Discussion 0B

1. Energy Disaggregation

(a) There are 3 unknowns (TV, air conditioner and refrigerator energy so we need to get 3 sets of measurements, where each one has to give us new information. One solution is to measure the power usage in the morning with the TV off (when only the refrigerator is running) and twice in the afternoon with the TV plugged and unplugged.

(b)
$$\begin{aligned} x_R &= T_1 \\ x_{AC} + x_{TV} + x_R &= T_2 \\ x_{AC} + x_R &= T_3 \end{aligned}$$

2. System of Equations:

(a) $\begin{cases} 2x + y = 6 \\ 3x - 2y = 2 \end{cases}$

$$\left[\begin{array}{cc|c} 2 & 1 & 6 \\ 3 & -2 & 2 \end{array}\right] \quad R_1 \leftarrow \frac{1}{2} R_1$$

$$\left[\begin{array}{cc|c} 1 & 1/2 & 3 \\ 3 & -2 & 2 \end{array}\right] \quad R_2 \leftarrow 3R_1 - R_2$$

$$\left[\begin{array}{cc|c} 1 & 1/2 & 3 \\ 0 & 7/2 & 7 \end{array}\right] \quad R_2 \leftarrow 2/7\, R_2$$

$$\left[\begin{array}{cc|c} 1 & 1/2 & 3 \\ 0 & 1 & 2 \end{array}\right] \quad R_1 \leftarrow R_1 - 1/2\, R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 2 \end{array}\right]$$

$\Rightarrow \quad x = 2,\ y = 2$

(b) $\begin{cases} x + y + z = 2 \\ x - y \quad\quad = 1 \\ \quad 2y + z = 1 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 0 & 1 \\ 0 & 2 & 1 & 1 \end{array}\right] \quad R_2 \leftarrow R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{array}\right] \quad R_2 \leftarrow 1/2\, R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 2 & 1 & 1 \end{array}\right] \quad R_3 \leftarrow R_3 - 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_1 \leftarrow R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 3/2 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

choose $z \rightarrow$ solve for x and y.

$x = 3/2 - 1/2\, z$

$y = 1/2 - 1/2\, z$

Infinite solutions

(c) $\begin{cases} 6x + 2y = 15 \\ 3x + y = 7 \end{cases}$

$$\left[\begin{array}{cc|c} 6 & 2 & 15 \\ 3 & 1 & 7 \end{array}\right] \xrightarrow{R_1 \leftarrow 1/6\, R_1} \left[\begin{array}{cc|c} 1 & 1/3 & 5/2 \\ 3 & 1 & 7 \end{array}\right] \xrightarrow{R_2 \leftarrow 3R_1 - R_2} \left[\begin{array}{cc|c} 1 & 1/3 & 5/2 \\ 0 & 0 & 1/2 \end{array}\right]$$

The second row implies that $0 = 1/2$, which is impossible.

No solutions