

# Fair New World

for the seminar: *Fairness in Machine Learning*,  
organized by M. Hardt, Fall 2017, UC Berkeley

Nima Hejazi

Division of Biostatistics  
University of California, Berkeley  
[stat.berkeley.edu/~nhejazi](http://stat.berkeley.edu/~nhejazi)



[nimahejazi.org](http://nimahejazi.org)  
[twitter/@onshejazi](https://twitter.com/onshejazi)  
[github/nhejazi](https://github.com/nhejazi)

[slides](https://slides.goo.gl/8RWEy5): [goo.gl/8RWEy5](https://goo.gl/8RWEy5)



# Preview: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Preview: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Preview: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Preview: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Preview: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Preliminaries: Notation

- ▶ Data  $\mathcal{D} = (Y, \mathbf{X})$ ; outcome  $Y$  and feature vector  $\mathbf{X}$ .
- ▶ Sensitive features:  $S \in \mathbf{X}$ , where inference on  $Y$  using  $S$  *might* result in discrimination.
- ▶ Treatment variable:  $A \in \mathbf{X}$ .
- ▶ Mediator variables:  $M \in \mathbf{X}$  or  $\mathbf{M} \subseteq \mathbf{X}$ .
- ▶ Potential outcome:  $Y(a)$ , realization of  $Y$  under  $A = a$ .

# Preliminaries: Notation

- ▶ Data  $\mathcal{D} = (Y, \mathbf{X})$ ; outcome  $Y$  and feature vector  $\mathbf{X}$ .
- ▶ Sensitive features:  $S \in \mathbf{X}$ , where inference on  $Y$  using  $S$  *might* result in discrimination.
- ▶ Treatment variable:  $A \in \mathbf{X}$ .
- ▶ Mediator variables:  $M \in \mathbf{X}$  or  $\mathbf{M} \subseteq \mathbf{X}$ .
- ▶ Potential outcome:  $Y(a)$ , realization of  $Y$  under  $A = a$ .



# Preliminaries: Notation

- ▶ Data  $\mathcal{D} = (Y, \mathbf{X})$ ; outcome  $Y$  and feature vector  $\mathbf{X}$ .
- ▶ Sensitive features:  $S \in \mathbf{X}$ , where inference on  $Y$  using  $S$  *might* result in discrimination.
- ▶ Treatment variable:  $A \in \mathbf{X}$ .
- ▶ Mediator variables:  $M \in \mathbf{X}$  or  $\mathbf{M} \subseteq \mathbf{X}$ .
- ▶ Potential outcome:  $Y(a)$ , realization of  $Y$  under  $A = a$ .

# Preliminaries: Notation

- ▶ Data  $\mathcal{D} = (Y, \mathbf{X})$ ; outcome  $Y$  and feature vector  $\mathbf{X}$ .
- ▶ Sensitive features:  $S \in \mathbf{X}$ , where inference on  $Y$  using  $S$  *might* result in discrimination.
- ▶ Treatment variable:  $A \in \mathbf{X}$ .
- ▶ Mediator variables:  $M \in \mathbf{X}$  or  $\mathbf{M} \subseteq \mathbf{X}$ .
- ▶ Potential outcome:  $Y(a)$ , realization of  $Y$  under  $A = a$ .

# Preliminaries: Notation

- ▶ Data  $\mathcal{D} = (Y, \mathbf{X})$ ; outcome  $Y$  and feature vector  $\mathbf{X}$ .
- ▶ Sensitive features:  $S \in \mathbf{X}$ , where inference on  $Y$  using  $S$  *might* result in discrimination.
- ▶ Treatment variable:  $A \in \mathbf{X}$ .
- ▶ Mediator variables:  $M \in \mathbf{X}$  or  $\mathbf{M} \subseteq \mathbf{X}$ .
- ▶ Potential outcome:  $Y(a)$ , realization of  $Y$  under  $A = a$ .

# Preliminaries: Mediation Analysis

- ▶ **Goal:** understand the mechanism by which  $A$  influences  $Y$ .
- ▶ Decompose the **ACE** into *direct* and *indirect* effects mediated by a variable  $M$ .
- ▶ Partition feature space  $\mathbf{X}$  into  $A$  (treatment),  $M$  (mediator), and  $\mathbf{C} = \mathbf{X} \setminus \{A, M\}$  (baseline factors).
- ▶ Counterfactual contrasts are expressed via *nested* potential outcomes (i.e.,  $Y(a, M(a'))$ ).

