Chapter 3 - Math 447

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3.11 The tensile strength of Portland cement is being studied. Four different mixing techniques can be used economically.A completely randomized experiment was conducted and the following data were collected:

```
my_data=read.table("/Users/nhivutu/Desktop/Math 447/dataset_chapter3/problem11.txt", header = TRUE)
my_data$Technique<-as.factor(my_data$Technique)
attach(my_data)
```

a) Test the hyphothesis that mixing techinques affect the strength of the cement. Use alpha= 0.05

H_0: Mixing techinques do not affect the strength of the cement vs H_1: Mixing techinques affect the strength of the cement.

```
model=aov(Tensile_Strength~Technique,data=my_data)
summary(model)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)    
## Technique    3 489740  163247   12.73 0.000489 ***
## Residuals   12 153908   12826                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since p_value is statistically significant (> 0.05), we reject H_0, then there is a sufficient evident to conlucde that mixing techinques affect the strength of the cement.

(b) Construct a graphical display as described in Section3.5.3 to compare the mean tensile strengths for the four mixing techniques. What are your conclusions?

Find the scale factor t:

```
MSe=summary(model)[[1]][2,3]
n=16
S_yibar=sqrt(MSe/n)
```

Find mean of each group

```
library(dplyr)
```

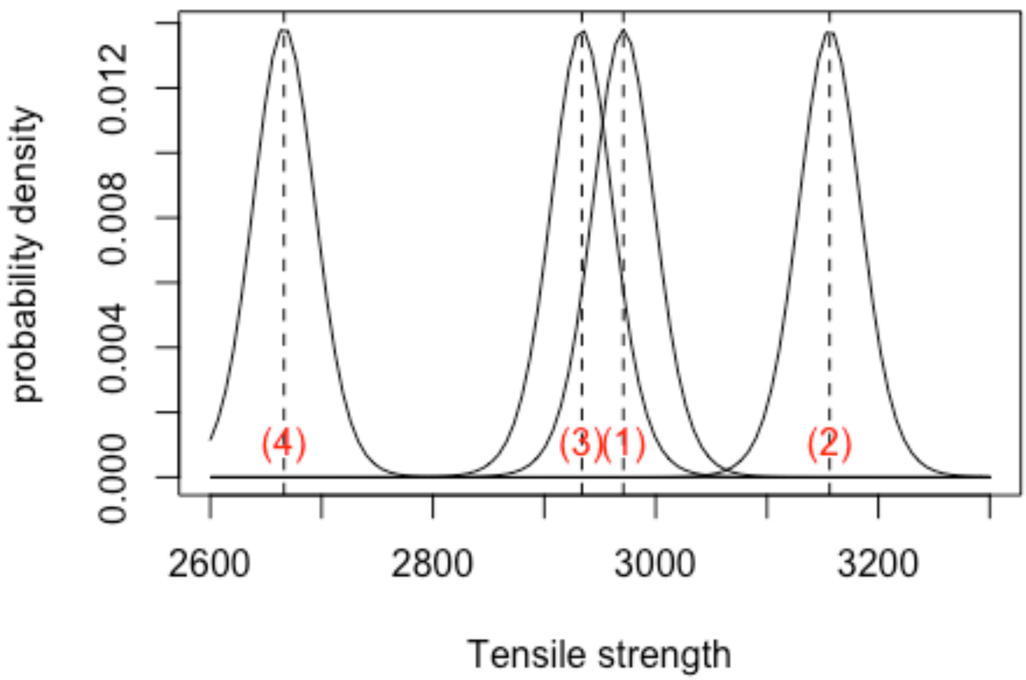
```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
group_by(my_data,Technique) %>%
  summarise(
    count = n(),
    mean = mean(Tensile_Strength, na.rm = TRUE),
    sd = sd(Tensile_Strength, na.rm = TRUE)
  )
```

```
## # A tibble: 4 x 4
##   Technique count  mean    sd
##   <fct>     <int> <dbl> <dbl>
## 1 1             4 2971  121.
## 2 2             4 3156. 136.
## 3 3             4 2934. 108.
## 4 4             4 2666.  81.0
```

Graph

```
#graphical comparisions means
# Technique 1
curve(dt((x - 2971)/S_yibar, df = 12)/S_yibar, from =
min(my_data$Tensile_Strength), to = max(my_data$Tensile_Strength),
ylab="probability density", xlab="Tensile strength")
abline(v = 2971, lty = 2)
text(x=2971,y=0.001,labels="(1)",col="red")
# Technique 2
curve(dt((x - 3156.25)/S_yibar, df = 12)/S_yibar,add = TRUE)
abline(v = 3156.25, lty = 2)
text(x=3156.25,y=0.001,labels="(2)",col="red")
# Technique 3
curve(dt((x - 2933.75)/S_yibar, df = 12)/S_yibar,add = TRUE)
abline(v = 2933.75, lty = 2)
text(x=2933.75,y=0.001,labels="(3)",col="red")
# Technique 4
curve(dt((x - 2666.25)/S_yibar, df = 16)/S_yibar, add = TRUE)
abline(v = 2666.25, lty = 2)
text(x=2666.25,y=0.001,labels="(4)",col="red")
```



There is a significant difference between the pairs “Mixing Technique 4 and Mixing Technique 3”, “Mixing Technique 4 and Mixing Technique 1” and “Mixing Technique 4 and Mixing Technique 2”.

There is a significant difference between the pair “Mixing Technique 3 and Mixing Techniques 2”

There is a significant difference between the pair “Mixing Technique 1 and Mixing Technique 2”.

c) Use the Fisher LSD method with alpha=0.05 to make comparisions between pairs of means.

Find LSD:

$$LSD = t_{\alpha/2,df}\sqrt{MSe(\frac{1}{n}+\frac{1}{a})}$$

