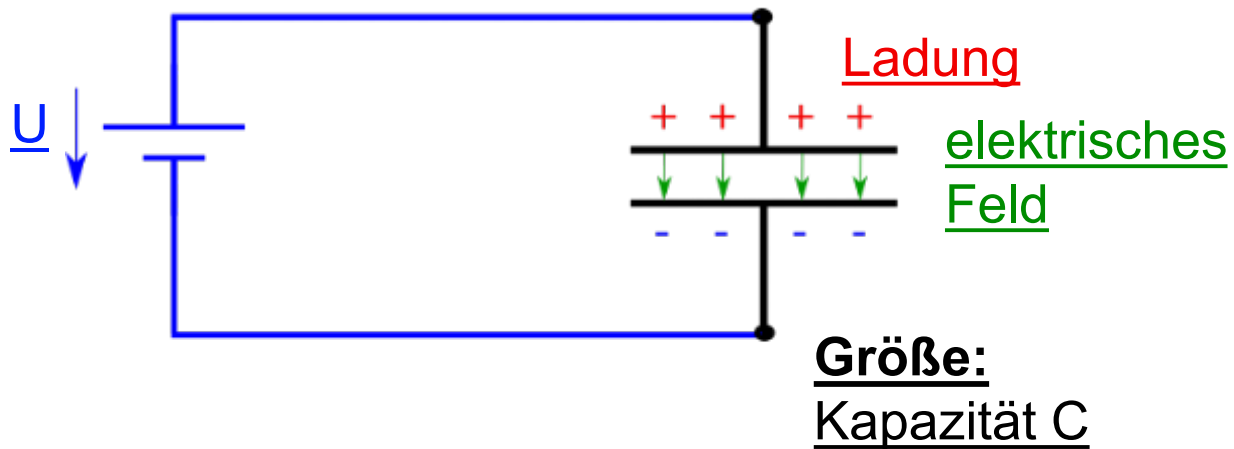


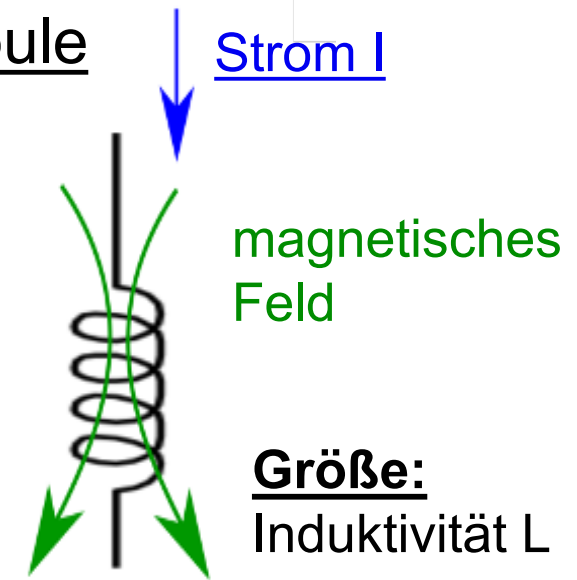
8. Schwingkreise

Elektrische Schwingkreise bestehen aus mindestens zwei Energiespeichern, deren gespeicherte Energie periodisch ausgetauscht wird.

1. Kondensator

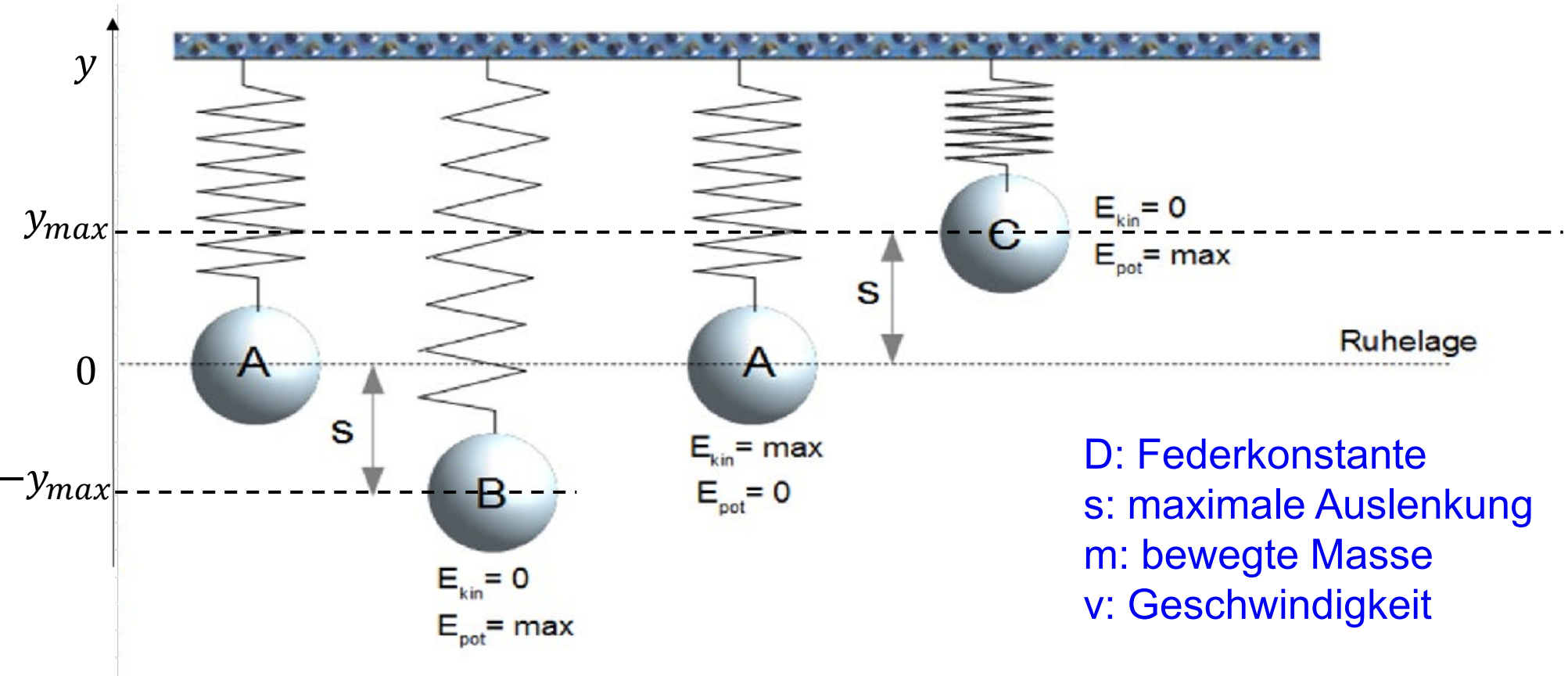
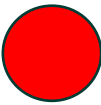


2. Spule



➡ Elektrische Schwingkreise verhalten sich gleichartig zu mechanischen Systemen!

Analogie zum Federpendel:



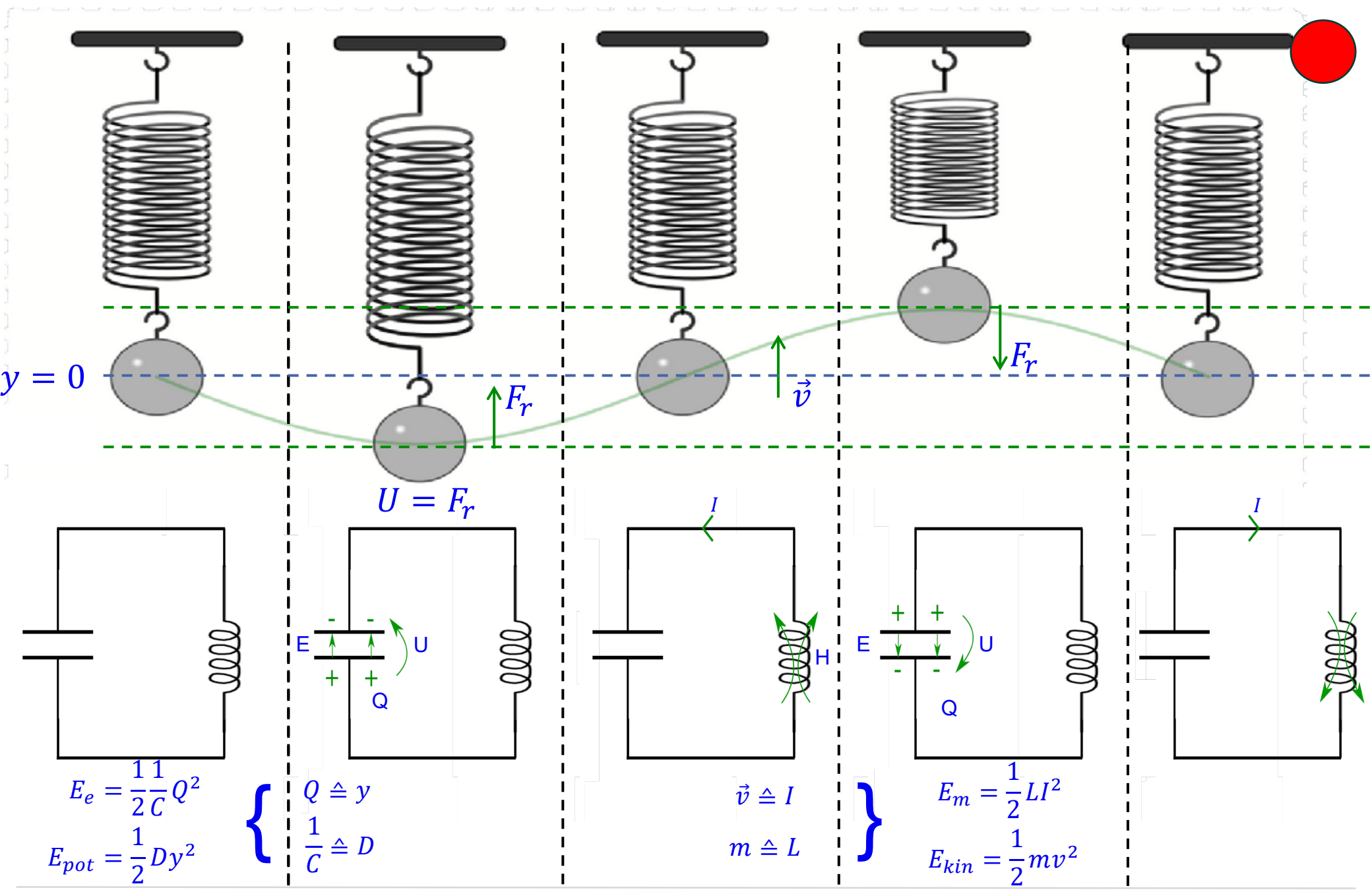
D: Federkonstante
s: maximale Auslenkung
m: bewegte Masse
v: Geschwindigkeit

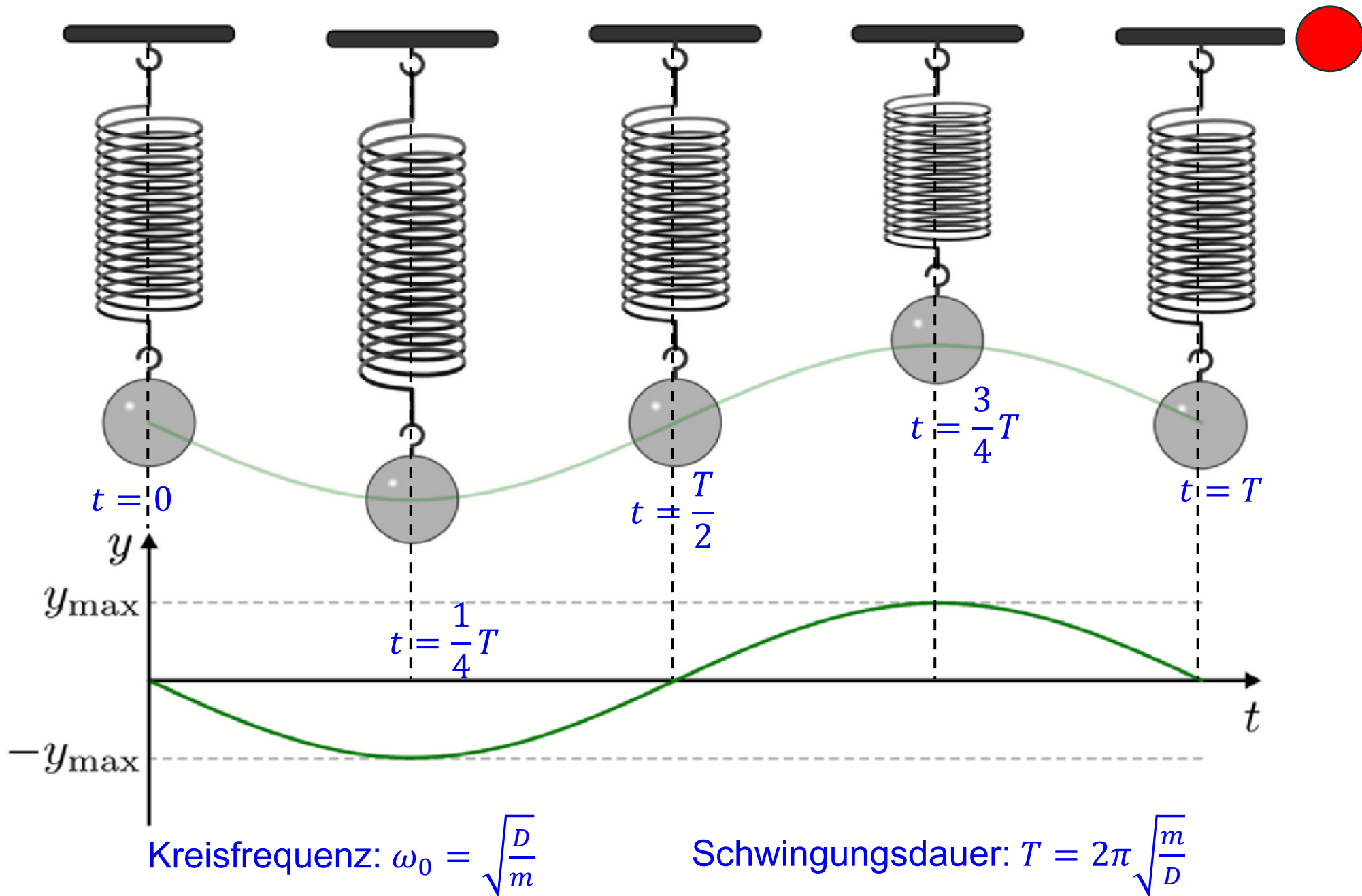
Spannarbeit:

$$E_{pot} = \frac{1}{2} D \cdot s^2$$

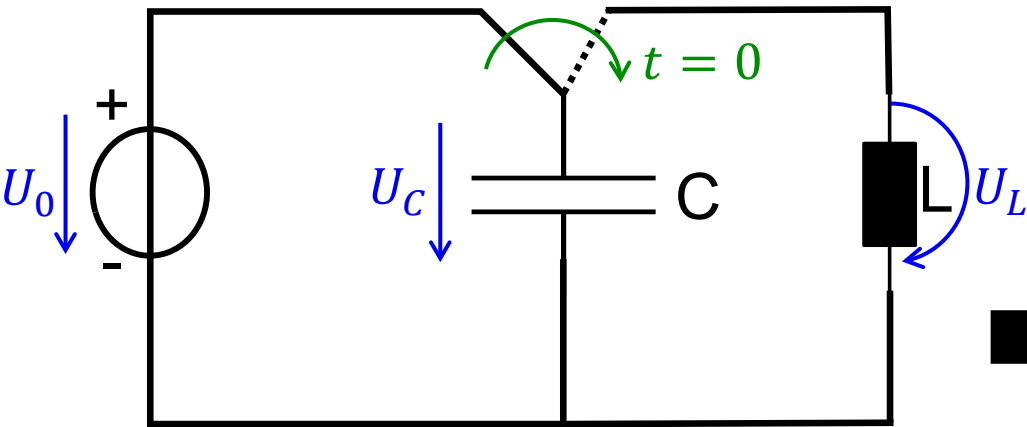
Kinetische Energie:

$$E_{kin} = \frac{1}{2} m \cdot v^2$$

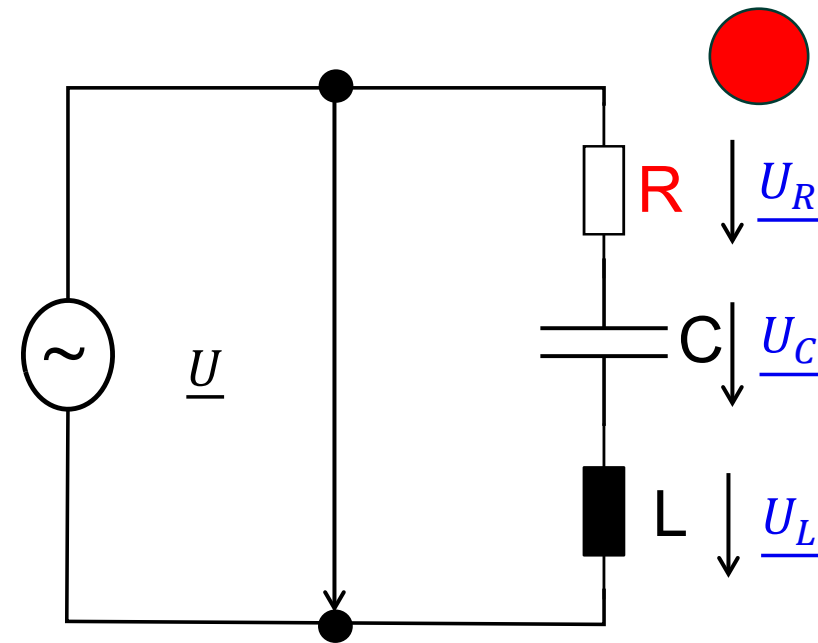




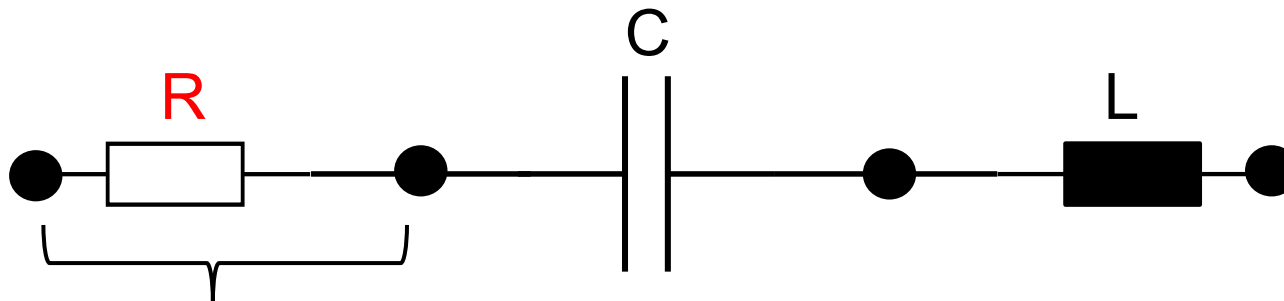
8.1 Serienschwingkreis



bisher: freischwinger idealer elektrischer Schwingkreis



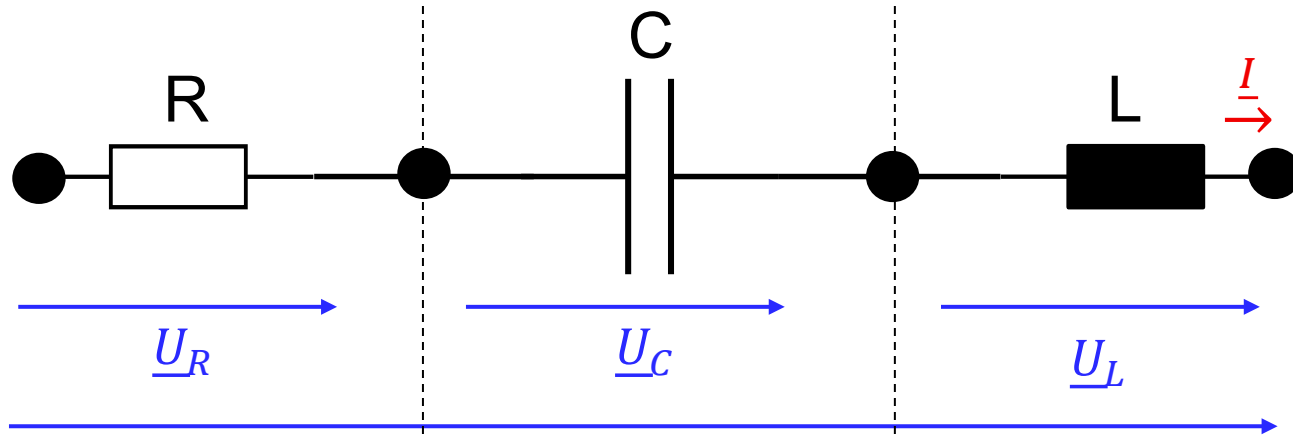
Serienschwingkreis



verlustbehafteter Widerstand

verlustbehafteter Serienschwingkreis

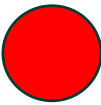
\underline{Z} Serienimpedanz



$$\underline{U}_R = R \cdot \underline{I}$$

$$\underline{U}_C = \frac{1}{j\omega C} \cdot \underline{I}$$

$$\underline{U}_L = j\omega L \cdot \underline{I}$$



$$\underline{U} = \underline{Z} \cdot \underline{I} = R \cdot \underline{I} + \frac{1}{j\omega C} \cdot \underline{I} + j\omega L \cdot \underline{I}$$

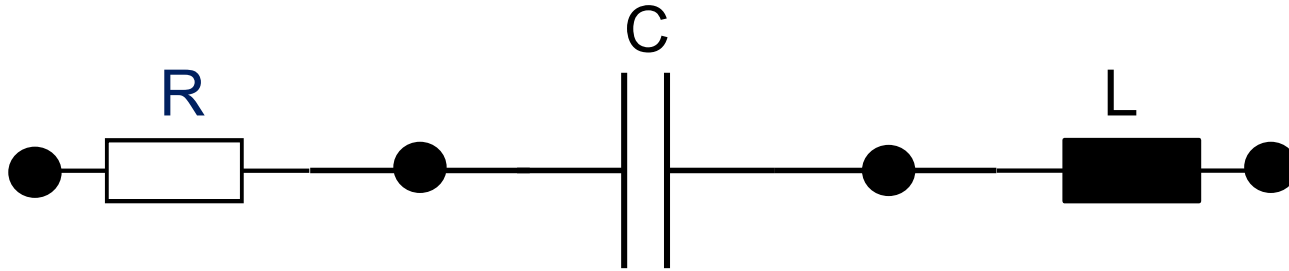
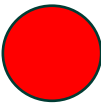
$$\underline{Z} = R + \frac{1}{j\omega C} + j\omega L$$

es gilt: $j = -\frac{1}{j}$!

aufteilen nach Real- und Imaginärteil

$$\underbrace{\underline{Z}}_{\text{Komplexe Impedanz}} = \underbrace{R}_{\substack{\text{Realteil} \\ \triangleq \\ \text{Wirkwiderstand}}} + j \underbrace{\left(-\frac{1}{\omega C} + \omega L \right)}_{\substack{\text{Imaginärteil} \\ \triangleq \\ \text{Blindwiderstand}}} = R + jX$$

Definition der Resonanzfrequenz f_0



Frage: Imaginärteil von $\underline{Z} \stackrel{!}{=} 0$?

$$X \stackrel{!}{=} \operatorname{Im}\{\underline{Z}(\omega_0)\} \stackrel{!}{=} 0 \text{ mit } \omega_0 = 2 \cdot \pi \cdot f_0$$

$$\leadsto X = \left(\omega_0 L - \frac{1}{\omega_0 C} \right) \stackrel{!}{=} 0 \quad \leadsto \boxed{\omega_0 L \stackrel{!}{=} \frac{1}{\omega_0 C}}$$

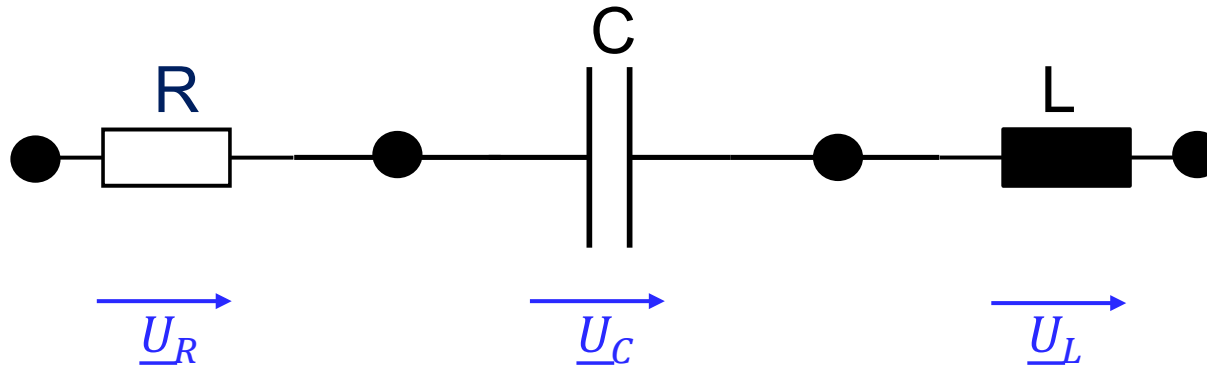
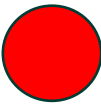
$$\leadsto \boxed{\omega_0 = \frac{1}{\sqrt{LC}} = 2 \cdot \pi \cdot f_0}$$

$$\omega \stackrel{!}{=} \omega_0 \leadsto \underline{U}_L = j\omega_0 L \cdot \underline{I}; \quad \underline{U}_C = -\frac{1}{j\omega_0 C} \cdot \underline{I} = 0 \quad \leadsto \boxed{\underline{U}_L + \underline{U}_C = 0}$$

$$\leadsto \boxed{\underline{U} = \underline{U}_R + \underline{U}_L + \underline{U}_C \stackrel{!}{=} \underline{U}_R \mid_{\omega = \omega_0}}$$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{\underline{U}_R}{\underline{I}} = \frac{R \cdot \underline{I}}{\underline{I}}$$