# Preliminaries: Mediation Analysis

- ▶ **Goal:** understand the mechanism by which  $A$  influences  $Y$ .
- ▶ Decompose the **ACE** into *direct* and *indirect* effects mediated by a variable  $M$ .
- ▶ Partition feature space  $\mathbf{X}$  into  $A$  (treatment),  $M$  (mediator), and  $\mathbf{C} = \mathbf{X} \setminus \{A, M\}$  (baseline factors).
- ▶ Counterfactual contrasts are expressed via *nested* potential outcomes (i.e.,  $Y(a, M(a'))$ ).

# Preliminaries: Mediation Analysis

- ▶ **Goal:** understand the mechanism by which  $A$  influences  $Y$ .
- ▶ Decompose the **ACE** into *direct* and *indirect* effects mediated by a variable  $M$ .
- ▶ Partition feature space  $\mathbf{X}$  into  $A$  (treatment),  $M$  (mediator), and  $\mathbf{C} = \mathbf{X} \setminus \{A, M\}$  (baseline factors).
- ▶ Counterfactual contrasts are expressed via *nested* potential outcomes (i.e.,  $Y(a, M(a'))$ ).

# Preliminaries: Mediation Analysis

- ▶ **Goal:** understand the mechanism by which  $A$  influences  $Y$ .
- ▶ Decompose the **ACE** into *direct* and *indirect* effects mediated by a variable  $M$ .
- ▶ Partition feature space  $\mathbf{X}$  into  $A$  (treatment),  $M$  (mediator), and  $\mathbf{C} = \mathbf{X} \setminus \{A, M\}$  (baseline factors).
- ▶ Counterfactual contrasts are expressed via *nested* potential outcomes (i.e.,  $Y(a, M(a'))$ ).

# The Average Causal Effect (ACE)

- ▶  $ACE = \mathbb{E}[Y(\mathbf{a})] - \mathbb{E}[Y(\mathbf{a}')] ]$
- ▶ Not computed via  $\mathbb{E}[Y \mid A]$ , as associations between  $A$  and  $Y$  may be “partly causal” or spurious.
- ▶ Decomposition:  $ACE = NDE + NIE$ , where **NDE** is the *Natural Direct Effect* and **NIE** is the *Natural Indirect Effect*.

$$\begin{aligned} ACE &= \mathbb{E}[Y(\mathbf{a})] - \mathbb{E}[Y(\mathbf{a}')] ] \\ &= \mathbb{E}[Y(\mathbf{a})] - \mathbb{E}[Y(\mathbf{a}, M(\mathbf{a}')) ] \\ &\quad + \mathbb{E}[(Y(\mathbf{a}, M(\mathbf{a}')) - \mathbb{E}[Y(\mathbf{a}')) ] \end{aligned}$$



# The Average Causal Effect (ACE)

- ▶  $ACE = \mathbb{E}[Y(a)] - \mathbb{E}[Y(a')]$
- ▶ Not computed via  $\mathbb{E}[Y | A]$ , as associations between  $A$  and  $Y$  may be “partly causal” or spurious.
- ▶ Decomposition:  $ACE = NDE + NIE$ , where **NDE** is the *Natural Direct Effect* and **NIE** is the *Natural Indirect Effect*.

$$\begin{aligned} ACE &= \mathbb{E}[Y(a)] - \mathbb{E}[Y(a')] \\ &= \mathbb{E}[Y(a)] - \mathbb{E}[Y(a, M(a'))] \\ &\quad + \mathbb{E}[(Y(a, M(a')) - \mathbb{E}[Y(a')])] \end{aligned}$$

# The Average Causal Effect (ACE)

- ▶  $ACE = \mathbb{E}[Y(a)] - \mathbb{E}[Y(a')]$
- ▶ Not computed via  $\mathbb{E}[Y | A]$ , as associations between  $A$  and  $Y$  may be “partly causal” or spurious.
- ▶ Decomposition:  $ACE = NDE + NIE$ , where **NDE** is the *Natural Direct Effect* and **NIE** is the *Natural Indirect Effect*.

$$\begin{aligned} ACE &= \mathbb{E}[Y(a)] - \mathbb{E}[Y(a')] \\ &= \mathbb{E}[Y(a)] - \mathbb{E}[Y(a, M(a'))] \\ &\quad + \mathbb{E}[(Y(a, M(a')) - \mathbb{E}[Y(a')]) \end{aligned}$$

# The Natural *Direct* Effect (NDE)

- ▶ Comparison of the mean outcome under only the part of the treatment that directly affects it ( $A = a$ ) and the placebo treatment (i.e.,  $A = a'$ ).
- ▶ Note that the *indirect* effect of the treatment (through the mediator  $M$ ) is “turned off” (i.e.,  $M(A = a')$ ).

