Imperial College
London

Department of Mathematics
Peer Tutorial, Group 47
2020-21, Information Sheet

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Session: Thursday 11am-12noon

Buddy Tutor: Zhe-Hui Lim (Group 8), Ken Lee (Group 44)

I am a third year MSCi mathematics student. I am currently participating a virtual exchange program between Imperial and MiT. I mainly work on Stochastic Analysis and Statistical Inference. The peer tutorial session has been set up to supplement your study. Feel free to ask for help if you need to.

Delivery: There will be **compulsory** weekly meetings every week in Microsoft Teams. We will go through lecture contents, problem sheets and possibly some extensions. I may occasionally record videos on unfinished contents in weekly meetings.

Attendance: Attendance of weekly meetings is compulsory - I am obligated to record your attendance. Please inform me if you cannot attend the weekly meetings.

Questions:

- When you have questions about your study and you want to ask me for help, **message in the MS Teams chat**. I will either answer your question in chat or defer to next regular peer tutorial.
- You are more than welcomed to ask ad-hoc questions during the peer tutorial, but bear in mind 1 hour is just not enough. (We (I) usually spend 15-20 minutes to go through 1 question so **we can only go through 3-4 questions in a session)**.

Survey: Please fill in the MS Team survey before first tutorial.

First Tutorial: The first tutorial will be different from others - it is intended for us to know each other. The plan is as followed:

- First 10 minutes: Self-introduction.
- For the next 20 minutes, I will break you into two groups, A and B.
  - Group A - further discussion on your background, academic interests and setting goals.
  - Group B - I will provide you a set of questions for you to discuss. (You only need to choose 2-3 questions to do).
- Two groups will swap for the next 20 minutes.
- For the final 10 minutes, we will go through some math together.

Disclaimer: I cannot do the coursework for you! We can discuss together, but I cannot give you answers!

Remember the more you tell, the more I can support. Good luck with your studies and see you in the weekly meetings.

The remaining of the sheet is general advice and overview. Please have a read.

Here are some general advice for you:

- What programme are you in? (G100, G103, G104, ...) Do you know your year weightings?
- Pass all your modules! You will be in huge trouble if you don't. **Try to maintain a first honour (70%)** – this can be done by doing enough exercises (not telling you to do all problems though), and **remember all the definitions and theorems.** Nothing extra (*probably)! (Extra: you will probably get into the Dean's List if you get 85% or more).
- **Do all coursework/tests seriously.** Even though their weightings are low, they may change your grade! You certainly don't want to get one mark off from your next grade (e.g. 69%) because you fail in one test.
- **Lectures:** Depends on your learning style. If you find lectures not useful or too boring you can just read books/notes – be sure that you are actually comfortable with the course contents and don't wait until the end of term to catch up! If you are really confident with the lecture content, why not take a break from math or **go ahead and watch lectures in upper years?** (You just need to ask permissions from lecturers and the Undergraduate Office).
- Try to struggle with the question ($\sim$15 min for first year, $\sim$30 min for second year, $>$1 hour for upper years.) After that, seek help from one (**or probably more**) the following (non-exhausive)
  - Ask the lecturer immediately after lecture. (Ideal)
  - Attend **office hours**.
  - Ask your friends.
  - Attend problem classes and ask the Graduate assistants (GTA). (You may join additional sessions for challenging problems).
  - Ask your personal tutor (**Note:** ask topics which are related to their research field only. They may not know questions from other field very well.)
  - Ask your peer tutor, right, that's me.

Here are some general resources you should look for:

- Official lecture notes
- Unofficial lecture notes (they are a bit old though...):
  - Imperial MathSoc `https://union.ic.ac.uk/rcsu/mathsoc/resources`
  - The source codes for some of the notes in Imperial MathSoc (those from Karim Bacchus) are available here: `https://github.com/martinogden/imperial-notes`)
  - MiT Open Courseware `https://ocw.mit.edu/courses/find-by-number/`
  - Oxford Course Management `https://courses.maths.ox.ac.uk/`
  - Cambridge Notes by Dexter Chua `http://dec41.user.srcf.net/`
- Recommended Textbooks: In most of the cases **(especially Year 1)** lecture notes are better than textbooks, but sometimes the textbooks are more organised than notes.
  - The *Schaum's Outline* Series (known as *SOS*) covers many Year 1/2 materials in great detail. (At least better than the *Dummies* series, I guess...)
- Don't forget to make the most of Imperial Library.
- Stack Exchange / Stack Overflow for quick questions.

This is an indicative (not exhausive) list of stuff you will expect throughout the year.

- 1D Analysis:
  - (T1) Countability: Can you compare size of infinite sets. Which of the following set is largest: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$?
  - (T1) Supremum and Infimum: What is the maximum of $\{0, 1\}$? How about $[0, 1]$ and $(0, 1)$? If there is no maximum/minimum, can we try to define some properties of sets which is similar to maximum/minimum? Can you construct roots like $\sqrt{2}$, $\sqrt[3]{4}$?
  - (T1) Sequences of real (and complex) numbers:
    - What does it mean for a sequence (a_n) to actually converge? Can you define a quantitative notion for this? Can you then prove standard results using our definition?
    - A sequence is Cauchy if two terms a_m and a_n become arbitrary closed. Is every convergent sequence Cauchy? **How about the converse?**
    - A sequence in $\mathbb{R}$ is bounded if $|a_m| < M$ for some $M > 0$ uniformly (i.e. holds for all m). Does it necessary converge? If not, can we extract some terms from (a_m) which forms a converging sequence? (We call them subsequences).
  - (T1) Series/Sum:
    - An infinite sum is the limit of partial sum? How do you know when a sum converges? Are there any routine to check if a sum converges?
    - Can we always re-arrange terms?
  - (T1/2) Continuity and Convergence of Function:
    - You should know that a function is continuous if its limit is equal to actual value. We therefore need to define another notion of limit. How is that related to notion of limit of sequences? Can we prove some standard properties of limits?
    - How do you know $f(x) = c$ always have a solution on some interval $[a, b]$? (Intermediate Value Theorem)