Definition des Kennwiderstands X_0



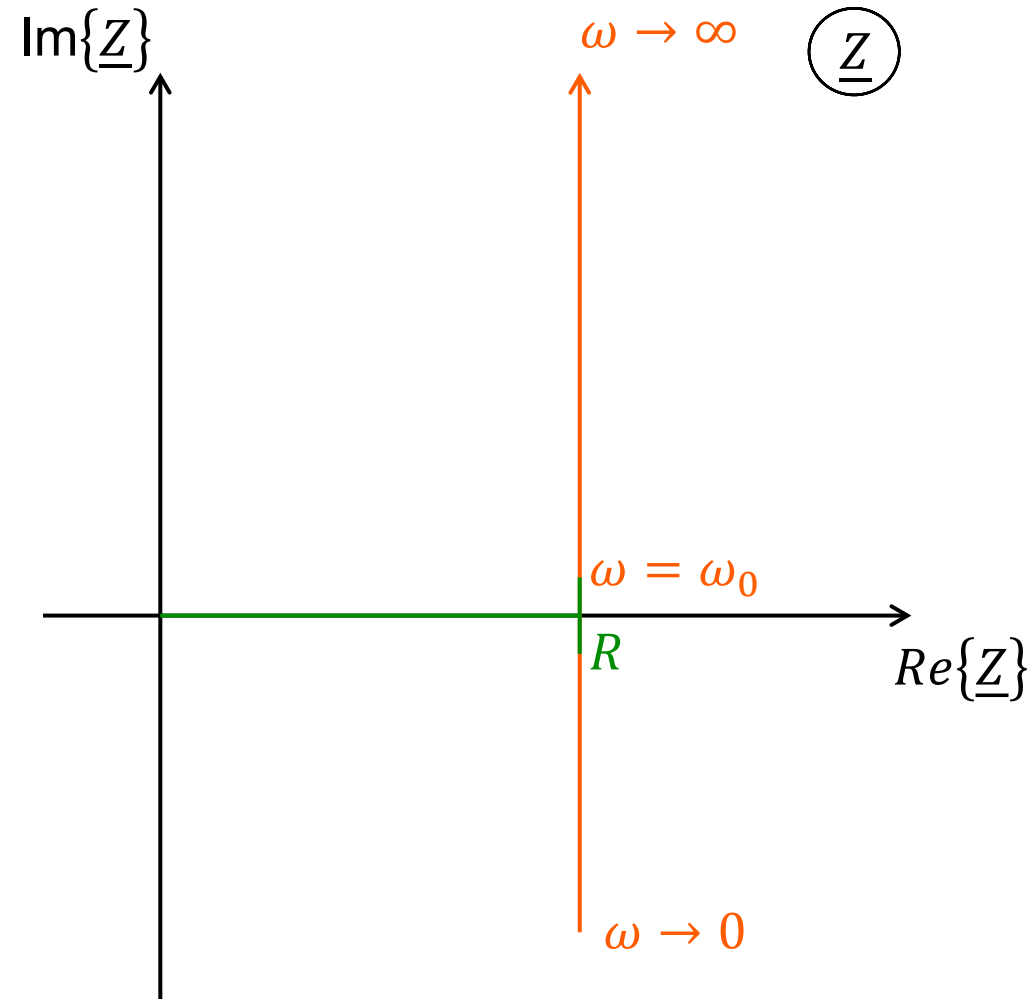
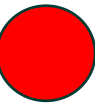
$$\omega = \omega_0 \quad \leadsto \quad -\underline{U_C} \stackrel{!}{=} \underline{U_L} \quad \leadsto \quad -j \frac{1}{\omega_0 C} \cdot \underline{I} \stackrel{!}{=} j \omega_0 L \cdot \underline{I}$$

mit $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\leadsto \boxed{X_L \Big|_{\omega_0} = \omega_0 L = \sqrt{\frac{L}{C}}} \quad \text{und} \quad \boxed{X_C \Big|_{\omega_0} = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}}$$

$$X_0 = \sqrt{\frac{L}{C}} \Rightarrow \boxed{\begin{aligned} L &= \frac{1}{\omega_0} \cdot X_0 \left[\frac{Vs}{A} = \Omega s \right] \\ C &= \frac{1}{\omega_0} \cdot \frac{1}{X_0} \left[\frac{As}{V} = \frac{s}{\Omega} \right] \end{aligned}} \quad \text{Kennwiderstand}$$

Ortskurve für die Serienimpedanz



$$\underline{Z} = R + j\underline{X}$$

$$= R + j \left(\omega L - \frac{1}{\omega C} \right)$$

$$\underline{Z}|_{\omega=0} = \textcircled{R} + j \left(\omega L - \frac{1}{\omega C} \right)$$

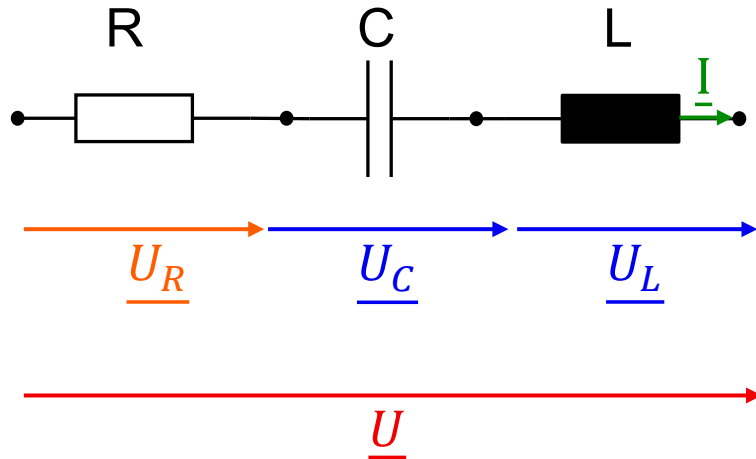
Diagram showing the limit as $\omega \rightarrow 0$. The term ωL approaches 0 (indicated by a blue arrow pointing to 0) and $\frac{1}{\omega C}$ approaches ∞ (indicated by a blue arrow pointing to ∞). The entire expression is circled in orange.

$$\underline{Z}|_{\omega=\omega_0} = R$$

$$\underline{Z}|_{\omega=\infty} = R + j \left(\omega L - \frac{1}{\omega C} \right)$$

Diagram showing the limit as $\omega \rightarrow \infty$. The term ωL approaches ∞ (indicated by a red arrow pointing to ∞) and $\frac{1}{\omega C}$ approaches 0 (indicated by a red arrow pointing to 0).

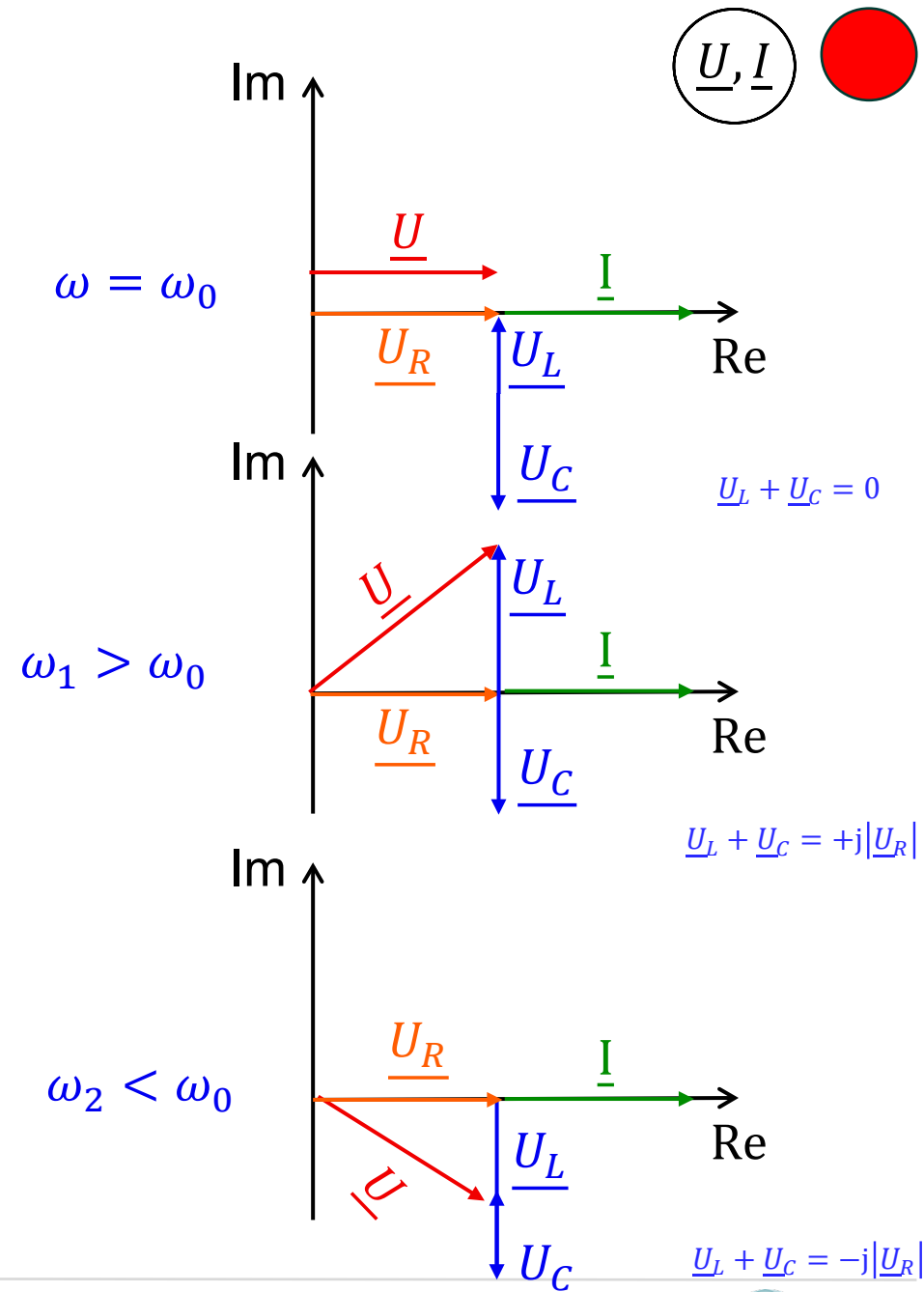
Zeigerdiagramm für \underline{U} und \underline{I}



Frage:

Bei Welcher Frequenz entspricht die Phasenverschiebung zwischen Spannung und Strom 45 Grad?

$$\angle \underline{U}, \underline{I} = 45^\circ \left(\frac{\pi}{4} \right) ?$$



$$\omega_1 > \omega_0 \angle (\underline{U}, \underline{I}) = +45^\circ$$

$$\rightarrow R = \left(\omega_1 L - \frac{1}{\omega_1 C} \right) \cdot \frac{\omega_1}{L}$$

$$\omega_1 \frac{R}{L} = \omega_1^2 - \frac{1}{LC}$$

$$0 = \omega_1^2 - \omega_1 \frac{R}{L} - \frac{1}{LC}$$

Lösen durch quadrat. Ergänzung

$$\begin{aligned} 0 &= \omega_1^2 - \omega_1 \frac{R}{L} + \left(\frac{R}{2L} \right)^2 - \left(\frac{R}{2L} \right)^2 - \frac{1}{LC} \\ &= \left(\omega_1 - \frac{R}{2L} \right)^2 - \left(\frac{R}{2L} \right)^2 - \frac{1}{LC} \end{aligned}$$

$$\rightarrow \boxed{\omega_1 = + \frac{R}{2L} \sqrt{\left(\frac{R}{2L} \right)^2 + \frac{1}{LC}}$$

Nur positive Frequenzen!

$$\omega_2 < \omega_0 \angle (\underline{U}, \underline{I}) = -45^\circ$$

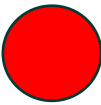


$$\rightarrow R = - \left(\omega_1 L - \frac{1}{\omega_1 C} \right) \cdot \frac{\omega_1}{L}$$



$$\rightarrow \boxed{\omega_2 = - \frac{R}{2L} \sqrt{\left(\frac{R}{2L} \right)^2 + \frac{1}{LC}}$$

Def.: „Frequenzbandbreite“, „Dämpfung“ und „Güte“



Frequenzbandbreite:

$$b_w := \frac{R}{L}$$

$$\begin{aligned} bw &:= \omega_1 - \omega_2 \\ &= +\frac{R}{2L} + \sqrt{\dots} - \left(-\frac{R}{2L} + \sqrt{\dots}\right) \end{aligned}$$

rel. Frequenzbandbreite. („Dämpfung“):

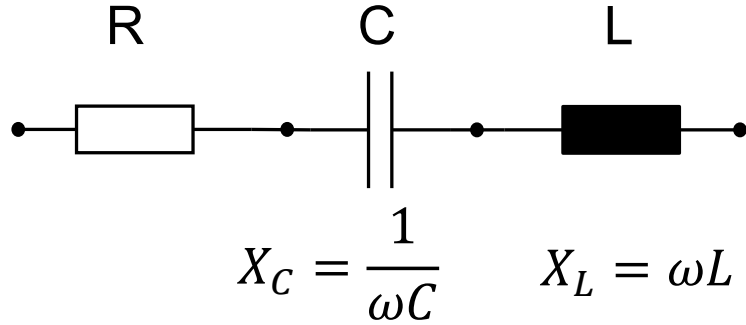
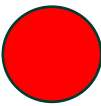
$$d := \frac{b_w}{\omega_0} = \frac{R}{L} \cdot \sqrt{LC} = R \cdot \sqrt{\frac{C}{L}} \quad \rightarrow \quad d := \frac{R}{X_0}$$

