Project 1

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Question 1. Random Number Generators

The empirical mean of the generated uniform random numbers is

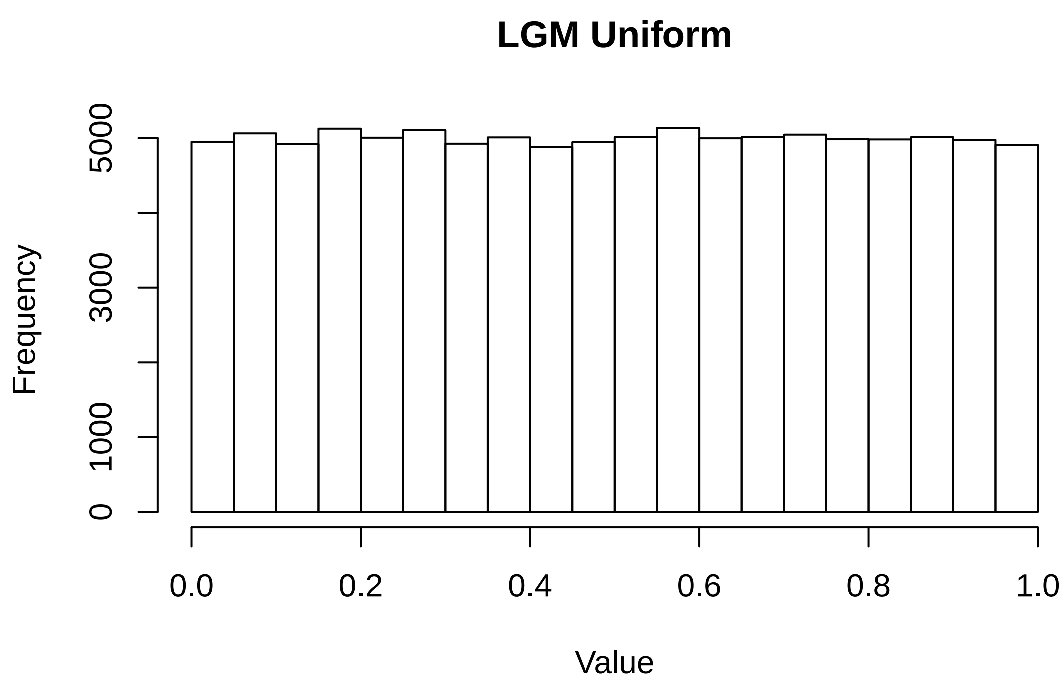
```
## [1] 0.4995044
```

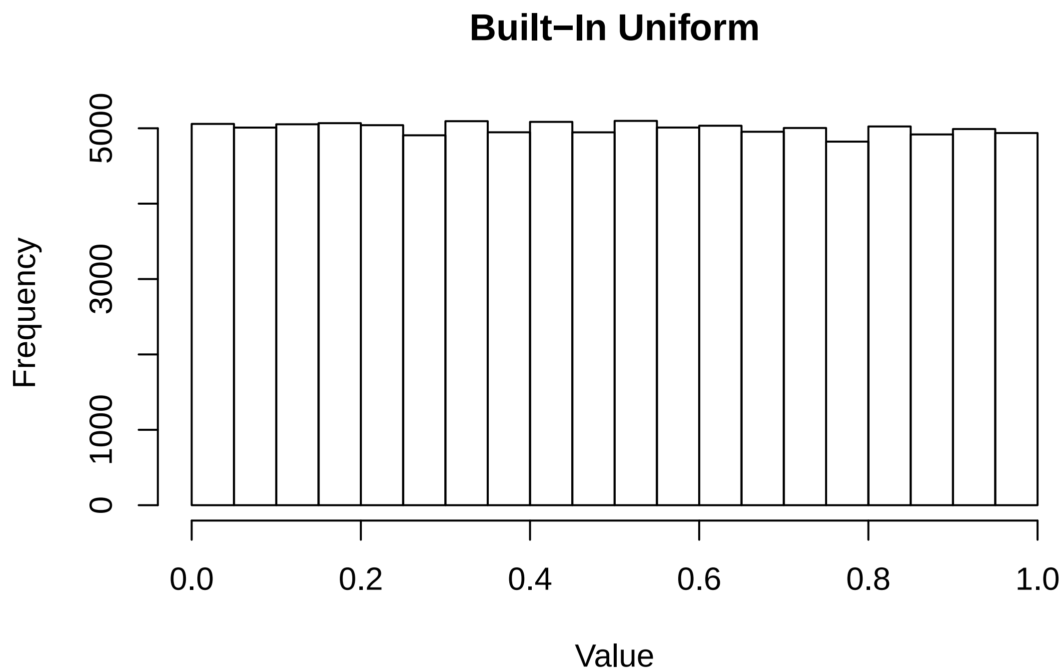
which is close to the theoretical value of 0.5 and the empirical standard deviation of the generated uniform random numbers is

```
## [1] 0.2883965
```

which is also pretty close to the theoretical value of $\sqrt{1/12}$.

