Terms $B_{\bar{N}}(2N)$ through $B_{\bar{N}}(2N+27)$ when $N \equiv 0 \pmod 7$

When $N \equiv 0 \pmod 7$ and $N \geq 72$, a pattern with 7 interleaved linear sequences lasts from index $N+67$ through $2N-1$. If $N \geq 196$, there are 28 terms after this pattern ends. Below are calculations of all of these terms along with the necessary lower bound on N for each calculation to be valid. Record large N bounds exceeding 72 are presented in bold.

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N)} &= B_{\bar{N}}(2N - B_{\bar{N}}(2N-1)) + B_{\bar{N}}(2N - B_{\bar{N}}(2N-2)) + B_{\bar{N}}(2N - B_{\bar{N}}(2N-3)) \\
&= B_{\bar{N}}(2N-(N-2)) + B_{\bar{N}}\left(2N - \left(\frac{15N}{7} - 8\right)\right) + B_{\bar{N}}\left(2N - \left(\frac{16N}{7} + 43\right)\right) \\
&= B_{\bar{N}}(N+2) + B_{\bar{N}}\left(-\frac{N}{7} + 8\right) + B_{\bar{N}}\left(-\frac{2N}{7} - 43\right) = (N+1) + 0 + 0 = \mathbf{N+1} \\
&(N \geq 70)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+1)} &= B_{\bar{N}}(2N+1 - B_{\bar{N}}(2N)) + B_{\bar{N}}(2N+1 - B_{\bar{N}}(2N-1)) + B_{\bar{N}}(2N+1 - B_{\bar{N}}(2N-2)) \\
&= B_{\bar{N}}(2N+1-(N+1)) + B_{\bar{N}}(2N+1-(N-2)) + B_{\bar{N}}\left(2N+1 - \left(\frac{15N}{7} - 8\right)\right) \\
&= B_{\bar{N}}(N) + B_{\bar{N}}(N+3) + B_{\bar{N}}\left(-\frac{N}{7} + 9\right) = N + (N+2) + 0 = \mathbf{2N+2} \\
&(N \geq 69)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+2)} &= B_{\bar{N}}(2N+2 - B_{\bar{N}}(2N+1)) + B_{\bar{N}}(2N+2 - B_{\bar{N}}(2N)) + B_{\bar{N}}(2N+2 - B_{\bar{N}}(2N-1)) \\
&= B_{\bar{N}}(2N+2-(2N+2)) + B_{\bar{N}}(2N+2-(N+1)) + B_{\bar{N}}(2N+2-(N-2)) \\
&= B_{\bar{N}}(0) + B_{\bar{N}}(N+1) + B_{\bar{N}}(N+4) = 0 + 6 + (N+3) = \mathbf{N+9} \\
&(N \geq 68)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+3)} &= B_{\bar{N}}(2N+3-B_{\bar{N}}(2N+2)) + B_{\bar{N}}(2N+3-B_{\bar{N}}(2N+1)) + B_{\bar{N}}(2N+3-B_{\bar{N}}(2N)) \\
&= B_{\bar{N}}(2N+3-(N+9)) + B_{\bar{N}}(2N+3-(2N+2)) + B_{\bar{N}}(2N+3-(N+1)) \\
&= B_{\bar{N}}(N-6) + B_{\bar{N}}(1) + B_{\bar{N}}(N+2) = (N-6)+1+(N+1) = \mathbf{2N-4} \\
&(N \geq 7)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+4)} &= B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+3)) + B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+2)) + B_{\bar{N}}(2N+4-B_{\bar{N}}(2N+1)) \\
&= B_{\bar{N}}(2N+4-(2N-4)) + B_{\bar{N}}(2N+4-(N+9)) + B_{\bar{N}}(2N+4-(2N+2)) \\
&= B_{\bar{N}}(8) + B_{\bar{N}}(N-5) + B_{\bar{N}}(2) = 8+(N-5)+2 = \mathbf{N+5} \\
