# CSE 301 – DATABASE

## *Lecture 10*

# Chapter 5: Relational Database Design

# Functional Dependencies

- A functional dependency (FD) is a relationship or association between two attributes (typically between the Primary Key and other non-key attributes within a table).
- For any relation R, attribute Y is functionally dependent on attribute X (usually the Primary Key), if for every valid instance of X, that value of X uniquely determines the value of Y.
- This relationship is indicated by $X \rightarrow Y$
- The left side of the above FD diagram is called the determinant, and the right side is the dependent.

# Examples of Functional Dependencies

R

| A | B |
|---|---|
| 1 | 6 |
| 2 | 7 |
| 3 | 8 |
| 4 | 9 |

- A → B, if for every valid instance of A, that value of A uniquely determines the value of B.
- $\{1 \rightarrow 6\}, \{2 \rightarrow 7\}, \{3 \rightarrow 8\}, \{4 \rightarrow 9\}$
- *B has the same value for the same value as A.*

R

| A | B |
|---|---|
| 1 | 6 |
| 2 | 7 |
| 3 | 8 |
| 2 | 9 |

- A $\not\rightarrow$ B, if for every valid instance of A, that value of A not uniquely determines the value of B.
- $\{1 \rightarrow 6\}, \{2 \rightarrow 7\}, \{3 \rightarrow 8\}, \{2 \rightarrow 9\}$
- *B has not the same value for the same value as A.*

# Functional Dependencies

- We can mathematically represent $X \rightarrow Y$,
  - When $X \subseteq R$ and $Y \subseteq R$
  - If $T_1[X] = T_2[X]$, then $T_1[Y] = T_2[Y]$

R

| | X | Y |
|---|---|---|
| T1 | 1 | 6 |
| | 2 | 7 |
| T2 | 1 | 6 |
| | 4 | 9 |

# Examples of Functional Dependencies

- Which of the following FD is not valid?

  a) $A \rightarrow B$

  b) $B \rightarrow C$

  c) $BC \rightarrow A$

  d) $AC \rightarrow B$

R

| A | B | C |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 2 | 3 |
| 5 | 3 | 3 |

- $A \rightarrow B$, $\{1 \rightarrow 2\}, \{4 \rightarrow 2\}, \{5 \rightarrow 3\}$ ***Valid***
- $B \rightarrow C$, $\{2 \rightarrow 3\}, \{2 \rightarrow 3\}, \{3 \rightarrow 3\}$ ***Valid***
- $BC \rightarrow A$, $\{2,3 \rightarrow 1\}, \{2,3 \rightarrow 4\}, \{3,3 \rightarrow 5\}$ ***Not Valid***
  - *Right hand side has not the same value for the same value as left hand side.*
- $AC \rightarrow B$, $\{1,3 \rightarrow 2\}, \{4,3 \rightarrow 2\}, \{5,3 \rightarrow 3\}$ ***Valid***

# Examples of Functional Dependencies

- Which of the following FD is valid?

  a) $XY \to Z,\ Z \to Y$

  b) $XZ \to X,\ Y \to Z$

  c) $YZ \to X,\ Z \to X$

  d) $XZ \to Y,\ Y \to Z$

R

| X | Y | Z |
|---|---|---|
| 1 | 4 | 3 |
| 1 | 5 | 3 |
| 4 | 6 | 3 |
| 3 | 2 | 2 |

c) $YZ \to X,\ Z \to X$

- $YZ \to X$
  - $\{4{,}3 \to 1\}, \{5{,}3 \to 1\}, \{6{,}3 \to 4\}, \{2{,}2 \to 3\}$
- $Z \to X$
  - $\{3 \to 1\}, \{3 \to 1\}, \{3 \to 4\}, \{2 \to 3\}$

**Option c is not valid**

a) $XY \to Z,\ Z \to Y$

- $XY \to Z$
  - $\{1{,}4 \to 3\}, \{1{,}5 \to 3\}, \{4{,}6 \to 3\}, \{3{,}2 \to 2\}$
- $Z \to Y$
  - $\{3 \to 4\}, \{3 \to 5\}, \{3 \to 6\}, \{2 \to 2\}$

**Option a is not valid**

b) $XZ \to X,\ Y \to Z$

- $XZ \to X$
  - $\{1{,}3 \to 1\}, \{1{,}3 \to 1\}, \{4{,}3 \to 4\}, \{3{,}2 \to 3\}$
- $Y \to Z$
  - $\{4 \to 3\}, \{5 \to 3\}, \{6 \to 3\}, \{2 \to 2\}$

**Option b is valid**

d) $XZ \to Y,\ Y \to Z$

- $XZ \to X$
  - $\{1{,}3 \to 4\}, \{1{,}3 \to 5\}, \{4{,}3 \to 6\}, \{3{,}2 \to 2\}$
- $Y \to Z$
  - $\{4 \to 3\}, \{5 \to 3\}, \{6 \to 3\}, \{2 \to 2\}$

