## Terms $B_{\bar{N}}(2N+3)$ through $B_{\bar{N}}(2N+20)$ when $N \equiv 4 \pmod 7$

When $N \equiv 4 \pmod 7$ and $N \geq 72$, a pattern with 7 interleaved linear sequences lasts from index $N+67$ through $2N+2$. If $N \geq 200$, there are 18 terms after this pattern ends. Below are calculations of all of these terms along with the necessary lower bound on $N$ for each calculation to be valid. Record large $N$ bounds exceeding 72 are noted with asterisks.

$$\begin{aligned} B_{\bar{N}}(2N+3) &= B_{\bar{N}}(2N+3-B_{\bar{N}}(2N+2)) + B_{\bar{N}}(2N+3-B_{\bar{N}}(2N+1)) + B_{\bar{N}}(2N+3-B_{\bar{N}}(2N)) \\ &= B_{\bar{N}}(2N+3-(N-2)) + B_{\bar{N}}\left(2N+3-\left(\frac{15N}{7}-\frac{53}{7}\right)\right) + B_{\bar{N}}\left(2N+3-\left(\frac{16N}{7}+\frac{307}{7}\right)\right) \\ &= B_{\bar{N}}(N+5) + B_{\bar{N}}\left(-\frac{N}{7}+\frac{74}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{286}{7}\right) = 9+0+0 = 9 \\ &(N \geq 74)* \end{aligned}$$

$$\begin{aligned} B_{\bar{N}}(2N+4) &= B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+3)) + B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+2)) + B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+1)) \\ &= B_{\bar{N}}(2N+4-9) + B_{\bar{N}}(2N+4-(N-2)) + B_{\bar{N}}\left(2N+4-\left(\frac{15N}{7}-\frac{53}{7}\right)\right) \\ &= B_{\bar{N}}(2N-5) + B_{\bar{N}}(N+6) + B_{\bar{N}}\left(-\frac{N}{7}+\frac{81}{7}\right) = (N-2)+(N+4)+0 = 2N+2 \\ &(N \geq 81)* \end{aligned}$$

$$\begin{aligned} B_{\bar{N}}(2N+5) &= B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+4)) + B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+3)) + B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+2)) \\ &= B_{\bar{N}}(2N+5-(2N+2)) + B_{\bar{N}}(2N+5-9) + B_{\bar{N}}(2N+5-(N-2)) \\ &= B_{\bar{N}}(3) + B_{\bar{N}}(2N-4) + B_{\bar{N}}(N+7) = 3+(N-2)+(N+5) = 2N+6 \\ &(N \geq 74) \end{aligned}$$

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\begin{aligned}
B_{\bar{N}}(2N+6) &= B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+5)) + B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+4)) + B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+3)) \\
&= B_{\bar{N}}(2N+6-(2N+6)) + B_{\bar{N}}(2N+6-(2N+2)) + B_{\bar{N}}(2N+6-9) \\
&= B_{\bar{N}}(0) + B_{\bar{N}}(4) + B_{\bar{N}}(2N-3) = 0+4+(2N-2) = 2N+2 \\
&(N \geq 73)
\end{aligned}
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\begin{aligned}
B_{\bar{N}}(2N+7) &= B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+6)) + B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+5)) + B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+4)) \\
&= B_{\bar{N}}(2N+7-(2N+2)) + B_{\bar{N}}(2N+7-(2N+6)) + B_{\bar{N}}(2N+7-(2N+2)) \\
&= B_{\bar{N}}(5) + B_{\bar{N}}(1) + B_{\bar{N}}(5) = 5+1+5 = 11 \\
&(N \geq 77)
\end{aligned}
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\begin{aligned}
B_{\bar{N}}(2N+8) &= B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+7)) + B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+6)) + B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+5)) \\
&= B_{\bar{N}}(2N+8-11) + B_{\bar{N}}(2N+8-(2N+2)) + B_{\bar{N}}(2N+8-(2N+6)) \\
&= B_{\bar{N}}(2N-3) + B_{\bar{N}}(6) + B_{\bar{N}}(2) = (2N-2)+6+2 = 2N+6 \\
&(N \geq 76)
\end{aligned}
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\begin{aligned}
B_{\bar{N}}(2N+9) &= B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+8)) + B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+7)) + B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+6)) \\
&= B_{\bar{N}}(2N+9-(2N+6)) + B_{\bar{N}}(2N+9-11) + B_{\bar{N}}(2N+9-(2N+2)) \\
&= B_{\bar{N}}(3) + B_{\bar{N}}(2N-2) + B_{\bar{N}}(7) = 3+2N+7 = 2N+10 \\
&(N \geq 105) *
\end{aligned}
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\begin{aligned}
B_{\bar{N}}(2N+10) &= B_{\bar{N}}(2N+10-B_{\bar{N}}(2N+9)) + B_{\bar{N}}(2N+10-B_{\bar{N}}(2N+8)) + B_{\bar{N}}(2N+10-B_{\bar{N}}(2N+7)) \\
&= B_{\bar{N}}(2N+10-(2N+10)) + B_{\bar{N}}(2N+10-(2N+6)) + B_{\bar{N}}(2N+10-11) \\
&= B_{\bar{N}}(0) + B_{\bar{N}}(4) + B_{\bar{N}}(2N-1) = 0+4+7 = 11 \\
&(N \geq 112) *
\end{aligned}
$$

