4.4. Let (a,b,c) denote that Goneril receives a, Regan receives b, and Cordelia receives c.

$\Omega = \{(12,3,0),(12,0,3),(13,2,0),(13,0,2),(23,1,0),(23,0,1),$
$(3,12,0),(0,12,3),(2,13,0),(0,13,2),(1,23,0),(0,23,1),$
$(0,3,12),(3,0,12),(2,0,13),(0,2,13),(1,0,23),(0,1,23)\}$

18 possibilities

4.7 $\frac{1}{38}$

4.8 a. assume e.g. $\subseteq$ a/c, then $A \cup B = \frac{1}{2}$, so $(A \cup B)^c = \boxed{\frac{1}{2}}$

b. $A \cup B = \frac{2}{3} = |A| + |B| - |A \cap B| = \frac{5}{6} - \cancel{A \cap B}$ $A \cap B = \boxed{\frac{1}{6}}$

4.9 The set needs the union and intersection of all of its elements, so {HH, HT, TT} needs to be added

4.13 a. $1 \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} = \frac{44}{5 \cdot 7^3} = \boxed{.0257}$

b. $\frac{1}{2} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} = \frac{8 \cdot 23}{4 \cdot 17 \cdot 49} = 5.5\%$ or $\boxed{.055}$

c. $1 \cdot \frac{39}{51} \cdot \frac{26}{50} \cdot \frac{13}{49} = .1055$ or $\boxed{10.6\%}$

4.19 $\frac{13}{39} \rightarrow \boxed{\frac{1}{3}}$

4.23 $\frac{2}{8}$

4.26 $p_t^E \cdot (1-p_t)^{T-E} \cdot \binom{T}{E}$

4.27 $\frac{(26C5)^2}{52C10} = \boxed{.273}$

4.28 a. $\frac{10C4 \cdot 30C3}{40C7} = .046$ for player 1

$\frac{20C4 \cdot 20C3}{40C7} = .296$ for player 2

$P1 \cap P2 = \boxed{.0136}$

b. $\frac{10C2 \cdot 30C5}{40C7} = .344$ for player 1

$\frac{20C3 \cdot 20C4}{40C7} = .296$ for player 2

$P1 \cap P2 = \boxed{.102}$

c

P(n lands)	P1	P2
0	10.9	.4
1	31.8	4.16
2	34.3	15.8
3	17.64	29.6
4	.046	.296
5	.59	15.8
6	≈0	4.16
7	≈0	.4

$P(P2 > P1) = .0416 \cdot .109$

$+ 15.8 + (.318 + 10.9)$

$+ .296 \cdot (.343 + .318 + .109)$

$+ .296 \cdot (.1764 + .343 + .318 + .109$

$+ .158(.994)$

$+ .046(1)$

$= \boxed{.914}$

4.30 independence $\Leftrightarrow P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = P(A) + P(A) - P(A \cup B)$

$= .7 - .65$

$= .05$

$P(A)P(B) = .1 \neq P(A \cap B)$

So A and B are NOT independent

4.31 a. $P(\text{red}) = \frac{24}{50}$

$P(q) = \frac{4}{50}$

$P(\text{red } q) = \frac{2}{50}$

$P(\text{red}) \cdot P(q) = \frac{96}{2500} \neq P(\text{red } q)$

No, they are not independent.

b. $P(\text{black}) = \frac{26}{50}$

$p(k) = \frac{2}{50}$

$P(b\,k) = \frac{2}{50} \neq \frac{52}{2500}$

No, not independent

4.37 a. $\frac{9C6}{34C6} = 2.6 \times 10^{-5}$

b. $\frac{30C6}{34C6} = .182$

4.40 a. $P(K) = .7$

b. $P(R|K) = 1$

c. $P(K|R) = \dfrac{P(R|K)P(K)}{P(R)} = \dfrac{.7 \cdot 1}{P(R)}$

$$P(R) = P(R \cap K) + P(R \cap K^c)$$
$$= .7 + P(R|K^c)P(K^c)$$
$$= .7 + \frac{1}{n} \cdot .3$$

$$P(K|R) = \frac{.7}{.7 + \frac{1}{n} \cdot .3}$$

d. $P(K|R) = \dfrac{.7}{.7 + \frac{1}{n} \cdot .3} \geq .99$

$$.7 \geq .693 + .297 \frac{1}{n}$$
$$\frac{1}{n} \leq .0236$$
$$n \geq 42.4$$
$$\boxed{n_{min} = 43 \text{ options}}$$

4.41 r_3 isn't random. By the definition of r_3, revealing a goat in door 2 guarantees a car behind door one. Thus, $P(C_1|G_2, r_3) = \boxed{1 \text{ or } 100\%}$

4.42 $P(C_1|G_2, r_4) = \dfrac{P(C_1 \cap G_2)}{P(G_2 \cap C_1) + P(G_2 \cap C_2) + P(G_2 \cap C_3)}$

$$= \frac{P(G_2|C_1) \cdot P(C_1)}{P(G_2|C_1)P(C_1) + 0 + P(G_2|C_3)P(C_3)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{6}} = \boxed{\frac{1}{2}}$$