Güte:

$$Q := \frac{1}{d} = \frac{X_0}{R} = \frac{\text{Kennwiderstand}}{\text{ohmscher Widerstand}}$$

$$R \downarrow + L \uparrow \leadsto Q \uparrow$$

Diagramme zur Beschreibung des Serienresonanzkreises



■ Impedanz

■ Betrag

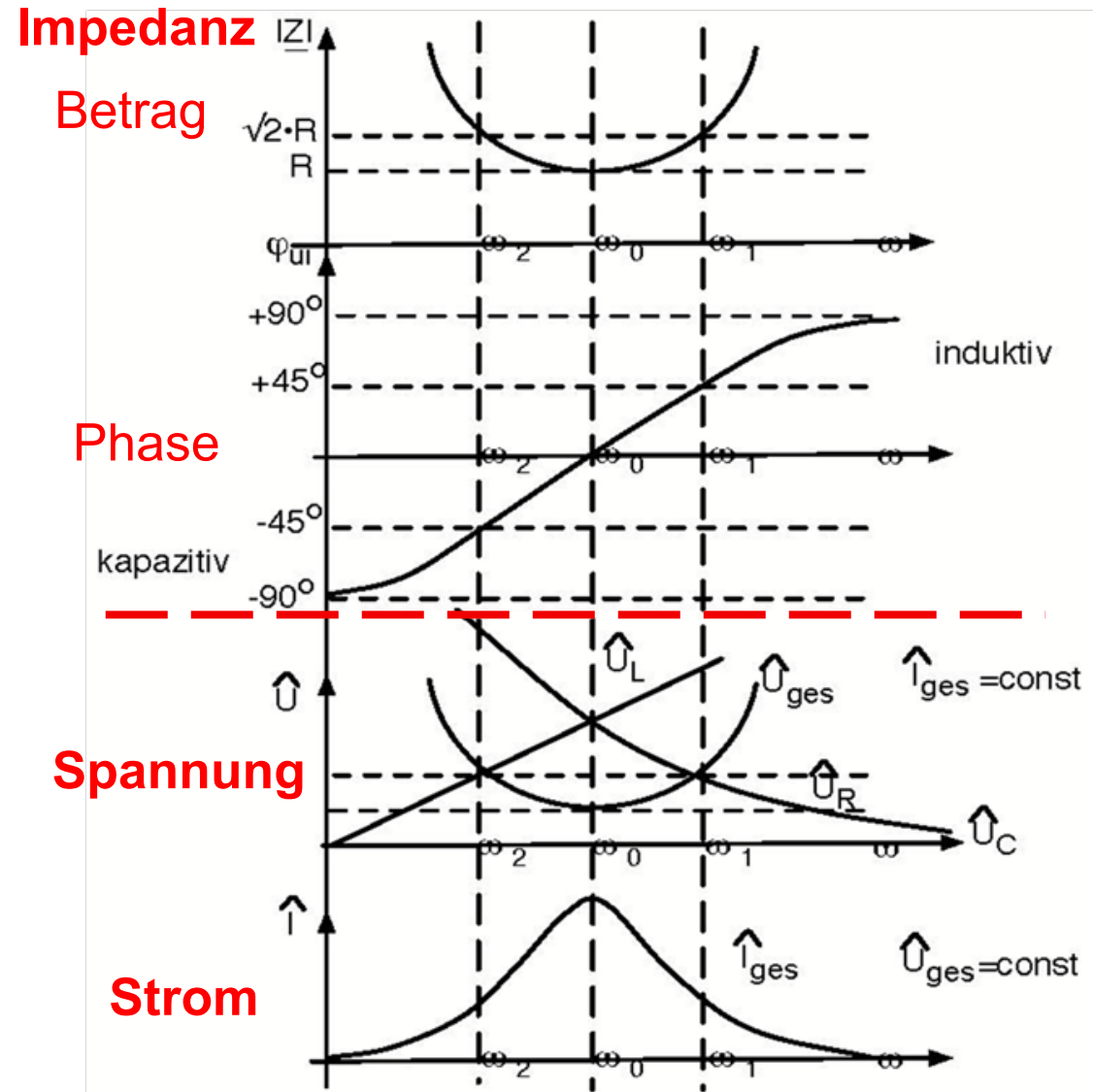
■ Phase

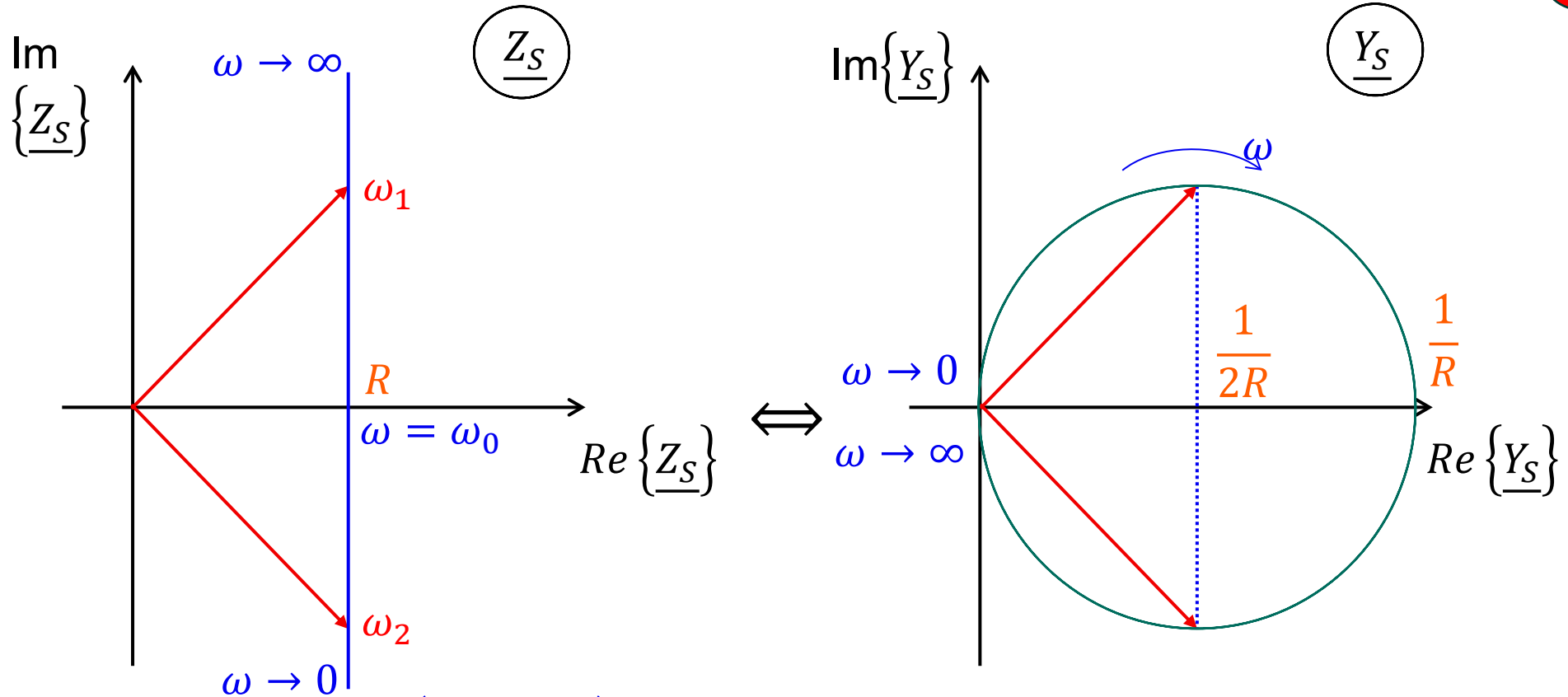
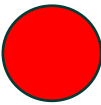
■ Strom

■ Betrag

■ Spannung

■ Betrag



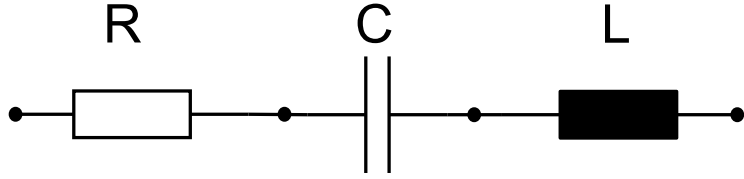


$$\underline{Z} = R + jX = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\left\{ \begin{array}{l} \omega \rightarrow 0: \quad \underline{Y} \sim \frac{1}{R - j\frac{1}{\omega C}} = \frac{R + j\frac{1}{\omega C}}{R^2 + \left(\frac{1}{\omega C}\right)^2} \\ \omega \gg \omega_0: \quad \underline{Y} \sim \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + (\omega L)^2} \end{array} \right.$$

Praktisches Beispiel:



gegeben:

$$R = 1 \, \Omega; \quad f_0 = 1 \, \text{MHz} = 1 \cdot 10^6 \frac{1}{s}$$

$$R = X_0; \quad \omega_0 = 2\pi \cdot 10^6 \frac{1}{s}$$

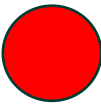
gesucht:

Impedanzverhalten, $L?$, $C?$

Lösung:

$$L = \frac{1}{\omega_0} \cdot X_0 = \frac{1}{2\pi \cdot 10^6 \frac{1}{s}} \cdot \Omega = \frac{1}{2\pi} \cdot 10^{-6} \Omega s \quad \left[H = \frac{Vs}{A} \right]$$

$$C = \frac{1}{\omega_0 X_0} = \frac{1}{2\pi \cdot 10^6 \frac{1}{s} \cdot \Omega} = \frac{1}{2\pi} \cdot 10^{-6} \frac{s}{\Omega} \quad \left[F = \frac{As}{V} \right]$$



$f = 100\text{kHz} \dots 5\text{MHz}$

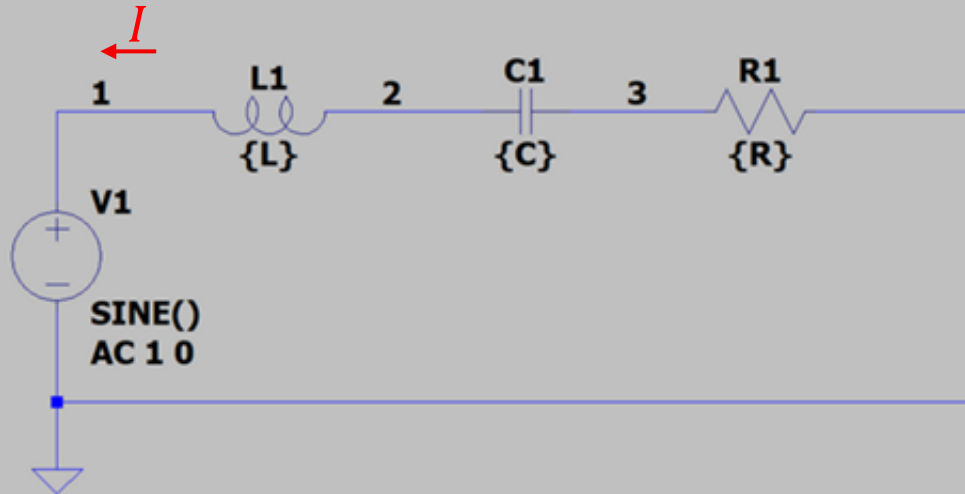
$L = 0,159\mu\text{H}$
 $C = 0,159\mu\text{F}$
 $R = 1\Omega$

```
.ac dec 100 0.01e6 5e6
```

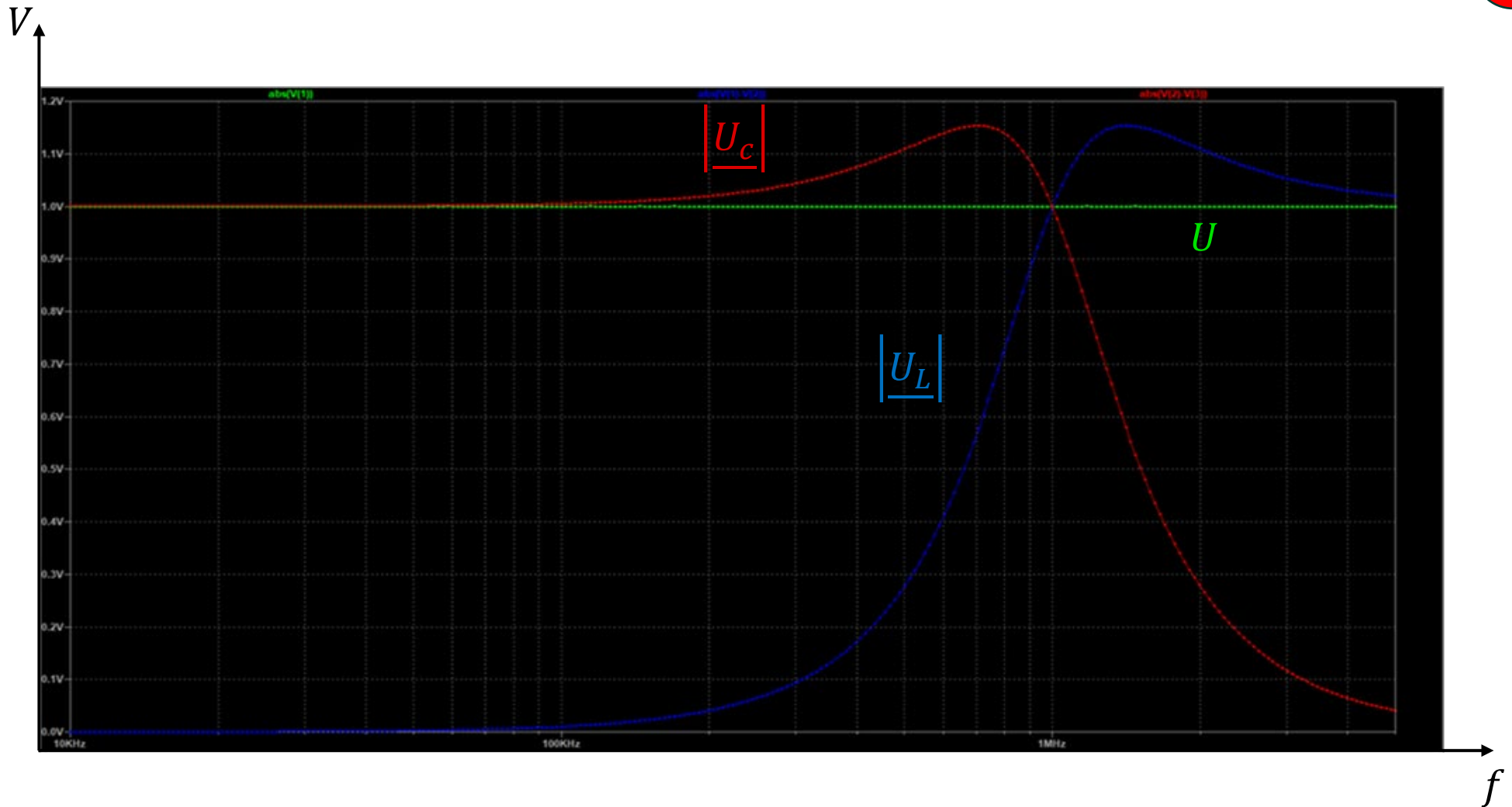
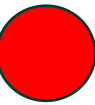
```
.param L 0.159e-6
```

