Name: ____________________, ____________________

PHYSICS 619 : SPRING SEMESTER 2019

Project #6: Numerical and MC Integrations

1) a) Evaluate the integral analytically:

$$I = \int_{-1}^{1} (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5) dx.$$

We have shown in class that the 3-point Gaussian quadrature formula reproduces the exact answer I. a) What is the result, I_{3S}, obtained by three-point Simpson's rule in evaluating this integral? What is the error $I_{3S} - I$? b) What is the result, I_{2G}, obtained by the two-point Gaussian rule in evaluating this integral? What is the error $I_{2G} - I$?

2) a) Now compute the integral

$$I = \int_0^1 dx f(x) = \int_0^1 dx \frac{4}{1+x^2} \tag{1}$$

by use of a 3-point Simpson rule, a 2-point Gaussian, and a 3-point Gaussian rule. Compare your answer for I to π. (DO NOT symmetrize the integral to $I = \frac{1}{2}\int_{-1}^{1} dx f(x)$, why?) b) Redo the integral as

$$I = \int_0^{1/2} dx \frac{4}{1+x^2} + \int_{1/2}^{1} dx \frac{4}{1+x^2} \tag{2}$$

by apply each rule twice. What are your results I for the three rules?

3) Compute the integral (1) again but this time uses the simple Monte Carlo method. Use 10,000 sampling points and plot a) $f(x_i)$ as a function of i from $i = 1$ to $i = 10,000$, where x_i is uniformly distributed over [0,1]. Set the vertical scale of this graph from 1.9 to 4.1 so that you can see how $f(x_i)$ fluctuates. b) Plot also the running average $I_n = (1/n)\sum_{i=1}^{n} f(x_i)$ as a function of n and the line π (use different colors if possible). Hand in this graph.

4) Compute the integral (1) again but this time factor $f(x) = h(x)p(x)$ and take $p(x) = A\mathrm{e}^{-ax}$. a) What must be A so that now $\int_0^1 p(x)dx = 1$? b) Choose a so that $f(x)/p(x)$ is as constant as possible and plot $h(x_i)$ as a function of i from $i = 1$ to $i = 10,000$, where x_i is now distributed according to $p(x)$. As in 2), plot both $h(x_i)$ and its average I_n. *Be sure to use the same vertical scale as in 2) so that you can see the improvment.* Hand in this plot. What value of a would minimize the fluctuations of $h(x_i)$? Hand in this plot.