Langevin algorithm $\longleftrightarrow$ Fokker-Planck Eq

↳ approaches to equilibrium

$$-\frac{\partial}{\partial t}\rho = \underbrace{-\frac{1}{2}\nabla^2\rho + \vec{\nabla}(\vec{v}\rho)}_{=0}$$

external

$$\vec{v} = \frac{\vec{\nabla}\phi}{\phi} \qquad \phi = e^{-S}$$

at equilibrium

$$\frac{\partial}{\partial t}\rho = 0$$

$$\vec{\nabla}\cdot\left(-\frac{1}{2}\vec{\nabla}\rho + \vec{v}\rho\right) = 0 \qquad = -\vec{\nabla}S(r) = -S'(r)\hat{r}$$

$$\frac{\vec{\nabla}\rho}{\rho} = 2\vec{v} = 2\frac{\vec{\nabla}\phi}{\phi} = -2\vec{\nabla}S$$

$$\rho = e^{-2S} = \phi^2$$

two terms

1) $-\frac{\partial}{\partial t}\rho = -\frac{1}{2}\nabla^2\rho$ ↙ diffusion

2) $-\frac{\partial}{\partial t}\rho = \vec{\nabla}(\vec{v}\rho)$ ↙ continuity

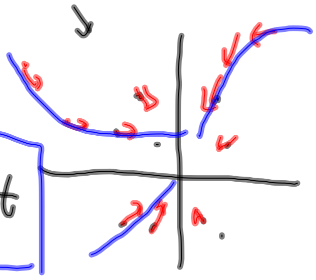
$\rho = \{x_i\}$ is the density of

1) $$\boxed{\vec{x}_i' = \vec{x}_i + \sqrt{\delta t}\,\vec{g}_i} \leftarrow \text{unit Gaussian}$$

$$\frac{d\vec{x}_i}{dt} = \vec{v}(\vec{x}_i)$$

to first order

$\parallel$



2) $$\boxed{\vec{x}_i' = x_i + \vec{v}(\vec{x}_i)\,\delta t}$$

First-order Langevin

$$\vec{x}_i' = \vec{x}_i + \vec{v}(\vec{x}_i)\,\delta t + \sqrt{\delta t}\,\vec{g}_i$$

transition prob.

$$p(x', x, \delta t) = e^{-\frac{1}{2\delta t}\left(\vec{x}_i' - \vec{x}_i - \vec{v}(x_i)\delta t\right)}$$

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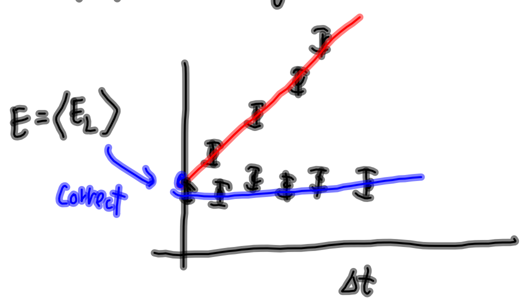
The first order Langevin has error $\propto \Delta t$



To eliminate this step-size error → use generalize Metropolis algorithm.

take $T(\tilde{x}',\tilde{x}) \propto e^{-\frac{1}{2\Delta t}\left(\tilde{x}'-\tilde{x}-\tilde{v}(x)\Delta t\right)^2} \Leftrightarrow x_i' = x_i + v(x)\Delta t + \sqrt{\Delta t}\, g_i$

to be the transition prob in the Metropolis algorithm.

then compute $R = \frac{P_e(x')}{P_e(x)} \frac{T(x,x')}{T(x',x)} = e^{-\Delta S} e^{-\tilde{\Delta S}}$

Δt ↳ is just a parameter to control the acceptance

$$\tilde{\Delta S} = \frac{1}{2\Delta t}\left[\left(x - (\tilde{x}' + \tilde{v}(x')\Delta t)\right)^2 - \left(x' - (x + v(x)\Delta t)\right)^2 \right]$$

accept + reject as before. $P_e = \phi^2$ $\sim r_{max}$

accept if $\Delta S + \tilde{\Delta S} \leq 0$

$$\tilde{v} = \frac{\vec{\nabla}\phi}{\phi}$$

if $\Delta S + \tilde{\Delta S} > 0$

accept with prob $e^{-(\Delta S + \tilde{\Delta S})}$

Your retirement! Basic saving in finance!

