Compton Scattering

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Abstract

In our experiment we use the radiation source cesium-137, an aluminum rod, and a NaI Scintillation Detector to measure the incident and reflected wavelengths of incoming and outgoing photons at various angles. Using this information and Compton's Scattering formula, we measured the mass of an electron. Our value came out to .514 MeV, only 0.59% off of the scientifically accepted value of .511 MeV. Our results both accurately match the accepted results of the scientific community, and confirm Compton's formula to be accurate.

1 Introduction

1.1 Motivation

The Compton Effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon. Light must behave as if it consists of particles to explain the low-intensity Compton scattering. [1] Compton's experiment convinced physicists that light can behave as a stream of particle-like objects (quanta) whose energy is proportional to the frequency.

Compton scattering is very important to radio biology, as it is the most probable interaction of gamma rays and high energy X-rays with atoms in living beings and is applied in radiation therapy.[2] In material physics, Compton scattering can be used to probe the wave function of the electrons in matter to give scientists a better idea of what electrons look like and how they work.

1.2 Background

Compton scattering is an inelastic scattering of a photon by a free charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X-ray or gamma ray photon), called the Compton effect. Part of the energy of the photon is transferred to the recoiling electron. The Compton effect was observed by Arthur Holly Compton in 1923 at Washington University in St. Louis and further verified by his graduate student Y. H. Woo in the years following. Compton earned the 1927 Nobel Prize in Physics for the discovery. [3]

Because the mass-energy and momentum of a system must both be conserved, it is not generally possible for the electron simply to move in the direction of the incident photon. The interaction between electrons and high energy photons results in the electron being given part of the energy (making it recoil), and a photon containing the remaining energy being emitted in a different direction from the original, so that the overall momentum of the system is conserved.

The derivation of Compton Scattering follows. A photon γ with wavelength λ collides with an electron e in an atom, which is treated as being at rest. The collision causes the electron to recoil, and a new photon γ' with wavelength λ' emerges at angle ϑ from the photon's incoming path.

The conservation of energy E simply requires that the energy before and after the collision is the same.

$$E_\gamma + E_e = E_{\gamma'} + E_{e'} \tag{1}$$

Compton postulated that photons carry momentum; [4] thus similarly from the conservation of momentum, the momentum's before and after should be the same.

$$P_\gamma = P_{\gamma'} + P_{e'} \tag{2}$$

Here we are assuming that $P_e = 0$

Substituting Planck's relation $E_\gamma = hf$, and the Einstein mass-energy equivalence equation $E = mc^2$, into equation 1 yields

$$hf + m_e c^2 = hf' + \sqrt{(p_{e'}c)^2 + (m_e c^2)^2}$$

or more conveniently,

$$p_{e'}^2 c^2 = (hf - hf' + m_e c^2)^2 - m_e^2 c^4 \tag{3}$$

Rearranging and squaring Eq. 2 yields

$$p_{e'}^2 = p_\gamma^2 + p_{\gamma'}^2 - 2p_\gamma p_{\gamma'} cos(\vartheta)$$

which can again use Planck's relation to be expressed as

$$p_{e'}^2 c^2 = (hf)^2 + (hf')^2 - 2(hf)(hf')cos(\vartheta) \tag{4}$$

Equating Eq. 3 and Eq 4 and simplifying yields

$$\frac{c}{f'} - \frac{c}{f} = \frac{h}{m_e c}(1 - cos(\vartheta))$$

Finally, using the fact that $f = \frac{v}{\lambda} = \frac{c}{\lambda}$ for photons, we obtain

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - cos(\vartheta)) \tag{5}$$

the so called Compton Scattering Formula.

In our experiment we are attempting to verify Compton's formula by using it to measure the already well known mass of the electron at .511 MeV. To do this we will measure the incident and reflected wavelengths of photons that are bounced off of an aluminum rod from a variety of angles using a scintillation detector. We will then use this data to calculate, via the compton formula, the mass of an electron. We will repeat this process over several trials and average our results together.

2 Experimental Setup

2.1 Apparatus

Our apparatus begins with a radiation source of cesium-137 surrounded by a lead sphere. The sphere has a hole punched in it allowing for a single, lazor-like beam of radiation that can be manipulated by the experimenter. The beam is directed through an aluminum rod and into a Bicron Saint Gobain NaI Scintillation Detector which is connected to a phototube multiplier. The detector is placed on a swivel which allows it to rotate freely.

