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Network Science

Measuring Facebook Influence Using Centrality Measures and Community Detection

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Abstract

Within the past couple of decades, social media has become a large part of everybody's life and our society. With the aid of social media, social influence can take place without physical human interaction and allow people to influence each other just by having social media accounts and the ability to access the Internet. After social media became popular, researchers and scientists have been studying the impact of social media using different methods such as surveys, trials, scientific experiments, and so on. Analyzing social media network graphs is another way of studying social media impact that this paper will cover. Given a public dataset of Page to Page network from Facebook in November 2017, multiple network graph measurements and a community detection method will help identify influential Facebook pages within a network. By understanding the differences between measurements and what information they provide, it will be beneficial to apply similar measurements and analysis methods on other social media networks to identify influential nodes

1 Introduction

With individuals having the ability to influence others anytime from any location anonymously, it is important to be able to identify their power and influential level within a social network to carefully access the impact. Without the ability to identify influential nodes, a network can be too large to study and time-consuming for anyone to get useful information.

1.1 Background

Social media impact has drastically increased its effect on life in the last 10-15 years, the diverse and far-reaching impacts cannot be overstated. Russian influence in the 2016 election, the Obama campaign utilizing social media to achieve a decisive victory in 2008, and social media fueling societal change and upheaval in the middle east and Hong Kong are some of the examples.

In another realm, corporations spend a massive 11.6 percent [1] average of their marketing through social media indicating a vast impact on society's choices and buying habits through social media.

Amid these large organizations and vast movements in social media, there is a fascinating and powerful impact propagated through lone individuals on social media. A single individual operating as a social media influencer is an existence that has become increasingly common in our societal vernacular. For many, a full-time job as a social influencer has become a viable career.

Understanding all of these social media forces and impacts is important on many levels. Devastating negative ramifications: a nation-state subverting democracy by impacting other countries' elections, hate groups spreading and inciting violence, and webs of misinformation leading to widespread distrust are all reasons alone to understand social media impact and influence.

1.2 Motivation and Objectives

The ultimate purpose of this paper is to analyze a public Facebook Page-to-Page network to identify important nodes and different communities.

The first objective is to answer the question that will observe the potential differences between multiple centrality measurements that help identify important nodes within a network - Can we simply use degree centrality to identify powerful nodes in a network?

The second objective is to answer the question that will identify the difference between a predetermined classification of a network and how likely would the nodes form a community within their own classification - Can we use page type to detect communities?

1.3 Existing Work

Social media is increasingly prevalent in commerce, politics, and all societal interaction. Unsurprising, research seeking to measure and understand social media influence and impact is prevalent.

There are several studies that are related to our project objectives. Some of them use similar centrality measures and some community detection techniques.

A Comparative Study of Modularity-Based Community Detection Methods for Online Social Networks [2] Karatas and Sahin also focus on communities in social networks using modularity. The study assesses datasets from multiple social media platforms using different static community detection algorithms and specifically focus on "modularity values, running time and accuracy" and compare and contrast these different algorithms for community detection assessing their strengths and weaknesses. The performance of the different algorithms was analyzed by measuring their modularity value, F1-score, and running time. Five different algorithms were used and the Louvain algorithm performed the best overall. The study suggests the following as problems with the currently available community detection algorithms: stability, scalability, refinement on computational complexity, dynamicity, and prediction. These ideas prompt and encourage future research on community network structure and the development of new community detection algorithms.

Social Circles in Facebook communities [3] The Social Circles in Facebook Communities study used the egonet dataset to assess specific subsets of the Facebook network which they termed social circles. Social circles are smaller groups within an individual's social network such as friends from high school or the workplace. The study utilized the egonet data set to construct various models of social circles and calculate Min Edit Distance and was purposed to: "predict what communities would form or what circles can be created using the given friendships" [3]. Initially Clique Percolation to model the social circle data, but later in the study, the researchers found that Spectral Clustering was more effective. "Spectral Clustering is a way to cluster based on 'Affinity' and connectivity" [3]. Our paper perhaps is a logical partner and expansion of this study. In the future perhaps integrating these two concepts would beneficial. The incorporation influence assessment might be a beneficial step towards identifying how communities would form around influential nodes.

Network Analysis of Page Likes from Facebook User Profiles [4] This study used the Python library NetworkX to analyze network data from Facebook in graphic form. Using the Facebook ego network which structures data along a central vertex, the exercise started by analyzing betweenness centrality to determine the most connected individuals present in the dataset. After calculating betweenness centrality, the highest scoring 10 in the dataset were placed as central hubs within the graph. The next different communities were identified using community detection algorithms focusing on the modularity of the network or the fraction of edges.

2 Process and Approaches

Figure 1 identifies the steps that are necessary to complete the objectives of this report. Data are collected from the Stanford Network Analysis Project. Once the data are collected, the next would be to clean up the network graph, perform high-level analysis, calculations, detection community method. Then, all findings will be analyzed, visualized, and compared to get final results.

