First 24 terms following initial conditions of B_N

Assuming $N \geq 9$, these are the first 24 terms of B_N following the initial conditions. Below are calculations of all of these terms along with the necessary lower bound on N for each calculation to be valid.

$$\begin{aligned} B_N(N+1) &= B_N(N+1-B_N(N)) + B_N(N+1-B_N(N-1)) \\ &\quad + B_N(N+1-B_N(N-2)) \\ &= B_N(N+1-N) + B_N(N+1-(N-1)) + B_N(N+1-(N-2)) \\ &= B_N(1) + B_N(2) + B_N(3) = 1+2+3 = 6 \\ &(N \geq 3) \end{aligned}$$

$$\begin{aligned} B_N(N+2) &= B_N(N+2-B_N(N+1)) + B_N(N+2-B_N(N)) \\ &\quad + B_N(N+2-B_N(N-1)) \\ &= B_N(N+2-6) + B_N(N+2-N) + B_N(N+2-(N-1)) \\ &= B_N(N-4) + B_N(2) + B_N(3) = (N-4)+2+3 = N+1 \\ &(N \geq 5) \end{aligned}$$

$$\begin{aligned} B_N(N+3) &= B_N(N+3-B_N(N+2)) + B_N(N+3-B_N(N+1)) \\ &\quad + B_N(N+3-B_N(N)) \\ &= B_N(N+3-(N+1)) + B_N(N+3-6) + B_N(N+3-N) \\ &= B_N(2) + B_N(N-3) + B_N(3) = 2+(N-3)+3 = N+2 \\ &(N \geq 4) \end{aligned}$$

