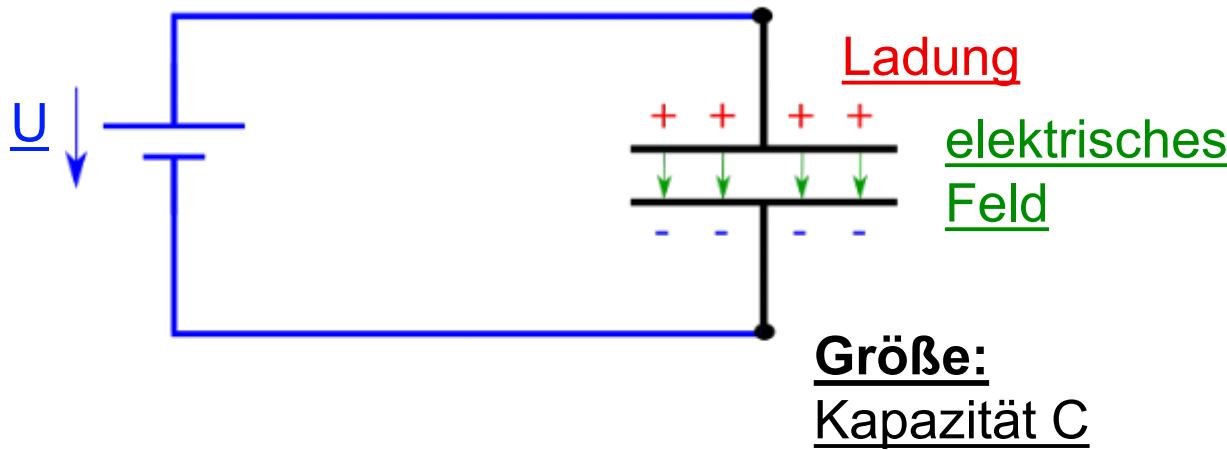


8. Schwingkreise

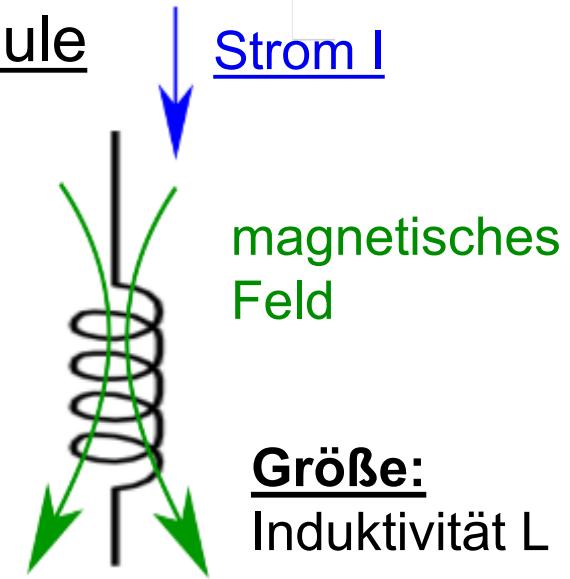


Elektrische Schwingkreise bestehen aus mindestens zwei Energiespeichern, deren gespeicherte Energie periodisch ausgetauscht wird.

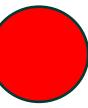
1. Kondensator



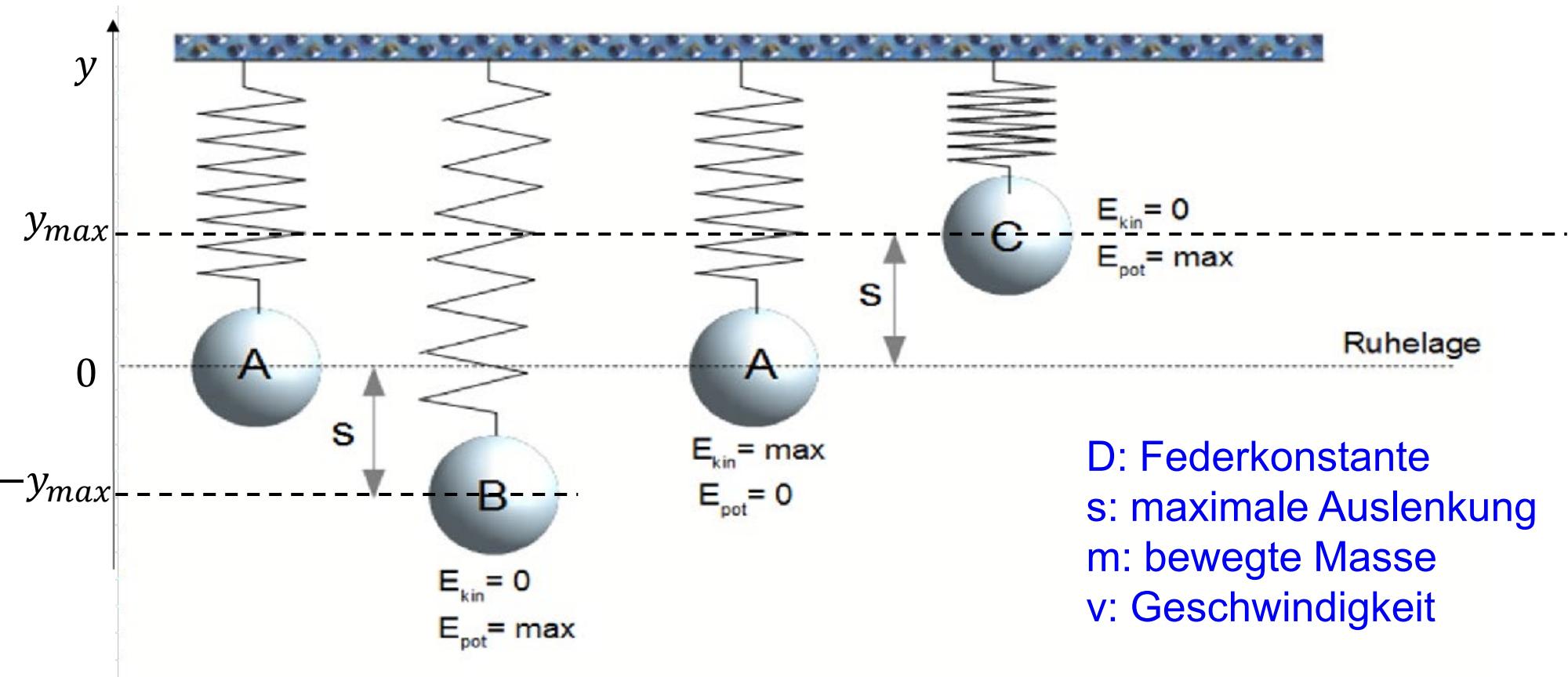
2. Spule



→ Elektrische Schwingkreise verhalten sich gleichartig zu mechanischen Systemen!



Analogie zum Federpendel:

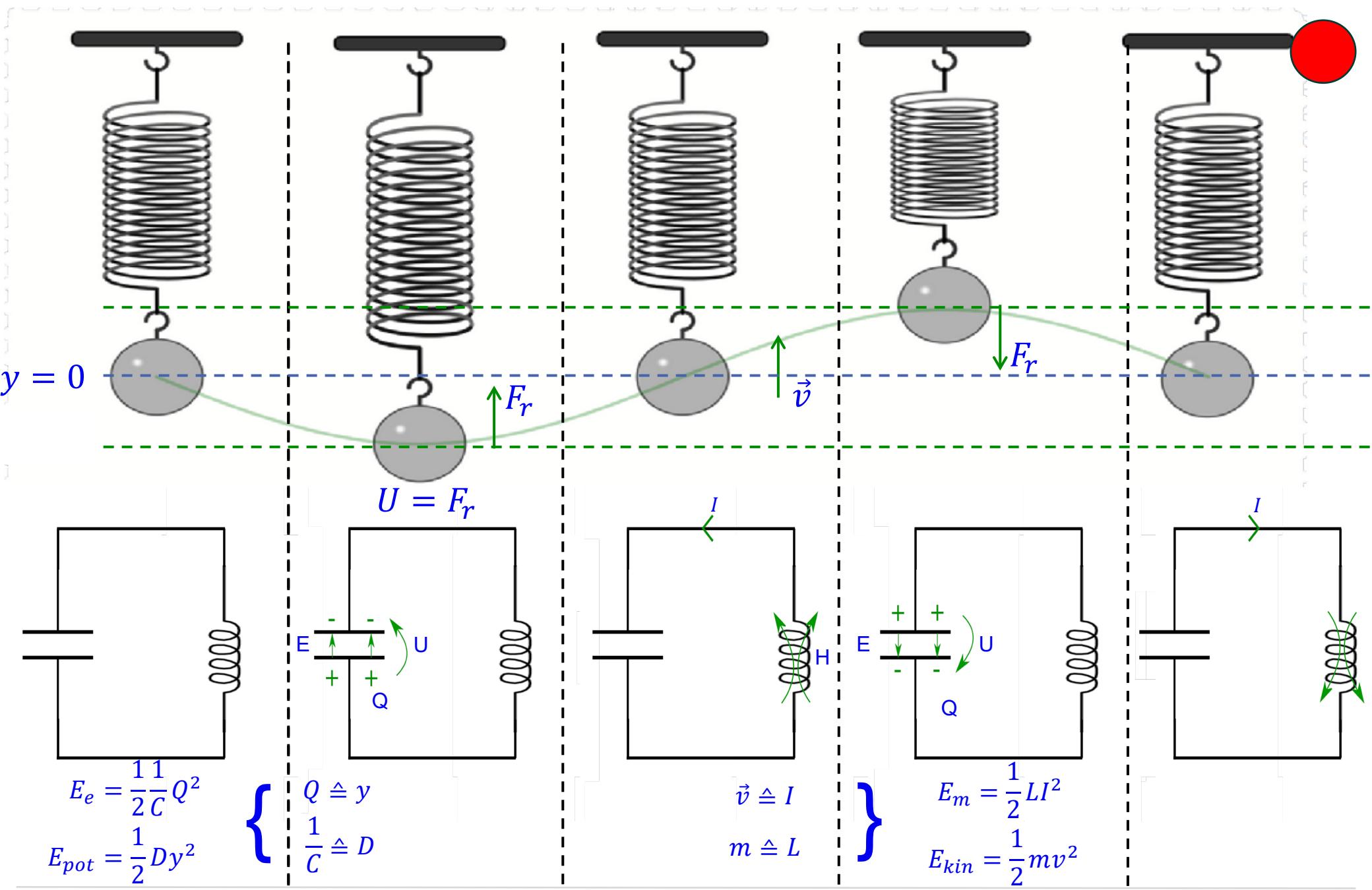


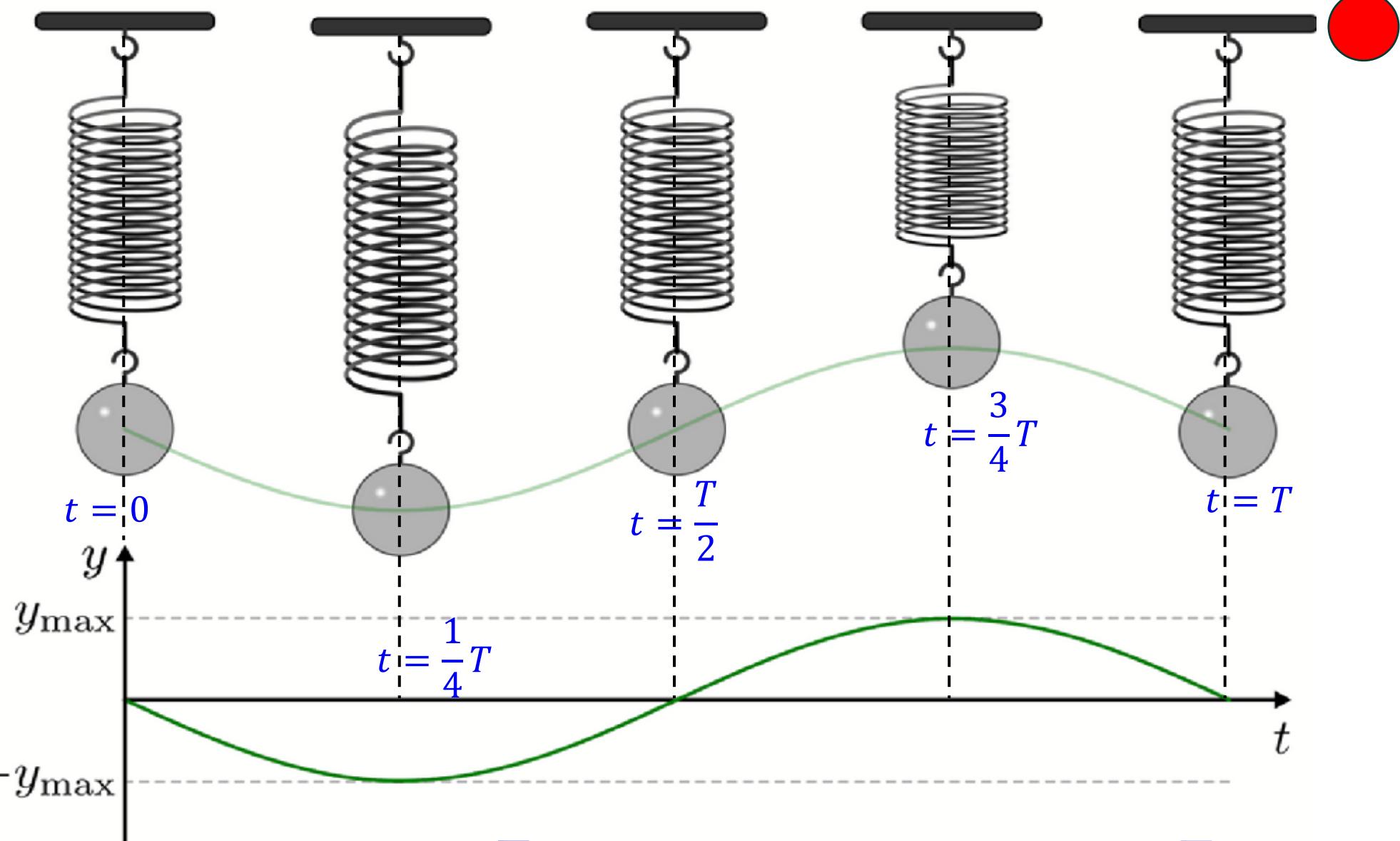
Spannarbeit:

$$E_{pot} = \frac{1}{2} D \cdot s^2$$

Kinetische Energie:

$$E_{kin} = \frac{1}{2} m \cdot v^2$$

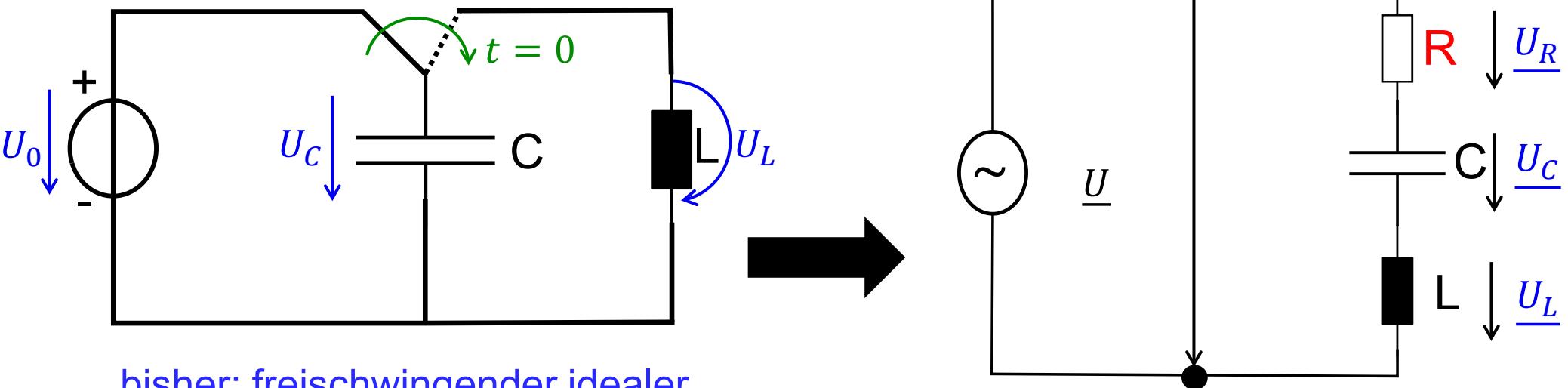




$$\text{Kreisfrequenz: } \omega_0 = \sqrt{\frac{D}{m}}$$

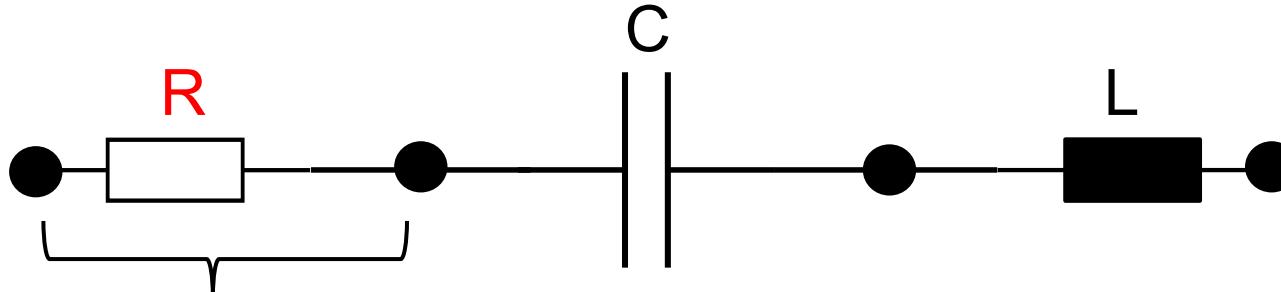
$$\text{Schwingungsdauer: } T = 2\pi \sqrt{\frac{m}{D}}$$

8.1 Serienschwingkreis



bisher: freischwingender idealer
elektrischer Schwingkreis

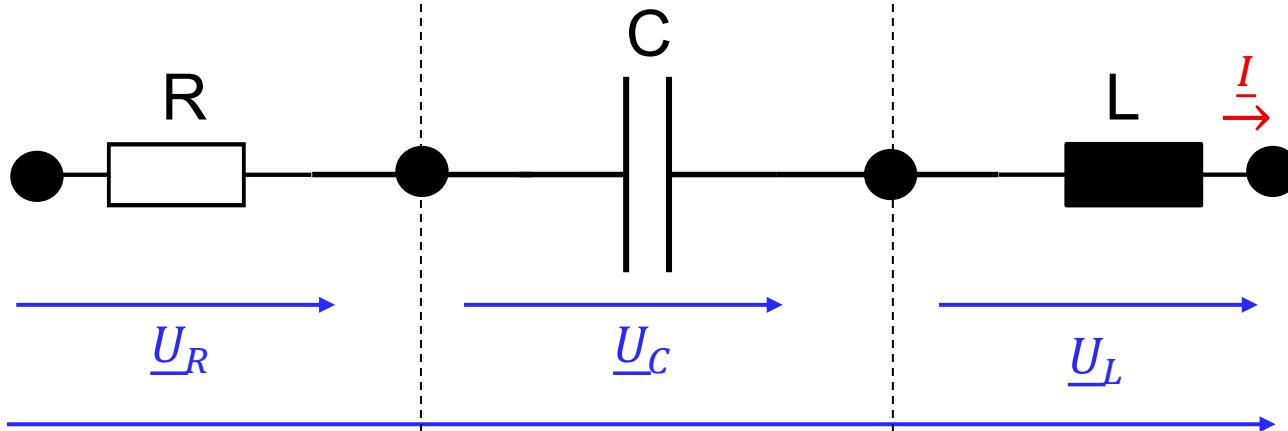
Serienschwingkreis



verlustbehafteter
Widerstand

verlustbehafteter
Serienschwingkreis

=: Z *Seriенimpedanz*



$$\underline{U}_R = R \cdot \underline{I}$$

$$\underline{U}_C = \frac{1}{j\omega C} \cdot \underline{I}$$

$$\underline{U}_L = j\omega L \cdot \underline{I}$$

$$\underline{U} = \underline{Z} \cdot \underline{I} = R \cdot \underline{I} + \frac{1}{j\omega C} \cdot \underline{I} + j\omega L \cdot \underline{I}$$

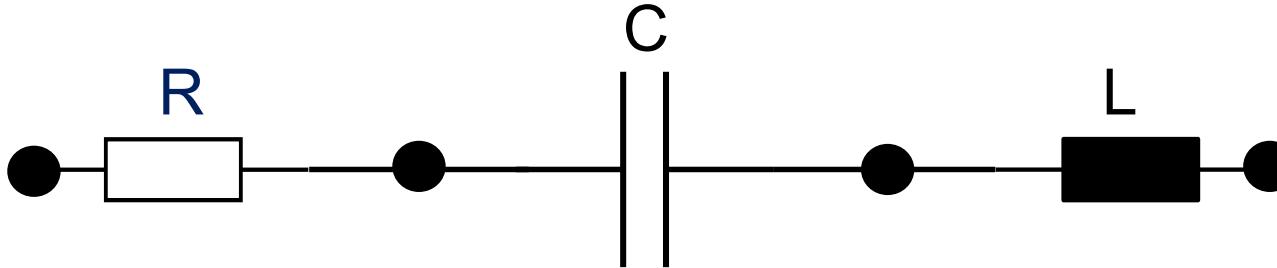
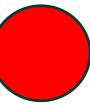
$$\underline{Z} = R + \frac{1}{j\omega C} + j\omega L$$

es gilt: $j = -\frac{1}{j}$!

aufteilen nach Real- und Imaginärteil

$$\underline{Z} = \underbrace{R}_{\substack{\text{Realteil} \\ \triangleq \\ \text{Wirkwiderstand}}} + j \left(\underbrace{-\frac{1}{\omega C}}_{\substack{\text{Imaginärteil} \\ \triangleq \\ \text{Blindwiderstand}}} + \omega L \right) = R + jX$$

Definition der Resonanzfrequenz f_0



Frage: Imaginärteil von $\underline{Z} \stackrel{!}{=} 0$?

$$X \stackrel{!}{=} \text{Im}\{\underline{Z}(\omega_0)\} \stackrel{!}{=} 0 \text{ mit } \omega_0 = 2 \cdot \pi \cdot f_0$$

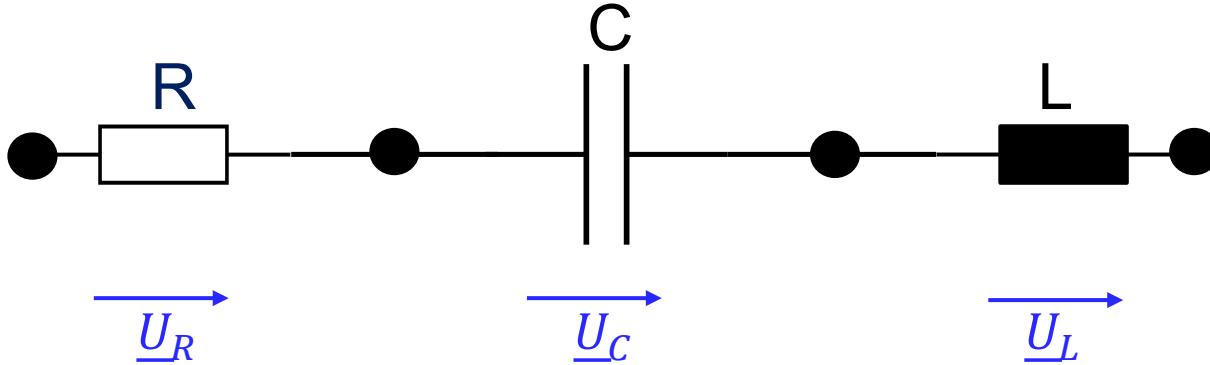
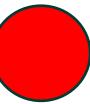
$$\leadsto X = \left(\omega_0 L - \frac{1}{\omega_0 C} \right) \stackrel{!}{=} 0 \quad \leadsto \quad \boxed{\omega_0 L \stackrel{!}{=} \frac{1}{\omega_0 C}}$$

$$\leadsto \boxed{\omega_0 = \frac{1}{\sqrt{LC}} = 2 \cdot \pi \cdot f_0}$$

$$\omega \stackrel{!}{=} \omega_0 \leadsto \underline{U_L} = j\omega_0 L \cdot \underline{I}; \quad \underline{U_C} = -\frac{1}{j\omega_0 C} \cdot \underline{I} = 0 \quad \leadsto \quad \boxed{\underline{U_L} + \underline{U_C} = 0}$$

$$\leadsto \boxed{\underline{U} = \underline{U_R} + \underline{U_L} + \underline{U_C} \stackrel{!}{=} \underline{U_R} \quad |_{\omega = \omega_0}} \quad \leadsto \quad \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{\underline{U_R}}{\underline{I}} = \frac{R \cdot \underline{Y}}{\underline{Y}}$$

Definition des Kennwiderstands X_0



$$\omega = \omega_0 \quad \sim \quad -\underline{U}_C \stackrel{!}{=} \underline{U}_L \quad \sim \quad -j \frac{1}{\omega_0 C} \cdot I \stackrel{!}{=} j\omega_0 L \cdot I$$

$$\text{mit } \omega_0 = \frac{1}{\sqrt{LC}}$$

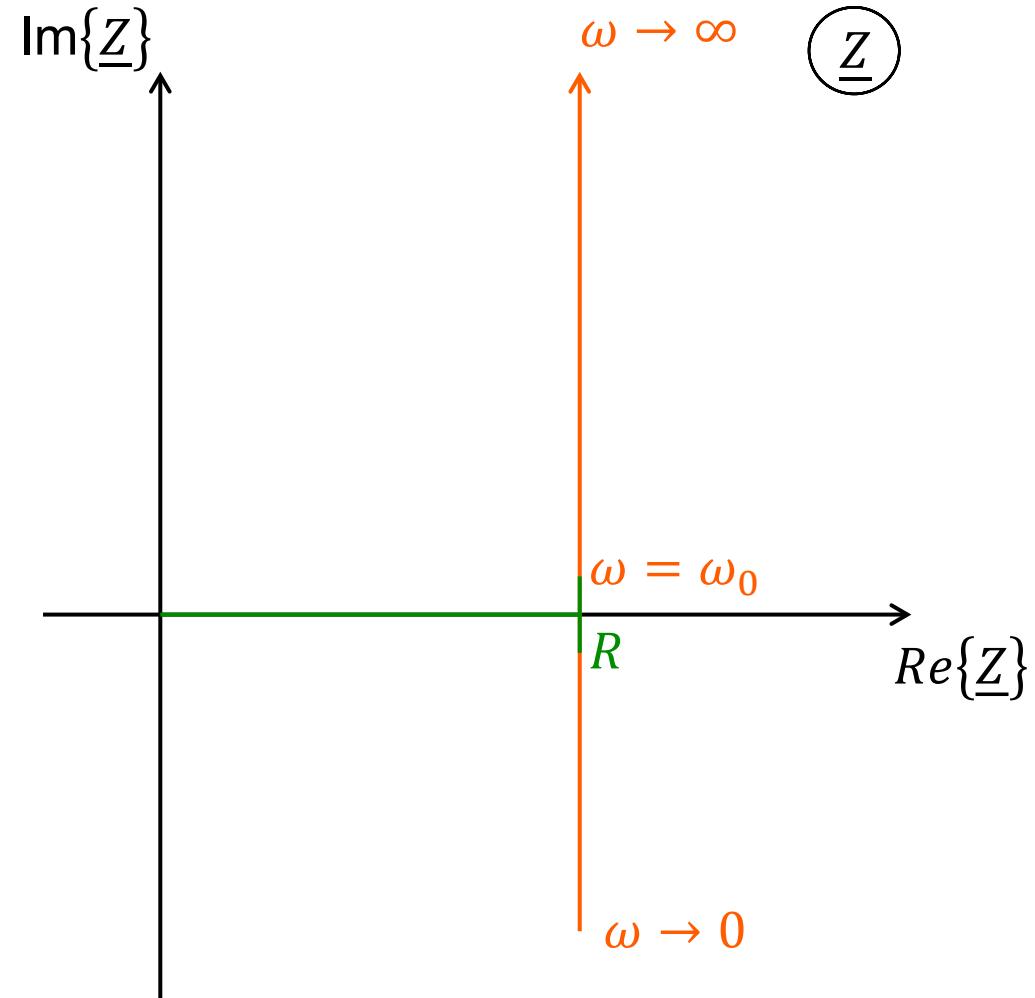
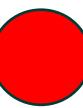
$$\sim \boxed{X_L \Big|_{\omega_0} = \omega_0 L = \sqrt{\frac{L}{C}}} \quad \text{und}$$

$$\boxed{X_C \Big|_{\omega_0} = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}}$$

