WimpSim Future Work/Handoff

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Simulations Meeting

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Table of Contents

1. Quick Overview - What is WIMPSim?

Quick Overview - What is WIMPSim?

Inputs

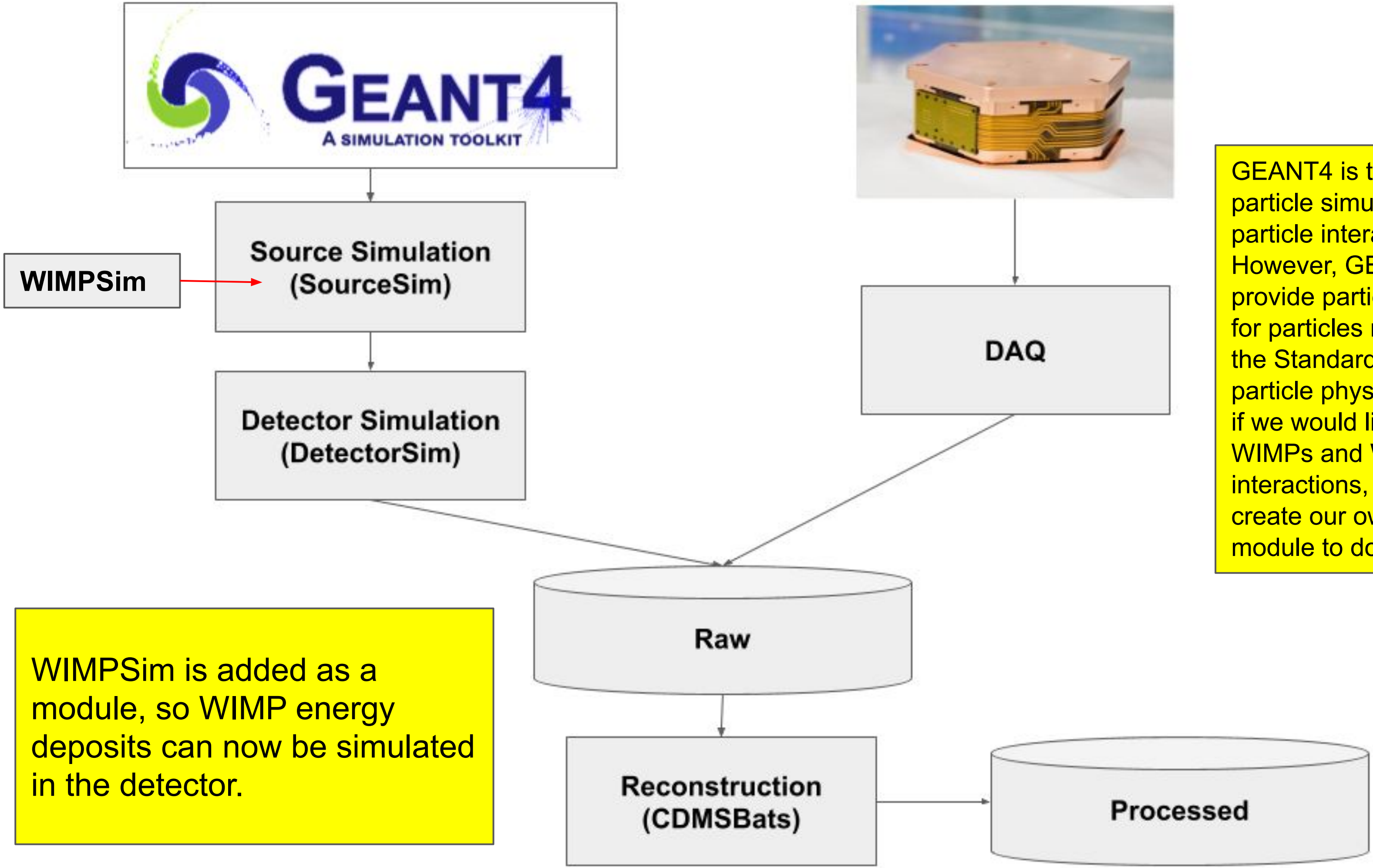
- WIMP mass
- Nuclei mass, A, Z

Outputs

- Nuclei recoil momentum 3-vector
- Nuclei recoil energy
- Extra information associated with the collision, such as earth phase, WIMP velocity, etc.

Runs WIMPSim 3 times

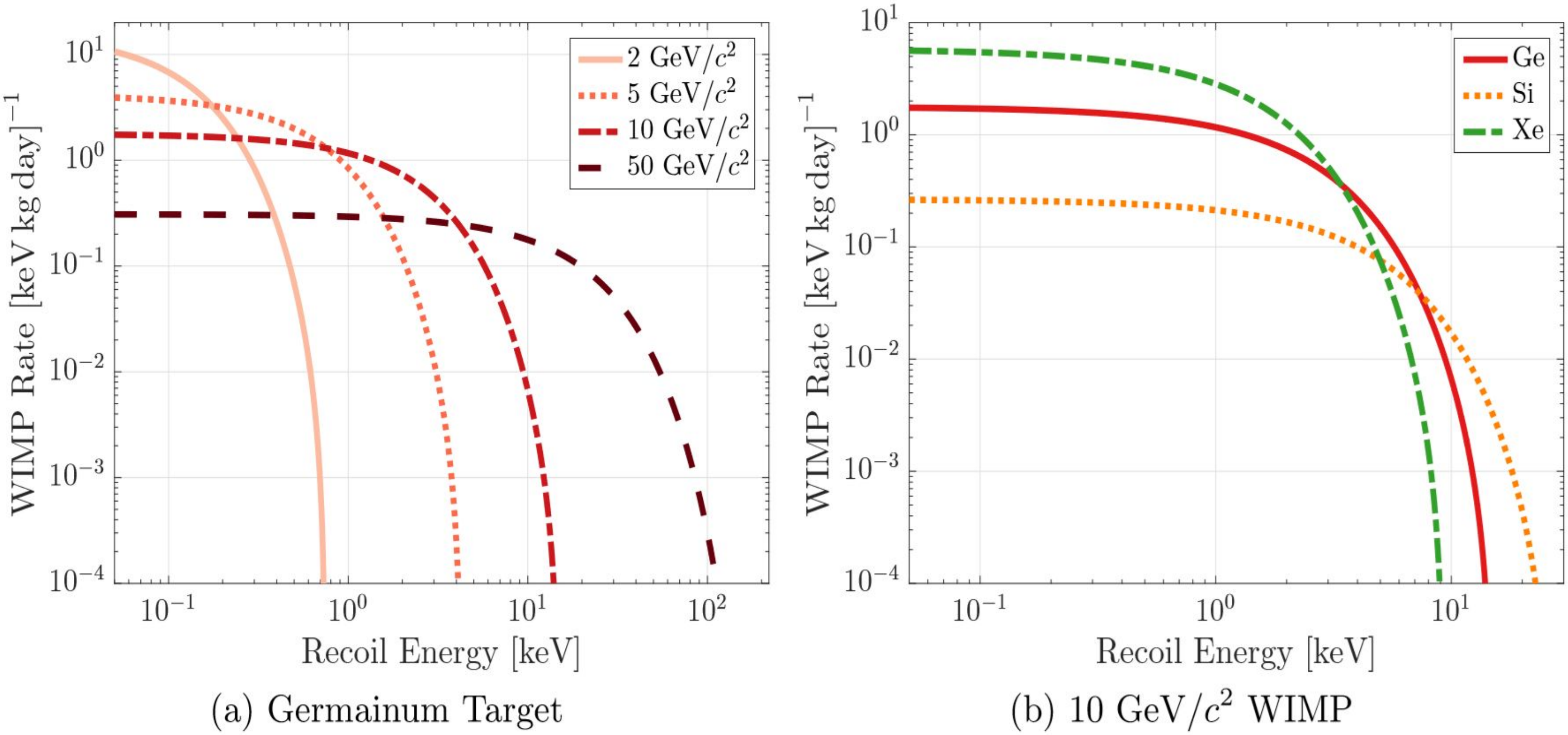
```
[nah588@login03 build]$ ls
CMakeCache.txt  CMakeFiles  cmake_install.cmake  install_manifest.txt  libwimp_sim.a  libwimp_sim.so  Makefile  wimp_sim_test
[nah588@login03 build]$ ./wimp_sim_test 3
2191.53,0.0563601,0.967968,0.29983,0.967968,-0.379873,159439,117058,-302131,130778,-123803,-302131,6882.46,0.0325395,0.0325395,0.0325395
552.011,0.028286,0.991847,0.636579,0.991847,0.564292,169189,249199,278644,145064,-633.977,278644,5490.19,0.016331,0.016331,0.016331
1946.78,0.0531199,0.971501,0.33281,0.971501,-0.633217,-204039,165583,-181754,-232392,-76217.8,-181754,5165.46,0.0306688,0.0306688,0.0306688
[nah588@login03 build]$
```



GEANT4 is the standard particle simulation tool for particle interactions. However, GEANT4 does not provide particle interactions for particles not contained in the Standard Model of particle physics. Therefore, if we would like to simulate WIMPs and WIMP interactions, we'll need to create our own custom module to do so.