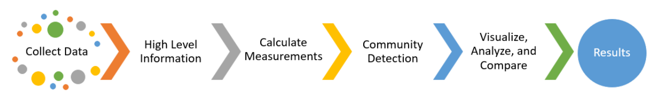


Figure 1: High Level Process

2.1 Collecting Data

Dataset [5] The dataset is November 2017 Facebook Large Page-Page Network from Stanford Network Analysis Project [5]. The dataset is a compressed zip file that contains the following files :

- `musae_facebook_edges.csv` – File that contains all edges between pages
- `musae_facebook_features.json` – JSON file that contains features of the nodes - this file will not be used.
- `musae_facebook_target.csv` - File that contains list of node ids, page names, and page types.

Each edge represents a mutual like between pages and each node is a Facebook page. There are four categories classifies for the Facebook pages: governmental organizations, politicians, television shows, and companies. The raw data contains 22,470 nodes and 171,002 edges.

2.2 Data Cleaning and High Level Information

Tools The following software and tools were used to perform the analysis:

- **Python and Python Packages** - Python packages include Networkx, Pandas, and Matplotlib
- **Jupyter Notebook**
- **Gephi** - visualization and analysis tool for network graph

Cleanup After using Pandas Python package to read and combine the edges list and pages categories information csv files, the next step would be converting the data into graph object and removing self-loops, edges that connect a node to itself. After removing self-loops, the total edge count reduced from 171,002 to 170,823.

High Level Information The average degree is 15.2045, which means that on average, each node connects to about 15 other nodes.

Figure 2 provides the composition of page category within the network.

The same composition can be shown and visualized using Gephi as shown in **Figure 3**. One advantage of using Gephi over the pie chart is the visual of clustered areas within a network. There are a highly connected area and some small clustered areas within governmental organization pages. Politicians and tv show pages also have more clustered areas toward the outer part of the graph and Company pages don't have any obvious cluster.

However, graphing the whole population makes it difficult to get more information out of the network. One way to work around that is to filter the network into a smaller network of high degree nodes for more observations. By filtering the whole network to only select giant components with higher-than-100-degree nodes, it resulted in a smaller network that only contains 314 nodes with 5,686 edges.

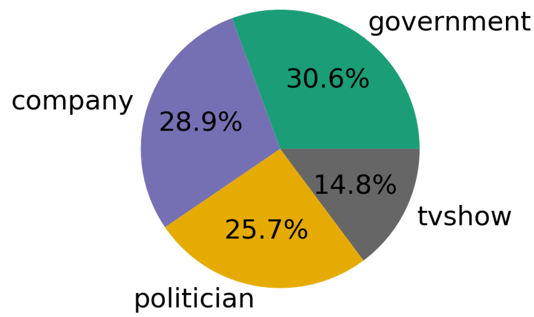


Figure 2: Percentage of Page Category

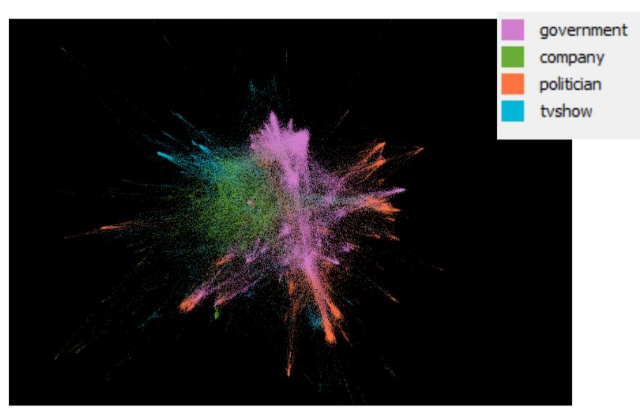


Figure 3: Whole Network Visualization by Page Category

Figure 4 shows that governmental organizations and politicians pages made of the majority

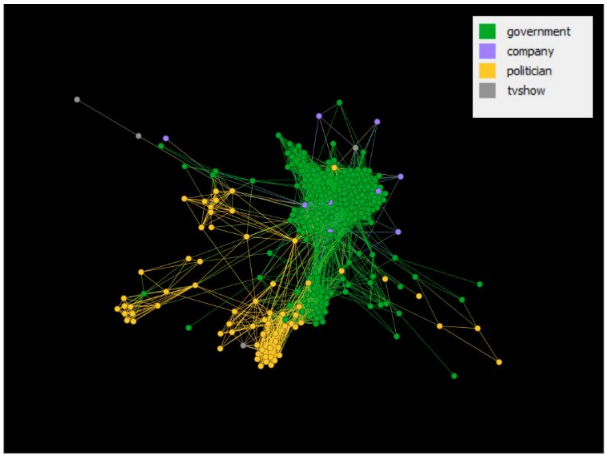


Figure 4: Visualization of Network with Nodes Greater Than 100 Degree

page_name	degree	page_type
U.S. Army	709	government
The White House	678	government
The Obama White House	659	government
U.S. Army Chaplain Corps	650	government
Honolulu District, U.S. Army Corps of Engineers	504	government
U.S. Department of State	468	government
FEMA Federal Emergency Management Agency	448	government
European Parliament	417	government
Army Training Network (ATN)	408	government
Defense Commissary Agency	387	government

Figure 5: Top Ten Highest Degree Pages

of the population in a network of higher-than-100-degree nodes. It is also quite easy to identify the cluster areas (about 5 areas).

