Waveguides, Optical Fibres and Photonic crystal fibres
Dispersive effects

3.1 Introduction and notations

Optical fibres are extensively used for telecommunications. Dispersion might yield distortion of the signal and it is therefore essential to look at their dispersion properties but most generally to study the dispersion of waveguides. This is what we would like to address in this chapter. At first, we will go quickly over the necessary concept of group velocity and group velocity dispersion. In a second time, we'll focus on optical fibre and see the origins of the dispersion in these waveguide.

As we know the dispersion effects are the results of the change of refractive index with the wavelength. The larger the bandwidth of the signal that is to be transported in the waveguide, the stronger may be the dispersion effects. This particularly apply to the propagation of pulses, the shorter temporally the broader spectrally. We will also see in a subsequent chapter that when the dispersion is associated to nonlinearity, this can yield generation of new frequencies in a control manner.

The electric field is written as a superposition of monochromatic waves by using a Fourier transform:

$$E(t) = \mathcal{F}\left[\widetilde{E}(\omega)\right](t) = \int \widetilde{E}(\omega) e^{-i\omega t} \frac{d\omega}{2\pi} \tag{3.1}$$

where

$$\widetilde{E}(\omega) = \mathcal{F}^{-1}\left[E(t)\right](\omega) = \int E(t) e^{i\omega t} dt \tag{3.2}$$

is a complex quantity. Since the electric field is a real quantity, its Fourier transform has to be symmetric and therefore $\tilde{E}^*(\omega) = \widetilde{E}(-\omega)$, where the * indicates the complex conjugate. The knowledge of the field for the positive frequencies is enought to reconstruct the whole electric field. We can then introduce the complex electric field by

$$\mathcal{E}(\omega) = 2\Theta(\omega)\widetilde{E}(\omega) \tag{3.3}$$

where the Heaviside function $\Theta(\omega)$ insure that we only consider the values corresponding to the positive frequencies. Inversely the electric field can be written from the complex field by

$$\widetilde{E}(\omega) = \frac{1}{2}\left[\mathcal{E}(\omega) + \mathcal{E}^*(-\omega)\right] \tag{3.4}$$

and using the inverse Fourier transform defined by eq. (3.2) we have the electric field given by

$$E(t) = \frac{1}{2}\left[\mathcal{E}(t) + \mathcal{E}^*(t)\right] = \mathrm{Re}\left[\mathcal{E}(t)\right] \tag{3.5}$$

Note that by using this notation we can easily introduce the amplitude $|\mathcal{E}(t)|$ and the phase $\phi(t)$ of the temporal signal such that

$$\mathcal{E}(t) = |\mathcal{E}(t)| e^{i\phi(t)} \tag{3.6}$$

Of course, in its most general form the electric field is a vectorial quantity which depends on both time and space:

$$\mathbf{E}(\mathbf{r}, t) = \int \widetilde{\mathbf{E}}(\mathbf{r}, \omega) e^{-i\omega t} \frac{d\omega}{2\pi} \tag{3.7}$$

or by using a plane-wave decomposition:

$$\mathbf{E}(\mathbf{r},t) = \int \widetilde{\mathbf{E}}(\mathbf{k},\omega) e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})} \frac{d\omega}{2\pi} \frac{d^3k}{(2\pi)^3} \tag{3.8}$$

where $\mathbf{k}$ is the wavevector. Its norm is the wavenumber $k = (n\omega/c)$ and n is a function of the frequency ω.

3.2 The influence of dispersion

3.2.1 Material dispersion and wavevector-frequency diagram

The theory is dispersion often starts with the bound electron model, yielding the susceptibility from which we extract the notion of refractive index and absorption band. Around this absorption band the polarisability has a Lorentzian-shape (Fig. 3.5b). Between two absorption band, the refractive index is a monotonically increasing function (Fig. 3.5a) for which the second derivative is negative, corresponding the *normal dispersion regime*.