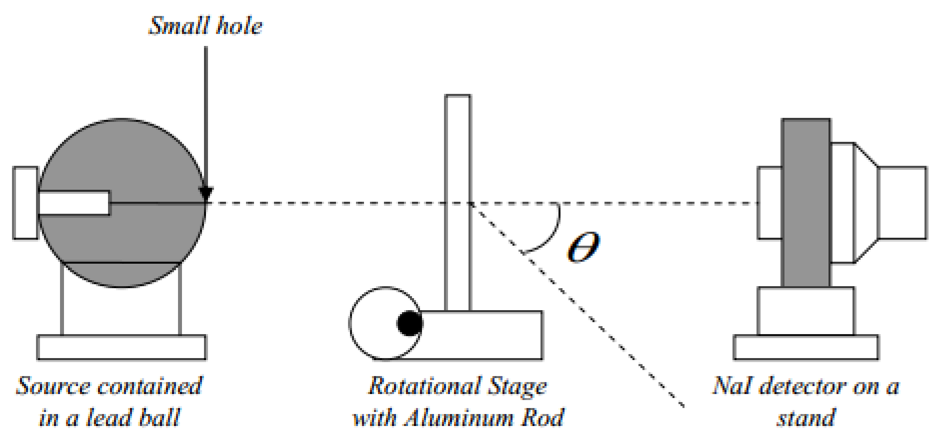

Figure 1: Main Part of Configuration

The signal detected by the scintillator is then fed through a series of electronics. First a 12V Ortec Model 113 Preamplifer, followed by a 12V Ortec Model 472 spectroscopy Amplifier connected to a 12V Ortec 927 Multichannel analyzer. All of which is finally connected to our computer which runs a program called Maestro, which allows for a graphical representation of the number of counts of photons obtained at specific energy levels. The entire apparatus is powered by a high voltage power supply.

A visual representation of the back of the apparatus can be observed in Fig. [2]

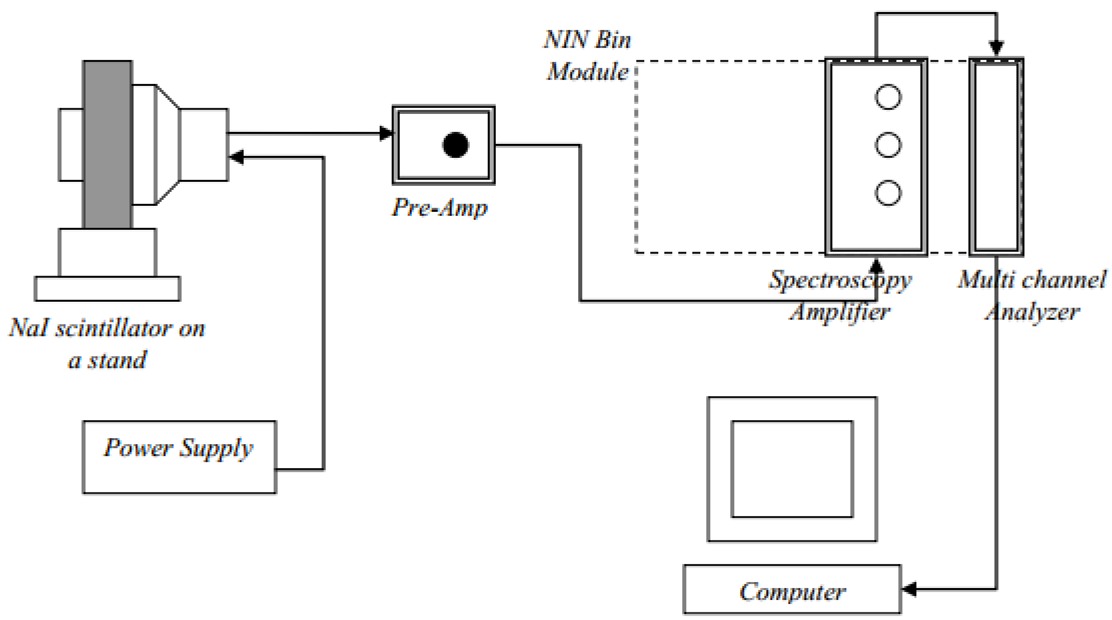

Figure 2: Back Part of Configuration

2.2 Data Collection and Proceedure

First counts were taken without a radiation source present in order to obtain a background radiation count, this was to be subtracted from any experimental data using the radiation source in order to make the affects of our radiation source more pronounced. Next measurements were taken with the NaI scintillator placed directly in line with the radiation beam, or 0^{o}. This gave us a very clear spike in photons of an energy of .669MeV. Using Planck's relation, we converted this energy into a wavelength which we then used as our initial wavelength. We then collected counts on energy levels of photons for a variety of angles up to 35 degrees in intervals of 5^{o}. Measuring these reflected wavelengths and applying them to Compton's formula yields us an estimated value of the mass of the electron. Each measurement was allowed to run for 18 hours, the maximum amount of time allowed by Maestro. The only exception to this was the 0^{o}, 5^{o}, and 10^{o} results. Since the angles were so low for these measurements it took very little time to get lots of hits on our computer. This turned out to be a mistake as results for these three measurements ended up being our most inaccurate.

