

Temperature Dependent Low Energy Electron Defraction

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Abstract

In our experiment, we used properties of low energy electron diffraction to verify that the classical spring constant k used to bind atoms together in a compound is independent of the total energy of the system. Using an electron gun and a cubic crystal lattice composed of tungsten in a vacuum, we measure the standard deviation of the displacement of each individual atom, $\sigma(T)$, and the maximum displacement of each atom, $x(T)_{max}$, as a function of temperature. A graph of these results show a slight positive correlation of .05% to the spring constant for each volt added to the system. As there is a large margin of error in the experiment, a small deviation from the theoretical value of 0% is acceptable.

1 Introduction

1.1 Background

A crystal lattice is an object whose atomic structure is composed of three-dimensional patterns. These patterns consist of atoms, or groups of atoms in ordered and symmetrical arrangements which are repeated at regular intervals, thus keeping the same orientations to one another. By replacing each atom or group of atoms by a representative point, a crystal lattice is obtained. Specifically, in an object made of pure tungsten, the crystal lattice structure can be thought of as a box comprised of equally spaced atoms held together by the electromagnetic force [1]. Classically, these forces can be modeled as a set of springs connecting each atom with the atoms nearest to it. As each atom is equally spaced and has identical force vectors, these springs would also be identically spaced, each observing the same length and spring constant k . A visual representation of a classical model of tungsten can be observed in Figure [1].

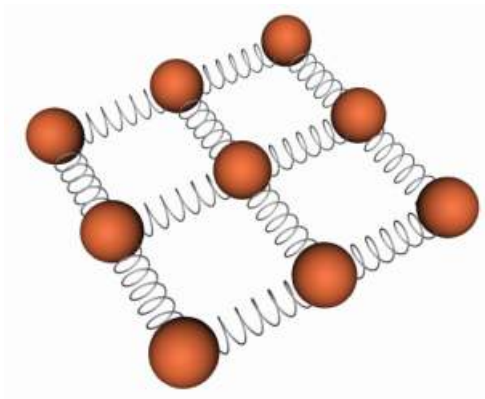


Figure 1: Classical Model of a Tungsten Crystal Lattice

As temperature increases, the atoms in the tungsten sample become excited and oscillate over larger distances. This phenomena can be observed through the use of electron diffraction. Plotting the maximum displacement, intensity, and standard deviation of an atom as a function of temperature, we find a relation for the spring constant k .

$$k(T) = \frac{4m\sigma(T)_x^4}{\pi^2 x(T)_{max}^4} \quad (1)$$

where m is the mass of the atom, $\sigma(T)_x$ is the standard deviation of the intensity of the atom, and $x(T)_{max}$ is the maximum displacement of the atom. For a full derivation of Eq. 1 please refer to appendix A.

However, according to the kinetic molecular theory [2], temperature is by definition some function of the average kinetic energy of a system. Therefore, since the classically modeled spring constant k is derived from the electromagnetic force, there should be no inherit dependence on the temperature of the system. This means that although the atoms will excite and oscillate faster, the spring constant k between each individual atom should remain constant, even though they are theoretically a function of temperature. Logically we can conclude that the slope of the spring constant as a function of temperature should be 0. In other words, the slope of σ^4/x_{max}^4 as a function of temperature should be 0.

$$\Delta k(T) = \frac{4m}{\pi^2} \left(\frac{\sigma_1(T)_x^4}{x_2(T)_{max}^4} - \frac{\sigma_2(T)_x^4}{x_2(T)_{max}^4} \right) \sim \frac{\Delta \sigma^4(T)}{\Delta x_{max}^4(T)} = 0 \quad (2)$$

In conclusion, we are attempting to show that the independence of the spring constant k as a function of temperature by showing that $\Delta \sigma^4/\Delta x_{max}^4 = 0$ as a function of temperature.

1.2 Motivation

Understanding the underlying mechanics of compounds at the molecular level is incredibly important to a wide range of applications. Though not immediately useful by itself, proving that the electromagnetic forces holding a compound together are independent of the temperature leads to a more in-depth and fundamental understanding of compounds and how they work. This leads to further developments, and is a fundamental steppingstone in the field of molecular sciences. Plain and simple, it is a confirmation of theory, aiming at furthering the research in the field of material sciences.

2 Experimental Setup

2.1 Apparatus

Our apparatus consists of an electron gun, a small sample of tungsten, and a florescent screen. The entire apparatus is placed in a vacuum chamber and the sample is connected to an external voltage source so that temperature can be freely adjusted. A camera is mounted on a platform directly behind the apparatus to record observed interference patterns, and an external pyrometer is used to record temperatures at any given voltage point. A visual representation of our setup can be observed in Fig 2.

