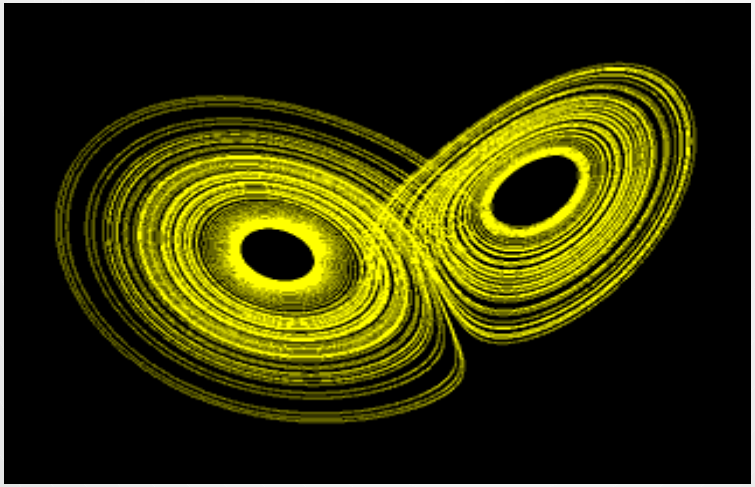
Observing Chaos Through a Forced Harmonic Pendulum

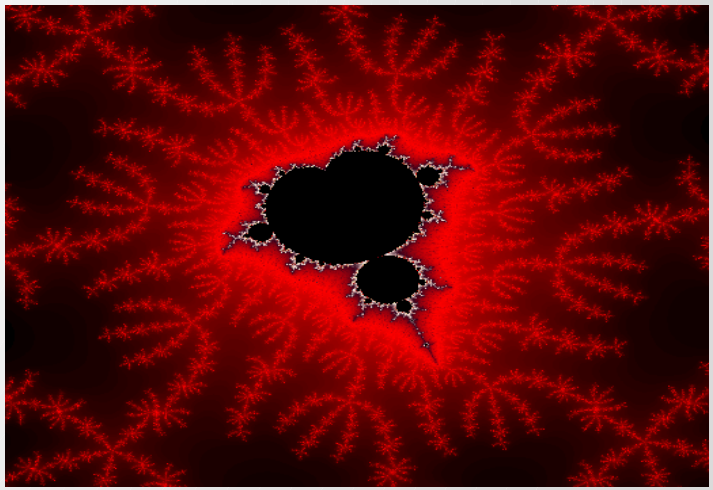
By: Nate Herbert

11/12/2012

Introduction

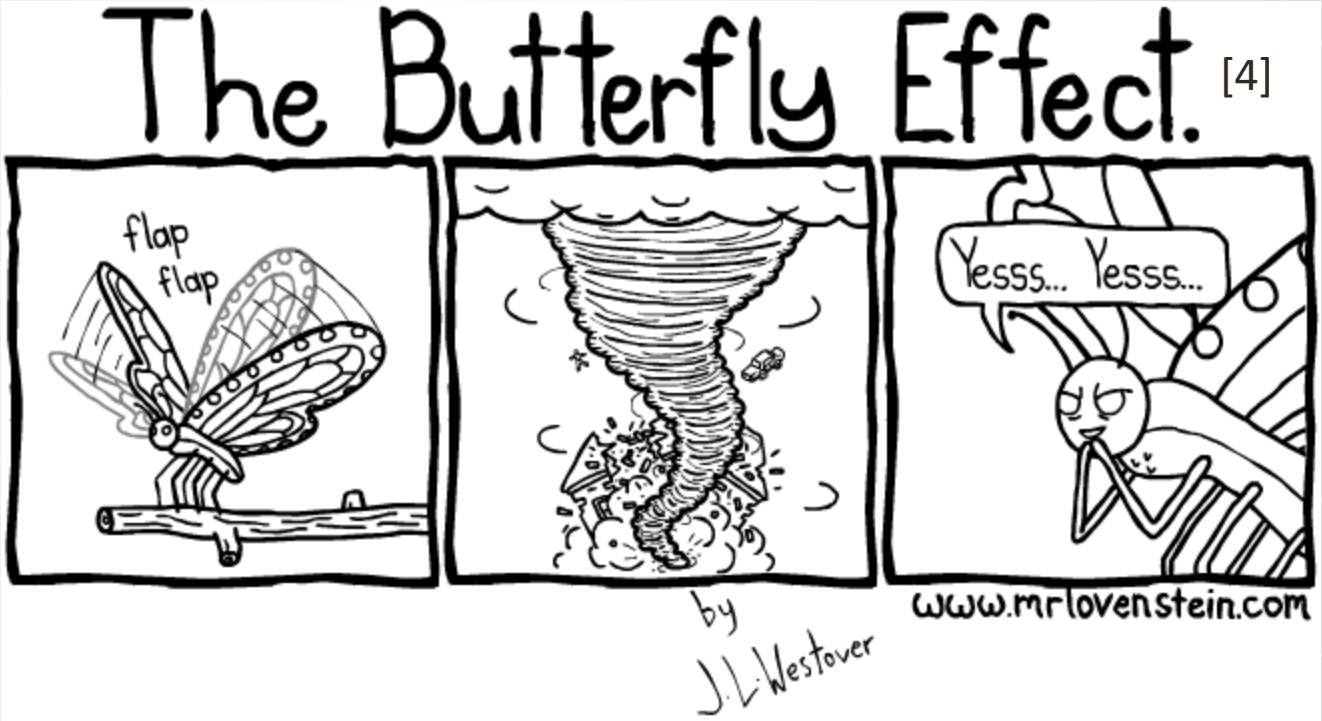
- Study of complex nonlinear dynamic systems
- Highly sensitive to initial conditions (butterfly effect) [1]
- The deterministic nature of these systems does not make them predictable.[2]





[3]