&(N \geq 8)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+5)} &= B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+4)) + B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+3)) + B_{\bar{N}}(2N+5-B_{\bar{N}}(2N+2)) \\
&= B_{\bar{N}}(2N+5-(N+5)) + B_{\bar{N}}(2N+5-(2N-4)) + B_{\bar{N}}(2N+5-(N+9)) \\
&= B_{\bar{N}}(N) + B_{\bar{N}}(9) + B_{\bar{N}}(N-4) = N+9+(N-4) = \mathbf{2N+5} \\
&(N \geq 9)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+6)} &= B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+5)) + B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+4)) + B_{\bar{N}}(2N+6-B_{\bar{N}}(2N+3)) \\
&= B_{\bar{N}}(2N+6-(2N+5)) + B_{\bar{N}}(2N+6-(N+5)) + B_{\bar{N}}(2N+6-(2N-4)) \\
&= B_{\bar{N}}(1) + B_{\bar{N}}(N+1) + B_{\bar{N}}(10) = 1+6+10 = \mathbf{17} \\
&(N \geq 10)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+7)} &= B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+6)) + B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+5)) + B_{\bar{N}}(2N+7-B_{\bar{N}}(2N+4)) \\
&= B_{\bar{N}}(2N+7-17) + B_{\bar{N}}(2N+7-(2N+5)) + B_{\bar{N}}(2N+7-(N+5)) \\
&= B_{\bar{N}}(2N-10) + B_{\bar{N}}(2) + B_{\bar{N}}(N+2) = \left(\frac{16N}{7}+41\right) + 2 + (N+1) = \mathbf{\frac{23N}{7}+44} \\
&(\mathbf{N \geq 77})
\end{aligned}$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+8)} &= B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+7)) + B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+6)) + B_{\bar{N}}(2N+8-B_{\bar{N}}(2N+5)) \\
&= B_{\bar{N}}\left(2N+8-\left(\frac{23N}{7}+44\right)\right) + B_{\bar{N}}(2N+8-17) + B_{\bar{N}}(2N+8-(2N+5)) \\
&= B_{\bar{N}}\left(-\frac{9N}{7}-36\right) + B_{\bar{N}}(2N-9) + B_{\bar{N}}(3) = 0 + \left(\frac{15N}{7}-9\right) + 3 = \mathbf{\frac{15N}{7}-6} \\
&(N \geq 76)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+9)} &= B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+8)) + B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+7)) + B_{\bar{N}}(2N+9-B_{\bar{N}}(2N+6)) \\
&= B_{\bar{N}}\left(2N+9-\left(\frac{15N}{7}-6\right)\right) + B_{\bar{N}}\left(2N+9-\left(\frac{23N}{7}+44\right)\right) + B_{\bar{N}}(2N+9-17) \\
&= B_{\bar{N}}\left(-\frac{N}{7}+15\right) + B_{\bar{N}}\left(-\frac{9N}{7}-35\right) + B_{\bar{N}}(2N-8) = 0 + 0 + (N-2) = \mathbf{N-2} \\
&(\mathbf{N \geq 105})