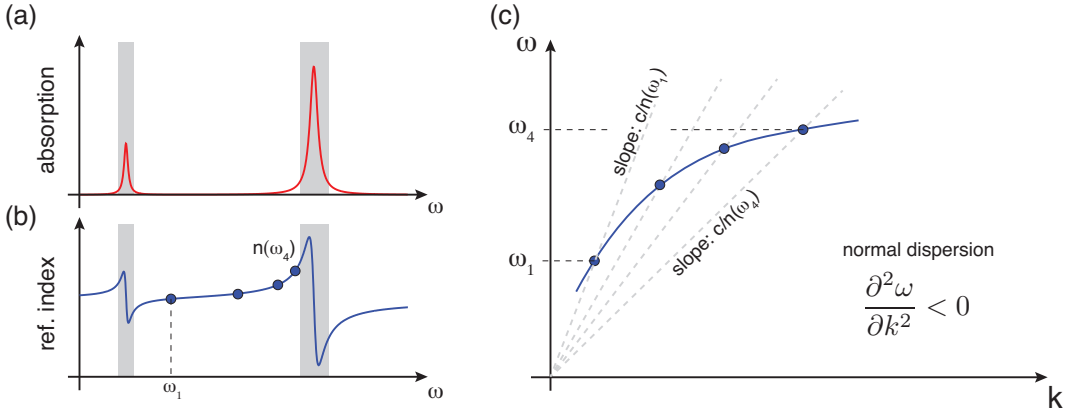

Figure 3.1: Material dispersion: (a) Schematic of the imaginary part (absorption) and (a) real part (refractive index) of the susceptibility of a material between two absorption bands and (c) dispersion diagram $\omega(k)$. The phase velocity is the slope of dashed grey line c/n and the local curvature is the group velocity v_g.

3.2.2 Group velocity

Let us consider a quasi-monochromatic pulse propagating along the $\hat{\mathbf{z}}-$direction. Such pulse can be described as the superposition of plane-waves:

$$E(z,t) = \int_{-\infty}^{+\infty} \widetilde{A}(\omega - \omega_0) e^{-i(\omega t - kz)} d\omega \tag{3.9}$$

where $A(\omega - \omega_0)$ is the complex amplitude of each spectral component of the pulse centred around $\omega = \omega_0$. Since we consider a quasi-monochromatic pulse, its spectral width $d\omega$ is much smaller than ω_0 its central frequency optical. If we do not know exactly the dispersion relation $k = k(\omega)$ we can use a Taylor expansion around the central frequency ω_0

$$k(\omega) = k_0 + \left(\frac{dk}{d\omega}\right)_{\omega_0} (\omega - \omega_0) + o(\omega^2) \tag{3.10}$$

where $k_0 = k(\omega_0)$ and the subscript in the derivative indicates that the derivative is calculated at $\omega = \omega_0$.

Inserting this relation of dispersion into eq. (3.9) yields

$$\begin{aligned}
E(z,t) &= \int_{-\infty}^{\infty} \widetilde{A}(\omega-\omega_0)\; e^{-i\left\{\omega t-\left[k_0+\left(\frac{dk}{d\omega}\right)_{\omega_0}(\omega-\omega_0)\right]z\right\}}d\omega \\
&= \int_{-\infty}^{\infty} \widetilde{A}(\omega-\omega_0)\; e^{-i\left\{(\omega-\omega_0+\omega_0)t-\left[k_0+\left(\frac{dk}{d\omega}\right)_{\omega_0}(\omega-\omega_0)\right]z\right\}}d\omega \\
&= e^{-i(\omega_0 t-k_0 z)}\int_{-\infty}^{\infty} \widetilde{A}(\omega-\omega_0)\; e^{-i\left\{(\omega-\omega_0)t-\left[\left(\frac{dk}{d\omega}\right)_{\omega_0}(\omega-\omega_0)\right]z\right\}}d\omega \\
E(z,t) &= e^{-i(\omega_0 t-k_0 z)}\int_{-\infty}^{\infty} \widetilde{A}(\Omega)\; e^{-i\left\{\Omega t-\left[\left(\frac{dk}{d\omega}\right)_{\omega_0}\Omega\right]z\right\}}d\Omega
\end{aligned} \tag{3.11}$$

with $\Omega = \omega - \omega_0$. Note that this integral is simply the Fourier transform[1] of the spectral amplitude $A(\Omega)$:

$$E(z,t) = A\left[t-\left(\frac{z}{v_g}\right)\right]e^{-i(\omega_0 t-k_0 z)} \tag{3.12}$$

where we have defined the group velocity[2] v_g by

$$v_g = \frac{1}{\left(dk/d\omega\right)_{\omega_0}} = \frac{c}{n+\omega\dfrac{dn}{d\omega}} = \frac{c}{n_g} \tag{3.13}$$

where n_g is the *group index*. The group velocity can be interpreted as the speed at which the wave-packet propagates. From the relation of dispersion this is also obvious that this the inverse of the slope of this relation. As we see for the eq. (3.12), the pulse does not change its shape but acquire a delay as it propagates along z. After a distance L in the disperse medium the pulse has accumulated a delay

$$\tau_G = \frac{L}{v_G} \tag{3.14}$$

Group velocity dispersion

In the previous section, we truncated the Taylor expansion at the first order. This is acceptable when the bandwidth of the propagating pulse is small enough with respect to the change of slope of the dispersion curve (Fig. 3.2a). However, for broader spectra (Fig. 3.2b) the truncation at the first order is no longer sufficient and we need to use the second term in the expansion of the dispersion relation

$$k(\omega) = k_0 + \left(\frac{dk}{d\omega}\right)_{\omega_0}(\omega-\omega_0) + \frac{1}{2}\left(\frac{d^2k}{d\omega^2}\right)_{\omega_0}(\omega-\omega_0)^2 + o(\omega^3) \tag{3.15}$$

and use the same type of calculation as we did previously. Alternatively, we can consider that different parts of the spectrum will experience different group velocity yielding a *dispersion of the group delay* $\Delta\tau_G$. This can be evaluated from the fastest (with frequency ω_1) and the slowest

[1] We remind that the Fourier transform is defined as

$$G(\omega) = \mathcal{F}\left[g(t)\right] = \int_{\infty}^{\infty} g(t)e^{i\omega t}d\omega \quad \text{and} \quad \mathrm{g(t)} = \frac{1}{2\pi}\int_{\infty}^{\infty}\mathrm{G}(\omega)\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}\mathrm{d}\omega$$

We also remind the shift properties of the Fourier transform:

$$\mathcal{F}\left[g(t-\tau)\right] = e^{t\omega\tau}\mathcal{F}\left[g(t)\right] = e^{t\omega\tau}G(\omega)$$

[2] The group velocity is in general different from the phase velocity.

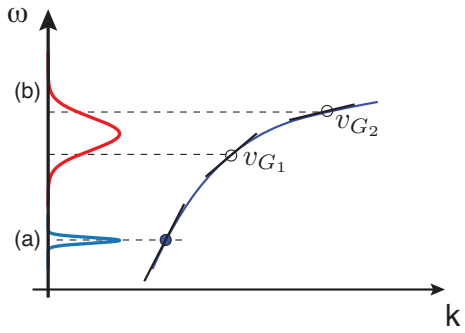

Figure 3.2: Relationship between the spectrum of the pulse and the normalised dispersion relation $k(\omega)$ in the case of (a) long and (b) short pulses.

(with a frequency ω_2) spectral component:

$$\Delta\tau_G = L\left(\frac{1}{v_G^{(2)}} - \frac{1}{v_G^{(1)}}\right) = L\left[\left(\frac{dk}{d\omega}\right)_{\omega_2} - \left(\frac{dk}{d\omega}\right)_{\omega_1}\right] \tag{3.16}$$

If we consider that the group-velocity $v_G^{(1)}$ is not too different from $v_G^{(2)}$ then we can use a Taylor expansion to re-calculate $\Delta\tau_G$

$$\Delta\tau_G = L\left[\frac{1}{v_G(\omega_2)} - \frac{1}{v_G(\omega_1)}\right] = L\left[\frac{1}{v_G(\omega_1)} + \frac{d}{d\omega}\left(\frac{1}{v_g}\right)_{\omega_1}(\omega_2 - \omega_1) - \frac{1}{v_G(\omega_1)}\right] \tag{3.17}$$