## Definition

Natural **Direct** Effect

$$\text{NDE} = \mathbb{E}[(Y(a, M(a')))] - \mathbb{E}[Y(a')]$$

# The Natural *Direct* Effect (NDE)

- ▶ Comparison of the mean outcome under only the part of the treatment that directly affects it ( $A = a$ ) and the placebo treatment (i.e.,  $A = a'$ ).
- ▶ Note that the *indirect* effect of the treatment (through the mediator  $M$ ) is “turned off” (i.e.,  $M(A = a')$ ).

## Definition

Natural **Direct** Effect

$$\text{NDE} = \mathbb{E}[(Y(a, M(a')))] - \mathbb{E}[Y(a')]$$

# The Natural *Direct* Effect (NDE)

- ▶ Comparison of the mean outcome under only the part of the treatment that directly affects it ( $A = a$ ) and the placebo treatment (i.e.,  $A = a'$ ).
- ▶ Note that the *indirect* effect of the treatment (through the mediator  $M$ ) is “turned off” (i.e.,  $M(A = a')$ ).

## Definition

### Natural **Direct** Effect

$$\text{NDE} = \mathbb{E}[Y(a, M(a'))] - \mathbb{E}[Y(a')]$$

# The Natural *Indirect* Effect (NIE)

- ▶ Comparison of the outcome affected by all treatment (both direct and indirect) and the outcome where the effect through the mediator ( $M$ ) is “turned off” (i.e.,  $M(A = a')$ ).
- ▶ Although in a roundabout manner, this quantity gets at the effect of the path-specific effect through the mediator on the outcome.

## Definition

Natural **Indirect** Effect

$$\text{NIE} = \mathbb{E}[Y(a)] - \mathbb{E}[(Y(a, M(a'))]$$

# The Natural *Indirect* Effect (NIE)

- ▶ Comparison of the outcome affected by all treatment (both direct and indirect) and the outcome where the effect through the mediator ( $M$ ) is “turned off” (i.e.,  $M(A = a')$ ).
- ▶ Although in a roundabout manner, this quantity gets at the effect of the path-specific effect through the mediator on the outcome.

## Definition

Natural **Indirect** Effect

$$\text{NIE} = \mathbb{E}[Y(a)] - \mathbb{E}[(Y(a, M(a'))]$$

# The Natural *Indirect* Effect (NIE)

- ▶ Comparison of the outcome affected by all treatment (both direct and indirect) and the outcome where the effect through the mediator ( $M$ ) is “turned off” (i.e.,  $M(A = a')$ ).
- ▶ Although in a roundabout manner, this quantity gets at the effect of the path-specific effect through the mediator on the outcome.

## Definition

### Natural **I**ndirect Effect

$$\text{NIE} = \mathbb{E}[Y(a)] - \mathbb{E}[(Y(a, M(a'))]$$



## Example: *Thank You for Smoking*

For a better intuition of  $Y(a, M(a'))$ , consider the following:

- ▶ Let  $Y$  be a health outcome (e.g., survival probability),  $A$  be a treatment (e.g., smoking).
- ▶ Consider a decomposition of the effect of  $A$  on  $Y$  — that is, let  $M$  be a mediator (e.g., cancer).
- ▶  $A$  affects  $Y$  directly (nicotine exposure) and indirectly (inducing lung cancer, through  $M$ ).
- ▶ Here,  $Y(a, M(a'))$  corresponds to “the response of  $Y$  to an intervention that sets the nicotine exposure (direct effect) to what it would be in smokers, and the smoke exposure (indirect effect) to what it would be in non-smokers” (e.g., nicotine patch).

## Example: *Thank You for Smoking*

For a better intuition of  $Y(a, M(a'))$ , consider the following:

- ▶ Let  $Y$  be a health outcome (e.g., survival probability),  $A$  be a treatment (e.g., smoking).
- ▶ Consider a decomposition of the effect of  $A$  on  $Y$  — that is, let  $M$  be a mediator (e.g., cancer).
- ▶  $A$  affects  $Y$  directly (nicotine exposure) and indirectly (inducing lung cancer, through  $M$ ).
- ▶ Here,  $Y(a, M(a'))$  corresponds to “the response of  $Y$  to an intervention that sets the nicotine exposure (direct effect) to what it would be in smokers, and the smoke exposure (indirect effect) to what it would be in non-smokers” (e.g., nicotine patch).