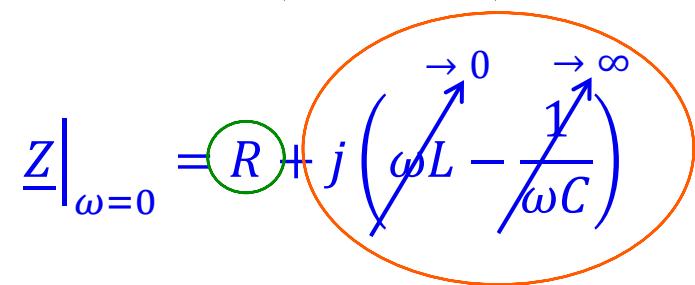
$$X_0 = \sqrt{\frac{L}{C}} \Rightarrow \boxed{\begin{aligned} L &= \frac{1}{\omega_0} \cdot X_0 \left[\frac{Vs}{A} = \Omega s \right] \\ C &= \frac{1}{\omega_0} \cdot \frac{1}{X_0} \left[\frac{As}{V} = \frac{s}{\Omega} \right] \end{aligned}}$$

Kennwiderstand

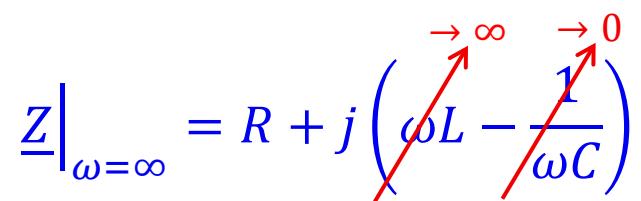
Ortskurve für die Serienimpedanz



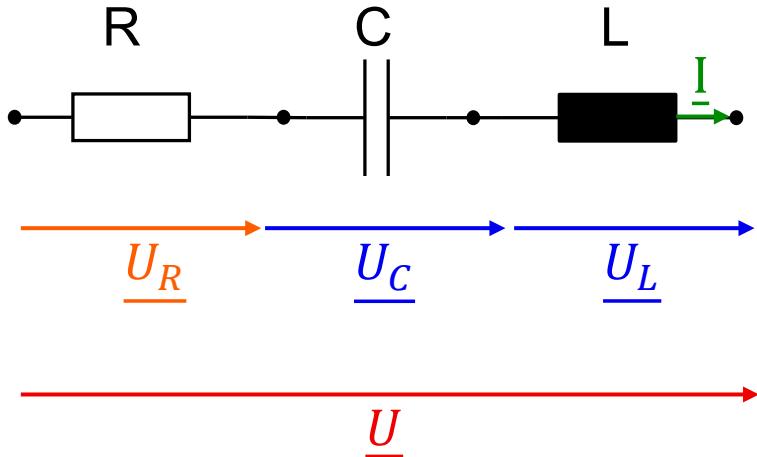
$$\begin{aligned}\underline{Z} &= R + j\underline{X} \\ &= R + j\left(\omega L - \frac{1}{\omega C}\right)\end{aligned}$$



$$\underline{Z}|_{\omega=\omega_0} = R$$



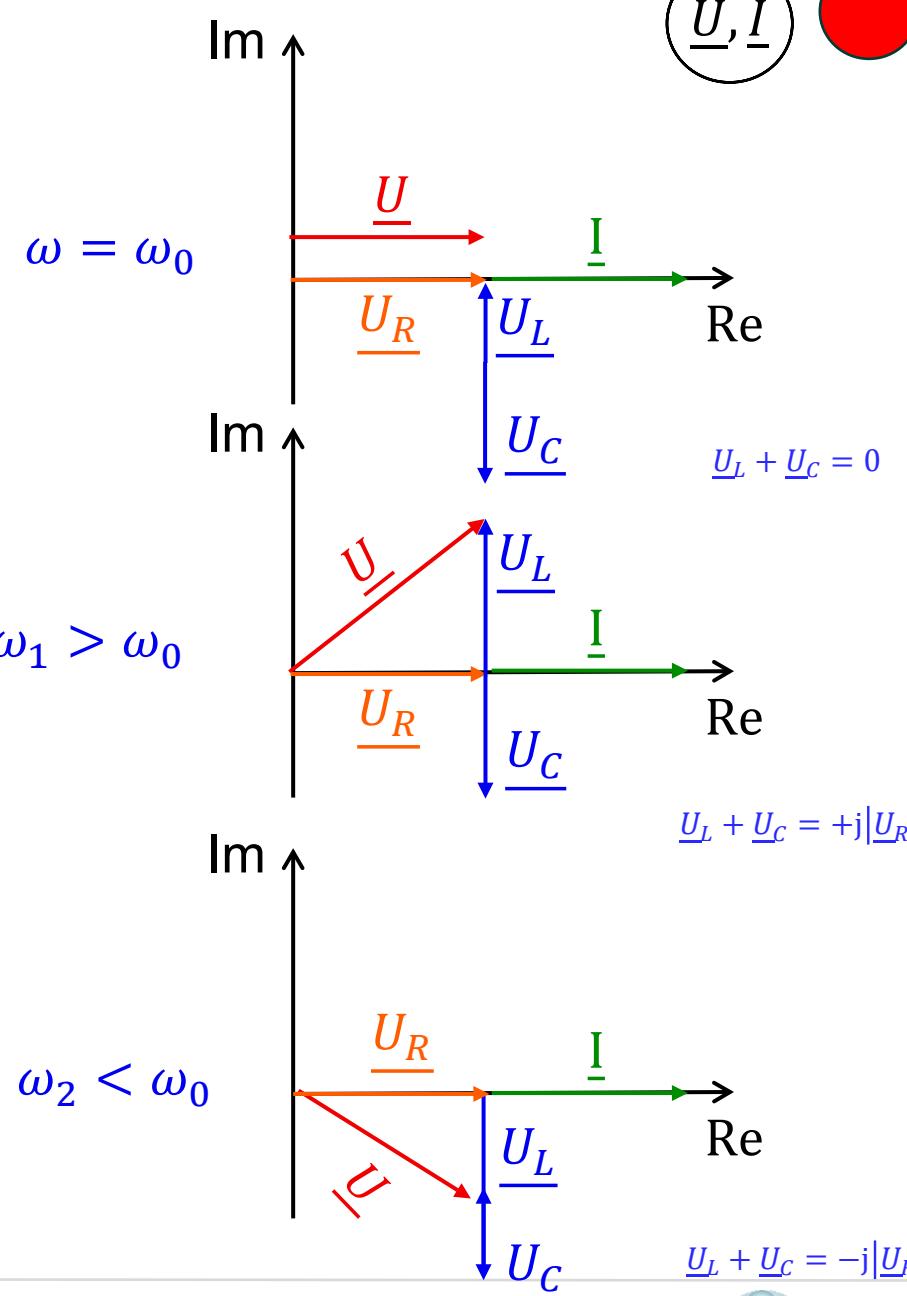
Zeigerdiagramm für \underline{U} und \underline{I}



Frage:

Bei Welcher Frequenz entspricht die Phasenverschiebung zwischen Spannung und Strom 45 Grad?

$$\angle \underline{U}, \underline{I} = 45^\circ \left(\frac{\pi}{4}\right) ?$$



$$\omega_1 > \omega_0 \Leftrightarrow (\underline{U}, \underline{I}) = +45^\circ$$

$$\rightsquigarrow R = \left(\omega_1 L - \frac{1}{\omega_1 C} \right) \quad \left| \cdot \frac{\omega_1}{L} \right.$$

$$\omega_1 \frac{R}{L} = \omega_1^2 - \frac{1}{LC}$$

$$0 = \omega_1^2 - \omega_1 \frac{R}{L} - \frac{1}{LC}$$

Lösen durch quadrat. Ergänzung

$$0 = \underbrace{\omega_1^2 - \omega_1 \frac{R}{L} + \left(\frac{R}{2L} \right)^2}_{= \left(\omega_1 - \frac{R}{2L} \right)^2} - \left(\frac{R}{2L} \right)^2 - \frac{1}{LC}$$

$$\rightsquigarrow \boxed{\omega_1 = + \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L} \right)^2 + \frac{1}{LC}}}$$

Nur positive Frequenzen!

$$\omega_2 < \omega_0 \Leftrightarrow (\underline{U}, \underline{I}) = -45^\circ$$

$$\rightsquigarrow R = - \left(\omega_1 L - \frac{1}{\omega_1 C} \right) \quad \left| \cdot \frac{\omega_1}{L} \right.$$



$$\rightsquigarrow \boxed{\omega_2 = - \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L} \right)^2 + \frac{1}{LC}}}$$



Def.: „Frequenzbandbreite“, „Dämpfung“ und „Güte“

Frequenzbandbreite:

$$b_w := \frac{R}{L}$$

$$\begin{aligned} bw &:= \omega_1 - \omega_2 \\ &= +\frac{R}{2L} + \sqrt{\dots} - \left(-\frac{R}{2L} + \sqrt{\dots} \right) \end{aligned}$$

rel. Frequenzbandbreite. („Dämpfung“):

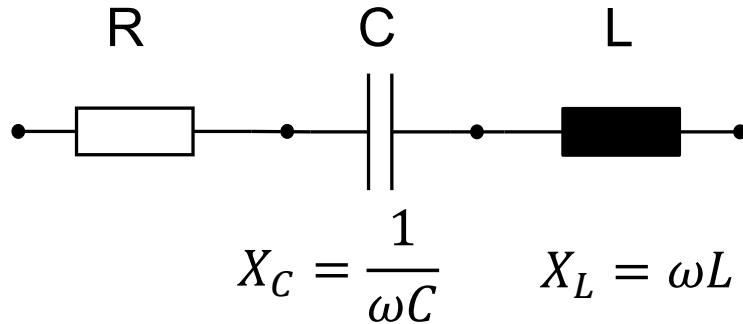
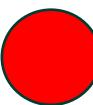
$$d := \frac{b_w}{\omega_0} = \frac{R}{L} \cdot \sqrt{LC} = R \cdot \sqrt{\frac{C}{L}} \quad \hookrightarrow \quad d := \frac{R}{X_0}$$

Güte:

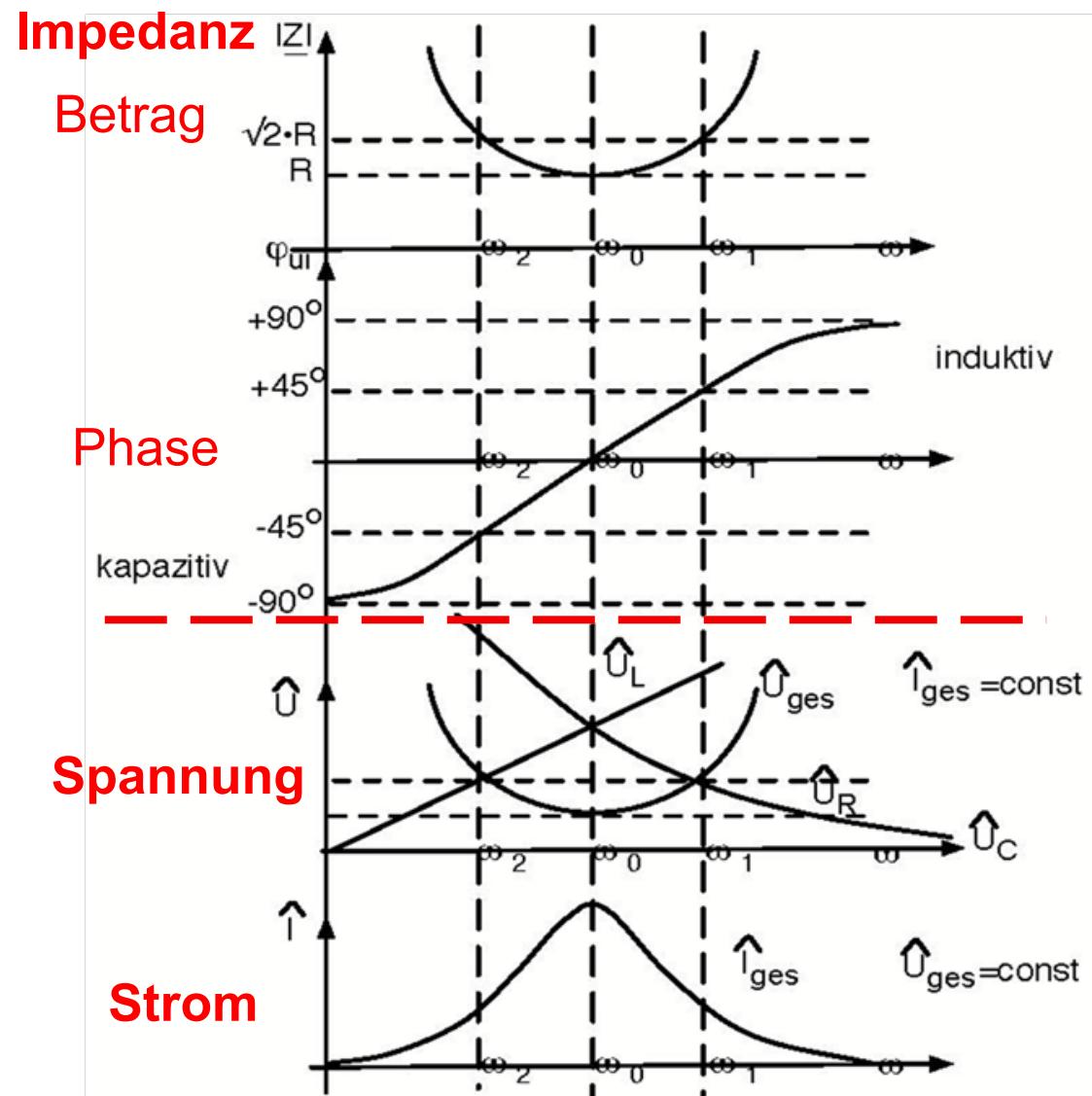
$$Q := \frac{1}{d} = \frac{X_0}{R} = \frac{\text{Kennwiderstand}}{\text{ohmscher Widerstand}}$$

