Discussion 1A

1. Span basics:

(a) $\text{span}\left\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}\right\}$ = a plane in $\mathbb{R}^3$

$\vec{v} = \begin{bmatrix}*\\*\\0\end{bmatrix}$

(b) $\begin{bmatrix}5\\5\\0\end{bmatrix} = 5/3\begin{bmatrix}1\\2\\0\end{bmatrix} + 5/3\begin{bmatrix}2\\1\\0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}5\\5\\0\end{bmatrix}$ is in $\text{span}\left\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}\right\}$

(c) possible choice of $\vec{v}$ $\begin{bmatrix}0\\0\\1\end{bmatrix}$

(d) $\begin{bmatrix}1&2\\2&1\\0&0\end{bmatrix}\vec{x} = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

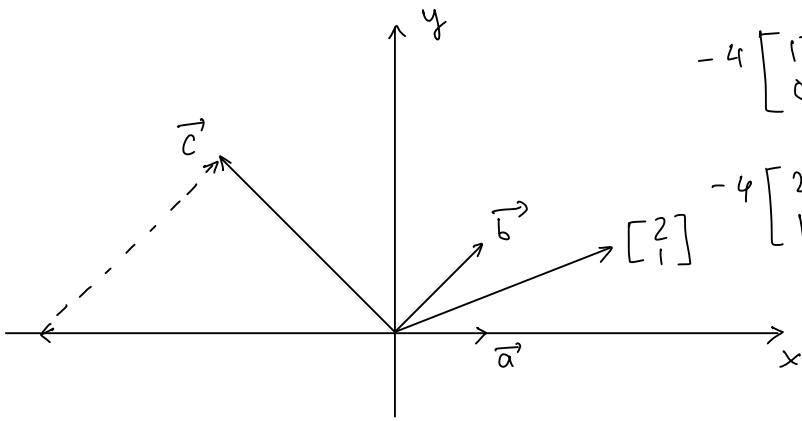
$$\left[\begin{array}{cc|c}1&2&b_1\\2&1&b_2\\0&0&b_3\end{array}\right] \to \left[\begin{array}{cc|c}1&2&b_1\\0&3&2b_1-b_2\\0&0&b_3\end{array}\right]$$

$$\to \left[\begin{array}{cc|c}1&0&\\0&1&(2b_1-b_2)/3\\0&0&b_3\end{array}\right]$$

Condition: $b_3 = 0$, $b_1, b_2 \in \mathbb{R}$

2. Visualising span:

(a) $\vec{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$



$-4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$-4\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 6\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} \vec{a} & \vec{b} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \vec{c}$