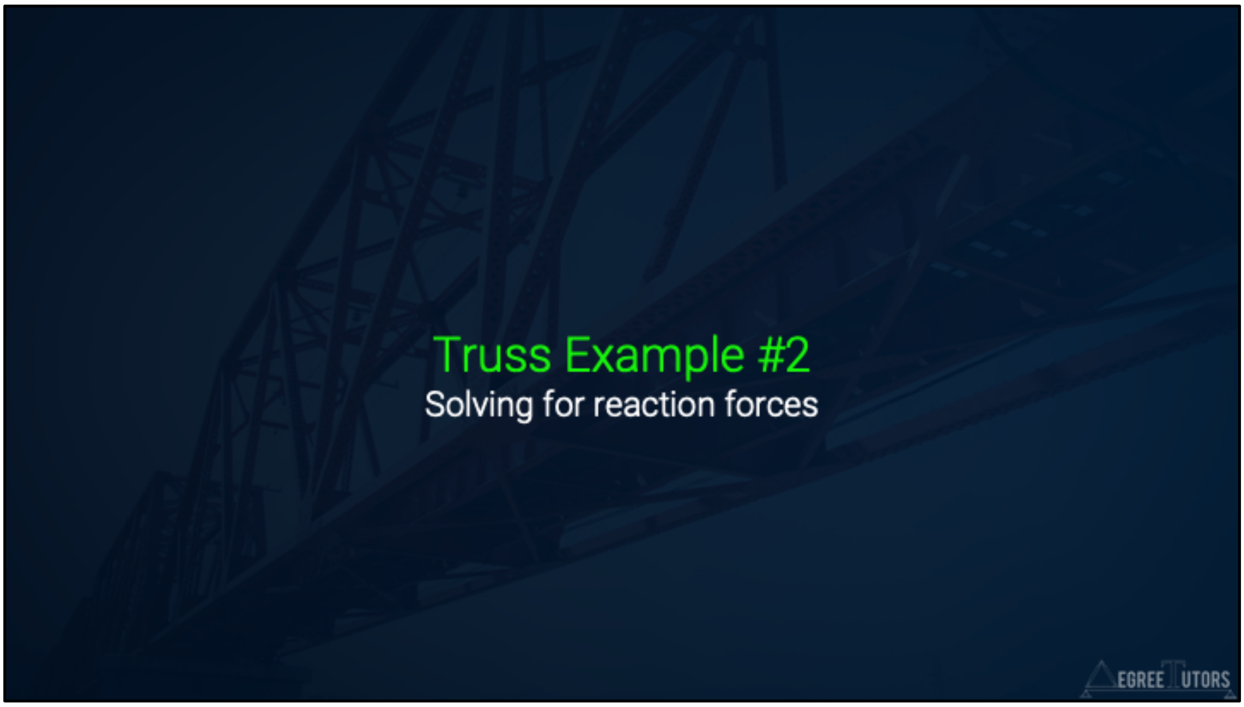
Solve for unknown displacements

$$[K_S]^{-1}\{F\} = \{U\}$$

$$\frac{1}{EA}\begin{bmatrix} 0.125 & 0.043 & -0.025 & -0.043 & 0 & 0 & -0.1 & 0 \\ 0.043 & 0.075 & -0.043 & -0.075 & 0 & 0 & 0 & 0 \\ -0.025 & -0.043 & 0.15 & 0 & -0.1 & 0 & -0.025 & 0.043 \\ -0.043 & -0.075 & 0 & 0.15 & 0 & 0 & 0.043 & -0.075 \\ 0 & 0 & -0.1 & 0 & 0.15 & 0 & -0.025 & -0.043 \\ 0 & 0 & 0 & 0 & 0 & 0.15 & -0.043 & -0.075 \\ -0.1 & 0 & -0.025 & 0.043 & -0.025 & -0.043 & 0.25 & 0 \\ 0 & 0 & 0.043 & -0.075 & -0.043 & -0.075 & 0 & 0.35 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_{x,1} \\ u_{y,1} \\ u_{x,2} \\ u_{y,2} \\ u_{x,3} \\ u_{y,3} \\ u_{x,5} \\ u_{y,5} \end{bmatrix}$$