```
LSD=qt(0.05/2,12,lower.tail = F)*sqrt(MSe*(1/4+1/4))
LSD

## [1] 174.4798
```

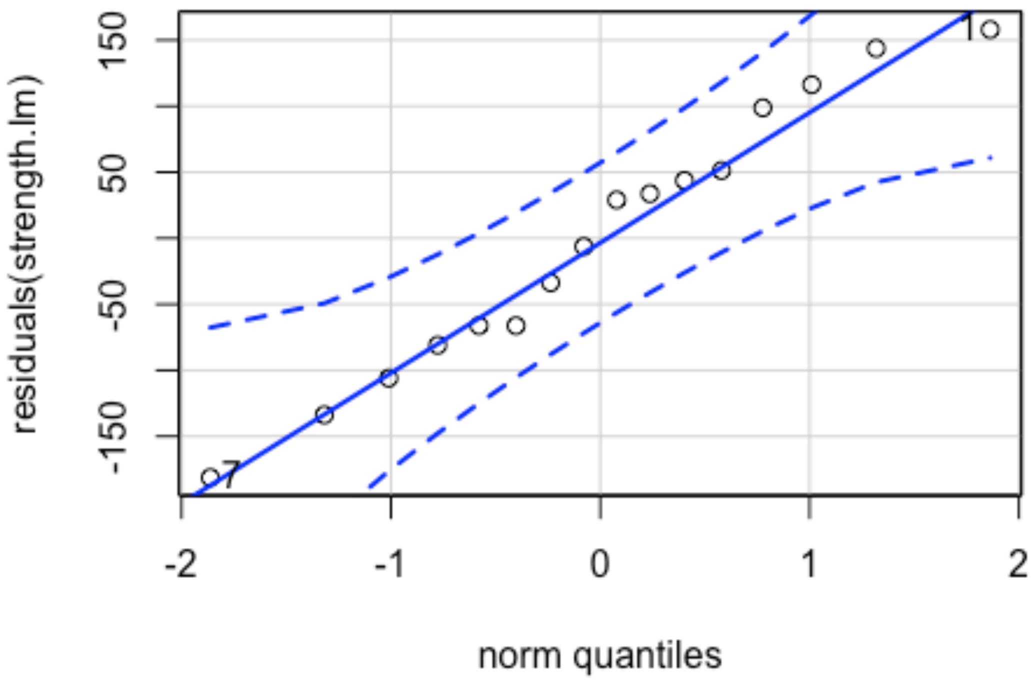
Comparison

```
# Mixing Technique 2 versus Mixing Technique 4
abs(3156.25-2666.25)>LSD

## [1] TRUE

# Mixing Technique 2 versus Mixing Technique 3
abs(3156.25-2933.75)>LSD

## [1] TRUE

# Mixing Technique 2 versus Mixing Technique 1
abs(3156.25-2971.00)>LSD

## [1] TRUE

# Mixing Technique 1 versus Mixing Technique 4
abs(2971.00-2666.25)>LSD

## [1] TRUE

# Mixing Technique 1 versus Mixing Technique 3
abs(2971.00-2933.75)>LSD

## [1] FALSE

# Mixing Technique 3 versus Mixing Technique 4
abs(2933.75-2666.25)>LSD

## [1] TRUE
```

From the comparison of the pair of means, it is clear that the pair "Mixing Technique 1 vs. Mixing Technique 3" is not significantly different. All the other pairs of means are significantly different.

(d) Construct a normal probability plot of the residuals. What conclusion would you draw about the validity of the normality assumption?

```
library(car)

## Loading required package: carData

##
## Attaching package: 'car'

## The following object is masked from 'package:dplyr':
##
##     recode

strength.lm=lm(Tensile_Strength~Technique,data=my_data)
qqPlot(residuals(strength.lm))
```

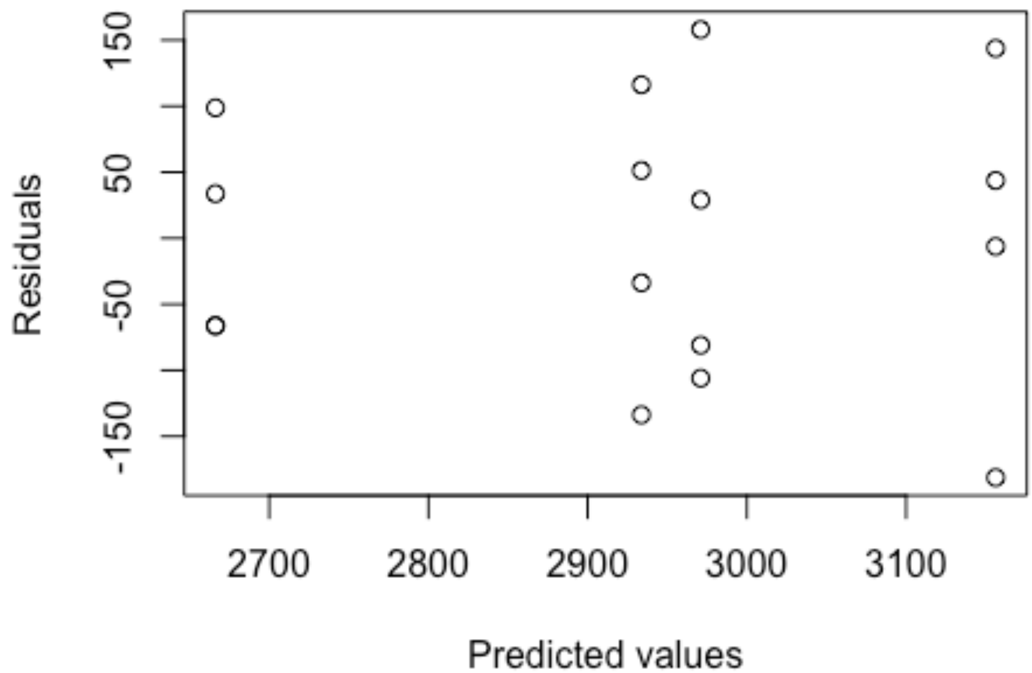



```
## [1] 7 1
```

As all the points fall approximately along this reference line, we can assume normality.

(e) Plot the residuals versus the predicted tensile strength. Comment on the plot.

