

GOETHE UNIVERSITY FRANKFURT

SPECIAL TOPICS IN DERIVATIVES AND ASSET PRICING

WINTER SEMESTER 2013/14

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**Risk-return trade-off with mixed data sampling**

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*Author:*

Diem Quynh TRAN THANH  
Friedberger Landstr. 134  
60316 Frankfurt am Main  
Tel.: 0176 99812268  
Student ID: 5179651  
Master in Money and Finance

*Submitted to:*

Prof. Dr. Christian SCHLAG  
Chair of Derivatives and  
Financial Engineering  
Faculty of Economics and  
Business Administration  
Johann Wolfgang  
Goethe-Universitat  
Frankfurt am Main

Frankfurt am Main, December 26, 2013

## **Abstract**

According to the fundamental theory in finance, investors require higher return on riskier investments, as a reward for bearing that higher risk. However, empirical results so far have not clearly proven that fact. This paper is on one hand a review of existing literature on risk-return trade-off models, on the other hand, an empirical replication of the paper "There is a risk-return trade-off after all" by Ghysels et al. (2004), which describes a model based on mixed frequency data, and weighted composition of the variance.

In the first section, variance models and their results are introduced and evaluated, in order to better understand later the significance of the results in the main (mixed data sampling) model. The second section is the empirical replication itself, which will show the most important implications of the model and prove the existence of the risk-return trade-off by confirming the results, which are significant and positive. The third section provides with the conclusion and suggestions for further research.

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# 1 Introduction

Sharpe (1964) introduced the capital asset pricing model (CAPM), which assumed a linear trade-off between excess return and market risk, described by the well known Sharpe-ratio. However, the standard model was highly criticised for its simplicity, also because it was a single-period model. Merton (1973) followed up by an intertemporal asset pricing model (ICAPM), which maximises utility over time and counts with the limited liability of assets. He finds that investors are indeed compensated for higher risk, and risk premia should vary with the conditional variance of the market.

$$E_t[R_{t+1}] = \mu + \gamma VAR_t[R_{t+1}] \quad (1)$$

In Equation (1)  $\gamma$  is interpreted as relative risk aversion, that is supposed to be positive. Many tried to find this result in the market data, but so far, findings did not clearly show that.

French et al. (1987) suggest that unexpected stock market returns are negatively correlated with unexpected change in volatility, from which they presume that there should be a positive relation between expected risk premia and volatility. Their empirical results show a positive gamma, however, it is insignificant. Campbell and Hentschel (1992) also arrived to a similar result. They emphasise, that volatility feedback is more important when volatility is high, furthermore, there might be a predictive asymmetry in expected return: when volatility is higher, there is a bigger chance for a big negative return afterwards.

Some others have found negative relationship between risk and return. Campbell (1987) shows that asset returns can be derived from a fixed weighted benchmark portfolio of bills, bonds and stocks, which is proportional to its conditional variance, but the direct relation between stock return and the benchmark variance is negative. Harvey (2001) finds that his linear model gives negative results on risk-return trade off, but a non-parametric model is more fitting to market data. Similarly to him, Baille and DeGennaro argue, that the traditional 2 parameter risk-return model is

not accurate.

Although the theory suggests, that a positive trade-off should exist, Glosten et al. (1993) argue, that a positive and negative relation can be also consistent. According to them, higher risk premium is not necessarily required when higher risk period coincides with a period, when investors can bear a particular type of risk better. Or, they want to save more in periods when the future is riskier. They also find that there is no high persistence for volatility.

In conclusion, the former models did not find significantly positive results, furthermore, they found the traditional 2-parametric model not accurate for forecasting expected returns and suggest to find another method to measure volatility. The mixed data sampling (MIDAS) model will try to overcome these problems by using different data frequency and a weighting function for variance composition.

## 2 The MIDAS test

Previous models on risk-return trade-off have not showed clear and significant results. The mixed data sampling method introduces a new estimate for conditional variance, and suggests to use different frequencies of returns in the model. The variance estimation is based on the weighting of lagged squared daily returns. The mixed frequency means using daily returns for variance estimation and monthly returns for the risk-return trade-off model estimation. The results show significant and positive relationship between excess return and variance estimation.

For further understanding of the MIDAS results, we will look into a rolling window model and a GARCH model. They will show either negative or insignificant results for risk-return trade-off. We will try to separate the two features of the MIDAS: the weighted composition of variance and the mixed frequency data sampling.

As of Nelson (1991), negative shocks on return have higher effect than positive shocks. He suggests an asymmetric GARCH model to replicate this feature. We will look into the asymmetric MIDAS model, and find that volatility is less persistent

when there is a negative shock, and the effect is much higher when a positive shock occurs.

## 2.1 Model setup

In this section, we replicate the model of Ghysels et al. We use Equation (1) for estimation of conditional variance, in such a way that for expected excess return we use monthly returns and for the composition of variance we use daily returns.