Another high-level measurement that is important and perhaps the most obvious measure of influence was a measurement of the degree of every node in the network. Degree centrality provides a simple yet telling quantitative data point for the number of edges or connections a given node has [6].

Figure 5 contains the list of the top ten nodes that have the highest degree centrality within the whole network. One observation is that all of them are governmental organization pages; it aligns with the high composition of governmental organization pages shown in **Figure 4** earlier. It is interesting to see that the top five highest nodes are the United State's related pages. Is it because the majority of Facebook users are in the United States or were the data skewed? Those are the questions that should be answered before making business or important decisions using this dataset.

page_name	degree	page_type
Facebook	380	company
NASA - National Aeronautics and Space Administ...	328	company
CNN	167	company
Walmart	124	company
Whole Foods Market	120	company
Microsoft	112	company
Army Knowledge Online/Defense Knowledge Online	112	company
Starbucks	106	company
Qantas	102	company
KLM Royal Dutch Airlines	97	company

Figure 6: Top Ten Highest Degree Company Pages

An additional filter of page type was added to get the top ten highest nodes of company pages, television shows, and politicians shown in **Figure 6**, **Figure 7**, and **Figure 8** respectively.

Both of the top highest nodes under company and politician pages are about half of the degree amount comparing to the highest node in governmental organization pages. Television show pages seem to have the least influential power

page_name	degree	page_type
Today Show	141	tvshow
Home & Family	137	tvshow
tagesschau	119	tvshow
The Simpsons	110	tvshow
Glee	101	tvshow
So You Think You Can Dance	99	tvshow
Family Guy	91	tvshow
Dancing with the Stars	90	tvshow
MasterChef	90	tvshow
New Girl	90	tvshow

Figure 7: Top Ten Highest Degree TV Show Pages

within this network.

2.3 Calculate Measurements

Other centrality measurements can be used to identify influential nodes within a network. It is important to understand the similarities and differences between these measurements and degree centrality to help with future network analysis. If degree centrality yields similar results to the rest of the other measurements, it might save time for researchers and scientists from having to calculate and perform deep-dive analysis.

2.3.1 Betweenness Centrality

One of the useful centrality measurements to analyze a network is betweenness centrality. Betweenness centrality finds the path between every node in the network, then finds the shortest path and the nodes that fall into most of these shortest paths will have the highest betweenness score [6].

page_name	degree	page_type
Barack Obama	341	politician
Manfred Weber	326	politician
Joachim Herrmann	320	politician
Martin Schulz	236	politician
Arno Klare MdB	226	politician
Katarina Barley	224	politician
Katja Mast	222	politician
Angela Merkel	217	politician
Niels Annen	199	politician
Sir Peter Bottomley MP	174	politician

Figure 8: Top Ten Highest Degree Politician Pages

Figure 9 contains the top ten highest betweenness centrality nodes. The top ten nodes as calculated by their betweenness scores provides six governmental organization nodes, two politicians, and two companies. The range of the scores is between 0.0155 for the 10th highest and 0.116 for the 1st highest score.

2.3.2 Closeness Centrality

The next centrality measurement to analyze is closeness centrality. Closeness centrality provides the average path length between one node to every other vertex [6].

Figure 10 provided two companies, one politician, and seven governmental organizations

page_name	betweenness	page_type
Facebook	0.115790	company
Barack Obama	0.089628	politician
The Obama White House	0.039820	government
The White House	0.039805	government
European Parliament	0.025954	government
CNN	0.022697	company
NATO	0.019557	government
Joachim Herrmann	0.019308	politician
U.S. Embassy Ottawa	0.017641	government
U.S. Department of State	0.015456	government

Figure 9: Top Ten Highest Betweenness Centrality

page_name	closeness	page_type
Facebook	0.324158	company
The Obama White House	0.317475	government
The White House	0.317413	government
Barack Obama	0.316438	politician
U.S. Embassy Ottawa	0.303557	government
CNN	0.302080	company
U.S. Department of State	0.302027	government
U.S. Army	0.298901	government
U.S. Embassy in Mozambique	0.297753	government
NATO	0.297469	government

Figure 10: Top Ten Highest Closeness Centrality

in the top ten overall nodes with the highest Closeness centrality scores. The 10th highest had 0.0297 and the highest had 0.324.