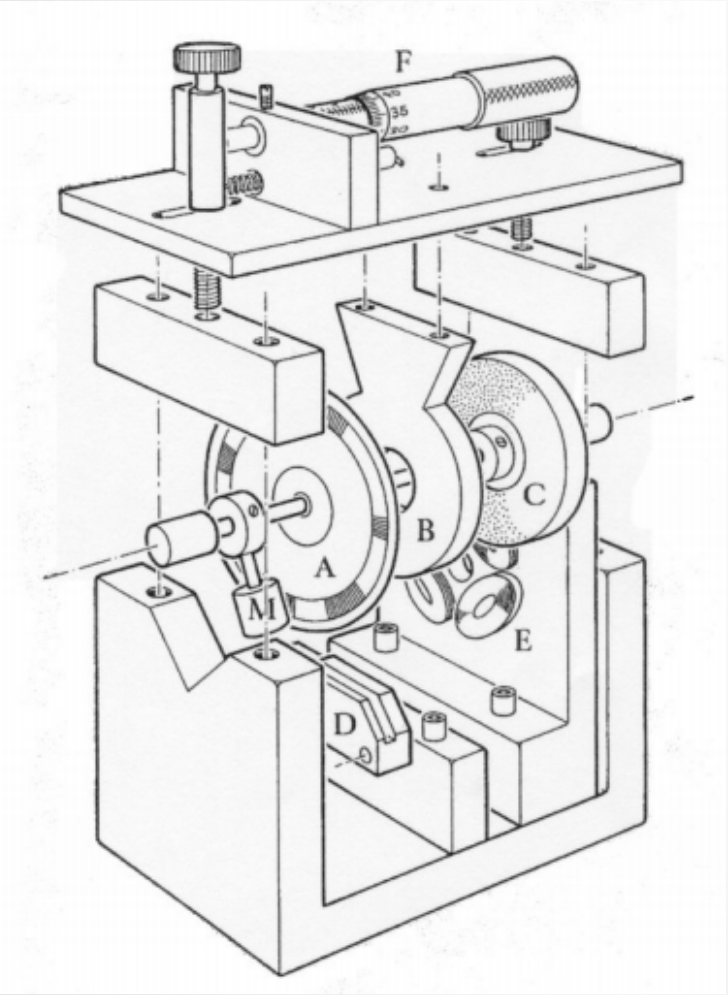
Background

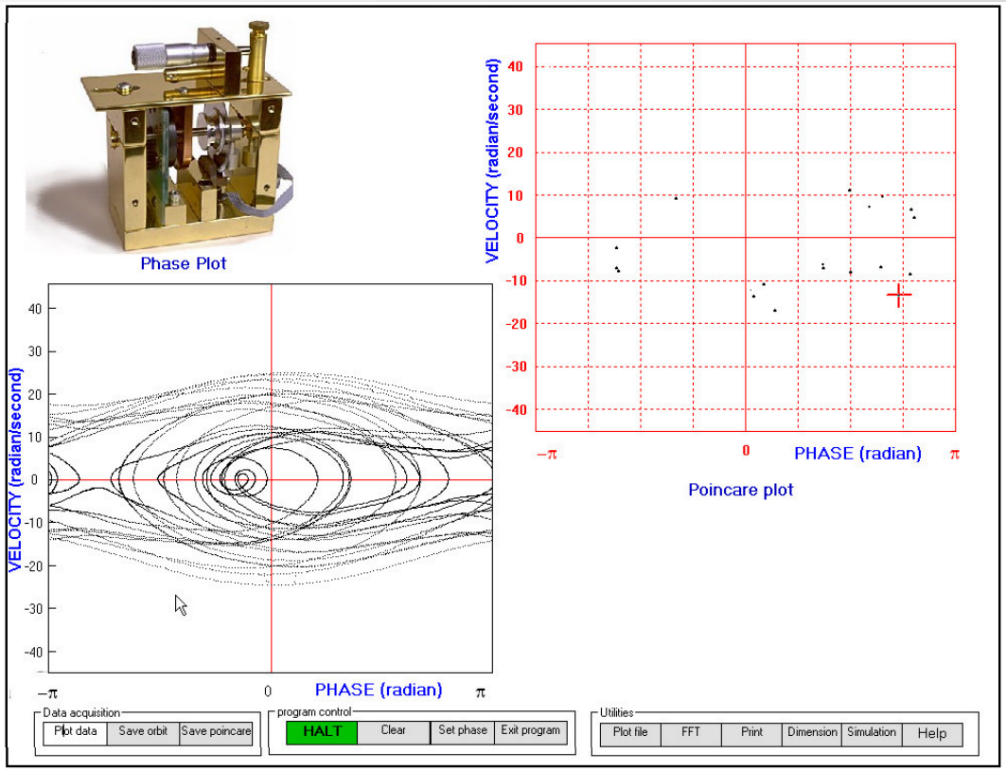


[4]

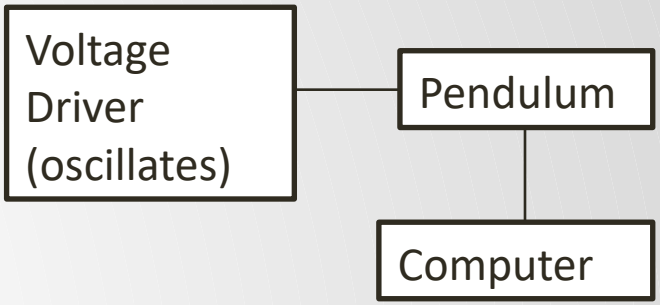
http://www.exploratorium.edu/complexity/java/lorenz.html

Apparatus


F
A
B
C
D
E
M


Phase Plot
VELOCITY (radian/second)
PHASE (radian)
Poincare plot
Data acquisition
Plot data
Save orbit
Save poincare
program control
HALT
Clear
Set phase
Exit program
Utilities
Plot file
FFT
Print
Dimension
Simulation
Help