For market returns, we use daily CRSP value-weighted portfolio dating from 1928 until 2000. Three-month Treasury bill rates are used as risk free rates, assuming that rates within month are constant. Returns are assumed to be continuously compounded, to make different frequency sorting simpler.

Equation (2) defines the composition of variance estimations, where MIDAS variance equals the sum of weighted lagged and squared daily returns.

$$V_t^{MIDAS} = 22 \sum_{d=0}^{251} w_d r_{t-d}^2 \quad (2)$$

Equation (3) defines the weight matched with the days lagged. It is difficult to interpret  $\kappa_1$  and  $\kappa_2$  separately, we can rather understand their joint effect on the weighting function through Figure 1. When both  $\kappa_1$  and  $\kappa_2$  are negative, the weighting function is decreasing with the number of days lagged,  $\kappa_1$  and  $\kappa_2$  being zero results in an equal weighting, and kappa2 being positive gives an increasing function.

$$w_d(\kappa_1, \kappa_2) = \frac{\exp(\kappa_1 d + \kappa_2 d^2)}{\sum_{i=0}^{251} \exp(\kappa_1 i + \kappa_2 i^2)} \quad (3)$$

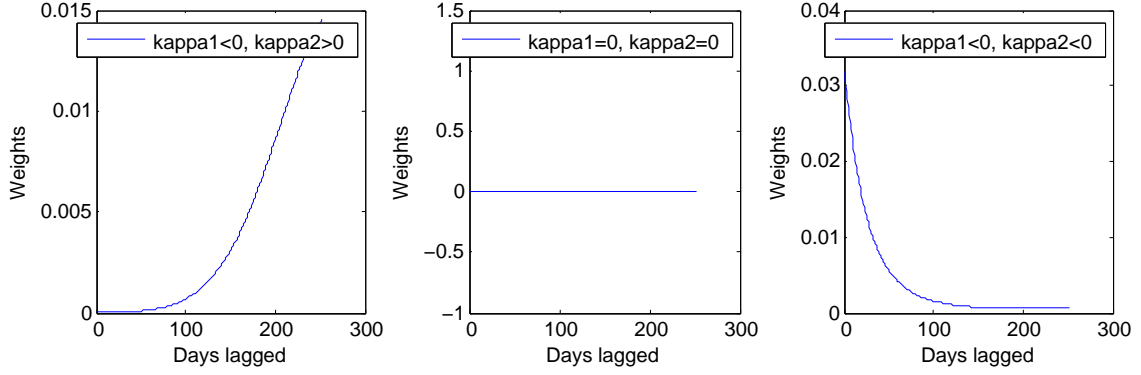


Figure 1: Weighting function for different kappa values

The parameters, namely  $\kappa_1, \kappa_2, \mu$ , and  $\gamma$  are obtained by maximising the likelihood function on Equation (1). Equation (4) is the objective log-likelihood function, assuming the following distribution for expected return:

$E_t[R_{t+1}] \sim N(\mu + \gamma V_t^{MIDAS}, V_t^{MIDAS})$ . In MATLAB, we will minimise the negative log-likelihood function.

$$LLF = -\frac{T}{2} \ln(2\pi) - \sum_{t=1}^T \ln(V_t^{MIDAS}) - \frac{1}{2} \sum_{t=1}^T \frac{(R_{t+1} - (\mu + \gamma V_t^{MIDAS}))^2}{V_t^{MIDAS}} \quad (4)$$

Error terms are calculated as follows:

$$\epsilon_t = R_{t+1} - (\mu + \gamma V_t^{MIDAS}) \quad (5)$$

In order to evaluate the significance of our results, we need t-statistics, which equals the ratio of the respective parameter and its standard error. Bollerslev and Wooldridge (2002) suggest a different way to calculate standard error to rule out heteroskedasticity. Andersen et al. (2009) find that given Gaussian distribution of the error, covariance matrix of the errors approximates the inverse Hessian of the log-likelihood. In our model, we are interested only in the diagonal of the Hessian for it captures the variance of parameters. Therefore our approximation for the t-statistic

is as follows, where  $\theta$  is the respective parameter:

$$t - stat = \frac{\theta}{\left(\frac{\partial^2 LLF}{\partial \theta^2}\right)^{-1}} \quad (6)$$

The goodness of fit measure is a good indicator to know, whether the model overall is accurate. We calculate  $R_R^2$  and  $R_{\sigma^2}^2$  from the following OLS regressions respectively:

$$\hat{R}_{t+1} = \beta_0^R + \beta_1^R V_t^{MIDAS} \quad (7)$$

$$\hat{V}_t^{Realized} = \beta_0^{\sigma^2} + \beta_1^{\sigma^2} V_t^{MIDAS} \quad (8)$$

Realized monthly variances from daily returns are calculated as follows:

$$V_t^{Realized} = \sum_{i=0}^{21} r_{t-i}^2 \quad (9)$$

This is very similar to the definition of MIDAS variance, except that MIDAS has a weighting function. With this model setup, we maximise the log-likelihood function, first for the full then the subsamples, and introduce the results for parameters in the next subsection.