We will now compare the uniform sequence generated through LGM and the uniform sequence generated by the built-in functions in R $runif$.

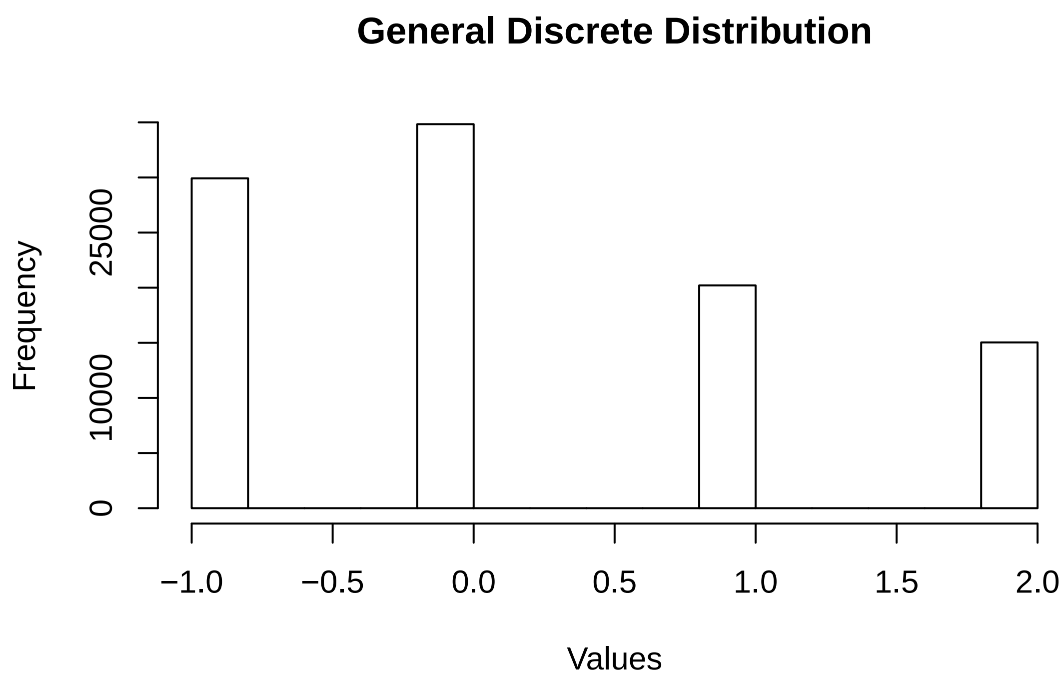




The uniform random numbers generated by the LGM method and by the built-in function in R are quite similar as seen by the two histograms plotted above.

Question 2. Discrete Probabilities

The histogram for the 10,000 generated sequence of discrete probabilities is shown below