2.3.3 Eigenvector Centrality

Lastly, Eigenvector centrality needs to be measured for this network. Eigenvector centrality measures the connections a node has, not just the quantity, but also the quality or the influence of those connected nodes. A high Eigenvector score would indicate that a node is connected to many other highly influential, high scoring nodes [6].

page_name	eigenvector	page_type
U.S. Army	0.177841	government
U.S. Army Chaplain Corps	0.160628	government
Honolulu District, U.S. Army Corps of Engineers	0.136369	government
Army Training Network (ATN)	0.121057	government
Defense Commissary Agency	0.120850	government
The White House	0.117068	government
The Obama White House	0.115851	government
U.S. Army Materiel Command	0.113773	government
United States Air Force	0.108163	government
U.S. Army Garrison Red Cloud	0.106441	government

Figure 11: Top Ten Highest Eigenvector Centrality

Similar to degree centrality, the top ten Eigenvector nodes are government-related page types. The range of the top ten eigenvector nodes ranges from 0.106 to 0.178 shown in **Figure 11**.

2.4 Community Detection - Modularity

The second objective evolves around the idea of community within a network and method to detect those communities. According to a research article called "Defining and Identifying Communities in Networks", a community is a group of nodes that have more dense connections between themselves comparing to the rest of the network [7]. One of the popular methods to detect community is by using Modularity measurement. Modularity is "the portion of the edge connections within the same cluster minus the expected portion if the connections were distributed randomly" [8].

Gephi can calculate and divide a network into each Modularity Class by providing a resolution number. According to Gephi's instruction, the higher the resolution value, the fewer the modularity classes there are. By providing a resolution value of 10, Gephi detected 8 modularity classes. Half of the modularity classes made up 98.37% of the entire population; therefore, they can be the main four communities to compare to the four categories of Facebook pages (see **Figure 12**).

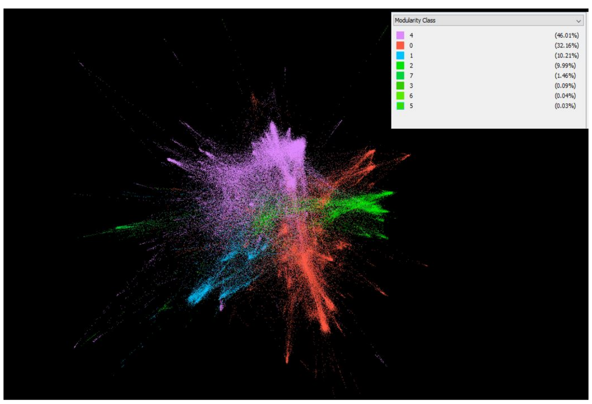


Figure 12: Modularity Classes Graph

3 Visualize, Analyze and Compare

3.1 Analysis and Comparison Methods

Network Science provides concepts and approaches to help measure influence from analysis, foremost of these approaches is an algorithmic tool to evaluate nodes, edges, communities in a network. Using these tools, the objective measurement was possible to evaluate the importance and influence in a network like the Facebook dataset that was selected for this study. The selection of the algorithms used to measure influence was done carefully in order to address the measure influence from as many objective angles as possible. The analysis was conducted to measure centrality, community structure, and overall importance of nodes in the Facebook dataset.

3.2 Comparing Degree Centrality to other Centrality Measurements

3.3 Visualizing and Analyzing Network of Each Page Category

One area that needs more exploration is identifying influential nodes per each page category. When exploring the entire network as a whole, government pages were the majority of the resulted influential nodes. However, in some cases, there might be a specific need to know the influential nodes of each individual category. Therefore, instead of exploring the entire network as a whole, it is beneficial to filter the network by page type to analyze the network of each page category using betweenness, closeness, and Eigenvector centrality measurements.

3.3.1 Governmental Organization Pages Network

By filtering the original network by page type of 'government' with nodes that are higher than 100 degrees, it resulted in a new network of 231 nodes and 4,869 edges. After recalculating all centrality measurements using the new government page network, Gephi can visualize and rank nodes with higher value by darkening node color and increasing node size.

Betweenness Centrality **Appendix A** is the visualization of the betweenness centrality ranking of the government pages network. The three nodes with the highest betweenness centrality value are 10379, 19743, 21729, which are the U.S Department of State, The White House, and The Obama White House, respectively. The result of this filtered population is similar to the result of the entire network as government pages have the highest population and high degree nodes. However, the order of ranking is different from the filtered government population to the entire network. Though being the 6th highest degree node, the U.S Department of State is the highest value of betweenness centrality within its own page type. In other words, the U.S Depart of State has the shortest-path to the rest of the governmental organization pages. This information is useful if the use case is to identify nodes that can influence the majority of the government pages.

Closeness Centrality For the closeness centrality of government pages, the ranking is shown in **Appendix F**. The closeness centrality graph is a lot different than the betweenness centrality graph of government pages network. There are a lot of darker nodes meaning there are many nodes with high closeness centrality value. Some of the nodes with high closeness centrality value are 1387 - Honolulu District, U.S. Army Corps of Engineers, 16895 - U.S. Army, 19743 - The White House, 21729 - The Obama White House, and so on. Some of the examples were also nodes with high betweenness centrality; however, some of them were not. Using closeness centrality, more influential nodes were identified and they are all efficient in spreading information, just not by using the same path measurements to other connected nodes.