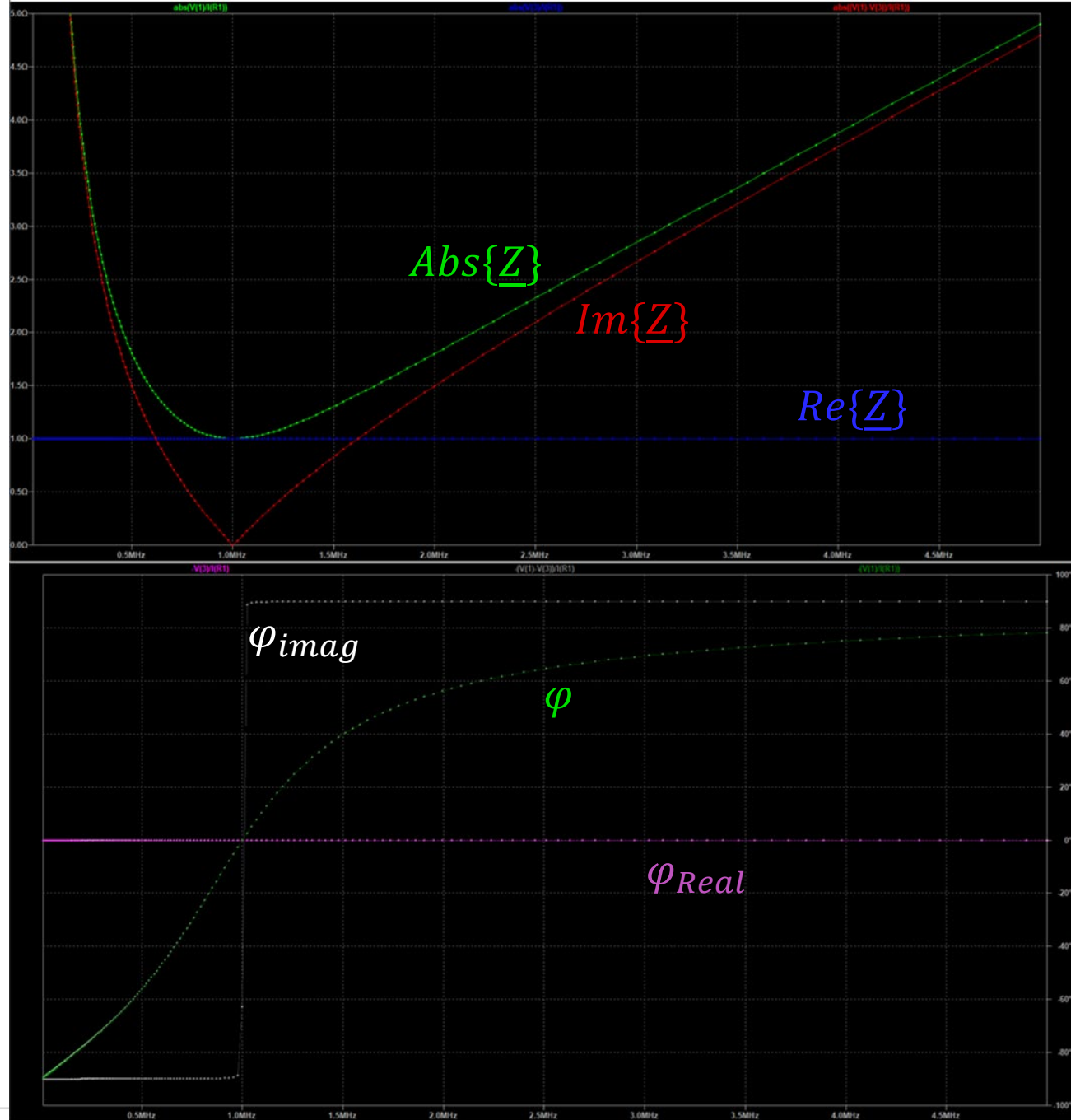
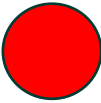
```
.param C 0.159e-6
```

```
.param R 1
```

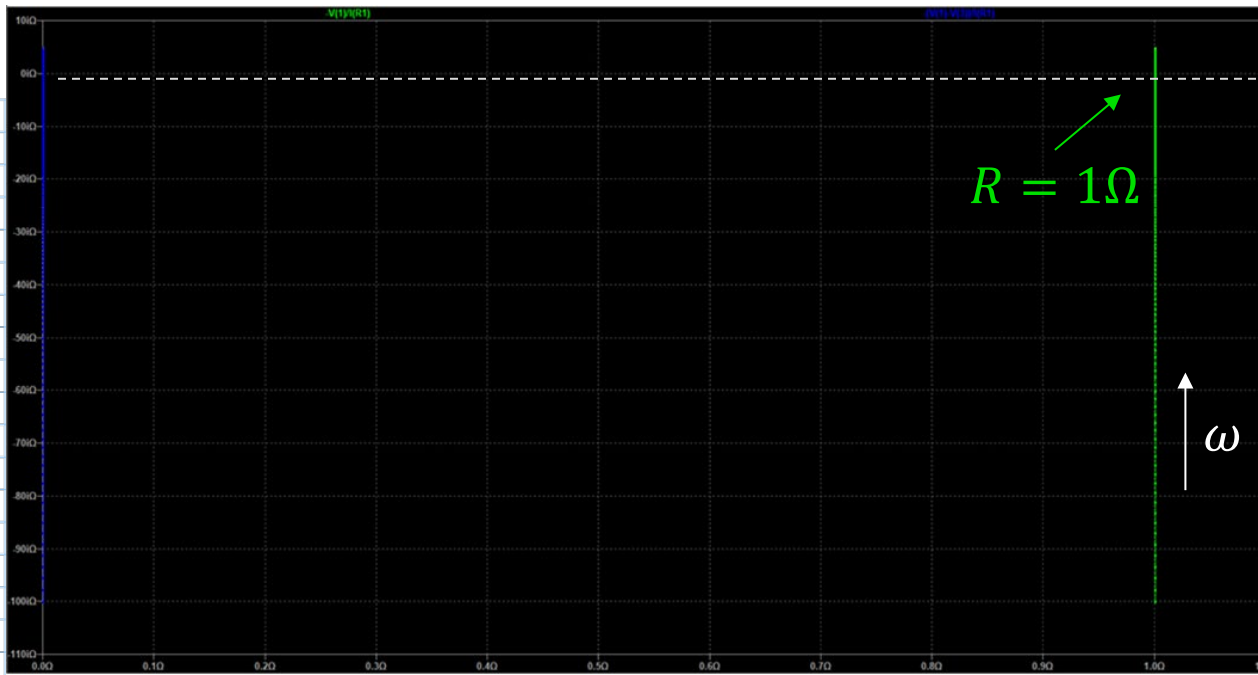


$$V_1 = 1\text{V} \cdot \sin(2\pi f t)$$

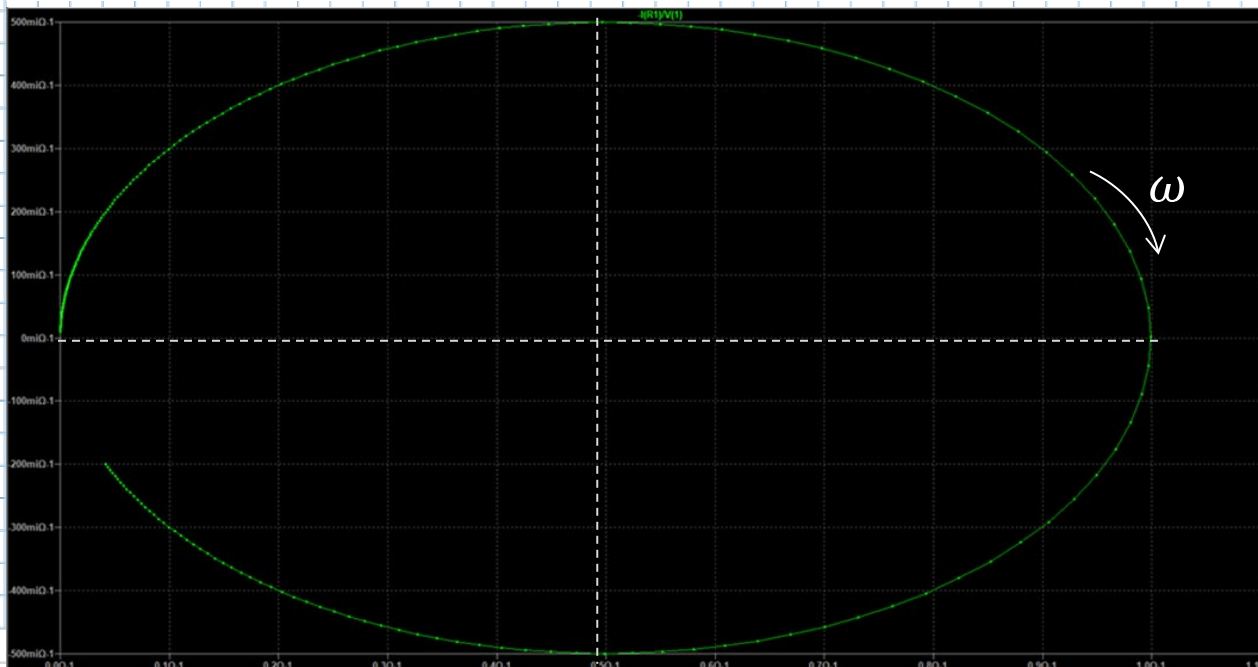




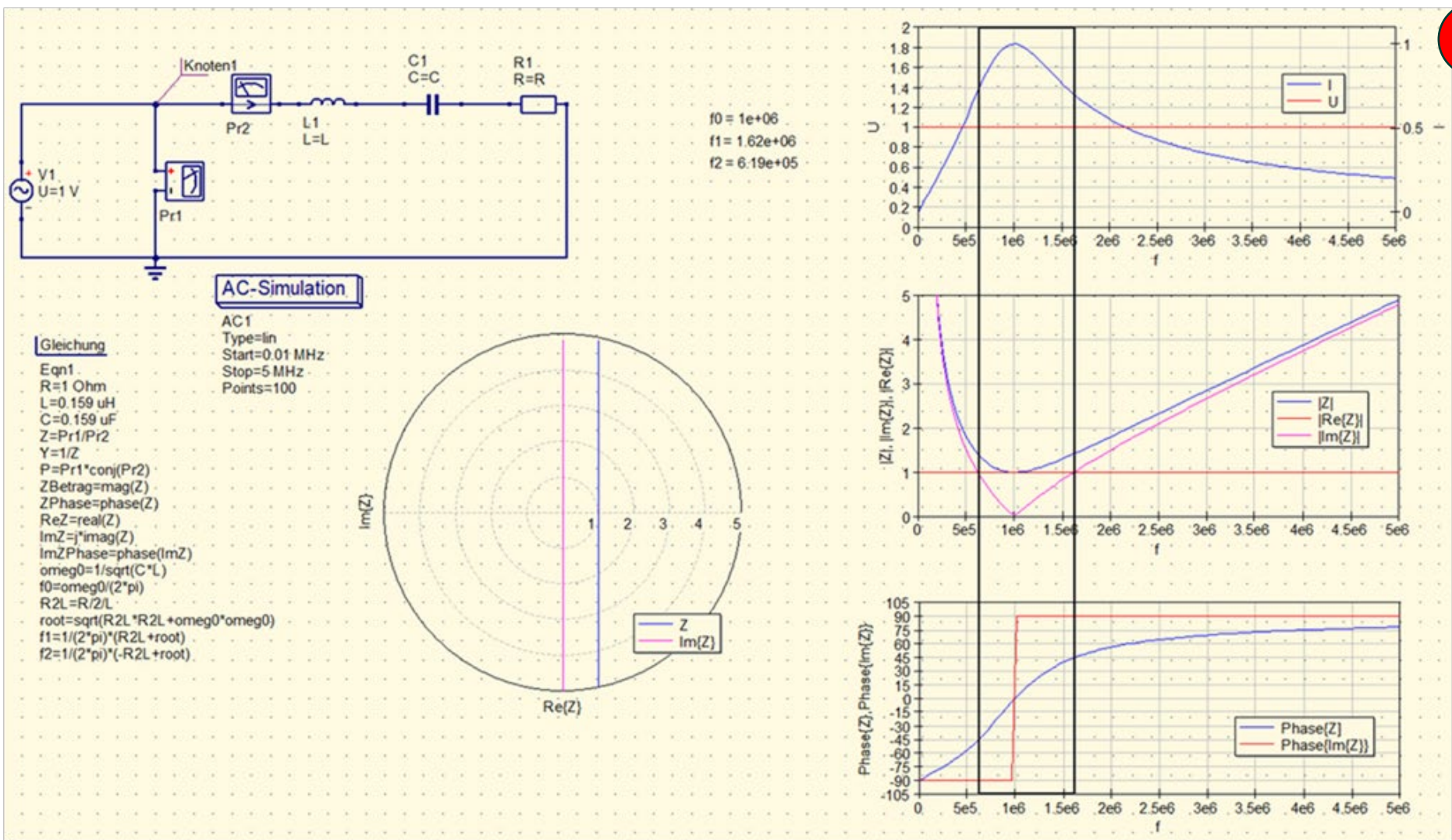
Ortskurven:



Impedanz



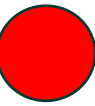
Admittanz



Programm: QucsStudio Ver 4.3.1

qucsstudio.de

Bemerkungen



Die Diagramme zur Beschreibung des Serienschwingkreises verlaufen im allgemeinen nicht symmetrisch um ω_0 .