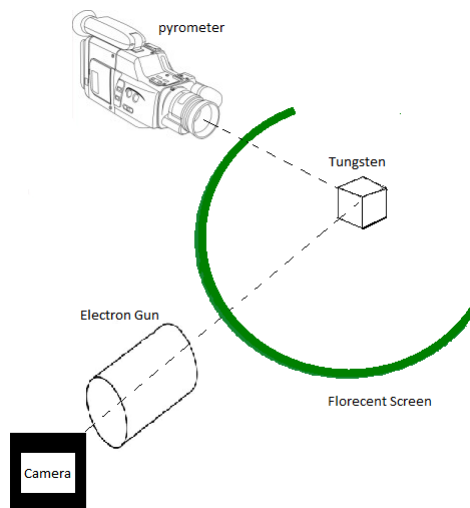


Figure 2: Apparatus

There are two vacuums connected to the apparatus, a roughing, mechanical pump which works on a series of springs to push large volumes of air out of

the chamber, and ion pump which ionizes molecules and pushes them to the sides of the apparatus so they don't interfere with the electron scattering. The ion pump is constantly running and the chamber is kept at a constant pressure of $1.99\text{E-}9$ mbar, insuring that it is, for all intensive purposes, a vacuum. The electron gun is also connected to a SpectaLEED control unit, allowing the user control over the electron energies, currents, and the lenses of the electron gun.

2.2 Data Collection and Procedure

First the tungsten sample is heated to well over 2000°C for several minutes to insure no debris is stuck to the surface. The pressure inside the test chamber is also measured to insure a vacuum tight seal. A current of roughly 1.5V is run through the electron gun to slowly increasing its emission current, this is done to insure that we do not harm the electron gun by accidentally applying too large of a current. When an emission current of roughly 0.1V is being achieved, the electron gun is set to an energy level of 400eV . As electrons scatter off of the tungsten and onto the florescent screen, interference patters are observed. The initial voltage on the tungsten is set to 0V , or room temperature. Voltage is increased from 0V to 50V in intervals of 10V . Several pictures are taken of the interference pattern at each interval.

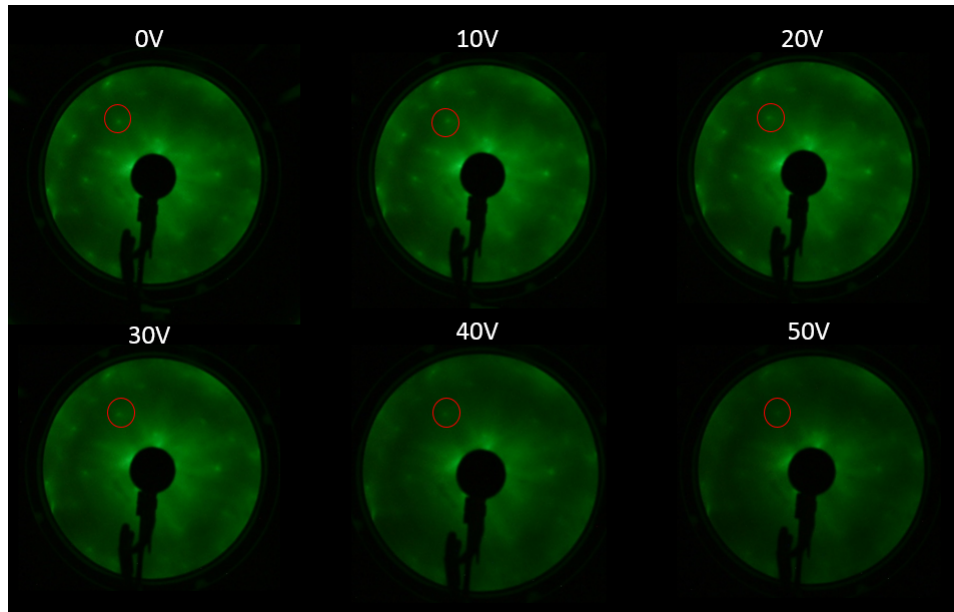


Figure 3: Interference Patterns of Electrons as a Function of Voltages

3 Data Analysis and Results

3.1 Data Processing

As our pyrometer could only read temperatures above 800°C, we first had to work backwards, finding several temperatures as a function of voltage and creating a fit plot to determine what the temperature would be at lower voltages. A full analysis can be found in appendix B. Next, assuming the intensity of each data point was a normal distribution, we measured the peak intensity and standard deviation of a chosen data point at voltages ranging from 0-50V in increments of 10V. Images were processed using ImageJ and intensities as a function of displacement and voltage were collected and fitted in MATLAB.