```
plot(fitted(strength.lm), residuals(strength.lm),
ylab="Residuals",xlab="Predicted values")
```



From the plot, it is observed that the equal standard deviation assumption is appropriate.

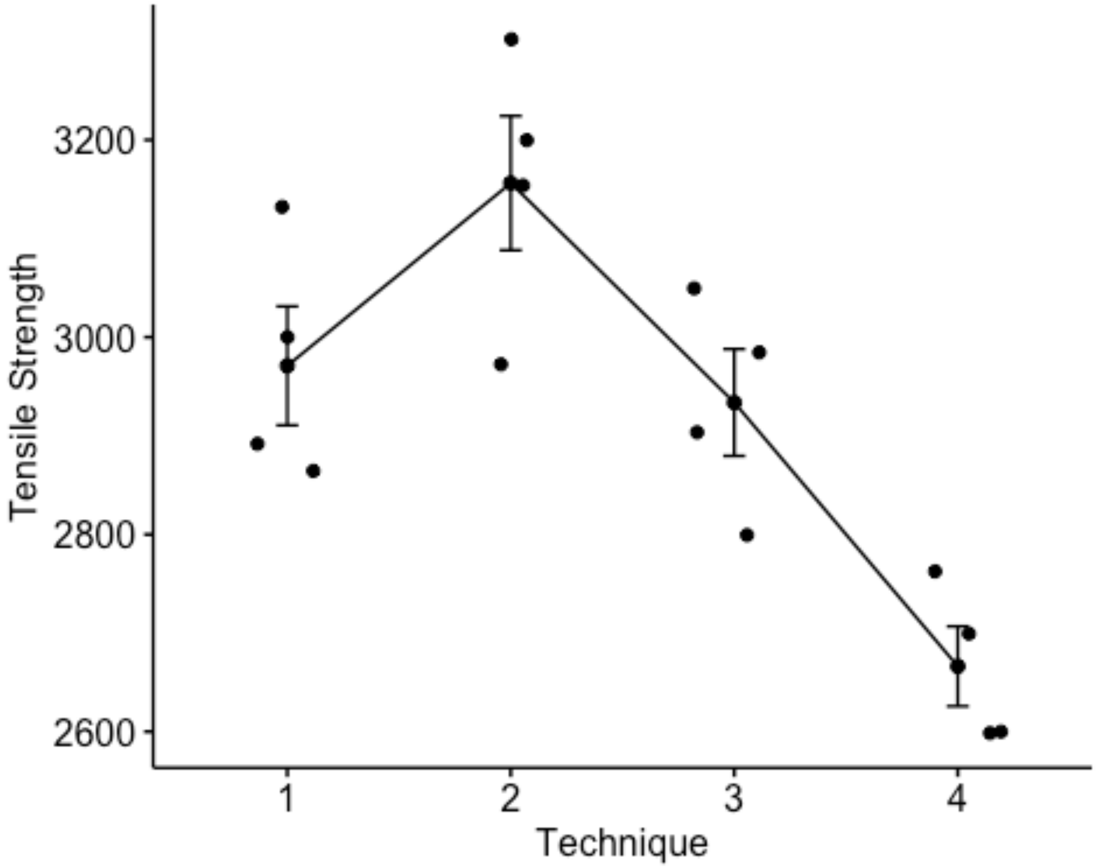
The ratio of largest and smallest standard deviation is,

$$\frac{135.97641}{80.97067} = 1.68$$

The ratio of largest and smallest standard deviation is less than 2. This indicates that the assumption of equal standard deviation is not violated.

(f) Prepare a scatter plot of the results to aid the interpretation of the results of this experiment.

```
library("ggpubr")

## Loading required package: ggplot2

ggline(my_data, x = "Technique", y = "Tensile_Strength",
       add = c("mean_se", "jitter"),
       order = c("1", "2", "3","4"),
       ylab = "Tensile Strength", xlab = "Technique")
```

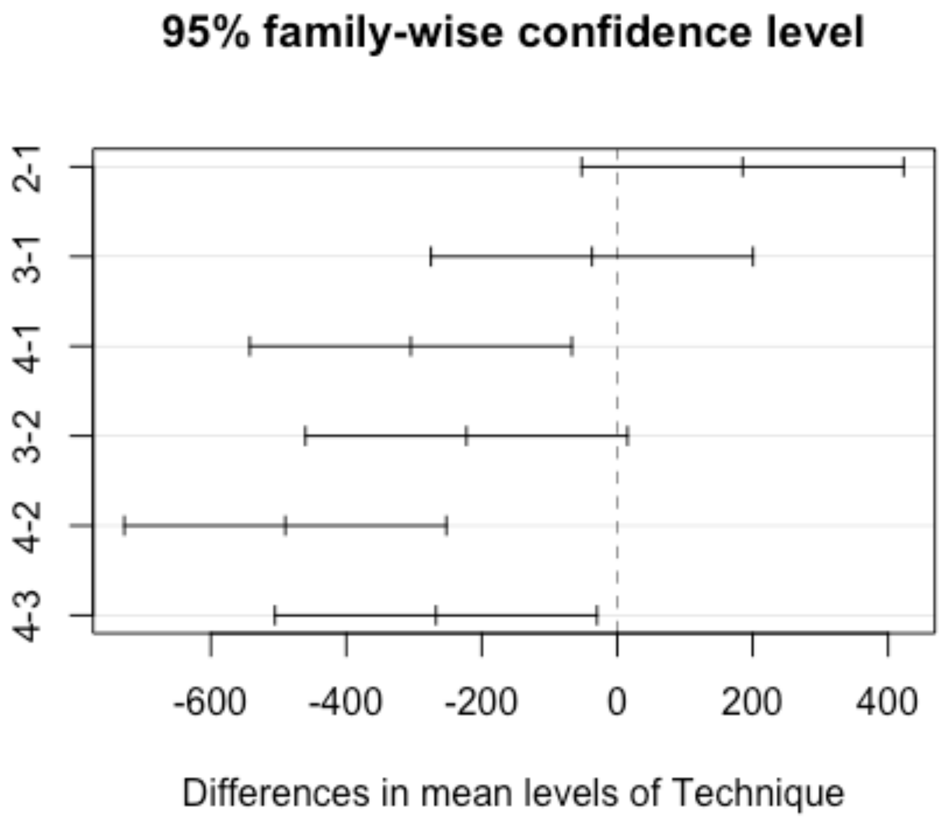


The mean tensile strength 4 seems to differ significantly from the means of the other three tensile strengths 1, 2, and 3. There is a less significant difference between the means of tensile strength 3 and tensile strength 1. There is a significant difference between tensile strength 3 and tensile strength 2. There is a significant difference between tensile strength 1 and tensile strength 2.

3.12 (a). Rework part (c) of Problem 3.11 using Tukey's test with α = 0.05. Do you get the same conclusions from Tukey's test that you did from the graphical proce- dure and/or the Fisher LSD method?

```
TukeyHSD(model)

##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = Tensile_Strength ~ Technique, data = my_data)
##
## $Technique
##        diff        lwr        upr     p adj
## 2-1  185.25  -52.50029  423.00029 0.1493561
## 3-1  -37.25 -275.00029  200.50029 0.9652776
## 4-1 -304.75 -542.50029  -66.99971 0.0115923
## 3-2 -222.50 -460.25029   15.25029 0.0693027
```

```
## 4-2 -490.00 -727.75029 -252.24971 0.0002622
## 4-3 -267.50 -505.25029  -29.74971 0.0261838

plot(TukeyHSD(model))
```



With alpha = 0.05, the difference between mixing technique 4 and 1, mixing technique 4 and 2, and mixing technique 4 and 3 are significant with all p_value < 0.05.

Thus, compare to the Fisher LSD method, the result is not the same. The mean of technique 4 is different from of 1,2,3 but the mean of technique 2 is not different from the means of techniques 1 and 3 according to the Tukey method, they were found to be different using the graphical and Fisher LSD method

(b) Explain the difference between the Tukey and Fisher procedures.

Although both Tukeu and Fisher utilize a single critical value, Tukey's is based on the studentized range statistic while Fisher's is based on t distribution.

3.13 Reconsider the experiment in Problem 3.11. Find a 95 percent confidence interval on the mean tensile strength of the Portland cement produced by each of the four mixing techniques. Also find a 95 percent confidence interval on the difference in means for techniques 1 and 3. Does this aid you in interpreting the results of the experiment?

We have:

$$\bar{y_i} \pm t_{\alpha/2,N-a}\sqrt{(\frac{MSe}{n})}$$