**Option d is not valid**

# Exercise on Functional Dependencies

- Which of the following FD is correct?
  - a) $A \rightarrow BC$
  - b) $DE \rightarrow C$
  - c) $C \rightarrow DE$
  - d) $BC \rightarrow A$

R

| A | B | C | D | E |
|---|---|---|---|---|
| A | 2 | 3 | 4 | 5 |
| 2 | A | 3 | 4 | 5 |
| A | 2 | 3 | 6 | 5 |
| a | 2 | 3 | 6 | 6 |

- Which of the following FD is not correct?
  - a) $XY \rightarrow Z,\ Z \rightarrow Y$
  - b) $YZ \rightarrow X,\ Y \rightarrow Z$
  - c) $XZ \rightarrow X,\ Z \rightarrow X$
  - d) $XZ \rightarrow Y,\ Y \rightarrow Z$

R

| X | Y | Z |
|---|---|---|
| 1 | 4 | 2 |
| 1 | 5 | 3 |
| 1 | 6 | 3 |
| 3 | 2 | 2 |

# Classification of Functional Dependencies

- Trivial functional dependency
  - ✓ A → B has trivial functional dependency if B is a subset of A (B ⊆ A ).
  - ✓ Examples: ***A* → *A*, *AB* → *B*, *{Employee_id, Employee_Name}* → *Employee_Id***
- Non-trivial functional dependency
  - ✓ A → B has a non-trivial functional dependency if B is not a subset of A.
  - ✓ If there is at least one attribute in right hand side that is not present in the left hand side.
  - ✓ When A intersection B is NULL, then A → B is called as complete non-trivial.
  - ✓ Examples: ***AB* → *BC*, *{ID}* → *{ID, DOB}*, *{Roll, Name}* → *{Roll, Name, Phone}***

# Classification of Functional Dependencies

- Fully functional dependency
  - ✓ Given R and A → B, then B is fully functional dependent on A if there is no Z where Z is a proper subset of A ($Z \subset A$) such that Z → B.
  - ✓ Examples: {*AB → C, A → D*} ***is a fully FD,*** {*AB → C, A → C*} ***is not a fully FD***
- Partial functional dependency
  - ✓ Given a relation R with FD F defined on the attributes of R and K as a candidate key, if A is a proper subset of K and if and only if X → A, then A said to be partially dependent on K.
  - ✓ Examples: *R(ABCD), C.K. → AB,* ***FD: {A → C}.*** *C is partially dependent on A.*

# Classification of Functional Dependencies

- Transitive functional dependency
  - ✓ If $A \rightarrow B$ and $B \rightarrow C$, then C is transitively functional dependent on A such that $A \rightarrow C$.
  - ✓ Examples: $\{AB \rightarrow C, C \rightarrow D\}$, **So,** $\{AB \rightarrow D\}$

# Armstrong's axioms / Inference Rules

- Reflexive Rule (IR1)
  - ✓ If Y is a subset of X (X ⊇ Y ), then X determines Y (X → Y).
  - ✓ Examples: {*AB → A*}, *{Employee_id, Employee_Name} → {Employee_Name}*
- Augmentation Rule (IR2)
  - ✓ If X determines Y (X → Y), then XZ determines YZ (XZ → YZ). for any Z.
  - ✓ Examples: *R(ABCD), if A → B then AC → BC*
- Transitive Rule (IR3)
  - ✓ If X determines Y (X → Y) and Y determine Z (Y → Z), then X must also determine Z (X → Z).

# Armstrong's axioms / Inference Rules

- Union Rule (IR1)
  - ✓ If X determines Y ($X \rightarrow Y$) and X determines Z ($X \rightarrow Z$), then X must also determine Y and Z ($X \rightarrow YZ$).
  - ✓ Examples: *R(ABCD), if $A \rightarrow B$ and $A \rightarrow C$ then $A \rightarrow BC$.*
- Decomposition Rule (IR2)
  - ✓ If X determines Y and Z ($X \rightarrow YZ$), then X determines Y ($X \rightarrow Y$) and X determines Z ($Y \rightarrow Z$) separately.
  - ✓ Examples: *R(ABCD), if $A \rightarrow BC$, then $A \rightarrow B$ and $A \rightarrow C$*
- Pseudo Transitive Rule (IR3)
  - ✓ If X determines Y ($X \rightarrow Y$) and YZ determines W ($YZ \rightarrow W$) , then XZ determines W ($XZ \rightarrow W$).

# Closure Set of Attributes

- $X^+$ is the set of all attributes that can be determine using the given set X(attributes).

Or

- The Closure Of Functional Dependency means the complete set of all possible attributes that can be functionally derived from given functional dependency using the inference rules known as Armstrong's Rules.

- *If "F" is a functional dependency then closure of functional dependency can be denoted using "{F}+".*

# Closure Set of Attributes

- There are three steps to calculate closure of functional dependency. These are:
  - ✓ Step-1 : Add the attributes which are present on Left Hand Side in the original functional dependency.
  - ✓ Step-2 : Now, add the attributes present on the Right Hand Side of the functional dependency.
  - ✓ Step-3 :
    - With the help of attributes present on Right Hand Side, check the other attributes that can be derived from the other given functional dependencies.
    - Repeat this process until all the possible attributes which can be derived are added in the closure.