$$R \downarrow + L \uparrow \curvearrowright Q \uparrow$$

Diagramme zur Beschreibung des Serienresonanzkreis



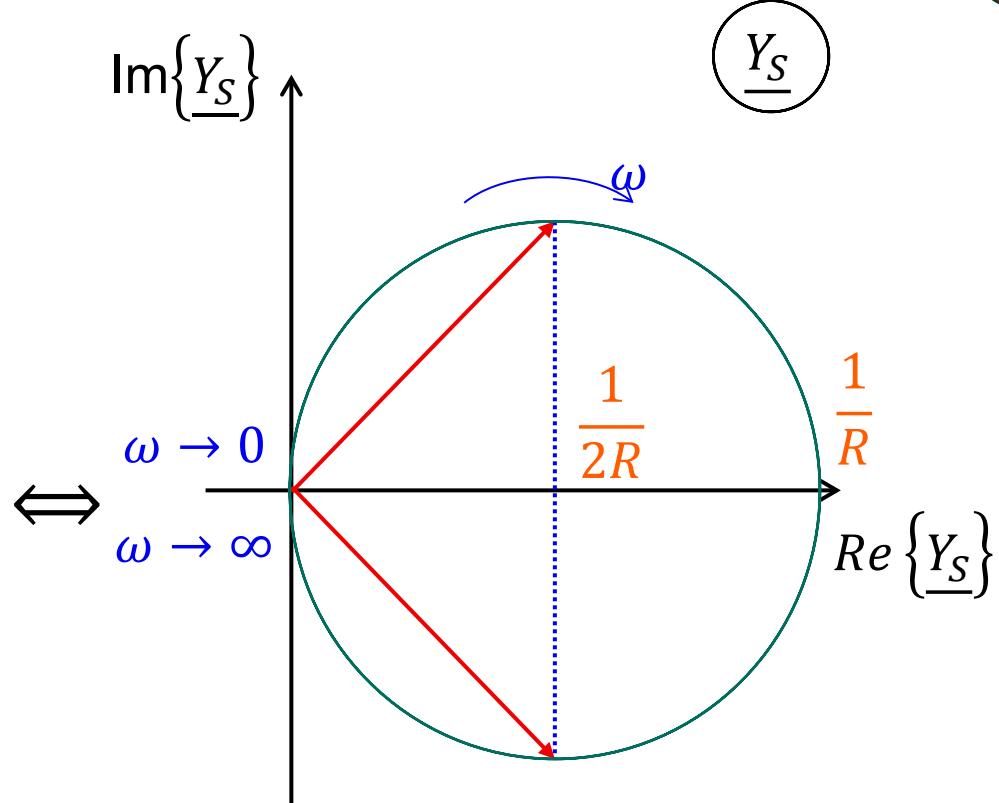
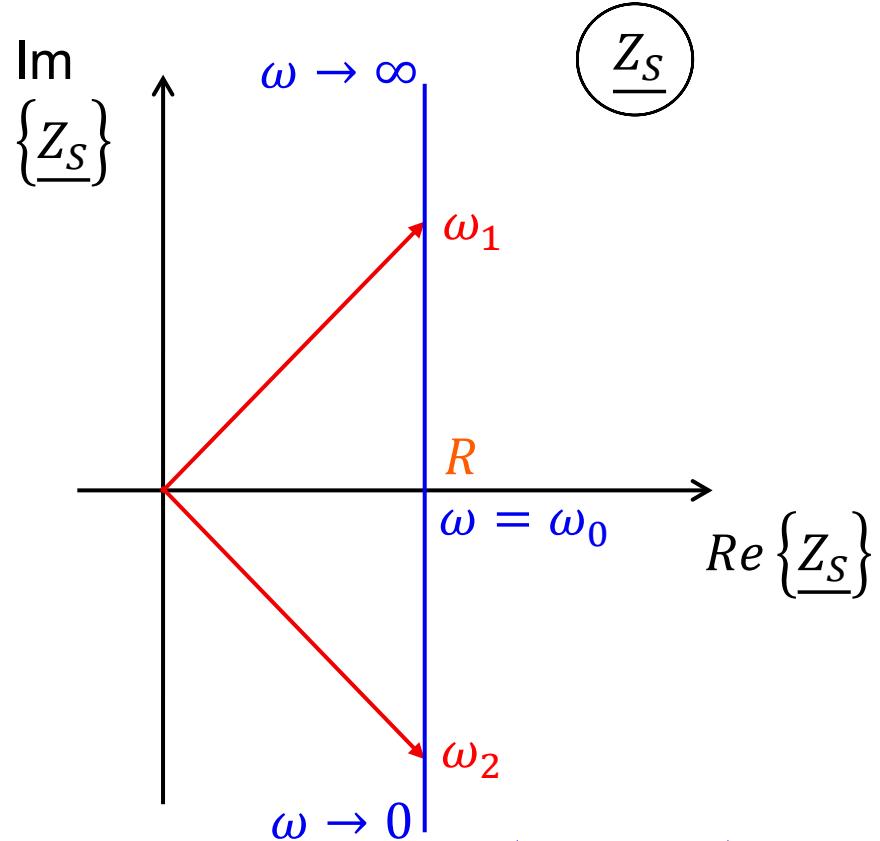
- Impedanz
 - Betrag
 - Phase
- Strom
 - Betrag
- Spannung
 - Betrag



Impedanzverhalten

\Leftrightarrow

Admittanzverhalten



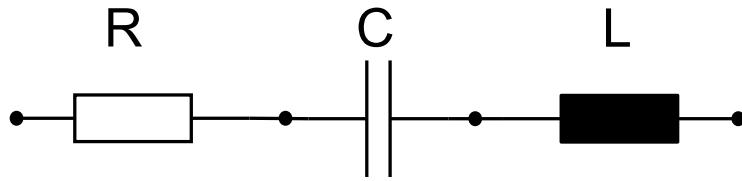
$$\underline{Z} = R + jX = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\begin{cases} \omega \rightarrow 0: & \underline{Y} \sim \frac{1}{R - j\frac{1}{\omega C}} = \frac{R + j\frac{1}{\omega C}}{R^2 + \left(\frac{1}{\omega C}\right)^2} \\ \omega \gg \omega_0: & \underline{Y} \sim \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + (\omega L)^2} \end{cases}$$



Praktisches Beispiel:



gegeben:

$$R = 1 \Omega; \quad f_0 = 1 \text{ MHz} = 1 \cdot 10^6 \frac{1}{\text{s}}$$

$$R = X_0; \quad \omega_0 = 2\pi \cdot 10^6 \frac{1}{\text{s}}$$

gesucht:

Impedanzverhalten, $L?$, $C?$

Lösung:

$$L = \frac{1}{\omega_0} \cdot X_0 = \frac{1}{2\pi \cdot 10^6 \frac{1}{\text{s}}} \cdot \Omega = \frac{1}{2\pi} \cdot 10^{-6} \Omega \text{s} \quad [H = \frac{Vs}{A}]$$

$$C = \frac{1}{\omega_0 X_0} = \frac{1}{2\pi \cdot 10^6 \frac{1}{\text{s}} \cdot \Omega} = \frac{1}{2\pi} \cdot 10^{-6} \frac{\text{s}}{\Omega} \quad [F = \frac{As}{V}]$$

