Measuring Stefan–Boltzmann Constant Using Light Bulbs

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Abstract

Our experiment is designed to pinpoint the exact value of the Stefan-Boltzmann constant. Using only a tungsten filament light bulb and some measuring devices we are able to achieve a value of $1.62 * 10^{-8} \pm 2.36 * 10^{-8} Wm^{-2}K^{-4}$ putting us within a factor of 1.5 away from the actual Stefan-Boltzmann constant of $5.6704 * 10^{-8} Wm^{-2}K^{-4}$ if we use our maximum error. Though we could not achieve an accurate result, with more time, more refined equipment, and better estimations I believe we could achieve a much more accurate result.

1 Introduction

1.1 Physics Motivation

The theory of black body radiation is extremely important in modern physics today. It describes many phenomena, from the glow of a star millions of light years away, to the thermal vision of a snake [1], to the hazardous gamma rays emitted by a nuclear power plant. As such a fundamental part of so many fields of science, it is extremely necessary to have an accurate measurement of the constant used to describe such a phenomenon, namely, the Stefan-Boltzmann constant.

1.2 Theoretical Background

The idea of a black body originally was introduced by Gustav Kirchhoff in 1860 as follows, "the supposition that bodies can be imagined which, for infinitely small thicknesses, completely absorb all incident rays, and neither reflect nor transmit any. I shall call such bodies perfectly black, or, more briefly, black bodies." [2] Planck later used the definition of a black body to define Planck's law which describes the amount of electromagnetic energy radiated by a black body at a specific temperature. In terms of frequency (v), Planck's law is written:

$$B_v(T) = \frac{2hv^3}{c^2} \frac{1}{e^{\frac{hv}{k_B T}} - 1}, \tag{1}$$

where B is the spectral radiance, T is the absolute temperature of the black body, k_B is the Boltzmann constant h is the Planck constant, and c is the speed of light. Integration of Planck's law of all frequencies provides an equation known as the Stefan-Boltzmann law written:

$$P = A_S \xi T^4 \sigma, \tag{2}$$

where P is power, A_S is the surface area of the object, ξ is the emissivity of the grey body, T is temperature, and σ is the Stefan-Boltzmann constant.

1.3 Our Approach

Resistivity is a property of a material; it quantifies how strongly the material opposes the flow of electric current and is usually denoted asρ. It is defined as

$$\rho = R\frac{A}{l}, \tag{3}$$

where R is the electrical resistance of a uniform specimen of the material, l is the length of the piece of material, and A is the cross-sectional area of the specimen. The resistivity of an object has a direct relationship with the temperature of the object. Using a chart comparing resistivity to temperature we can deduce an equation relating the two. Using our measured power and our extrapolated temperatures, we can solve for the Stefan-Boltzmann constant.

Our experiment consists of a light bulb, a power supply, a voltmeter and an ammeter. We did this because it is the simplest setup. Using the simplest setup means we are taking the least amount of measurements, which in turn leads to the smallest number of errors and thus to the smallest propagation of errors.

2 Experimental setup

2.1 Apparatus

In order to measure the voltage and current through our light bulb we had to set up a circuit. We attached a Tenma DC Power Oscillator to our light bulb and two BK Tool Kit 2707A's. One as an ammeter in series to measure current, and the other as a voltmeter in parallel to measure the voltage. See figure 1.

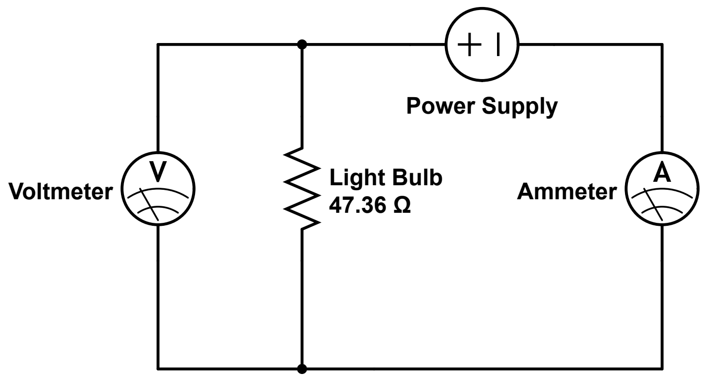


Figure 1: Circuit Setup

2.2 Data Collection

First we measured the dimensions to the filament of a normal, unfrosted, GE Specialty 25W aquarium light bulb with a tungsten filament. To do this we used a caliper accurate to .05mm. We measured the length of the filament using a light bulb which had previously had the glass broken, though the filament was intact. To measure the density of the coils, width, and thickness of the filament, we used an SEM micrograph courtesy of Dr. DeLozanne's lab. The dimensions of the filament were calculated using a caliber accurate to ±.05mm. Data collection for the current and voltage running through the lightbulb was collected straight from the ammeter and voltmeter connected to our circuit. After allowing the light bulb's temperature to steady after several minutes of having it turned on, we took measurements of current by varying the voltage. This provided accurate results of current to ±.05Am percentage while varying the voltage accurately from 1V to 60V with an error of ±.005V.

3 Data Analysis and Results

3.1 Data Processing and Hypothesis Testing

First we measured the resistivity of our filament as a function of voltage. See Eq. 3. Using our measured dimensions, we calculated the cross sectional area and the total length of the light bulb's filament. The coiled length of the filament we measured using our calibrator to be l=72.91±.05mm. The diameter of the wire (D_v), the diameter of the coil (D_H), and the density of the coils (u) were all measured off of an SEM. Using the scale on the picture, and measuring the dimensions using the same caliper gives measurements of D_v=29.0698±.23µm, D_H=93.95±.23µm, and u=38.75±.23µm.