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+10)} &= B_{\bar{N}}(2N+10-B_{\bar{N}}(2N+9)) + B_{\bar{N}}(2N+10-B_{\bar{N}}(2N+8)) + B_{\bar{N}}(2N+10-B_{\bar{N}}(2N+7)) \\
&= B_{\bar{N}}(2N+10-(N-2)) + B_{\bar{N}}\left(2N+10-\left(\frac{15N}{7}-6\right)\right) + B_{\bar{N}}\left(2N+10-\left(\frac{23N}{7}+44\right)\right) \\
&= B_{\bar{N}}(N+12) + B_{\bar{N}}\left(-\frac{N}{7}+16\right) + B_{\bar{N}}\left(-\frac{9N}{7}-34\right) = (N+9) + 0 + 0 = \mathbf{N+9} \\
&(\mathbf{N \geq 112})
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+11)} &= B_{\bar{N}}(2N+11-B_{\bar{N}}(2N+10)) + B_{\bar{N}}(2N+11-B_{\bar{N}}(2N+9)) + B_{\bar{N}}(2N+11-B_{\bar{N}}(2N+8)) \\
&= B_{\bar{N}}(2N+11-(N+9)) + B_{\bar{N}}(2N+11-(N-2)) + B_{\bar{N}}\left(2N+11-\left(\frac{15N}{7}-6\right)\right) \\
&= B_{\bar{N}}(N+2) + B_{\bar{N}}(N+13) + B_{\bar{N}}\left(-\frac{N}{7}+17\right) = (N+1) + 15 + 0 = \mathbf{N+16} \\
&(\mathbf{N \geq 119})
\end{aligned}
$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+12)} &= B_{\bar{N}}(2N+12-B_{\bar{N}}(2N+11)) + B_{\bar{N}}(2N+12-B_{\bar{N}}(2N+10)) + B_{\bar{N}}(2N+12-B_{\bar{N}}(2N+9)) \\
&= B_{\bar{N}}(2N+12-(N+16)) + B_{\bar{N}}(2N+12-(N+9)) + B_{\bar{N}}(2N+12-(N-2)) \\
&= B_{\bar{N}}(N-4) + B_{\bar{N}}(N+3) + B_{\bar{N}}(N+14) = (N-4) + (N+2) + (N+10) = \mathbf{3N+8} \\
&(N \geq 6)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+13)} &= B_{\bar{N}}(2N+13-B_{\bar{N}}(2N+12)) + B_{\bar{N}}(2N+13-B_{\bar{N}}(2N+11)) + B_{\bar{N}}(2N+13-B_{\bar{N}}(2N+10)) \\
&= B_{\bar{N}}(2N+13-(3N+8)) + B_{\bar{N}}(2N+13-(N+16)) + B_{\bar{N}}(2N+13-(N+9)) \\
&= B_{\bar{N}}(-N+5) + B_{\bar{N}}(N-3) + B_{\bar{N}}(N+4) = 0 + (N-3) + (N+3) = \mathbf{2N} \\
&(N \geq 5)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+14)} &= B_{\bar{N}}(2N+14-B_{\bar{N}}(2N+13)) + B_{\bar{N}}(2N+14-B_{\bar{N}}(2N+12)) + B_{\bar{N}}(2N+14-B_{\bar{N}}(2N+11)) \\
&= B_{\bar{N}}(2N+14-2N) + B_{\bar{N}}(2N+14-(3N+8)) + B_{\bar{N}}(2N+14-(N+16)) \\
&= B_{\bar{N}}(14) + B_{\bar{N}}(-N+6) + B_{\bar{N}}(N-2) = 14 + 0 + (N-2) = \mathbf{N+12} \\
&(N \geq 14)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+15)} &= B_{\bar{N}}(2N+15-B_{\bar{N}}(2N+14)) + B_{\bar{N}}(2N+15-B_{\bar{N}}(2N+13)) + B_{\bar{N}}(2N+15-B_{\bar{N}}(2N+12)) \\
&= B_{\bar{N}}(2N+15-(N+12)) + B_{\bar{N}}(2N+15-2N) + B_{\bar{N}}(2N+15-(3N+8)) \\
&= B_{\bar{N}}(N+3) + B_{\bar{N}}(15) + B_{\bar{N}}(-N+7) = (N+2) + 15 + 0 = \mathbf{N+17} \\