Eigenvector Centrality The ranking of Eigenvector centrality of government pages is displayed in **Appendix J**. Two nodes are darker than the rest which are 14497 and 16895. Node 14497 is U.S. Army Chaplain Corps and node 16895 is U.S. Army. Again, the U.S. Army was also one of the top nodes for betweenness centrality and closeness centrality. U.S Army Chaplain Corps is a new node that stands out a lot more by using eigenvector centrality.

In summary, U.S Army seems to be the most obvious and safe influential node pick since it has the highest ranking in every centrality measurement. However, other nodes are also useful for different scenarios and needs. However, all of these nodes are not new information, they were already discovered by looking at the top ten degree nodes of the whole network.

3.3.2 Politician Pages Network

The next filtered network is by selecting nodes that have page types of 'politicians' and have higher than 100 degrees. This politician network contains 64 nodes and 379 edges.

Betweenness Centrality **Appendix B** is the graph of betweenness centrality ranking for politician pages network. Some of the more visible nodes are 11611, 11003, and 3906. Node 11611 is Justin Trudeau, node 11003 is Barack Obama, and node 3906 is Michael Roth. Besides Barack Obama being mentioned in the top ten closeness and between the centrality of the whole network, the other two nodes are completely new findings. This information can be used for those who are interested in influencing or spreading information within specifically politician pages using the shortest-path measurements.

Closeness Centrality In **Appendix F**, node 18819, 3906, and 11003 were the highest closeness centrality nodes within the politician network. They are Niels Annen, Michael Roth, and Barack Obama respectively. Niels Annen is a discovery using closeness centrality measurement.

Eigenvector Centrality **Appendix J** is the graph of Eigenvector centrality ranking for politician network. It does not show any nodes that stand out with a specifically higher value of eigenvector comparing to the rest of the nodes.

3.3.3 Television Show Pages Network

By selecting the page type of 'tvshow' and higher-than-100-degree nodes, it results in a new filtered television show network. This network contains 90 nodes and 1917 edges.

Betweenness Centrality Appendix C shows the ranking of betweenness centrality of television show pages network. Interestingly, it is quite obvious that there are two separate clusters within this network and the three highest between centrality nodes are the connection points between the two clusters. It is still unclear and requires more analysis of what the differences between the two clusters are. The three highest betweenness centrality nodes are 18952, 11248, 4296 which represents Access, Empire, and Home & Family, respectively. All of these nodes are new information is essential for the use case of finding influential nodes within television shows network and can be overseen by just looking at the network as a whole.

Closeness Centrality For closeness centrality ranking of television show network, **Appendix G** shows that the majority of the nodes in the bigger cluster have high closeness centrality comparing to the other cluster. Some of the darker nodes such as 11428, 13140, and 1517 have higher closeness centrality than the rest. Node 11428 is the show Empire, node 13140 is New Girl, and node 1517 is Brooklyn Nine-Nine. None of them were highlighted using betweenness centrality.

Eigenvector Centrality Appendix K shows that no node has specifically higher eigenvector centrality comparing to the rest of the television show pages.

3.3.4 Company Pages Network

The company pages network has lower degree nodes as shown in **Figure 7**; therefore, the filter criteria are different than the rest of the networks. The filtered company network is using nodes that are higher than 50 degrees with page type of 'company'.

Betweenness Centrality Appendix D shows the betweenness centrality ranking of the company pages network. Some noticeable nodes are 20083 - DIRECTV Puerto Rico and 5114 - Cartoon Network. This information is really interesting because big companies like Facebook or Walmart showed up in the highest degree company pages list but did not make it to the highest betweenness centrality list. This is another example of how degree centrality can be different than betweenness centrality.

Closeness Centrality Appendix H is the ranking visualization of closeness centrality for the company pages network. Some larger size nodes are 14228, 701, and 20083. Node 20083 represents DIRECTV Puerto Rico was already discovered using betweenness centrality, node 701 is Facebook which is the highest degree node within company pages, and 14228 is Oreo company. Oreo is a new finding that did not show up in the top ten highest degree centrality list nor the highest betweenness centrality list for company pages.

Eigenvector Centrality Appendix L shows no node with a higher eigenvector comparing to the rest of the company pages.

3.4 Comparing Community Detection Results to Page Types

By comparing between **Figure 3** and **Figure 12**, there are some similarities but some differences. A portion of government pages stays within the same community (modularity class 4) while the rest are in a different community (modularity class 0) that contains mostly politicians. The majority of company pages are actually in the same community as government pages. Some government and politician pages are also in the same new community (modularity class 1).

