Microwave Project

Project Topic : **Mode Visualiser Series**

Pictorial Representation of EM Modes in Rectangular and Cylindrical waveguides

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Acknowledgement:

We’d like to thank our respected **Prof. Dr. S Raghavan** for encouraging us to take up this challenging project, constantly motivating us to finish it and providing valuable supporting material for reference. It would not have been possible without his guidance. We also would like to thank our classmates for their appreciation and support.

Main Contributions:

While many find it difficult to pictorially represent modes in a waveguide, which form the basis of design and for understanding the design principles, this is a very novel attempt to visualize any mode (User needs to enter ‘m’ and ‘n’, and an interesting visualization is shown through CAD. It involves the deep understanding of E and H expressions for both rectangular and cylindrical waveguides) and other planar transmission line modes. This is a first of the kind work using Python (void of any commercial software, in pure code, which will make every student love EM FIELDS which otherwise is considered a nightmare.

Summary of the Work Done -

- A web application is presented, for pictorial representation of fields in waveguides.
- The application requires the user to select the mode, type of transmission line, as well as the m and n values for that mode.
- Corresponding to the selections made by the user, the Electric and Magnetic Field representations are displayed.
- Along with this, we also display the parameters of waveguide transmission for that particular mode.
- This is compacted into a Graphical User Interface with interactive tools/widgets for the user to play around with.
- This can be treated as a Field Playground for EM Theory enthusiasts.
- We provide novelty in the application, access to visualisation of higher order modes (not common in existing literature/software) and also presentability with unique colors for each of the field patterns.
- The application is made open to all for access and experimentation, hosted using Heroku.

Abstract:

In this report, we present the procedure and results of a web application made to visualise field lines of Electric and Magnetic waves inside a waveguide. After an extensive analysis to locate related works on the internet, we propose a first of the kind Graphical User Interface for waveguide visualisation, as a public resource. Here we provide the user with options to visualise upto (X) modes, which does not exist in any current literature due to field complexity. Additionally, all the parameters related to the selected mode, including the cut off frequency, cut off wave number will be calculated and displayed. **(Quasi-TEM mode will be unveiled in PART 2 of the mode visualiser series)**

Introduction:

Energy can propagate through a medium or a vacuum. Propagation modes vary depending upon the form of energy and the nature of the medium.

Electromagnetic energy is transmitted in **Waves**. Like sound, it may propagate equally in all directions **(omni-directional)**, the wave energy radiates in concentric shells corresponding to amplitude. In some cases, it may be directional, focused to form a tight beam.

Electromagnetic waves can travel along waveguides using a number of different modes. The propagation occurs due to **internal reflection.**

The different waveguide modes have varied properties and therefore it is necessary to ensure that the correct mode for any waveguide is excited and others are suppressed as far as possible, if they are even able to be supported.

Waveguide modes

There are various formats in which the wave can propagate in the waveguide. These different types of waves correspond to the different elements within an electromagnetic wave.

- **TE mode:** This waveguide mode is dependent upon the transverse electric waves, sometimes called H waves, characterised by the fact that the **electric field vector (E)** being always **perpendicular** to the direction of propagation.
- **TM mode:** Transverse magnetic waves, also called E waves are characterised by the fact that the **magnetic field vector (H vector)** is always **perpendicular** to the direction of propagation.

- **TEM mode:** The Transverse electromagnetic wave **cannot** be propagated within a waveguide. It is commonly used in coaxial cables and open wire feeders. The TEM wave is characterised by the fact that both the **electric vector (E vector) and the magnetic vector (H vector) are perpendicular to the direction of propagation**.

The notation used to denote TE and TM modes, are with integers after them: **$TE_{m,n}$**. The numerals M and N are always integers that can take on separate values from 0 or 1 to infinity. These indicate the wave modes within the waveguide.

Only a limited number of different m, n modes can be propagated along a waveguide dependent upon the waveguide dimensions and format.