## Example: *Thank You for Smoking*

For a better intuition of  $Y(a, M(a'))$ , consider the following:

- ▶ Let  $Y$  be a health outcome (e.g., survival probability),  $A$  be a treatment (e.g., smoking).
- ▶ Consider a decomposition of the effect of  $A$  on  $Y$  — that is, let  $M$  be a mediator (e.g., cancer).
- ▶  $A$  affects  $Y$  directly (nicotine exposure) and indirectly (inducing lung cancer, through  $M$ ).
- ▶ Here,  $Y(a, M(a'))$  corresponds to “the response of  $Y$  to an intervention that sets the nicotine exposure (direct effect) to what it would be in smokers, and the smoke exposure (indirect effect) to what it would be in non-smokers” (e.g., nicotine patch).

## Example: *Thank You for Smoking*

For a better intuition of  $Y(a, M(a'))$ , consider the following:

- ▶ Let  $Y$  be a health outcome (e.g., survival probability),  $A$  be a treatment (e.g., smoking).
- ▶ Consider a decomposition of the effect of  $A$  on  $Y$  — that is, let  $M$  be a mediator (e.g., cancer).
- ▶  $A$  affects  $Y$  directly (nicotine exposure) and indirectly (inducing lung cancer, through  $M$ ).
- ▶ Here,  $Y(a, M(a'))$  corresponds to “the response of  $Y$  to an intervention that sets the nicotine exposure (direct effect) to what it would be in smokers, and the smoke exposure (indirect effect) to what it would be in non-smokers” (e.g., nicotine patch).

# Path-Specific Effects

- ▶ A more general idea than the NDE and NIE — such effects are easily formulated as nested counterfactuals.
- ▶ *Intuition:* along a path of interest, all nodes behave as if the active rule were imposed (i.e.,  $A = a$ ) while, along all other paths, nodes behave as though the alternative were the case (i.e.,  $A = a'$ ).

## Definition

### Path-Specific Effect (PSE)

(Along a path, say  $A \rightarrow W \rightarrow Y$ )

$$\mathbb{E}[Y(a', W(M(a'), a), M(a'))] - \mathbb{E}[Y(a')]$$

# Path-Specific Effects

- ▶ A more general idea than the NDE and NIE — such effects are easily formulated as nested counterfactuals.
- ▶ *Intuition:* along a path of interest, all nodes behave as if the active rule were imposed (i.e.,  $A = a$ ) while, along all other paths, nodes behave as though the alternative were the case (i.e.,  $A = a'$ ).

## Definition

### Path-Specific Effect (PSE)

(Along a path, say  $A \rightarrow W \rightarrow Y$ )

$$\mathbb{E}[Y(a', W(M(a'), a), M(a'))] - \mathbb{E}[Y(a')]$$

# Path-Specific Effects

- ▶ A more general idea than the NDE and NIE — such effects are easily formulated as nested counterfactuals.
- ▶ *Intuition:* along a path of interest, all nodes behave as if the active rule were imposed (i.e.,  $A = a$ ) while, along all other paths, nodes behave as though the alternative were the case (i.e.,  $A = a'$ ).

## Definition

### Path-Specific Effect (PSE)

(Along a path, say  $A \rightarrow W \rightarrow Y$ )

$$\mathbb{E}[Y(a', W(M(a'), a), M(a'))] - \mathbb{E}[Y(a')]$$

# Finding Fairness

- ▶ Much work has focused on defining fairness via associative relationships (including equalized odds). Such criteria provided unintuitive results when the sensitive feature is not randomly assigned.
- ▶ Here, an approach that ought to provide intuitive results (wrt fairness), even when the sensitive attribute is associated with the outcome (perhaps by way of an unobserved feature), is proposed.
- ▶ Associative fairness metrics fail to properly model sources of confounding (between  $S$  and  $Y$ ).
- ▶ Generally, this failure is rooted in the fact that “counterfactual probabilities are complex functions of the observed data, not just conditional densities.”



# Finding Fairness

- ▶ Much work has focused on defining fairness via associative relationships (including equalized odds). Such criteria provided unintuitive results when the sensitive feature is not randomly assigned.
- ▶ Here, an approach that ought to provide intuitive results (wrt fairness), even when the sensitive attribute is associated with the outcome (perhaps by way of an unobserved feature), is proposed.
- ▶ Associative fairness metrics fail to properly model sources of confounding (between  $S$  and  $Y$ ).
- ▶ Generally, this failure is rooted in the fact that “counterfactual probabilities are complex functions of the observed data, not just conditional densities.”