WIMPSim is added as a module, so WIMP energy deposits can now be simulated in the detector.

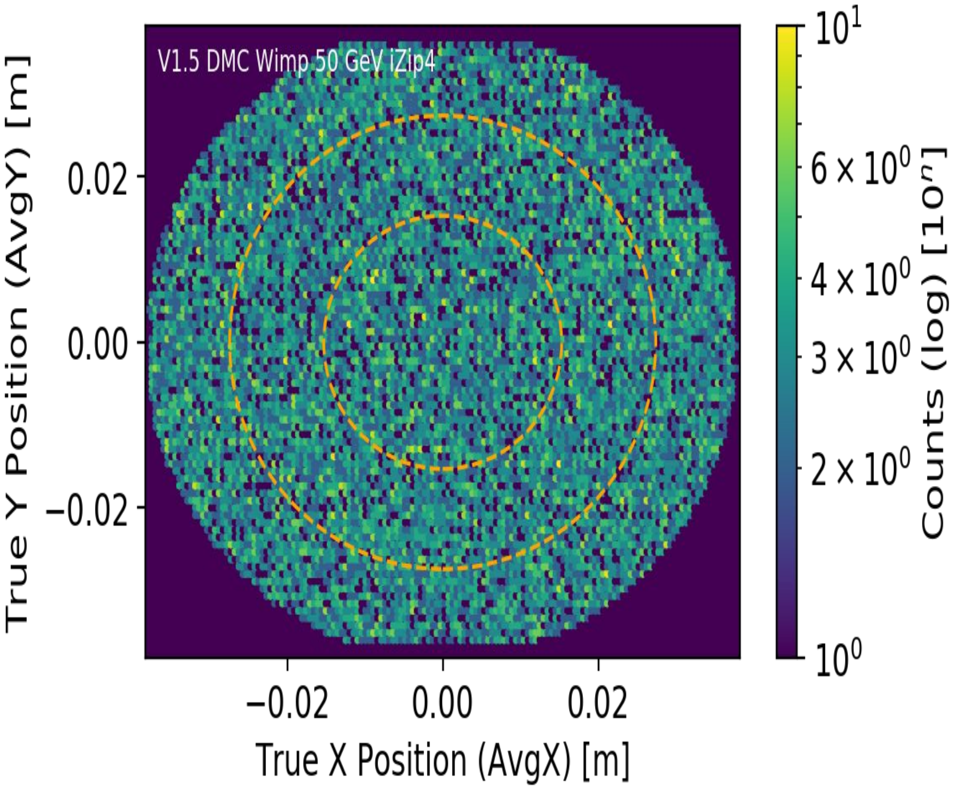
Results from Previous Calculations as a Function of Wimp Mass and Detector Type

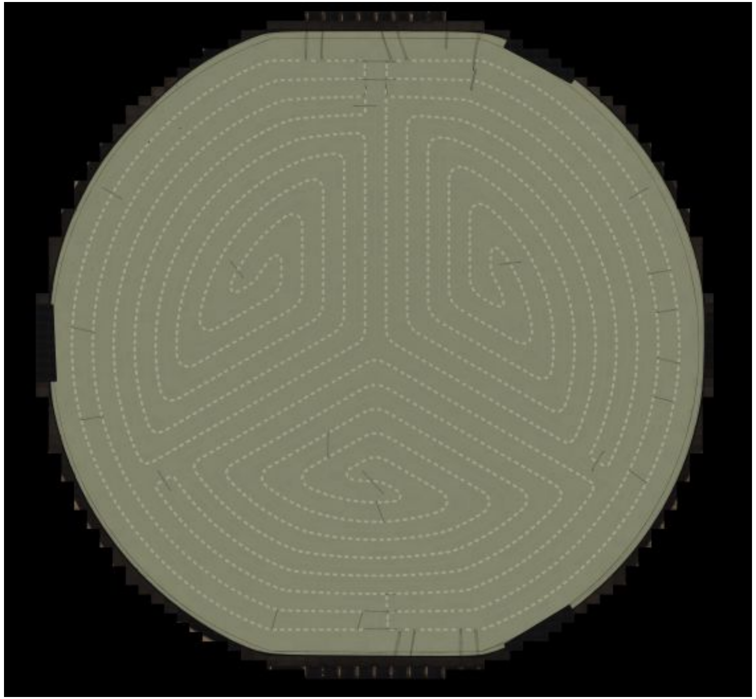


Source: “Low-Mass Dark Matter Search Results and Radiogenic Backgrounds for the Cryogenic Dark Matter Search” by Mark Pepin; Figure 3.3 [4]

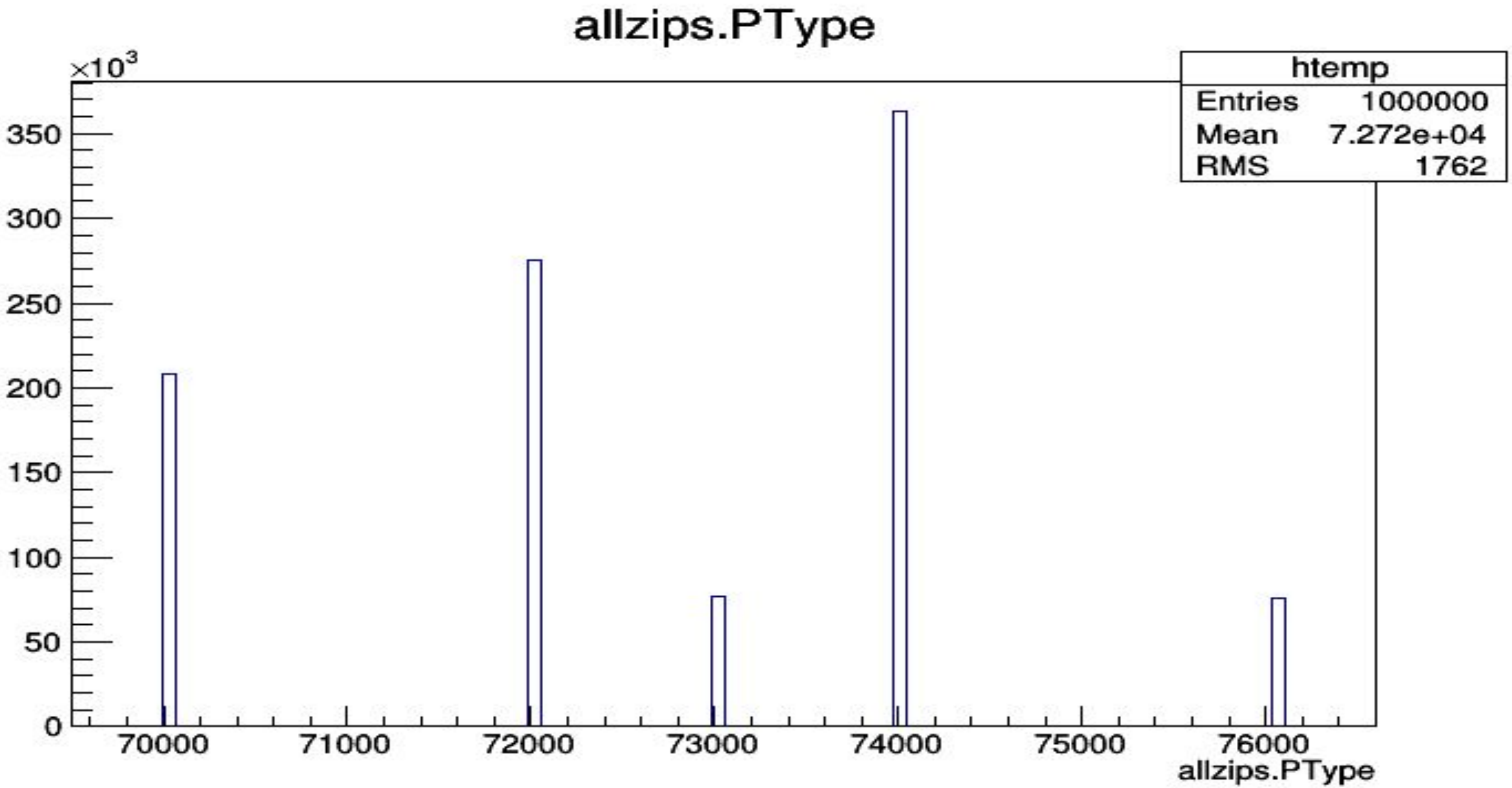
2. Current Results

Simulation ran through a germanium detector from the CDMS Soudan experiment. Collisions are uniformly distributed throughout the detector. Here we show the location of 40k collisions for WIMPs at a mass of 50GeV.