The mean of the general discrete sequence of 10,000 numbers is

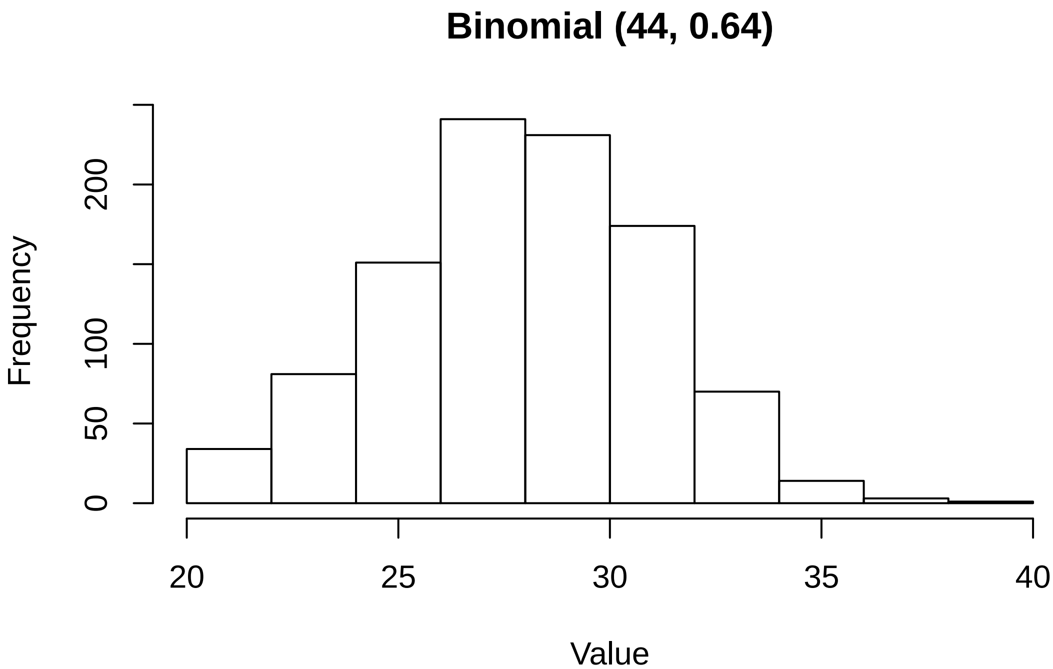
```
## [1] 0.1992
```

and the empirical standard deviation of the sequence is

```
## [1] 1.023633
```

Question 3. Binomial Distribution

The histogram of the 1000 binomial distribution is shown below



The exact probability P(X >= 40) of a Binom(44, 0.64) is

```
## [1] 4.823664e-05
```

and the empirical probability using 1,000 samples of binomial distribution is

```
## [1] 0
```

The actual probability is very close to 0, the empirical probability found using 1,000 samples is 0 because the sample of the binomial distribution is too small. If we try again with 1,000,000 samples we get that the empirical probability is

```
## [1] 5.8e-05
```

which is close to the theoretical value.

Question 4. Exponential Distribution

The empirical probability $P(X \geq 1)$ is

```
## [1] 0.2193
```

compared to the theoretical value:

```
## [1] 0.2231302
```

they are close to each other

and the empirical probability $P(X \geq 4)$ is

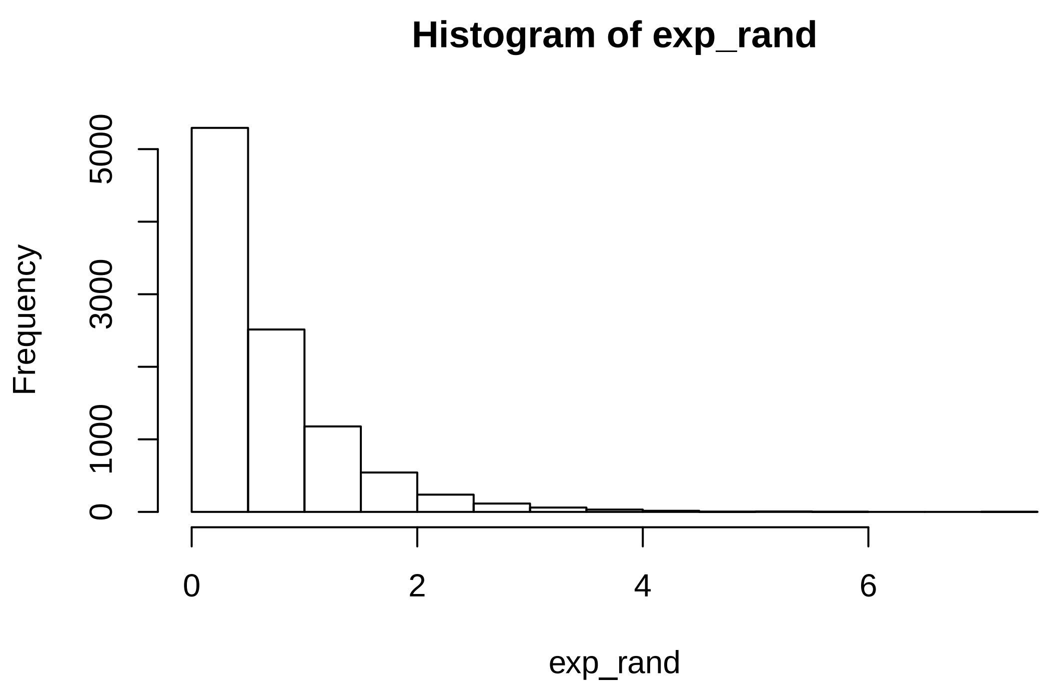
```
## [1] 0.0027
```

compared to the theoretical value:

```
## [1] 0.002478752
```

they are also quite close to each other.

The 10,000 generated exponential distributed random numbers with $\lambda = 1.5$ is shown by the histogram below:



The empirical mean is

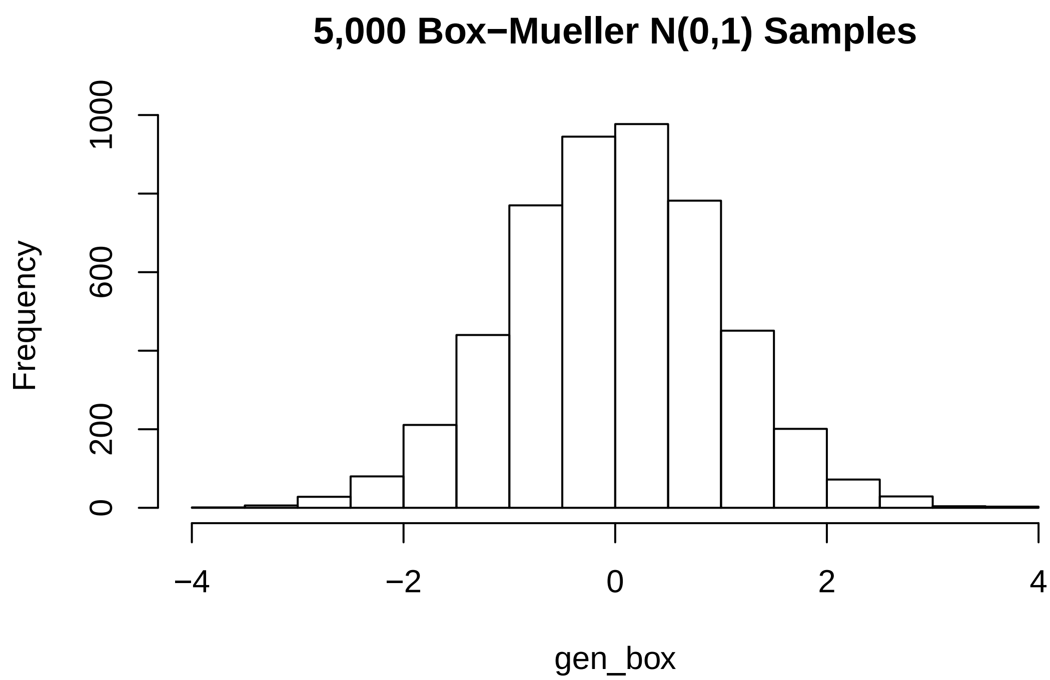
```
## [1] 0.6617143
```

and the empirical sandard deviation of the generated sequence is

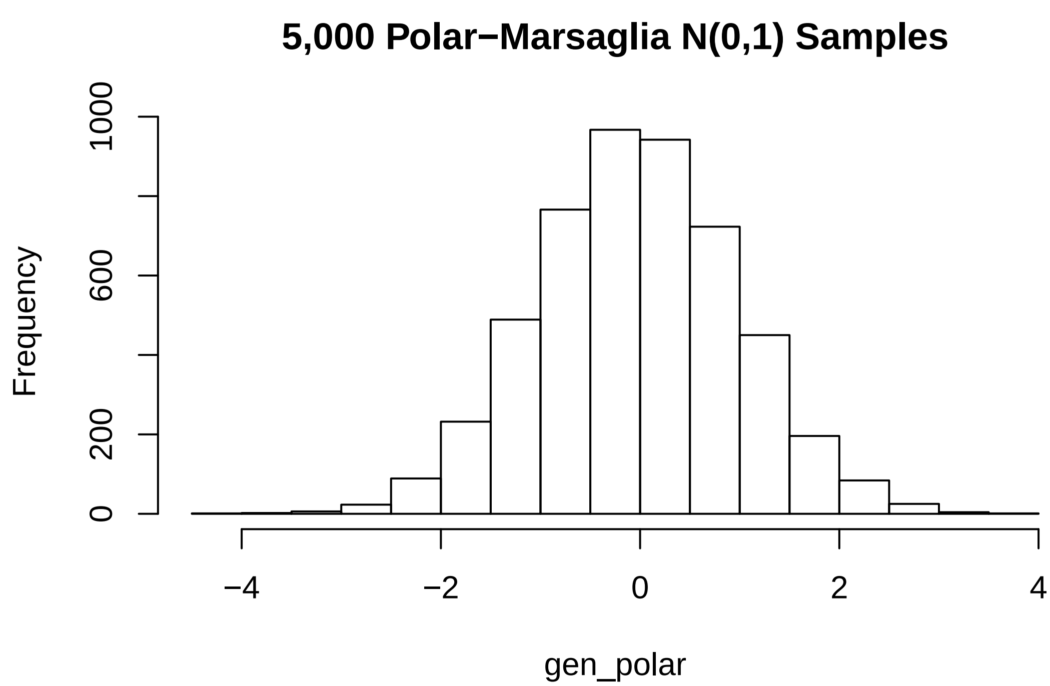
```
## [1] 0.6657191
```

Question 5. Normal Distribution

The histogram of the generated 5,000 normals using Box-Mueller is shown below



and the one generated using Polar-Marsaglia is shown below



The time required to generate 5,000 normals using Box-Mueller is

```
## elapsed
##    0.03
```

whereas using the Polar-Marsaglia method is

```
## elapsed
##    0.09
```

From the results above we can see that the Box-Mueller method can generate sequences faster than the Polar-Marsaglia method. Hence, the time taken to evaluate sin and cos is less than to evaluate the log function and check the unit circle (Polar-Marsaglia).