Simple compound interest:

x dollar per period for N periods at an interest rate a per period.

total after N period $= \left[(x(1+a)+x)(1+a)+x\right](1+a) \ldots N$ times

$$= x(1+a)^N + x(1+a)^{N-1} + \ldots \; x(1+a)^1$$

$$= x(1+a)^N \left[1 + \frac{1}{1+a} + \frac{1}{(1+a)^2} \cdots \frac{1}{(1+a)^{N-1}}\right] \qquad z = \frac{1}{1+a}$$

$$\left[1 + z + z^2 \cdots z^{N-1}\right]$$

$$S = 1 + z + z^2 \cdots z^{N-1}$$

$$zS = z + z^2 \cdots z^N$$

$$S(1-z) = 1 - z^N$$

$$S = \frac{1-z^N}{1-z}$$

$$T = x(1+a)^N \frac{1-\left(\frac{1}{1+a}\right)^N}{\cancel{\left(1-\frac{1}{1+a}\right)}} \; \frac{1+a}{a} \qquad \frac{a}{1+a}$$

$$\boxed{T = x\left[(1+a)^N - 1\right]\frac{1+a}{a}}$$

Ex. \$1,200 a year

$a = 0.12 \qquad N = 30$

$\hookrightarrow$ \$36,000

$T =$ \$324,351

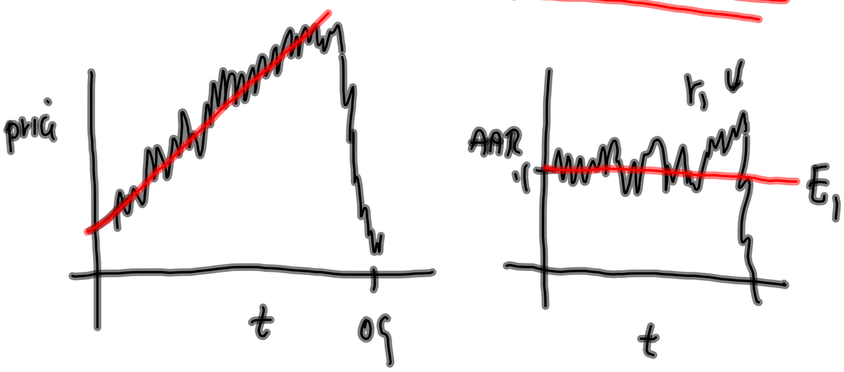
Ex 2: \$100 per month

$a = 0.01 \qquad N = 12 \cdot 30$

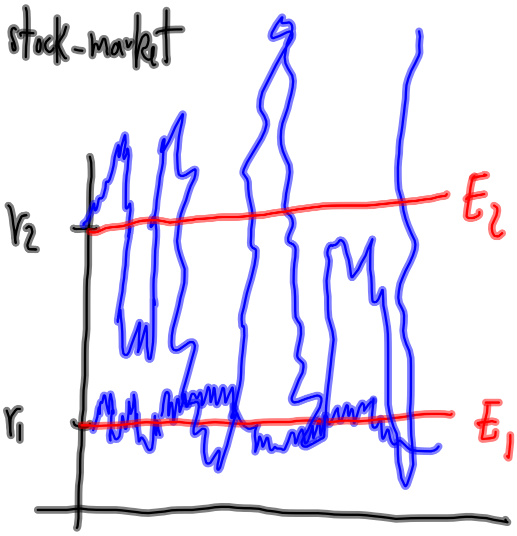
$T =$ \$353,000 etc.

Go to stock-market

Let r_1, r_2 denote the average annual return ← random variables



Law of the stock-market



high returns for high risk ← variance

Low returns for low risk

or standard deviation

with expectation values $E_1 = \langle r_1 \rangle$

and risk $E_2 = \langle r_2 \rangle$

↪ $\sigma_{1,2} = \sqrt{\langle r_i^2 \rangle - \langle r_i \rangle^2}$

"Portfolio" ⇒ collection of stocks

↪ 2 stocks, r_1, r_2

↓

allocation $1 = x_1 + x_2$ ⇐ % of allocation

Portfolio return $R = x_1 r_1 + x_2 r_2$

$$E = \langle R \rangle = x_1 \langle r_1 \rangle + x_2 \langle r_2 \rangle$$

$$= x_1 E_1 + x_2 E_2$$

↓

What is the portfolio risk?

$$\sigma^2 = \langle R^2 \rangle - \langle R \rangle^2$$

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