# Example of Closure Set of Attributes

- Consider the table Student_details having (Roll_no, Name, Marks, Location) as the attributes and having two functional dependencies.

  FD1 : Roll_no → Name, Marks

  FD2 : Name → Marks, Location

- Now, we will calculate the closure of all the attributes present in the relation using the three steps mentioned below. Find closure set of attributes of{Roll_no}+
- Step-1 : add attributes present on the LHS of the first functional dependency to the closure.

  {Roll_no}+ = {Roll_no}

- Step-2 : add attributes present on the RHS of the original functional dependency to the closure.

  {Roll_no}+ = {Roll_no, Name, Marks}

# Example of Closure Set of Attributes

- Step-3 : Add the other possible attributes which can be derived using attributes present on the RHS of the closure.
  - So, Roll_No attribute cannot functionally determine any attribute, but Name attribute can determine other attributes such as Marks and Location using 2nd Functional Dependency.
  - Therefore, complete closure of Roll_No will be :

    {Roll_no}+ = {Roll_No, Marks, Name, Location}

# Example of Closure Set of Attributes

- Example 1: R (ABCDEFG)
  - $\{A \rightarrow B,\ BC \rightarrow DE,\ AEG \rightarrow G\}$
  - Find $(AC)^+ = ?$

- $(AC)^+ = AC$

$= ABC \quad (A \rightarrow B)$

$= ABCDE \quad (BC \rightarrow DE)$

$= ABCDE$

- Example 2: R (ABCDE)
  - $\{A \rightarrow BC,\ CD \rightarrow E,\ B \rightarrow D,\ E \rightarrow A\}$
  - Find $(B)^+ = ?$

- $(B)^+ = B$

$= BD \quad (B \rightarrow D)$

$= BD$

- Example 3: R (ABCDEF)
  - $\{AB \rightarrow C,\ CD \rightarrow E,\ DE \rightarrow B\}$
  - Find $(AB)^+ = ?$

- $(AB)^+ = AB$

$= ABC \quad (AB \rightarrow C)$

# Example of Closure Set of Attributes

- Example 1: R (ABCDEFGH)
  - {A $\rightarrow$ BC, CD $\rightarrow$ E, E $\rightarrow$ C, D $\rightarrow$ AEH, ABH $\rightarrow$ BD, DH $\rightarrow$ BC}
  - Find $(BCD)^+ \rightarrow H$ ?
  - $(BCD)^+ = BCD$

  = BCDE (CD $\longrightarrow$ E)

  = ABCDEH (D $\rightarrow$ AEH)

  So, $(BCD)^+ \rightarrow H$ is valid

- Exercise 1: R (ABCDEF)
  - {A $\rightarrow$ BC, CD $\rightarrow$ EF, B $\rightarrow$ D, E $\rightarrow$ A}
  - Find $(AE)^+$ = ?

- Exercise 2: R (ABCDEF)
  - {AB $\rightarrow$ C, CD $\rightarrow$ E, DE $\rightarrow$ B}
  - Find $(AB)^+$ = ?

- Exercise 3: Student (Roll, Name, DoB, Phone, Course)
  - {Roll, Name} $\rightarrow$ Phone, {Course, DoB} $\rightarrow$ Roll, Course $\rightarrow$ Name, Roll $\rightarrow$ Name}
  - Find $(Course)^+$ = ?

# Equivalence Of Functional Dependencies

- R (ACDEH)

- F: {$A \to C$, $AC \to D$, $E \to AD$, $E \to H$}
- G: {$A \to CD$, $E \to AH$}

- $(A)^+ = ACD$
- $(AC)^+ = ACD$
- $(E)^+ = EAH = EAHCD$
- So, $F \subseteq G$

- $(A)^+ = ACD$
- $(E)^+ = EADCH$

So, $G \subseteq F$

- Find the correct option

a) $F \subseteq G$

b) $G \subseteq F$

c) $F = G$

d) $F \neq G$

- $F \subseteq G$ and $G \subseteq F$, so, $F = G$

# Equivalence Of Functional Dependencies

- R (ACDEH)
- F: $\{A \rightarrow C,\ AC \rightarrow D,\ E \rightarrow AD,\ E \rightarrow H\}$
- G: $\{A \rightarrow CD,\ E \rightarrow AH\}$
- Check both FD's are equivalent or not.

- R (ABC)
- F: $\{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}$
- G: $\{A \rightarrow BC,\ B \rightarrow A,\ C \rightarrow A\}$
- Find the correct option

a) $F \subseteq G$

b) $G \subseteq F$

c) $F = G$

d) $F \neq G$

- R (VWXYZ)
- F: $\{W \rightarrow X,\ WX \rightarrow Y,\ Z \rightarrow WY,\ Z \rightarrow V\}$
- G: $\{W \rightarrow XY,\ Z \rightarrow WX\}$
- Find the correct option

a) $F \subseteq G$

b) $G \subseteq F$

c) $F = G$

d) $F \neq G$