## 2.2 Empirical results

Table 1. shows the descriptive statistics of the monthly data. We break the full sample into two, nearly equal subsamples: from 1928 until 1963 and from 1964 until 2000. The average monthly return in the second sample is lower than that in the first one, this is mainly due to the financial market crash in 1987. The density function of returns is slightly left-tailed, according to the negative skewness factor, and more peaked than a normal density function<sup>1</sup>. Realised variances have autocorrelation

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<sup>1</sup>A normal density function has skewness of 0 and kurtosis of 3.



with the first lag of 0.61965 for the full sample, 0.65102 for the first and 0.63862 for the second subsample.

Table 1: Descriptive statistics

	Mean	Variance	Skeweness	Kurtosis	AR(1)	AR(1-12)	T
Panel A :monthly excess returns							
1928-2000	0.005723	0.002783	-0.310730	8.835690	0.098911	0.158958	1046
1928-1963	0.007204	0.003369	-0.044042	9.532350	0.143032	0.286414	486
1964-2000	0.004425	0.002317	-0.397227	5.684860	0.029042	-0.038580	560
Panel B:monthly realized variance							
1928-2000	0.00250	0.00002	6.00452	50.13220	0.61965	3.89114	1046
1928-1963	0.00288	0.00003	4.37394	27.38530	0.65102	4.98426	486
1964-2000	0.00218	0.00002	8.05139	87.18740	0.63862	2.43840	560

Table 2. reveals the main results for the MIDAS test on conditional variance. Most importantly, we have positive  $\gamma$  of 2.55033 for the full sample, a bit lower, 1.86743 for the first subsample and higher, 2.91472 for the second subsample. In all cases  $\gamma$  is significant at 95% level<sup>2</sup>. These results are consistent with those described in the paper of Ghysels et al. Therefore, we can confirm that the MIDAS method gives consistent results with the risk-return trade-off.

Goodness of fit measures show, MIDAS values explain 30.568% of all variance in realised variance, and only 0.736% of variance in returns. Andersen et al. (2004) finds that maximum  $R^2_{\sigma_2}$  obtainable is 40%. Therefore we can conclude that our MIDAS values perform well.

As for the weighting function, we have negative values for both  $\kappa_1$  and  $\kappa_2$ , which means, the weighs are decreasing, recent data have more importance than older ones.

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<sup>2</sup>t-value=1.96

Table 2: MIDAS test

Sample	$\kappa_1$	$\kappa_2$	$\mu$	$\gamma$	$R_R^2$	$R_{\sigma^2}^2$	LLF
1928-2000	-0.00578	-0.00109	0.01678	2.55033	0.00736	0.30568	1860
t-statistic	[-0.191143]	[-2.11859]	[17.1701]	[3.20366]			
1928-1963	-0.00263	-0.00170	0.01971	1.86743	0.02239	0.48650	883
t-statistic	[-0.0508636]	[-1.40298]	[20.4604]	[2.3475]			
1964-2000	-0.00437	0.00000	0.00126	2.91472	0.00494	0.19290	676
t-statistic	[-1.02749]	[-0.113953]	[1.1515]	[3.67193]			

To understand the power of different frequencies, we run the same model on weekly, bi-monthly and quarterly returns. We modify the left side of Equation (1) by the respective frequency returns. Table 3. shows the results, which differ from those described in the paper. We found that  $\gamma$  and its significance increases along with the increase in frequency. Ghysels et al. found monthly returns to be most accurate for the trade-off model, however, weekly returns performed the best in our model.

For  $\mu$ , we have significantly different results from zero, we can conclude, that expected excess returns are positive in all cases. Goodness of fit values are low for expected returns, and higher for realised variance. In the weekly case, it exceeds the limit found by Andersen et al. (2004).

Table 3: MIDAS test with different frequencies

	$\mu$	$\gamma$	$R_R^2$	$R_{\sigma^2}^2$	LLF
Sample:1928:01-2000:12					
Weekly	0.00575 [32.5195]	6.19214 [9.98675]	0.03398	0.77650	13443
Monthly	0.01678 [17.1701]	2.55033 [3.20366]	0.00736	0.30568	1860
BiMonthly	0.03059 [16.6343]	0.72818 [1.16487]	0.03364	0.37365	904
Quarterly	0.02945 [11.9414]	1.72053 [2.84791]	0.12657	0.46458	534
Sample:1928:01-1963:12					
Weekly	0.00601 [22.8107]	5.73113 [6.78267]	0.02247	0.76926	6135
Monthly	0.01971 [20.4604]	1.86743 [2.3475]	0.02239	0.48650	884
BiMonthly	0.03580 [14.178]	0.52075 [0.606336]	0.00548	0.32325	405
Quarterly	0.04818 [13.1807]	0.13372 [0.144385]	0.02179	0.34209	234
Sample:1964:01-2000:12					
Weekly	0.00585 [22.9393]	6.38105 [6.91173]	0.02955	0.80337	7118
Monthly	0.00126 [1.1515]	2.91472 [3.67193]	0.00494	0.19290	676
BiMonthly	0.01916 [7.76529]	2.41679 [2.72954]	0.18857	0.78503	494
Quarterly	0.02307 [6.56116]	2.44880 [2.65117]	0.15384	0.55806	295