And since

$$\frac{d}{d\omega}\left(\frac{1}{v_G}\right) = \frac{d}{d\omega}\left(\frac{dk}{d\omega}\right) = \frac{d^2k}{d\omega^2} \tag{3.18}$$

we can rewrite broadening $\Delta\tau_G$ as

$$\Delta\tau_G = L\underbrace{\left|\left(\frac{d^2k}{d\omega^2}\right)_{\omega_0}\right|}_{\text{GVD}}(\omega - \omega_0) \tag{3.19}$$

Regarding the units of the different variable that we just introduced. We see from eq. (3.19) that the *group velocity dispersion* (GVD) has the unit of [time]^2/[length] whilst the group delay dispersion has the unit of [time]^2.

3.2.3 Pulse propagating in a purely dispersive medium

Since the dispersion is defined in the spectral domain, it is convinient to express the field in the frequency domain. Moreover, we can use the properties of the Fourier transform by considering that a change $\Delta\omega = \omega - \omega_0$ yields a change of wavenumber $\Delta k = k - k_0$ so that from the expression of the electric field in the space-time domain and its reciprocal $\{k - \omega\}$ domain:

$$\tilde{E}(\Delta k, \Delta\omega) \propto \iint E(z,t)e^{+i(\Delta\omega t - \Delta k z)}dzdt \tag{3.20a}$$

$$E(z,t) \propto \iint \tilde{E}(\Delta k, \Delta\omega)e^{-i(\Delta\omega t - \Delta k z)}d\omega dk \tag{3.20b}$$

we can identify that

$$\Delta k = (k - k_0) \longleftrightarrow -i\partial_z \tag{3.21a}$$

$$\Delta\omega = (\omega - \omega_0) \longleftrightarrow i\partial_t \tag{3.21b}$$

The operator equation (eq. (3.15) applied to the envelope of the input field is straightforwardly given by

$$-i\frac{\partial E}{\partial z} = ik_1\frac{\partial E}{\partial t} - \frac{1}{2}k_2\frac{\partial^2 E}{\partial t^2} \tag{3.22}$$

Neglecting the second order dispersion this simply becomes

$$-i\left(\frac{\partial}{\partial_z} + k_1\frac{\partial}{\partial z}\right)E = 0 \tag{3.23}$$

which has a trivial solution that can be expressed by any function of the variable $(z - t/k_1)$, corresponding to the situation when the pulse propagates at the group velocity without changing its shape. A good practise is to introduce the change of variable $T = t - z/v_g$, which means that the observer is travelling at the same velocity of the pulse. The eq. (3.22) then becomes

$$\boxed{i\frac{\partial E}{\partial z} = \frac{1}{2}k_2\frac{\partial^2 E}{\partial T^2}} \tag{3.24}$$

Using the so-called *Fourier-transform method*, which is based on the definition of the pulse from its spectrum

$$E(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\widetilde{E}(z,\omega)e^{-i\omega T}d\omega \tag{3.25}$$

and allows transforming the eq. (3.24) in the frequency domain by using

$$\mathcal{F}\left[\partial_z E(z,T)\right] = \partial_z\widetilde{E}(z,T) \tag{3.26a}$$

$$\mathcal{F}\left[(\partial_T)^n E(z,T)\right] = (-i\omega)^n\,\widetilde{E}(z,T) \tag{3.26b}$$

The equation (3.24) is then

$$i\frac{\partial\widetilde{E}(z,\omega)}{\partial z} = -\frac{1}{2}k_2\,\omega^2\widetilde{E}(z,\omega) \tag{3.27}$$

which is readily solved as

$$\widetilde{E}(z,\omega) = \widetilde{E}(0,\omega)\exp\left(\frac{i}{2}k_2\,\omega^2 z\right) \tag{3.28}$$

where $\widetilde{U}(0,\omega)$ is the spectrum of the pulse at $z = 0$. It is clear from the eq. (3.28) that the spectrum in the initial pulse is ***not modified*** during the propagation. As this equation suggests each spectral component ω acquires a different phase during the propagation. The temporal shape of the pulse after a distance z is then

$$E(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\widetilde{E}(0,\omega)\exp\left(\frac{i}{2}k_2\,\omega^2 z - i\omega T\right)d\omega \tag{3.29}$$

where $\widetilde{E}(0,\omega)$ is the inverse Fourier transform of the pulse at $z = 0$

$$\widetilde{E}(0,\omega) = \int_{-\infty}^{+\infty}E(0,T)e^{i\omega T}dT \tag{3.30}$$

