5.1 $\boxed{\forall x \in (0, 1, \ldots, 35, 36) \quad P(X=x) = \frac{1}{37}}$

5.2 a $P(\{x \leq 2\}) = \boxed{\frac{2}{13}}$ (ace or two)

b $P(\{x \geq 10\}) = \boxed{\frac{4}{13}}$ (10, J, Q or K)

c $\boxed{\frac{1}{2}}$ (not red)

d

$Y-X$	P
-1	$\frac{1}{2}$
$+1$	$\frac{1}{2}$

e $p(\{y \geq 12\}) = \boxed{\frac{3}{26}}$ (black J, Q, K)

5.3 a

X	$P(x)$
1	$\frac{1}{2}$
2	$\frac{1}{6}$
3	$\frac{1}{3}$

b

x	$p(X \leq x)$
1	$\frac{1}{2}$
2	$\frac{2}{3}$
3	1

5.6 a. $P(\{S=0\}) = P(\{L_1=0\}) \cap P(\{L_2=0\}) = \frac{30C7}{40C7} \cdot \frac{20C7}{40C7}$

bc L_1 and L_2 are indep.

$= .000454$

b. $P(\{D=0\}) = P(\{L_1 = L_2\})$ or $\sum_i P(L_1=i) \cap P(L_2=i)$

n	$L_1=n$	$L_2=n$	$L_1 \cap L_2 = L_1 L_2$
0	$\frac{30C7}{40C7}$	$\frac{20C7}{40C7}$	.000454
1	$\frac{30C6 \cdot 10}{40C7}$	$\frac{20C6 \cdot 20}{40C7}$	.0132
2	$\frac{30C5 \cdot 10C2}{40C7}$	$\frac{20C5 \cdot 20C2}{40C7}$	.0543
3	$\frac{30C4 \cdot 10C3}{40C7}$	$\frac{20C4\ 20C3}{40C7}$	.0522
4	$\frac{30C3 \cdot 10C4}{40C7}$	$\frac{20C4\ 20C3}{40C7}$	.0135
5	$\frac{30C2\ 10C5}{40C7}$	$\frac{20C2\ 20C5}{40C7}$	.000929
6	$\frac{30 \cdot 10C6}{40C7}$	$\frac{20C6 \cdot 20}{40C7}$	≈ 0
7	$\frac{10C7}{40C7}$	$\frac{20C7}{40C7}$	$+ \approx 0$

$\boxed{.135} \approx$

5.6 c, d, e.

$$P(L_1=n) = \frac{(10CN)(30C7-n)}{40C7}$$

if $L_t=10$, there cannot be any lands in hand b/c that's all the lands

$$P(L_1=n \mid L_t=5) = \frac{(5CN)(30C7-N)}{35C7}$$

n	c. $P(L_1=n)$	d. $P(L_1=n \mid L_t=10)$	e. $P(L_1=n \mid L_t=10)$
0	.109	1	.303
1	.318	0	.441
2	.344	0	.212
3	.176	0	.041
4	.046	0	.003
5	.006	0	≈ 0
6	.0003	0	0
7	≈ 0	0	0

5.7.a. $\int_{-\infty}^{\infty} p(x) = 1$, so $c = \frac{1}{\int_{-\infty}^{\infty} g(x)\,dx}$

$= \frac{1}{\sin x \big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}$

$= \boxed{\frac{1}{2}}$

b. $\boxed{\frac{1}{2}}$

c $\frac{1}{2}\left|\int_{-\frac{\pi}{2}}^{1} \cancel{\cos} x \; dx\right|$

$\frac{1}{2} \sin x \big|_{\frac{\pi}{2}}^{1} = \frac{1.84}{2} = \boxed{.92}$

5.14 a. $P(x) = 1 - P(x^c)$

$p(HHH) \cup p(TTT) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.