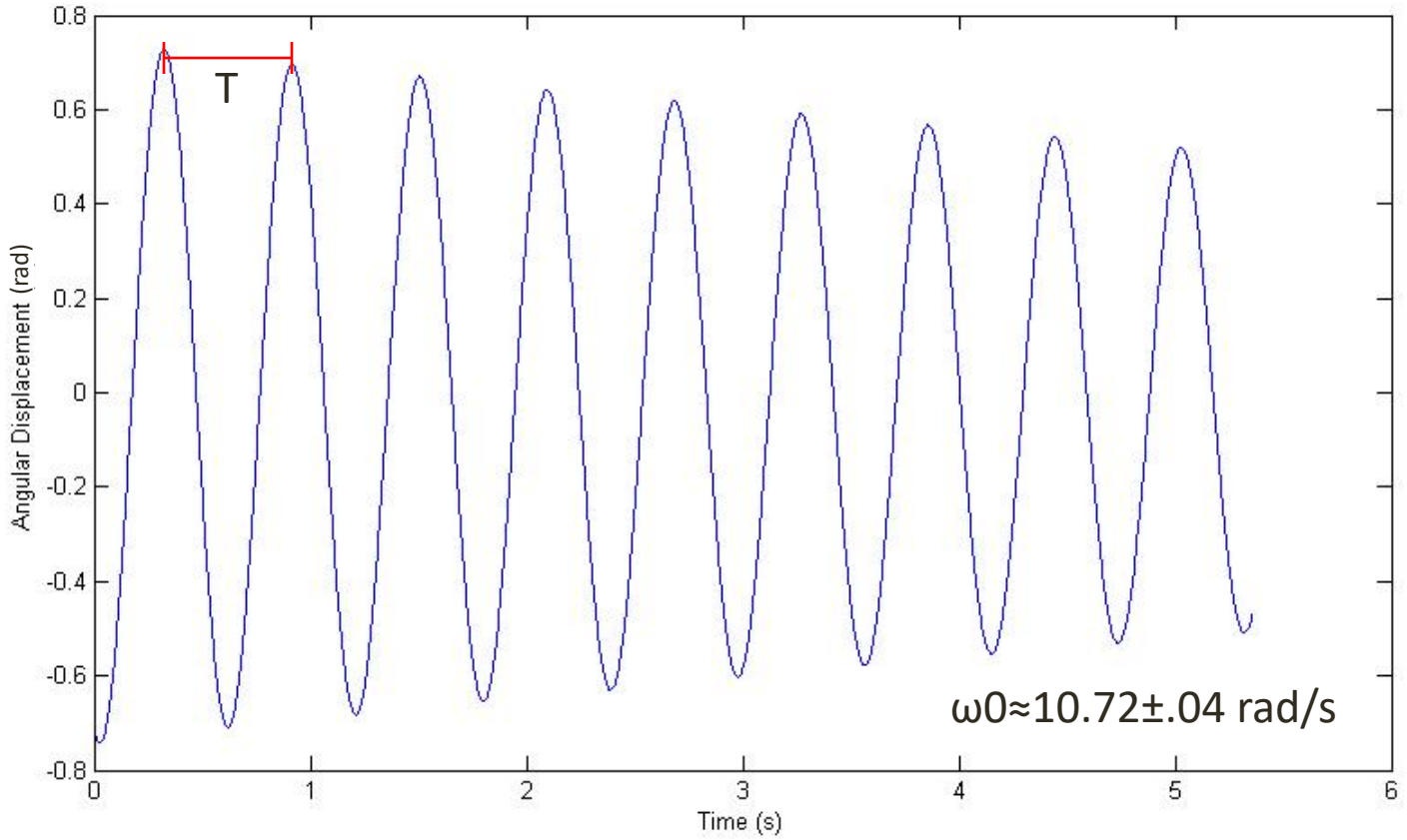
Procedure



- Natural frequency (ω0)
- Damping constant (β)
- Input voltage vs. torque (V vs. T)
- Drive frequency vs. angular amplitude (ω vs. θ)
- Hysteresis

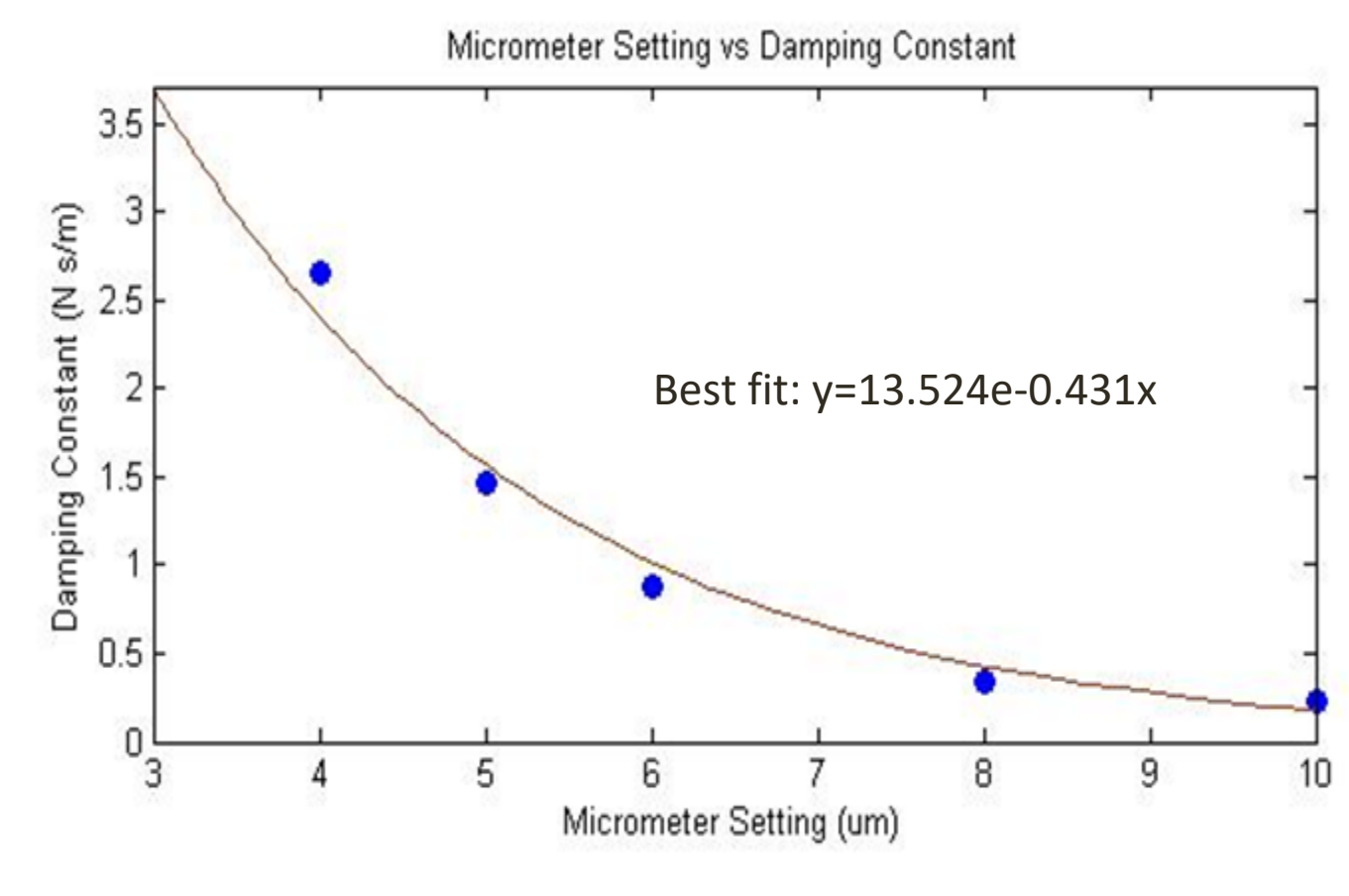
Natural Frequency (ω0)