$$\begin{bmatrix} u_{x,1} \\ u_{y,1} \\ u_{x,2} \\ u_{y,2} \\ u_{x,3} \\ u_{y,3} \\ u_{x,5} \\ u_{y,5} \end{bmatrix} = \frac{10^6}{EA}\begin{bmatrix} 2.3 \\ -18.7 \\ -8.1 \\ -10 \\ -5.8 \\ -0.7 \\ 1.2 \\ -2 \end{bmatrix}$$

DEGREE TUTORS


Truss Example #2
Solving for reaction forces
EGREE UTORS

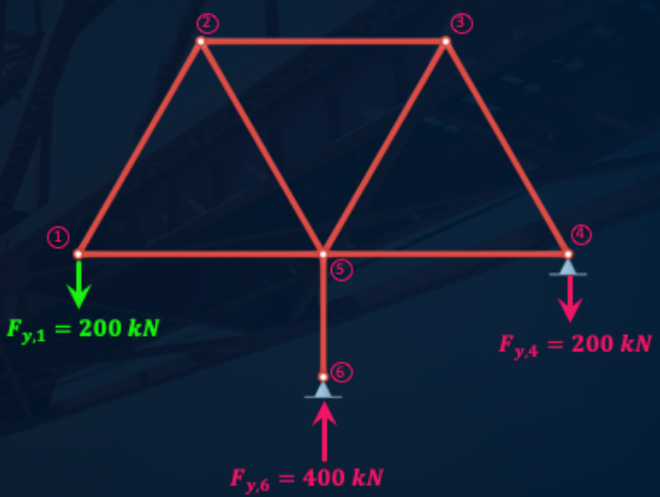
Solve for reaction forces

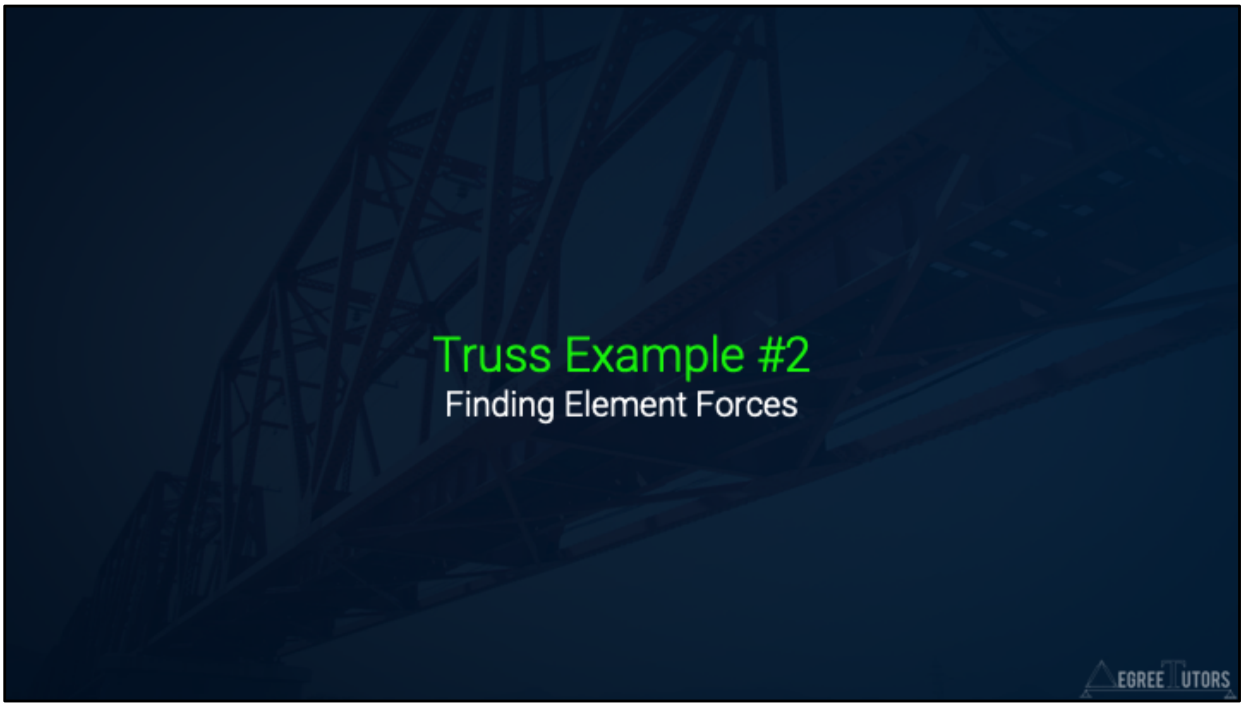
$$\begin{bmatrix} F_{x,1} \\ F_{y,1} \\ F_{x,2} \\ F_{y,2} \\ F_{x,3} \\ F_{y,3} \\ F_{x,4} \\ F_{y,4} \\ F_{x,5} \\ F_{y,5} \\ F_{x,6} \\ F_{y,6} \end{bmatrix} = EA \underbrace{\begin{bmatrix} 0.125 & 0.043 & -0.025 & -0.043 & 0 & 0 & 0 & 0 & -0.1 & 0 & 0 & 0 \\ 0.043 & 0.075 & -0.043 & -0.075 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.025 & -0.043 & 0.15 & 0 & -0.1 & 0 & 0 & 0 & -0.025 & 0.043 & 0 & 0 \\ -0.043 & -0.075 & 0 & 0.15 & 0 & 0 & 0 & 0 & 0.043 & -0.075 & 0 & 0 \\ 0 & 0 & -0.1 & 0 & 0.15 & 0 & -0.025 & 0.043 & -0.025 & -0.043 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.15 & 0.043 & -0.075 & -0.043 & -0.075 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.025 & 0.043 & 0.125 & -0.043 & -0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.043 & -0.075 & -0.043 & 0.075 & 0 & 0 & 0 & 0 \\ -0.1 & 0 & -0.025 & 0.043 & -0.025 & -0.043 & -0.1 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0.043 & -0.075 & -0.043 & -0.075 & 0 & 0 & 0 & 0.35 & 0 & -0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 00 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0.2 \end{bmatrix}}_{\text{Primary stiffness matrix}} \underbrace{\frac{10^6}{EA} \begin{bmatrix} 2.3 \\ -18.7 \\ -8.1 \\ -10 \\ -5.8 \\ -0.7 \\ 0 \\ 0 \\ 1.2 \\ -2 \\ 0 \\ 0 \end{bmatrix}}_{\text{Known displacements}}$$

$$\{F\} = \{0 \quad -200 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 - 200 \quad 0 \quad 0 \quad 0 \quad 0 \quad 400\}^T$$

Solve for reaction forces

$$\begin{bmatrix} F_{x,1} \\ F_{y,1} \\ F_{x,2} \\ F_{y,2} \\ F_{x,3} \\ F_{y,3} \\ F_{x,4} \\ F_{y,4} \\ F_{x,5} \\ F_{y,5} \\ F_{x,6} \\ F_{y,6} \end{bmatrix} = \begin{bmatrix} 0 \\ -200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -200 \\ 0 \\ 0 \\ 0 \\ 400 \end{bmatrix}$$