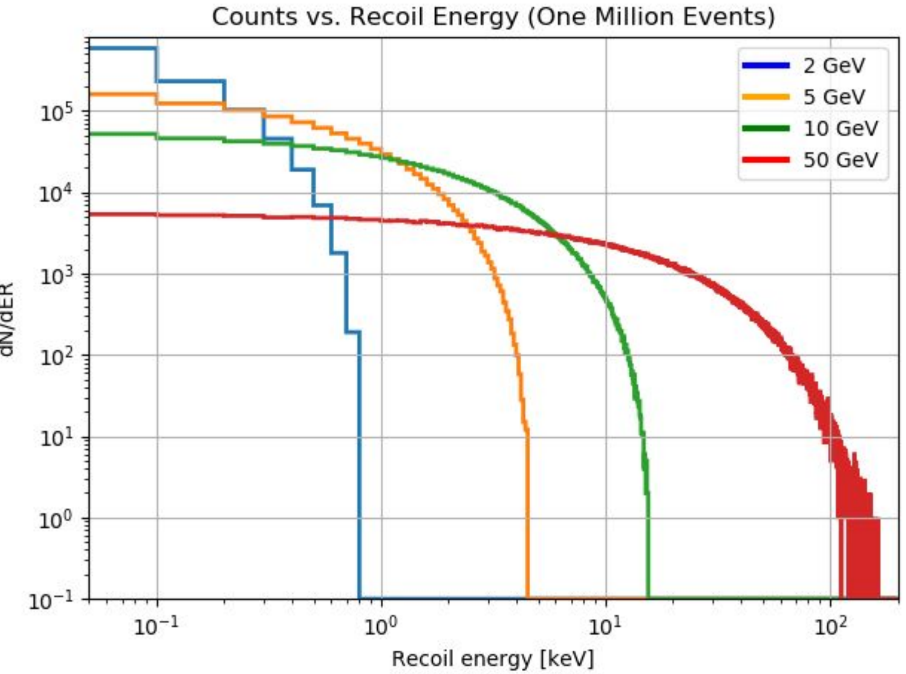




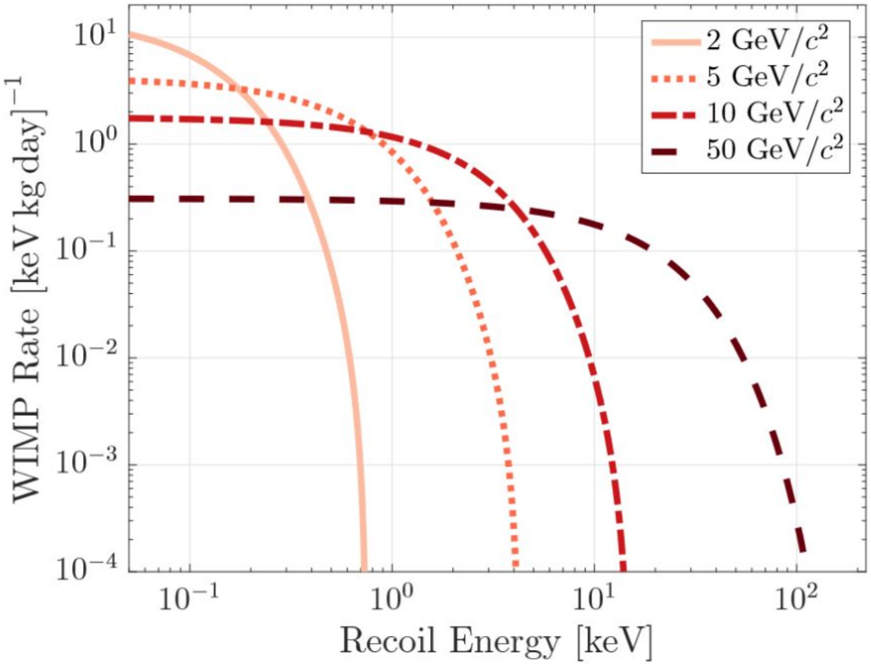
Counts vs Nuclei Type - For izip14, these are all isotopes of Germanium (1e6 hits)



Simulation results; Counts vs Energy



Theory results; Rate vs Energy

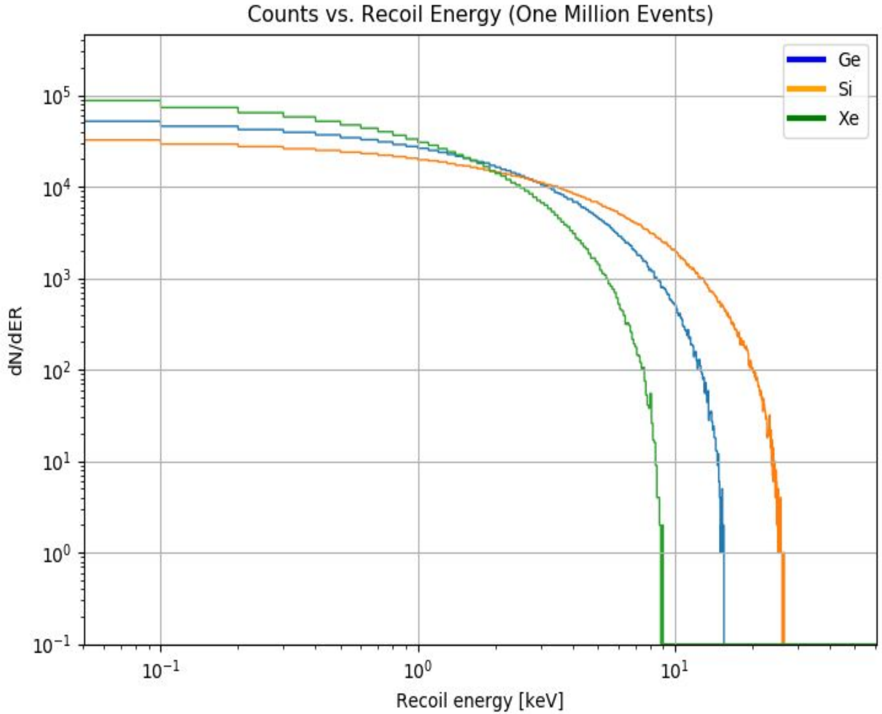


Germainum Target

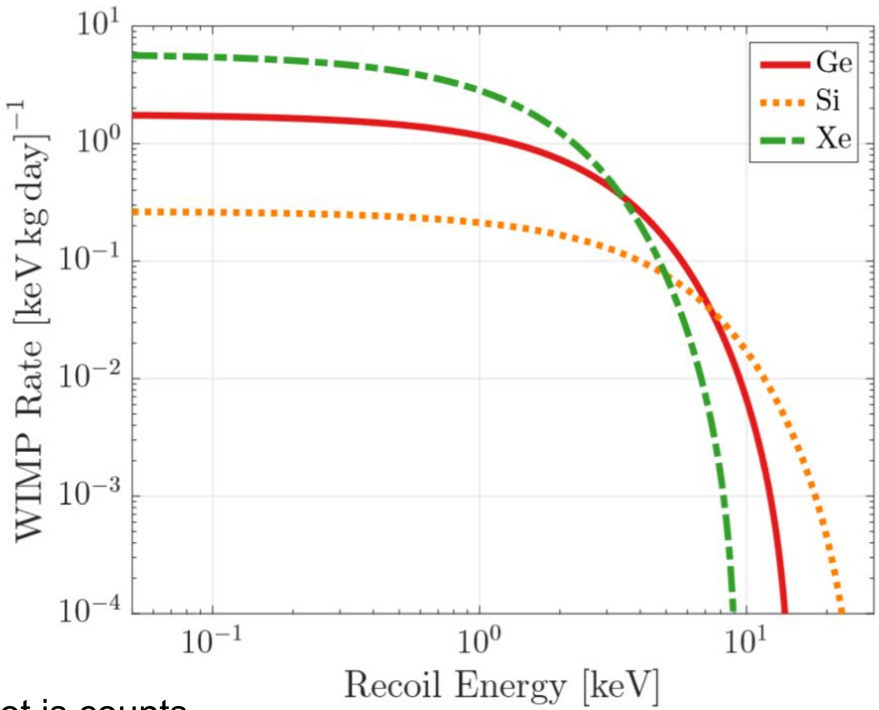
Mark Pepin; Figure 3.3 [4]

These are not directly comparable as the theoretical plot is counts per unit time, per unit mass of detector. Left hand side is a Monte-Carlo simulation of distributions at a large number of counts.

