

## Supplementary Materials

**Algorithm S1:** Transform each sequence into a chaos game representation.

Input:

$S$  - A sequence such as that  $S = s_1, s_2, \dots, s_i \in \{A, C, G, T/U\}$

$P_{i-1}$  - The  $x$  and  $y$  coordinates of the nucleotide found at position  $i-1$  in sequence  $S$ .

For  $s_i$  in  $S$ :

    If  $i = 1$ :

$$P_i = 0.5[(0, 0) + g(s_i)]$$

    Else:

$$P_i = 0.5[P_{i-1} + g(s_i)]$$

Where:

$g(s)$  is  $(1, 1)$  if  $s = A$ ,  $(-1, 1)$  if  $s = C$ ,  $(-1, -1)$  if  $s = G$ , or  $(1, -1)$  if  $s = T/U$

Output:

A chaos game representation of  $S$

**Algorithm S2:** Pseudo-code for the FFT block

```
def FFTBlock(x):
    x = RFFT2(x)

    x = proj(x, d)
    x = reshape(b*h, n, 32)
    x = x + attn_dropout(efficient_attention(x), 0.125)
    x = RMSNorm(x)
    x = NoisyFactoriedLinear(x, 32) + x
    x = reshape(b, n, d)
    x = merge(b, n, d//2+1)
    x = shrinkage(x)

    x = IRFFT2(x)
    x = scale(x, -5, 5)
    x = FFN(x)
    x = Dropout1d(0.125)

    return x
```

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### Algorithm S3: Pseudo-code for the Wavelet Transform block

```
def DWTBlock(x):  
  
    x = init_proj(x, d)  
    x = reshape(b, h, n, 32)  
    y_L, y_H = DWT(x)  
  
    x_L = StartNet(y_L)  
    x_L = RMSNorm(x_L)  
    x_L = FFN(x_L) + x_L  
    x_H = StarNet(y_H)  
    x_H = RMSNorm(x_H)  
    x_H = Linear(x_H) + x_H  
    x_H = shrinkage(x_H)  
  
    x = iDWT(x_L, x_H)  
    x = reshape(b, n, d)  
    x = FFN(x)  
    x = Dropout1d(0.125)  
  
    return x
```

### Algorithm S4: Calculation of the Router Z-loss

To ensure that experts are utilized efficiently, we make use of the router z-loss (Equation 1), which penalizes large logits before the router in the ST-MoE network. This helps increase the utilization of experts at each layer [1]. In this equation  $x$  represents the logits,  $T$  are the number of tokens, and  $N$  is the number of experts. A scaling of  $\frac{1}{10}$  is used so that this loss does not overwhelm the cross-entropy loss.

$$Z\ Loss(x) = \frac{\frac{1}{T} \sum_{i=1}^T \left( \log \sum_{j=1}^N e^{x_i^{(j)}} \right)}{10} \quad (1)$$

### Algorithm S5: Calculation of the KAN Regularization Loss

L1 regularization can be used to favor sparsity and simplicity within feed-forward networks. This helps prevent overfitting and generally can improve the robustness of models. Within the KAN, linear weights are replaced with learnable activation functions [2] and regularization forces the network to select the most appropriate activation functions.

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The first part of the KAN loss involves calculating the L1-norms of all the activation functions [2]. The L1-norm of the spline weights vector,  $|W|_1$ , can be approximated by taking the absolute value of the spline weights tensor,  $W$ , and then dividing this by the size of the last dimension of the spline weight tensor,  $d_{spline}$  (Equation 2).

$$|W|_1 = \sum_{i=1}^{d_{in}} \sum_{j=1}^{d_{out}} \frac{\sum_{k=1}^{d_{spline}} |W_{i,j,k}|}{d_{spline}} \quad (2)$$

To induce sparsity, L1 regularization is not sufficient on its own and entropy regularization is also required [2]. This can be calculated according to Equation 3.

$$S(W) = - \sum_{i=1}^{d_{in}} \sum_{j=1}^{d_{out}} \frac{\sum_{k=1}^{d_{spline}} |W_{i,j,k}|}{|W|_1} \log \left( \frac{\sum_{k=1}^{d_{spline}} |W_{i,j,k}|}{\sum_{k=1}^{d_{spline}} |W_{i,j,k}|} \right) \quad (3)$$

The final KAN loss is then calculated according to Equation 4, where  $L$  is the number of KAN layers in the classification head. A scaling of  $\frac{1}{100}$  is used so that this loss does not overwhelm the cross-entropy loss.

$$KAN \text{ Loss} = \frac{\sum_{l=1}^L (|W_l|_1 + S(W_l))}{100L} \quad (4)$$

## References

- [1] Zoph B, Bello I, Kumar S, Du N, Huang Y, Dean J, et al. ST-MoE: Designing Stable and Transferable Sparse Expert Models 2022. <https://doi.org/10.48550/arXiv.2202.08906>.
- [2] Liu Z, Wang Y, Vaidya S, Ruehle F, Halverson J, Soljačić M, et al. KAN: Kolmogorov-Arnold Networks 2024. <https://doi.org/10.48550/arXiv.2404.19756>.