```
me=c(-1,1)*qt(0.05/2,12)*sqrt(MSe/4)
```

```
library(dplyr)
group_by(my_data,Technique) %>%
  summarise(
    count = n(),
    mean = mean(Tensile_Strength, na.rm = TRUE),
    sd = sd(Tensile_Strength, na.rm = TRUE)
  )
```

```
## # A tibble: 4 x 4
##   Technique count  mean    sd
##   <fct>     <int> <dbl> <dbl>
## 1 1             4 2971   121.
## 2 2             4 3156. 136.
## 3 3             4 2934. 108.
## 4 4             4 2666.  81.0
```

CI for technique 1

```
2971.00 +me
```

```
## [1] 3094.376 2847.624
```

CI for mean of technique 2

```
3156.25 +me
```

```
## [1] 3279.626 3032.874
```

CI for mean of technique 3

```
2933.75 +me
```

```
## [1] 3057.126 2810.374
```

CI for mean of technique 4

```
2666.25
```

```
## [1] 2666.25
```

find a 95 percent confidence interval on the difference in means for techniques 1 and 3.

We have :

$$\bar{y_i} - \bar{y_j} \pm t_{\alpha/2,N-a}\sqrt{(\frac{2 \times MSe}{n})}$$

```
(2971.00-2933.75)+ c(1,-1)*qt(0.05/2,12)*sqrt(2*MSe/4)

## [1] -137.2298  211.7298
```

3.16 A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

```
problem16=read.table("/Users/nhivutu/Desktop/Math
447/dataset_chapter3/problem16.txt", header = TRUE)
attach(problem16)
```

(a) Is there evidence to indicate that dosage level affects bioactivity? Use α = 0.05.

```
act.aov=aov(Activity~Dosage,data=problem16)
summary(act.aov)

##             Df Sum Sq Mean Sq F value Pr(>F)
## Dosage       2  450.7  225.33   7.036 0.0145 *
## Residuals    9  288.2   32.03
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the output, p_value < 0.05 then we reject H_0. Thus, the data provides sufficient evident to conclude that dosage level affects bioactivity.

(b) If it is appropriate to do so, make comparisons between the pairs of means. What conclusions can you draw?

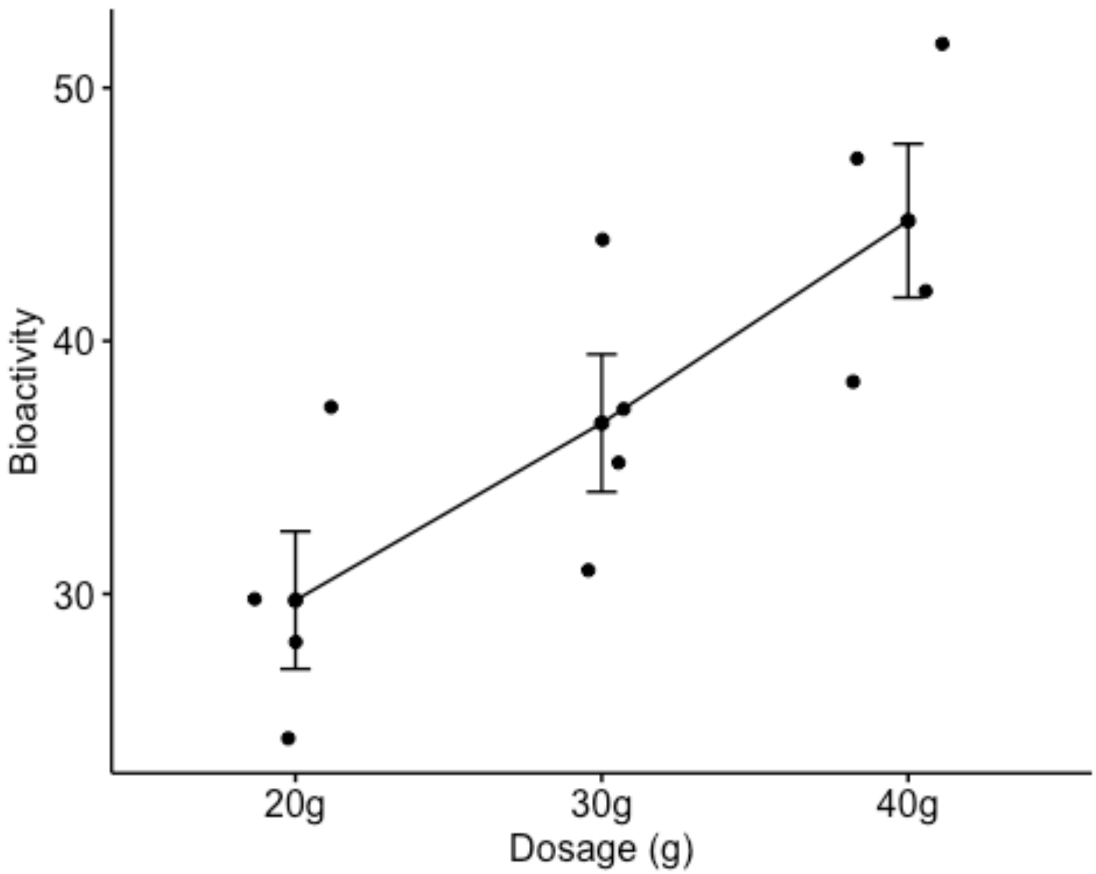
Summary by each treatment

```
library(dplyr)
group_by(problem16,Dosage)%>%
  summarise(
    count = n(),
    mean = mean(Activity, na.rm = TRUE),
    sd = sd(Activity, na.rm = TRUE)
  )

## # A tibble: 3 x 4
##   Dosage count  mean    sd
##   <chr>  <int> <dbl> <dbl>
## 1 20g        4  29.8  5.44
## 2 30g        4  36.8  5.44
## 3 40g        4  44.8  6.08
```

There are difference between their means.

Using graph

```
library("ggpubr")
ggline(problem16, x = "Dosage", y = "Activity",
       add = c("mean_se", "jitter"),
       order = c("20g", "30g", "40g"),
       ylab = "Bioactivity", xlab = "Dosage (g)")
```



It's clear to see the differences between means among different dosage, but there are significant difference between means of 20g and 40g.