Verifikation mit LTspice



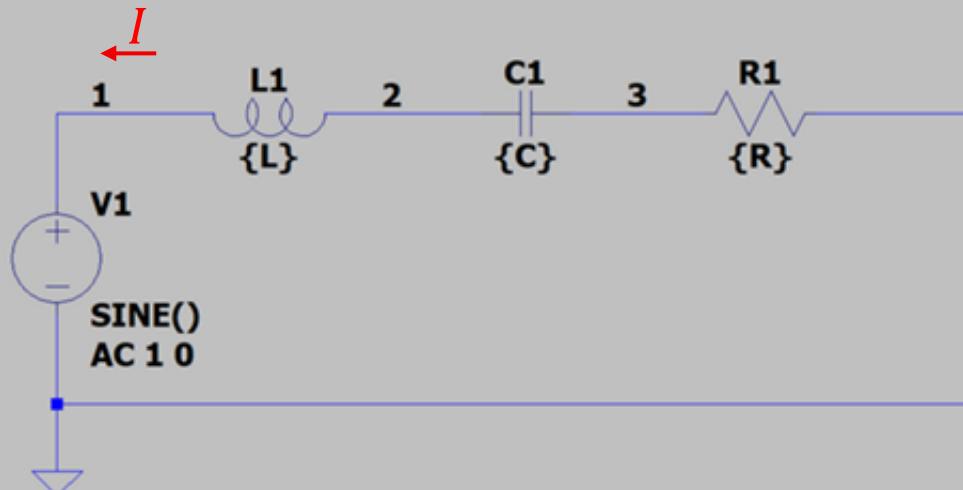
$$f = 100\text{kHz} \dots 5\text{MHz}$$

$$L = 0,159\mu\text{H}$$

$$C = 0,159\mu\text{F}$$

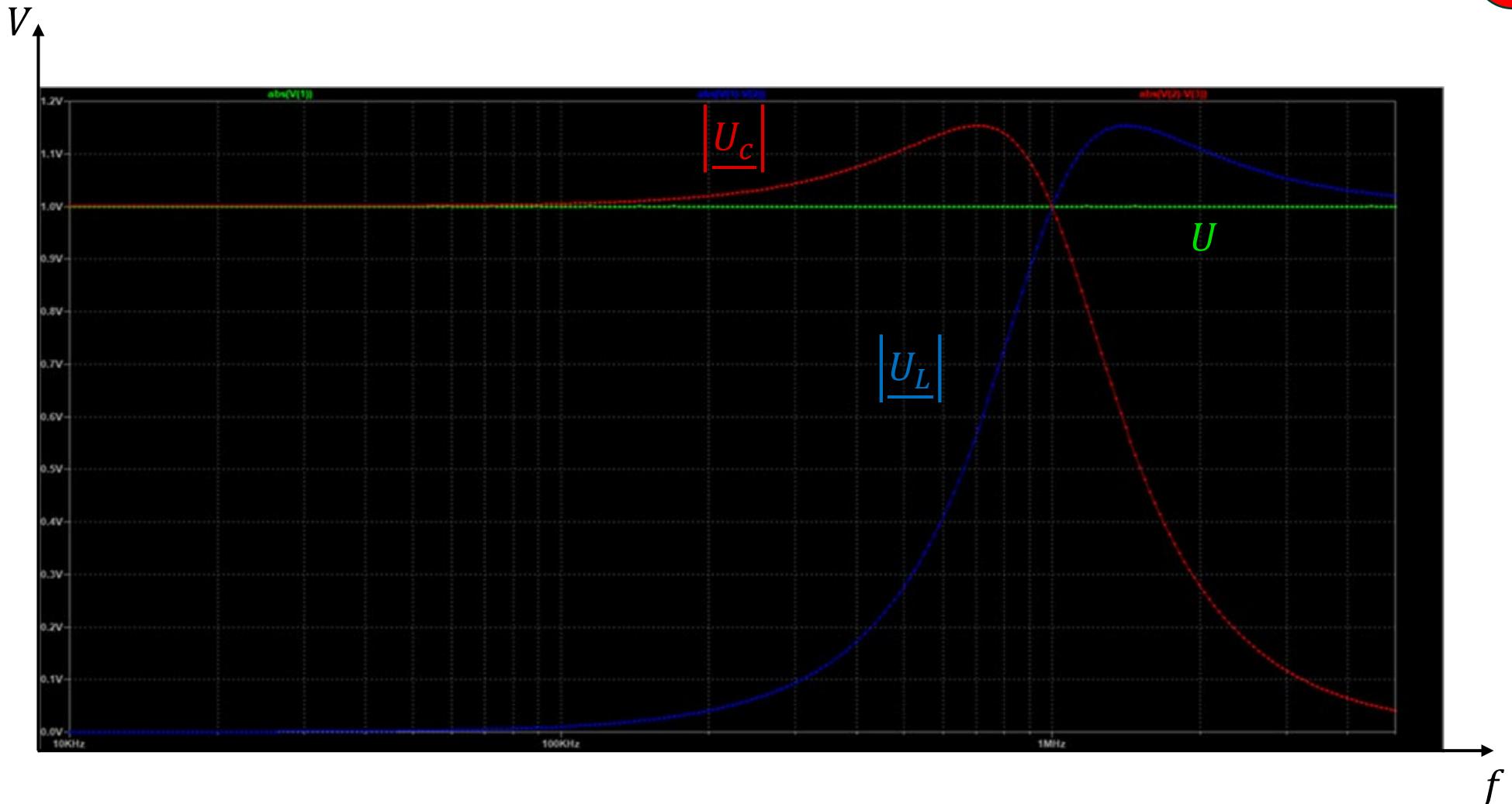
$$R = 1\Omega$$

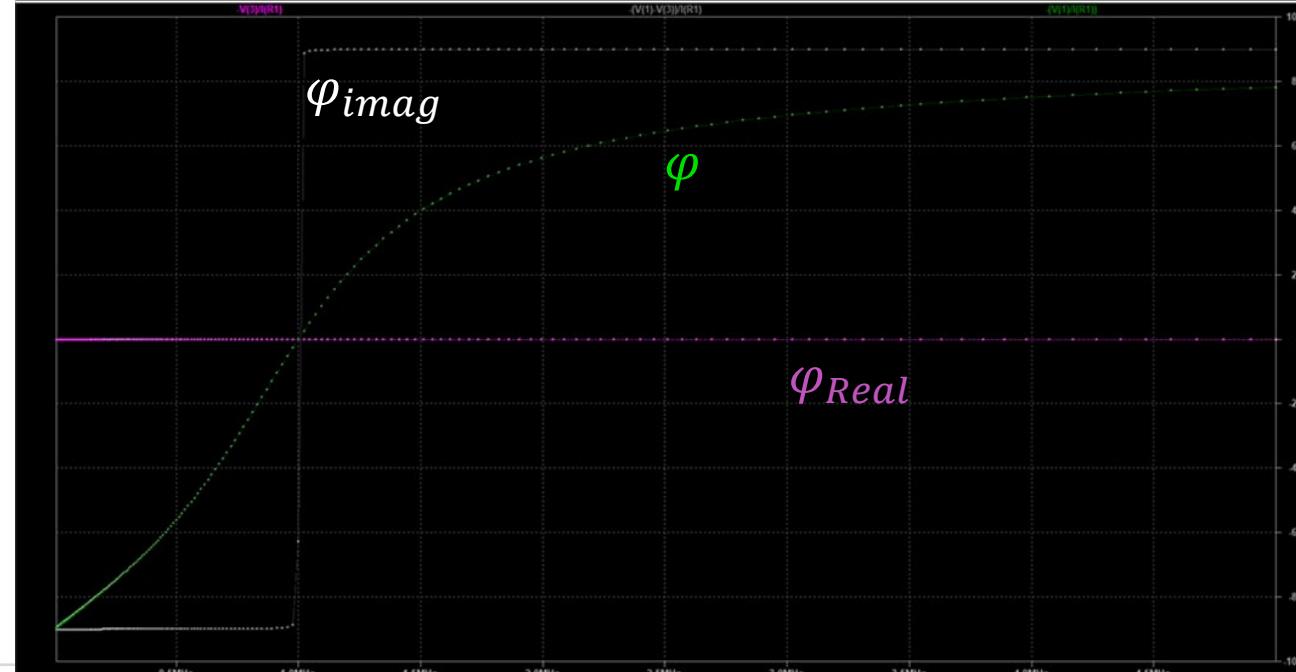
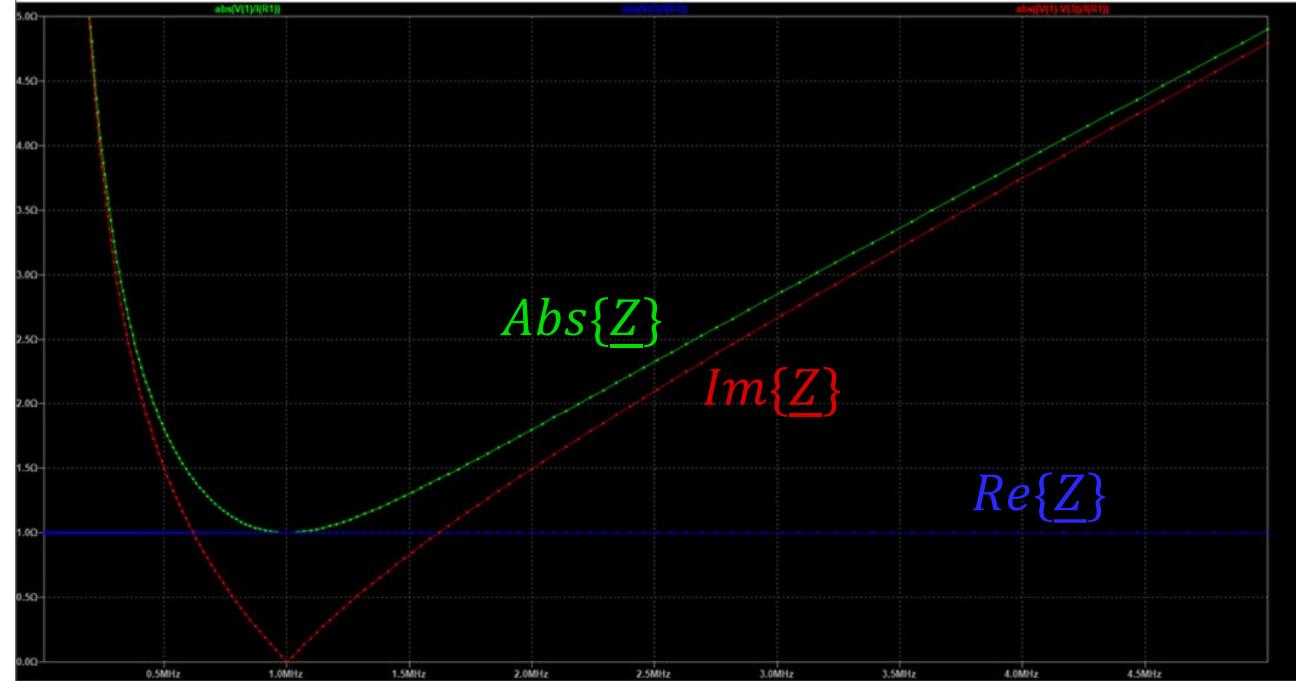
```
.ac dec 100 0.01e6 5e6
.param L 0.159e-6
.param C 0.159e-6
.param R 1
```

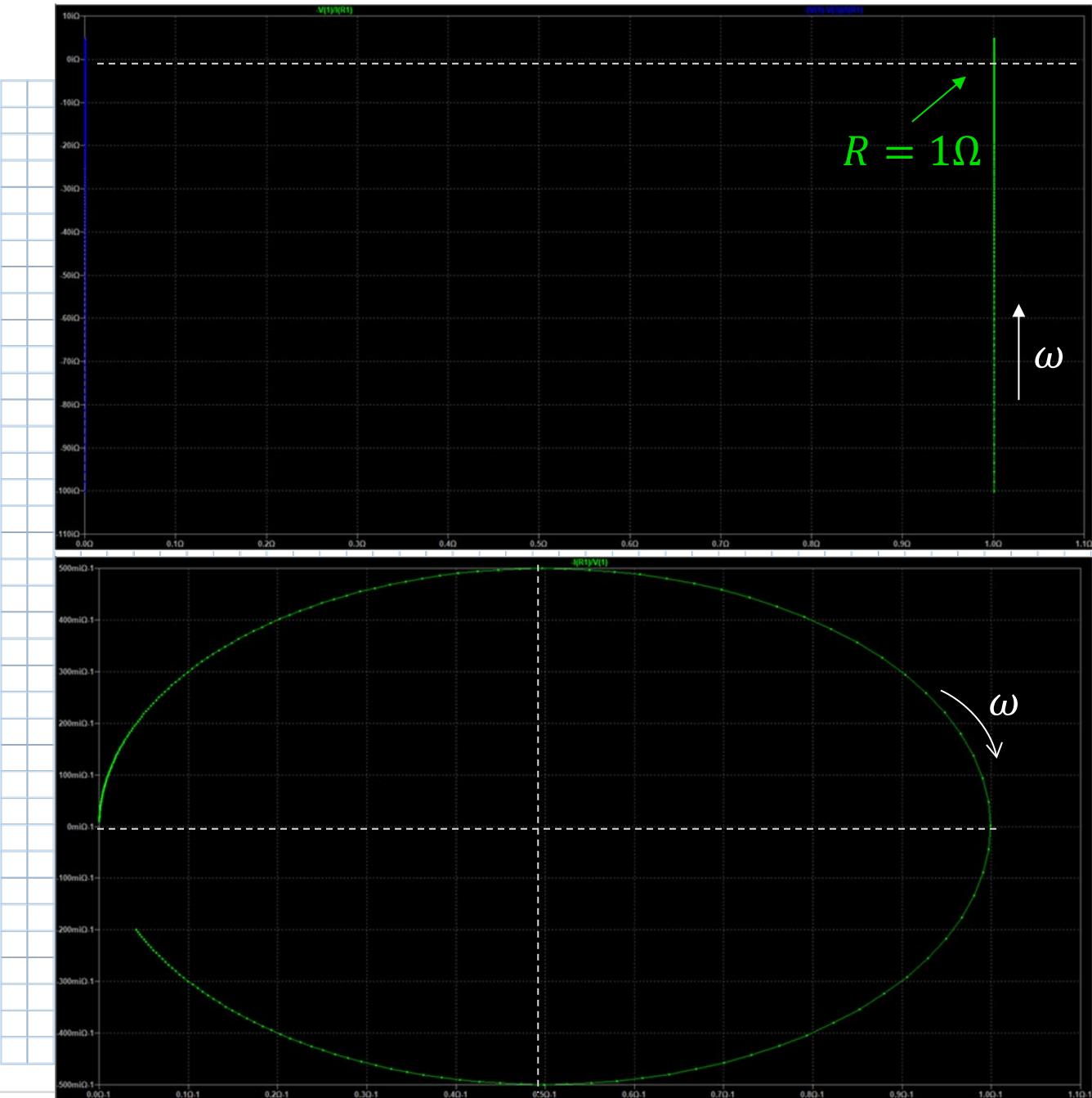


$$V_1 = 1\text{V} \cdot \sin(2\pi ft)$$

Spannungen



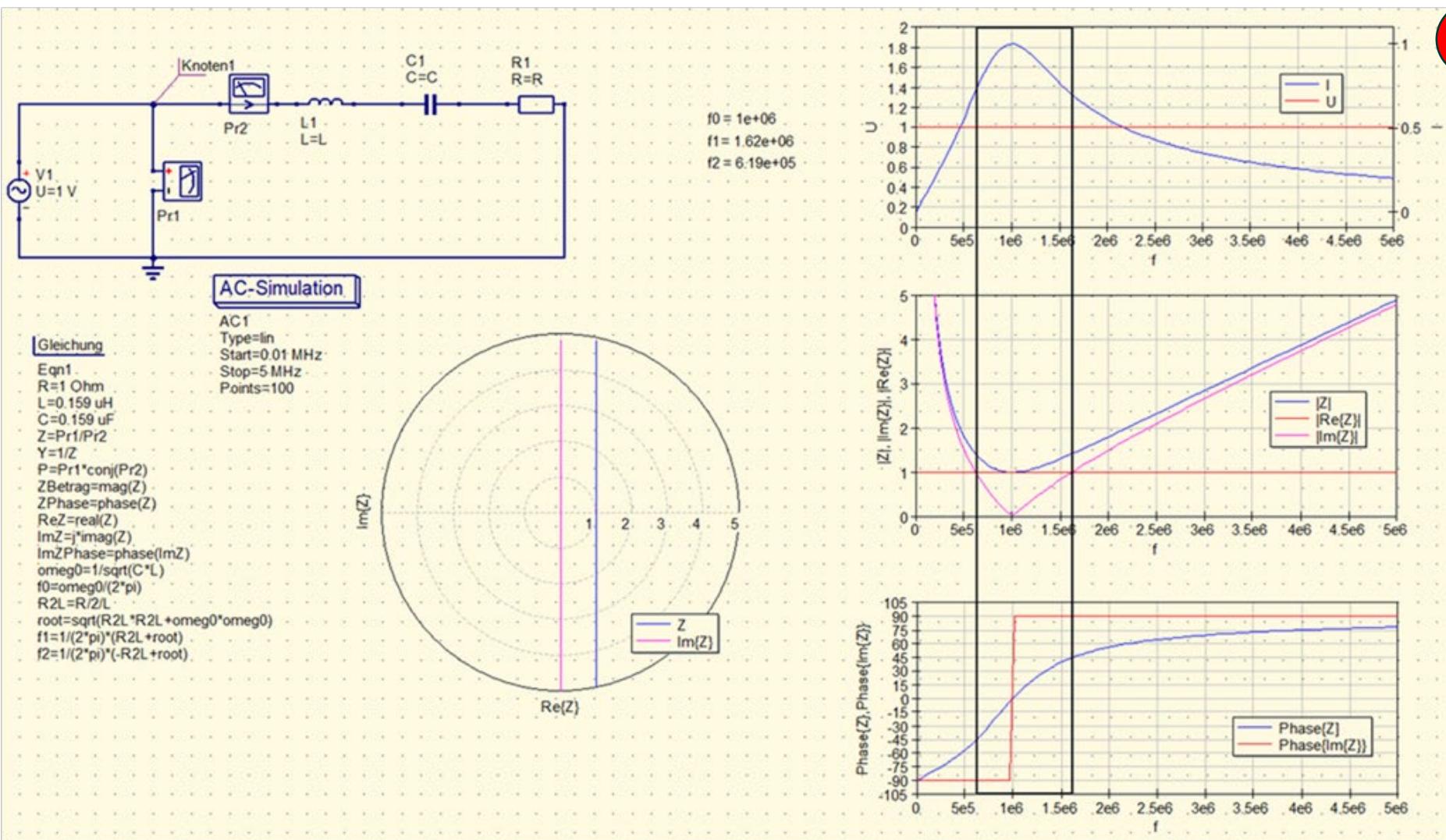




Ortskurven:

Impedanz

Admittanz



Programm: QucsStudio Ver 4.3.1
qucsstudio.de

Bemerkungen



Die Diagramme zur Beschreibung des Serienschwingkreises verlaufen im allgemeinen nicht symmetrisch um ω_0 .