# Finding Fairness

- ▶ Much work has focused on defining fairness via associative relationships (including equalized odds). Such criteria provided unintuitive results when the sensitive feature is not randomly assigned.
- ▶ Here, an approach that ought to provide intuitive results (wrt fairness), even when the sensitive attribute is associated with the outcome (perhaps by way of an unobserved feature), is proposed.
- ▶ Associative fairness metrics fail to properly model sources of confounding (between  $S$  and  $Y$ ).
- ▶ Generally, this failure is rooted in the fact that “counterfactual probabilities are complex functions of the observed data, not just conditional densities.”

# Finding Fairness

- ▶ Much work has focused on defining fairness via associative relationships (including equalized odds). Such criteria provided unintuitive results when the sensitive feature is not randomly assigned.
- ▶ Here, an approach that ought to provide intuitive results (wrt fairness), even when the sensitive attribute is associated with the outcome (perhaps by way of an unobserved feature), is proposed.
- ▶ Associative fairness metrics fail to properly model sources of confounding (between  $S$  and  $Y$ ).
- ▶ Generally, this failure is rooted in the fact that “counterfactual probabilities are complex functions of the observed data, not just conditional densities.”

# In Pursuit of “Fair Inference”

- ▶ Fairness is, at its core, rooted in counterfactuals. Thus, we can see “*fair inference*” as a branch of causal inference wherein the counterfactuals to be considered are with respect to a “fair” world.
- ▶ *Discrimination* may be expressed as the presence of a particular PSE, with choice of the specific PSE left as a domain-specific issue.
- ▶ Thus, minimization of specific PSEs corresponds to minimizing discrimination and is a problem of constrained inference on statistical models.

# In Pursuit of “Fair Inference”

- ▶ Fairness is, at its core, rooted in counterfactuals. Thus, we can see “*fair inference*” as a branch of causal inference wherein the counterfactuals to be considered are with respect to a “fair” world.
- ▶ *Discrimination* may be expressed as the presence of a particular PSE, with choice of the specific PSE left as a domain-specific issue.
- ▶ Thus, minimization of specific PSEs corresponds to minimizing discrimination and is a problem of constrained inference on statistical models.

# In Pursuit of “Fair Inference”

- ▶ Fairness is, at its core, rooted in counterfactuals. Thus, we can see “*fair inference*” as a branch of causal inference wherein the counterfactuals to be considered are with respect to a “fair” world.
- ▶ *Discrimination* may be expressed as the presence of a particular PSE, with choice of the specific PSE left as a domain-specific issue.
- ▶ Thus, minimization of specific PSEs corresponds to minimizing discrimination and is a problem of constrained inference on statistical models.

# Fairness as PSE Minimization

- ▶ Let  $p(Y, \mathbf{X})$  be a statistical model, assumed to be induced by a *causal model*.
- ▶ Discrimination (wrt  $Y$  based on  $S \in \mathbf{X}$ ) in this model is a PSE, identified as the functional  $f(p(Y, \mathbf{X}))$ .
- ▶ Let  $(\epsilon_l, \epsilon_u)$  be lower and upper bounds on the PSE, giving the degree of unfairness considered tolerable (n.b., the PSE is removed in the special case  $\epsilon_l = \epsilon_u$ ).
- ▶ **Proposal**: transform  $p(Y, \mathbf{X})$  into  $p^*(Y, \mathbf{X})$  under the constraint that the PSE of interest lies within  $(\epsilon_l, \epsilon_u)$ , where the two distributions are close in the sense of KL-divergence.

# Fairness as PSE Minimization

- ▶ Let  $p(Y, \mathbf{X})$  be a statistical model, assumed to be induced by a *causal model*.
- ▶ Discrimination (wrt  $Y$  based on  $S \in \mathbf{X}$ ) in this model is a PSE, identified as the functional  $f(p(Y, \mathbf{X}))$ .
- ▶ Let  $(\epsilon_l, \epsilon_u)$  be lower and upper bounds on the PSE, giving the degree of unfairness considered tolerable (n.b., the PSE is removed in the special case  $\epsilon_l = \epsilon_u$ ).
- ▶ **Proposal:** transform  $p(Y, \mathbf{X})$  into  $p^*(Y, \mathbf{X})$  under the constraint that the PSE of interest lies within  $(\epsilon_l, \epsilon_u)$ , where the two distributions are close in the sense of KL-divergence.