Nehmen wir dies jedoch hier an: $\omega_0 = \frac{1}{\sqrt{LC}}$; $\omega_{1,2} \cong \omega_0 \pm \frac{R}{2L}$

Dann bedeutet dies: $\left(\frac{R}{2L}\right)^2 \ll \frac{1}{LC} \rightarrow 1 \ll \frac{1}{\left(\frac{R}{2L}\right)^2} \frac{1}{LC}$ mit $Q = \frac{X_0}{R} = \frac{\sqrt{L}}{R\sqrt{C}}$

↷ Güte ist groß!

mit

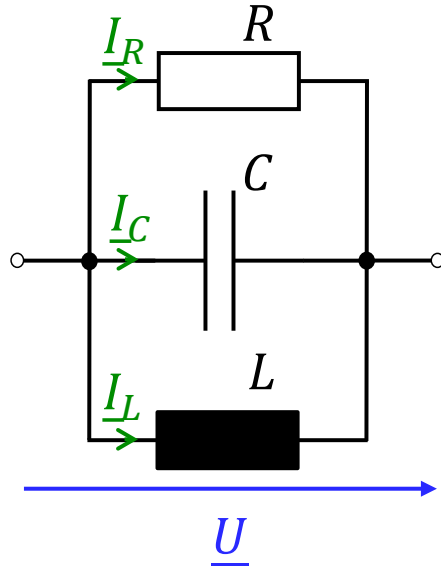
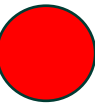
$$X_L \Big|_{\omega_0} = \omega_0 L = \sqrt{\frac{L}{C}}$$

und

$$X_C \Big|_{\omega_0} = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

$$\rightarrow \operatorname{Im}\{\underline{U}_{L_0}\} \cong \operatorname{Im}\{\underline{U}_{C_0}\} \approx Q \cdot \underline{U}_R \rightarrow \operatorname{Im}\{\underline{U}_{L_0}\} \cong \operatorname{Im}\{\underline{U}_{C_0}\} \gg \underline{U}_R$$

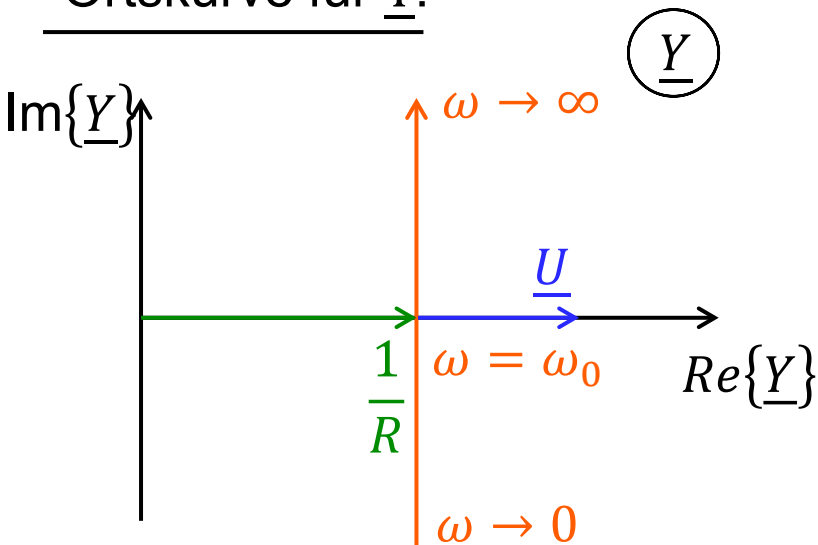
8.2 Parallelschwingkreis



$$\underline{Y} = \underbrace{\frac{1}{R}}_{\text{Realteil}} + \underbrace{j\omega C + \frac{1}{j\omega L}}_{\text{Imaginärteil}}$$

Realteil Imaginärteil

Ortskurve für \underline{Y} :



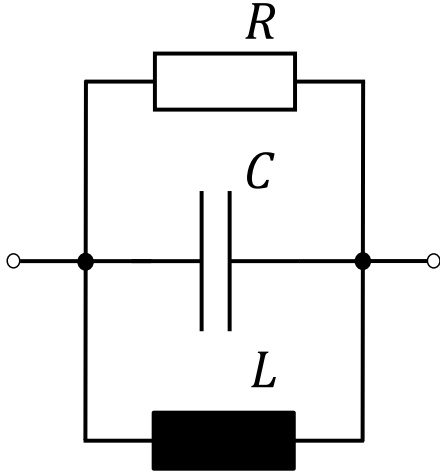
$$\underline{\omega} \rightarrow 0: \quad \underline{Y} \sim \frac{1}{R} + j \frac{1}{\cancel{\omega L}} + j \cancel{\omega} C \quad \begin{matrix} \nearrow \sim \infty \\ \nearrow 0 \end{matrix}$$

$$\underline{\omega} = \omega_0: \quad \underline{Y}_0 = \frac{1}{R} \text{ mit } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\underline{\omega} \rightarrow \infty: \quad \underline{Y} \sim \frac{1}{R} + j\omega C + \frac{j}{\cancel{j\omega L}} \quad \begin{matrix} \nearrow 0 \\ \nearrow \sim \infty \end{matrix}$$

Bei Resonanz $\omega = \omega_0$:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$$\underline{I}_{R_0} = \frac{\underline{U}}{R}$$

$$\underline{I}_{C_0} = j\omega_0 C \cdot \underline{U}$$

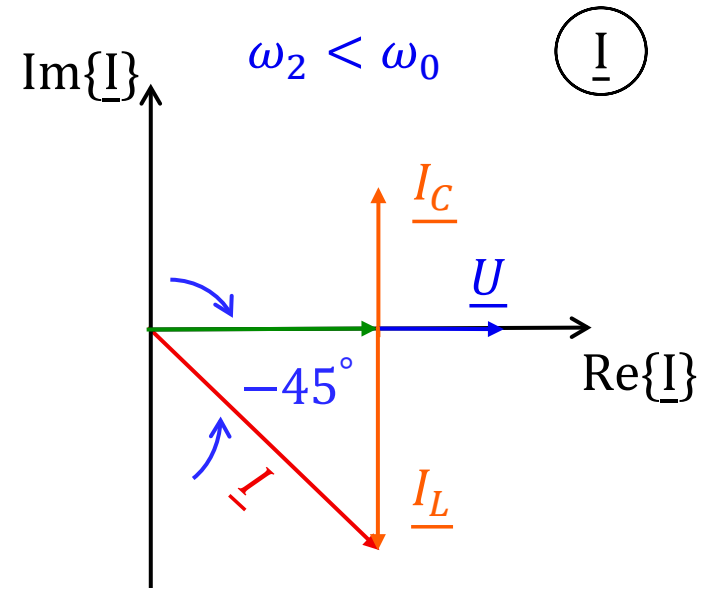
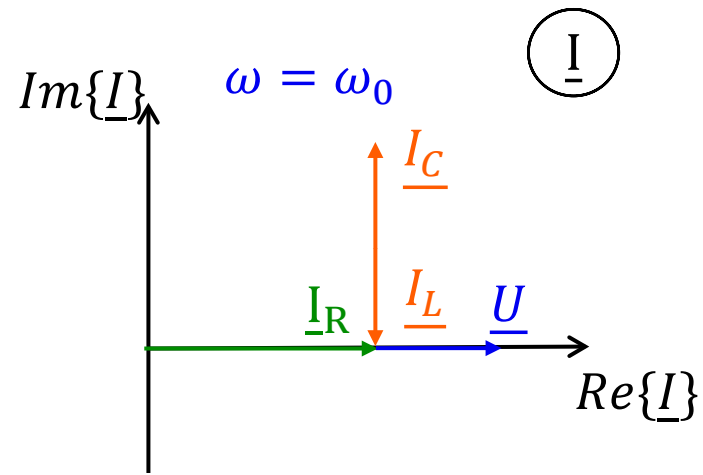
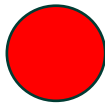
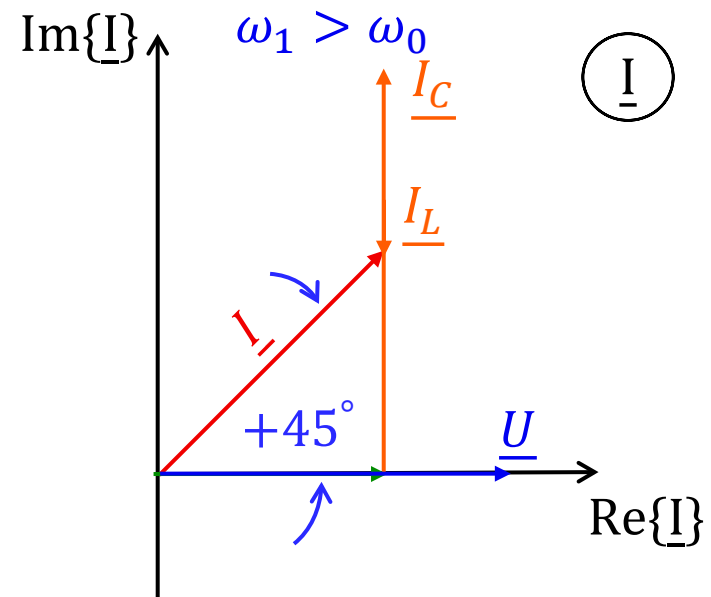
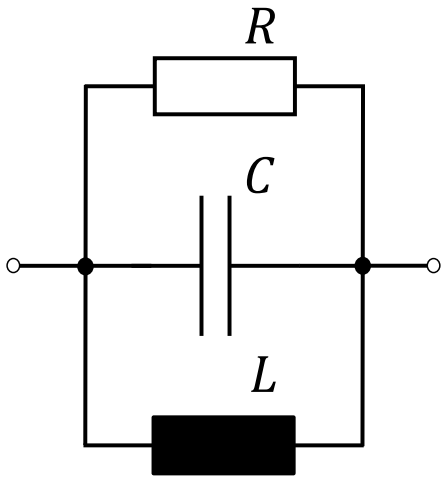
$$\underline{I}_{L_0} = \frac{\underline{U}}{j\omega_0 L} = -j \frac{\underline{U}}{\omega_0 L}$$

$$\left. \begin{array}{l} \underline{I}_{C_0} \\ \underline{I}_{L_0} \end{array} \right\} \underline{I}_{C_0} = -\underline{I}_{L_0}$$

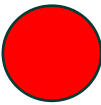
Def.: Kennleitwert

$$B_0 = \omega_0 C = \frac{1}{\omega_0 L} \leadsto B_0 := \sqrt{\frac{C}{L}} = \frac{1}{X_0}$$

Zeigerdiagramm für \underline{I}



Frequenzbandbreite der Parallelschaltung:



$$\frac{1}{R} = \omega_1 C - \frac{1}{\omega_1 L} \quad \leadsto$$

$$\omega_1 = \frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} + \frac{1}{LC}}$$

$$b_w := \omega_1 - \omega_2 = \underline{\underline{\frac{1}{RC}}}$$

$$\omega_2 = -\frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} + \frac{1}{LC}}$$

■ Dämpfung

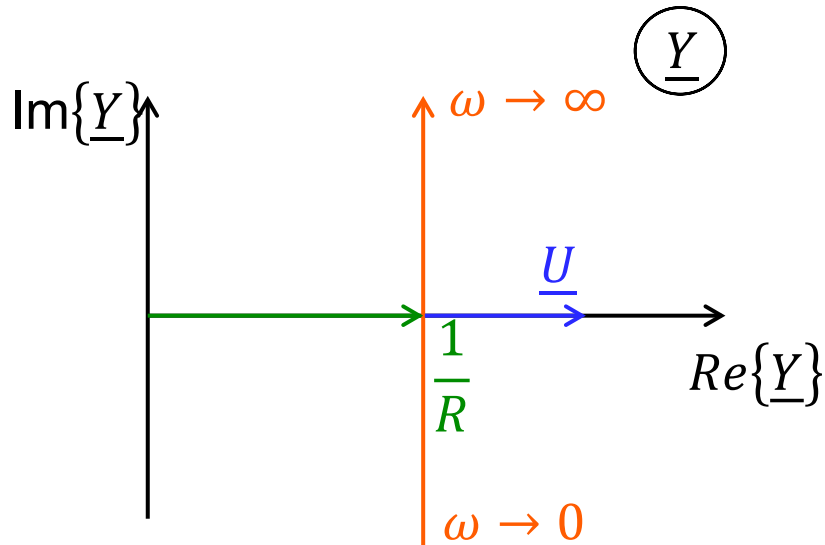
$$d_p := \frac{\omega_1 - \omega_2}{\omega_0} = \frac{1}{RC} \cdot \sqrt{LC} = \underline{\underline{\sqrt{\frac{L}{C}} \cdot \frac{1}{R}}}$$

■ Güte

$$Q_p := \frac{1}{d_p} = R \cdot \sqrt{\frac{C}{L}} = \underline{\underline{R \cdot B_0}} \quad \leadsto \quad R \uparrow \text{ und } C \uparrow \leadsto Q_p \uparrow$$