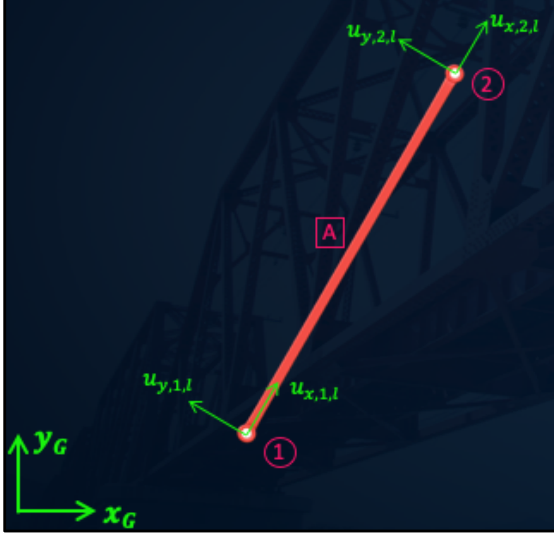



Truss Example #2
Finding Element Forces

Member A

$L_A = 10\,m,$

$\theta = 1.047\ rads\ (60°)$



$$\begin{bmatrix} u_{x,1,l} \\ u_{x,2,l} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} u_{x,1,G} \\ u_{y,1,G} \\ u_{x,2,G} \\ u_{y,2,G} \end{bmatrix}$$

$$\begin{bmatrix} u_{x,1,l} \\ u_{x,2,l} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} 2.31 \\ -18.7 \\ -8.08 \\ -10 \end{bmatrix} \frac{10^6}{EA}$$

$$\begin{bmatrix} u_{x,1,l} \\ u_{x,2,l} \end{bmatrix} = \frac{10^6}{EA} \begin{bmatrix} -15 \\ -12.7 \end{bmatrix}$$

$$F_{1,2} = \frac{EA}{L_A}\left(u_{x,2,l} - u_{x,1,l}\right)$$

$$F_{1,2} = \frac{10^6 EA}{10}\left(\frac{-12.7}{EA} - \frac{-15}{EA}\right)$$

$$F_{1,2} = 230\ \text{kN}$$

Member B

$L_A = 10\,m,$

$\theta = 0\ rads\ (0°)$

$$\begin{bmatrix} u_{x,2,l} \\ u_{x,3,l} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} u_{x,2,G} \\ u_{y,2,G} \\ u_{x,3,G} \\ u_{y,3,G} \end{bmatrix}$$

$$\begin{bmatrix} u_{x,2,l} \\ u_{x,3,l} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} -8.08 \\ -10 \\ -5.77 \\ -0.67 \end{bmatrix} \frac{10^6}{EA}$$

$$\begin{bmatrix} u_{x,2,l} \\ u_{x,3,l} \end{bmatrix} = \frac{10^6}{EA} \begin{bmatrix} -8.08 \\ -5.77 \end{bmatrix}$$

$$F_{2,3} = \frac{EA}{L_A}\left(u_{x,3,l} - u_{x,2,l}\right)$$

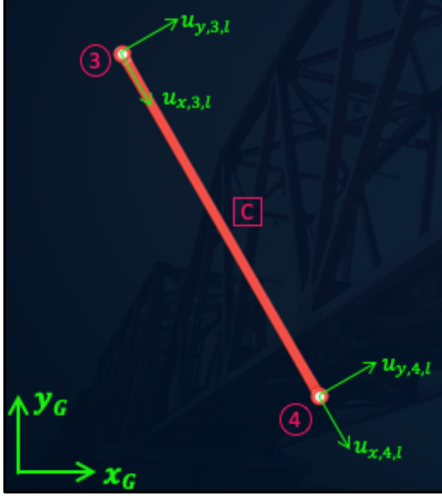
$$F_{2,3} = \frac{10^6 EA}{10}\left(\frac{-5.77}{EA} - \frac{-8.08}{EA}\right)$$

$F_{2,3} = 231$ kN

Member C

$L_A = 10\ m,$

$\theta = 5.236rads\ (300°)$



$$\begin{bmatrix} u_{x,3,l} \\ u_{x,4,l} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} u_{x,3,G} \\ u_{y,3,G} \\ u_{x,4,G} \\ u_{y,4,G} \end{bmatrix}$$