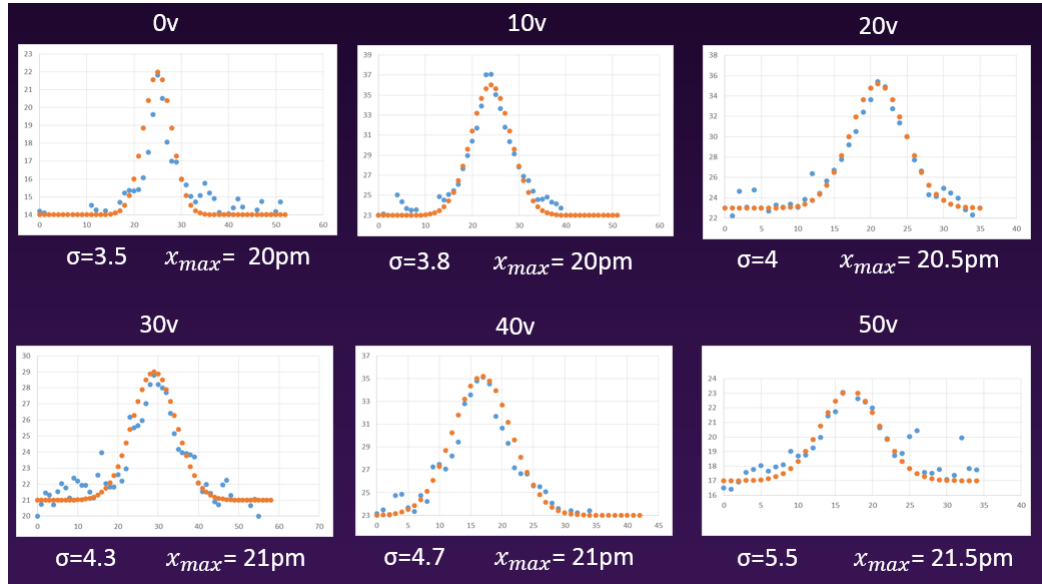


Figure 4: Standard Deviations and Maximum Displacements as a Function of Voltage

Using our results from Fig 4, a graph relating σ^4/x_{max}^4 as a function of voltage is plotted.

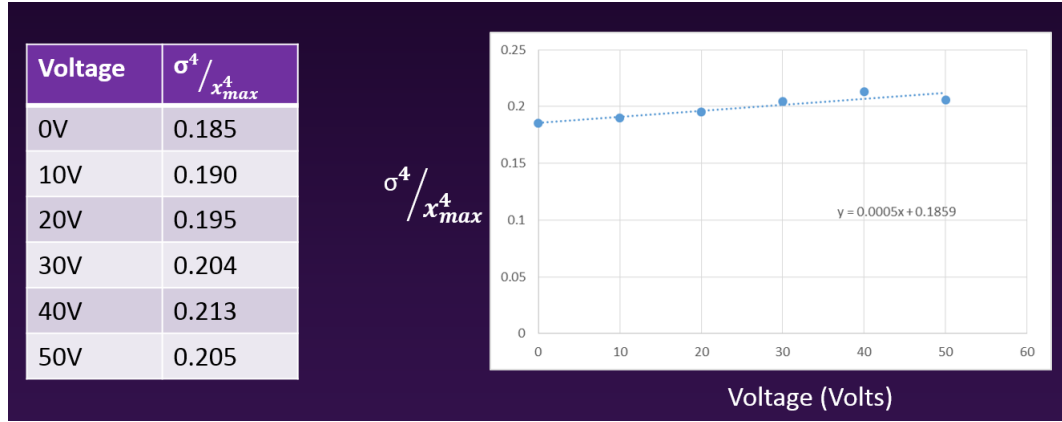


Figure 5: σ^4/x_{max}^4 as a Function of Voltage

A slope of .0005 $1/v$ is obtained for σ^4/x_{max}^4 as a function of voltage.

4 Summary and Conclusion

Our data yields a very slight positive trend in the spring constant k with a ratio of .0005 $1/v$. In other words, for every 10V added to the tungsten, we observed a 0.5% increase to the spring constant. As this is a very loose fit, it is not unreasonable to justify the discrepancy as a product of error in the experiment.

There are several sources of error in the experiment, and although most are systematic, it is impossible to attach a value to them. The two largest sources of error come from finding the intensity distribution and from the ambiguity of determining $x(T)_{max}$. The intensity distributions were found through ImageJ by drawing a line through the atom and displaying the color intensities through the line as a list of numbers. However, the lines had to be drawn by hand, and thus since each line is slightly different, an inherent discrepancy is observed. Secondly, since the displacement of each atom does not end abruptly and there is a large amount of noise in the pictures, choosing $x(T)_{max}$ was difficult and inconsistent. Though each $x(T)_{max}$ was averaged over several pictures, it is not enough to remove human error.