Propagation of a Gaussian pulse As an example we consider here that we have a Gaussian pulse propagating in a dispersive medium. Its envelope is given by

$$E(0,T) = \exp\left(-\frac{T^2}{2\,T_0^2}\right) \tag{3.31}$$

where T_0 is the half-width (at $1/e$ -intensity point). Although this definition makes the calculations easier but it is in practice more convenient to work with the full-width at half-maximum (FWHM) instead of T_0

$$T_{\mathrm{FWHM}} = 2\sqrt{\ln 2}\;T_0 \approx 1.665\;T_0 \tag{3.32}$$

By inserting the eq. (3.31) into eq. (3.29) and calculating the integration we obtain the shape of the pulse at a distance z:

$$E(z,T) = \frac{T_0}{\sqrt{T_0^2 - ik_2 z}} \exp\left[-\frac{T^2}{2\left(T_0^2 - ik_2 z\right)}\right] \tag{3.33}$$

which is still a Gaussian pulse. However the amplitude and the duration of the pulse are modified. In particular the exponential can be written as

$$\begin{aligned}\exp\left[\frac{T^2}{2\left(T_0^2 - ik_2 z\right)}\right] &= \exp\left[-\frac{T^2\left(t_0^2 + ik_2 z\right)}{2\left(T_0^4 + k_2^2 z^2\right)}\right] \\ &= \exp\left[-\frac{T_0^2 T^2}{2\left(T_0^4 + k_2^2 z^2\right)}\right] \exp\left[-i\frac{k_2 z T^2}{2\left(T_0^4 + k_2^2 z^2\right)}\right]\end{aligned} \tag{3.34}$$

From the first exponential we find that the pulse is actually getting broader. Its duration is now given by

$$T^2(z) = \frac{T_0^4 + k_2^2 z^2}{T_0^2} \quad \rightarrow \quad \boxed{T(z) = T_0\sqrt{1 + \left(\frac{z}{L_D}\right)^2}} \tag{3.35}$$

where the *dispersion length* $L_D = T_0^2/|k_2|$. This length is a sort of scale for the minimal length necessary to observe the effect of dispersion. For a propagation distance $L > L_D$ the effect of dispersion can be observed. We could notice that this scaling factor was somehow already visible from the eq. (3.22) by analysing the dimensions:

$$\begin{aligned} i\frac{\partial E}{\partial z} &= -\frac{1}{2}k_2\frac{\partial^2 E}{\partial T^2} \\ \Rightarrow \quad \frac{[E]}{[\text{length}]} &= [k_2]\frac{[E]}{[\text{time}^2]} \;\Rightarrow\; \text{typical length} \propto \frac{[\text{time}^2]}{k_2}\end{aligned} \tag{3.36}$$

It is clear that the shorter the pulse the shorter the dispersion length. Therefore, for a given physical length of the dispersion element, short pulses will be more affected by dispersion than long pulses. In the context of laser cavity, this means that the effect of dispersion will be more severe when the targeted pulses are short.

Obviously the appearance of complex number in the expression of $E(z,T)$ in both the amplitude and the exponential part (eq. (3.33)) implies that the phase is modified as the pulse propagates. We can rewrite $E(z,T)$ as

$$E(z,T) = |E(z,T)| \exp\left[i\phi(z,T)\right] \tag{3.37}$$

where the phase $\phi(z,T)$ is then given by

$$\phi(z,T) = \frac{-\text{sgn}(\beta_2)\left(z/L_D\right)}{1 + \left(z/L_D\right)^2}\frac{T^2}{2T_0^2} + \frac{1}{2}\tan^{-1}\left(\frac{z}{L_D}\right) \tag{3.38}$$