# Fairness as PSE Minimization

- ▶ Let  $p(Y, \mathbf{X})$  be a statistical model, assumed to be induced by a *causal model*.
- ▶ Discrimination (wrt  $Y$  based on  $S \in \mathbf{X}$ ) in this model is a PSE, identified as the functional  $f(p(Y, \mathbf{X}))$ .
- ▶ Let  $(\epsilon_l, \epsilon_u)$  be lower and upper bounds on the PSE, giving the degree of unfairness considered tolerable (n.b., the PSE is removed in the special case  $\epsilon_l = \epsilon_u$ ).
- ▶ **Proposal:** transform  $p(Y, \mathbf{X})$  into  $p^*(Y, \mathbf{X})$  under the constraint that the PSE of interest lies within  $(\epsilon_l, \epsilon_u)$ , where the two distributions are close in the sense of KL-divergence.

# Fairness as PSE Minimization

- ▶ Let  $p(Y, \mathbf{X})$  be a statistical model, assumed to be induced by a *causal model*.
- ▶ Discrimination (wrt  $Y$  based on  $S \in \mathbf{X}$ ) in this model is a PSE, identified as the functional  $f(p(Y, \mathbf{X}))$ .
- ▶ Let  $(\epsilon_l, \epsilon_u)$  be lower and upper bounds on the PSE, giving the degree of unfairness considered tolerable (n.b., the PSE is removed in the special case  $\epsilon_l = \epsilon_u$ ).
- ▶ **Proposal**: transform  $p(Y, \mathbf{X})$  into  $p^*(Y, \mathbf{X})$  under the constraint that the PSE of interest lies within  $(\epsilon_l, \epsilon_u)$ , where the two distributions are close in the sense of KL-divergence.

# Finding Fair Worlds I

- ▶ **Proposal**: We can make *any* function of  $p$  **fair**, merely by computing it from  $p^*$  (instead of from  $p$ ).
- ▶ To ensure fairness, we must make inference only in the “fair world”, just as we only perform inference on counterfactuals in causal inference.
- ▶ To do this, map any  $x^i$  from  $p$  to a sensible version of it drawn from  $p^*$  — i.e., find a  $g : x_p^i \mapsto x_{p^*}^i$ .
- ▶ I want to be fair, so what exactly do I do?

# Finding Fair Worlds I

- ▶ **Proposal**: We can make *any* function of  $p$  ***fair***, merely by computing it from  $p^*$  (instead of from  $p$ ).
- ▶ To ensure fairness, we must make inference only in the “fair world”, just as we only perform inference on counterfactuals in causal inference.
- ▶ To do this, map any  $x^i$  from  $p$  to a sensible version of it drawn from  $p^*$  — i.e., find a  $g : x_p^i \mapsto x_{p^*}^i$ .
- ▶ I want to be fair, so what exactly do I do?

# Finding Fair Worlds I

- ▶ **Proposal**: We can make *any* function of  $p$  **fair**, merely by computing it from  $p^*$  (instead of from  $p$ ).
- ▶ To ensure fairness, we must make inference only in the “fair world”, just as we only perform inference on counterfactuals in causal inference.
- ▶ To do this, map any  $x^i$  from  $p$  to a sensible version of it drawn from  $p^*$  — i.e., find a  $g : x_p^i \mapsto x_{p^*}^i$ .
- ▶ I want to be fair, so what exactly do I do?

# Finding Fair Worlds I

- ▶ **Proposal**: We can make *any* function of  $p$  ***fair***, merely by computing it from  $p^*$  (instead of from  $p$ ).
- ▶ To ensure fairness, we must make inference only in the “fair world”, just as we only perform inference on counterfactuals in causal inference.
- ▶ To do this, map any  $x^i$  from  $p$  to a sensible version of it drawn from  $p^*$  — i.e., find a  $g : x_p^i \mapsto x_{p^*}^i$ .
- ▶ I want to be fair, so what exactly do I do?

# Finding Fair Worlds II

- ▶ Consider the following general setup:
  - finite samples  $\mathcal{D}$  drawn from  $p(Y, \mathbf{X})$
  - a likelihood function  $\mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha)$
  - a discriminative PSE  $f(p(Y, \mathbf{X}))$  with bounds  $(\epsilon_l, \epsilon_u)$
  - an estimator of the PSE  $g(\mathcal{D})$ .
- ▶ We obtain fairness by solving:

$$\hat{\alpha} = \arg \max_{\alpha} \mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha),$$

subject to  $\epsilon_l \leq g(\mathcal{D}) \leq \epsilon_u$ .