- $\omega = 2\pi f$
- $f = \frac{1}{T}$



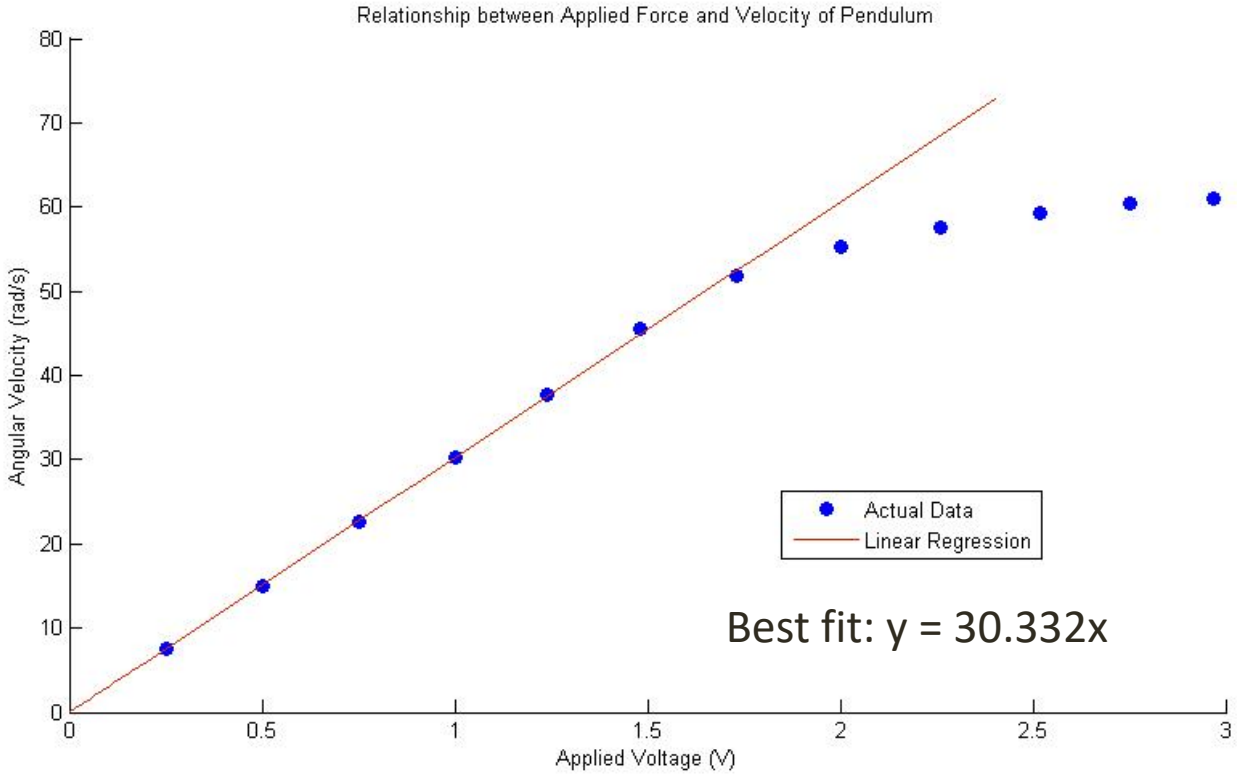
Damping Constant (β)

- $\ln\frac{A_n}{A_{n+1}}=\frac{\beta T}{2}$



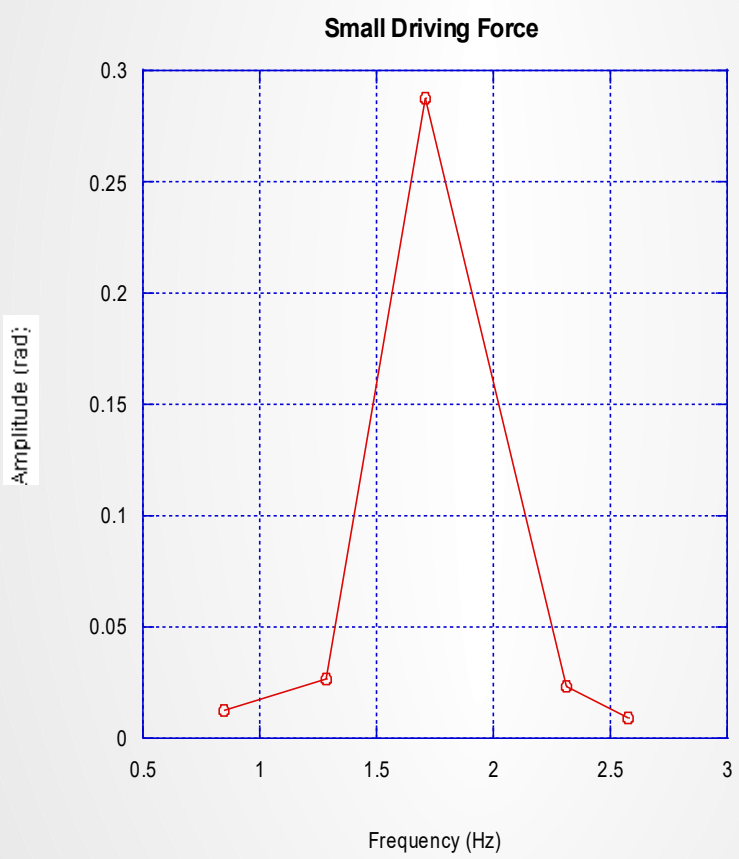
Input Voltage VS. Torque

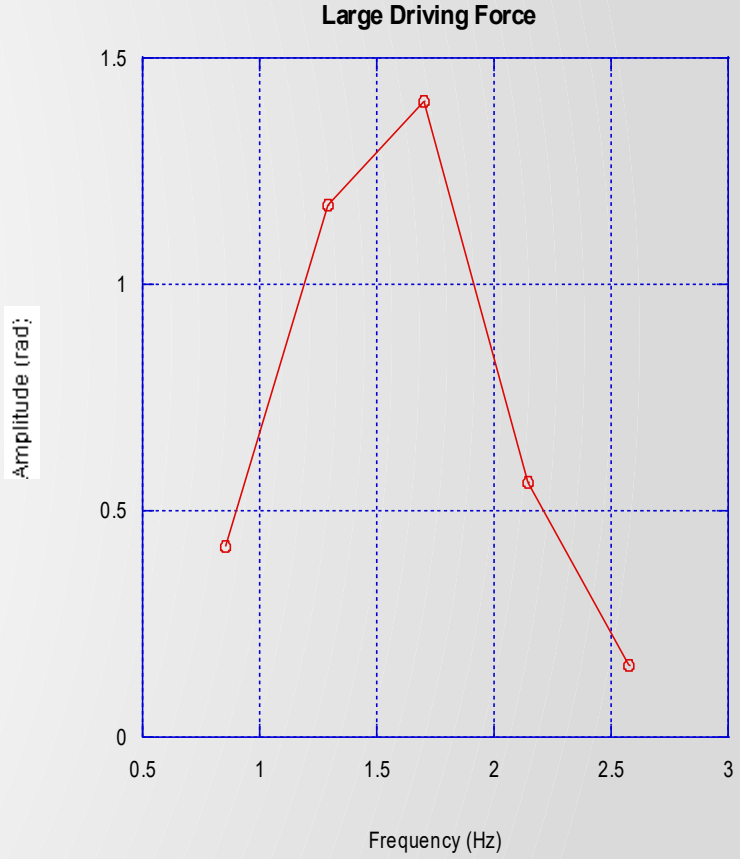
- By measuring the terminal velocity for several different applied voltages, it is possible to construct a plot of voltage vs torque. Such a plot will reveal deviations from linearity in the drive circuit; these typically will appear at larger applied voltages.



