8.1a θ is μ

$$L(\theta) = \prod_{i\in D} P(d_i|\theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} \exp \frac{-(x_i-\theta)^2}{2\sigma^2}$$

$$\underset{\theta}{\text{argmax}}\, \cancel{L}(\theta) = \underset{\theta}{\text{argmax}} \log(L(\theta))$$

$$\frac{\partial}{\partial\theta} \log(L(\theta)) = 0$$

$$\log(L(\theta)) = \sum_{i\in D} \log \frac{1}{\sqrt{2\pi}\sigma} + \frac{-(x_i-\theta)^2}{2\sigma^2}$$

$$\frac{\partial}{\partial\theta} \log(L(\theta)) = \sum_{i\in D} \frac{\cancel{2}(x_i-\theta)}{\cancel{\sigma}} = 0$$

$$\sum_{i\in D} x_i = n\theta$$

$$\theta = \frac{\sum_{i=0}^{n} x_i}{n} = \text{mean}(\{x\})$$

b. θ is $\cancel{\sigma}$, μ constant.

$$L(\theta) = \prod_{i\in D} P(d_i|\theta) = \prod_{i=0}^{n} \frac{1}{\sqrt{2\pi}\,\theta} \exp \frac{-(x_i-\mu)^2}{2\theta^2}$$

$$\underset{\theta}{\text{arg max}}\, L(\theta) = \underset{\theta}{\text{argmax}} \log(L(\theta))$$

$$\log(L(\theta)) = \sum_{i=0}^{n} \log \frac{1}{\sqrt{2\pi}\theta} + \sum_{i=0}^{n} \frac{-(x_i-\mu)^2}{2\theta^2}$$

$$= \sum_{i=0}^{n} -\log\sqrt{2\pi} - \log\theta + \frac{1}{2\theta^2} \cdot \sum_{i=0}^{n} -(x_i-\mu)^2$$

$$\frac{\partial}{\partial\theta} \log L(\theta) = \frac{-n}{\cancel{\theta}} + \frac{1}{\theta^{\cancel{3}2}} \sum_{i=0}^{n} (x_i-\mu)^2 = 0$$

$$n = \frac{1}{\theta^2} \sum_{i=0}^{n} (x_i-\mu)^2, \quad \theta^2 = \frac{\sum_{i=0}^{n} (x_i-\mu)^2}{n}$$

$$\theta = \sqrt{\frac{\sum_{i=0}^{n} (x_i-\mu)^2}{n}} = \text{std}(\{x\})$$

c. If all numbers equal some value x, then $\mu = x$

$$L(\theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\theta} \cdot \exp \frac{-(x_i-\mu)^2}{2\theta^2}$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\theta} \cdot \exp 0$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\theta} = \frac{n}{\sqrt{2\pi}\theta},$$ which is maximized with $\boxed{\theta = 0}$

8.2a $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $L(\theta) = \prod_{i \in D} P(d_i \mid \theta)$

$$\operatorname*{argmax}_\theta L(\theta) = \operatorname{argmax} \log(L(\theta))$$

$$\log(L(\theta)) = \sum_{i=0}^{n} \left(\log \lambda^{x_i} + \log e^{-\lambda} - \log x_i!\right)$$

θ is λ

$$\log(L(\theta)) = \sum_{i=0}^{n} \left(x_i \log\theta + -\theta - \log x_i!\right)$$

$$\frac{\partial}{\partial\theta} \log(L(\theta)) = \sum_{i=0}^{n} \left(\frac{x_i}{\theta} + -1\right) = 0$$

$$\frac{1}{\theta} \sum_{i=0}^{n} x_i = n.$$

$$\theta = \frac{\sum_{i=0}^{n} x_i}{n} = \text{mean}(\{x\})$$ ✓

day 1: mean({3,1,4,2}) = $\boxed{2.5}$

day 2: mean({2,1,2}) = $\frac{5}{3}$ = $\boxed{1.67}$

day 3: mean({3,2,2,1,4}) = $\frac{12}{5}$ = $\boxed{2.4}$

b. day 4: mean({x}) = $\frac{13}{6}$ = $\boxed{2.17}$

8.2 c. $\text{mean}(\{\text{total}\}) = \frac{\sum_{i \in D} x_i}{n} = \frac{5+12+13+10}{4+3+5+6} = \boxed{2.22}$

8.3a. $P(D|\theta) = \prod_{i \in D} (d_i | \theta) = \left(1 - \frac{\theta - 36}{\theta}\right)^r \cdot \left(1 - \frac{36}{\theta}\right)$

$$= \left(\frac{36}{\theta}\right)^r \cdot \left(\frac{\theta - 36}{\theta}\right)$$

$$\frac{\partial}{\partial \theta} L(\theta) = 0 =$$

$$\frac{-r \cdot 36^r}{\theta^{r+1}} \cdot \left(1 - \frac{36}{\theta}\right) + \frac{36}{\theta^2} \cdot \frac{36^r}{\theta^r} = 0$$

$$\frac{36^{\cancel{r+1}}}{\theta^{\cancel{r+2}}} = \frac{r \cdot \cancel{36^r}}{\cancel{\theta^{r+1}}} \cdot \left(1 - \frac{36}{\theta}\right)$$

$$\frac{36}{\theta} = r - \frac{36r}{\theta}$$

$$(36 + 36r)\frac{1}{\theta} = r$$

$$\boxed{\theta = \frac{36}{r} + 36}$$

b. This isn't very reliable because only 1 series was observed, indicating high variance. You should watch the wheel for several series of zeroes before making an estimate to improve accuracy (as is done in part c)