Simulation results; Counts vs Energy



Theory results; Rate vs Energy



10 GeV/c^2 WIMP

Mark Pepin; Figure 3.3 [4]

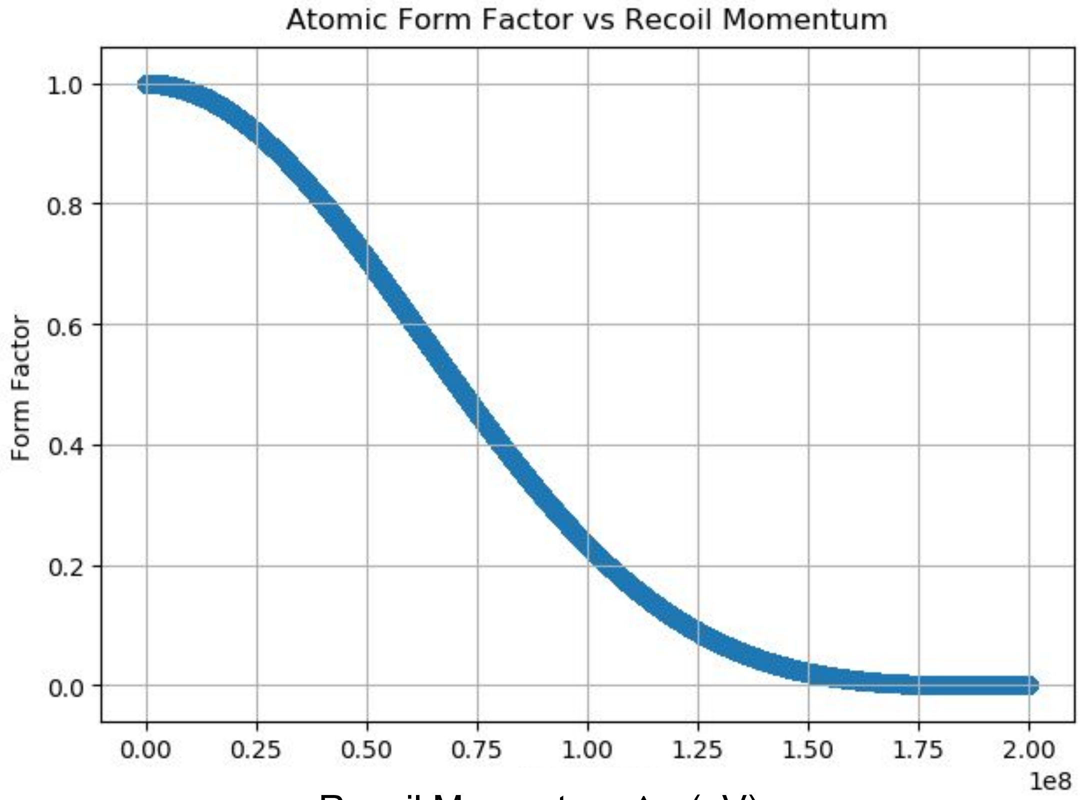
These are not directly comparable as the theoretical plot is counts per unit time, per unit mass of detector. Left hand side is a Monte-Carlo simulation of distributions at a large number of counts.

3. Anomalies/Issues with Data

List of Issues

- Form Factor Optimization
- Form Factor is not weighing results when ran through SuperSim
- Center of mass scattering angle is skewed after being weighted by the form factor

What is the Atomic Form Factor?



Recoil Momentum Δp (eV)

The atomic form factor depends on the type of nuclei and recoil energy/momentum. The higher the relative recoil momentum of a particular collision, the less likely it is to occur.

This graph is specifically the form factor for a Germanium isotope - atomic weight A = 70

$$F_{SI}(\Delta p) = 3\frac{j_1(\Delta p, r_n)}{\Delta p r_n} e^{-\frac{1}{2}(\Delta p s)^2}$$

Form Factor Explanation

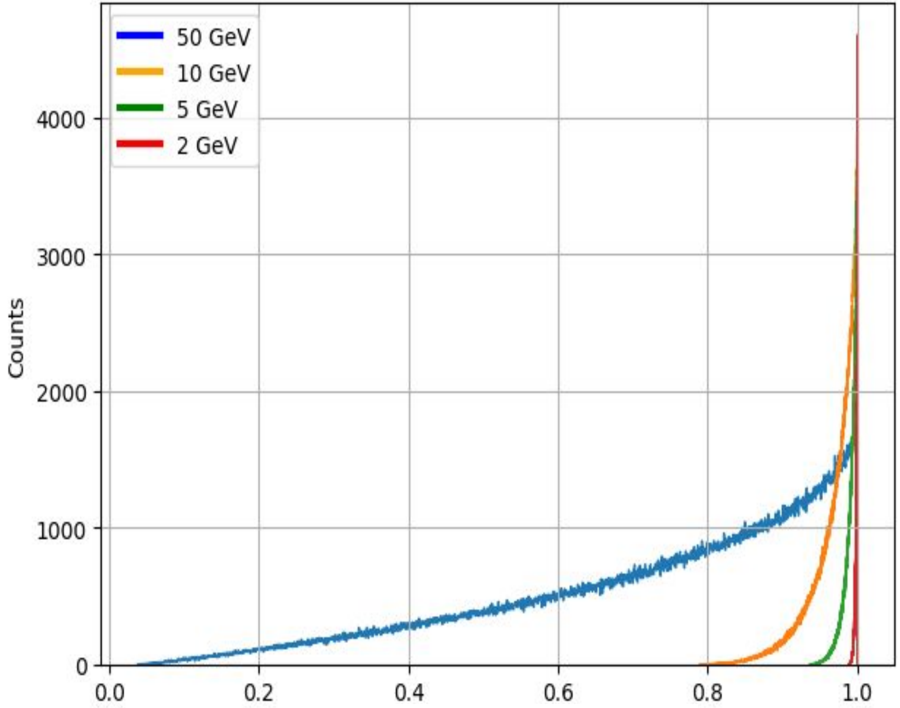
WIMP are generated in WIMPSim with predetermined mass and velocity

WIMP hits a nucleus and recoil momentum/energy are calculated. **The recoil momentum is weighted by the form factor to produce the correct distribution on hits.** For example, for a given collision, let's say F(q) = 0.5 meaning the probability of that collision relative to other collisions is 0.25. We only keep 25% of collisions with this momentum transfer in order to match analytical calculations.

This is fine for low mass WIMPs, as most collisions are low recoil momentum, but is **extremely inefficient for high mass WIMPs**, as the calculated recoil momentum goes up significantly on average.