$$\begin{aligned}
B_{\bar{N}}(2N+11) &= B_{\bar{N}}(2N+11-B_{\bar{N}}(2N+10)) + B_{\bar{N}}(2N+11-B_{\bar{N}}(2N+9)) + B_{\bar{N}}(2N+11-B_{\bar{N}}(2N+8)) \\
&= B_{\bar{N}}(2N+11-11) + B_{\bar{N}}(2N+11-(2N+10)) + B_{\bar{N}}(2N+11-(2N+6)) \\
&= B_{\bar{N}}(2N) + B_{\bar{N}}(1) + B_{\bar{N}}(5) = \left(\frac{16N}{7} + \frac{307}{7}\right) + 1 + 5 = \frac{16N}{7} + \frac{349}{7} \\
&(N \geq 136)*
\end{aligned}$$

$$\begin{aligned}
B_{\bar{N}}(2N+12) &= B_{\bar{N}}(2N+12-B_{\bar{N}}(2N+11)) + B_{\bar{N}}(2N+12-B_{\bar{N}}(2N+10)) + B_{\bar{N}}(2N+12-B_{\bar{N}}(2N+9)) \\
&= B_{\bar{N}}\left(2N+12-\left(\frac{16N}{7} + \frac{349}{7}\right)\right) + B_{\bar{N}}(2N+12-11) + B_{\bar{N}}(2N+12-(2N+10)) \\
&= B_{\bar{N}}\left(-\frac{2N}{7} - \frac{265}{7}\right) + B_{\bar{N}}(2N+1) + B_{\bar{N}}(2) = 0 + \left(\frac{15N}{7} - \frac{53}{7}\right) + 2 = \frac{15N}{7} - \frac{39}{7} \\
&(N \geq 143)*
\end{aligned}$$

$$\begin{aligned}
B_{\bar{N}}(2N+13) &= B_{\bar{N}}(2N+13-B_{\bar{N}}(2N+12)) + B_{\bar{N}}(2N+13-B_{\bar{N}}(2N+11)) + B_{\bar{N}}(2N+13-B_{\bar{N}}(2N+10)) \\
&= B_{\bar{N}}\left(2N+13-\left(\frac{15N}{7} - \frac{39}{7}\right)\right) + B_{\bar{N}}\left(2N+13-\left(\frac{16N}{7} + \frac{349}{7}\right)\right) + B_{\bar{N}}(2N+13-11) \\
&= B_{\bar{N}}\left(-\frac{N}{7} + \frac{130}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7} - \frac{258}{7}\right) + B_{\bar{N}}(2N+2) = 0 + 0 + (N-2) = N-2 \\
&(N \geq 150)*
\end{aligned}$$

$$\begin{aligned}
B_{\bar{N}}(2N+14) &= B_{\bar{N}}(2N+14-B_{\bar{N}}(2N+13)) + B_{\bar{N}}(2N+14-B_{\bar{N}}(2N+12)) + B_{\bar{N}}(2N+14-B_{\bar{N}}(2N+11)) \\
&= B_{\bar{N}}(2N+14-(N-2)) + B_{\bar{N}}\left(2N+14-\left(\frac{15N}{7} - \frac{39}{7}\right)\right) + B_{\bar{N}}\left(2N+14-\left(\frac{16N}{7} + \frac{349}{7}\right)\right) \\
&= B_{\bar{N}}(N+16) + B_{\bar{N}}\left(-\frac{N}{7} + \frac{137}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7} - \frac{251}{7}\right) = 17 + 0 + 0 = 17 \\
&(N \geq 137)
\end{aligned}$$