Finding Resonant Frequency

$f_0 = 1.706 \pm .006$ Hz





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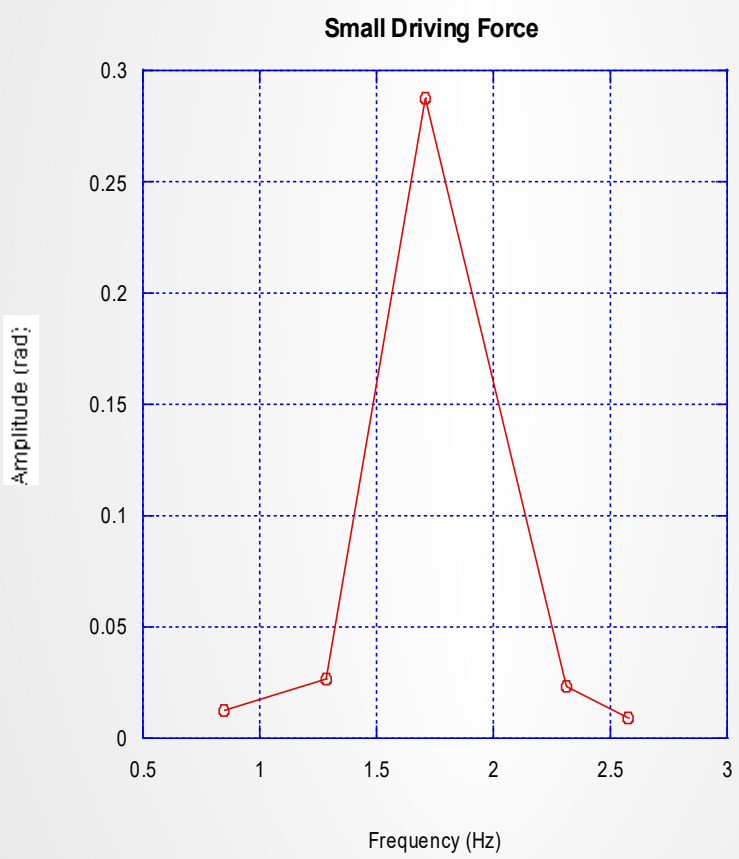
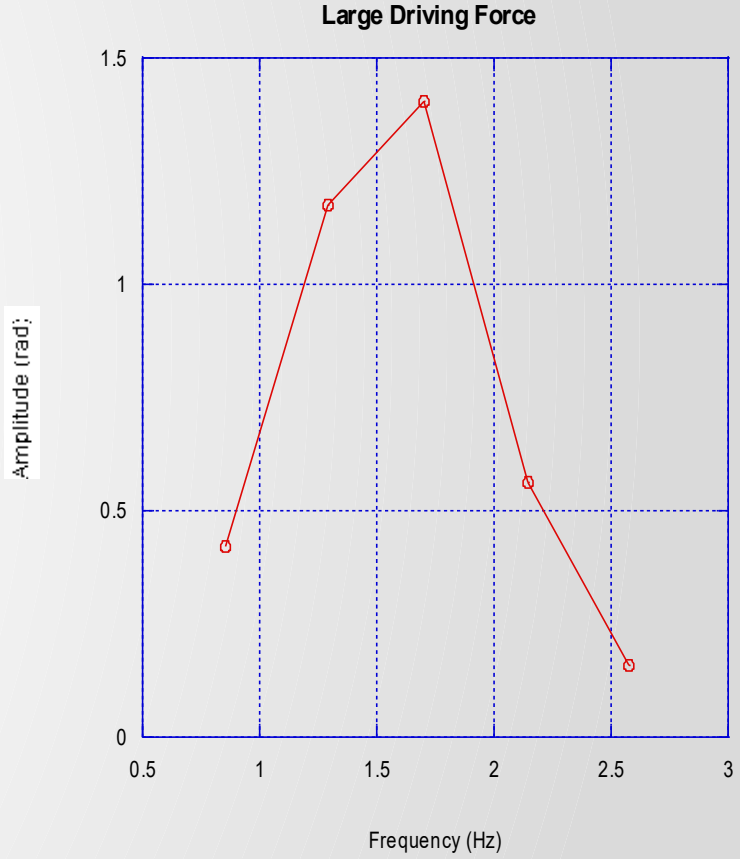
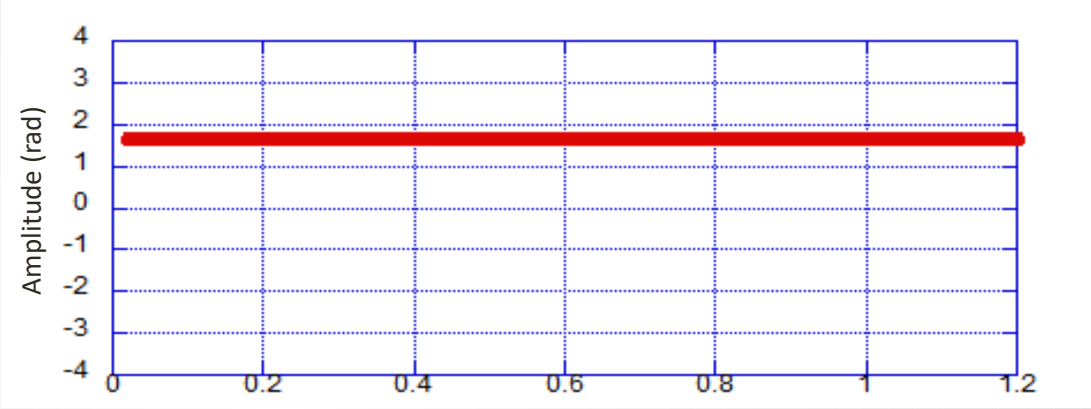
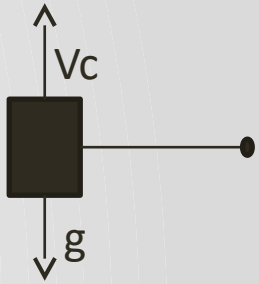