Nehmen wir dies jedoch hier an: $\omega_0 = \frac{1}{\sqrt{LC}}$; $\omega_{1,2} \cong \omega_0 \pm \frac{R}{2L}$

Dann bedeutet dies: $\left(\frac{R}{2L}\right)^2 \ll \frac{1}{LC} \rightarrow 1 \ll \frac{1}{\left(\frac{R}{2L}\right)^2} \frac{1}{LC}$ mit $Q = \frac{X_0}{R} = \frac{\sqrt{\frac{L}{C}}}{R}$

≈ Güte ist groß!

mit

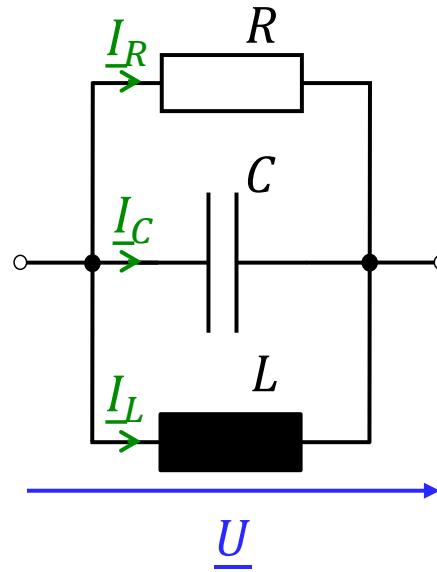
$$X_L \Big|_{\omega_0} = \omega_0 L = \sqrt{\frac{L}{C}}$$

und

$$X_C \Big|_{\omega_0} = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

→ $\text{Im}\{\underline{U}_{L_0}\} \cong \text{Im}\{\underline{U}_{C_0}\} \approx Q \cdot \underline{U}_R \rightarrow \text{Im}\{\underline{U}_{L_0}\} \cong \text{Im}\{\underline{U}_{C_0}\} \gg \underline{U}_R$

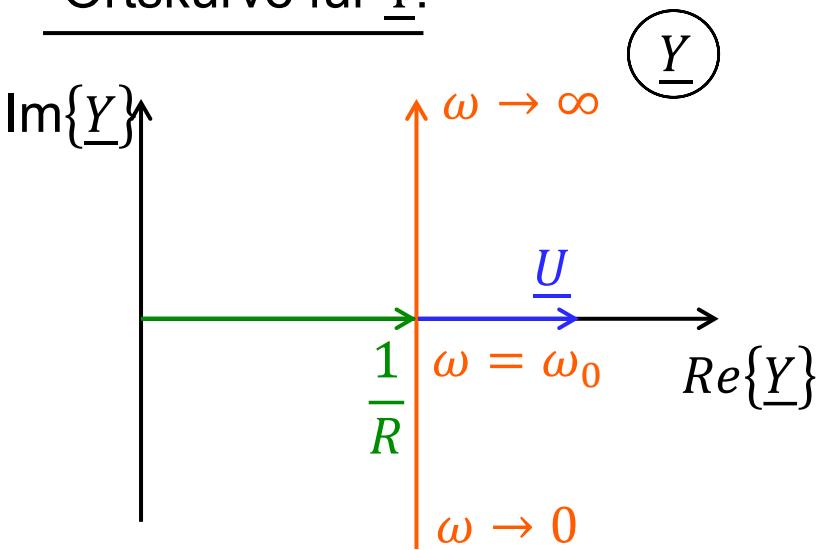
8.2 Parallelschwingkreis



$$\underline{Y} = \underbrace{\frac{1}{R}}_{\text{Realteil}} + j\omega C + \underbrace{\frac{1}{j\omega L}}_{\text{Imaginärteil}}$$

Realteil Imaginärteil

Ortskurve für \underline{Y} :

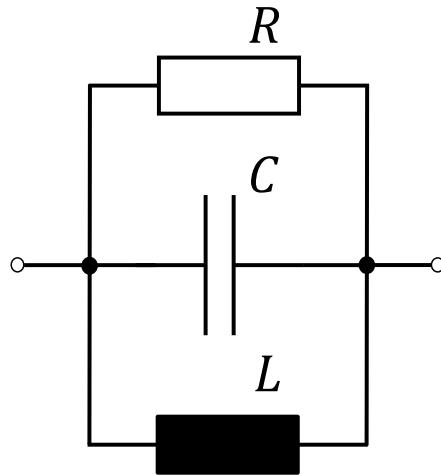
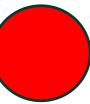


$$\underline{\omega \rightarrow 0}: \underline{Y} \sim \frac{1}{R} + j \frac{1}{\omega L} + j \phi C \xrightarrow{\omega \rightarrow 0} \infty$$

$$\underline{\omega = \omega_0}: \underline{Y}_0 = \frac{1}{R} \quad \text{mit } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\underline{\omega \rightarrow \infty}: \underline{Y} \sim \frac{1}{R} + j\omega C + \frac{j}{\omega L} \xrightarrow{\omega \rightarrow \infty} 0$$

Bei Resonanz $\omega = \omega_0$:



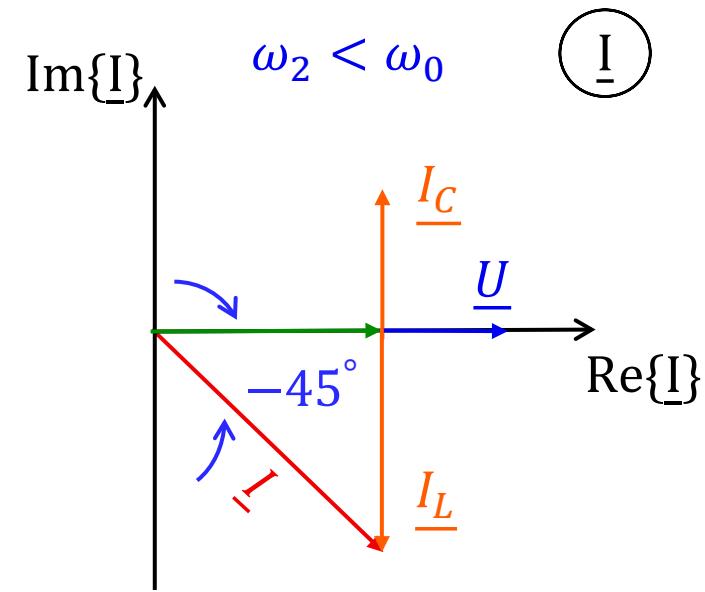
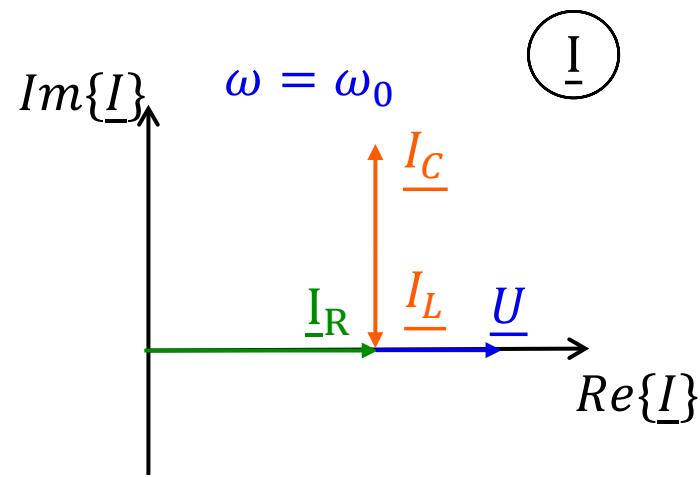
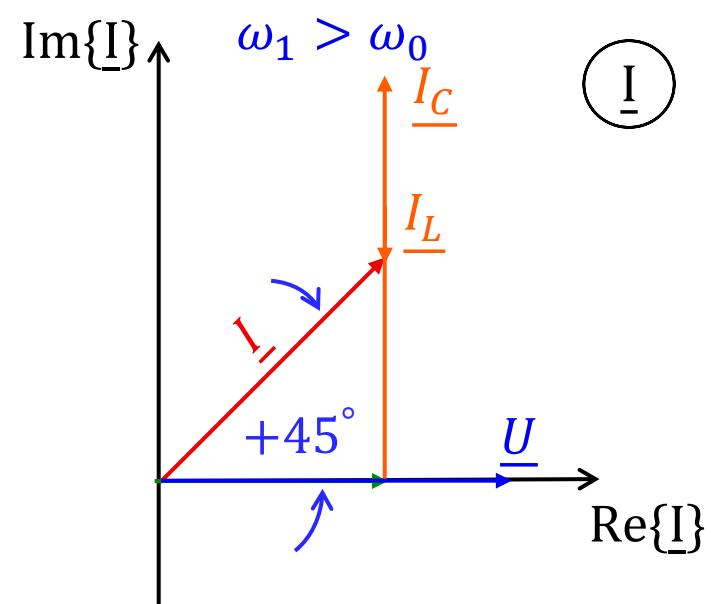
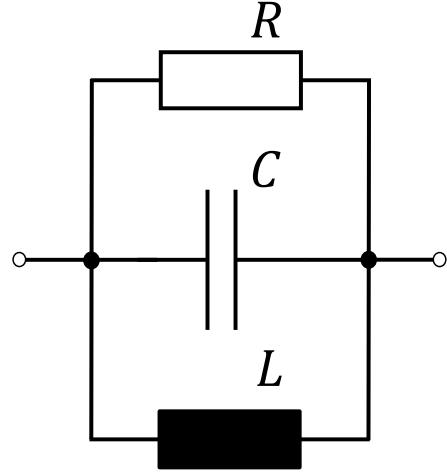
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\left. \begin{aligned} I_{R_0} &= \underline{\underline{U}} / R \\ I_{C_0} &= j\omega_0 C \cdot \underline{\underline{U}} \\ I_{L_0} &= \frac{\underline{\underline{U}}}{j\omega_0 L} = -j \frac{\underline{\underline{U}}}{\omega_0 L} \end{aligned} \right\} I_{C_0} = -I_{L_0}$$

Def.: Kennleitwert

$$B_0 = \omega_0 C = \frac{1}{\omega_0 L} \sim B_0 := \sqrt{\frac{C}{L}} = \frac{1}{X_0}$$

Zeigerdiagramm für \underline{I}





Frequenzbandbreite der Parallelschaltung:

$$\frac{1}{R} = \omega_1 C - \frac{1}{\omega_1 L}$$

~

$$\omega_1 = \frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} + \frac{1}{LC}}$$

$$b_w := \omega_1 - \omega_2 = \underline{\underline{\frac{1}{RC}}}$$

$$\omega_2 = -\frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} + \frac{1}{LC}}$$

■ Dämpfung

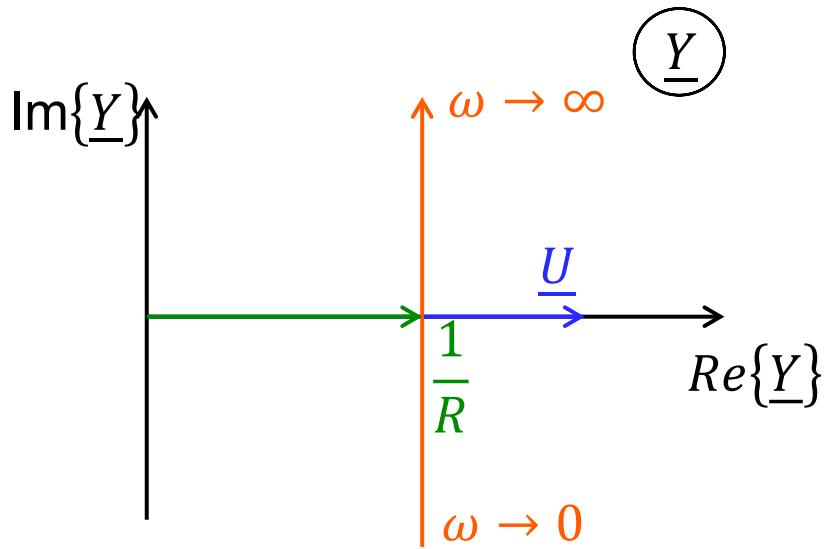
$$d_p := \frac{\omega_1 - \omega_2}{\omega_0} = \frac{1}{RC} \cdot \sqrt{LC} = \underline{\underline{\sqrt{\frac{L}{C} \cdot \frac{1}{R}}}}$$

■ Güte

$$Q_p := \frac{1}{d_p} = R \cdot \sqrt{\frac{C}{L}} = \underline{\underline{R \cdot B_0}} \quad \sim \quad R \uparrow \text{ und } C \uparrow \sim Q_p \uparrow$$