Using Tukey

```
TukeyHSD(act.aov)

##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = Activity ~ Dosage, data = problem16)
##
## $Dosage
##         diff        lwr      upr     p adj
## 30g-20g    7 -4.172869 18.17287 0.2402975
```

```
## 40g-20g   15  3.827131 26.17287 0.0114434
## 40g-30g    8 -3.172869 19.17287 0.1680265
```

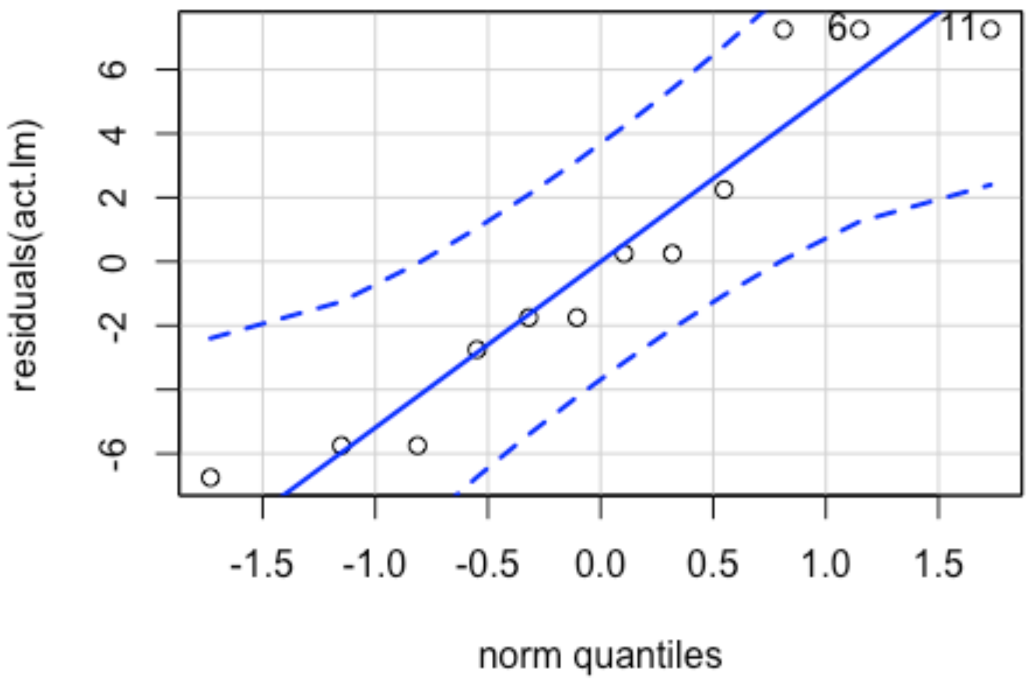
With alpha=0.05, there is significant difference between 40g and 20g.

(c) Analyze the residuals from this experiment and comment on model adequacy.

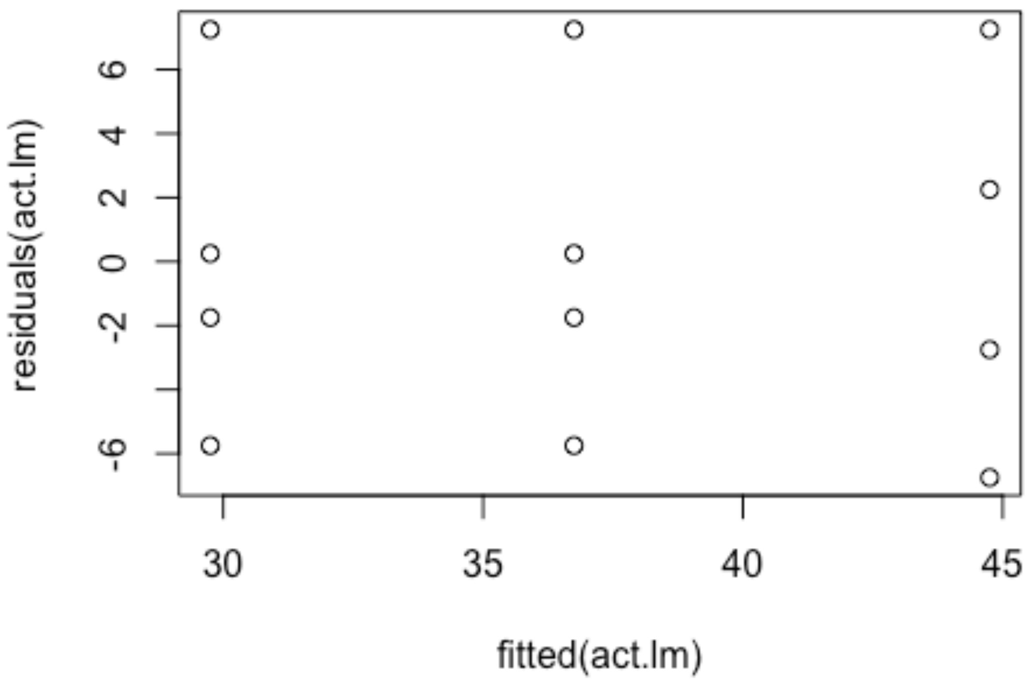
```
act.lm=lm(Activity~Dosage)
summary(act.lm)
```

```
##
## Call:
## lm(formula = Activity ~ Dosage)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -6.75  -3.50  -0.75   3.50   7.25
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   29.750      2.830  10.514 2.35e-06 ***
## Dosage30g      7.000      4.002   1.749  0.11418
## Dosage40g     15.000      4.002   3.748  0.00457 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.659 on 9 degrees of freedom
## Multiple R-squared:  0.6099, Adjusted R-squared:  0.5232
## F-statistic: 7.036 on 2 and 9 DF,  p-value: 0.01446
```

```
# check normaility
library(car)
qqPlot(residuals(act.lm))
```




```
## [1] 11  6

# check structureless(independence)
plot(fitted(act.lm), residuals(act.lm))
```



There is nothing unusual about the residual plots. Also, we found the R-squared = 0.6099 which means approximately 60.99% variation in acitivity can be explained by dosage.

3.20 An experiment was run to determine whether four specific firing temperatures affect the density of a certain type of brick. A completely randomized experiment led to the following data:

```
problem20=read.table("/Users/nhivutu/Desktop/Math
447/dataset_chapter3/problem20.txt", header = TRUE)
problem20$Temp<-as.factor(problem20$Temp)
attach(problem20)
```

(a) Does the firing temperature affect the density of the? Use α = 0.05.

```
density.aov=aov(Density~Temp,data=problem20)
summary(density.aov)

##             Df Sum Sq Mean Sq F value Pr(>F)
## Temp         3 0.1561 0.05204   2.024  0.157
## Residuals   14 0.3600 0.02571
```

Since p_value is large (>0.05) then we fail to reject H_0, thus the firing temperature does not affect the density of the brick.