$p(\text{odd person}) = \boxed{\frac{3}{4}}$

b. $P(3H, 1T) \cup P(3T, 1H)$

$= \frac{1}{16} \cdot (4C1 + 4C1) = \frac{8}{16} = \boxed{\frac{1}{2}}$

c. $P(4H, 1T) \cup P(1H, 4T)$

$= \frac{1}{32} \cdot (5C1 + 5C1) = \frac{10}{32} = \frac{5}{16}$

n = number of games played total

n	$P(n)$	$n \cdot P(n)$
1	$\frac{5}{16}$	$\frac{5}{16} \approx .3125$
2	$\left(\frac{11}{16}\right)^1 \left(\frac{5}{16}\right)^1$	.43
3	$\left(\frac{11}{16}\right)^2 \left(\frac{5}{16}\right)$	.443
4	$\left(\frac{11}{16}\right)^3 \left(\frac{5}{16}\right)$	.406
5	$\left(\frac{11}{16}\right)^4 \left(\frac{5}{16}\right)$	.35
6	$(1-p)^5 p$	.288
7	$(1-p)^6 p$	.23
8	$(1-p)^7 p$	.18
9	$(1-p)^8 p$	.14
10		.107
11		.08
12		.06
13		.048

Continued on reverse

n	$n(P(n))$
14	.034
15	.025
16	.018
17	.013
18	.0096
19	.007
20	.005

$\sum_{i \in \mathbb{N}} n \approx 3.2$ also, mean $= \frac{1}{p}$ for geometric series

$p = \frac{5}{16}$ mean $= \frac{16}{5} = 3.2$ ✓

5.16 · $\text{var}[X+Y] = E[((X+Y) - E[(X+Y)])^2]$

Expectations are linear, so $E[(X+Y)] = E[X] + E[Y]$

$\text{var}[x+y] = E[(x+y-(E[(x)] + E[y]))^2]$

$= E[((x - E[x]) + (y - E[y]))^2]$

$=$ [redacted]

$= E[(x-E[x])^2] + E[(y-E[y])^2] + E[2(x-E[x])(y-E[y])]$

$= \text{var}[x] + \text{var}[y] + 2E[xy - E[x]y - xE[y] + E[x]E[y]]$

$= \text{var}[x] + \text{var}[y] + 2(E[xy] - E[x]E[y] - E[x]E[y] + E[x]E[y])$

b/c x and y are independent, $E[xy] = E[x]E[y]$

$= \text{var}[x] + \text{var}[y] + 2(0)$

$= \text{var}[x] + \text{var}[y]$ ✓

QED

5.17 markov's inequality: $P(|x| \ge a) \le \frac{E[|x|]}{a}$

$$P(\{x=0\}) = 1 - P(\{|x|=1\}) - P(\{|x|=2\})$$

$$= 1-(P\{|x|=1\} + P\{|x|=2\})$$

$$P\{|x|=1\} + P\{|x|=2\} \le P(|x| \ge 1) \le \frac{.2}{1} = .2$$

so $1-(p\{|x|=1\} + p(\{|x|=2\})) \ge 1 - .2 \ge \boxed{.8}$

$$P(\{x=0\}) \ge .8$$

5.18 Chebyshev's inequality: $P(|x-E[x]| \ge a) \le \frac{var(x)}{a}$

if $P(x) \le C$, $P(x^c) \ge 1-C$

$$p(|x-E[x]| \ge 1) \le \frac{.01}{1} \le .01$$

so $p(|x-E[x]| < 1) \ge 1 - .01 \ge .99$

and $p(|X-2| < 1) \ge .99$

$$p(X=2) \ge .99$$

5.19 $p(|x-E[x]| \ge 1) \le \frac{.5}{1} \le .5$

$$p(x-E[x] > 1) = p(x-E[x] < -1)$$

so $p(x-E[x] > 1) = p(|x-E[x]| \ge 1)/2$

$$0 \le p(X \ge 0) \le .25$$

5.20a mean $= 0$ sd $= 1$, var $=$ sd$^2 = 1$

$p(|x - E[x]| \geq 1) \leq \frac{1}{1} \leq 100\% \leq 1$

$p(x \neq 0) \leq 1$

b $p(|x - E[x]| \geq 2) \leq \frac{1}{2} \leq .5$

$p(x = 2 \text{ or } x = -2) \leq \boxed{.5}$