Squaring the Form Factor gives the probability of a particular collision

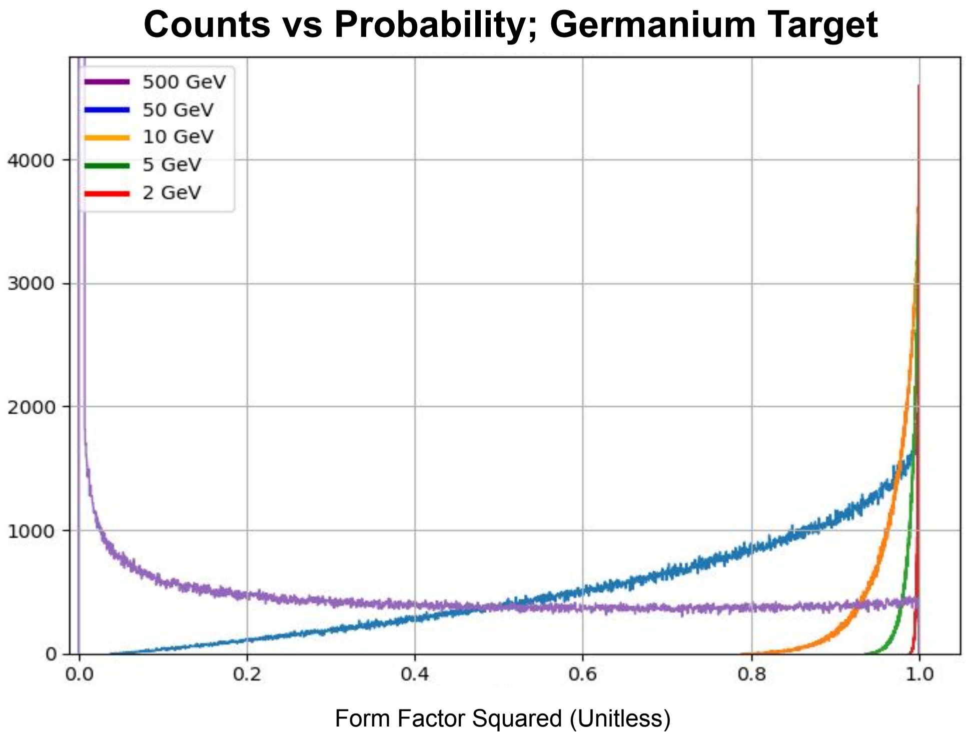
Counts vs Probability; Germanium Target



Form Factor Squared (Unitless)

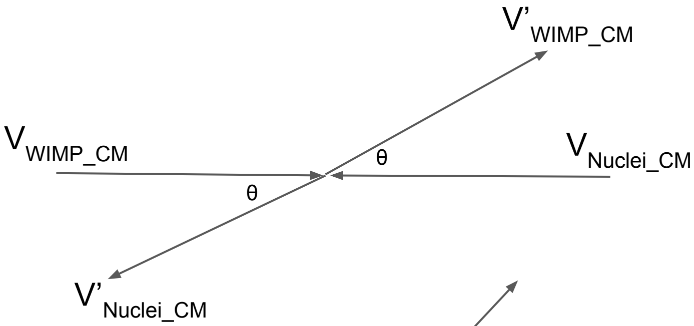
Collision counts per type (1e6 runs)

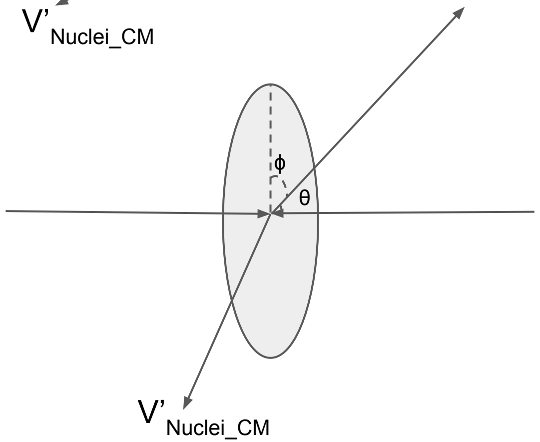
WIMP Mass; Target	Events	% Calc
2 GeV; Germanium	998365	99.8%
5 GeV; Germanium	990646	99.1%
10 GeV; Germanium	967952	96.8%
10 GeV; Silicon	987017	98.7%
10 GeV; Xenon	944313	94.4%
50 GeV; Germanium	733118	73.3%
500 GeV; Germanium	391052	39.1%



Calculating Recoil 3-Momentum

Let's define a coordinate system where all of the WIMP's momentum is in the x-direction. For simplicity's sake, let's also assume the recoil will always occur in the x-y plane. In the CM frame of this coordinate system:





Calculating 3-momentum is a 5 step process.

1. Rotate to WIMP coordinate system
2. Boost to CM frame of reference
3. Calculate recoil momentum 3-vector using a given θ and ϕ

- We now have a recoiled nucleus momentum 3-vector in the WIMP's coordinate system, in the center of mass frame of reference.

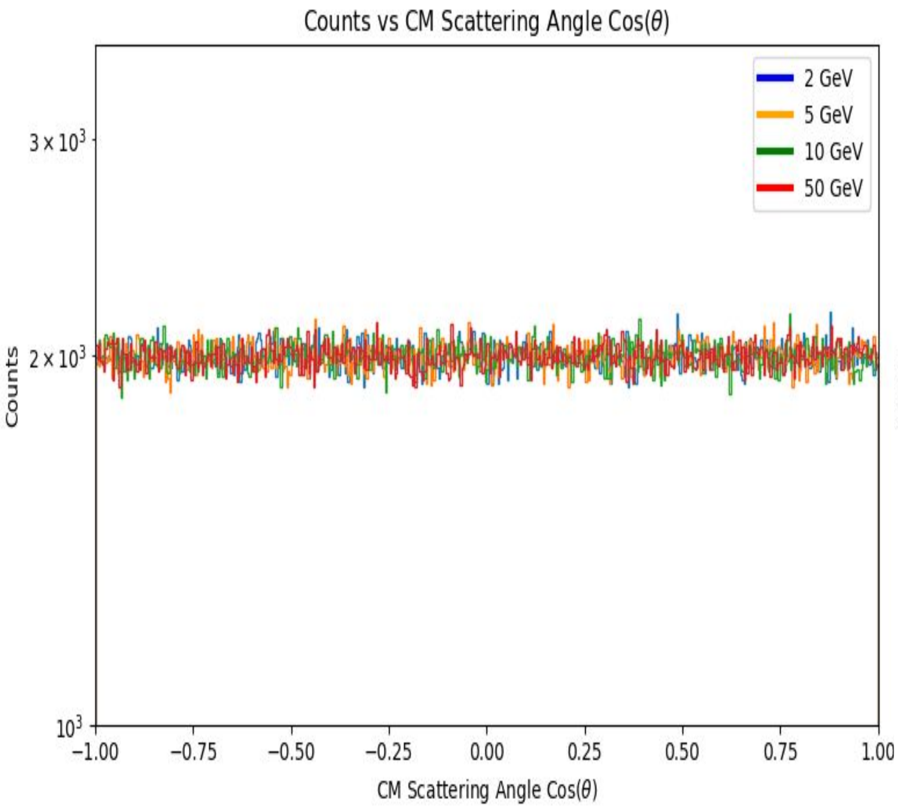
4. Boost back to the detector frame of reference
5. Inverse the rotation to rotate back into the detector coordinate system

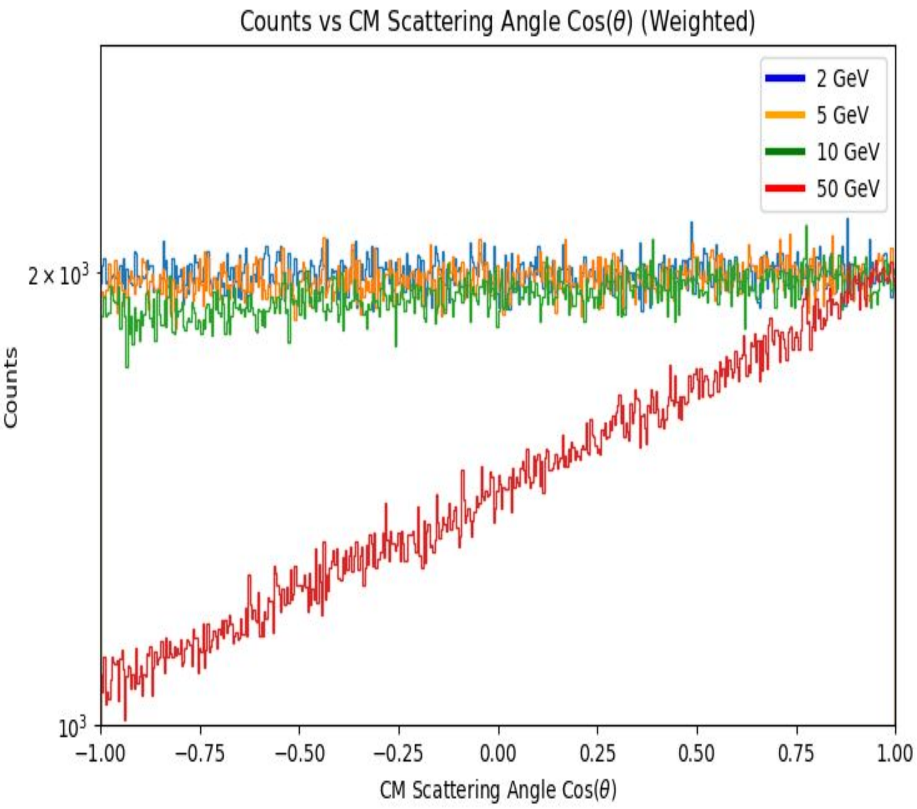
Given a unit-axis of rotation **k** and an angle of rotation θ, we can rotate an arbitrary vector **v** using Rodrigues' rotation formula.