## 2.3 Alternative models: Rolling window and GARCH

To understand, why MIDAS gives these results, whereas other models in the literature are conflicting, we look into a rolling window and a GARCH model.

In the rolling window model, variances are estimated similarly to that of the MIDAS model, except for, lagged squared returns are equally weighted. In Equation (10)  $D$  stands for the width of the window. Table 4. reports the result on this model.

$$V_t^{RW} = 22 \sum_{d=0}^D \frac{1}{D} r_{t-d}^2 \quad (10)$$

Of different window widths (1 month, 2 months and 3 months), we get positive  $\gamma$  only for the 1 month horizon, but it is not significant. In the other two cases, only negative relation can be shown. The one month rolling window with 22 days width is the realised variance itself. The wider the rolling window, the more significant is the  $\gamma$  parameter. This also strengthens the use of MIDAS length of lag, which is 1 year.

Table 4: Rolling window test

Horizon (month)	$\mu$	$\gamma$	$R_R^2$	$R_{\sigma^2}^2$	LLF
1	0.006187	0.001259	0.004609	1.000000	1365
t-statistics	[6.62818]	[0.00160855]			
2	0.008235	-1.466160	0.007855	0.446790	1339
t-statistics	[8.38596]	[-1.86024]			
3	0.008670	-1.723250	0.005672	0.354280	1331
t-statistics	[8.82571]	[-2.20794]			

The GARCH model is a generalised autoregressive model on conditional heteroskedasticity, which captures the heterogeneity of volatility by generating variance estimates depending on its lags and its lagged error terms. In our model, we use

only the first lag, as Equation (11) shows:

$$V_t^{GARCH} = \omega + \alpha\epsilon_t^2 + \beta V_{t-1}^{GARCH} \quad (11)$$

where  $\epsilon_t = R_t - \mu - \gamma V_{t-1}^{GARCH}$ . Similarly to the previous models, we estimate the parameters by maximising the log-likelihood function.

Furthermore, we will also look into the absolute GARCH model, which uses the absolute value of errors, instead of their squares.

$$(V_t^{ABSGARCH})^{1/2} = \omega + \alpha|\epsilon_t| + \beta(V_{t-1}^{ABSGARCH})^{1/2} \quad (12)$$

Table 5. reports the parameter estimations on GARCH and ABSGARCH model. Unfortunately, the ABSGARCH model did not produce the expected results with its negative and insignificant  $\gamma$ . The original GARCH is consistent with the findings of French et al. (1987), we got positive but small, and insignificant value for  $\gamma$ .

Table 5: GARCH test

Model	$\omega$	$\alpha$	$\beta$	$\mu$	$\gamma$	$R_R^2$	$R_{\sigma^2}^2$	LLF
GARCH	0.000600 [13.0988]	0.119078 [15.118]	0.265130 [6.59304]	0.010122 [7.31156]	0.718522 [0.618613]	0.057939	0.288146	2404
ABS-G	0.009963 [130.412]	0.488443 [127.844]	-0.186792 [-162.787]	0.001665 [9.28779]	-0.031722 [-0.685206]	0.447088	0.027287	3270

## 2.4 Comparison of conditional variance models

In the previous section, we generated variance estimates by rolling window and GARCH method. Figure 2. shows how MIDAS estimates and the above mentioned two alternative models fit realised variance. The most poorly, GARCH performs, it cannot capture the high peaks of realised variance, while rolling window and MIDAS estimates fits more smoothly.

As in Table 6. MIDAS show the best performance among the three conditional variance models. Standard error variance is the lowest with 1.56195, and errors' distribution is the closest to a normal distribution, with skewness of -0.4481 and kurtosis of 3.96454. The goodness of fit measure, which results from a zero constant regression with realised variance, MIDAS also scores highest with 52.9256%.

The correlation matrix in Panel C also shows that MIDAS correlates more with the other models, than the other models with each other. Correlation value between MIDAS-GARCH is 0.550079, between MIDAS-RW is 0.840884 (comparing with RW-GARCH of 0.465853). Also, MIDAS correlates the most with realised variance, as we could already see on Figure 2.

