Empirical relations for simple design of photonic crystal fibers

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Abstract: In order to simply design a photonic crystal fiber (PCF), we provide numerically based empirical relations for V parameter and W parameter of PCFs only dependent on the two structural parameters – the air hole diameter and the hole pitch. We demonstrate the accuracy of these expressions by comparing the proposed empirical relations with the results of full-vector finite element method. Through the empirical relations we can easily evaluate the fundamental properties of PCFs without the need for numerical computations.

OCIS codes: (060.2280) Fiber design and fabrication; (060.2400) Fiber properties

References and links

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1. Introduction

Photonic crystal fibers (PCFs) [1], also called holey fibers or microstructured optical fibers, have been under intensive study as they offer design flexibility in controlling the modal properties. Index-guiding PCFs are usually formed by a central solid defect region surrounded by multiple air holes in a regular triangular lattice. These fibers have some extraordinary properties, such as wide single-mode wavelength range, unusual chromatic dispersion, and high or low non-linearity.

Theoretical descriptions of PCFs have traditionally been based on numerical approaches, such as the plane wave expansion method [2], the multipole method (MM) [3], the finite element method (FEM) [4, 5], and so on, because of the relatively complex cross section of a PCF for which rotational symmetry is absent. However, numerical simulations are, in general, time-consuming and costly. Recently, an analytical approach based on the *V* parameter (normalized frequency) frequently used in the design of conventional optical fibers has been developed for index-guiding PCFs [6]. By appropriately defining the *V* parameter, various unique properties of PCFs can be qualitatively understood within the framework of well-established classical fiber theories without heavy numerical computations [6]. Although the *V* parameter offers a simple way to design a PCF, a limiting factor is that a numerical method is still required for obtaining the accurate effective cladding index. If we can get empirical relations for not only *V* parameter but also *W* parameter (normalized transverse attenuation constant) only dependent on the wavelength and the structural parameters, they would be very useful for simple design of PCFs.

The aim of this work is to provide the empirical relations for *V* parameter and *W* parameter of PCFs based on the fundamental geometrical parameters – the air hole diameter and the hole pitch. We demonstrate the accuracy of these expressions by comparing the proposed empirical relations with the results of full-vector FEM. Through the empirical relations we can easily evaluate the fundamental properties of PCFs without the need for numerical computations.

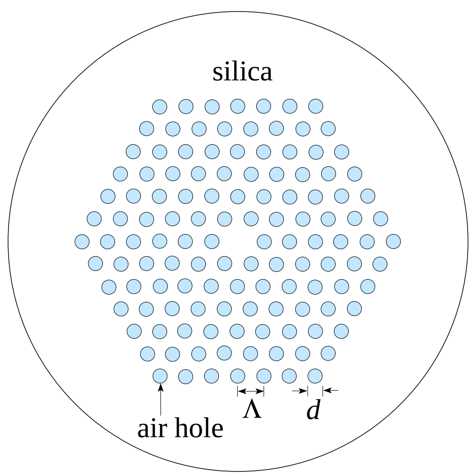

Fig. 1. Index-guiding photonic crystal fiber.

2. The *V* parameter expression

We consider a PCF with a triangular lattice of holes as shown in Fig. 1, where d is the hole diameter, Λ is the hole pitch, and the refractive index of silica is 1.45. In the center an air hole is omitted creating a central high index defect serving as the fiber core.

Recently, we have claimed that the triangular PCFs can be well parameterized in terms of the *V* parameter [6] that is given by

$$V = \frac{2\pi}{\lambda} a_{eff} \sqrt{n_{co}^2 - n_{FSM}^2} = \sqrt{U^2 + W^2} \tag{1}$$

with

$$U = \frac{2\pi}{\lambda} a_{eff} \sqrt{n_{co}^2 - n_{eff}^2} \tag{2}$$

$$W = \frac{2\pi}{\lambda} a_{eff} \sqrt{n_{eff}^2 - n_{FSM}^2} \tag{3}$$

where λ is the operating wavelength, n_{co} is the core index, n_{FSM} is the cladding index, defined as the effective index of the so-called fundamental space-filling mode in the triangular air-hole lattice [7], n_{eff} is the effective index of the fundamental guided mode, and a_{eff} is the effective core radius that here is assumed to be $\Lambda/\sqrt{3}$ [4, 6]. The parameters U and W are called, respectively, the normalized transverse phase and attenuation constants. Mortensen *et al*. proposed the following effective V parameter [8] for triangular PCFs:

$$V_{eff} = \frac{2\pi}{\lambda} \Lambda \sqrt{n_{eff}^2 - n_{FSM}^2} \tag{4}$$

and reported the empirical relation for V_{eff} of Eq. (4) [9]. However, this definition is intrinsically different from the original V parameter definition in step-index fiber (SIF) theory and corresponds to the W parameter. Therefore it seems to be difficult to apply the design principle of SIFs straightforwardly to PCFs. So we adopt the V parameter definition of Eq. (1). Although we can estimate the fundamental properties of PCFs using the V parameter in Eq. (1) [6], a limiting factor for using Eq. (1) is that a numerical method is required for obtaining the accurate effective cladding index n_{FSM}.

Figure 2 shows V values calculated through vector FEM [4] as a function of λ/Λ for d/Λ ranging from 0.20 to 0.80 in steps of 0.05, where data are shown by open circles. By trial and error, we find that each data set in Fig. 2 can be fitted to a function of the form

$$V\left(\frac{\lambda}{\Lambda}, \frac{d}{\Lambda}\right) = A_1 + \frac{A_2}{1 + A_3 \exp(A_4 \lambda / \Lambda)} \tag{5}$$

and the results are indicated by the solid curves. For accurate fitting, the data sets are truncated at $V = 0.85$. In Eq. (5) the fitting parameters A_i (i = 1 to 4) depend on d/Λ only. The data are well described by the following expression

$$A_i = a_{i0} + a_{i1}\left(\frac{d}{\Lambda}\right)^{b_{i1}} + a_{i2}\left(\frac{d}{\Lambda}\right)^{b_{i2}} + a_{i3}\left(\frac{d}{\Lambda}\right)^{b_{i3}} \tag{6}$$

and the coefficients a_{i0} to a_{i3} and b_{i1} to b_{i3} are given in Table 1. For $\lambda/\Lambda < 2$ and $V > 0.85$ the expression of Eq. (5) gives values of V which deviates less than 1.3% from the corrected values obtained from Eq. (1).

Using the effective V parameter in Eq. (5), the effective cladding index n_{FSM} can be obtained without the need for numerical computations. Figure 3 shows n_{FSM} as a function of λ/Λ for d/Λ ranging from 0.2 to 0.8 in steps of 0.1, where the open circles show numerical results from a fully vectorial FEM [4] and the solid curves show the results from Eqs. (1) and (5) with $a_{eff} = \Lambda/\sqrt{3}$. Mortensen *et al*. proposed the empirical expression for the value of n_{FSM} [10] to directly fit the effective cladding index, however, the results are not so accurate. On the other hand, the expression of Eq. (5) gives values of n_{FSM} which deviates less than 0.25% from the values obtained through vector FEM for $\lambda/\Lambda<1.5$ and $V>0.85$.

As in Ref. [6], from Eq. (5) the cutoff condition is given by V=2.405, as in conventional SIFs. Using the empirical relation of Eq. (5) and various formulas in terms of the V parameter found for SIFs, we can easily estimate the fundamental properties of PCFs, such as mode field diameter, beam divergence, splice loss, and so on [6].

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Table 1. Fitting coefficients in Eq. (6).

	$i = 1$	$i = 2$	$i = 3$	$i = 4$
a_{i0}	0.54808	0.71041	0.16904	–1.52736
a_{i1}	5.00401	9.73491	1.85765	1.06745
a_{i2}	–10.43248	47.41496	18.96849	1.93229
a_{i3}	8.22992	–437.50962	–42.4318	3.89
b_{i1}	5	1.8	1.7	–0.84
b_{i2}	7	7.32	10	1.02
b_{i3}	9	22.8	14	13.4