3.5 Overall Weighted Influence Score

Cenrality Measures and Node Degree Combining all the centrality measures and the node degree an overall weighted influence score was derived. The weighted score was established by giving each node a 1 through 10 scores based on each individual centrality measure. Adding these scores, an overall weighted influence score across all measures was derived.

Page Name	Page Type	Weighted Influence Score
The Obama White House	government	29
The White House	government	29
U.S. Army	government	23
Facebook	company	20
U.S. Army Chaplain Corps	government	16
Barack Obama	politician	16
Honolulu District, U.S. Army Corps of Engineers	government	14
U.S. Department of State	government	10
CNN	company	10
European Parliament	government	9
Army Training Network (ATN)	government	9
U.S. Embassy Ottawa	government	8
Defense Commissary Agency	government	7
NATO	government	5
FEMA Federal Emergency Management Agency	government	4
Joachim Herrmann	politician	3
U.S. Army Materiel Command	government	3
U.S. Embassy in Mozambique	government	2
United States Air Force	government	2
U.S. Army Garrison Red Cloud	government	1

Figure 13: Weighted Influence

4 Discussion

This study attempts to derive and understand how to determine influence in a social media network. The research found that several nodes were high scoring in terms of both connectivity to other nodes: degree centrality, but also in most of the algorithmic centrality methods. In particular: “The Obama White House”, “The White House”, “U.S. Army”, and “Facebook” appear to be highly influential in our network. Further study could be done to discover how these nodes grew to be influential nodes overtime.

The study looked closely at communities and found variability in community formation as defined by page type. The research found that government and company nodes were found in a large community but on the whole, the overall influential category of government nodes was found spread throughout various communities. A future study could look into how communities change over time with the emergence of new influential nodes. The node “The Obama White House” and “The White House” may be of particular interest because of the nature of the US presidency being of limited duration. A hypothesis would be that an updated dataset would find that the node “The Obama White House” would have reduced influences based on this study’s metrics but of even more interest would be to see how the communities around these nodes evolved.

More centrality methods and community detection algorithms and methods could be useful in the long term to derive an understanding of influence in the network. But the methods used in this study appear to be a good baseline.

Besides, while this study focused on a social media dataset, the network science principles of centrality and communities could be applied to many other network types to identify how events or static ongoing influence impacts a network.

5 Conclusion

Understanding influence in a network is important; groups spreading misinformation or the ability of a node to provide health information to the community at risk of a pandemic are several of many reasons among many to understand and utilize tools to identify and influence and communities in a social network. This study found that tools like Networkx package and Gephi are excellent tools to utilize to achieve the ends of this study as we seek a better understanding of social and other networks all around us.

6 Acknowledgment

Special thanks to professor Yong-Yeol Ahn and Jaehyuk Park for introducing multiple methods, concepts, and measurements in network science used in this paper.

7 References

References

[1] WebFX, “How much does social media marketing cost in 2020?.” https://www.webfx.com/Social-Media-Pricing.html. Access Online on 2020-07-03.

[2] A. Karataş and S. Şahin, “A comparative study of modularity-based community detection methods for online social networks,” http://ceur-ws.org/Vol-2201/UYMS_2018_paper_68.pdf, 2018.

[3] M. Nandi, “Social network analysis with NetworkX.” https://blog.dominodatalab.com/social-network-analysis-with-networkx, July 2015. Access Online on 2020-07-03.

[4] K. Brauner, A. Kocheturov, and P. M. Pardalos, “Network analysis of page likes from facebook user profiles,” ȟttps://ufdc.ufl.edu/UF00091523/00851.

[5] S. N. A. Project, “Facebook Large Page-Page Network.”

https://snap.stanford.edu/data/facebook-large-page-page-network.html. Accessed Online on 2020-07-03.

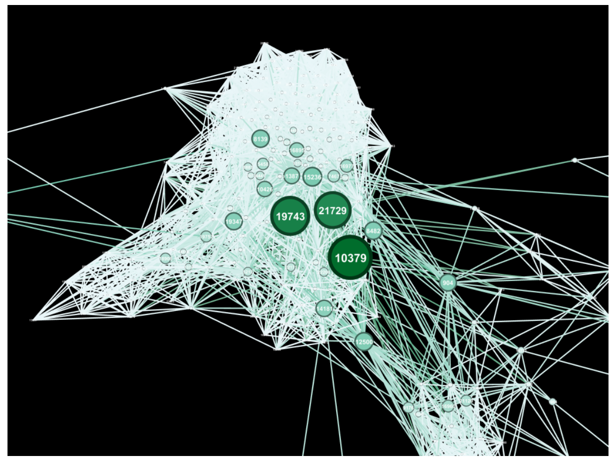
[6] M. E. Newman, "The mathematics of networks," *The new palgrave encyclopedia of economics*, vol. 2, no. 2008, pp. 1–12, 2008.

[7] F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, and D. Parisi, "Defining and identifying communities in networks," *Proceedings of the National Academy of Sciences*, vol. 101, no. 9, pp. 2658–2663, 2004.