Table 6: Comparison of conditional variance models

PANEL A Summary statistics				
	Mean	Variance	AR(1)	AR(12)
MIDAS	0.00182708	3.24E-06	0.990728	0.744684
GARCH	0.00120498	4.48E-07	0.541216	0.318232
RW	0.00184495	4.91E-06	0.751231	0.508727
PANEL B Performance of conditional variance models				
	Var of std.error	Skewness	Kurtosis	Goodness of fit
MIDAS	1.56195	-0.4481	3.96454	0.529256
GARCH	1.89605	-0.579118	4.09048	0.363861
RW	1.94116	-0.514057	4.1204	0.44679
PANEL C Correlation matrix				
	MIDAS	GARCH	RW	Realized
MIDAS	1	0.530079	0.840884	0.773596
GARCH	0.530079	1	0.465853	0.67406
RW	0.840884	0.465853	1	0.667228
Realized	0.773596	0.67406	0.667228	1

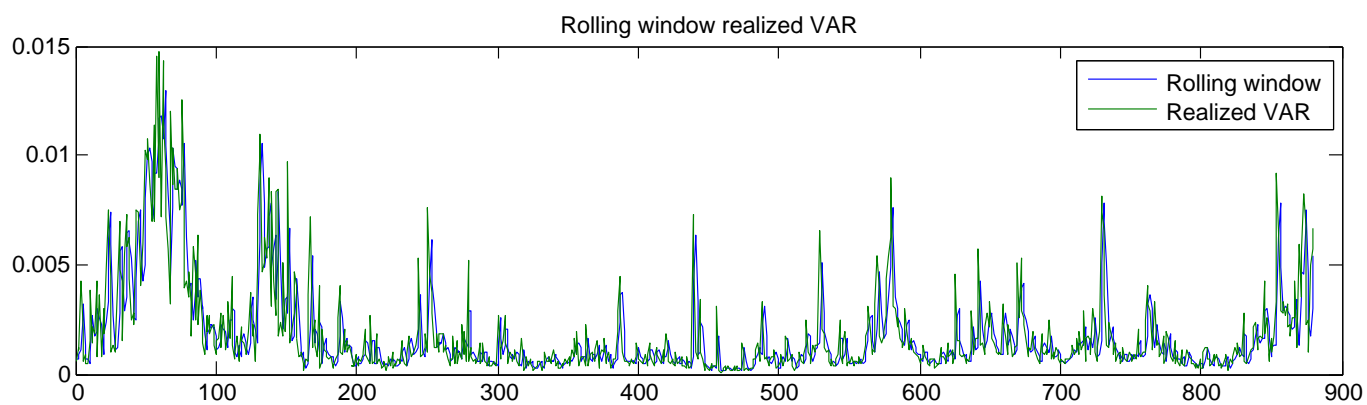
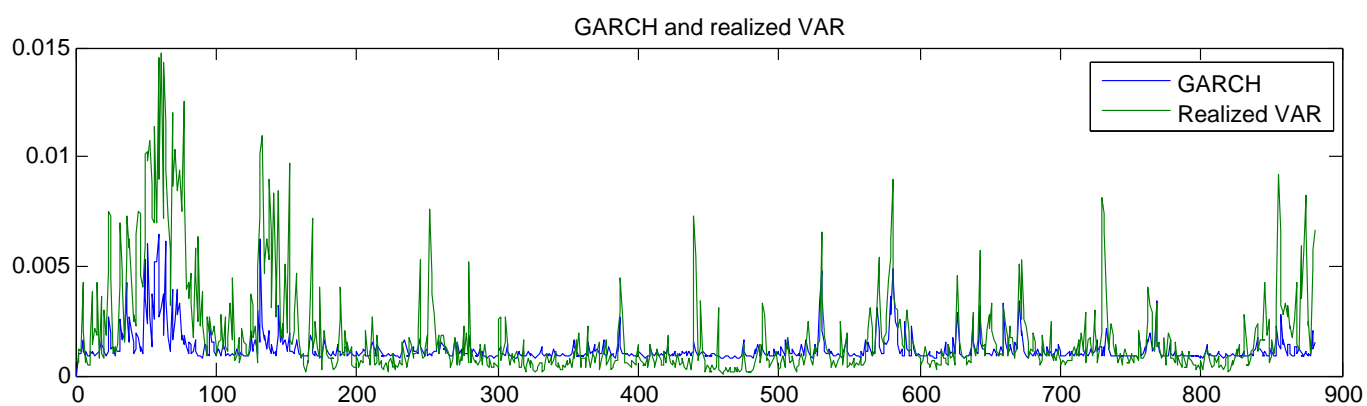
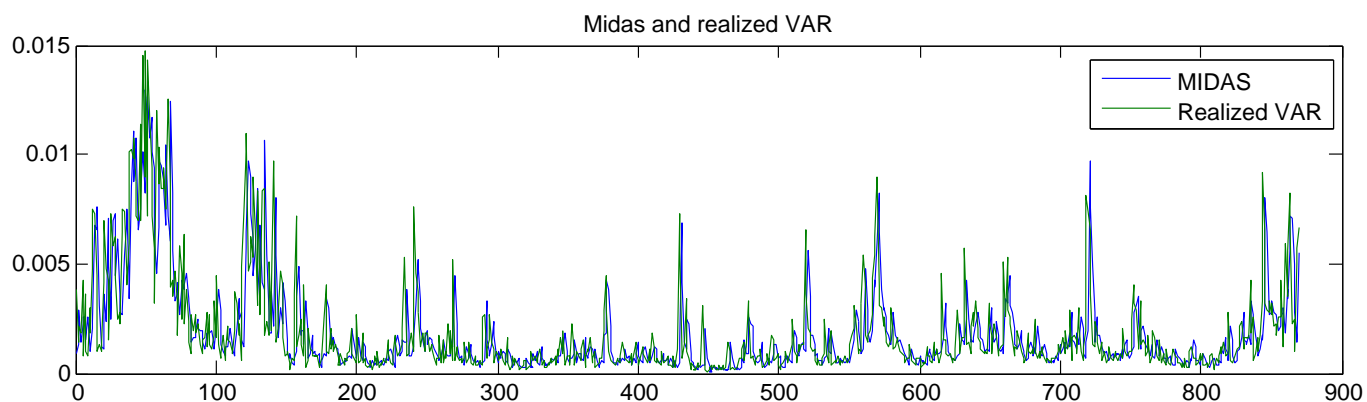


