Terms $B_{\bar{N}}(2N+1)$ through $B_{\bar{N}}(2N+9)$ when $N \equiv 6 \pmod 7$

When $N \equiv 6 \pmod 7$ and $N \geq 72$, a pattern with 7 interleaved linear sequences lasts from index $N+67$ through $2N$. If $N \geq 118$, there are 9 terms after this pattern ends. Below are calculations of all of these terms along with the necessary lower bound on N for each calculation to be valid. Record large N bounds exceeding 72 are presented in bold.

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+1)} &= B_{\bar{N}}(2N+1-B_{\bar{N}}(2N)) + B_{\bar{N}}(2N+1-B_{\bar{N}}(2N-1)) + B_{\bar{N}}(2N+1-B_{\bar{N}}(2N-2)) \\
&= B_{\bar{N}}(2N+1-(N-2)) + B_{\bar{N}}\left(2N+1-\left(\frac{15N}{7}-\frac{55}{7}\right)\right) + B_{\bar{N}}\left(2N+1-\left(\frac{16N}{7}+\frac{303}{7}\right)\right) \\
&= B_{\bar{N}}(N+3) + B_{\bar{N}}\left(-\frac{N}{7}+\frac{62}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{296}{7}\right) = (N+2)+0+0 = \mathbf{N+2} \\
&(N \geq 72)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+2)} &= B_{\bar{N}}(2N+2-B_{\bar{N}}(2N+1)) + B_{\bar{N}}(2N+2-B_{\bar{N}}(2N)) + B_{\bar{N}}(2N+2-B_{\bar{N}}(2N-1)) \\
&= B_{\bar{N}}(2N+2-(N+2)) + B_{\bar{N}}(2N+2-(N-2)) + B_{\bar{N}}\left(2N+2-\left(\frac{15N}{7}-\frac{55}{7}\right)\right) \\
&= B_{\bar{N}}(N) + B_{\bar{N}}(N+4) + B_{\bar{N}}\left(-\frac{N}{7}+\frac{69}{7}\right) = N+(N+3)+0 = \mathbf{2N+3} \\
&(N \geq 71)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+3)} &= B_{\bar{N}}(2N+3-B_{\bar{N}}(2N+2)) + B_{\bar{N}}(2N+3-B_{\bar{N}}(2N+1)) + B_{\bar{N}}(2N+3-B_{\bar{N}}(2N)) \\
&= B_{\bar{N}}(2N+3-(2N+3)) + B_{\bar{N}}(2N+3-(N+2)) + B_{\bar{N}}(2N+3-(N-2)) \\
&= B_{\bar{N}}(0) + B_{\bar{N}}(N+1) + B_{\bar{N}}(N+5) = 0+6+9 = \mathbf{15} \\
&(\mathbf{N \geq 75})
\end{aligned}$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+4)} &= B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+3)) + B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+2)) + B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+1)) \\
&= B_{\bar{N}}(2N+4-15) + B_{\bar{N}}(2N+4-(2N+3)) + B_{\bar{N}}(2N+4-(N+2)) \\
&= B_{\bar{N}}(2N-11) + B_{\bar{N}}(1) + B_{\bar{N}}(N+2) = (2N-9)+1+(N+1) = \mathbf{3N-7} \\
&(\mathbf{N \geq 81})
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+5)} &= B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+4)) + B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+3)) + B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+2)) \\
&= B_{\bar{N}}(2N+5-(3N-7)) + B_{\bar{N}}(2N+5-15) + B_{\bar{N}}(2N+5-(2N+3)) \\
&= B_{\bar{N}}(-N+12) + B_{\bar{N}}(2N-10) + B_{\bar{N}}(2) = 0+7+2 = \mathbf{9} \\
&(N \geq 77)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+6)} &= B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+5)) + B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+4)) + B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+3)) \\
&= B_{\bar{N}}(2N+6-9) + B_{\bar{N}}(2N+6-(3N-7)) + B_{\bar{N}}(2N+6-15) \\
&= B_{\bar{N}}(2N-3) + B_{\bar{N}}(-N+13) + B_{\bar{N}}(2N-9) = 7+0+\left(\frac{16N}{7}+\frac{289}{7}\right) = \mathbf{\frac{16N}{7}+\frac{338}{7}} \\
&(N \geq 76)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+7)} &= B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+6)) + B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+5)) + B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+4)) \\
&= B_{\bar{N}}\left(2N+7-\left(\frac{16N}{7}+\frac{338}{7}\right)\right) + B_{\bar{N}}(2N+7-9) + B_{\bar{N}}(2N+7-(3N-7)) \\
&= B_{\bar{N}}\left(-\frac{2N}{7}-\frac{289}{7}\right) + B_{\bar{N}}(2N-2) + B_{\bar{N}}(-N+14) = 0+\left(\frac{16N}{7}+\frac{303}{7}\right)+0 = \mathbf{\frac{16N}{7}+\frac{303}{7}} \\
&(N \geq 77)
\end{aligned}
$$

$$\begin{aligned}\mathbf{B_{\bar{N}}(2N+8)} &= B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+7)) + B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+6)) + B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+5)) \\ &= B_{\bar{N}}\left(2N+8-\left(\frac{16N}{7}+\frac{303}{7}\right)\right) + B_{\bar{N}}\left(2N+8-\left(\frac{16N}{7}+\frac{338}{7}\right)\right) + B_{\bar{N}}(2N+8-9) \\ &= B_{\bar{N}}\left(-\frac{2N}{7}-\frac{247}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{282}{7}\right) + B_{\bar{N}}(2N-1) = 0+0+\left(\frac{15N}{7}-\frac{55}{7}\right) = \mathbf{\frac{15N}{7}-\frac{55}{7}} \\ &(N \geq 76)\end{aligned}$$

$$\begin{aligned}\mathbf{B_{\bar{N}}(2N+9)} &= B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+8)) + B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+7)) + B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+6)) \\ &= B_{\bar{N}}\left(2N+9-\left(\frac{15N}{7}-\frac{55}{7}\right)\right) + B_{\bar{N}}\left(2N+9-\left(\frac{16N}{7}+\frac{303}{7}\right)\right) + B_{\bar{N}}\left(2N+9-\left(\frac{16N}{7}+\frac{338}{7}\right)\right) \\ &= B_{\bar{N}}\left(-\frac{N}{7}+\frac{118}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{240}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{275}{7}\right) = 0+0+0 = \mathbf{0} \\ &(\mathbf{N \geq 118})\end{aligned}$$