Technologies Used:

The web application is written in **Python** programming language. It uses **Streamlit** for basic user interface. It also uses **matplotlib** streamplot function for the actual generation of plots. **Scipy** library was used for calculation of bessel functions.

Theory:

TM Modes

Consider the shape of the rectangular waveguide above with dimensions **a and b** (assume a>b) and the parameters **e** and **m.** For TM waves **H_z = 0** and **E_z** should be solved from equation for TM mode;

$$\textbf{Ñ}^2_{xy} E_z^0 + h^2 E_z^0 = 0$$

Since $\mathbf{E_z(x,y,z) = E_z^0(x,y)e^{-gz}}$, we get the following equation,

$$\boxed{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) E_z^0(x,y) = 0}$$

If we use the method of separation of variables, that is $\mathbf{E_z^0(x,y)=X(x).Y(y)}$ we get,

$$\boxed{-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + h^2}$$

Since the right side contains **x** terms only and the left side contains **y** terms only, they are both equal to a constant. Calling that constant as $\mathbf{k_x^2}$, we get;

$$\boxed{\frac{d^2X(x)}{dx^2} + k_x^2 X(x) = 0}$$

$$\boxed{\frac{d^2Y(y)}{dy^2} + k_y^2 Y(y) = 0}$$

where $\boldsymbol{k_y^2 = h^2 - k_x^2}$

Now, we should solve for X and Y from the preceding equations. Also we have the boundary conditions of;

$$\mathbf{E_z^0(0,y)=0}$$

$$\mathbf{E_z^0(a,y)=0}$$

$$\mathbf{E_z^0(x,0)=0}$$

$$\mathbf{E_z^0(x,b)=0}$$

From all these, we conclude that

X(x) is in the form of **sin k_xx**, where **k_x=mp/a**, m=1,2,3,...

Y(y) is in the form of **sin k_yy**, where **k_y=np/b**, n=1,2,3,...

So the solution for **E_z^0(x,y)** is

$$\boxed{E_z^0(x,y) = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)} \text{ (V/m)}$$

From $k_y^2=h^2-k_x^2$, we have;

$$\boxed{h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For **TM waves**, we have

$$\boxed{H_x^0 = \frac{jw\varepsilon}{h^2}\frac{\partial E_z^0}{\partial y}}$$

$$\boxed{H_y^0 = -\frac{jw\varepsilon}{h^2}\frac{\partial E_z^0}{\partial x}}$$

$$\boxed{E_x^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial x}}$$

$$\boxed{E_y^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial y}}$$

From these equations, we get

$$E_x^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_y^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_x^0(x,y) = \frac{jw\varepsilon}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_y^0(x,y) = -\frac{jw\varepsilon}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

where

$$\gamma = j\beta = j\sqrt{w^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Here, **m and n represent possible modes** and it is designated as the **TM_{mn}** mode. **m** denotes the number of half cycle variations of the fields in the x-direction and **n** denotes the number of half cycle variations of the fields in the y-direction.

When we observe the above equations we see that *for TM modes in rectangular waveguides, neither m nor n can be zero*. **This is because of the fact that the field expressions are identically zero if either m or n is zero**. Therefore, the lowest mode for rectangular waveguide TM mode is **TM_{11}** .

Here, the **cut-off wave number** is

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

and therefore,

$$\boxed{\beta=\sqrt{k^2-k_c^2}}$$

The cut-off frequency is at the point where **g** vanishes. Therefore,

$$\boxed{f_c=\frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\ (Hz)}$$

Since ***l=u/f***, we have the cut-off wavelength,

$$\boxed{\lambda_c=\frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}\ (m)}$$

At a given operating frequency **f**, only those frequencies, which have **f_c<f will propagate**. The modes with **f<f_c** will lead to an imaginary **b** which means that the field components will decay exponentially and **will not propagate**. Such modes are called ***cut-off*** **or** ***evanescent*** **modes.**

The mode with the lowest cut-off frequency is called the ***dominant mode***. Since TM modes for rectangular waveguides start from TM_{11} mode, the **dominant frequency** is

$$\boxed{(f_c)_{11}=\frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}\ (Hz)}$$