Figure 2: Variance estimations with realised variance

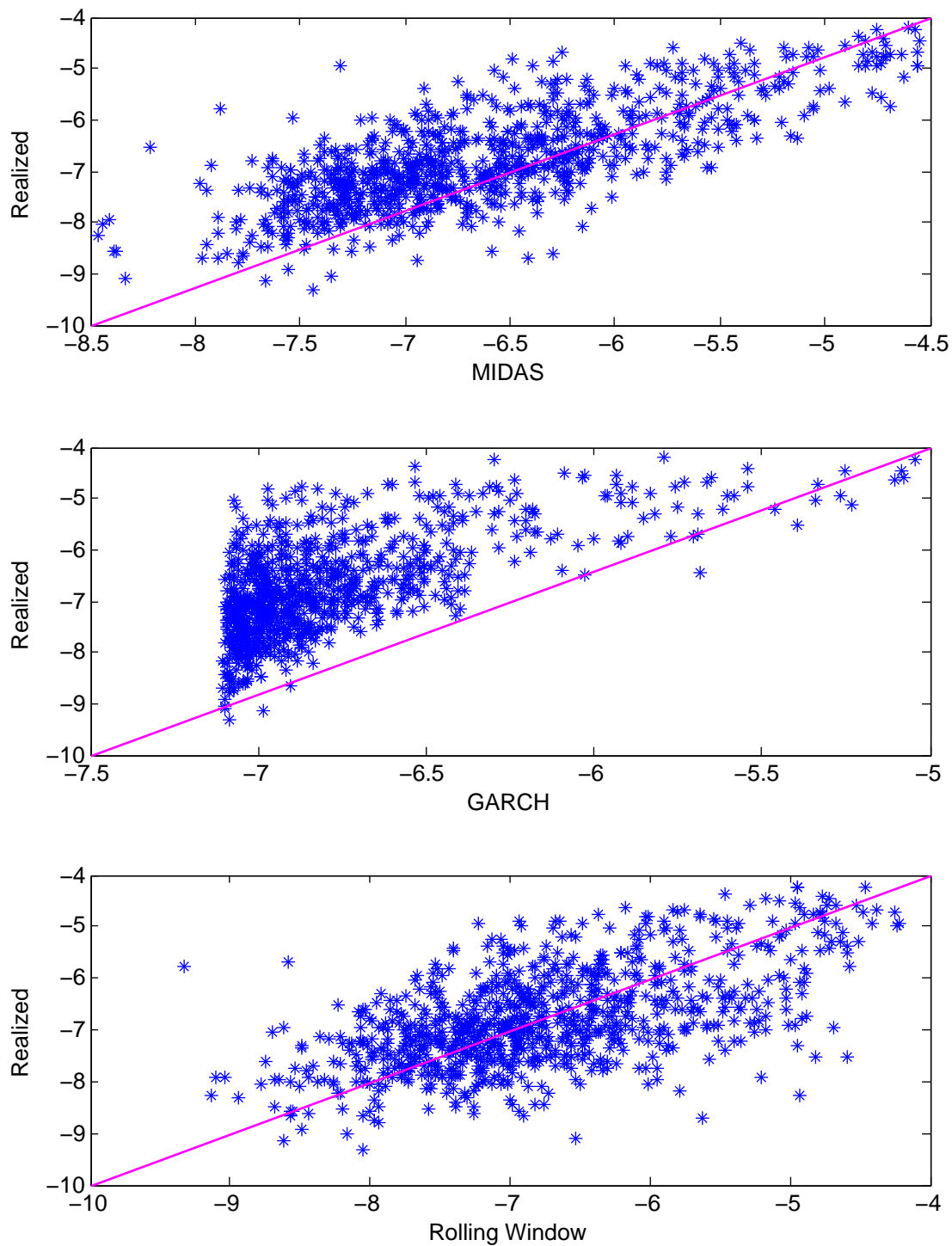


Figure 3: Scatterplot of log forecasted variances versus log realised variance



Figure 3. is a scatterplot on log forecasted variances versus log realised variances. GARCH variances systematically underestimate realised variances, while MIDAS and Rolling window variances fit the 1:1 line better. Rolling window variances are, however, a bit more dispersed than those of MIDAS.