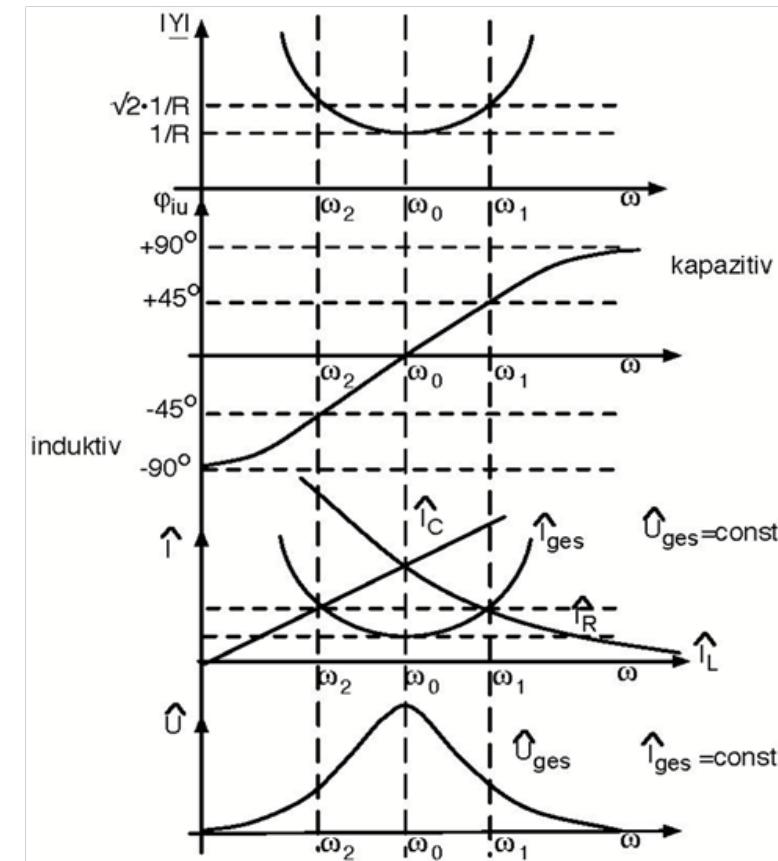
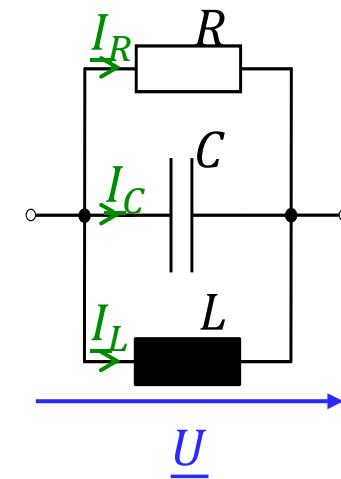
Diagramme für Parallelschwingkreis

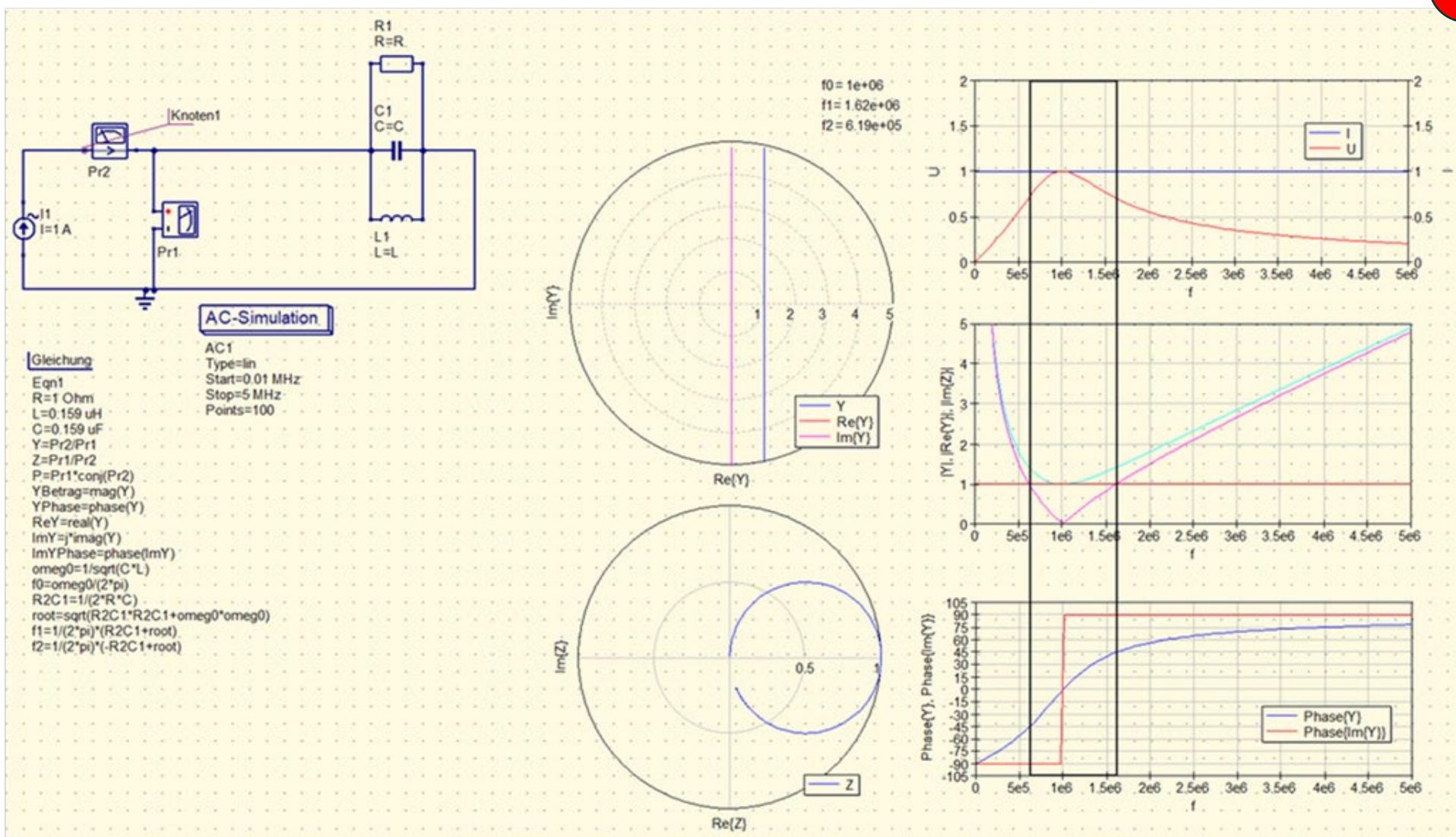
$$\underline{Y} = \frac{1}{R} + j \left(\omega C - \frac{1}{\omega L} \right)$$



Für $Q \gg 1$:

$$I_{C_0} = I_{L_0} = Q \cdot I_R \quad \curvearrowright \quad I_{C_0} = I_{L_0} \gg I_R$$





8.3 Verallgemeinerte Definition der Güte



- Serienschwingkreis:

$$Q_R = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$$

Kennwiderstand

- Parallelschwingkreis:

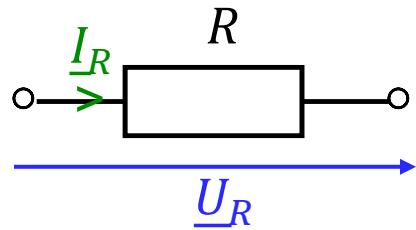
$$Q_p = R \cdot \sqrt{\frac{C}{L}}$$

Kennleitwert

- Für jedes schwingungsfähige System gilt:

$$Q = 2\pi \cdot \frac{\text{gesamte gespeicherte Energie}}{\text{Energieverlust pro Periode}}$$

Leistung und Energie: Widerstand R



$$i_R = \hat{I}_R \cdot \sin(\omega t) = \sqrt{2} \cdot |I_R| \cdot \sin(\omega t)$$

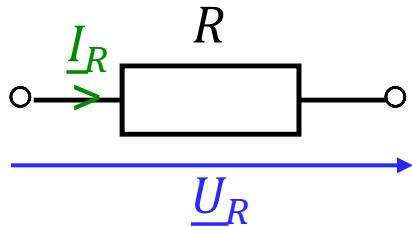
$$u_R = R \cdot i_R = R \cdot \hat{I}_R \cdot \sin(\omega t)$$

$$P_R = u_R \cdot i_R = R \cdot \hat{I}_R^2 \cdot \sin^2(\omega t)$$

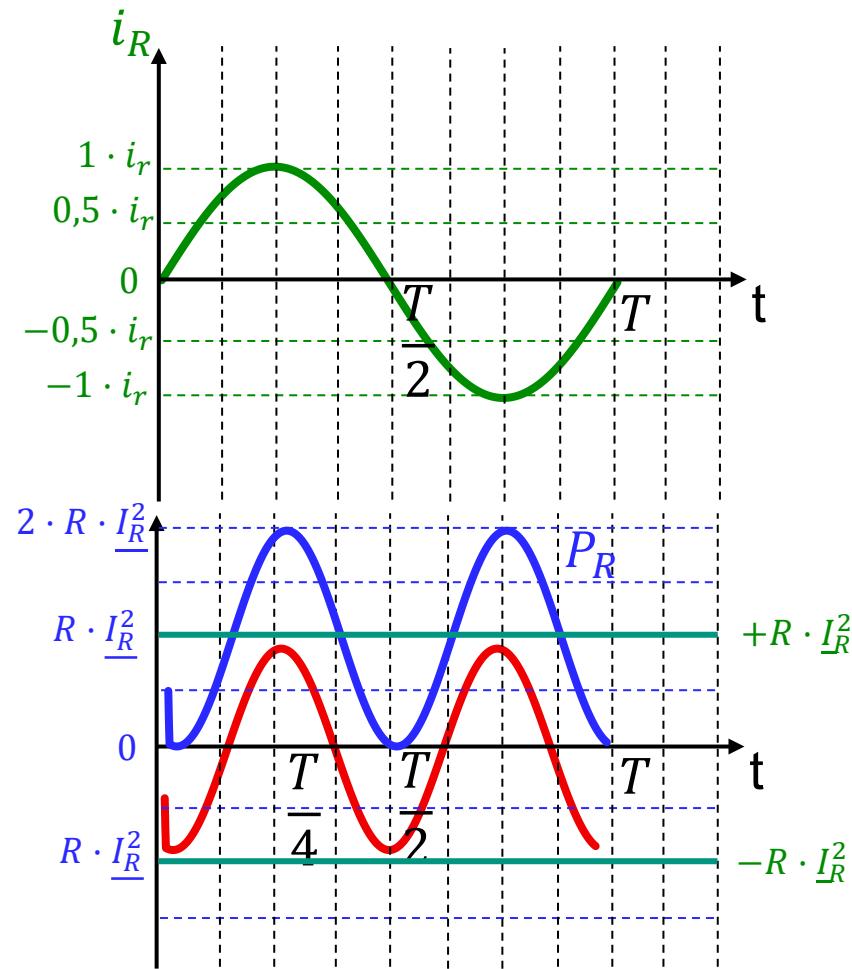
$$\begin{aligned} P_R &= R \cdot \hat{I}_R^2 \cdot \frac{1}{2} \cdot (1 - \cos(2\omega t)) \\ &= R \cdot (\sqrt{2} \cdot |I_R|)^2 \cdot \frac{1}{2} \cdot (1 - \cos(2\omega t)) \end{aligned}$$

$$P_R = R \cdot |I_R|^2 \cdot (1 - \cos(2\omega t))$$

Leistung und Energie: Widerstand R



$$P_R = u_R \cdot i_R = R \cdot I_R^2 \cdot (1 - \cos(2\omega t))$$

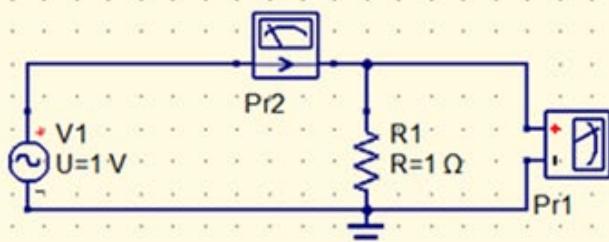


Einschub
 $\cos(2\omega t) \doteq 0$

$$\approx 2\omega t = \frac{\pi}{2} \pm h \cdot \pi$$

$$\omega = \frac{2\pi}{T}$$

$$\approx t = \left(\frac{1}{8} \pm \frac{h}{4} \right) T$$

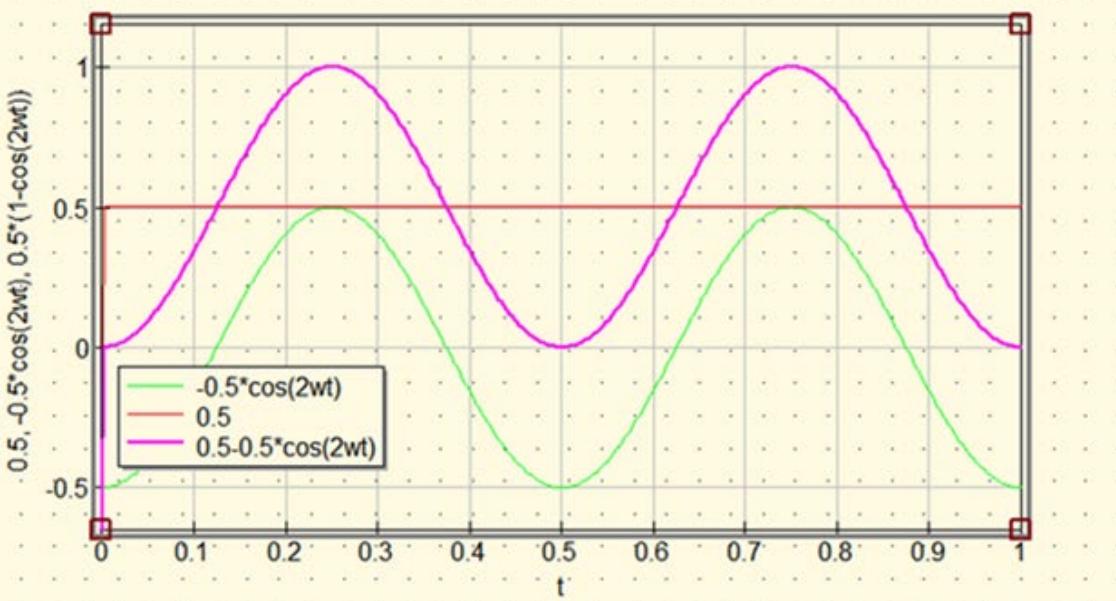
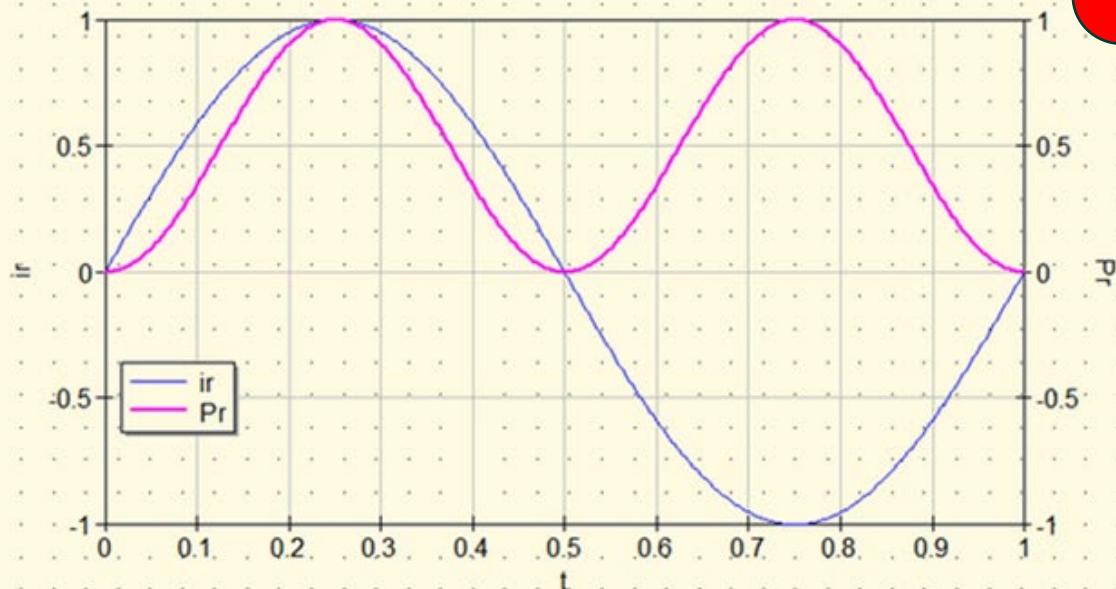


