Measuring Heat Capacity Ratio Using Properties of Adiabatic Systems

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Abstract

Our experiment was designed to measure values of the heat capacity ratio (γ) for both monatomic and triatomic gases. Using a chamber filled with a pure gas, we measured the variations in pressure and temperature by varying the volume of the chamber. The gas was compressed rapidly so as to prevent heat from escaping into the surrounding environment, thus simulating an adiabatic process. Using theoretical properties of adiabatic systems, we measured our γ's to be 1.337 and 1.206 respectively, only 20% and 10% off of the theoretical values. Though we did not achieve a perfect result, our results do support the theory of thermodynamics and help shape the model for future experiments.

1 Introduction

1.1 Physics Motivation

Thermodynamics is an extremely important concept in our modern world. This phenomena describes the flow of heat through objects which, as one can imagine, has many important uses. Engines, heat pumps, refrigerators, rockets, air-planes, even black holes [1], can be described using the theory of thermodynamics. As such a fundamental part of so many fields of science, it is extremely necessary to have accurate results to match the predictions made by this theoretical phenomena. For our experiment, we will be confirming the heat capacity ratio for both monatomic, and triatomic gases.

1.2 Theoretical Background

The heat capacity ratio, denoted γ, is defined as the ratio of the heat capacity of a gas at a constant pressure (C_P) to heat capacity at constant volume (C_V) [2].

$$\gamma = \frac{C_P}{C_V} \tag{1}$$

A degree of freedom is any form of energy in which heat transferred into an object can be stored [3]. This can be in translational kinetic energy, rotational

kinetic energy, or other forms such as potential energy in vibrational modes. Only three translational degrees of freedom (corresponding to the x,y, and z axis) are available for any individual atom, whether it is free, as a monatomic molecule, or bound into a polyatomic molecule. The heat capacity ratio (γ) for an ideal gas can be related to the degrees of freedom (f) of a molecule by:

$$\gamma = 1 + \frac{2}{f}. \tag{2}$$

Thus, for a monatomic gas, with three degrees of freedom such as argon:

$$\gamma = \frac{5}{3}$$

and for a triatomic gas, with six degrees of freedom such as carbon:

$$\gamma = \frac{4}{3}.$$

One common method of measuring the ratio of heat capacities of gases, is Ruchardt's method. [4] This experiment was done by Dr. Eric Ayars at California State in 2007. This method uses the oscillation of a mass supported by the pressure of the gas. Traditionally, this is done with a steel bearing oscillating in a precisely- tted glass tube attached to a gas reservoir. We will use a cruder method of determaining the heat capacity ratio using properties of adiabatic processes.

An adiabatic process is defined by allowing some thermodynamic process to occur without allowing heat to enter or leave the system ($\delta Q = 0$). [5] A transformation of a thermodynamic system can be considered adiabatic when it is quick enough that no significant heat is transferred between the system and the outside. Variations in temperature occur due to a change in pressure within the system while not allowing the diffusion of heat with the outside environment.

For a closed system, the total change in energy is the sum of the work done and the heat added,

$$dU = \delta W + \delta Q. \tag{3}$$

The change in enthalpy is given by

$$dH = \delta W + \delta Q + P dV + V dP \tag{4}$$

where P is pressure and V is volume.

Next, it is always true for any transformation of an ideal gas that $dU = nC_v dT$ and $dH = nC_p dT$. Thus:

$$nC_v dT = -PdV$$

and

$$nC_p dT = VdP$$

using the facts that δQ=0 and δW=-PdV for adiabatic processes. Solving for C_v and C_p and plugging into Eq. 1, we can see that:

$$\gamma = -\frac{dP/P}{dV/V}. \tag{5}$$

for an ideal gas γ is constant, hence an integration of Eq. 5 provides

$$PV^\gamma = constant,$$

or written in a more convenient form,

$$\frac{P_2}{P_1} = (\frac{V_1}{V_2})^\gamma \tag{6}$$

1.3 Our Approach

Using the ratio defined in Eq. 6, we compared the pressures and volumes at different points of an adiabatic system to determine the heat capacity ratio for two specific monatomic and triatomic gases, namely argon and carbon.

Our experiment consists of an isolated chamber filled with gas, capable of measuring volume, pressure, and temperature. The various measuring devices were hooked up to a computer and Data Studio was used to record data. Rapid compression of the chamber simulates an adiabatic system. Using this fact we plugged in various values for volume and pressure to obtain a measurement of the heat capacity ratio in Eq. 6. This measurement was then compared with the theoretical value of γ given by Eq. 2. We did this because it is a fairly simple setup which should theoretically give very practical results. Using a simple setup means we are taking the least amount of measurements, which in turn leads to the smallest number of errors, and thus to the smallest propagation of errors.

2 Experimental setup

2.1 Apparatus

First, an ideal gas is pumped into an Adiabatic Gas-Law Apparatus. Ours was a model 1000 manufactured by A.U. Physics Enterprises. This is essentially an empty chamber for holding gas with a manual pump allowing one to vary the volume of the chamber. The chamber was measured to have a diameter of 4.4±.05cm, a max height of 15±.05cm and a minimum height of 6.1±.05cm using a combination of a simple ruler attached to the side of the apparatus for the height and a caliper for the width.

Either carbon or argon were then pumped into this apparatus at roughly 5kPa using a standard gas tank. The gas was inserted into the apparatus and then the chamber was opened to allow all the gas to escape. This process was repeated about 10 times to ensure that only the specified gas was left in the chamber.

Next, the Adiabatic Gas-Law Apparatus was attached to a Pasco Science Workshop 750 Interface model CI-7599 with three voltage readers. These three readers measured the pressure, volume, and temperature inside the tube. Graphs and data was collected using Data Studio.