[8] W. Li and D. Schuurmans, "Modular community detection in networks." https://www.ijcai.org/Proceedings/11/Papers/231.pdf. Accessed Online on 2020-05-03.

Appendices

Appendix A



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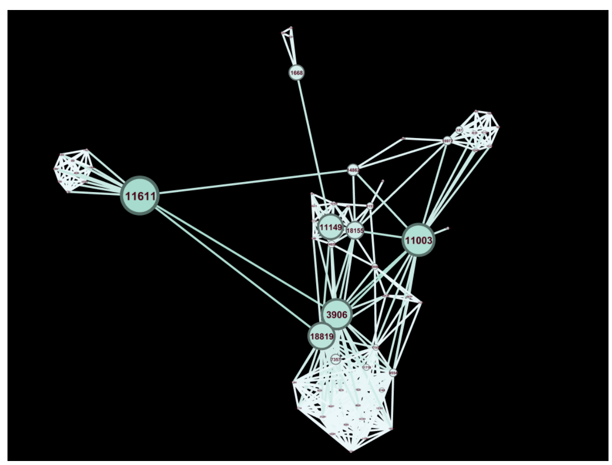
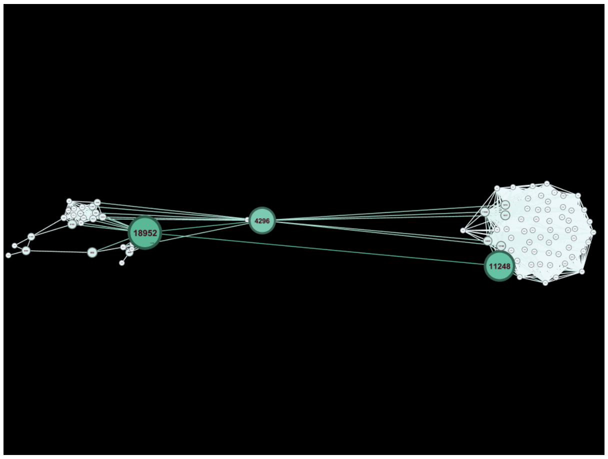


Appendix C



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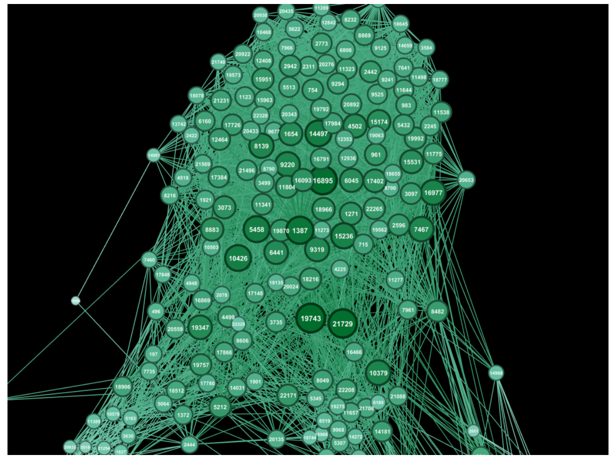
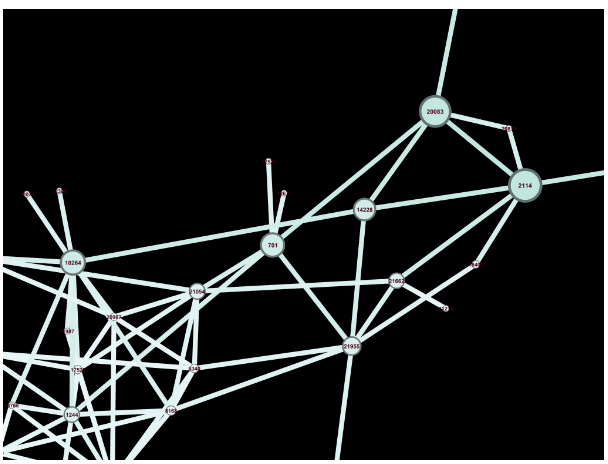
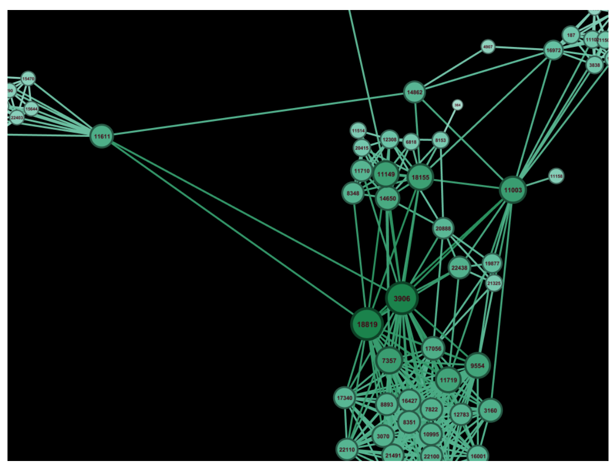