$$\mathbf{v}_{\mathrm{rot}} = \mathbf{v}\cos\theta + (\mathbf{k}\times\mathbf{v})\sin\theta + \mathbf{k}\,(\mathbf{k}\cdot\mathbf{v})(1-\cos\theta)\,.$$

To rotate back, simply replace θ → -θ

Scattering angle is completely skewed after being weighted by the atomic form factor. Asymmetry appears to increase with WIMP mass. At minimum it should be symmetric. (Below is 1e6 collisions)





4. Work to Be Done

WimpSim

- Form Factor is unoptimized for high energy recoils (i.e. high mass WIMPs)
- Collision angle anomaly
- Momentum 3-vector calculation is broken; (potentially not an issue as SuperSim can calculate this result)
- Carefully consider difference between galactic and detector frame of reference, WimpSim may need an altitude variable
- Multithreading
- Major Spaghetti Code (mixture of C and C++ arrays)
- Rewrite as a class with methods instead of functions
- RNG needs to be pulled from SuperSim (so the same seed produces the same results)
- Inelastic Scattering?
- Consistent units need to be used or assumed throughout, such that the various kinematic calculations can all be done without requiring extra factors of c or resulting in nonsensical mixed units.

SuperSim

- Form Factor does not weigh simulation results when ran through SuperSim
- Multiple types of WIMPs (Spin 0, Spin ½, Spin 1) allow for distinguishing between higher order collision terms in WIMP interactions
- Adding hooks for annual modulation studies. i.e. Adding user defined data to root output
- A template UI command class, CDMSWimpSimMessenger was created, but it does not contain relevant commands. It should be populated with appropriate macro commands to set the configurable parameters used by wimp_sim. The generator class will need data members for those parameters to be set, and which it will pass through to wimp_sim.

Analysis

- Compare Monte Carlo with analytic results
- Additional quantitative validations and comparisons
  - (Residual plot?)

Conclusion

WIMPSim is very lightweight tool currently integrated into SuperSim. Since it does not have any dependencies on external libraries, it can be run independently as extremely quickly tool, making it easy to modify, update, and analyze.

While WIMPSim is a good start, there are many things that need to be updated.

- Develop tools to validate these results are correct.
- Add tools to SuperSim (mainly hooks for additional data to be sent to root).
- Most likely WIMPSim will have to be rewritten from the ground up with multithreading in mind.

How to get involved with WIMPSim

At the time of writing 8/2/2019 information can be found at the following:

Confluence page on WIMPSim: https://confluence.slac.stanford.edu/display/CDMS/WIMPs

The repository can be found in GitBlit on SUF at Simulations/wimp_sim

References

[1] Images credit: Van Albada et al. (L), A. Carati, via arXiv:1111.5793 (R). Observed velocities versus distance from the center of galaxy NGC 3198. The theoretical prediction before observations followed the trend labeled “disk”, but observations (black squares) showed constant, rather than decreasing velocity. Adding a contribution from a dark matter halo (center line) makes the theory match predictions.

[2] Review of mathematics, numerical factors, and corrections dark matter experiments based on elastic nuclear recoil [J.D. Lewin, RF. Smith]

[3] Model Independent Form Factors for Spin Independent Neutralino-Nucleon Scattering from Elastic Electron Scattering Data [Gintaras Duda, Ann Kemper, Paolo Gondolo]

[4] Low-Mass Dark Matter Search Results and Radiogenic Backgrounds for the Cryogenic Dark Matter Search [Mark David Pepin]

[5] The Inelastic Frontier: Discovering Dark Matter at High Recoil Energy [Joseph Bramante, Patrick J. Fox, Graham D. Kribs, Adam Martin]

Backup Slides

Parameters in WimpSim

Scattering angle (Θ): -π/2 to π/2

Velocity of WIMP in galactic frame: norm(0, 220x10^3 km s^{-1})†

Earth Phase: 0 - 2π

Nuclear skin thickness (s): 0.9e-15 m

Radius to half maxima of charge distribution for an atom (c): 1.23$A^{-1/3}$ - 0.6e-15 m

Proton Nuclear Radius (a): 0.52e-15 m

†Drukier et al. argue that v_0 = V_r (the galactic rotation velocity) for a galaxy with a flat rotation curve.
Reported values for V_r, are: 243 ± 20 km s^{-1}; 222 ± 20 km s^{-1}; and 228 ± 19 km s^{-1}. I used V_0 = V_r = 230 km s^{-1}

Weights

Helm Form Factor:

$$F_{SI}(\Delta p) = 3\frac{j_1(\Delta p, r_n)}{\Delta p r_n} e^{-\frac{1}{2}(\Delta p s)^2}$$

$$r_n^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2$$

$$c \simeq 1.23A^{1/3} - 0.60 \text{ fm}$$

$$s \simeq 0.9 \text{ fm}$$

$$a \simeq 0.52 \text{ fm}.$$

Wimp Escape Velocity: $v_{esc} = 544 \times 10^3$ m/s

WIMP recoil probability weighted by Helm form factor, and if velocity of wimp is greater than the escape velocity the weight is set to 0

Theory Portion

What we expect a count rate vs eV distribution to look like:

$$\frac{\mathrm{d}R}{\mathrm{d}E_\mathrm{r}} = \underbrace{\frac{N_T m_T}{2m_\chi \mu_T^2}}_{\text{Detector}} \cdot \underbrace{\left[\sigma_0^{\mathrm{SI}} F_{\mathrm{SI}}^2(E_\mathrm{r}) + \sigma_0^{\mathrm{SD}} F_{\mathrm{SD}}^2(E_\mathrm{r})\right]}_{\text{Particle and Nuclear}} \cdot \underbrace{\rho_0 \int_{v_{\min}}^{v_{\max}} \frac{1}{k} \frac{f(\boldsymbol{v}, \boldsymbol{v}_E)}{v} \mathrm{d}^3\boldsymbol{v}}_{\text{Astro}}$$

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Focus on spin independent WIMPs

Why is there a minimum velocity in $\frac{\mathrm{d}R}{\mathrm{d}E_{\mathrm{r}}}$?

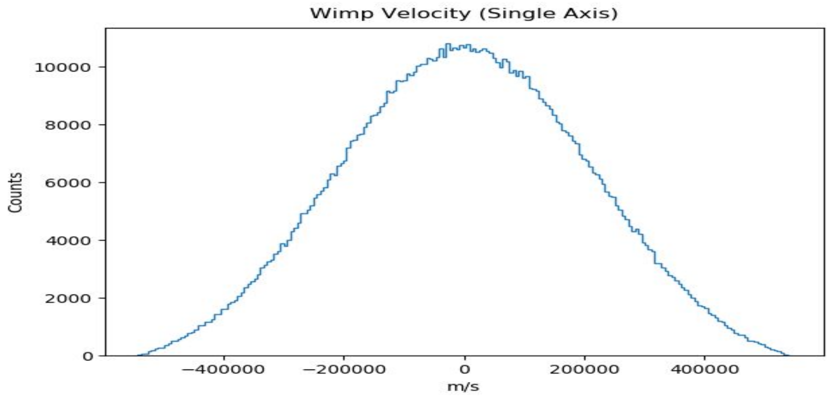
Since recoil energy is given by: $\Delta E = \frac{\mu^2}{m_2} V_1^2 (1 - cos(\theta))$

