



Department of Mathematics
Peer Tutorial, Group 21
2021 Spring, Information Sheet

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Session: Monday 10.30am - 11.30am

I am a third year MSCi mathematics student. I am currently participating a virtual exchange program between Imperial and MIT. I mainly work on Stochastic Analysis and Statistical Inference. The peer tutorial session has been set up to supplement your study. Feel free to ask for help if you need to.

Delivery: There will be **compulsory** weekly meetings every week in Microsoft Teams. We will go through lecture contents, problem sheets and possibly some extensions. I may occasionally record videos on unfinished contents in weekly meetings.

Attendance: Attendance of weekly meetings is compulsory - I am obligated to record your attendance. Please inform me if you cannot attend the weekly meetings.

Questions:

- When you have questions about your study and you want to ask me for help, **message in the MS Teams chat**. I will either answer your question in chat or defer to next regular peer tutorial.
- You are more than welcomed to ask ad-hoc questions during the peer tutorial, but bear in mind 1 hour is just not enough. (We (I) usually spend 15-20 minutes to go through 1 question so **we can only go through 3-4 questions in a session**).

Survey: Please fill in the MS Team survey before first tutorial.

Tentative Schedule: You can change this whenever you want!

- (22/01) Introduction to linear algebra using numpy.
- (25/01) QR Factorisation I (Derivation via Classical Gram Schmidt)
- (01/02) QR Factorisation II (Examples and Least Square Problem)
- (08/02) Analysis on monotone functions in \mathbb{R}
- (15/02) Midterm (we might have to reschedule this meeting)
- (22/02) Least Square Estimation using numpy and R.
- (01/03) Fixed Point Theorems
- (08/03) Bifurcation in practice
- (15/03) Problems in Group Theory
- (22/03) Sendoff
- Additional workshop in LaTeX

Disclaimer: I cannot do the coursework for you! We can discuss together, but I cannot give you answers!

Remember the more you tell, the more I can support. Good luck with your studies and see you in the weekly meetings.

The remaining of the sheet is general advice and overview. Please have a read.

Here are some general advice for you:

- What programme are you in? (G100, G103, G104, ...) Do you know your year weightings?
- Pass all your modules! You will be in huge trouble if you don't. **Try to maintain a first honour (70%)** – this can be done by doing enough exercises (not telling you to do all problems though), and **remember all the definitions and theorems**. Nothing extra (*probably)! (Extra: you will probably get into the Dean's List if you get 85% or more).
- **Do all coursework/tests seriously.** Even though their weightings are low, they may change your grade! You certainly don't want to get one mark off from your next grade (e.g. 69%) because you fail in one test.
- **Lectures:** Depends on your learning style. If you find lectures not useful or too boring you can just read books/notes – be sure that you are actually comfortable with the course contents and don't wait until the end of term to catch up! If you are really confident with the lecture content, why not take a break from math or **go ahead and watch lectures in upper years?** (You just need to ask permissions from lecturers and the Undergraduate Office).
- Try to struggle with the question (~15 min for first year, ~30 min for second year, >1 hour for upper years.) After that, seek help from one (**or probably more**) the following (non-exhaustive)
 - Ask the lecturer immediately after lecture. (Ideal)
 - Attend **office hours**.
 - Ask your friends.
 - Attend problem classes and ask the Graduate assistants (GTA). (You may join additional sessions for challenging problems).
 - Ask your personal tutor (**Note:** ask topics which are related to their research field only. They may not know questions from other field very well.)
 - Ask your peer tutor, right, that's me.

Here are some general resources you should look for:

- Official lecture notes
- Unofficial lecture notes (they are a bit old though...):
 - Imperial MathSoc <https://union.ic.ac.uk/rcsu/mathsoc/resources>
 - The source codes for some of the notes in Imperial MathSoc (those from Karim Bacchus) are available here: <https://github.com/martinogden/imperial-notes>
 - MIT Open Courseware <https://ocw.mit.edu/courses/find-by-number/>
 - Oxford Course Management <https://courses.maths.ox.ac.uk/>
 - Cambridge Notes by Dexter Chua <http://dec41.user.srcf.net/>
- Recommended Textbooks: In most of the cases (**especially Year 1**) lecture notes are better than textbooks, but sometimes the textbooks are more organised than notes.
 - The *Schaum's Outline* Series (known as *SOS*) covers many Year 1/2 materials in great detail. (At least better than the *Dummies* series, I guess...)
- Don't forget to make the most of Imperial Library.
- Stack Exchange / Stack Overflow for quick questions.