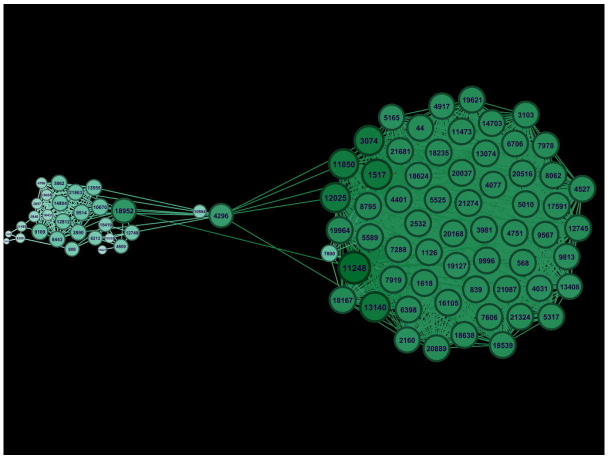
Appendix F



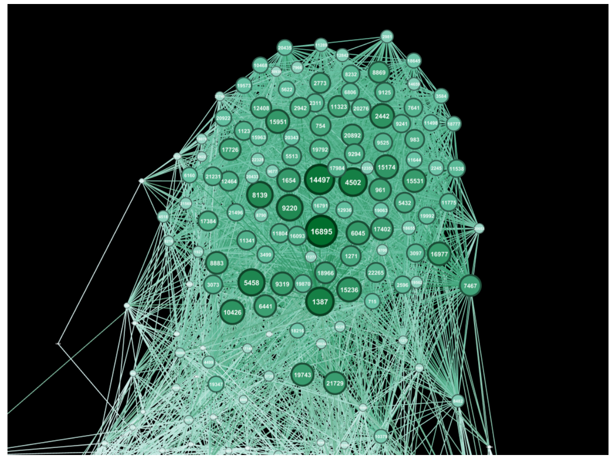
Appendix E



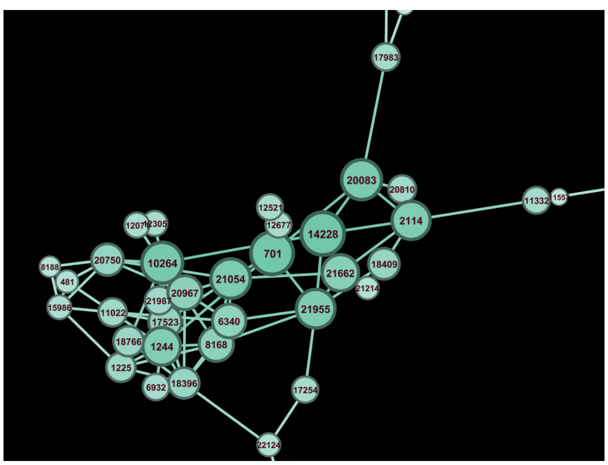
Appendix G



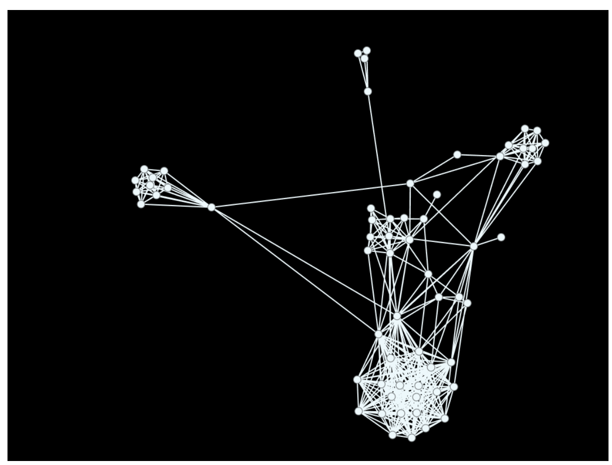
Appendix H



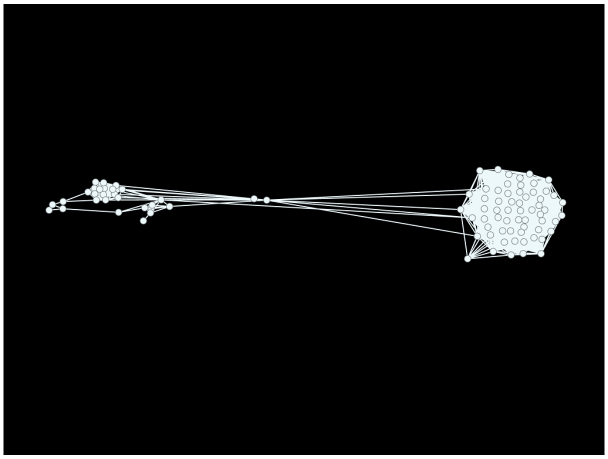
Appendix J



Appendix I



Appendix K



Appendix L