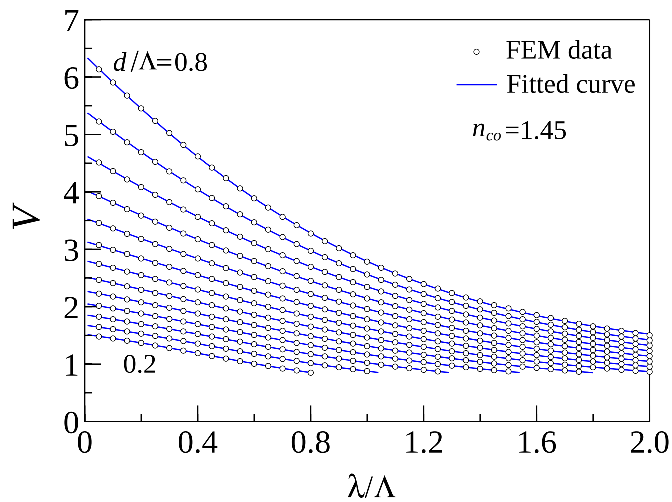

Fig. 2. Effective V parameter as a function of λ/Λ.

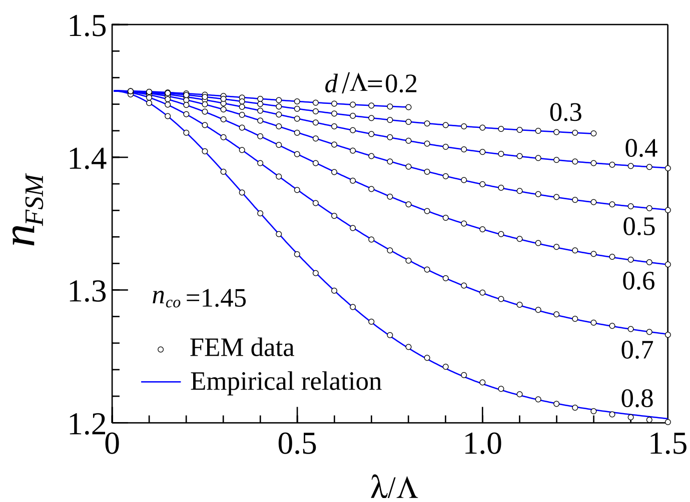

Fig. 3. Effective cladding index as a function of λ/Λ.

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3. The *W* parameter expression

In the previous section we provided the empirical relation for the V parameter of PCFs. Using Eq. (5) we can easily obtain the effective cladding index n_{FSM}, however, we usually need heavy numerical computations to obtain the accurate values of n_{eff} in Eq. (3). It would be more convenient to have an empirical relation for the W parameter of PCFs. Nielsen *et al*. have reported the empirical relation for the W parameter [9], however, we can not obtain the value of n_{eff} from the W parameter only. In order to obtain n_{eff}, we need the empirical relations for both the V and W parameters.

Figure 4 shows W values calculated through vector FEM [5] as a function of λ/Λ for d/Λ ranging from 0.20 to 0.80 in steps of 0.05, where data are shown by open circles. Again, by trial and error, we find that each data set in Fig. 4 can be fitted to the same function in Eq. (5) as

$$W\left(\frac{\lambda}{\Lambda}, \frac{d}{\Lambda}\right) = B_1 + \frac{B_2}{1 + B_3 \exp(B_4 \lambda / \Lambda)} \tag{7}$$

and the results are indicated by the solid curves. For accurate fitting, the data sets are truncated at $W = 0.1$. In Eq. (7) the fitting parameters B_i (i = 1 to 4) depend on d/Λ only. The data are well described by the following expression

$$B_i = c_{i0} + c_{i1}\left(\frac{d}{\Lambda}\right)^{d_{i1}} + c_{i2}\left(\frac{d}{\Lambda}\right)^{d_{i2}} + c_{i3}\left(\frac{d}{\Lambda}\right)^{d_{i3}} \tag{8}$$

and the coefficients c_{i0} to c_{i3} and d_{i1} to d_{i3} are given in Table 2. For $\lambda/\Lambda < 2$ and $W > 0.1$ the expression of Eq. (7) gives values of W which deviates less than 0.015 from the corrected values obtained from Eq. (3).