The **wave impedance** is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for **E_x and H_y** (see the equations above);

$$\boxed{Z_{TM} = \frac{E_x}{H_y} = \frac{\gamma}{jw\varepsilon} = \frac{j\beta}{jw\varepsilon} \Rightarrow} \boxed{Z_{TM} = \frac{\beta\eta}{k}}$$

The **guide wavelength** is defined as the **distance between two equal phase planes along the waveguide** and it is equal to

$$\boxed{\lambda_g = \frac{2\pi}{\beta}} \boxed{> \frac{2\pi}{k} = \lambda}$$

which is thus greater than **l**, the wavelength of a plane wave in the filling medium.

The **phase velocity** is

$$\boxed{u_p = \frac{w}{\beta}} \boxed{> \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}}}$$

which is **greater than the speed of light** (plane wave) in the filling material.

Attenuation for propagating modes results when there are losses in the dielectric and in the imperfectly conducting guide walls. The **attenuation constant** due to the **losses in the dielectric** can be found as follows:

$$\boxed{\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

TE Modes

Consider again the rectangular waveguide below with dimensions **a and b (assume a>b)** and the parameters **e** and **m.**

For TE waves $\mathbf{E_z = 0}$ and H_z should be solved from equation for TE mode;

$$\tilde{N}^2_{xy} H_z + h^2 H_z = 0$$

Since $\mathbf{H_z(x,y,z) = H_z^0(x,y)e^{-gz}}$**,** we get the following equation,

$$\boxed{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) H_z^0(x,y) = 0}$$

If we use the **method of separation of variables**, that is $\mathbf{H_z^0(x,y)=X(x).Y(y)}$ we get,

$$\boxed{-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + h^2}$$

Since the right side contains **x** terms only and the left side contains **y** terms only, they are both equal to a constant. Calling that constant as $\mathbf{k_x^2}$, we get;

$$\boxed{\frac{d^2X(x)}{dx^2} + k_x^2 X(x) = 0}$$

$$\boxed{\frac{d^2Y(y)}{dy^2} + k_y^2 Y(y) = 0}$$

where $\boldsymbol{k_y^2=h^2-k_x^2}$

Here, we must **solve for X and Y** from the preceding equations. Also we have the following boundary conditions:

$$\boxed{\frac{\partial H_z^0}{\partial x} = 0(E_y = 0)} \text{ at } x=0$$

$$\boxed{\frac{\partial H_z^0}{\partial x} = 0(E_y = 0)} \text{ at } x=a$$

$$\boxed{\frac{\partial H_z^0}{\partial y} = 0(E_x = 0)} \text{ at } y=0$$

$$\boxed{\frac{\partial H_z^0}{\partial y} = 0(E_x = 0)} \text{ at } y=b$$

From all these, we get

$$\boxed{H_z^0(x,y) = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)} \text{(A/m)}$$

From $\boldsymbol{k_y^2=h^2-k_x^2}$, we have;

$$\boxed{h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For **TE waves**, we have

$$\boxed{H_x^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x}}$$

$$H_y^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_x^0 = -\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_y^0 = -\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial x}$$

From these equations, we obtain

$$E_x^0(x,y) = \frac{jw\mu}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_y^0(x,y) = -\frac{jw\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_x^0(x,y) = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_y^0(x,y) = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

where

$$\gamma = j\beta = j\sqrt{w^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

As explained before, **m and n** represent **possible modes** and it is shown as the **TE_{mn}** mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

Here, the **cut-off wave number** is

$$\boxed{k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

and therefore,

$$\boxed{\beta = \sqrt{k^2 - k_c^2}}$$

The **cut-off frequency** is at the point where **g** vanishes. Therefore,

$$\boxed{f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\ (Hz)}$$

Since ***l=u/f***, we have the **cut-off wavelength**,

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}\ (m)}$$

At a given operating frequency f, only those frequencies, which have **$f>f_c$ will propagate**. The modes with **$f<f_c$ will not propagate.**