- ▶ In this setup, fairness is achieved by constraining parts of  $p(Y, \mathbf{X}; \alpha)$ , with the choice of  $g$  determining exactly what is constrained.

# Finding Fair Worlds II

- ▶ Consider the following general setup:
  - finite samples  $\mathcal{D}$  drawn from  $p(Y, \mathbf{X})$
  - a likelihood function  $\mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha)$
  - a discriminative PSE  $f(p(Y, \mathbf{X}))$  with bounds  $(\epsilon_l, \epsilon_u)$
  - an estimator of the PSE  $g(\mathcal{D})$ .
- ▶ We obtain fairness by solving:

$$\hat{\alpha} = \arg \max_{\alpha} \mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha),$$

subject to  $\epsilon_l \leq g(\mathcal{D}) \leq \epsilon_u$ .

- ▶ In this setup, fairness is achieved by constraining parts of  $p(Y, \mathbf{X}; \alpha)$ , with the choice of  $g$  determining exactly what is constrained.



# Finding Fair Worlds II

- ▶ Consider the following general setup:
  - finite samples  $\mathcal{D}$  drawn from  $p(Y, \mathbf{X})$
  - a likelihood function  $\mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha)$
  - a discriminative PSE  $f(p(Y, \mathbf{X}))$  with bounds  $(\epsilon_l, \epsilon_u)$
  - an estimator of the PSE  $g(\mathcal{D})$ .
- ▶ We obtain fairness by solving:

$$\hat{\alpha} = \arg \max_{\alpha} \mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha),$$

subject to  $\epsilon_l \leq g(\mathcal{D}) \leq \epsilon_u$ .

- ▶ In this setup, fairness is achieved by constraining parts of  $p(Y, \mathbf{X}; \alpha)$ , with the choice of  $g$  determining exactly what is constrained.

# Finding Fair Worlds II

- ▶ Consider the following general setup:
  - finite samples  $\mathcal{D}$  drawn from  $p(Y, \mathbf{X})$
  - a likelihood function  $\mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha)$
  - a discriminative PSE  $f(p(Y, \mathbf{X}))$  with bounds  $(\epsilon_l, \epsilon_u)$
  - an estimator of the PSE  $g(\mathcal{D})$ .
- ▶ We obtain fairness by solving:

$$\hat{\alpha} = \arg \max_{\alpha} \mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha),$$

subject to  $\epsilon_l \leq g(\mathcal{D}) \leq \epsilon_u$ .

- ▶ In this setup, fairness is achieved by constraining parts of  $p(Y, \mathbf{X}; \alpha)$ , with the choice of  $g$  determining exactly what is constrained.

# Finding Fair Worlds II

- ▶ Consider the following general setup:
  - finite samples  $\mathcal{D}$  drawn from  $p(Y, \mathbf{X})$
  - a likelihood function  $\mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha)$
  - a discriminative PSE  $f(p(Y, \mathbf{X}))$  with bounds  $(\epsilon_l, \epsilon_u)$
  - an estimator of the PSE  $g(\mathcal{D})$ .
- ▶ We obtain fairness by solving:

$$\hat{\alpha} = \arg \max_{\alpha} \mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha),$$

subject to  $\epsilon_l \leq g(\mathcal{D}) \leq \epsilon_u$ .

- ▶ In this setup, fairness is achieved by constraining parts of  $p(Y, \mathbf{X}; \alpha)$ , with the choice of  $g$  determining exactly what is constrained.

# Finding Fair Worlds II

- ▶ Consider the following general setup:
  - finite samples  $\mathcal{D}$  drawn from  $p(Y, \mathbf{X})$
  - a likelihood function  $\mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha)$
  - a discriminative PSE  $f(p(Y, \mathbf{X}))$  with bounds  $(\epsilon_l, \epsilon_u)$
  - an estimator of the PSE  $g(\mathcal{D})$ .
- ▶ We obtain fairness by solving:

$$\hat{\alpha} = \arg \max_{\alpha} \mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha),$$

subject to  $\epsilon_l \leq g(\mathcal{D}) \leq \epsilon_u$ .

- ▶ In this setup, fairness is achieved by constraining parts of  $p(Y, \mathbf{X}; \alpha)$ , with the choice of  $g$  determining exactly what is constrained.