    - * Does x, y close always mean $f(x), f(y)$ close? (Think about $f(x) = 1/x$.
  - (T2) Differentiation:
    - * You know that a derivative of function $f(x)$ is some sort of drawing tangent lines. How can you formalise the limit? Can you prove standard properties?
    - * Can you prove Mean Value Theorem, Extreme Value Theorem and Fixed Point Theorem...
    - * Can you find minimum/maximum of functions? Can you ensure that the Taylor Series of a function actually agree with the function itself? Does it even converge?
    - * What does it mean for a function to converge? (Hint: There are AT LEAST TWO WAYS).
  - (T2) Integration:
    - * You know that integration is in some sense finding area under the curves. We may estimate the area by chopping it into rectangles? Does the estimation change when we chop in another way? Can we formalise integrability here?
    - * Can we formalise the fundamental theorem of calculus and link integration with differentiation? Can we formalise calculus tricks like integration by parts and integration by substitution?
    - * Can we exchange limits with differentiation/integration?
- Algebra
  - Do you still remember the notion of linear independence, span and basis?
  - (T1) What is actually a matrix? A matrix is not just an array of numbers! It encodes all information of linear transformations - how a basis of vectors mapped to another basis! Can we always construct a matrix of linear transformation with respect to a basis? Can we always change the underlying basis? Can we use this information to solve linear equations / check if a set of vectors are linear independent and/or span. What is the kernel and image of a linear transformation?

  - (T2) Can we make sense of determinant of matrices?
  - (T2) Can you make sense of eigenvalues? Is a matrix always diagonisable?
  - (T2) What can yo say about symmetric, orthogonal matrices?
  - (T2) Group $(G, +)$ is a set with an operation that is similar to addition. Can you give some examples other than $(\mathbb{Z}, +)$? (Concept check: vector space is actually a group!)
  - (T2) What can you say about the subsets/subgroup of groups? (Think about Lagrange Theorem). Can you apply that to simple theorems in number theory (e.g. Fermat Little Theorem)?
  - (T2) Are two groups essentially the same? (Isomorphism)

- Calculus
  - (T1) Are you a calculus fans? Do you know how to differentiate, integrate, plot graphs, do Taylor expansion, ... There might be some tricks you have missed out!
  - (T1/T2) You probably know how to expand a function with Taylor Series. How about sines/cosines? (Fourier Series/Transform) Can we study the amplitude/frequencies of square waves etc.?
  - (T2) Can you generalise all those tricks to higher dimension?
  - (T2) Do you know how to solve 1D differential equations? Sometimes we can include parameters in differential equations – how does the behavior of solution change w.r.t. parameters?
  - (T2) How about 2D? Sometimes you can't write the solution in closed form, but can you understand qualitative behavior of solutions?

- Probability and Statistics
  - (T1) How do we formalise the concept of probability you have learnt in A-Level? What is outcomes and events in mathematical language? Look at the Kolmogorov's Axioms to Probability Measures (it is a function of sets?!) Can we always define a measure on any sets of sets?
  - (T1) How about random variables? Can you understand the moments (e.g. means and variance) of random variables by

looking at the Laplace Transform/Fourier Transform of random variables? (Does those transform always exists?)

- (T1) Can you generalise those ideas to 2D (or higher dimensions)? Can you talk about independence of random variables?
- (T1) Can you transform random variables?
- (T2) Given a set of data, can you estimate parameters of random variables (those are called estimators)? You may think of sample mean, sample variance etc. How good are your estimates? What are the distributions of those estimators?
- (T2) What is simple linear regression? Once again, how do you estimate the parameters? How good are your estimates? What are the distributions of those estimators?
- (T2) You have learnt how to do Hypothesis Tests? Can you formalise it? We will introduce more hypothesis tests, focusing on the mean of data. How do you use those hypothesis tests in your experiments?
- (T2) There are three large sample results you should bear in mind (they will simplify your calculation): Weak/Strong Law of Large Numbers, Central Limit Theorem. What are they all about? How do you use them to simplify your estimation of parameters?
- We will teach you how to use R language to perform statistical analysis.

• Introduction to Applied Mathematics (T2)

- Do you find analysing forces annoying? How can you analyse forces of truss system in a systematic way?
- Do you find formulae for resistor / spring system similar? How do you generalise them to any complex system using matrix tricks?
- Some Markov Chain - when probability and linear algebra comes together? (Warning: This is a simplified version!)
- How do you understand the conductance of materials? How do you understand harmonic function? Complex Analysis may help here...

- Introduction to Computation: We will teach you how to use Python to perform complex calculation. How do you instruct a computer to perform complex calculation (e.g. if, logic, for/while loop)? What data types does Python have? (e.g. integer, float, array (matrix), list, tuple, dictionary...) We will also walk you through some libraries (e.g. math, cmath, numpy, sympy, scipy, matplotlib, ...) How do you incorporate the programmes in a notebook?
- Additional stuff: Maybe try typesetting in LaTeX.