4.4. Let (a,b,c) denote that Goneril receives a, Regan receives b, and Cordelia receives c.

$\Omega =$ [redacted] $\{(12,3,0),(12,0,3),(13,2,0),(13,0,2),(23,1,0),(23,0,1),$
$(3,12,0),(0,12,3),(2,13,0),(0,13,2),(1,23,0),(0,23,1),$
$(0,3,12),(3,0,12),(2,0,13),(0,2,13),(1,0,23)(0,1,23)\}$

18 possibilities.

4.7 $\frac{1}{38}$

4.8 a. assume $e \cap g \subseteq a \cap c$, then $A \cup B = \frac{1}{2}$, so $(A\cup B)^c = \boxed{\frac{1}{2}}$

b $A \cup B = \frac{2}{3} = |A| + |B| - |A \cap B| = \frac{5}{6} - A\cap B$ $A \cap B = \boxed{\frac{1}{6}}$

4.13 a. $1 \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} = \frac{44}{5 \cdot 7^3} = .0257$

b. $\frac{1}{2} \cdot \frac{26}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} = \frac{8 \cdot 23}{4 \cdot 17 \cdot 49} = 5.5\%$ or $\boxed{.055}$

c. $1 \cdot \frac{39}{51} \cdot \frac{26}{50} \cdot \frac{13}{49} = .1055$ or $\boxed{10.6\%}$

4.19 $\frac{13}{39} \rightarrow \boxed{\frac{1}{3}}$

4.23 $\frac{2}{8}$

4.26 $p_t^E \cdot (1-p_t)^{T-E} \cdot \binom{T}{E}$

4.27 $\frac{(26C5)^2}{52C10} = \boxed{.273}$

4.28 a. $\frac{10C4 \cdot 30C3}{40C7} = .046$ for player 1

$\frac{20C4 \cdot 20C3}{40C7} = .296$ for player 2

$P1 \cap P2 = \boxed{.0136}$

b. $\frac{10C2 \cdot 30C5}{40C7} = .344$ for player 1

$\frac{20C3 \cdot 20C4}{40C7} = .296$ for player 2

$P1 \cap P2 = \boxed{.102}$

c

P(n lands)	P1	P2
0	10.9	.4
1	31.8	4.16
2	34.3	15.8
3	17.64	29.6
4	.046	.296
5	.59	15.8
6	≈0	4.16
7	≈0	.4

$P(P2 > P1) = .0416 \cdot .109$
$+ 15.8 + (.318 + 10.9)$
$+ .296 \cdot (.343 + .318 + .109)$
$+ .296 \cdot (.1764 + .343 + .318 + .109$
$+ .158 (.994)$
$+ .046 (1)$
$= \boxed{.914}$

4.30 independence $\Leftrightarrow P(A\cap B) = P(A) \cdot P(B)$

$$P(A\cap B) = P(A) + P(A) - P(A\cup B)$$
$$= .7 - .65$$
$$= .05$$
$$P(A)\,P(B) = .1 \neq P(A\cap B)$$

So A and B are NOT independent

4.31 a. $P(\text{red}) = \frac{24}{50}$

$$P(q) = \frac{4}{50}$$
$$P(\text{red } q) = \frac{2}{50}$$
$$P(\text{red}) \cdot P(q) = \frac{96}{2500} \neq P(\text{red } q)$$

No, they are not independent.

b. $P(\text{black}) = \frac{26}{50}$

$$p(k) = \frac{2}{50}$$
$$P(b\,k) = \frac{2}{50} \neq \frac{52}{2500}$$

No, not independent

4.37 a. $\frac{9C6}{39C6} = 2.6 \times 10^{-5}$

b. $\frac{30C6}{39C6} = .182$

4.40 a. $P(K) = .7$

b. $P(R|K) = 1$

c. ~~[illegible]~~

$P(R) = .7 + .3 \cdot \frac{1}{n}$

$P(K|R) = \frac{.7}{.7 + .3\frac{1}{n}}$

d. $\frac{.7}{.7 + .3\frac{1}{n}} = .99$

$.7 = .693 + .297\frac{1}{n}$

$.297\frac{1}{n} = .007$

$\frac{1}{n} = .0236$

$n \geq 42$

minimum $\boxed{n = 43 \text{ options}}$

4.41 $P(C_1 | G_2, r_3) = \frac{P(C_1 \cap G_2)}{P(G_2)}$

But r_3 isn't random. Revealing door 2 means that the car is behind door 1 for sure

Thus $P(C_1 | G_2, r_3) = \boxed{1 \text{ or } 100\%}$

4.42 $P(C_1 | G_2, r_4) = \frac{P(C_1 \cap G_2)}{P(G_2)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$ or 50%.