Since the phase depends on time the instantaneous frequency will differ at different location in the pulse from its central frequency ω_0. This difference of instantaneous frequency ($\delta\omega$) is called *the chirp* and is defined (using eq. (3.38)) as[3]

$$\delta\omega = -\frac{\partial\phi}{\partial T} = \frac{\text{sgn}(\beta_2)\left(z/L_D\right)}{1 + \left(z/L_D\right)^2}\frac{T}{T_0^2} \tag{3.39}$$

It is clear that the instantaneous frequency changes linearly across the pulse. This is a *linear chirp*. For normal dispersion regime ($\beta_2 > 0$) $\delta\omega$ increases from negative ($T < 0$) value to positive ones. Obviously in anomalous dispersion regime the situation is reversed.

[3]The minus sign in the definition of the instantaneous frequency - and therefore the chirp - comes from the choice of definition for the electric field eq. (3.1).

3.3 Dispersion of optical fibres

As we saw, the constant β characterised the propagation of one specific mode, and it is a solution of one eigenvalue problem with a closing relation $V^2 = k_0^2 a^2 \left(n_{co}^2 - n_{cl}^2\right)$ that highly depends on the dimension of the considered waveguide, but also on the operating wavelength. Not only the wavelength appears in k_0 but also through the respective refractive indices of the core and cladding materials since these are wavelength-dependent. Such dependence leads to dispersion, which is a very common phenomena in many fields of physics such as acoustics, plasma physics... Back a few years in your study, you certainly studied the dispersion of white light by a prism.

In the context of waveguide, since $\beta = \beta(\omega)$ strongly depends on which mode to consider, the group velocity

$$v_G = \left(\frac{\partial \beta}{\partial \omega}\right)^{-1} \tag{3.40}$$

will differs from one mode to another[4] yielding **modal dispersion**. this property can be used in nonlinear optics to fulfil desired phase-matching conditions in order to observe one or another nonlinear effect (see exercise).

In the present case, we will focus on a simpler situation by considering a single-mode fibre, operating in the weak-guidance approximation. Hopefully, this will help us to apprehend the origin of the dispersion in a waveguide as a result of the waveguide dispersion and the material dispersion.

3.3.1 Dispersion of single-mode fibre

Following the approach from Gloge [2] we introduce the normalised propagation constant

$$b = 1 - \frac{U^2}{V^2} = \frac{n_{\text{eff}}^2 - n_{cl}^2}{n_{co}^2 - n_{cl}^2} = \frac{\beta^2 - k_0^2 n_{cl}^2}{k_0^2 \left(n_{co}^2 - n_{cl}^2\right)} \tag{3.41}$$

As it can be seen from the second expression of $b(v)$, this parameter indicates how the effective index of the considered mode n_{eff} is located between n_{co} and n_{cl}. Since $n_{cl} \leq n_{eff} \leq n_{co}$ then $0 \leq b(V) \leq 1$ and it approaches 0 when $n_{\text{eff}} \to n_{cl}$ corresponding to the cutoff frequency of that specific mode. From eq. 3.41 we have

$$\beta^2 = \left(\frac{\omega}{c}\right)^2 \left[n_{cl}^2 + \left(n_{co}^2 - n_{cl}^2\right) b\right] \tag{3.42}$$

which becomes in the weak-approximation limit[5]:

$$\beta \simeq kn\,[1 + \Delta b] \tag{3.43}$$

which we can use to evaluate the group delay $\tau_g = \dfrac{L}{c}\left(\dfrac{\partial \beta}{\partial k}\right)$:

$$\tau_g = \frac{L}{c}\left[\frac{d(nk)}{dk} + n\Delta \frac{d(bV)}{dV}\right] \tag{3.44}$$

which is remarkable since the first term is only linked to the dispersion of the materials, while the second origins from the waveguide itself. Note that the first term is called the *group index*:

$$n_g = \frac{d(nk)}{dk} = n - \lambda \frac{dn}{d\lambda} \tag{3.45}$$

[4] the value of β can be calculated by solving the eigenvalue equation for the considered mode. See previous chapter. You may also visit the online solver at `https://pulse-app-mpsp.herokuapp.com/apps/results`

[5] Since we are in the weak approximation we have $n_{co} \simeq n_{cl}$ and we can drop the index.