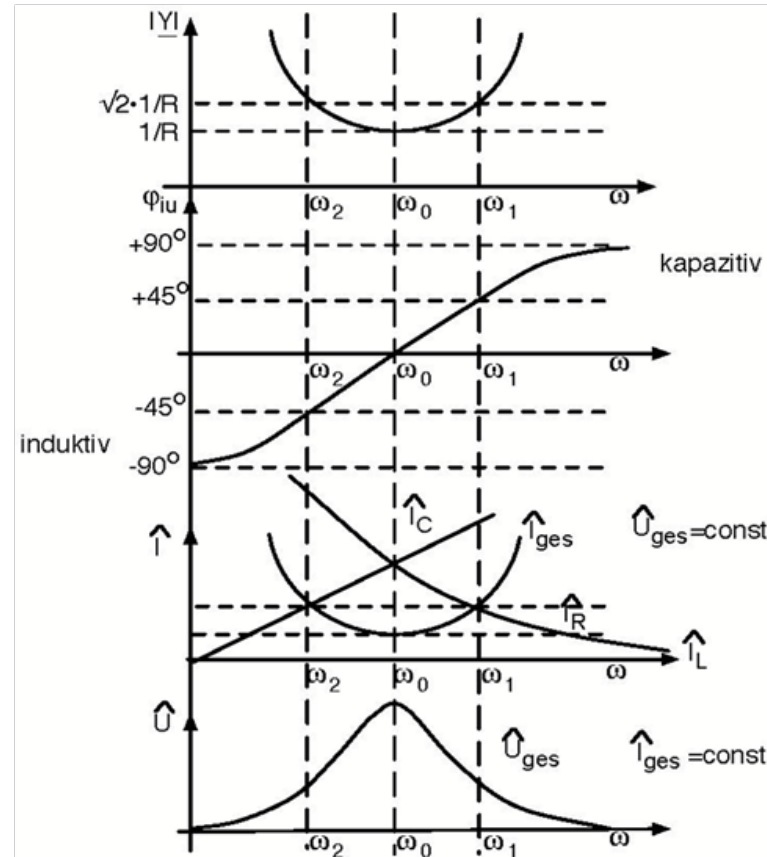
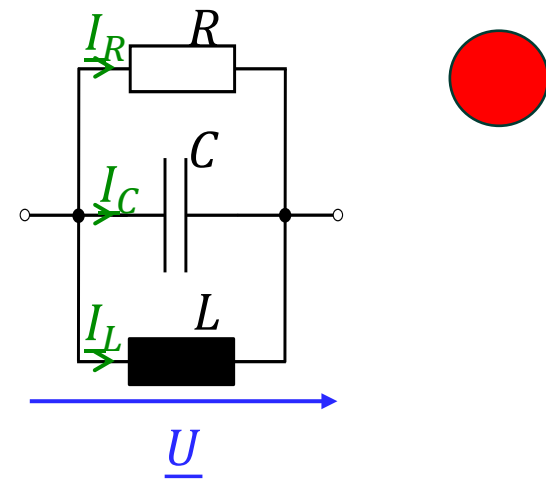
Diagramme für Parallelschwingkreis

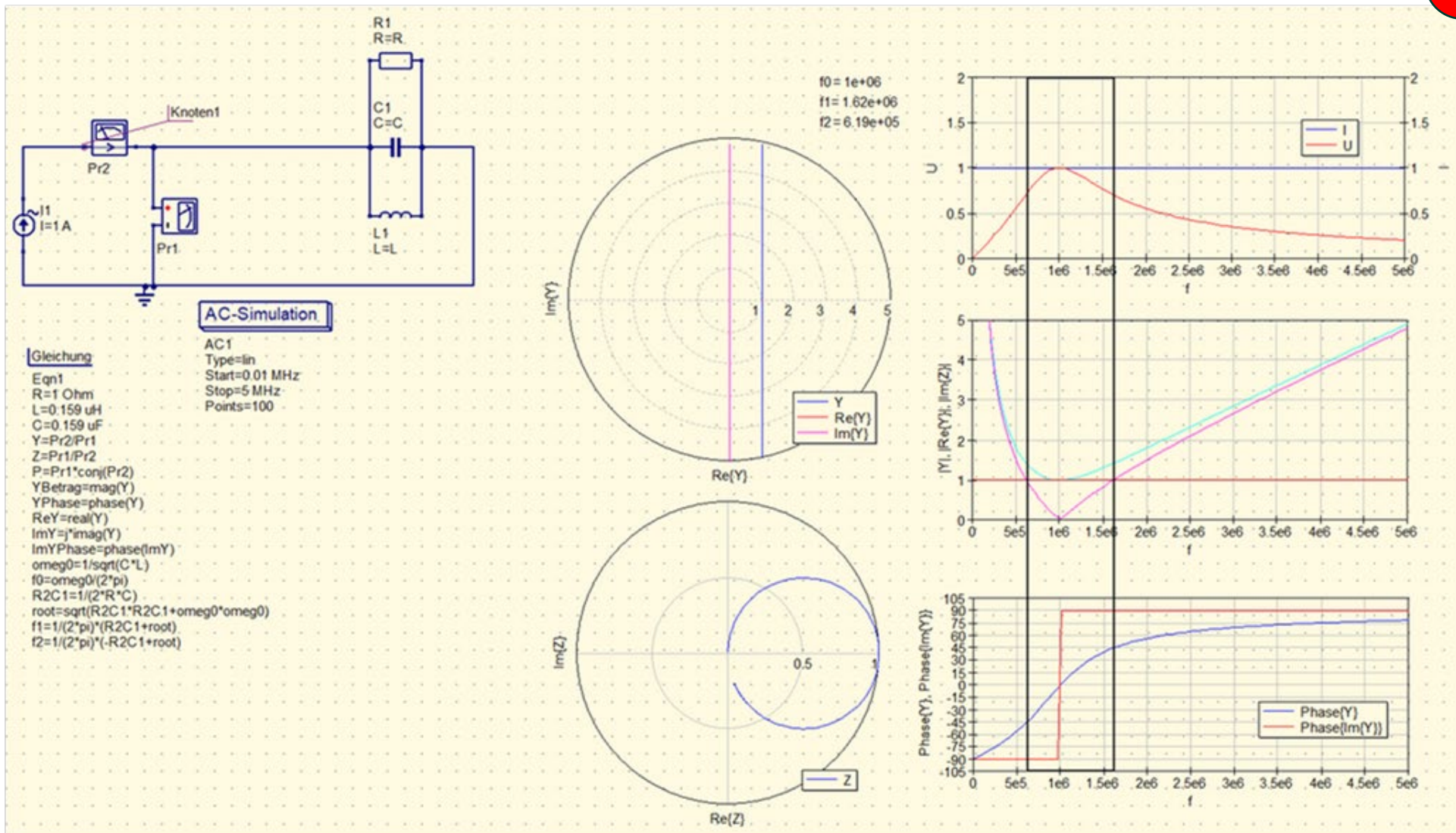
$$\underline{Y} = \frac{1}{R} + j \left(\omega C - \frac{1}{\omega L} \right)$$



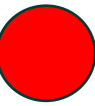
Für $Q \gg 1$:

$$\underline{I}_{C_0} = \underline{I}_{L_0} = Q \cdot \underline{I}_R \quad \sim \quad \underline{I}_{C_0} = \underline{I}_{L_0} \gg \underline{I}_R$$





8.3 Verallgemeinerte Definition der Güte



■ Serienschwingkreis:

$$Q_R = \frac{1}{R} \cdot \underbrace{\sqrt{\frac{L}{C}}}$$

Kennwiderstand

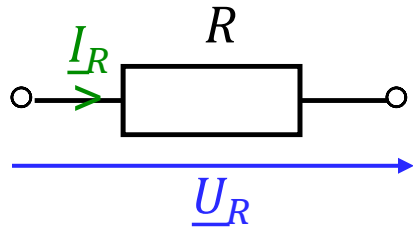
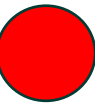
■ Parallelschwingkreis:

$$Q_p = R \cdot \underbrace{\sqrt{\frac{C}{L}}}$$

Kennleitwert

■ Für jedes schwingungsfähige System gilt:

$$Q = 2\pi \cdot \frac{\text{gesamte gespeicherte Energie}}{\text{Energieverlust pro Periode}}$$



$$i_R = \hat{I}_R \cdot \sin(\omega t) = \sqrt{2} \cdot |\underline{I}_R| \cdot \sin(\omega t)$$

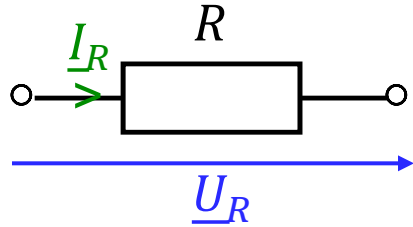
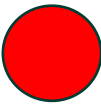
$$u_R = R \cdot i_R = R \cdot \hat{I}_R \cdot \sin(\omega t)$$

$$P_R = u_R \cdot i_R = R \cdot \hat{I}_R^2 \cdot \sin^2(\omega t)$$

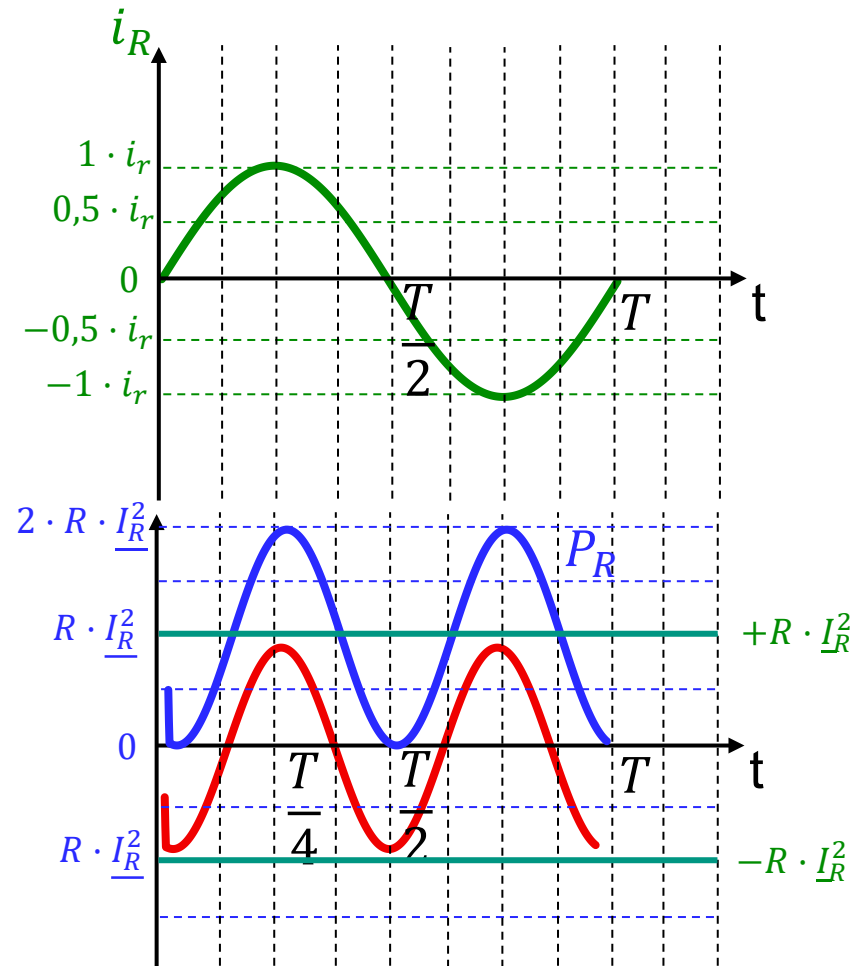
$$\begin{aligned} P_R &= R \cdot \hat{I}_R^2 \cdot \frac{1}{2} \cdot (1 - \cos(2\omega t)) \\ &= R \cdot (\sqrt{2} \cdot |\underline{I}_R|)^2 \cdot \frac{1}{2} \cdot (1 - \cos(2\omega t)) \end{aligned}$$

$$P_R = R \cdot |\underline{I}_R|^2 \cdot (1 - \cos(2\omega t))$$

Leistung und Energie: Widerstand R



$$P_R = u_R \cdot i_R = R \cdot \underline{I_R}^2 \cdot \underline{(1 - \cos(2\omega t))}$$



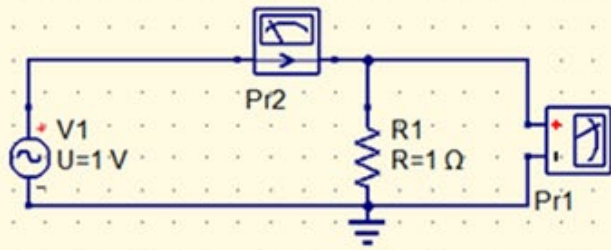
Einschub

$$\cos(2\omega t) \stackrel{!}{=} 0$$

$$\leadsto 2\omega t = \frac{\pi}{2} \pm h \cdot \pi$$

$$\omega = \frac{2\pi}{T}$$

$$\leadsto t = \left(\frac{1}{8} \pm \frac{h}{4} \right) T$$

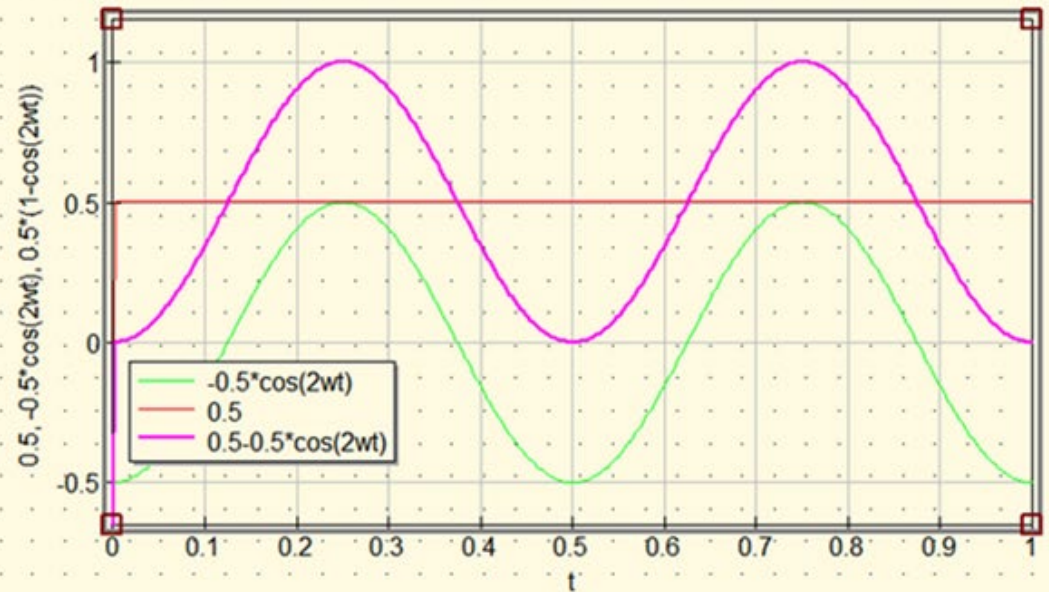
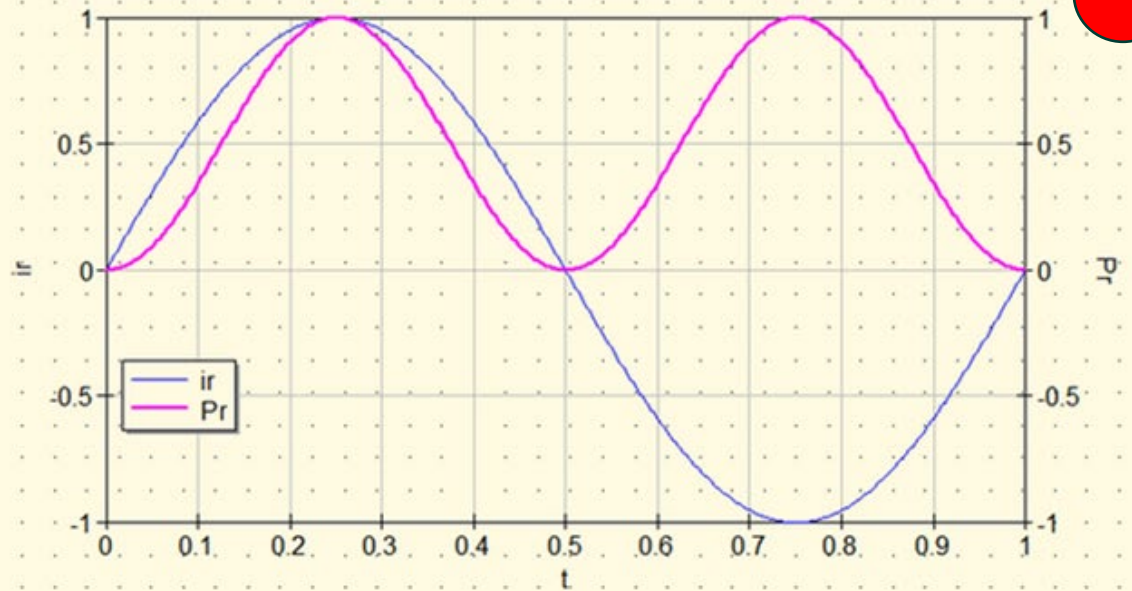


