Discussion 1B

1. Visualising matrices as operations

Part 1: Rotation Matrices as Rotations:

(a) Rotate the unit square by 45°: $T_1 T_2$

" " " " 30°: $T_2 T_2$

(b) $T_3 = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\alpha - \sin\alpha\sin\theta \\ \sin\theta\cos\alpha + \cos\theta\sin\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\theta) \\ \sin(\alpha+\theta) \end{bmatrix}$

(d) Rotate using: $\begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) \\ \sin(-60^\circ) & \cos(60^\circ) \end{bmatrix}$

(e) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(f) Matrix that reflect a vector about:

(i) x-axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(ii) y-axis $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(iii) $x = y$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Part 2: Commutativity of Operations

(a) (b) The two operations are not the same

(c) The resulting matrices that are obtained are different depending on the order of multiplication

(d) The reflections are not commutative unless the two reflection are perpendicular to each other.