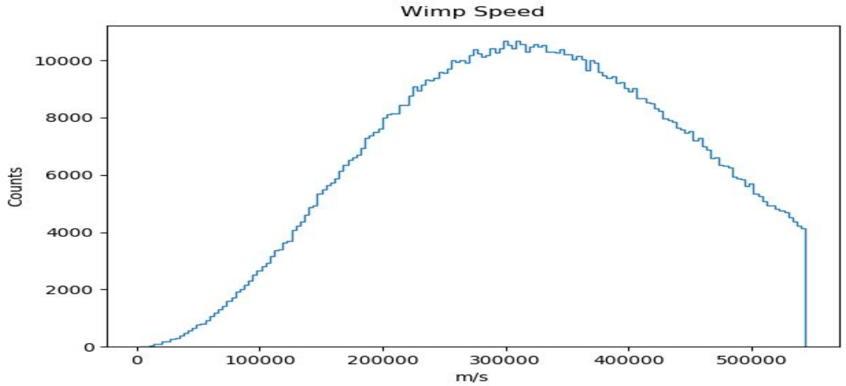
There is only a specific range of wimp energies (and hence velocities) that will produce a given recoil energy

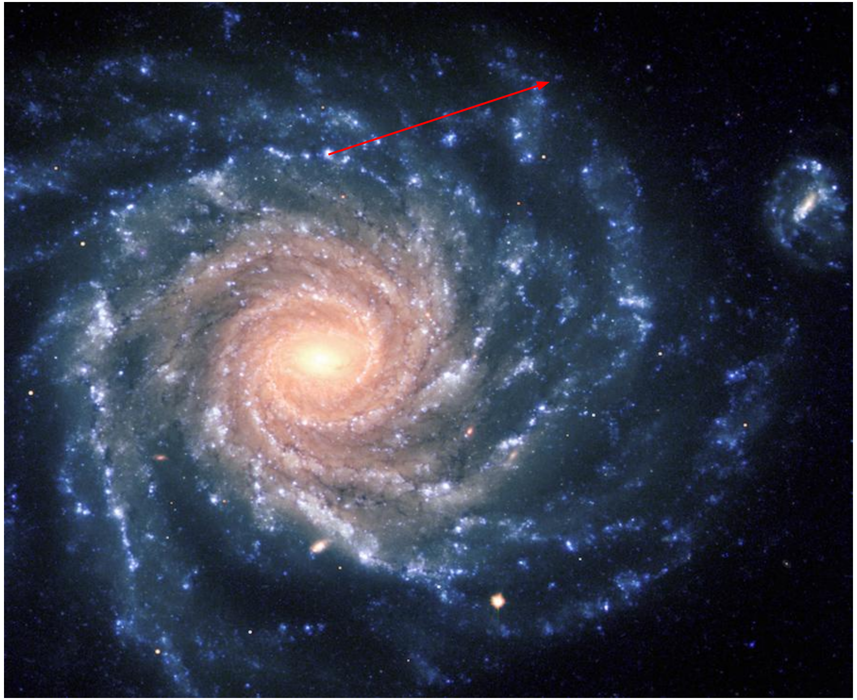
$$v_{\min} = \sqrt{\frac{m_T E_{\mathrm{r}}}{2\mu_T^2}}$$

Astrophysics: Velocity Distribution

Wimp Escape Velocity: v_{esc}= 544x10^3 m/s







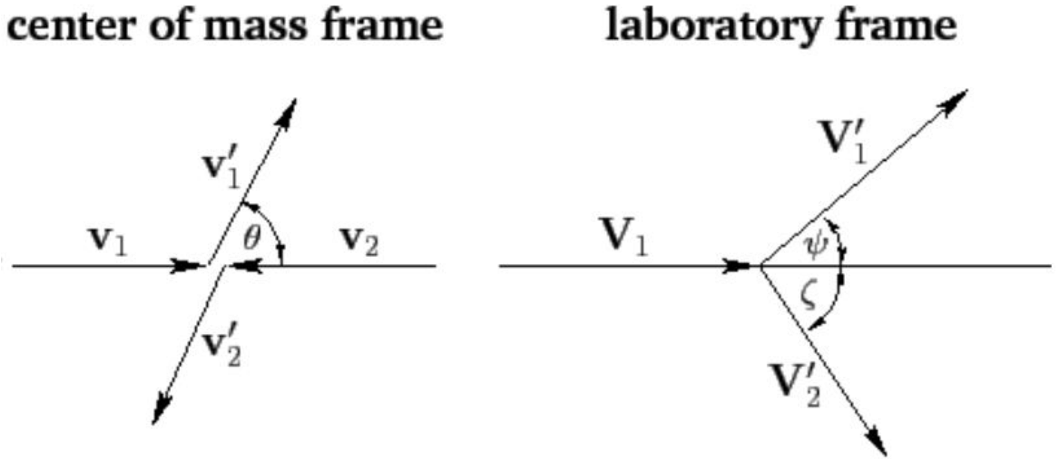
- WIMPs are modeled to be a non-interacting gas at thermal equilibrium, hence a Maxwell–Boltzmann distribution
- We use the simplifying assumption that WIMPs are normally distributed throughout the galaxy, with a hard cutoff at the galactic escape velocity.

Kinematics: Nuclear Recoil Energies

An elastic collision between a wimp and nucleus is the leading-order contribution to the energy transfer.

The experiment is chilled to a temperature of 40mK, making it safe to model the target nucleus as stationary compared to the WIMP.

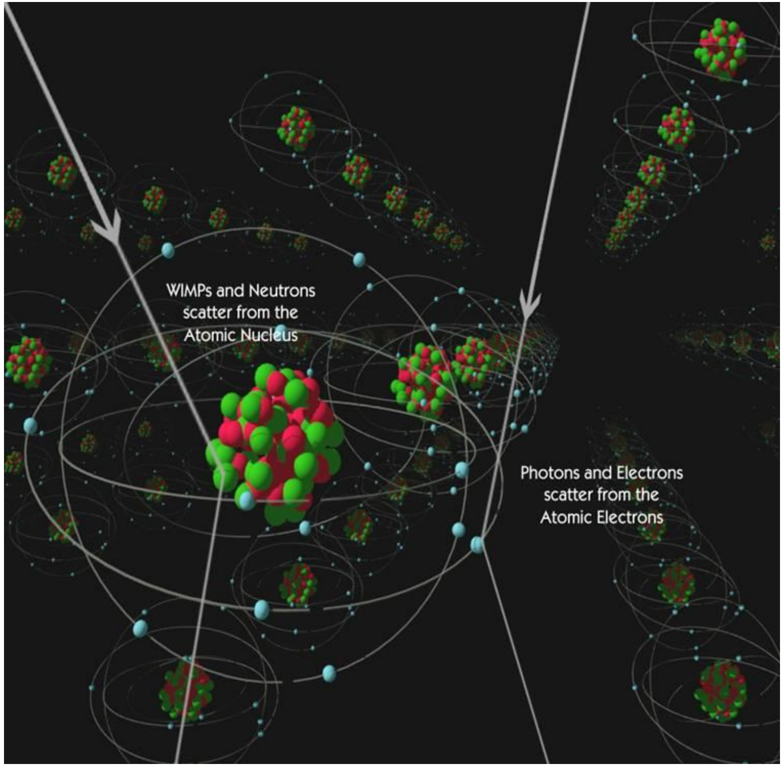
Given the two masses, WIMP velocity, and scattering angle, the energy and momentum transferred to the recoiled nucleus are given by:



$$E_{recoil} = \frac{\mu^2}{m_N} V_1^2 [1 - cos(\theta)] \qquad p_{recoil} = \sqrt{2 m_N E_{recoil}}$$

