Discussion 0D

1. Solving systems of equations

i) $\begin{cases} 49x + 7y = 49 \\ 42x + 6y = 42 \end{cases} \Leftrightarrow \begin{cases} 7x + y = 7 \\ 7x + y = 7 \end{cases}$

Infinite sols: $\begin{cases} y = t \\ x = 1 - t/7 \end{cases}$

ii) $\begin{cases} 5x + 3y = -21 \quad (1) \\ 2x + y = -9 \quad (2) \end{cases}$

From (2): $y = -9 - 2x$. Substitute to (1), we get:

$5x + 3(-9 - 2x) = -21$

$5x - 27 - 6x = -21$

$x = -6$

$y = -9 - 2x = -9 - 2(-6) = -9 + 12 = 3$

Hence $x = -6$, $y = 3$

iii) $\begin{cases} 49x + 7y = 60 \\ 42x + 6y = 30 \end{cases} \Leftrightarrow \begin{cases} 49x + 7y = 60 \\ 7x + y = 5 \end{cases}$

$\Leftrightarrow \begin{cases} 49x + 7y = 60 \\ 49x + 7y = 35 \end{cases} \Rightarrow 60 = 35$ (contradiction)

The equation has no solution.

iv) $\begin{cases} 2x + 2y + 4z = -1 \\ y + z = -2 \\ x + 2y + 3z = 2 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & -1 \\ 0 & 1 & 1 & -2 \\ 1 & 2 & 3 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & -2 \\ 0 & 2 & 2 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 9 \end{array}\right]$$

The last row implies $0 = 9$ which is impossible.
Thus, the equation has no solution.

v.) $\begin{cases} 2x + 2y + 4z = 6 \\ y + z = 1 \\ x + 2y + 3z = 4 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 6 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

choose $z \to$ solve for x, y

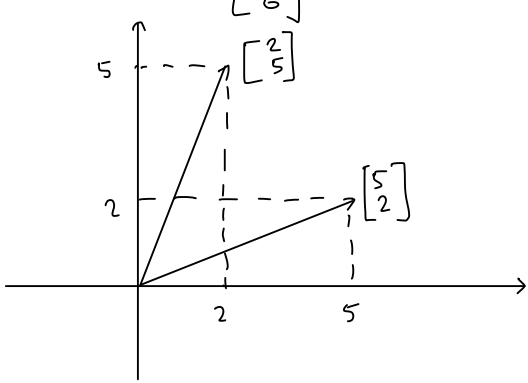
$\begin{cases} x = 2 - z \\ y = 1 - z \end{cases}$

vi) $\begin{cases} x + y + z = 4 \\ 3z = 6 \\ y + z = 3 \end{cases} \iff \begin{cases} x = 1 \\ z = 2 \\ y = 1 \end{cases}$

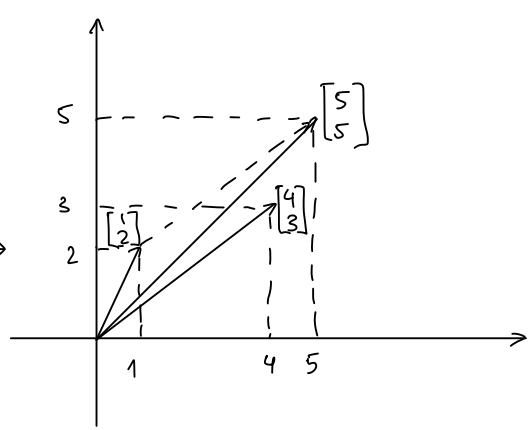
2. Vectors

(a) Vector in $\mathbb{R}^2$: $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$

(b)



(c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$



3. Matrix multiplication:

(a) $A_1 B_1 = \begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 11$

(b) $AB = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 6 \\ 12 & 7 \end{bmatrix}$

(c) $BA = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 18 \\ 4 & 11 \end{bmatrix}$

(d) $AC = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 21 & 13 & 15 \\ 14 & 27 & 16 & 20 \end{bmatrix}$

(e) DC: the product does not exist because $\dim_D = 4 \times 3$, $\dim_C = 2 \times 4$

(f) $CD = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 100 & 33 & 75 \\ 52 & 29 & 56 \end{bmatrix}$

(g) $EF = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 53 & 50 & 64 \\ 34 & 70 & 57 \\ 33 & 90 & 44 \end{bmatrix}$

(h) $FE = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 65 & 56 & 59 \\ 40 & 59 & 66 \\ 45 & 62 & 43 \end{bmatrix}$