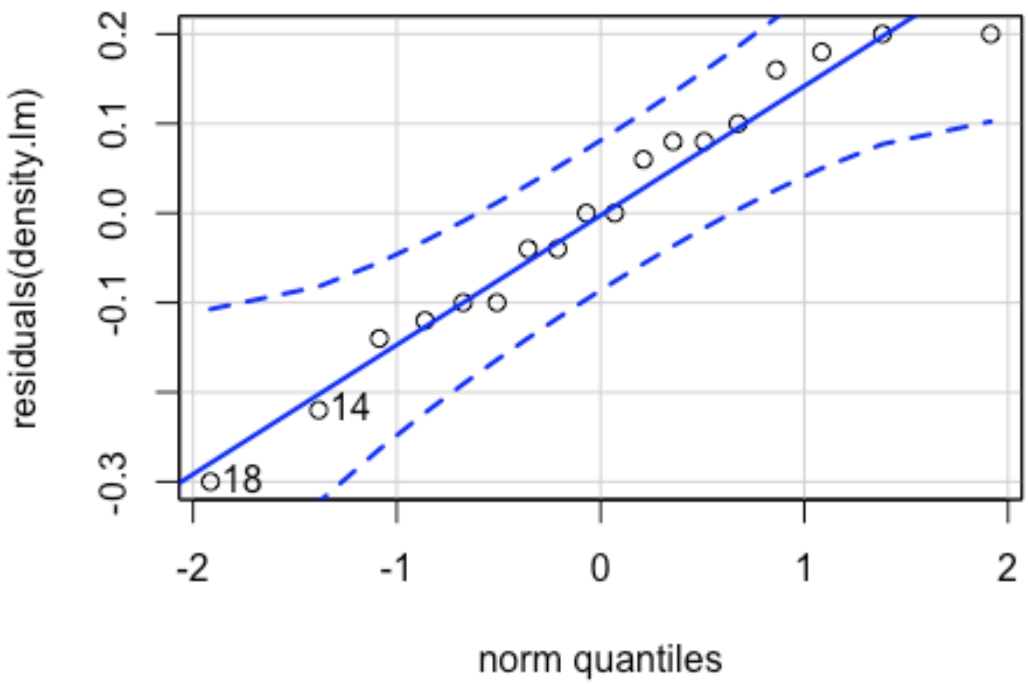
(b) Is it appropriate to compare the means using the Fisher LSD method (for example) in this experiment?

```
library("dplyr")
group_by(problem20,Temp) %>%
  summarise(
    count = n(),
    mean = mean(Density, na.rm = TRUE),
    sd = sd(Density, na.rm = TRUE)
  )

## # A tibble: 4 x 4
##   Temp  count  mean    sd
##   <fct> <int> <dbl> <dbl>
## 1 100       5  21.7 0.114
## 2 125       4  21.5 0.141
## 3 150       5  21.7 0.164
## 4 175       4  21.7 0.216
```

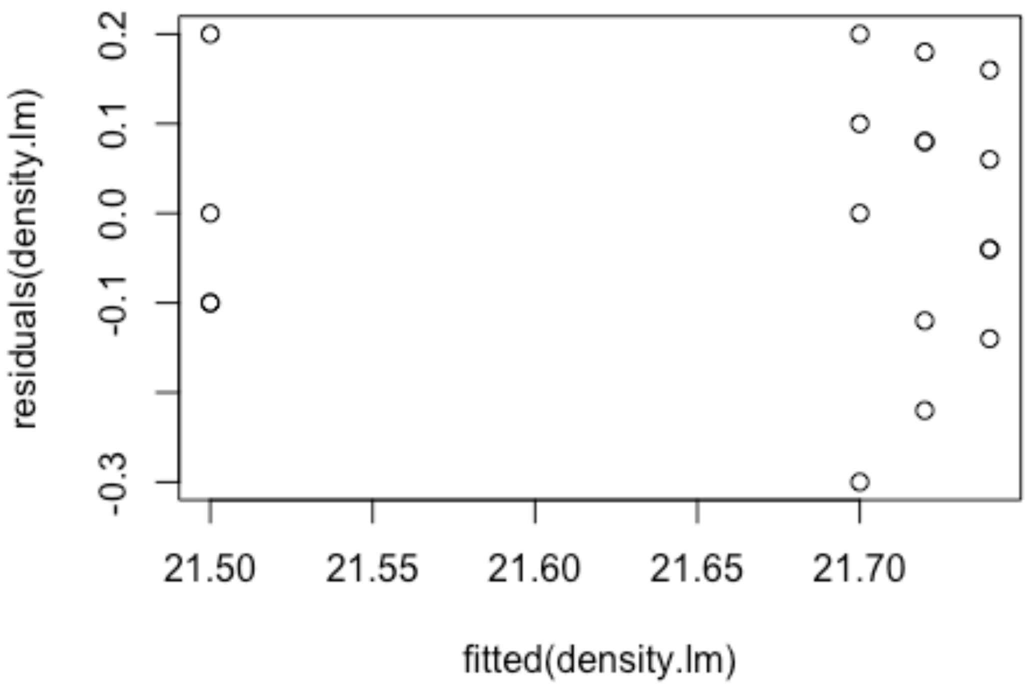
It is very clear that there are not any significant difference between their means, they are approximately equal which are around 22. Thus There is no need to proceed with the Fisher LSD method to decide which mean is difference.

(c) Analyze the residuals from this experiment. Are the analysis of variance assumptions satisfied?

```
density.lm<-lm(Density~Temp,data=problem20)
# check normaility
library(car)
qqPlot(residuals(density.lm))
```




```
## [1] 18 14

# check structureless(independence)
plot(fitted(density.lm), residuals(density.lm))
```