This is an indicative (not exhaustive) list of stuff you will expect throughout the year.

- 1D Analysis:

- (T1/2) Continuity and Convergence of Function:

- * You should know that a function is continuous if its limit is equal to actual value. We therefore need to define another notion of limit. How is that related to notion of limit of sequences? Can we prove some standard properties of limits?
 - * How do you know $f(x) = c$ always have a solution on some interval $[a, b]$? (Intermediate Value Theorem)
 - * Does x, y close always mean $f(x), f(y)$ close? (Think about $f(x) = 1/x$.)

- (T2) Differentiation:

- * You know that a derivative of function $f(x)$ is some sort of drawing tangent lines. How can you formalise the limit? Can you prove standard properties?
 - * Can you prove Mean Value Theorem, Extreme Value Theorem and Fixed Point Theorem...
 - * Can you find minimum/maximum of functions? Can you ensure that the Taylor Series of a function actually agree with the function itself? Does it even converge?
 - * What does it mean for a function to converge? (Hint: There are AT LEAST TWO WAYS).

- (T2) Integration:

- * You know that integration is in some sense finding area under the curves. We may estimate the area by chopping it into rectangles? Does the estimation change when we chop in another way? Can we formalise integrability?
 - * Can we formalise the fundamental theorem of calculus and link integration with differentiation? Can we formalise calculus tricks like integration by parts and integration by substitution?
 - * Can we exchange limits with differentiation/integration?

- Algebra

- Do you still remember the notion of linear independence, span and basis?
- (T2) Can we make sense of determinant of matrices?
- (T2) Can you make sense of eigenvalues? Is a matrix always diagonalisable?
- (T2) What can you say about symmetric, orthogonal matrices?
- (T2) Group $(G, +)$ is a set with an operation that is similar to addition. Can you give some examples other than $(\mathbb{Z}, +)$? (Concept check: vector space is actually a group!)
- (T2) What can you say about the subsets/subgroup of groups? (Think about Lagrange Theorem). Can you apply that to simple theorems in number theory (e.g. Fermat Little Theorem)?
- (T2) Are two groups essentially the same? (Isomorphism)

- Calculus

- (T1/T2) You probably know how to expand a function with Taylor Series. How about sines/cosines? (Fourier Series/Transform) Can we study the amplitude/frequencies of square waves etc.?
- (T2) Can you generalise all those tricks to higher dimension?
- (T2) Do you know how to solve 1D differential equations? Sometimes we can include parameters in differential equations – how does the behavior of solution change w.r.t. parameters?
- (T2) How about 2D? Sometimes you can't write the solution in closed form, but can you understand qualitative behavior of solutions?

- Probability and Statistics

- (T2) Given a set of data, can you estimate parameters of random variables (those are called estimators)? You may think of sample mean, sample variance etc. How good are your estimates? What are the distributions of those estimators?
- (T2) What is simple linear regression? Once again, how do you estimate the parameters? How good are your estimates? What are the distributions of those estimators?

- (T2) You have learnt how to do Hypothesis Tests? Can you formalise it? We will introduce more hypothesis tests, focusing on the mean of data. How do you use those hypothesis tests in your experiments?
- (T2) There are three large sample results you should bear in mind (they will simplify your calculation): Weak/Strong Law of Large Numbers, Central Limit Theorem. What are they all about? How do you use them to simplify your estimation of parameters?
- We will teach you how to use R language to perform statistical analysis.
- Introduction to Applied Mathematics (T2)
 - Do you find analysing forces annoying? How can you analyse forces of truss system in a systematic way?
 - Do you find formulae for resistor / spring system similar? How do you generalise them to any complex system using matrix tricks?
 - Some Markov Chain - when probability and linear algebra comes together? (Warning: This is a simplified version!)
 - How do you understand the conductance of materials? How do you understand harmonic function? Complex Analysis may help here...
- Introduction to Computation: We will teach you how to use Python to perform complex calculation. How do you instruct a computer to perform complex calculation (e.g. if, logic, for/while loop)? What data types does Python have? (e.g. integer, float, array (matrix), list, tuple, dictionary...) We will also walk you through some libraries (e.g. math, cmath, numpy, sympy, scipy, matplotlib, ...) How do you incorporate the programmes in a notebook?
- Additional stuff: Maybe try typesetting in \LaTeX .