Using the V parameter in Eq. (5) and the W parameter in Eq. (7), the effective index of the fundamental mode n_{eff} can be obtained without the need for numerical computations. Figure 5 shows n_{eff} as a function of λ/Λ for d/Λ ranging from 0.2 to 0.8 in steps of 0.1, where the open circles show numerical results from a fully vectorial FEM [5] and the solid curves show the results from Eqs. (1), (3), (5), and (7) with $a_{eff} = \Lambda/\sqrt{3}$. For $\lambda/\Lambda<1.5$ and $W>0.1$ the expressions of Eqs. (5) and (7) give values of n_{eff} which deviates less than 0.15% from the values obtained through vector FEM.

Table 2. Fitting coefficients in Eq. (8).

	i = 1	i = 2	i = 3	i = 4
c_{i0}	–0.0973	0.53193	0.24876	5.29801
c_{i1}	–16.70566	6.70858	2.72423	0.05142
c_{i2}	67.13845	52.04855	13.28649	–5.18302
c_{i3}	–50.25518	–540.66947	–36.80372	2.7641
d_{i1}	7	1.49	3.85	–2
d_{i2}	9	6.58	10	0.41
d_{i3}	10	24.8	15	6

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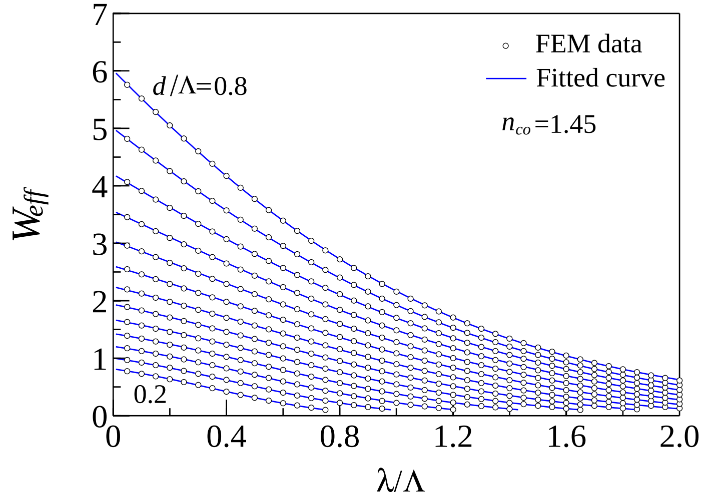

Fig. 4. Effective W parameter as a function of λ/Λ.

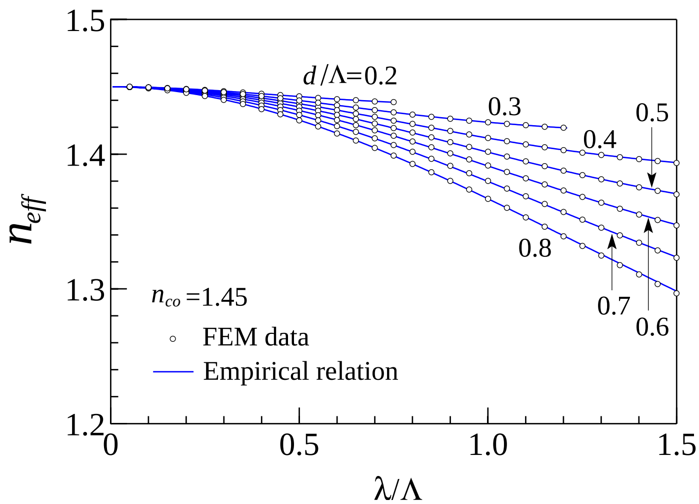

Fig. 5. Effective index of the fundamental mode n_{eff} as a function of λ/Λ.

Next, using the empirical relations of Eqs. (5) and (7) we calculate the chromatic dispersion in PCFs. In order to use universal data for the effective index of the fundamental mode n_{eff}, we assume that the waveguide contribution to the dispersion parameter D is independent of material dispersion D_m, namely

$$D = -\frac{\lambda}{c}\frac{d^2 n_{eff}}{d\lambda^2} + D_m \tag{9}$$

where c is the light velocity in a vacuum and D_m is given by the Sellmeier relation. Figure 6 shows the dispersion parameter D as a function of wavelength for d/Λ ranging from 0.2 to 0.8 in steps of 0.1, where the background index of silica is assumed to be 1.45, namely, n_{co} = 1.45. The results based on the empirical relations of Eqs. (5) and (7) agree well with the numerical results obtained by FEM [5]. It is worth noting that, in Ref. [6], the chromatic dispersion of PCFs was calculated by using the Gloge formula [11] and V values, while, here, the expressions of Eqs. (5) and (7) are used for direct calculation of the chromatic dispersion.