8.3c $L(\theta) = \prod_{i \in D} P(d_i \mid \theta) = \prod_{i=1}^{k} \left(\frac{36}{\theta}\right)^{r_i} \left(1 - \frac{36}{\theta}\right)$

Let $R = \sum_{i=1}^{k} r_i$

$$= \left(\frac{36}{\theta}\right)^{R} \left(1 - \frac{36}{\theta}\right)^{k}$$

$$\operatorname{argmax} L(\theta) = \operatorname{argmax} \operatorname{Log}(L(\theta))$$

$$\log L(\theta) = \log \left(\frac{36}{\theta}\right)^{R} + \log \left(1 - \frac{36}{\theta}\right)^{k}$$

$$= R(\log 36 - \log \theta) + k\left(\log\left(1 - \frac{36}{\theta}\right)\right)$$

$$\frac{\partial}{\partial \theta} \log L(\theta) = \frac{-R}{\theta} + k\left(\frac{36}{\theta^2} \cdot \frac{1}{1 - \frac{36}{\theta}}\right) = 0$$

$$\frac{36}{\theta^2} \cdot \frac{1}{1 - \frac{36}{\theta}} = \frac{R}{k\theta}$$

$$\frac{1}{1 - \frac{36}{\theta}} = \frac{\theta R}{36k}$$

$$1 - \frac{36}{\theta} = \frac{36k}{\theta R}$$

$$\theta - 36 = \frac{36k}{R}$$

$$\theta = 36 + \frac{36k}{\sum_{i=1}^{k} r_i}$$

$$= 36 + \frac{36}{\text{Mean}(\{r\})}$$

8.4 a. $P(D|\theta) = \theta^b \cdot (1-\theta)^y \binom{b+y}{b}$

$$= 1 \cdot 1 \cdot (1-\theta)$$

$$= 1 - \theta$$

$$\underset{\theta}{\operatorname{argmax}} \; P(D|\theta) = 0$$

$$\boxed{\theta = 0}$$

b. $P(D|\theta) = \binom{10}{3} \theta^3 (1-\theta)^7$

$$\frac{\partial}{\partial\theta} L(\theta) =$$

$$= \binom{10}{3}\left(3\theta^2 (1-\theta)^7 + -7(1-\theta)^6 \theta^3\right) = 0$$

$$3\cancel{\theta^2}(1-\theta)^{\cancel{7}} = 7\cancel{(1-\theta)^6}\,\theta^{\cancel{3}}$$

$$3 - 3\theta = 7\theta$$

$$\boxed{\theta = \frac{3}{10}}$$

8.5 a. $P(n|z) = \frac{z}{36+z}$

$P(n|z=0) = 0$ $\qquad$ $P(n|z=2) = \frac{1}{19}$

$P(n|z=1) = \frac{1}{37}$ $\qquad$ $P(n|z=3) = \frac{1}{13}$

b. When you have observed no zeroes, $P(z=0|obs)$ is non-zero

c. $P(z|n) = \frac{P(n|z) \cdot p(z)}{p(n)}$

$P(z=0|obs) = 0$

$P(z=1|obs) = \frac{36C2 \left(\frac{1}{37}\right)^2 \left(\frac{36}{37}\right)^{34} \cdot .2}{P(n)} = \frac{.0363}{P(n)}$

$P(z=2|obs) = \frac{36C2 \left(\frac{2}{38}\right)^2 \left(\frac{36}{38}\right)^{34} \cdot .4}{P(n)} = \frac{.111}{P(n)}$

$P(z=3|obs) = \frac{36C2 \left(\frac{3}{39}\right)^2 \left(\frac{36}{39}\right)^{34} \cdot .3}{P(n)} = \frac{.073}{P(n)}$

Probabilites need to sum to 1 b/c definition of distribution

so $P(n) = .0363 + .111 + .073 = .2203$

$P(z=0|obs) = 0$

$P(z=1|obs) = .165$

$P(z=2|obs) = .504$

$P(z=3|obs) = .331$

8.7 a. estimated popmean $\{x\} = E(\{X^{(k)}\}) = \frac{\sum_{i=1}^{k} x_i}{k}$

$= \frac{199}{10} = \boxed{19.9}$

b. standard error $= \frac{\text{popsd}\{x\}}{\sqrt{k}}$

$\text{popsd}\{x\} \approx \text{std}(x)$

$= \sqrt{\frac{\sum_{i=1}^{k}(x_i - x^{(k)})^2}{k-1}} = 3.51$

$\text{std}(\{X^{(k)}\}) = \frac{3.51}{\sqrt{10}} = \boxed{1.11}$

c. $\text{std}(x^{(k)}) = \frac{3.51}{\sqrt{k}} \leq .1$

$k \geq 35.1^2 \geq \boxed{1232}$ mice

8.8a. $\text{popsd}(\{x\}) = \text{std}(\{x\}) = \sqrt{\frac{\sum_{i=1}^{k}(x_i - .3)^2}{k-1}} = .483$

$\text{std}(x^{(k)}) = \frac{.483}{\sqrt{10}} = \boxed{.153}$

b. $\frac{.483}{\sqrt{n}} \leq .05$

$\sqrt{n} \geq 9.66$

$n \geq 93.3$

or $\boxed{94 \text{ repeats}}$