$$
\begin{aligned}
B_N(N+4) &= B_N(N+4-B_N(N+3)) + B_N(N+4-B_N(N+2)) \\
&\quad + B_N(N+4-B_N(N+1)) \\
&= B_N(N+4-(N+2)) + B_N(N+4-(N+1)) + B_N(N+4-6) \\
&= B_N(2) + B_N(3) + B_N(N-2) = 2+3+(N-2) = N+3 \\
&(N \geq 3)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+5) &= B_N(N+5-B_N(N+4)) + B_N(N+5-B_N(N+3)) \\
&\quad + B_N(N+5-B_N(N+2)) \\
&= B_N(N+5-(N+3)) + B_N(N+5-(N+2)) + B_N(N+5-(N+1)) \\
&= B_N(2) + B_N(3) + B_N(4) = 2+3+4 = 9 \\
&(N \geq 4)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+6) &= B_N(N+6-B_N(N+5)) + B_N(N+6-B_N(N+4)) \\
&\quad + B_N(N+6-B_N(N+3)) \\
&= B_N(N+6-9) + B_N(N+6-(N+3)) + B_N(N+6-(N+2)) \\
&= B_N(N-3) + B_N(3) + B_N(4) = (N-3)+3+4 = N+4 \\
&(N \geq 4)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+7) &= B_N(N+7-B_N(N+6)) + B_N(N+7-B_N(N+5)) \\
&\quad + B_N(N+7-B_N(N+4)) \\
&= B_N(N+7-(N+4)) + B_N(N+7-9) + B_N(N+7-(N+3)) \\
&= B_N(3) + B_N(N-2) + B_N(4) = 3+(N-2)+4 = N+5 \\
&(N \geq 4)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+8) &= B_N(N+8-B_N(N+7)) + B_N(N+8-B_N(N+6)) \\
&\quad + B_N(N+8-B_N(N+5)) \\
&= B_N(N+8-(N+5)) + B_N(N+8-(N+4)) + B_N(N+8-9) \\
&= B_N(3) + B_N(4) + B_N(N-1) = 3+4+(N-1) = N+6 \\
&(N \geq 4)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+9) &= B_N(N+9-B_N(N+8)) + B_N(N+9-B_N(N+7)) \\
&\quad + B_N(N+9-B_N(N+6)) \\
&= B_N(N+9-(N+6)) + B_N(N+9-(N+5)) + B_N(N+9-(N+4)) \\
&= B_N(3) + B_N(4) + B_N(5) = 3+4+5 = 12 \\
&(N \geq 5)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+10) &= B_N(N+10-B_N(N+9)) + B_N(N+10-B_N(N+8)) \\
&\quad + B_N(N+10-B_N(N+7)) \\
&= B_N(N+10-12) + B_N(N+10-(N+6)) + B_N(N+10-(N+5)) \\
&= B_N(N-2) + B_N(4) + B_N(5) = (N-2)+4+5 = N+7 \\
&(N \geq 5)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+11) &= B_N(N+11-B_N(N+10)) + B_N(N+11-B_N(N+9)) \\
&\quad + B_N(N+11-B_N(N+8)) \\
&= B_N(N+11-(N+7)) + B_N(N+11-12) + B_N(N+11-(N+6)) \\
&= B_N(4) + B_N(N-1) + B_N(5) = 4+(N-1)+5 = N+8 \\
&(N \geq 5)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+12) &= B_N(N+12-B_N(N+11)) + B_N(N+12-B_N(N+10)) \\
&\quad + B_N(N+12-B_N(N+9)) \\
&= B_N(N+12-(N+8)) + B_N(N+12-(N+7)) + B_N(N+12-12) \\
&= B_N(4) + B_N(5) + B_N(N) = 4+5+N = N+9 \\
&(N \geq 5)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+13) &= B_N(N+13-B_N(N+12)) + B_N(N+13-B_N(N+11)) \\
&\quad + B_N(N+13-B_N(N+10)) \\
&= B_N(N+13-(N+9)) + B_N(N+13-(N+8)) + B_N(N+13-(N+7)) \\
&= B_N(4) + B_N(5) + B_N(6) = 4+5+6 = 15 \\
&(N \geq 6)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+14) &= B_N(N+14-B_N(N+13)) + B_N(N+14-B_N(N+12)) \\
&\quad + B_N(N+14-B_N(N+11)) \\
&= B_N(N+14-15) + B_N(N+14-(N+9)) + B_N(N+14-(N+8)) \\
&= B_N(N-1) + B_N(5) + B_N(6) = (N-1)+5+6 = N+10 \\
&(N \geq 6)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+15) &= B_N(N+15-B_N(N+14)) + B_N(N+15-B_N(N+13)) \\
&\quad + B_N(N+15-B_N(N+12)) \\
&= B_N(N+15-(N+10)) + B_N(N+15-15) + B_N(N+15-(N+9)) \\
&= B_N(5) + B_N(N) + B_N(6) = 5+N+6 = N+11 \\
&(N \geq 6)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+16) &= B_N(N+16-B_N(N+15)) + B_N(N+16-B_N(N+14)) \\
&\quad + B_N(N+16-B_N(N+13)) \\
&= B_N(N+16-(N+11)) + B_N(N+16-(N+10)) + B_N(N+16-15) \\
&= B_N(5) + B_N(6) + B_N(N+1) = 5+6+6 = 17 \\
&(N \geq 6)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+17) &= B_N(N+17-B_N(N+16)) + B_N(N+17-B_N(N+15)) \\
&\quad + B_N(N+17-B_N(N+14)) \\
&= B_N(N+17-17) + B_N(N+17-(N+11)) + B_N(N+17-(N+10)) \\
&= B_N(N) + B_N(6) + B_N(7) = N+6+7 = N+13 \\
&(N \geq 7)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+18) &= B_N(N+18-B_N(N+17)) + B_N(N+18-B_N(N+16)) \\
&\quad + B_N(N+18-B_N(N+15)) \\
&= B_N(N+18-(N+13)) + B_N(N+18-17) + B_N(N+18-(N+11)) \\
&= B_N(5) + B_N(N+1) + B_N(7) = 5+6+7 = 18 \\
&(N \geq 7)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+19) &= B_N(N+19-B_N(N+18)) + B_N(N+19-B_N(N+17)) \\
&\quad + B_N(N+19-B_N(N+16)) \\
&= B_N(N+19-18) + B_N(N+19-(N+13)) + B_N(N+19-17) \\
&= B_N(N+1) + B_N(6) + B_N(N+2) = 6+6+(N+1) = N+13 \\
&(N \geq 6)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+20) &= B_N(N+20-B_N(N+19)) + B_N(N+20-B_N(N+18)) \\
&\quad + B_N(N+20-B_N(N+17)) \\
&= B_N(N+20-(N+13)) + B_N(N+20-18) + B_N(N+20-(N+13)) \\
&= B_N(7) + B_N(N+2) + B_N(7) = 7 + (N+1) + 7 = N+15 \\
&(N \geq 7)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+21) &= B_N(N+21-B_N(N+20)) + B_N(N+21-B_N(N+19)) \\
&\quad + B_N(N+21-B_N(N+18)) \\
&= B_N(N+21-(N+15)) + B_N(N+21-(N+13)) + B_N(N+21-18) \\
&= B_N(6) + B_N(8) + B_N(N+3) = 6+8+(N+2) = N+16 \\
&(N \geq 8)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+22) &= B_N(N+22-B_N(N+21)) + B_N(N+22-B_N(N+20)) \\
&\quad + B_N(N+22-B_N(N+19)) \\
&= B_N(N+22-(N+16)) + B_N(N+22-(N+15)) \\
&\quad + B_N(N+22-(N+13)) \\
&= B_N(6) + B_N(7) + B_N(9) = 6+7+9 = 22 \\
&(N \geq 9)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+23) &= B_N(N+23-B_N(N+22)) + B_N(N+23-B_N(N+21)) \\
&\quad + B_N(N+23-B_N(N+20)) \\
&= B_N(N+23-22) + B_N(N+23-(N+16)) + B_N(N+23-(N+15)) \\
&= B_N(N+1) + B_N(7) + B_N(8) = 6+7+8 = 21 \\
&(N \geq 8)
\end{aligned}
$$

$$
\begin{aligned}
B_N(N+24) &= B_N(N+24-B_N(N+23)) + B_N(N+24-B_N(N+22)) \\
&\quad + B_N(N+24-B_N(N+21)) \\
&= B_N(N+24-21) + B_N(N+24-22) + B_N(N+24-(N+16)) \\
&= B_N(N+3) + B_N(N+2) + B_N(8) = (N+2) + (N+1) + 8 = 2N+11 \\
&(N \geq 8)
\end{aligned}
$$