The better performance of MIDAS is due to its two main differences: the mixed frequency data, and its flexible weighting function. Therefore, we try to look into each effect separately, by controlling the other. We will compare the original MIDAS with a mixed frequency GARCH, and then the original GARCH with a non-mixed frequency MIDAS.

Table 7. reports the results on these controlled models. To examine the advantage of the flexible weighting function, we compare a daily GARCH function with the original MIDAS model. From daily GARCH variances we estimate monthly variances by summing up 22 days, this way we can produce a mixed frequency GARCH model, and control for the frequency to see the separate effect of the weighting function. The daily GARCH model is obtained by modifying Equation (11) by daily returns. We get positive but insignificant  $\gamma$  of 1.00927, which is a poorer result than that of the original MIDAS.

Controlling for the weighting function, we modify the MIDAS model and use monthly returns to estimate variance. The lag this time is again one year, more exactly 12 months. This way, we eliminated the weighting function, although mixing frequency has also disappeared, we still consider this model as a MIDAS model.

$$V_t^{MIDAS} = \sum_{d=0}^{11} w_d r_{t-d}^2 \quad (13)$$

Comparing this model to the original GARCH model, we can assess the gains of mixing frequency. We get significant and positive  $\gamma$  of 2.05778, which is definitely a better result than the small and insignificant  $\gamma$  of the GARCH model. We can confirm, that the two main features: mixed frequency and flexible weighting function are both playing key roles in getting the expected positive and significant relationship of risk-return trade-off.

Table 7: Daily GARCH and monthly MIDAS test

MODEL: Daily GARCH							
$\omega$	$\alpha$	$\beta$	$\mu$	$\gamma$	$R_R^2$	$R_{\sigma^2}^2$	LLF
0.00020	0.12186	0.12664	-0.01373	1.00927	0.05924	0.66280	1822
[17.8441]	[3.60632]	[3.03306]	[-19.635]	[0.518328]			
MODEL: Monthly MIDAS							
$\kappa_1$	$\kappa_2$	$\mu$	$\gamma$	$R_R^2$	$R_{\sigma^2}^2$	LLF	
0.11794	-0.05703	0.01646	2.05778	0.01326	0.24807	1796	
[0.916336]	[-3.53491]	[13.837]	[2.9762]				

## 2.5 Model extension

In this section we will discuss several ways to extend the MIDAS model. As of Campbell and Hentschel (1992), they emphasise that there is an asymmetry in negative and positive returns regarding the effect of volatility. Given decreasing trend, there is a higher probability of a big negative jump, when volatility is high, than given increasing trend, to have a big positive jump. Therefore, Ghysels et al. (2004) suggest an asymmetric MIDAS model, which allows to capture the effect of negative and positive returns separately, with separate parameters.

$$V_t^{ASYMIDAS} = 22 \left[ \phi \sum_{d=0}^{251} w_d(\kappa_1^-, \kappa_2^-) \mathbf{1}_{\mathbf{t-d}}^- r_{t-d}^2 + (2 - \phi) \sum_{d=0}^{251} w_d(\kappa_1^+, \kappa_2^+) \mathbf{1}_{\mathbf{t-d}}^+ r_{t-d}^2 \right] \quad (14)$$

In Equation (14),  $\phi$ 's role is similar to a weight of negative shocks, it stays in the interval of (0, 2). To separate the positive and negative returns, we introduce the  $\mathbf{1}_{\mathbf{t-d}}^-$  and  $\mathbf{1}_{\mathbf{t-d}}^+$  vectors, which are indicator functions for negative and positive returns respectively.