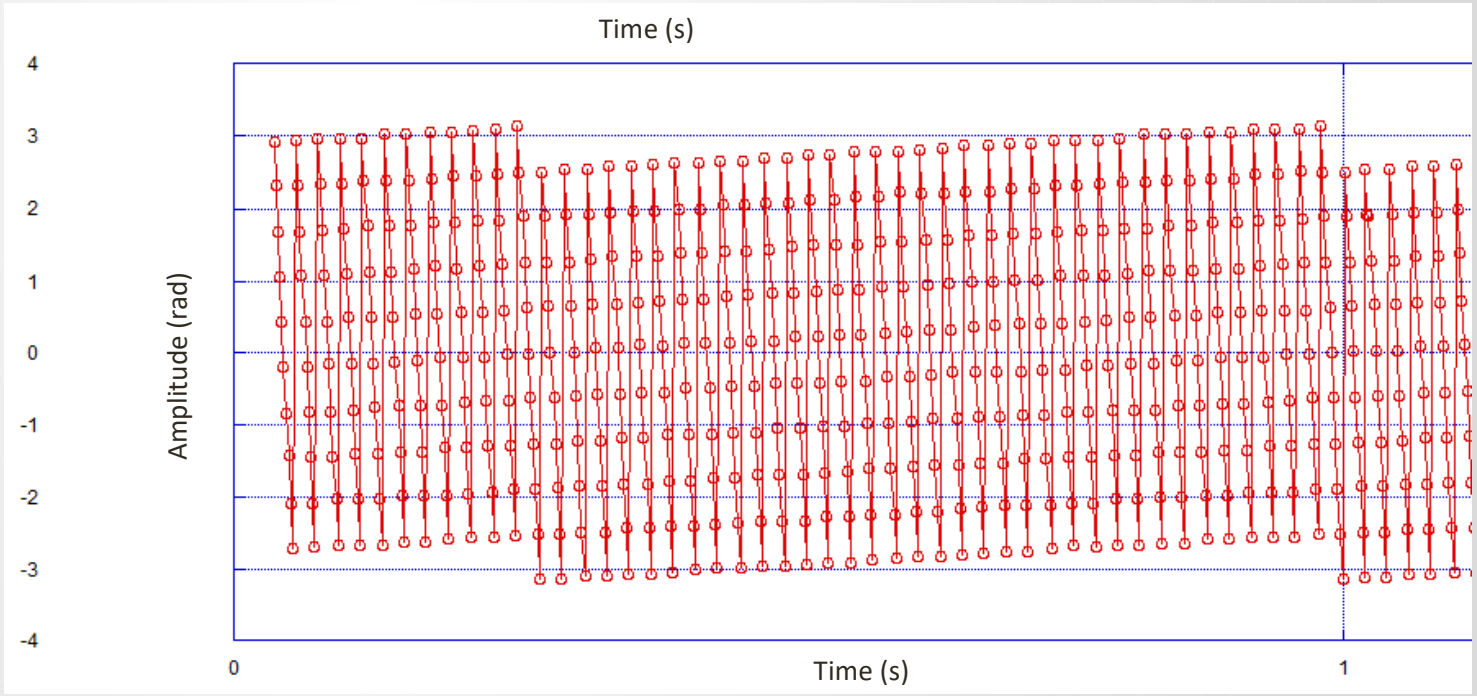
Hysteresis

Vc=1.95V

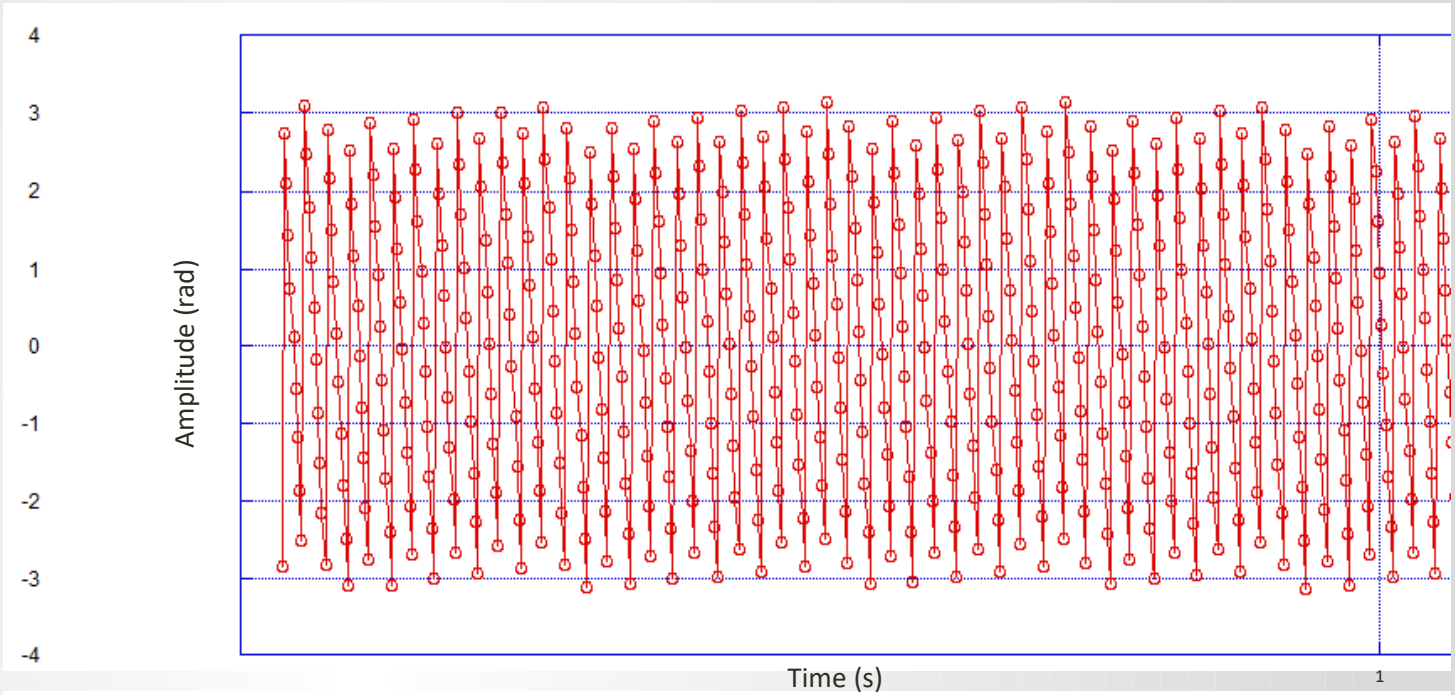




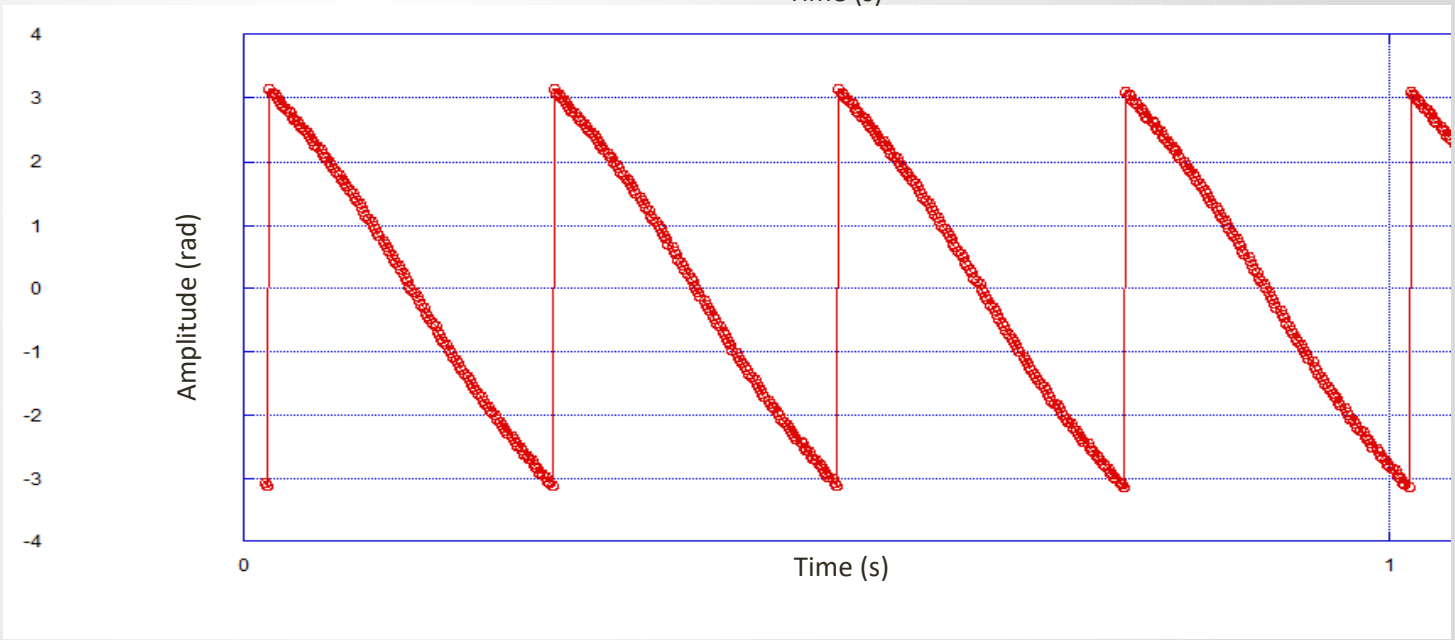
V=2.29V



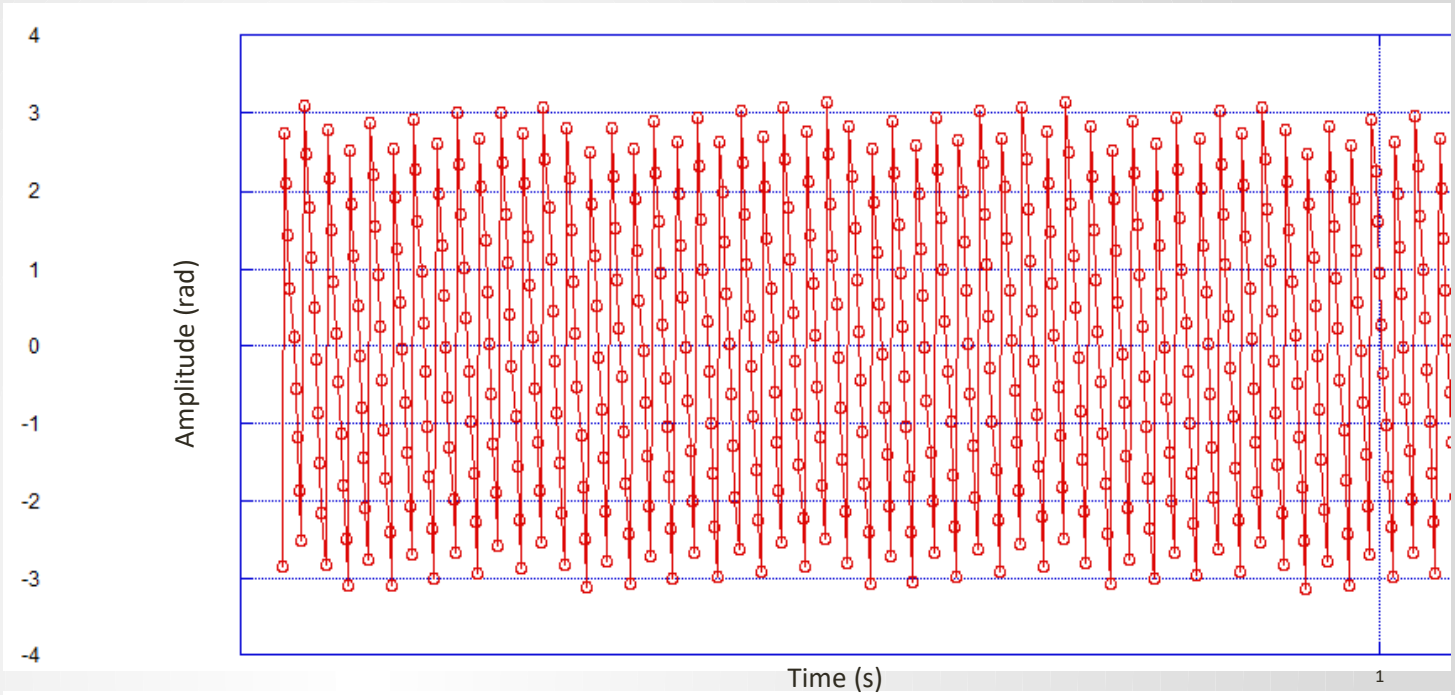
V=3.82V


4
3
2
1
0
-1
-2
-3
-4
Amplitude (rad)
Time (s)
1

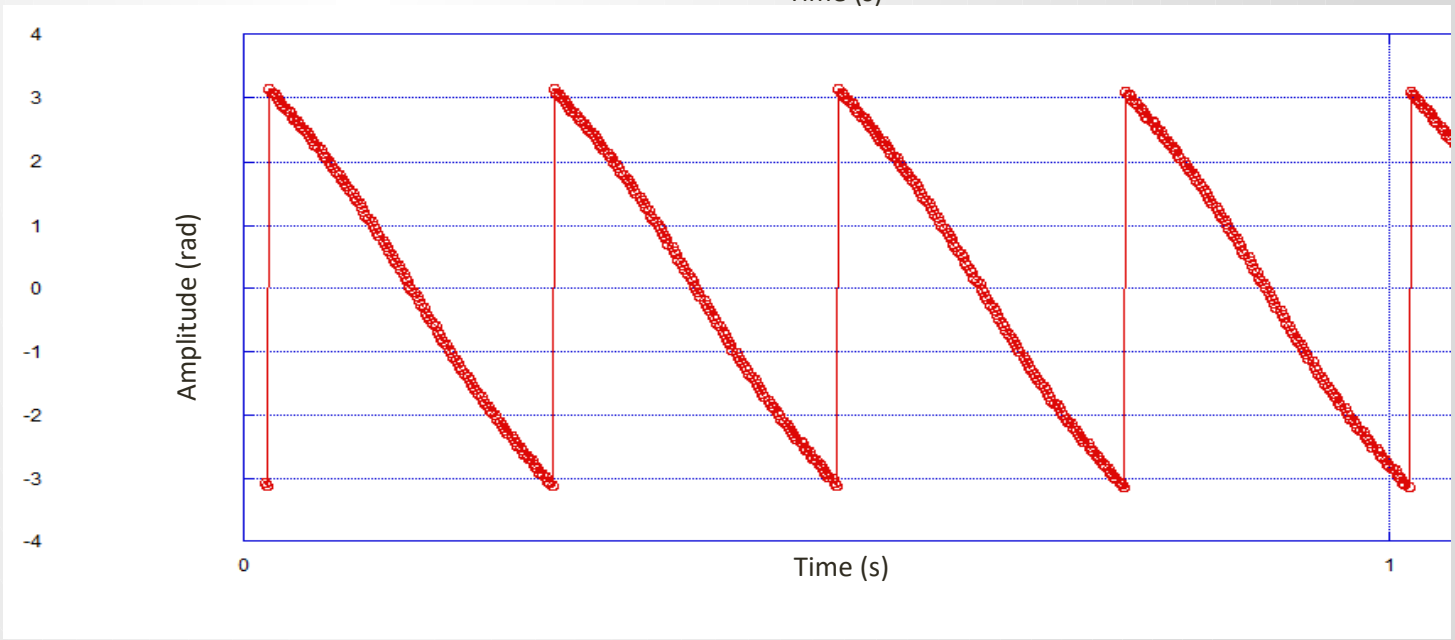
V=.1V


4
3
2
1
0
-1
-2
-3
-4
Amplitude (rad)
0
Time (s)
1

V=3.82V


4
3
2
1
0
-1
-2
-3
-4
Amplitude (rad)