Material dispersion The term linked to the material dispersion (τ_m) is

$$\tau_g^{(mat.)} = \frac{L}{c}\frac{dn}{dk} = \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right) \tag{3.46}$$

And for a source with a central wavelength λ and a bandwidth $\delta\lambda$, we have the dispersion $\Delta\tau_m$ of τ_m as

$$\Delta\tau^{(mat.)} = \frac{\partial}{\partial\lambda}\left[\tau_g^{(mat.)}\right]\delta\lambda = -\frac{L}{c}\lambda\frac{d^2n}{d\lambda}\delta\lambda \tag{3.47}$$

yielding the dispersion per unit length and per unit spectral width caused by the material :

$$D_m = \frac{\Delta\tau_m}{L\delta\lambda} = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2} \tag{3.48}$$

and to evaluate $\left(d^2n/d\lambda^2\right)$ we use the Sellmeier expression

$$n^2(\lambda) = 1 + \sum_{j=1}^{3}\frac{a_j\lambda^2}{\lambda^2 - b_j^2} \tag{3.49}$$

The table 3.1 shows the coefficients for the Sellmeier equation for pure silica as well as for silica doped with different oxides - the concentration of the oxides is in mode %. On fig. 3.3, we plot the refractive index of those materials as well as the corresponding dispersion (eq. 3.48). As we can see,

dopant	a1	a2	a3	b1 [μm]	b2 [μm]	b3 [μm]
pure SiO_2	0.6961663	0.4079426	0.8974794	0.068404	0.116241	9.896161
GeO_2 (6.3%)	0.7083952	0.4203993	0.8663412	0.08538421	0.10248385	9.89617502
GeO_2 (19.3%)	0.77347008	0.4461191	0.8081698	0.07646793	0.12460807	9.89620331
B_2O_3	0.6910021	0.4022430	0.9439644	0.07058214	0.1172887	9.89613713
P_2O_5	0.7058489	0.4176021	0.8952753	0.07212788	0.11347819	9.89616138
Fluorine (1%)	0.69325	0.39720	0.86008	0.06724	0.11714	9.7761
Fluorine (2%)	0.67744	0.40101	0.87193	0.06135	0.12030	9.8563

Table 3.1: Coefficients of the Sellmeier equation for silica doped with different materials

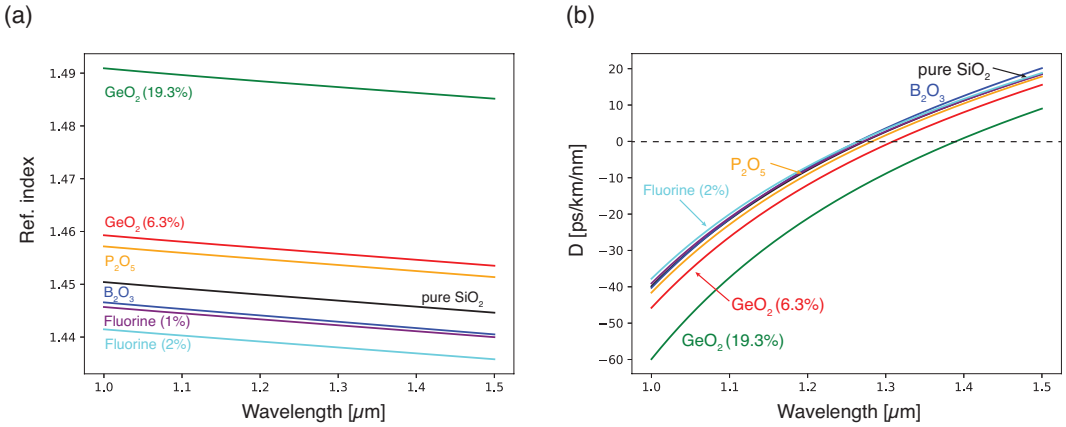

Figure 3.3: (a) Refractive index and (b) dispersion for pure silica and silica doped with different common oxides.

