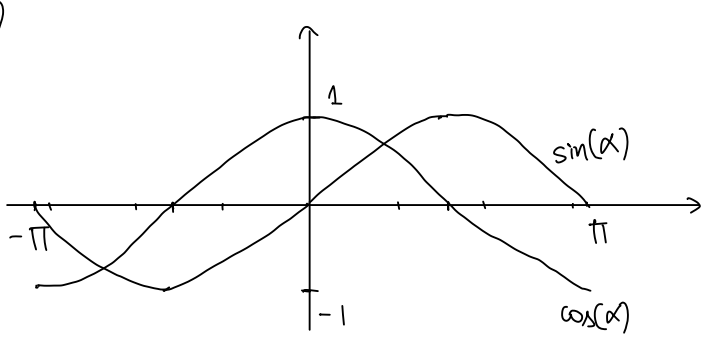
Homework 1A

1. Kinetic model for a simple car

(a)



Using this graph, we can justify that for small values of α ($\alpha \simeq 0$), we have:

$\sin(\alpha) \simeq 0$

$\cos(\alpha) \simeq 1$

$\tan(\alpha) \simeq 0$

(b) From part (a):

$$x(k+1) = x(k) + v(k)\cos(\theta(k))\Delta t$$
$$\simeq x(k) + v(k)\Delta t$$
$$y(k+1) \simeq y(k) + v(k)\sin(\theta(k))\Delta t$$
$$\simeq y(k)$$
$$\theta(k+1) \simeq \theta(k) + \frac{v(k)}{L}\tan(\phi(k))\Delta t$$
$$\simeq \theta(k)$$
$$v(k+1) \simeq v(k) + \alpha(k)\Delta t$$

$$A = \begin{bmatrix} 1 & 0 & 0 & \Delta t \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \Delta t & 0 \end{bmatrix}$$

(c) The linear system is a good approximation. This is because the angle is small (0.0001 rads) as our assumption in part (a).

(d) The linear system is different from the real circumstance. The reason for this is that the angle is larger than the one in part (c) (0.5 vs 0.0001) so the graph is not close to a straight line (linear) anymore.