3 Data Analysis and Results

3.1 Data Processing

Finding the peaks in our data was surprisingly messy. It turns out that Maestro changed the bin sizes every time we took our measurements, giving us random peaks in places where there shouldn't have been, and not being accurate enough in places it should have. For example, for each calculation, after subtracting out our background radiation I would obtain a graph that looked like Fig. [3].

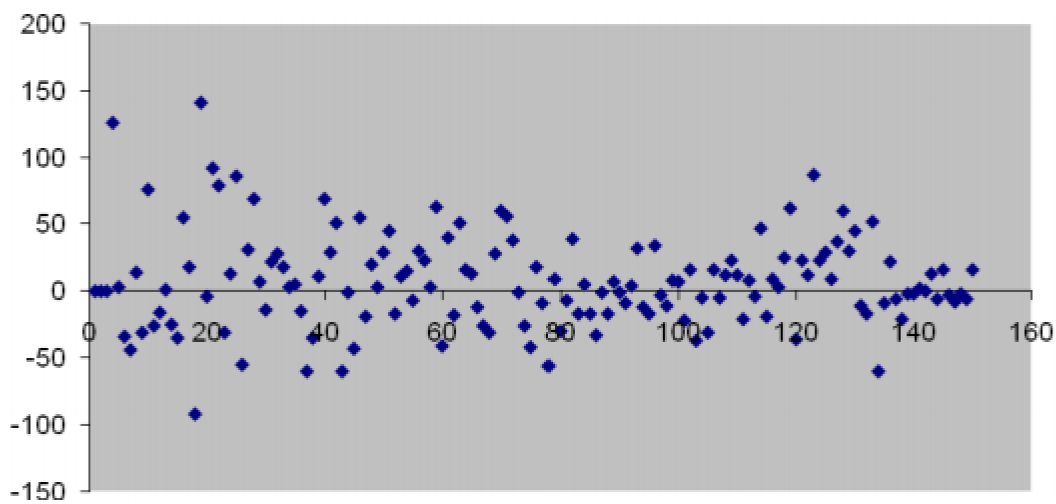

Figure 3: Before Bin Correction

My lab partner, Will Beason, informed me that the peculiar handling of the bins tends to occur when you have old equipment or a bad voltage supply. Correcting the bins ended up being complicated, and a brief description of what we did can be found in Appendix A.

After correcting the bins, it is much easier to spot and estimate the peaks. For example, after correcting the bins of Fig. [3] I obtained Fig. [4].

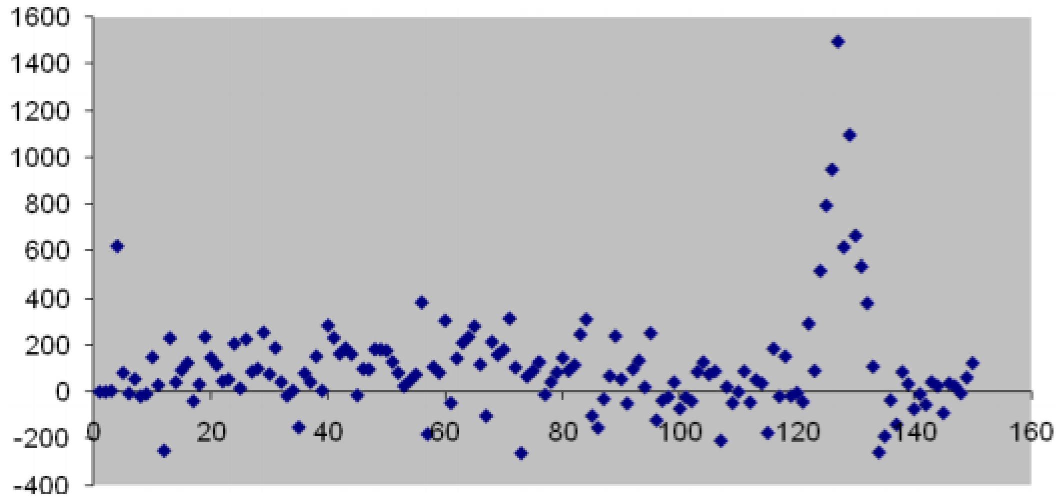

Figure 4: After Bin Correction

This process was repeated 8 times for each of our 8 calculations. Energy values were estimated graphically and the reflected wavelengths and the corresponding estimated values of the mass of an electron were calculated from the energy values using Planck's relation and the Compton Scattering formula. A summary of our results are listed in Table [1].