Table 8: Asymmetric MIDAS test

	$\mu$	$\gamma$	$\kappa_1^-$	$\kappa_2^-$	$\kappa_1^+$	$\kappa_2^+$	$\phi$	$R_R^2$	$R_{\sigma^2}^2$	LLF
Full	0.006464	6.37	0.376	-4.72E-03	0.618	-0.13030	0.0423	0.0519	0.3839	2338
	[11.209]	[10.05]	[4.7553]	[-4.85114]	[10.334]	[-14.865]	[3.1]			
1st	9.42E-03	5.53	6.79E	-0.655	0.784	-0.157	0.028	0.0855	0.3169	1056
	[10.4951]	[6.17]	[9.8531]	[-9.68706]	[9.9175]	[-12.537]	[2.24]			
2nd	5.73E-03	7.029	1.50	-0.0116	0.749	-0.136	0.0499	0.041	0.1724	1257
	[6.45036]	[7.45]	[11.4686]	[-11.8525]	[7.0985]	[-11.843]	[2.49]			

In our model, as Table 8. reports, we find  $\phi$  is 0.04226 for the full sample and significant, that is, positive shocks have much higher effect on conditional variance, than negative shocks. Also, as Figure 4 shows, negative shocks have most of the weights on the most recent returns, and are less persistent than positive shocks. Furthermore,  $\gamma$  is higher in this model, it is 6.37 and significant for the full sample.

Another way to extend the MIDAS model is to include other variables in the ICAPM equation, such as variables related to business cycle. In Equation (15),  $Z$  stands for the predictive variables, and  $\theta$  denotes their coefficients. Unfortunately, in this paper, specific models will not be discussed.

$$E_t[R_{t+1}] = \mu + \gamma VAR_t[R_{t+1}] + \theta^T Z_t \quad (15)$$

MIDAS also appears in other models, since it is a flexible and easily applicable model. Colacito et al. (2010) use MIDAS for multi-asset portfolios to describe a model for dynamic correlations, but it is also commonly used when sampled data frequency differs within the model.

Ghysels et al. (2006) suggest several structures for the weighting function and also verify the results in Ghysels (2004) on a different, shorter database. They find, similarly to the previous paper, significant and positive  $\gamma$ .

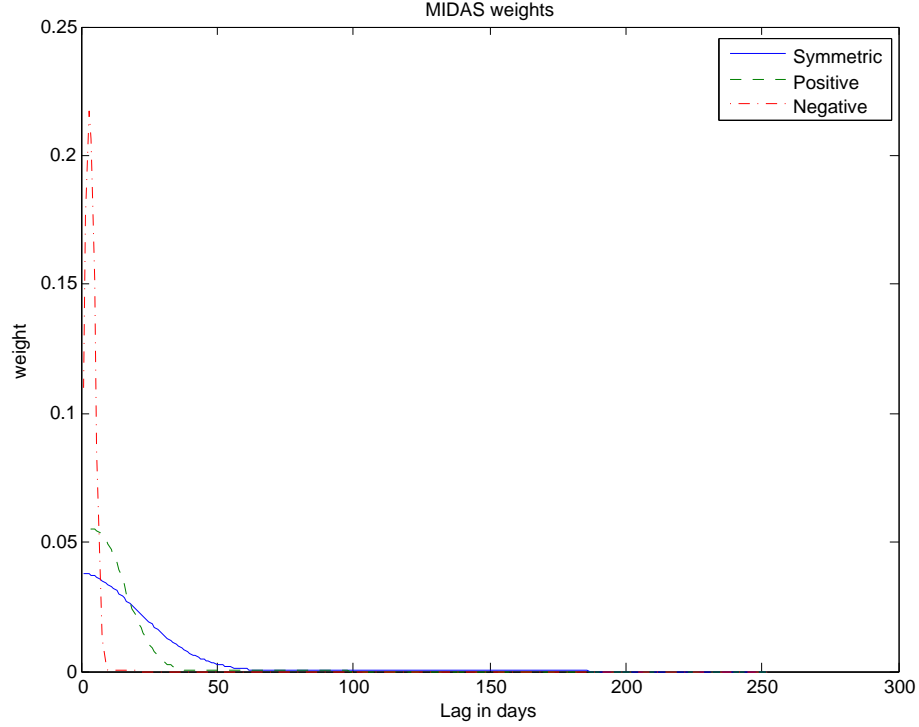


Figure 4: Weighting function for symmetric and asymmetric MIDAS

### 3 Conclusion

The MIDAS model provides with a powerful method to confirm the risk-return trade-off described by the standard model, the ICAPM. In this paper, we confirmed the results of Ghysels et al. (2004), that is, conditional variance is a positive and significant factor in forecasting excess return. The magnitude of the risk aversion factor,  $\gamma$ , increases with the increase of frequency of data sampling.

Comparing with similar models, the rolling window and the GARCH model, we find that MIDAS performs the best, since rolling window variance estimation gives negative result for  $\gamma$ , while GARCH gives positive but insignificant value for the risk factor.

The advantages of MIDAS originates from its two features: the data frequency

mixing and the flexible weighting function. Controlling for one to understand the effect of the other, we find that both features are powerful and result in a significant and positive  $\gamma$ .

In order to separate the effects of negative and positive shocks, we replicated the asymmetric MIDAS model. We can conclude, that positive shocks have much bigger impact on the conditional variance, however, negative shocks have big, immediate effect.

We confirm, that the MIDAS model is powerful and its results are consistent with expectations of the standard model. The mixed data sampling is therefore an important method for conditional variance estimation.

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