Where μ is the reduced mass of the system: $\mu = \frac{m_W m_N}{m_W + m_N}$

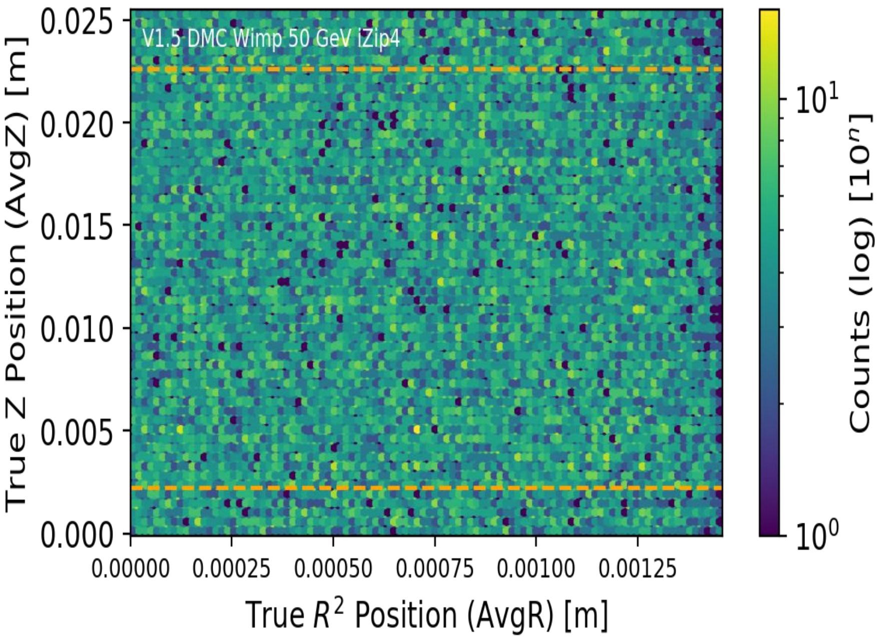
Particle Physics: Nuclear Parameters

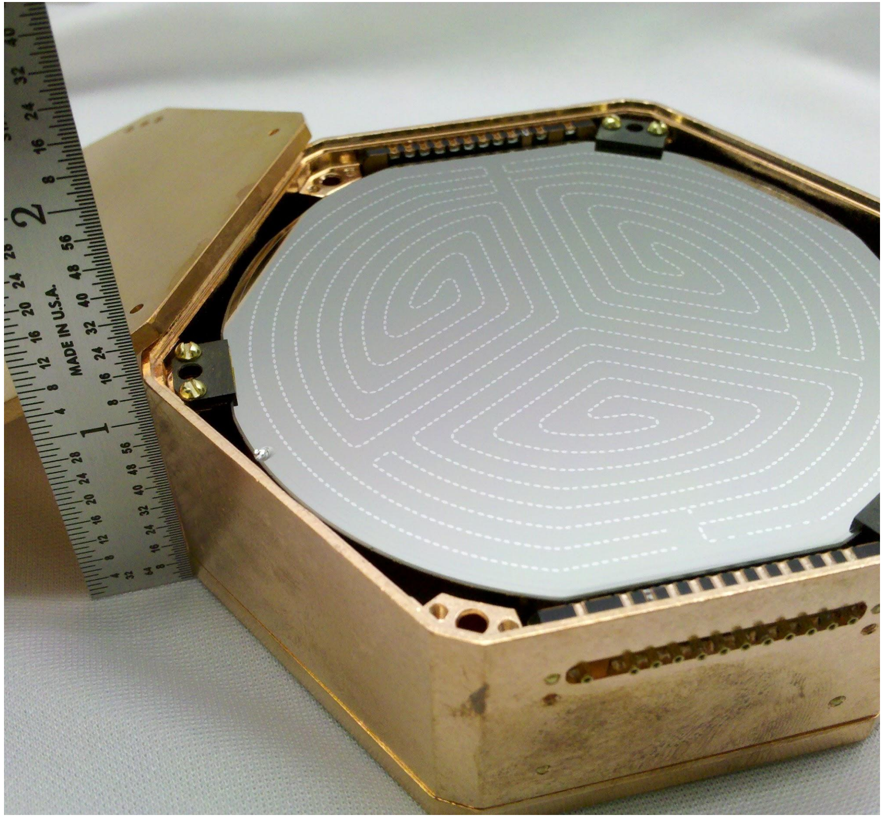


Nucleus specific parameters such as the effective nuclear radius are needed to produce correct hit distributions.

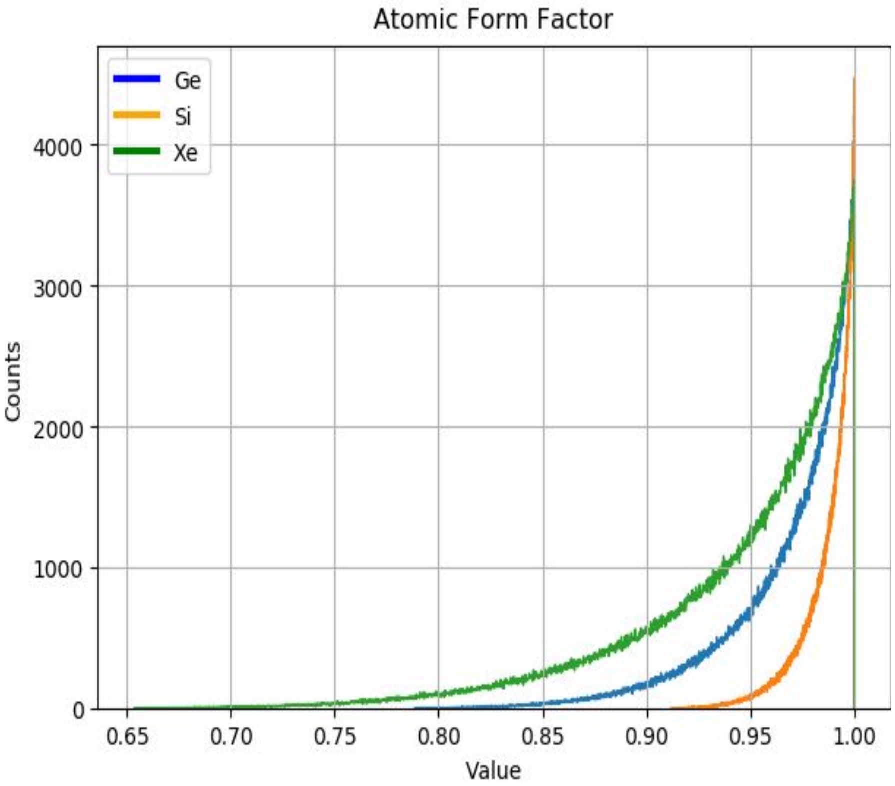
- A WIMP is more likely to interact with a larger nucleus.
- A weight is added to each collision as a WIMP is more likely to graze a nucleus rather than hit it head on.

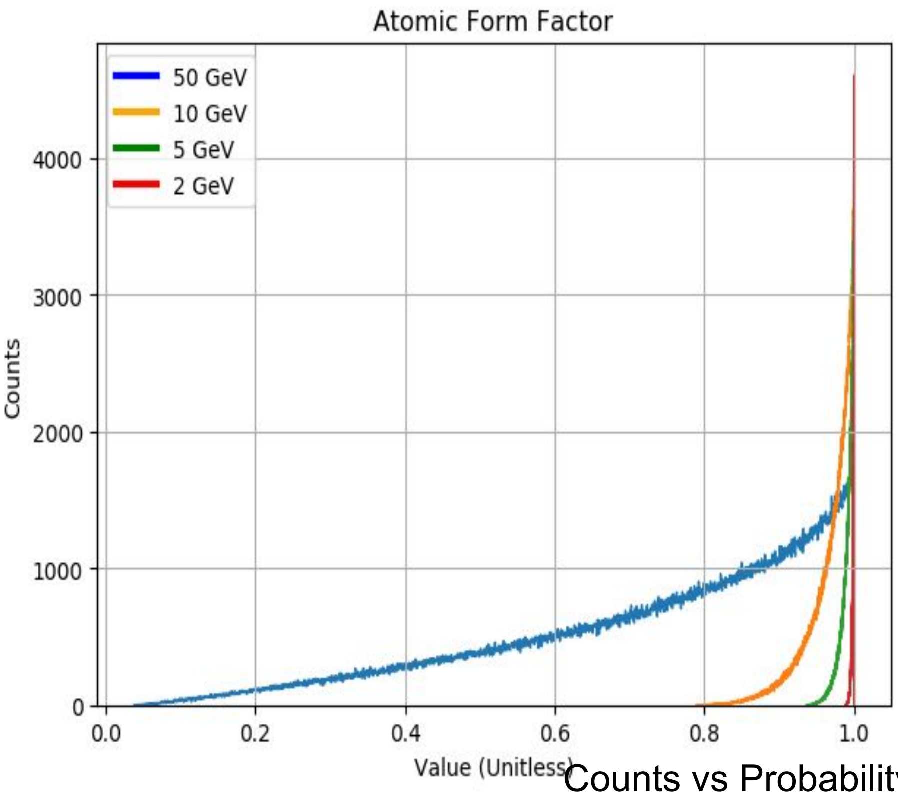
Simulation ran through a germanium detector from the CDMS Soudan experiment. Collisions are uniformly distributed throughout the detector. Here we show the location of 40k collisions for WIMPs at a mass of 50GeV.





Squaring the Form Factor gives the probability of a particular collision occurring.





Counts vs Probability