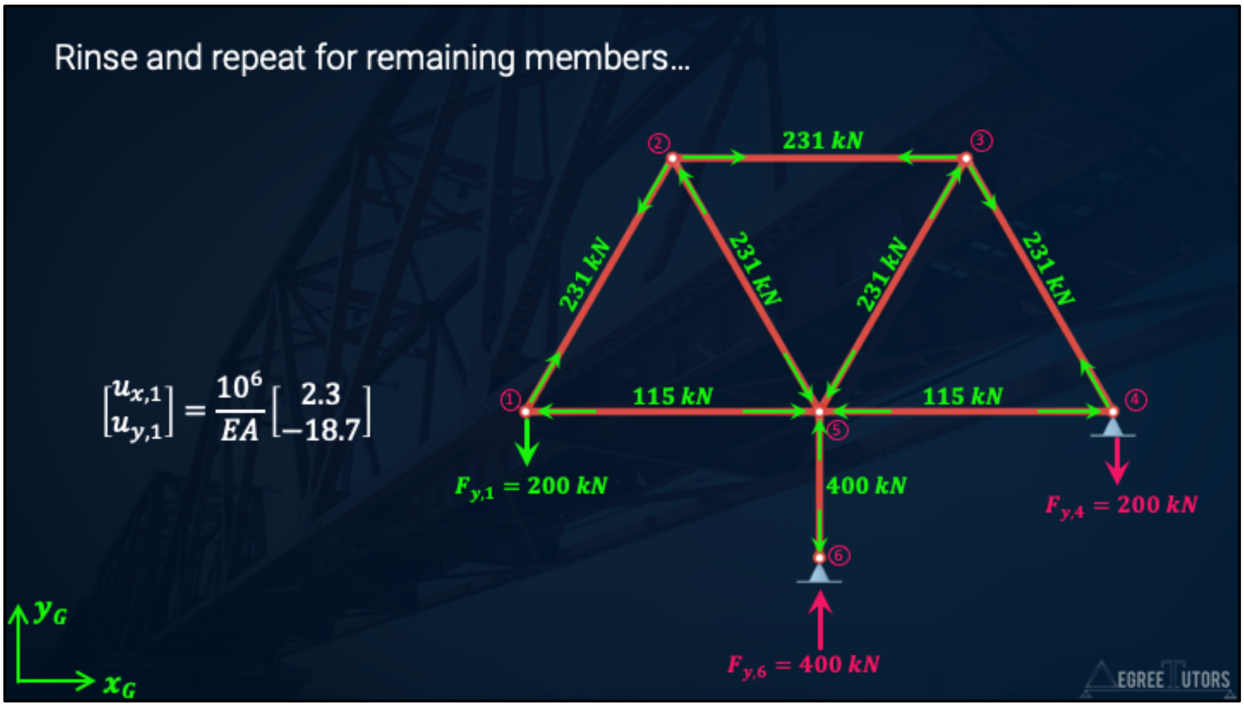
$$\begin{bmatrix} u_{x,3,l} \\ u_{x,4,l} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} -5.77 \\ -0.67 \\ 0 \\ 0 \end{bmatrix} \frac{10^6}{EA}$$

$$\begin{bmatrix} u_{x,3,l} \\ u_{x,4,l} \end{bmatrix} = \frac{10^6}{EA} \begin{bmatrix} -2.31 \\ 0 \end{bmatrix}$$

$$F_{3,4} = \frac{EA}{L_A}\left(u_{x,4,l} - u_{x,3,l}\right)$$

$$F_{3,4} = \frac{10^6 EA}{10}\left(0 - \frac{-2.31}{EA}\right)$$

$F_{3,4} = 231$ kN


Rinse and repeat for remaining members...
$\begin{bmatrix} u_{x,1} \\ u_{y,1} \end{bmatrix} = \frac{10^6}{EA}\begin{bmatrix} 2.3 \\ -18.7 \end{bmatrix}$
231 kN
231 kN
231 kN
231 kN
231 kN
115 kN
115 kN
400 kN
$F_{y,1} = 200\ kN$
$F_{y,4} = 200\ kN$
$F_{y,6} = 400\ kN$
y_G
x_G
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