&(N \geq 15)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+16)} &= B_{\bar{N}}(2N+16-B_{\bar{N}}(2N+15)) + B_{\bar{N}}(2N+16-B_{\bar{N}}(2N+14)) + B_{\bar{N}}(2N+16-B_{\bar{N}}(2N+13)) \\
&= B_{\bar{N}}(2N+16-(N+17)) + B_{\bar{N}}(2N+16-(N+12)) + B_{\bar{N}}(2N+16-2N) \\
&= B_{\bar{N}}(N-1) + B_{\bar{N}}(N+4) + B_{\bar{N}}(16) = (N-1) + (N+3) + 16 = \mathbf{2N+18} \\
&(N \geq 16)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+17)} &= B_{\bar{N}}(2N+17-B_{\bar{N}}(2N+16)) + B_{\bar{N}}(2N+17-B_{\bar{N}}(2N+15)) + B_{\bar{N}}(2N+17-B_{\bar{N}}(2N+14)) \\
&= B_{\bar{N}}(2N+17-(2N+18)) + B_{\bar{N}}(2N+17-(N+17)) + B_{\bar{N}}(2N+17-(N+12)) \\
&= B_{\bar{N}}(-1) + B_{\bar{N}}(N) + B_{\bar{N}}(N+5) = 0 + N + 9 = \mathbf{N+9} \\
&(N \geq 1)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+18)} &= B_{\bar{N}}(2N+18-B_{\bar{N}}(2N+17)) + B_{\bar{N}}(2N+18-B_{\bar{N}}(2N+16)) + B_{\bar{N}}(2N+18-B_{\bar{N}}(2N+15)) \\
&= B_{\bar{N}}(2N+18-(N+9)) + B_{\bar{N}}(2N+18-(2N+18)) + B_{\bar{N}}(2N+18-(N+17)) \\
&= B_{\bar{N}}(N+9) + B_{\bar{N}}(0) + B_{\bar{N}}(N+1) = 12 + 0 + 6 = \mathbf{18} \\
&(N \geq 1)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+19)} &= B_{\bar{N}}(2N+19-B_{\bar{N}}(2N+18)) + B_{\bar{N}}(2N+19-B_{\bar{N}}(2N+17)) + B_{\bar{N}}(2N+19-B_{\bar{N}}(2N+16)) \\
&= B_{\bar{N}}(2N+19-18) + B_{\bar{N}}(2N+19-(N+9)) + B_{\bar{N}}(2N+19-(2N+18)) \\
&= B_{\bar{N}}(2N+1) + B_{\bar{N}}(N+10) + B_{\bar{N}}(1) = (2N+2) + (N+7) + 1 = \mathbf{3N+10} \\
&(N \geq 1)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+20)} &= B_{\bar{N}}(2N+20-B_{\bar{N}}(2N+19)) + B_{\bar{N}}(2N+20-B_{\bar{N}}(2N+18)) + B_{\bar{N}}(2N+20-B_{\bar{N}}(2N+17)) \\
&= B_{\bar{N}}(2N+20-(3N+10)) + B_{\bar{N}}(2N+20-18) + B_{\bar{N}}(2N+20-(N+9)) \\
&= B_{\bar{N}}(-N+10) + B_{\bar{N}}(2N+2) + B_{\bar{N}}(N+11) = 0 + (N+9) + (N+8) = \mathbf{2N+17} \\
&(N \geq 10)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B_{\bar{N}}(2N+21)} &= B_{\bar{N}}(2N+21-B_{\bar{N}}(2N+20)) + B_{\bar{N}}(2N+21-B_{\bar{N}}(2N+19)) + B_{\bar{N}}(2N+21-B_{\bar{N}}(2N+18)) \\
&= B_{\bar{N}}(2N+21-(2N+17)) + B_{\bar{N}}(2N+21-(3N+10)) + B_{\bar{N}}(2N+21-18) \\
&= B_{\bar{N}}(4) + B_{\bar{N}}(-N+11) + B_{\bar{N}}(2N+3) = 4 + 0 + (2N-4) = \mathbf{2N} \\
&(N \geq 11)
\end{aligned}$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+22)} &= B_{\bar{N}}(2N+22-B_{\bar{N}}(2N+21)) + B_{\bar{N}}(2N+22-B_{\bar{N}}(2N+20)) + B_{\bar{N}}(2N+22-B_{\bar{N}}(2N+19)) \\
&= B_{\bar{N}}(2N+22-2N) + B_{\bar{N}}(2N+22-(2N+17)) + B_{\bar{N}}(2N+22-(3N+10)) \\