$$\begin{aligned}
B_{\bar N}(2N+15) &= B_{\bar N}(2N+15-B_{\bar N}(2N+14)) + B_{\bar N}(2N+15-B_{\bar N}(2N+13)) + B_{\bar N}(2N+15-B_{\bar N}(2N+12)) \\
&= B_{\bar N}(2N+15-17) + B_{\bar N}(2N+15-(N-2)) + B_{\bar N}\left(2N+15-\left(\frac{15N}{7}-\frac{39}{7}\right)\right) \\
&= B_{\bar N}(2N-2) + B_{\bar N}(N+17) + B_{\bar N}\left(-\frac{N}{7}+\frac{144}{7}\right) = 2N+(N+13)+0 = 3N+13 \\
&(N \geq 144)
\end{aligned}$$

$$\begin{aligned}
B_{\bar N}(2N+16) &= B_{\bar N}(2N+16-B_{\bar N}(2N+15)) + B_{\bar N}(2N+16-B_{\bar N}(2N+14)) + B_{\bar N}(2N+16-B_{\bar N}(2N+13)) \\
&= B_{\bar N}(2N+16-(3N+13)) + B_{\bar N}(2N+16-17) + B_{\bar N}(2N+16-(N-2)) \\
&= B_{\bar N}(-N+3) + B_{\bar N}(2N-1) + B_{\bar N}(N+18) = 0+7+18 = 25 \\
&(N \geq 68)
\end{aligned}$$

$$\begin{aligned}
B_{\bar N}(2N+17) &= B_{\bar N}(2N+17-B_{\bar N}(2N+16)) + B_{\bar N}(2N+17-B_{\bar N}(2N+15)) + B_{\bar N}(2N+17-B_{\bar N}(2N+14)) \\
&= B_{\bar N}(2N+17-25) + B_{\bar N}(2N+17-(3N+13)) + B_{\bar N}(2N+17-17) \\
&= B_{\bar N}(2N-8) + B_{\bar N}(-N+4) + B_{\bar N}(2N) = 7+0+\left(\frac{16N}{7}+\frac{307}{7}\right) = \frac{16N}{7}+\frac{356}{7} \\
&(N \geq 75)
\end{aligned}$$

$$\begin{aligned}
B_{\bar N}(2N+18) &= B_{\bar N}(2N+18-B_{\bar N}(2N+17)) + B_{\bar N}(2N+18-B_{\bar N}(2N+16)) + B_{\bar N}(2N+18-B_{\bar N}(2N+15)) \\
&= B_{\bar N}\left(2N+18-\left(\frac{16N}{7}+\frac{356}{7}\right)\right) + B_{\bar N}(2N+18-25) + B_{\bar N}(2N+18-(3N+13)) \\
&= B_{\bar N}\left(-\frac{2N}{7}-\frac{230}{7}\right) + B_{\bar N}(2N-7) + B_{\bar N}(-N+5) = 0+\left(\frac{16N}{7}+\frac{293}{7}\right)+0 = \frac{16N}{7}+\frac{293}{7} \\
&(N \geq 74)
\end{aligned}$$

$$
\begin{aligned}
B_{\bar{N}}(2N+19) &= B_{\bar{N}}(2N+19-B_{\bar{N}}(2N+18)) + B_{\bar{N}}(2N+19-B_{\bar{N}}(2N+17)) + B_{\bar{N}}(2N+19-B_{\bar{N}}(2N+16)) \\
&= B_{\bar{N}}\left(2N+19-\left(\frac{16N}{7}+\frac{293}{7}\right)\right) + B_{\bar{N}}\left(2N+19-\left(\frac{16N}{7}+\frac{356}{7}\right)\right) + B_{\bar{N}}(2N+19-25) \\
&= B_{\bar{N}}\left(-\frac{2N}{7}-\frac{160}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{223}{7}\right) + B_{\bar{N}}(2N-6) = 0+0+\left(\frac{15N}{7}-\frac{60}{7}\right) = \frac{15N}{7}-\frac{60}{7} \\
&(N \geq 77)
\end{aligned}
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$$
\begin{aligned}
B_{\bar{N}}(2N+20) &= B_{\bar{N}}(2N+20-B_{\bar{N}}(2N+19)) + B_{\bar{N}}(2N+20-B_{\bar{N}}(2N+18)) + B_{\bar{N}}(2N+20-B_{\bar{N}}(2N+17)) \\
&= B_{\bar{N}}\left(2N+20-\left(\frac{15N}{7}-\frac{60}{7}\right)\right) + B_{\bar{N}}\left(2N+20-\left(\frac{16N}{7}+\frac{293}{7}\right)\right) + B_{\bar{N}}\left(2N+20-\left(\frac{16N}{7}+\frac{356}{7}\right)\right) \\
&= B_{\bar{N}}\left(-\frac{N}{7}+\frac{200}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{153}{7}\right) + B_{\bar{N}}\left(-\frac{2N}{7}-\frac{216}{7}\right) = 0+0+0 = 0 \\
&(N \geq 200)*
\end{aligned}
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