Several smaller factors include fluctuation in voltages and intensities of the colors in the photographs. I included a very small error in assuming a ± 1 in our voltages, but in reality the voltages were very consistent and I would consider any deviation from our two significant figures negligible.

Finally random error in the inherent quantum nature of the atoms allows for discrepancies in the normal distributions of intensities. However, since the intensities were averaged over several pictures and each distribution fits to a normal distribution well, this term is negligible.

5 Acknowledgements

There are several people I would like to thank in this creation of this experiment. Firstly would be my lab partner, Will Beason, who helped me both in the experiment and with the correction to MATLAB. Secondly, I would like to thank our two TA's, Emanuel Lissek and Chris Reilly who were willing to take the time out of their day to help setup our experiments and explain the equipment to us. Lastly, I would like to thank the university for allowing me the opportunity to take this course and Professor Jack Ritchie for guiding us through the semester.

6 References

- [1] Boca, Raton. "Introduction to Tungsten." <http://ed.augie.edu/~awaspaas/inorg/tungsten.pdf>. Captain Obvious Inc., 1 Sep. 2012. Web. 3 Oct. 2013
- [2] Arnold, Thompson. "The Kinetic Molecular Theory." <http://chemed.chem.purdue.edu/genchem/topicreview/bp/ch4/kinetic4.html>. Reading Physics Inc., 1 Sep. 2018. Web. 3 Sep. 2008
- [3] J.R. Hook, "Solid State Physics (2nd Edition)," H.E. Hall, Manchester Physics Series, John Wiley & Sons, 2010, ISBN 978-0-471-92804-1

7 Appendix A - Derivation of Spring Constant Formula

The atoms inside our tungsten sample are modeled as a ball attached to 2 identical springs. The atoms are assumed to oscillate in harmonic motion as a function of temperature, the intensities we observe at each pixel represent how often the particle is at that location.

As the springs are identical and connected in series, it is trivial to reduce the two spring system into a one spring system with spring constant $k = \frac{k_1}{2}$.

When the particle is maximally displaced,

$$U_{max} = E = .5kx_{max}^2 \quad (3)$$

where x_{max} is the maximum displacement of the particle. Kinetic energy can then be solved as follows

$$KE = E - U \rightarrow .5mv(x)^2 = .5k(x_{max}^2 - x^2) \quad (4)$$

The standard deviation of the displacement of a normal distribution is

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \int_{-x_{max}}^{x_{max}} \frac{x^2}{v(x)} dx - 0 \quad (5)$$

solving Eq. 4 for $v(x)$ and substituting into Eq. 5 yields

$$\sigma_x^2 = .5\sqrt{\frac{m}{k}}\pi x_{max}^2 \quad (6)$$

or, solving for k

$$k(T) = \frac{4m\sigma(T)_x^4}{\pi x(T)_{max}^4} \quad (7)$$

8 Appendix B - Finding Temperature as a Function of Voltage

Starting with the equation for power and assuming some constant relating to room temperature.

$$P = C + \frac{V^2}{R} = kT^4 \quad (8)$$

Setting two arbitrary temperatures as a function of voltage equal to each other and collecting the constant k into the constant C allows us to solve for our new constant, C'

$$\frac{T_1^4}{T_2^4} = \frac{C' + V_1^2}{C' + V_2^2} \rightarrow C' = \frac{T_2^4 V_1^2 - T_1^4 V_2^2}{T_1^4 - T_2^4} \approx 45$$

Measuring temperature as a function of voltage in the lab yields the following table.

Volts	Temp ($^{\circ}\text{C}$)
0 \pm 1	23 \pm 30
100 \pm 1	875 \pm 30
125 \pm 1	950 \pm 30
150 \pm 1	1000 \pm 30
175 \pm 1	1200 \pm 30
200 \pm 1	1300 \pm 30

Table 1: Temperature ($^{\circ}\text{C}$) as a Function of Voltage (V)

This gives us an experimental value of our constant $C' \approx 45$.

We can now solve for T at any given V

$$T(^{\circ}\text{C}) = 293 \frac{\sqrt[4]{C' + V^2}}{\sqrt[4]{C'}} - 272 \quad (9)$$

Using Eq. 9, we can now estimate the temperatures in our experiment as a function of V.

Volts	Temp ($^{\circ}\text{C}$)
0	21 \pm 1
10 \pm 1	120 \pm 8
20 \pm 1	247 \pm 15
30 \pm 1	355 \pm 27
40 \pm 1	448 \pm 55
50 \pm 1	530 \pm 116

Table 2: Temperature ($^{\circ}\text{C}$) as a Function of Voltage (V) for Voltages Used in Experiment