&= B_{\bar{N}}(22) + B_{\bar{N}}(5) + B_{\bar{N}}(-N+12) = 22+5+0 = \mathbf{27} \\
&(N \geq 22)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+23)} &= B_{\bar{N}}(2N+23-B_{\bar{N}}(2N+22)) + B_{\bar{N}}(2N+23-B_{\bar{N}}(2N+21)) + B_{\bar{N}}(2N+23-B_{\bar{N}}(2N+20)) \\
&= B_{\bar{N}}(2N+23-27) + B_{\bar{N}}(2N+23-2N) + B_{\bar{N}}(2N+23-(2N+17)) \\
&= B_{\bar{N}}(2N-4) + B_{\bar{N}}(23) + B_{\bar{N}}(6) = 7+23+6 = \mathbf{36} \\
&(N \geq 71)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+24)} &= B_{\bar{N}}(2N+24-B_{\bar{N}}(2N+23)) + B_{\bar{N}}(2N+24-B_{\bar{N}}(2N+22)) + B_{\bar{N}}(2N+24-B_{\bar{N}}(2N+21)) \\
&= B_{\bar{N}}(2N+24-36) + B_{\bar{N}}(2N+24-27) + B_{\bar{N}}(2N+24-2N) \\
&= B_{\bar{N}}(2N-12) + B_{\bar{N}}(2N-3) + B_{\bar{N}}(24) = (2N-10) + \left(\frac{16N}{7}+43\right) + 24 = \mathbf{\frac{30N}{7}+57} \\
&(N \geq 79)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+25)} &= B_{\bar{N}}(2N+25-B_{\bar{N}}(2N+24)) + B_{\bar{N}}(2N+25-B_{\bar{N}}(2N+23)) + B_{\bar{N}}(2N+25-B_{\bar{N}}(2N+22)) \\
&= B_{\bar{N}}\left(2N+25-\left(\frac{30N}{7}+57\right)\right) + B_{\bar{N}}(2N+25-36) + B_{\bar{N}}(2N+25-27) \\
&= B_{\bar{N}}\left(-\frac{16N}{7}-32\right) + B_{\bar{N}}(2N-11) + B_{\bar{N}}(2N-2) = 0+7+\left(\frac{15N}{7}-8\right) = \mathbf{\frac{15N}{7}-1} \\
&(N \geq 78)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+26)} &= B_{\bar{N}}(2N+26-B_{\bar{N}}(2N+25))+B_{\bar{N}}(2N+26-B_{\bar{N}}(2N+24))+B_{\bar{N}}(2N+26-B_{\bar{N}}(2N+23)) \\
&= B_{\bar{N}}\left(2N+26-\left(\frac{15N}{7}-1\right)\right)+B_{\bar{N}}\left(2N+26-\left(\frac{30N}{7}+57\right)\right)+B_{\bar{N}}(2N+26-36) \\
&= B_{\bar{N}}\left(-\frac{N}{7}+27\right)+B_{\bar{N}}\left(-\frac{16N}{7}-31\right)+B_{\bar{N}}(2N-10)=0+0+\left(\frac{16N}{7}+41\right)=\mathbf{\frac{16N}{7}+41} \\
&(\mathbf{N \geq 189})
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B_{\bar{N}}(2N+27)} &= B_{\bar{N}}(2N+27-B_{\bar{N}}(2N+26))+B_{\bar{N}}(2N+27-B_{\bar{N}}(2N+25))+B_{\bar{N}}(2N+27-B_{\bar{N}}(2N+24)) \\
&= B_{\bar{N}}\left(2N+27-\left(\frac{16N}{7}+41\right)\right)+B_{\bar{N}}\left(2N+27-\left(\frac{15N}{7}-1\right)\right)+B_{\bar{N}}\left(2N+27-\left(\frac{30N}{7}+57\right)\right) \\
&= B_{\bar{N}}\left(-\frac{2N}{7}-14\right)+B_{\bar{N}}\left(-\frac{N}{7}+28\right)+B_{\bar{N}}\left(-\frac{16N}{7}-30\right)=0+0+0=\mathbf{0} \\
&(\mathbf{N \geq 196})
\end{aligned}
$$