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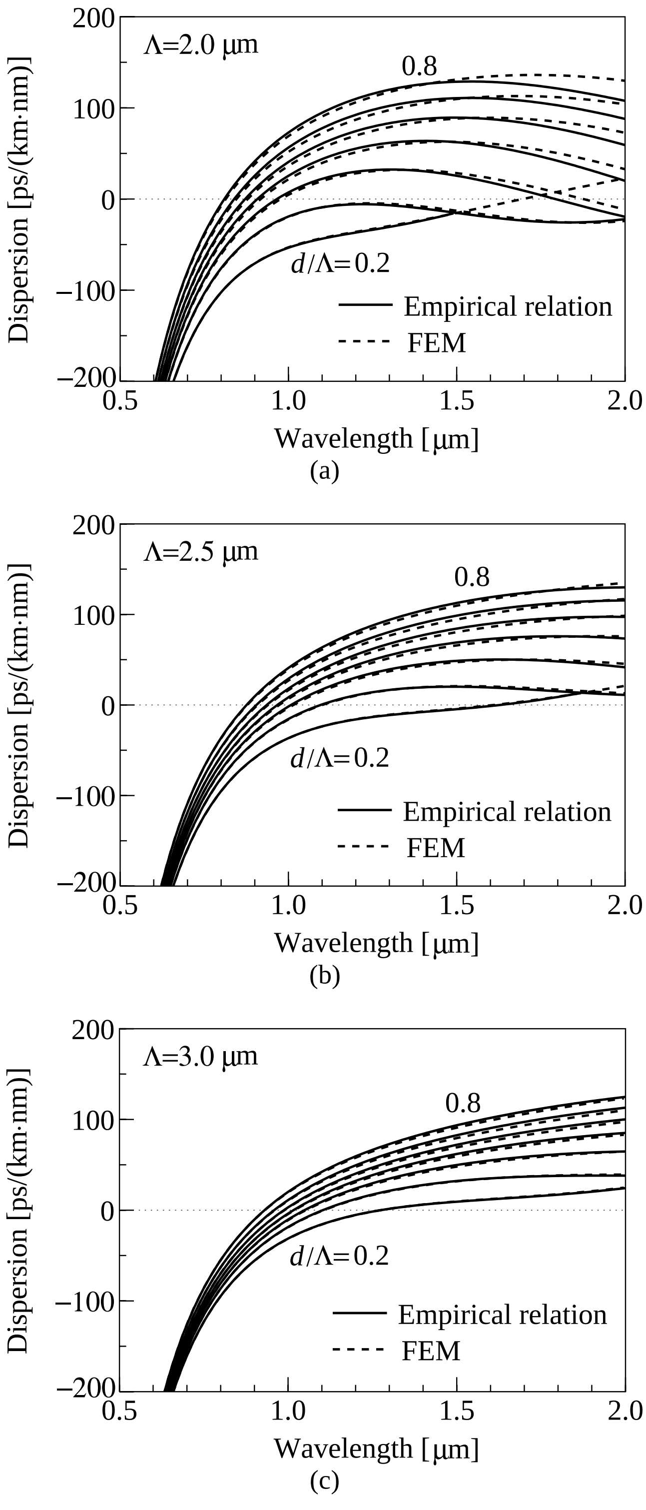

Fig. 6. Chromatic dispersion as a function of wavelength for (a) Λ = 2.0 μm, (b) Λ = 2.5 μm, and (c) Λ = 3.0 μm. Solid curves, results of empirical relations; dashed curves, results of vector FEM.

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4. Conclusions

In order to simply design a PCF, we provided the empirical relations for both *V* parameter and *W* parameter of PCFs only dependent on the air hole diameter and the hole pitch. We demonstrated the accuracy of these expressions by comparing the proposed empirical relations with the results of full-vector FEM. Through the empirical relations the fundamental properties of PCFs could be easily estimated without the need for numerical computations.

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