2.2 Data Collection

Data was collected through Data Studio. Voltage readings with an error of $\pm.05$V were measured for pressure, volume, and temperature and then converted using equations found on the side of the apparatus.

$$V_{ol} = 3.19 * 10^{-5}V_V + 8.22 * 10^{-5}\pm.06 * 10^{-5} \tag{7}$$

$$P = 100V_P\pm5.0 \tag{8}$$

$$T = 41.1V_T + 259.65\pm2.1 \tag{9}$$

Where V_{ol} is the volume of the gas chamber and was measured in m^3, V_V is the voltage measurement for the volume, P is the pressure measured in kPa, V_P is the voltage measurement for the preasure, T is the temperature measured in ^{o}K, and V_T is the voltage measurement for the temperature.

Two types of data were taking for each gas. A fast, adiabatic pull so that heat had no chance to escape the apparatus and a slow, isothermal pull allowing for constant temperature. Each experiment was repeated 3 times, giving us 12 independent trials with 3 variables in each trial.

3 Data Analysis and Results

3.1 Data Processing and Hypothesis Testing

Reading were taken in voltages for pressure, volume, and temperature and then converted using equations 7, 8, and 9. Data was acquired through Data Studio and converted into tables in Microsoft Excel. First, argon is pumped into the gas chamber and filtered out approximately 10 times to flush out any contaminants. We acquired readings simulating an adiabatic system by rapidly compressing the chamber. This only allows an insignificant amount of heat to disperse to the outside environment. Doing this essentially set $\delta Q = 0$, the definition of an adiabatic system. Pressure readings were acquired just before compression and just after compression, before heat had a chance to dissipate. See figure 1.

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Figure 1: Pressures for Adiabatic Systems vs Time
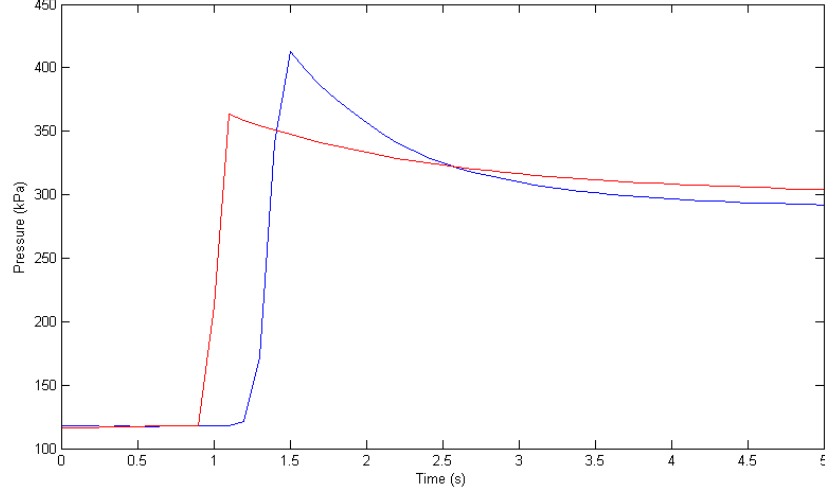
Note that this graph was only used to find the peak pressures. The tail end of the graph continues to decrease because the heat of the system is dissipating into the outside environment. This reduction in heat in turn leads to a decrease in pressure. This therefore, no longer simulates an adiabatic system. This procedure is repeated two more times for argon and then the entire experiment is repeated for carbon.

Volume readings for the chamber before and after compression were very consistent. Uncompressed, the volume averages to be $2.34 \pm 0.016 * 10^{-4} m^3$, and compressed the volume is $9.25 \pm .16 * 10^{-5} m^3$.

We now had readings for uncompressed volume and pressure, and for compressed volume and pressure. Using Eq. 6, we solved for gamma for each trial.

	Argon	Carbon
Trial 1	1.319	1.190
Trial 2	1.338	1.199
Trial 3	1.353	1.229
Avg γ	1.337	1.206

Table 1: Measured values of γ for Argon and Carbon

3.2 Results and Brief Discussion

Our result for the heat capacity ratio of argon and carbon are $1.337 \pm .185$ and $1.206 \pm .124$. This is comparable with the ideal results of 1.666 for argon and

1.333 for carbon. Our results are off for argon by 20.8% and our results for carbon are off by 9.5%

There were many sources of error in this experiment. The largest of which, I believe is the accuracy of the measuring devices. The apparatus claims to have a voltage reading of $\pm.05$, this gives as an incredibly accurate reading of $\pm 1.6*10^{-6} m^3$ for the volume of the gas chamber. Using the voltage readings for an open chamber and Eq 7, the volume comes out to be $2.612\pm0.016*10^{-4}m^3$ using one of our particularly high voltage readings. However, upon measuring the chamber ourselves we found the chamber to be $2.29 \pm 0.13*10^{-4}m^3$. This is off by a factor of 13% even with maximum error values!

Another large source of error is the fact that these are not ideal gases. The entire theory of the adiabatic process is only a crude model of what is actually happening at the atomic level of an atom. It is entirely possible the model is not completely accurate.

Lastly, there is always a chance of impurities in the gases. Filtering a gas chamber 10 times is not nearly as sufficient as completely vaccumizing the chamber before filling it with pure argon or carbon. I believe that other molecules such as oxygen or H_2O may have interfered with our results.

4   Summary and Conclusion

Though our values of the the heat capacity ratio of argon and carbon are off by roughly 20% and 10% respectively, we are very close to being within the range of γ with our maximum error bar analysis. With more professional equipment and error analysis I believe a better construction of these values could be constructed.

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5   References

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