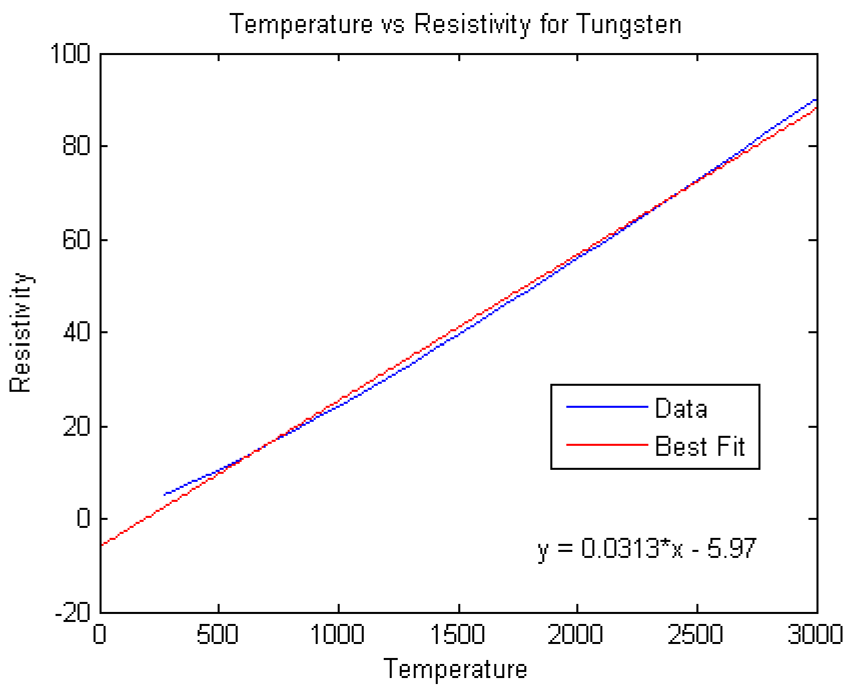
Figure 2: Temperature Relating Resistivity

The total length of the filament l_T is simply

$$l_T = \frac{l}{u} * \pi D_H$$

Giving a total length of l_T=566191.65±7536μm. Then, the cross sectional area A is calculated as $A = \pi(D_v/2)^2$ to be 663.702±23μm. Finally, using these two values and substituting $R = \frac{V}{I}$ given by Ohm's law, we have a table of resistivites as a function of voltage.

Using a table of experimental data relating resistivities to temperature [3] we came up with a linear relationship between temperature and resistivity. Using this relationship and our values for resistivity, we were able to come up with a chart relating voltage to temperature. We best fit a function of resistivity and temperature (in Kelvin) to find that ρ=.0313*T-5.97.



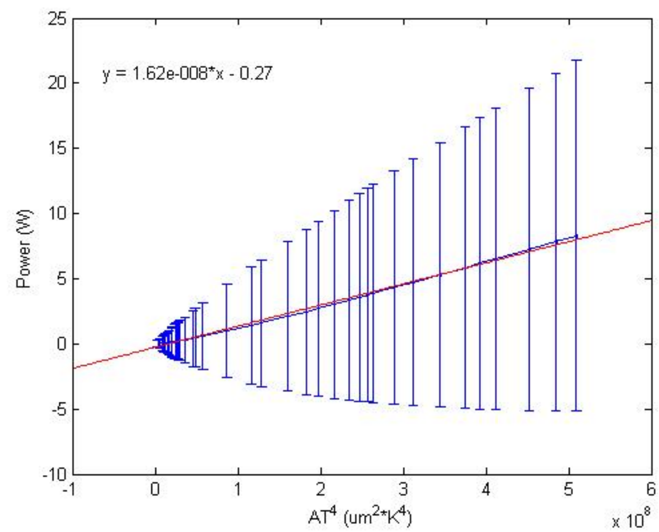
Next, we measured the power output using the fact that $P = IV$. This gave us a list of powers dependent on voltage.

Finally, we can assume the surface area of the filament is a perfect cylinder with surface area equal to

$$A_s = \pi D_H + l D_H.$$

Using our calculated values for T, P, and A_s and assuming our light blub is a black body of ξ=1, we can imput our values into Eq. 2 yielding a value for σ.

Figure 3: Graphical Representation of Eq. 3 with Data



3.2 Results and Brief Discussion

Our result for the Stefan-Boltzmann constant is $1.62*10^{-8} \pm 2.36*10^{-8} W m^{-2} K^{-4}$ This is comparable with the real result of $5.6704 * 10^{-8} W m^{-2} K^{-4}$. There are a multitude of sources of error in this experiment. Of course, a light bulb is not an ideal black body so there is some factor of emissivity. The surface area of the light bulb is not a perfect sphere resulting in a change of surface area. The purity of the tungsten is also an issue, our resistivity to temperature relationship is made on the assumption that the filament is made out of pure tungsten but that is simply not the case. There is internal resistance in the wires resulting in a systematic error. These errors along with several others left unmentioned result in a huge error as the voltage increases.

4 Summary and Conclusion

Though our value of the Stefan-Boltzmann constant is off by a factor of 3.5, with a maximum error we are only off by approximately a factor of 1.5 meaning though obviously are results are inconsistent with the current understanding of Stefan's Law, we are in the ballpark. With more professional equipment and error analysis I believe a better construction of this value could be constructed.

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5 References

[1] http://phys.org/news76249412.html

[2] (1860): G. Kirchhoff (July, 1860). The London, Edinburgh and Dublin philosophical magazine and journal of science

[3] http://sampa.if.usp.br/~suaide/LabFlex/blog/files/tung.pdf Resistance and Radiation of Tungsten as a Function of Temperature, (April, 1934)