Waveguide dispersion For the second term of eq. 3.44 we proceed the same way to evaluate first $\Delta\tau_g^{(wav.)}$ the dispersion of the group delay originating from the waveguide:

$$\Delta\tau_g^{(wav.)} = \frac{\partial}{\partial\lambda}\left[\tau_g^{(wav.)}\right]\delta\lambda \simeq n\Delta\frac{L}{c}\frac{d^2(bV)}{dV^2}\cdot\frac{dV}{d\lambda}\delta\lambda \simeq \frac{-n\,\Delta}{c\,\lambda}V\frac{d^2(bV)}{dV^2}\cdot(\delta\lambda\times L) \tag{3.50}$$

And the dispersion per unit length and per unit spectral width due to the waveguide:

$$D_w = \frac{\Delta\tau_g^{(wav.)}}{L\delta\,\lambda} = \frac{-n\,\Delta}{c\,\lambda}V\frac{d^2(bV)}{dV^2} \tag{3.51}$$

In the case of a single-mode fibre, the waveguide dispersion can be approximated by [3]:

$$D_w \simeq \frac{-n_{cl}\Delta}{3\lambda_0}\left[0.008 + 0.549\cdot(2.834 - V)^2\right] \tag{3.52}$$

which is strictly negative since $V < 2.405$ for single-mode operation.

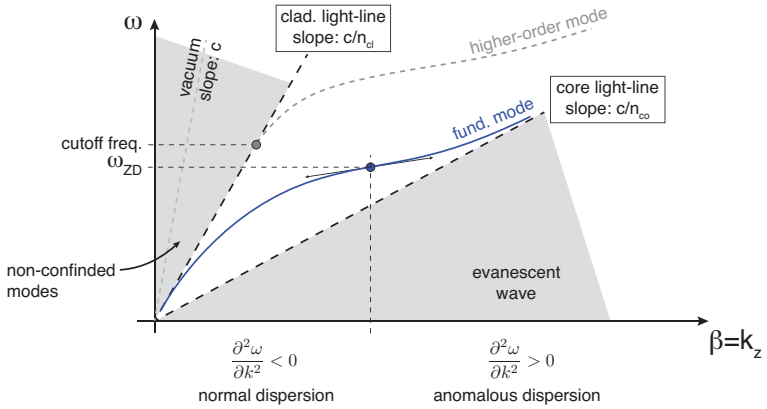

Figure 3.4: Material dispersion: (a) Schematic of the imaginary part (absorption) and (a) real part (refractive index) of the susceptibility of a material between two absorption bands and (c) dispersion diagram $\omega(k)$. The phase velocity is the slope of dashed grey line c/n and the local curvature is the group velocity v_g.

1. Standard single mode fibre (SMF): ZDW at 1.3 μm and $17ps/km/nm$ at 1550 nm. ($a = 4.1$ μm, $\Delta = 2.7\times10^{-3}$)

2. Dispersion shifted fibre (DSF): ZDW shifted to 1550nm
   ($a = 2.6\ \mu$m, $\Delta = 7.5 \times 10^{-3}$)

3. Dispersion compensated fibre (DCF) with strong negative dispersion at 1550 nm.
   ($a = 1.5\ \mu$m, $\Delta = 2 \times 10^{-2}$)

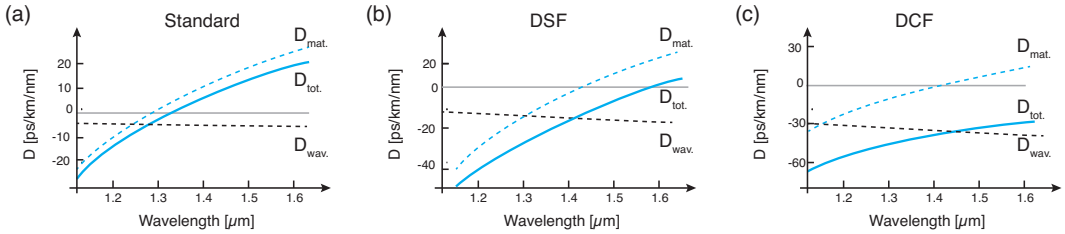

Figure 3.5: Typical dispersion for (a) standard fibre, (b) dispersion-shifted fibre and (c) dispersion compensated fibre.

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