Transienten Simulation

TR1
Stop=1000 ms
Points=501

Gleichung:

- Eqn1
- $Pr = Pr1.dVt * Pr2.lt$
- $t = \text{time}$
- $T = 1$
- $w = 2\pi/T$
- $m\cos(2wt) = -0.5\cos(2w*t)$
- $c05 = 0.5*t/t$
- $PrRechnung = c05 + m\cos(2wt)$



Mittlere Energie (Verlust) über eine Schwingungsperiode



$$P_R = R \cdot \hat{I}_R^2 \cdot \frac{1}{2} (1 - \cos(2\omega t))$$

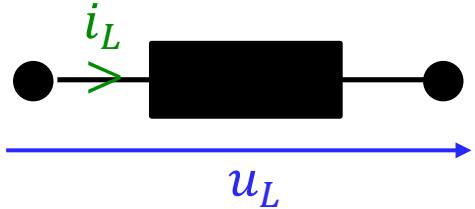
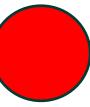
$$W_R = \int_0^T P_R dt = R \cdot \hat{I}_R^2 \cdot \frac{1}{2} \cdot T - \frac{1}{2} R \cdot \hat{I}_R^2 \cdot \left[\frac{1}{2\omega} \cdot \sin(2\omega t) \right]_0^T$$

$$\leadsto W_R = \frac{1}{2} \cdot R \cdot \hat{I}_R^2 \cdot T \stackrel{\triangleq}{=} \underline{\underline{\frac{1}{2} \hat{U}_R^2 \cdot \frac{1}{R} \cdot \frac{2\pi}{\omega}}}$$

- ~ Im Widerstand wird über eine Schwingungsperiode Energie verbraucht!

$$W_R = P_R \cdot T$$

Leistung, Energie bei der Induktivität (Spule)



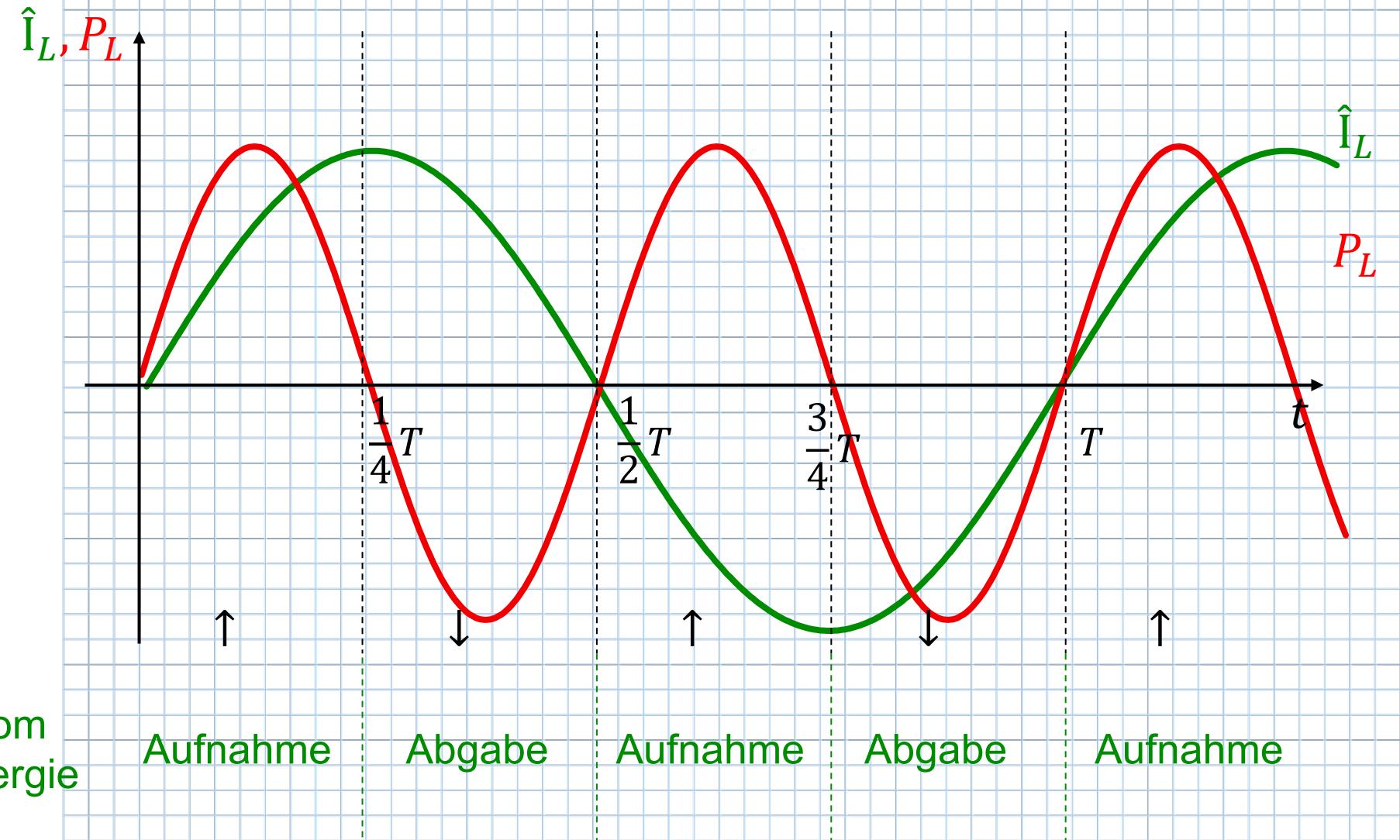
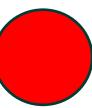
$$u_L = L \cdot \frac{di_L}{dt} \quad i_L = \hat{I}_L \cdot \sin(\omega t)$$
$$\approx u_L = L \cdot \omega \hat{I}_L \cos(\omega t)$$

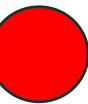
$$\rightarrow P_L = u_L \cdot i_L = \omega \cdot L \cdot \hat{I}_L^2 \cdot \cos(\omega t) \cdot \sin(\omega t)$$
$$\qquad\qquad\qquad \underbrace{\qquad\qquad\qquad}_{\frac{1}{2} \sin(2\omega t)}$$

$$P_L = \underline{\underline{\omega L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \sin(2\omega t)}}$$

\approx über eine Schwingungsperiode gemittelt, ist die gesamte von der Spule aufgenommene Leistung $P_L = 0$!

$$P_L = \frac{1}{2} \omega L \hat{I}^2 \cdot \sin(2\omega t)$$





Mittlere Energie (**gespeicherte**) einer idealen verlustlosen Spule über eine Schwingungsperiode

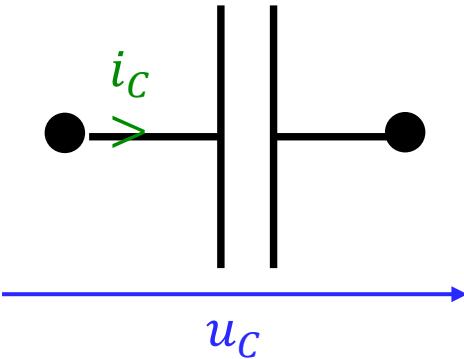
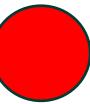
$$P_L = \omega L \cdot i_L = \omega L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \sin(2\omega t)$$

$$\begin{aligned} W_L &= \int_0^T P_L dt = \omega L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \int_0^T \sin(2\omega t) dt \\ &= \cancel{\omega} \cdot L \cdot \hat{I}_L^2 \cdot \frac{1}{2} \cdot \left[-\frac{1}{2\cancel{\omega}} \cdot \cos(2\omega t) \right]_0^T \\ &= \frac{1}{2} \omega L \cdot \hat{I}_L^2 \cdot \left(-\frac{1}{2} \omega \right) \cdot \left[\cos\left(2\frac{2\pi}{T} T\right) - \cos(0) \right]_0^T \end{aligned}$$

$$= \underline{\underline{0}}! \quad \text{Über eine Periode } (T) \quad \simeq \quad W_L = 0$$

Aber! Integration $\int_0^T dt \simeq W_L = \frac{1}{2} L \hat{I}_L^2$!

Mittlere Leistung, Energie bei einem Kondensator C



$$u_C = \hat{U}_C \cdot \sin(\omega t)$$

$$i_C = C \cdot \frac{du_C}{dt} = \omega C \cdot \hat{U}_C \cdot \cos(\omega t)$$

■ $P_C = u_C \cdot i_C = \hat{U}_C \cdot \sin(\omega t) \cdot \omega \cdot C \cdot \hat{U}_C \cdot \cos(\omega t)$

$$\rightarrow P_C = \omega \cdot C \cdot \hat{U}_C^2 \cdot \frac{1}{2} \cdot \sin(2\omega t)$$

■ $W_C = \int_0^T P_C dt = \omega \cdot C \cdot \hat{U}_C \cdot \frac{1}{2} \cdot \int_0^T \sin(2\omega t) dt$

$$= \cancel{\omega} C \cdot \hat{U}_C \cdot \frac{1}{2} \cdot \frac{1}{2\cancel{\omega}} \cdot [-\cos(2\omega t)]_0^T$$

$$= C \cdot \hat{U}_C \cdot \frac{1}{2}$$

Zusammenfassung

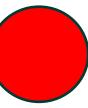


$$Q = 2\pi \cdot \frac{\text{gesamte gespeicherte Energie}}{\text{Verlust über eine Schwingungsperiode}}$$

$$\text{Verlust: } U_R = \frac{1}{2} R \cdot \hat{I}_R^2 \cdot T \quad T = \frac{1}{f} [\text{s}]$$

$$\text{Gesamte gespeicherte Energie: } W_C = \frac{1}{2} C \cdot \hat{U}_C^2$$

$$W_L = \frac{1}{2} L \cdot \hat{I}_L^2$$



■ Für Serienkreis gilt: $\hat{I}_L = \hat{I}_R =: \hat{I}$

$$\begin{aligned} \text{■ } Q_R &= \cancel{2\pi} \cdot \frac{\cancel{\frac{1}{2}} L \hat{I}^2}{\cancel{\frac{1}{2}} R \cdot \cancel{\hat{I}^2} \cdot \cancel{\frac{2\pi}{\omega_0}}} = \\ &= \frac{L}{R} \cdot \omega_0 = \frac{L}{R} \cdot \frac{1}{\sqrt{LC}} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}} = \frac{X_0}{R} ! \end{aligned}$$

■ Für Parallelkreis gilt: $\hat{U}_C = \hat{U}_R =: \hat{U}$

$$\text{■ } Q_R = \frac{\cancel{\frac{1}{2}} C \hat{U}^2 \cdot \cancel{2\pi}}{\cancel{\frac{1}{2}} \hat{U}^2 \cdot \cancel{\frac{2\pi}{\omega_0}}} = R \cdot C \cdot \frac{1}{\sqrt{LC}} = R \cdot \sqrt{\frac{C}{L}} = \frac{B_0}{\frac{1}{R}} !$$

8.4 Verallgemeinerte Resonanzkurven



Definition der Transformation

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

~

$$\vartheta := \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

~

~ Frequenzzentriert um ω_0

ω	ϑ
0	- ∞
ω_0	0
$+\infty$	- ∞

Für Serienresonanzkreis

$$\begin{aligned}\underline{Z} &= R + j \left(\omega L - \frac{1}{\omega C} \right) = R + j \omega_0 L \left(\frac{\omega L}{\omega_0 L} - \frac{1}{\omega_0 L \omega C} \right) \\ &= R + j \omega_0 L \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = R + j \omega_0 L \cdot \vartheta \\ &= R \cdot \left(1 + j \omega_0 \frac{L}{R} \cdot \vartheta \right) = R \cdot (1 + j Q_R \cdot \vartheta)\end{aligned}$$

Achtung! Fehler im Skript auf S.145 Gl. 8.39