# Finding Fair Worlds II

- ▶ Consider the following general setup:
  - finite samples  $\mathcal{D}$  drawn from  $p(Y, \mathbf{X})$
  - a likelihood function  $\mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha)$
  - a discriminative PSE  $f(p(Y, \mathbf{X}))$  with bounds  $(\epsilon_l, \epsilon_u)$
  - an estimator of the PSE  $g(\mathcal{D})$ .
- ▶ We obtain fairness by solving:

$$\hat{\alpha} = \arg \max_{\alpha} \mathcal{L}_{Y, \mathbf{X}}(\mathcal{D}; \alpha),$$

subject to  $\epsilon_l \leq g(\mathcal{D}) \leq \epsilon_u$ .

- ▶ In this setup, fairness is achieved by constraining parts of  $p(Y, \mathbf{X}; \alpha)$ , with the choice of  $g$  determining exactly what is constrained.

# Fairness is (Partial?) (Un)Awareness

- ▶ Since using all of the information contained in  $p$  leads to unfairness, this approach amounts to discarding information that is exclusively in  $p$ , relative to  $p^*$ .
- ▶ The goal of this approach is to use the available information as well as possible, but only in so far as our inferences are drawn from the “fair world.”
- ▶ In this approach, fairness is characterized as the *a priori* inadmissability of certain paths in the DAG of interest — that is, paths other than a single edge path might cause discrimination.

# Fairness is (Partial?) (Un)Awareness

- ▶ Since using all of the information contained in  $p$  leads to unfairness, this approach amounts to discarding information that is exclusively in  $p$ , relative to  $p^*$ .
- ▶ The goal of this approach is to use the available information as well as possible, but only in so far as our inferences are drawn from the “fair world.”
- ▶ In this approach, fairness is characterized as the *a priori* inadmissability of certain paths in the DAG of interest — that is, paths other than a single edge path might cause discrimination.

# Fairness is (Partial?) (Un)Awareness

- ▶ Since using all of the information contained in  $p$  leads to unfairness, this approach amounts to discarding information that is exclusively in  $p$ , relative to  $p^*$ .
- ▶ The goal of this approach is to use the available information as well as possible, but only in so far as our inferences are drawn from the “fair world.”
- ▶ In this approach, fairness is characterized as the *a priori* inadmissability of certain paths in the DAG of interest — that is, paths other than a single edge path might cause discrimination.



# Review: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Review: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Review: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Review: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# Review: Summary

- ▶ Mediation analysis provides a framework under which intuitive definitions of fairness may be expressed.
- ▶ “Fair inference” is analogous to causal inference, except in that the counterfactuals explored refer to a “fair” world (n. b., intentionally vague).
- ▶ Fairness may be characterized as the absence (or dampening) of a **path-specific effect (PSE)**.
- ▶ Restriction of a PSE is easily expressed as a likelihood maximization problem that features constraining the magnitude of the undesirable PSE.
- ▶ This approach to fairness avoids throwing away information (i.e., “fairness through unawareness”) but leaves the definition of fairness to the analyst.

# References I

- Hardt, M., Price, E., Srebro, N., et al. (2016). Equality of opportunity in supervised learning. In *Advances in Neural Information Processing Systems*, pages 3315–3323.
- Miles, C. H., Kanki, P., Meloni, S., and Tchetgen, E. J. T. (2015). On partial identification of the pure direct effect. *arXiv preprint arXiv:1509.01652*.
- Nabi, R. and Shpitser, I. (2017). Fair Inference On Outcomes. *ArXiv e-prints*.
- Pearl, J. (2001). Direct and indirect effects. In *Proceedings of the seventeenth conference on uncertainty in artificial intelligence*, pages 411–420. Morgan Kaufmann Publishers Inc.

## References II

- Pearl, J. (2009). *Causality: Models, Reasoning, and Inference*. Cambridge University Press.
- Robins, J. M. (2000). Marginal structural models versus structural nested models as tools for causal inference. In *Statistical models in epidemiology, the environment, and clinical trials*, pages 95–133. Springer.
- Tchetgen, E. J. T. and Shpitser, I. (2012). Semiparametric theory for causal mediation analysis: efficiency bounds, multiple robustness, and sensitivity analysis. *Annals of statistics*, 40(3):1816.

# Thank you.

Slides: [goo.gl/i3CxL9](https://goo.gl/i3CxL9)



Notes: [goo.gl/8RWEy5](https://goo.gl/8RWEy5)

Source (repo): [goo.gl/qJSoz6](https://goo.gl/qJSoz6)

[stat.berkeley.edu/~nhejazi](https://stat.berkeley.edu/~nhejazi)

[nimahejazi.org](https://nimahejazi.org)

[twitter/@nshejazi](https://twitter.com/nshejazi)

[github/nhejazi](https://github.com/nhejazi)