Differential Equation Python Quiz 4

1) Consider the system of first order differential equations $\mathbf{x}' = \begin{pmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix} \mathbf{x}$

From your understanding of the Euler function, write a script to approximate the solution to this system as follows

$$x_n^{(1)} = x_{n-1}^{(1)} + h(-3x_{n-1}^{(1)} + \sqrt{2}x_{n-1}^{(2)})$$
$$x_n^{(2)} = x_{n-1}^{(2)} + h(\sqrt{2}x_{n-1}^{(1)} - 2x_{n-1}^{(2)})$$
$$t_n = t_{n-1} + h$$

where $\mathbf{x} = \begin{pmatrix} x^{(1)} \\ x^{(2)} \end{pmatrix}$ and $x_n^{(1)}$ is the value of $x^{(1)}$ at time step n and t_n is time step n

Take the initial values at $t = 0$ as $x_0^{(1)} = 2$, $x_0^{(2)} = 2\sqrt{2}$

Choose the step size $h = 0.01$

(a) Plot the trajectory of $x^{(1)}$ (y-axis) vs time (x-axis) and $x^{(2)}$(y-axis) v/s time (x-axis) on the same figure. (Use $0 \leq t \leq 5$ for time values with increment of h in steps)

(b) Plot the trajectory for the following exact solution to this problem, on the **same figure**

$$\tilde{x}^{(1)}(t) = 2e^{-t}$$
$$\tilde{x}^{(2)}(t) = 2\sqrt{2}e^{-t}$$

Use the same time vector you create in part (a)

(c) Calculate the distance between the 2 trajectories of part (a) and (b) as follows

$$d = \sum_{i=1}^{i=n} \sqrt{(\tilde{x}_i^{(1)} - x_i^{(1)})^2 + (\tilde{x}_i^{(2)} - x_i^{(2)})^2}$$

where $\tilde{x}_i^{(1)}$ is the value of the exact solution $\tilde{x}^{(1)}$ at time step i and $x_i^{(1)}$ is the value of the approximation $x^{(1)}$ (obtained from part (a)) at time step i and n is the number of time steps in your time vector.

2) Suppose that the Spring model can be reduced to the following equation:

$$mu''(t) + ru'(t) + ku(t) = F(t)$$

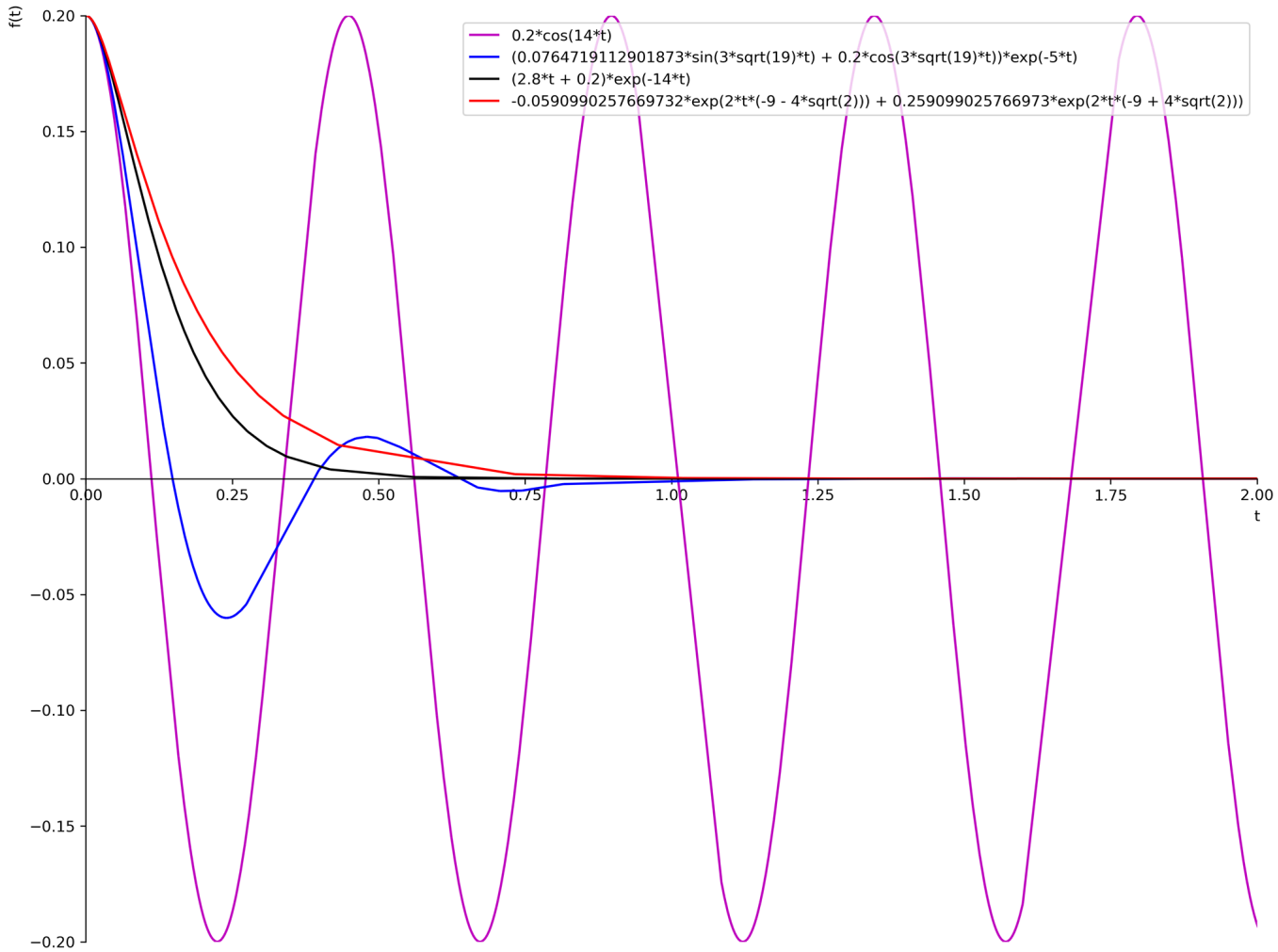
with initial value

$$u(0) = u, u'(0) = v_0$$

Here we assume that $F(t) = 0$

a) Write a function Q4 that will take m, r, k, u_0, v_0 as parameters and return symbolic solution and category of that equation i.e. undamped, critically damped and so on.

b) Plot figure like the one you see down below.



Upload your Python script as "StudentID_python_quiz4.py" to new E3 before 12:00 AM.
No late submissions will be possible.