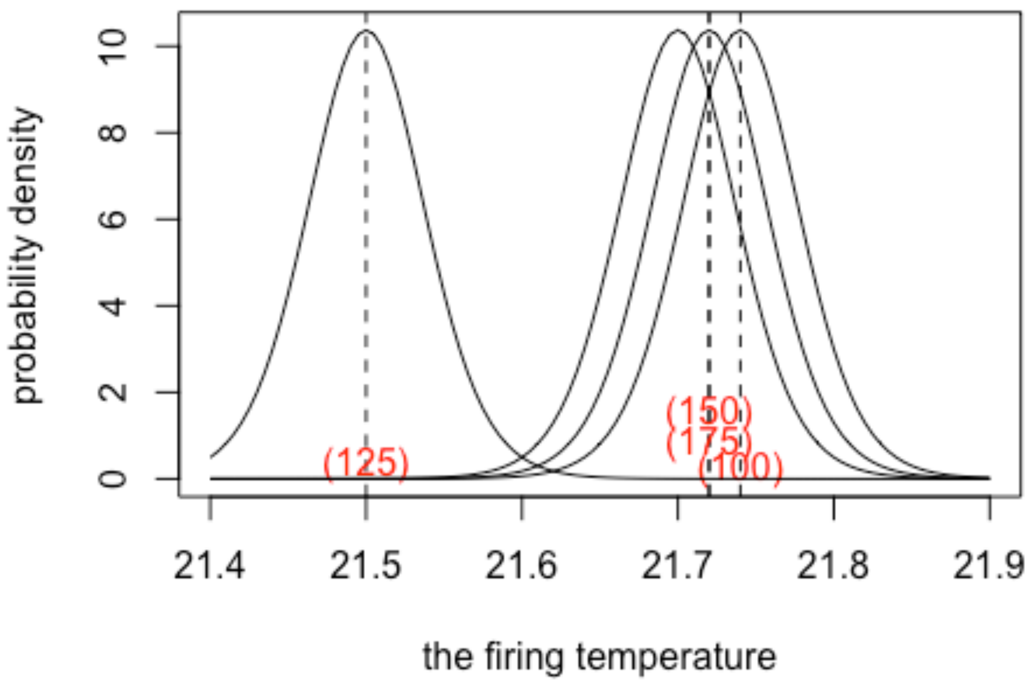
There is nothing unusual about the residual plots.

(d) Construct a graphical display of the treatment as described in Section 3.5.3. Does this graph adequately summarize the results of the analysis of variance in part (a)?

Find scaled t

```
MSe=summary(density.aov)[[1]][2,3]
n=18
S_yibar=sqrt(MSe/n)

curve(dt((x -21.74 )/S_yibar, df = 14)/S_yibar, from =
min(problem20$Density), to = max(problem20$Density), ylab="probability
density", xlab="the firing temperature")
abline(v = 21.74 , lty = 2)
text(x=21.74,y=0.2,labels="(100)",col="red")

curve(dt((x - 21.50)/S_yibar, df = 14)/S_yibar,add = TRUE)
abline(v = 21.50, lty = 2)
text(x=21.50,y=0.3,labels="(125)",col="red")

curve(dt((x - 21.72)/S_yibar, df = 14)/S_yibar,add = TRUE)
```

```
abline(v = 21.72, lty = 2)
text(x=21.72,y=1.5,labels="(150)",col="red")

curve(dt((x - 21.70)/S_yibar, df = 14)/S_yibar, add = TRUE)
abline(v = 21.72, lty = 2)
text(x=21.72,y=0.8,labels="(175)",col="red")
```



Yes, this graph adequately summarize the results of the analysis of variance in part (a).

3.21 Rework part (d) of Problem 3.20 using the Tukey method. What conclusions can you draw? Explain carefully how you modified the technique to account for unequal sample sizes.

Because the design is unbalance

```
# mu1 vs mu2
t0=(21.74-21.50)/sqrt(2*MSe*(1/5+1/4))
pvalue=2*pt(abs(t0),14,lower.tail = F)
pvalue

## [1] 0.1369748
```

```r
#mu1vs mu3
t0=(21.74-21.72)/sqrt(2*MSe*(1/5+1/5))
pvalue=2*pt(abs(t0),14,lower.tail = F)
pvalue
```

```
## [1] 0.891086
```

```r
# mu1 vs mu4
t0=(21.74-21.7)/sqrt(2*MSe*(1/5+1/4))
pvalue=2*pt(abs(t0),14,lower.tail = F)
pvalue
```

```
## [1] 0.7964292
```

```r
#mu2 vs mu3
t0=(21.5-21.72)/sqrt(2*MSe*(1/5+1/4))
pvalue=2*pt(abs(t0),14,lower.tail = F)
pvalue
```

```
## [1] 0.1701492
```

```r
#mu2 vs mu4
t0=(21.5-21.7)/sqrt(2*MSe*(1/4+1/4))
pvalue=2*pt(abs(t0),14,lower.tail = F)
pvalue
```

```
## [1] 0.2327788
```

```r
#mu3 vs mu4
t0=(21.72-21.7)/sqrt(2*MSe*(1/5+1/4))
pvalue=2*pt(abs(t0),14,lower.tail = F)
pvalue
```

```
## [1] 0.8972754
```

Using the Tukey method, with alpha=0.05, we fail to reject all H_0 then there are not any significant differences between means of treatment.

mean of temp100 vs of temp125 is unbalance

mean of temp100 vs of temp175 is unbalance

mean of temp125 vs of temp150 is unbalance

mean of temp100 vs of temp150 is balance

mean of temp125 vs of temp175 is balance.

3.33 A semiconductor manufacturer has developed three different methods for reducing particle counts on wafers. All three methods are tested on five different wafers and the after treatment particle count obtained. The data are shown below:

```
problem33=read.table("/Users/nhivutu/Desktop/Math 
447/dataset_chapter3/problem33.txt", header = TRUE)
problem33$Method<-as.factor(problem33$Method)
```