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Degrees (ϑ)	Scattered Wavelength (λ')	Calculated Mass of Electron	% Error
0	1850.21 fm	NA	NA
5	1860.56 fm	0.463 MeV	9.39%
10	1910.17 fm	0.317 MeV	37.96%
15	1926.20 fm	0.555 MeV	8.61%
20	1993.00 fm	0.521 MeV	1.96%
25	2086.89 fm	0.490 MeV	4.11%
30	2163.96 fm	0.528 MeV	3.33%
35	2271.78 fm	0.531 MeV	3.91%

Table 1: Summary of Experimental Results

Immediately there is something very peculiar about our data. For starters, the theoretical wavelength of celsium-137 is 1869.77 fm where our calculated value is 1850.21 fm. This is not an insignificant difference. Luckily however, throughout the entire experiment we are subtracting the reflected wavelength from the incident, and assuming they both include this error of 69 fm, the difference should cancel out. Secondly notice the huge difference in errors as we get to higher and higher angels. I believe this can be attributed to the fact that we took very short measurements for trials 0^{o}, 5^{o}, and 10^{o}. Since our error for 10^{o} was so huge, I will be omitting it from our results. Thus, averaging our values of the mass of the electron and excluding the major outlier of 10^{o} yields an average value of $m_e = \frac{\sum m_{e_j}}{j} = .514$ MeV, a remarkably close calculation to the accepted value of .511 MeV.

4 Summary and Conclusion

In conclusion I am very happy with our results. We set out to use Compton's formula in an attempt to measure the mass of the electron. Our data gave us an average mass of .514 MeV for the electron while the accepted value is .511 MeV. Our results differ from the current scientifically accepted value by 0.59%.

I under no circumstances assume that since we achieved an excellent answer, our experiment was without flaw and our calculations are accurate. For starters, the % error on each individual measurement was on average 5.22%, and although averaging them gave a very good result, since we only had 8 non outlier data points, it may have simply been luck that they averaged out so accurately.

There are several ways to improve this experiment. First and most easily attainable is time. Had we taken our data points for 5 and 10 degrees longer we would have obtained much better results for them. More data points in general, and repeating each degree multiple times would most likely have increased accuracy significantly.

Secondly a more accurate tool for measuring the degree of scattering may have been in order. The 360 protractor we were using was notoriously inaccurate. We ended up relying on a system where we would simply turn the protractor 5 degrees from where we had it, but zeroing it was difficult. This fact is most obviously observed in the large difference between our measured and experimental values of incident wavelength of cesium-137.

Lastly Maestro is an old and completely outdated program. A more sophisticated counting program that relays quick and accurate information would have been a huge help in this project. It is possible I may have made some errors in correcting for the mistakes of Maestro.

There are several people I would like to thank in this creation of this experiment. Firstly would be my lab partner, Will Beason, who helped me both in the experiment and with the correction to Maestro. Secondly I would like to thank our two TA's, Emanuel Lissek and Chris Reilly who were willing to take the time out of their day to help setup our experiments and allow us access to the labs after hours. Lastly I would like to thank the university for allowing me the opportunity to take this course and I feel honored that I am finally about to graduate with my BS in physics.

5 References

[1] http://www.astro.utu.fi/~cflynn/astroII/l7.pdf

[2] Camphausen KA, Lawrence RC. "Principles of Radiation Therapy" in Pazdur R, Wagman LD, Camphausen KA, Hoskins WJ (Eds) Cancer Management: A Multidisciplinary Approach. 11 ed. 2008.

[3] http://en.wikipedia.org/wiki/Compton_scattering

[4] Taylor, J.R.; Zafiratos, C.D.; Dubson, M.A. (2004). Modern Physics for Scientists and Engineers (2nd ed.). Prentice Hall. pp. 136–9. ISBN 0-13-805715-X.

6 Appendix A

First, there is a bump appearing on every graph at about 1700. Use a fit to find the bump locations. The baseline bump is at about 1648, and the others have the bump anywhere between 1720 and 1735.

Now we need to scale the baseline graph so the bump appears in the same place as it does on the data points. Here is a code-y representation:

```
baselinebump = 1647.958
curbump = 1735.141 #the bump at 15 degrees
function morphbaseline(thebump)
{
factor = thebump / baselinebump
newbase = double(4096) #the column vector which will hold the new baseline
i = 1
while((i-1)/factor <= 4096 && i <= 4096) {
realindex = (i-1)/factor
```

```
left = floor(realindex)
right = floor(realindex) + 1
c = realindex - left
newbase[i] = (1 - c) * baseline[left] + c * baseline[right]
i = i + 1
}
result = newbase
return(result)
}
base15 = morphbaseline(curbump)
difffrombase15 = data15 - base15
```

That algorithm makes difffrombase15 the data we want to analyze. It should have the peak we are looking for.