Time (s)
1

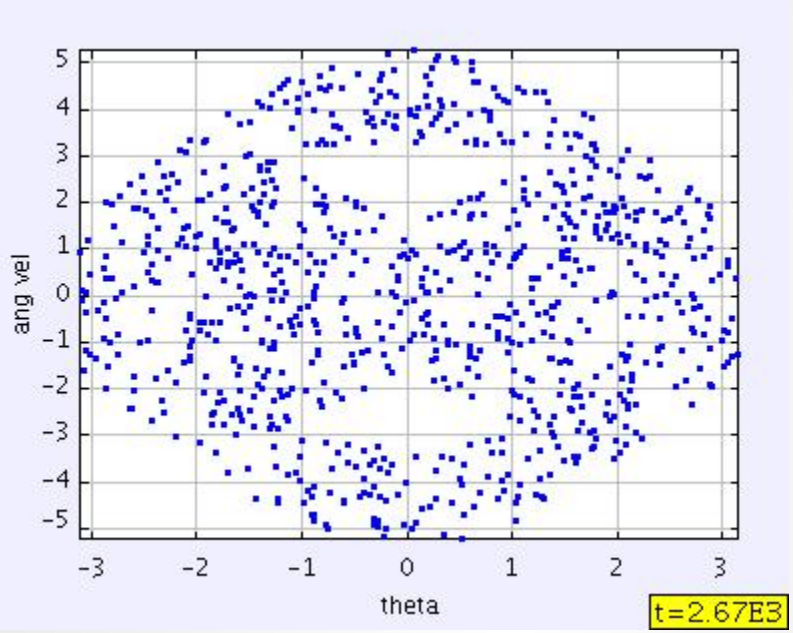
V=.1V

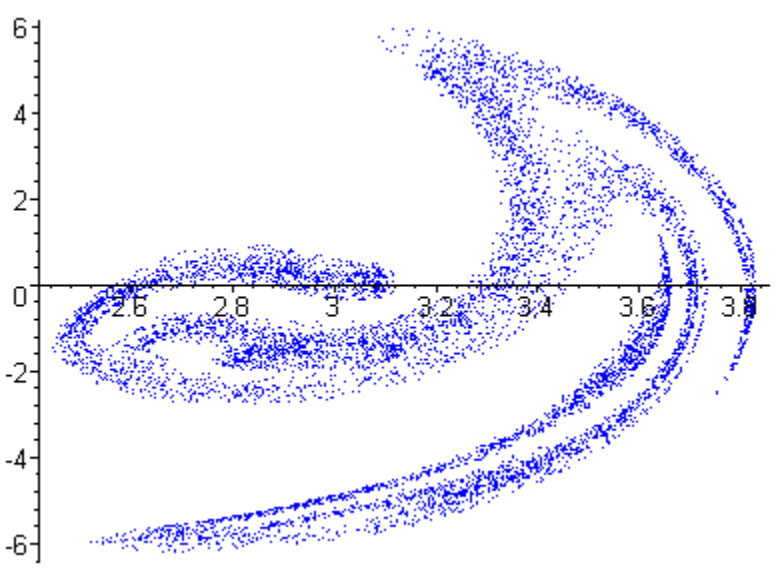

4
3
2
1
0
-1
-2
-3
-4
Amplitude (rad)
0
Time (s)
1

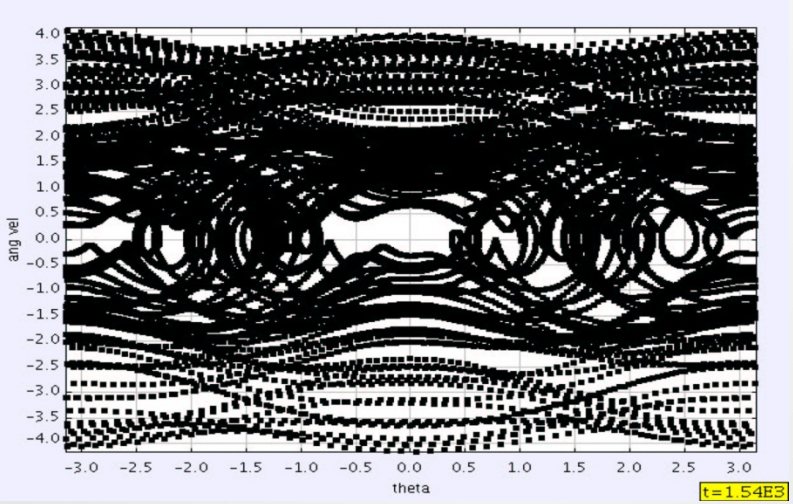
Conclusion

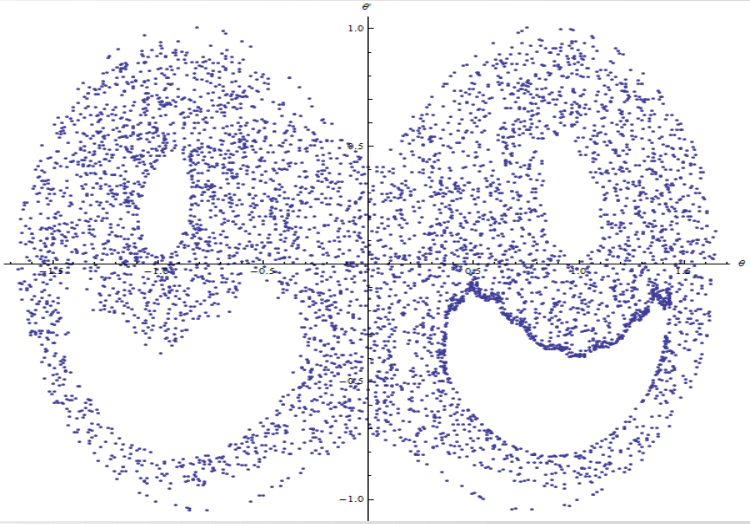
Chaos when:

- driven V > Vc
- driven ω < ω0









References

- [1] Kellert, Stephen H. (1993). *In the Wake of Chaos: Unpredictable Order in Dynamical Systems*. University of Chicago Press. p. 32. ISBN 0-226-42976-8.
- [2] Werndl, Charlotte (2009). "What are the New Implications of Chaos for Unpredictability?". *The British Journal for the Philosophy of Science* **60** (1): 195–220. doi:10.1093/bjps/axn053
- [3] http://sprott.physics.wisc.edu/chaos/manchaos.htm
- [4] http://suite101.com/article/math-and-chaos---sisters-under-the-skin-a245354