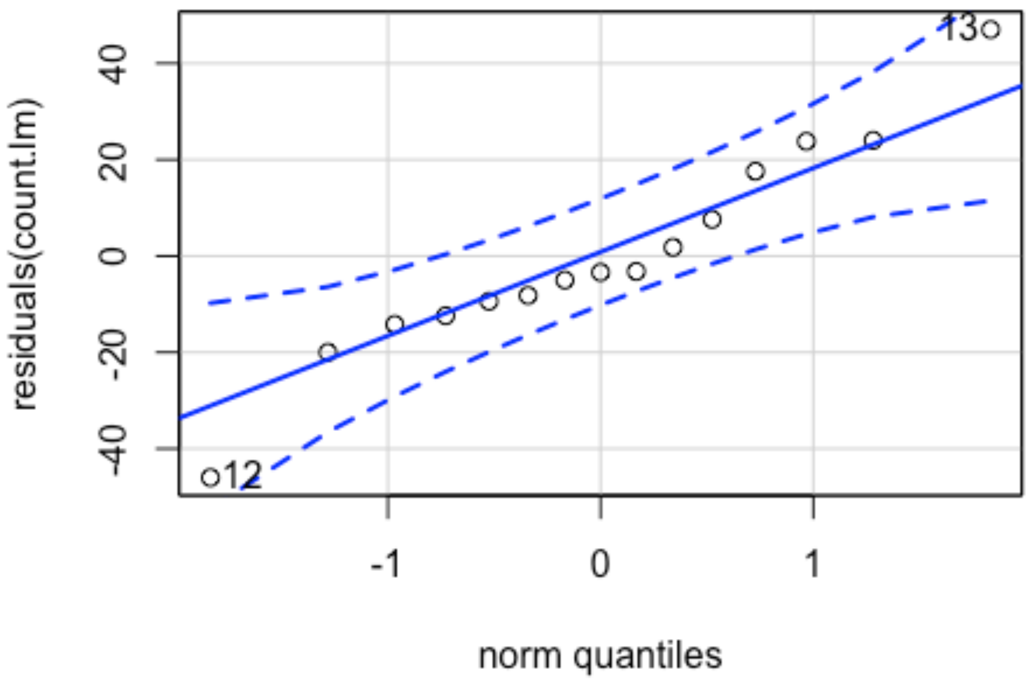
(a) Do all methods have the same effect on mean particle count?

```
count.aov=aov(problem33$Count~problem33$Method)
summary(count.aov)

##                  Df Sum Sq Mean Sq F value  Pr(>F)   
## problem33$Method  2   8964    4482   7.914 0.00643 **
## Residuals        12   6796     566                   
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since pvalue is small (<0.05), then we reject H_0: all methods have the same effect on mean particle count. Thus, there is sufficient evident to conclude that at least one method has different effect on mean particle count.

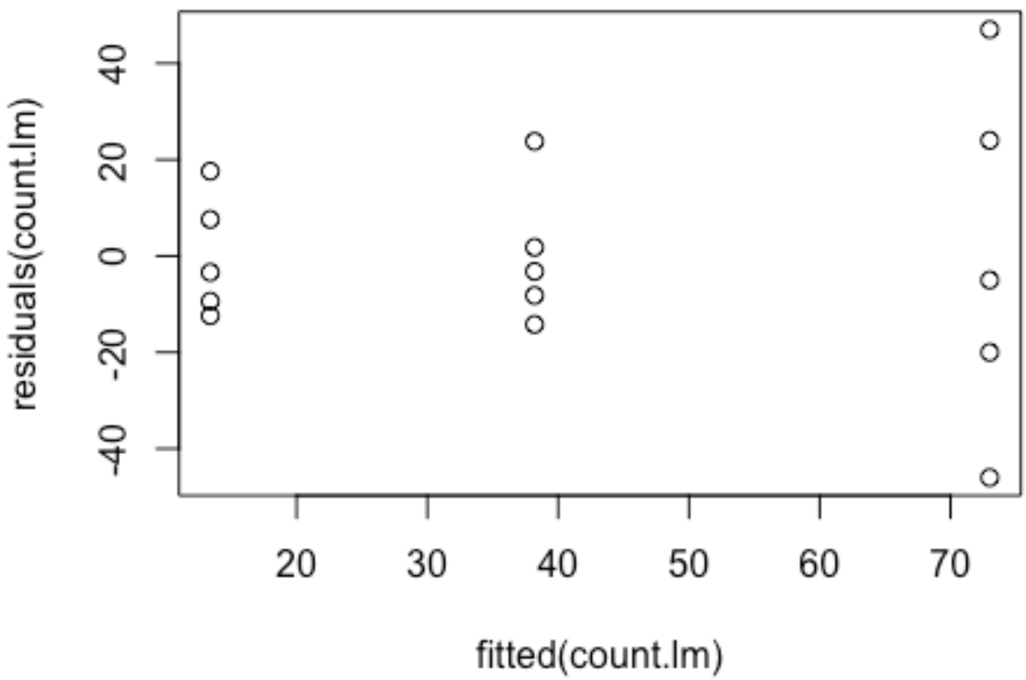
(b) Plot the residuals versus the predicted response. Construct a normal probability plot of the residuals. Are there potential concerns about the validity of the assumptions?

```
count.lm=lm(problem33$Count~problem33$Method)
library(car)
# check normaility
qqPlot(residuals(count.lm))
```




```
## [1] 13 12
```

```r
# check structureless(independence)
plot(fitted(count.lm), residuals(count.lm))
```



From the plot, the residuals are not scatter and the residuals plotted in the normal probability plot do not fall along a straight line, which suggests that the normality assumption is not valid. Thus, assumption of residuals are not satisfied.

(c) Based on your answer to part (b), conduct another analysis of the particle count data and draw appropriate conclusions.

Since, the assumption of residuals does not hold, a non-parametric Kruskal-Wallis test cam be used to check whether the methods have same effect on mean particle count or not.

H_0: All mean are equal. vs H_1: At least one of them differs from other.

```
kruskal.test(problem33$Count~problem33$Method, problem33)

##
##  Kruskal-Wallis rank sum test
##
## data:  problem33$Count by problem33$Method
## Kruskal-Wallis chi-squared = 8.54, df = 2, p-value = 0.01398
```

We get p_value is small so we can reject H_0, then there is sufficient evidence to conclude that the methods do not have same effect on mean particle count.

3.51 Consider the experiment in Problem 3.30. If we wish to construct a 95 percent confidence interval on the difference in two mean battery lives that has an accuracy of ±2 weeks, how many batteries of each brand must be tested?

```
problem30=read.table("/Users/nhivutu/Desktop/Math 447/dataset_chapter3/problem30.txt", header = TRUE)

life.aov=aov(problem30$Life~factor(problem30$Brand))
```

We have:

$$\text{width} = t_{0.025,N-a}\sqrt{(\frac{2MSe}{n})}$$

```
n=seq(4,100, by =1)
MSe=summary(life.aov)[[1]][2,3]
a=3
N=n*a
df=N-a
t=qt(0.025,df,lower.tail = F)
width=sqrt(2*MSe/n)*t
width

##  [1] 6.317861 5.442673 4.860453 4.435455 4.106908 3.842770 3.624255
3.439495
##  [9] 3.280557 3.141910 3.019554 2.910519 2.812542 2.723868 2.643107
2.569146
## [17] 2.501079 2.438161 2.379773 2.325395 2.274588 2.226976 2.182239
2.140096
## [25] 2.100308 2.062661 2.026971 1.993073 1.960822 1.930089 1.900759
1.872727
## [33] 1.845901 1.820196 1.795536 1.771853 1.749084 1.727171 1.706062
1.685708
## [41] 1.666067 1.647096 1.628760 1.611023 1.593853 1.577221 1.561099
1.545462
## [49] 1.530286 1.515548 1.501228 1.487307 1.473766 1.460588 1.447758
1.435260
## [57] 1.423080 1.411206 1.399623 1.388322 1.377290 1.366516 1.355992
1.345707
## [65] 1.335653 1.325821 1.316203 1.306792 1.297579 1.288558 1.279724
1.271068
## [73] 1.262586 1.254271 1.246119 1.238123 1.230280 1.222583 1.215030
1.207614
## [81] 1.200333 1.193182 1.186157 1.179255 1.172472 1.165805 1.159250
1.152805
## [89] 1.146466 1.140230 1.134095 1.128058 1.122117 1.116268 1.110510
1.104840
## [97] 1.099256
```

Therefore, to get an accuracy of ±2 weeks, we find a most appropriate the value of n is 29 then $N = 29 \times 3 = 87$