Transient Simulation

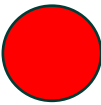
TR1
Stop=1000 ms
Points=501

Gleichung

Eqn1
Pr=Pr1.dVt*Pr2.It
t=time
T=1
w=2*pi/T
mcos2wt=-0.5*cos(2*w*t)
c05=0.5*t/t
PrRechnung=c05+mcos2wt.



Mittlere Energie (Verlust) über eine Schwingungsperiode



$$P_R = R \cdot \hat{I}_R^2 \cdot \frac{1}{2} (\textcircled{1} - \cos(2\omega t))$$

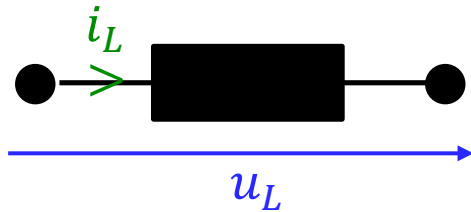
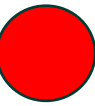
$$W_R = \int_0^T P_R dt = R \cdot \hat{I}_R^2 \cdot \frac{1}{2} \cdot \textcircled{T} - \frac{1}{2} R \cdot \hat{I}_R^2 \cdot \left[\frac{1}{2\omega} \cdot \sin(2\omega t) \right]_0^T$$

$$\leadsto \boxed{W_R = \frac{1}{2} \cdot R \cdot \hat{I}_R^2 \cdot T} \quad \underline{\underline{\triangleq \frac{1}{2} \hat{U}_R^2 \cdot \frac{1}{R} \cdot \frac{2\pi}{\omega}}}}$$

↪ Im Widerstand wird über eine Schwingungsperiode Energie verbraucht!

$$\boxed{W_R = P_R \cdot T}$$

Leistung, Energie bei der Induktivität (Spule)



$$u_l = L \cdot \frac{di_L}{dt}$$

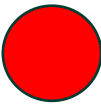
$$i_L = \hat{I}_L \cdot \sin(\omega t)$$

$$\leadsto u_L = L \cdot \omega \hat{I}_L \cos(\omega t)$$

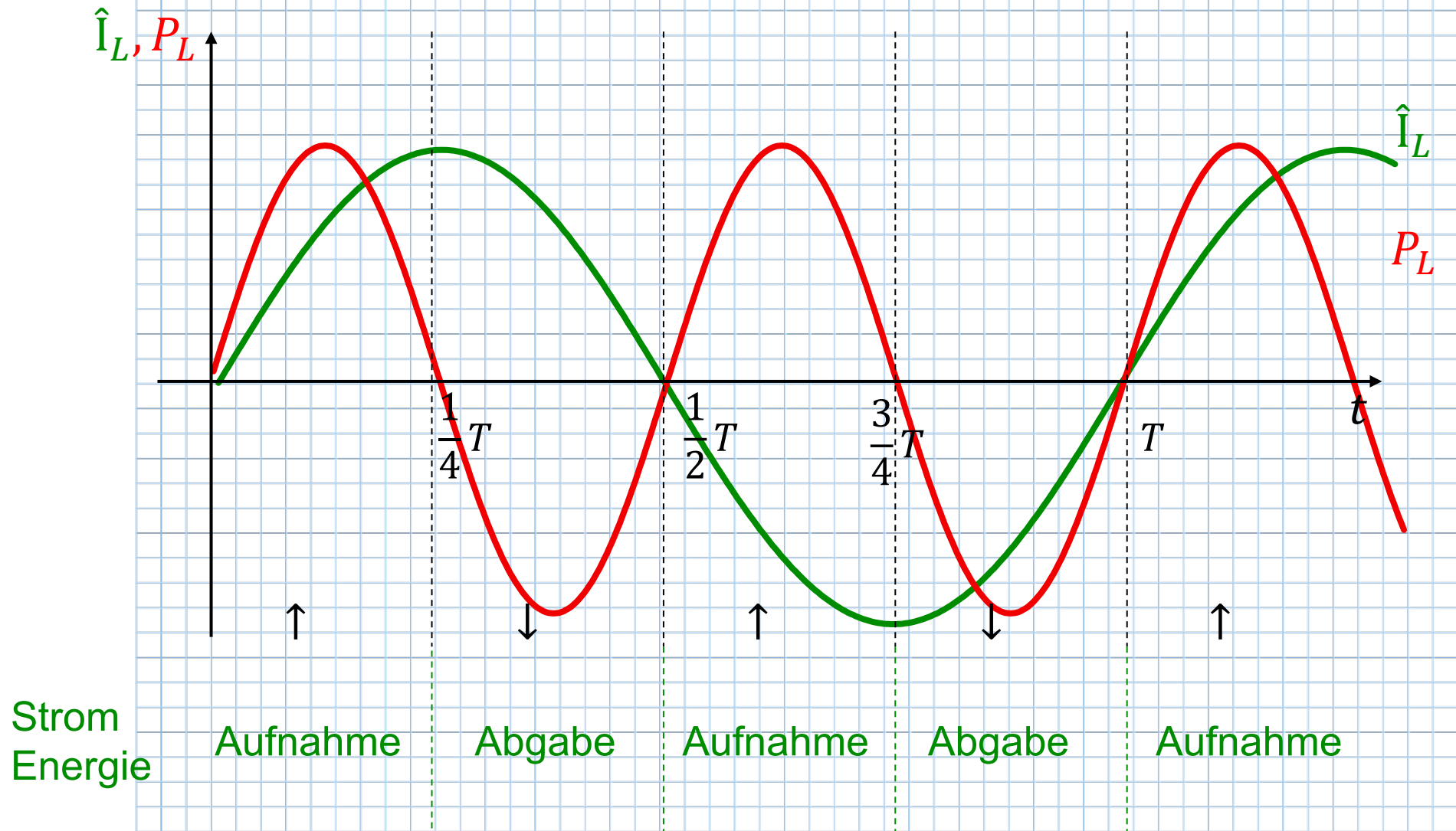
$$\rightarrow P_L = u_L \cdot i_L = \omega \cdot L \cdot \hat{I}_L^2 \cdot \underbrace{\cos(\omega t) \cdot \sin(\omega t)}_{\frac{1}{2} \sin(2\omega t)}$$

$$\underline{\underline{P_L = \omega L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \sin(2\omega t)}}$$

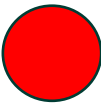
\leadsto über eine Schwingungsperiode gemittelt, ist die gesamte von der Spule aufgenommene Leistung $P_L = 0!$



$$P_L = \frac{1}{2} \omega L \hat{I}^2 \cdot \sin(2\omega t)$$



Mittlere Energie (**gespeicherte**) einer idealen verlustlosen Spule über eine Schwingungsperiode



$$P_L = \omega L \cdot i_L = \omega L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \sin(2\omega t)$$

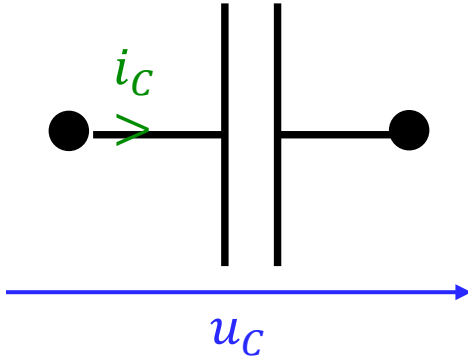
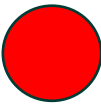
$$W_L = \int_0^T P_L dt = \omega L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \int_0^T \sin(2\omega t) dt$$

$$\begin{aligned} &= \cancel{\omega} \cdot L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \left[-\frac{1}{\cancel{2\omega}} \cdot \cos(2\omega t) \right]_0^T \\ &= \frac{1}{2} \omega L \cdot \hat{I}_L^2 \cdot \left(-\frac{1}{2} \omega \right) \cdot \left[\cancel{\cos\left(2\frac{2\pi}{T}T\right)} - \cancel{\cos(0)} \right]_0^T \end{aligned}$$

$$= \underline{\underline{0!}} \quad \text{Über eine Periode (T)} \quad \leadsto \quad W_L = 0$$

$$\text{Aber! Integration } \int_0^T dt \leadsto W_L = \frac{1}{2} L \hat{I}_L^2 \quad !$$

Mittlere Leistung, Energie bei einem Kondensator C



$$u_C = \hat{U}_C \cdot \sin(\omega t)$$

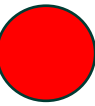
$$i_C = C \cdot \frac{du_C}{dt} = \omega C \cdot \hat{U}_C \cdot \cos(\omega t)$$

$$\blacksquare P_C = u_C \cdot i_C = \hat{U}_C \cdot \sin(\omega t) \cdot \omega \cdot C \cdot \hat{U}_C \cdot \cos(\omega t) \\ \rightarrow P_C = \omega \cdot C \cdot \hat{U}_C^2 \cdot \frac{1}{2} \cdot \sin(2\omega t)$$

$$\blacksquare W_C = \int_0^T P_C dt = \omega \cdot C \cdot \hat{U}_C \cdot \frac{1}{2} \cdot \int_0^T \sin(2\omega t) dt$$

$$= \cancel{\omega C} \cdot \hat{U}_C \cdot \frac{1}{2} \cdot \frac{1}{\cancel{2\omega}} \cdot [-\cos(2\omega t)]_0^{\frac{T}{2}}$$

$$= C \cdot \hat{U}_C \cdot \frac{1}{2}$$



$$Q = 2\pi \cdot \frac{\text{gesamte gespeicherte Energie}}{\text{Verlust über eine Schwingungsperiode}}$$

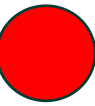
$$\text{Verlust: } U_R = \frac{1}{2} R \cdot \hat{I}_R^2 \cdot T \qquad T = \frac{1}{f} [s]$$

Gesamte gespeicherte Energie:

$$W_C = \frac{1}{2} C \cdot \hat{U}_C^2$$

$$W_L = \frac{1}{2} L \cdot \hat{I}_L^2$$

■ Für Serienkreis gilt: $\hat{I}_L = \hat{I}_R =: \hat{I}$

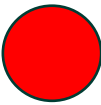


$$\begin{aligned} \text{■ } Q_R &= \cancel{2\pi} \cdot \frac{\frac{1}{2} L \hat{I}^2}{\frac{1}{2} R \cdot \hat{I}^2 \cdot \cancel{2\pi} \omega_0} = \\ &= \frac{L}{R} \cdot \omega_0 = \frac{L}{R} \cdot \frac{1}{\sqrt{LC}} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}} = \frac{X_0}{R} ! \end{aligned}$$

■ Für Parallelkreis gilt: $\hat{U}_C = \hat{U}_R =: \hat{U}$

$$\text{■ } Q_R = \frac{\frac{1}{2} C \hat{U}^2 \cdot \cancel{2\pi}}{\frac{1}{2} \hat{U}^2 \cdot \cancel{2\pi} \omega_0 R} = R \cdot C \cdot \frac{1}{\sqrt{LC}} = R \cdot \sqrt{\frac{C}{L}} = \frac{B_0}{\frac{1}{R}} !$$

8.4 Verallgemeinerte Resonanzkurven



Definition der Transformation

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \leadsto \quad \boxed{\vartheta := \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}} \quad \leadsto$$

\leadsto Frequenzzentriert um ω_0

ω	ϑ
0	$-\infty$
ω_0	0
$+\infty$	$+\infty$

Für Serienresonanzkreis

$$\begin{aligned}\underline{Z} &= R + j \left(\omega L - \frac{1}{\omega C} \right) = R + j \omega_0 L \left(\frac{\omega L}{\omega_0 L} - \frac{1}{\omega_0 L \omega C} \right) \\ &= R + j \omega_0 L \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = R + j \omega_0 L \cdot \vartheta \\ &= R \cdot \left(1 + j \omega_0 \frac{L}{R} \cdot \vartheta \right) = R \cdot (1 + j Q_R \cdot \vartheta)\end{aligned}$$

Achtung! Fehler im Skript auf S.145 Gl. 8.39

