Modeling Stock Trades Using Poisson Point Processes

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1 Introduction

Financial econometrics is the study of stock market price movements using statistical methods. An important aspect of price movement is the random spacing of upward and downward events. We can analyze the irregular occurrences of such events by modeling them as Poisson point processes. This article describes a method for executing trades based on a such models. We utilize the property that market events tend to be clustered over time to seek opportunities for upward price movement.

2 Definitions

Consider a price signal $(O_t, C_t) = f(t)$ where $t \in (0, n)$ is the discrete time index of the price data and $f(t)$ is a function that maps the time index to a pair consisting of the open price o_t and close price c_t. Let $\{t_i\}_{i \in \{1,\ldots,n\}}$ denote a sequence of increasing event times $t_0 < \ldots < t_n$ associated with a point process. In this case, each data point in the price signal is an event, and we will classify them into several types. Then $N(a, b) := \sum 1_{\{a \leq t_i \leq b\}}$ defines the counting function for the events within times a and b. Furthermore, let $\{W_i\}_{i \in \{1,\ldots,n\}}$ denote a sequence with K possible values representing K types of events. Thus we call the process $\{t_i, W_i\}_{i \in \{1,\ldots,n\}}$ a K-variate *marked* point process with counting functions $N^k(s,t) := \sum 1_{\{s \leq t_i \leq t\}} 1_{\{W_i = k\}}$. An expectation of $N^k(s,t)$ implies some rate λ^k such that $E[N^k(a,b)] = \lambda^k(b-a)$. Additionally, $Var[N^k(a,b)] = \lambda^k(b-a)$.

3 Method

The steps of the trading method are as follows:

- Define the size of the discrete time step.
- Define the sizes of the analysis and trading windows.
- Define the threshold for an upward or downward price movement event as a percentage of the current stock price.
- Count the events within the analysis time window.
- Compute the expected number of events within the trading window and use the expected variance to select trades that have a positive return with high confidence.

We now discuss each step in more detail. We first select the granularity of the stock price data to work with. The size of the time step could range from daily to minute or even second level price data. This choice largely depends on access to data, as high frequency real-time stock price data is usually expensive to obtain. Regardless if the selection, the following process and formulas remain the same.

Given the discrete time indices $t \in (0, n)$ and a pair (O_t, C_t) representing the open and close prices of the stock, define Δ as the size of the analysis window. Then an analysis window can be represented as a pair of values $(a_i, a_i + \Delta)$ where i is the start time of the analysis window. 0 is the time associated with the first data point of the initial analysis window and n is the current time. Therefore initially we have $a_0 = 0$ and $\Delta = n$. We also select a time frame $\theta \leq \Delta$ to execute the trade. Selecting θ requires some trial and error but in general it should be much shorter than Δ.

Next let's define the events that we will use to model the trades. We will categorize each $t \in (0, n)$ into one of three bins based on (O_t, C_t), essentially generating an event for each t. To differentiate "significant" price movements from smaller ones, let's define a delta $\delta \in (0, 1)$ as a threshold for relative price movement. For example, a δ of 0.01 means a 1% increase or decrease. Then we can label each event as follows:

$$|C_i - O_i| < O_i * \delta \rightarrow W_i = 1$$
$$C_i - O_i > 0; C_i - O_i \geq O_i * \delta \rightarrow W_i = 2$$
$$C_i - O_i \leq 0; C_i - O_i \leq O_i * \delta \rightarrow W_i = 3$$

Now we have the events labeled, and can use the counting function to determine the number of events within the analysis window. This gives three values $N^1(a_i, a_i + \Delta)$, $N^2(a_i, a_i + \Delta)$, $N^3(a_i, a_i + \Delta)$, which we will write shorthand as $N^1_{a_i}$, $N^2_{a_i}$, and $N^3_{a_i}$.

Using the formula $E[N^k(a, b)] = \lambda^k(b - a)$, we now compute the rates λ^2 and λ^3 as follows:

$$E[N^k(a_i, a_i + \Delta)] = N^k(a_i, a_i + \Delta) = \lambda^k(a_i, a_i + \Delta) \Rightarrow$$
$$\lambda^k = \frac{N^k_{a_i}}{\Delta}$$

In simple terms, the rate of each type of event is equal to the number of times the event occurred within the analysis window divided by the length of the window. Using these rates we can compute the expected number of events within the trading window:

$$E[N^k(a_i + \Delta, a_i + \Delta + \theta)] = \lambda^k \theta$$

We will denote this results shorthand as $E[N^k_\theta]$. Taking $E[N^2_\theta - N^3_\theta] = E[N^2_\theta] - E[N^3_\theta]$ gives the total expected number of significant upward price movements within the trading window, subtracting the significant downward price movements. Finally, we compute the variance as follows:

$$Var[N^2_\theta - N^3_\theta] = Var[N^2_\theta] + Var[N^3_\theta] = \lambda^2\theta + \lambda^3\theta$$

This assumes that the random variables N_2 and N_3 are independent. Now with the expectation and the variance of significant price movements within the trading window, we can decide whether to place a trade. We can use the variance to compute the likelihood that a profit is made with a certain level of confidence. Suppose a buy order is placed, then we expect hold the stock for δ time. However we can also update the analysis window as new data comes in, and if we still expect to make a profit, we can update the trading window forward in time.

4 Conclusion

In this article we outlined a framework for modeling stock trades using poisson point processes. We limited our method to three types of events in a marked point process, however that number could be expanded to cover a wider range of price movement magnitudes. There is also the possibility of computing the covariance of the events types to gain a more accurate measure of confidence.

5 References

BAUWENS, LUC, AND HAUTSCH, NIKOLAUS (2007): "Modelling Financial Figh Frequency Data Using Point Processes," *Humboldt University of Berlin*.

6 Conflicts of Interest

None declared.

7 Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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