Name: ________________________________

PHYSICS 619 : SPRING SEMESTER 2019

Project #1: Keplerian Orbits

Please note: For all future project assignments, please write your name above, use this assignment sheet as your cover sheet and staple this to the front of your work. Answer questions, DO NOT just throw in graphs without stating which part of the assignment you are doing.

Compute the orbit of the Kepler problem in 2-D defined by the equation of motion

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\mathbf{r}}{r^3}.$$

Start the motion at $\mathbf{r}_0 = (x_0, 0)$ with $\mathbf{v}_0 = (0, v_0)$, where $x_0 = 10$, $v_0 = 1/10$.

1. First, learn to output your data graphically by plotting out the exact orbit given by

   $$r = \frac{p}{1 \pm e\cos(\theta)}$$

   where $p = h^2$, $h = x_0 v_0$ and $e = \sqrt{1 - p/a}$. Use the *minus sign* in the present case. The initial energy is $E_0 = (1/2)v_0^2 - 1/x_0$, the semi-major axis $a = -1/(2E_0)$ and the period $P = 2\pi a^{3/2}$. To convert the above trajectory in polar coordinates back to Cartesian coordinates, use $x = r\cos(\theta)$ and $y = r\sin(\theta)$. (This is already done for you if you run my "kepler.f90" program. The reslt is then contained in the file "ellipse.pl".

2. Next, directly integrate the equation of motion by use of a) the so called "Cromer" algorithm, b) the second order Runge-Kutta algorithm and c) the velocity Verlet algorithm. Integrate for 10 complete periods with time step size $\Delta t = P/1000$. For each algorithm plot the resulting trajectory as a solid line and the exact trajectory as just hollow circles. Hand in your three programs and the three plots. **Save paper** by printing your program listings on both sides using the double-column format.

3. For each algorithm, plot $E(t)/E_0 - 1$ as a function of time, in unit of the period, *i.e.*, as a function of t/P. To see the shape of the energy error function, plot only from t/P=0.45 to t/P=0.55 . $E(t) = (1/2)\mathbf{v}^2(t) - 1/r(t)$ is the algorithm's energy along the trajectory. Plot all three curves in the same plot. Comment on how well each algorithm conserves energy.
4. Comment on how well each algorithm conserves angular momentum $\mathbf{r} \times \mathbf{v}$.