The mode with the **lowest cut-off frequency** is called the ***dominant mode***. Since **TE_{10}** mode is the **minimum** possible mode that gives non zero field expressions for rectangular waveguides, it is the dominant mode of a rectangular waveguide with a>b and so the **dominant frequency** is

$$\left(f_c\right)_{10} = \frac{1}{2a\sqrt{\mu\varepsilon}} (Hz)$$

The **wave impedance** is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for **E_x and H_y** (see the equations above);

$$Z_{TE} = \frac{E_x}{H_y} = \frac{jw\mu}{\gamma} = \frac{jw\mu}{j\beta} \Rightarrow Z_{TE} = \frac{k\eta}{\beta}$$

The **guide wavelength** is defined as the distance between two equal phase planes along the waveguide and it is equal to

$$\lambda_g = \frac{2\pi}{\beta} > \frac{2\pi}{k} = \lambda$$

which is thus greater than **l**, the wavelength of a plane wave in the filling medium.

The **phase velocity** is

$$u_p = \frac{w}{\beta} > \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$

which is greater than the speed of the plane wave in the filling material.

The **attenuation constant** due to the losses in the dielectric is obtained as follows:

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

After some manipulation, we get

$$\alpha_d = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{k^2 \tan\delta}{2\beta}$$

TEM Modes

$$E_r = -\frac{\partial V}{\partial r} = \begin{cases} 0 & (r > b), \\ \frac{V_0}{r\ln(b/a)} e^{i(kz-\omega t)} & (a < r < b), \\ 0 & (r < a), \end{cases}$$

$$E_\phi = -\frac{1}{r}\frac{\partial V}{\partial \phi} = 0,$$

$$E_z = -\frac{\partial V}{\partial z} - \frac{\partial A_z}{\partial t} = -ikV + i\omega A_z = 0,$$

$$B_r = \frac{1}{r}\frac{\partial A_z}{\partial \phi} = 0,$$

$$B_\phi = -\frac{\partial A_z}{\partial r} = -\frac{k}{\omega}\frac{\partial V}{\partial r} = \frac{k}{\omega}E_r = \frac{E_r}{v},$$

$$B_z = 0.$$

Equations:

The final equations which determine the field pattern in corresponding waveguides are as follows.

TE Rectangular:

$$E_x \sim \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$
$$E_y \sim \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$
$$H_x \sim \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$
$$H_y \sim \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$

TM Rectangular:

$$E_x = E_{0x}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_g Z}$$
$$E_y = E_{0y}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_g Z}$$
$$E_z = \text{Eq. (4-1-61)}$$
$$H_x = H_{0x}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_g Z}$$
$$H_y = H_{0y}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_g Z}$$
$$H_z = 0$$

TE Circular: (PTO)

$$E_\phi = E_{0\phi} J_n'\left(\frac{X'_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g Z}$$

$$E_z = 0$$

$$H_r = -\frac{E_{0\phi}}{Z_g} J_n\left(\frac{X'_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g Z}$$

$$H_\phi = \frac{E_{0r}}{Z_g} J_n\left(\frac{X'_{np} r}{a}\right) \sin(n\phi) e^{-j\beta_g Z}$$

$$H_z = H_{0z} J_n\left(\frac{X'_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g Z}$$

TM Circular:

$$E_r = E_{0r} J_n'\left(\frac{X_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g Z}$$

$$E_\phi = E_{0\phi} J_n\left(\frac{X_{np} r}{a}\right) \sin(n\phi) e^{-j\beta_g Z}$$

$$E_z = E_{0z} J_n\left(\frac{X_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g Z}$$

$$H_r = \frac{E_{0\phi}}{Z_g} J_n\left(\frac{X_{np} r}{a}\right) \sin(n\phi) e^{-j\beta_g Z}$$

$$H_\phi = \frac{E_{0r}}{Z_g} J_n'\left(\frac{X_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g Z}$$

$$H_z = 0$$

TEM:

$$E_r = -\frac{\partial V}{\partial r} = \begin{cases} 0 & (r > b), \\ \frac{V_0}{r \ln(b/a)} e^{i(kz-\omega t)} & (a < r < b), \\ 0 & (r < a), \end{cases}$$

$$E_\phi = -\frac{1}{r}\frac{\partial V}{\partial \phi} = 0,$$

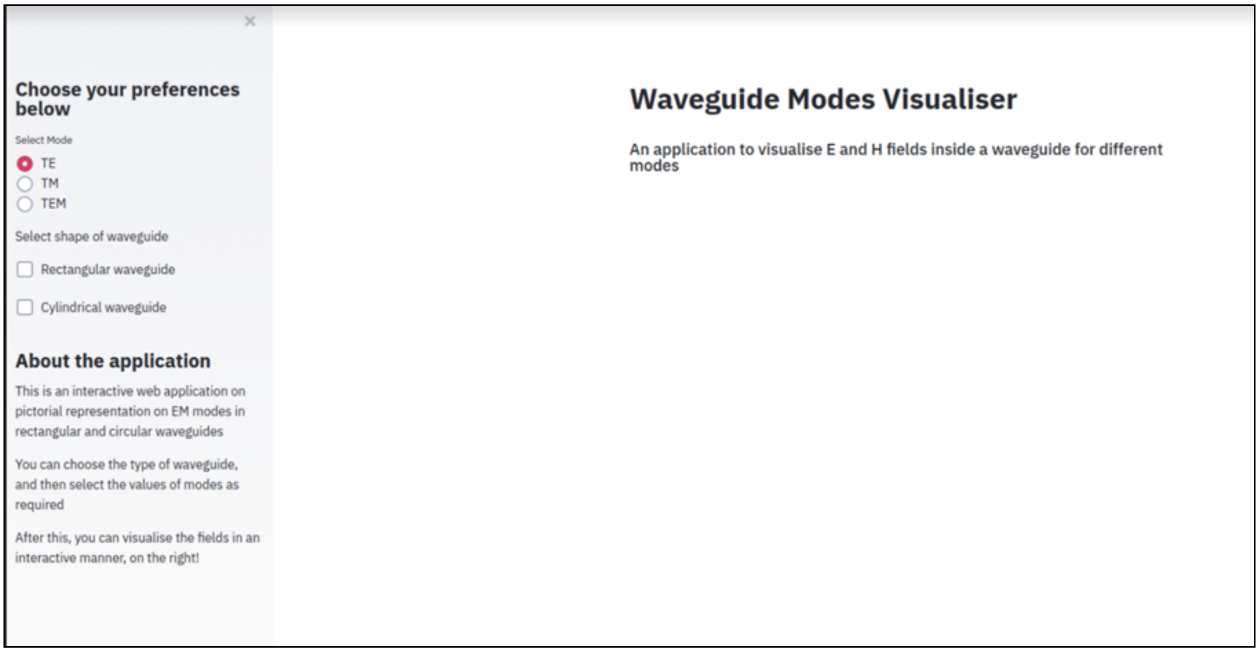
$$E_z = -\frac{\partial V}{\partial z} - \frac{\partial A_z}{\partial t} = -ikV + i\omega A_z = 0,$$

$$B_r = \frac{1}{r}\frac{\partial A_z}{\partial \phi} = 0,$$

$$B_\phi = -\frac{\partial A_z}{\partial r} = -\frac{k}{\omega}\frac{\partial V}{\partial r} = \frac{k}{\omega} E_r = \frac{E_r}{v},$$

$$B_z = 0.$$

The Graphical User Interface:



The attached image shows the basic **User Interface**. In the **sidebar** section of the website, the user will be able to select the mode of transmission **(TE, TM or TEM)** and the shape of waveguide (rectangular or cylindrical). Once they do that they would be asked to fill the values of m and n (n and p in case of cylindrical) and the field lines will be immediately generated.

Illustration 1 - TE11 Mode - Rectangular Waveguide

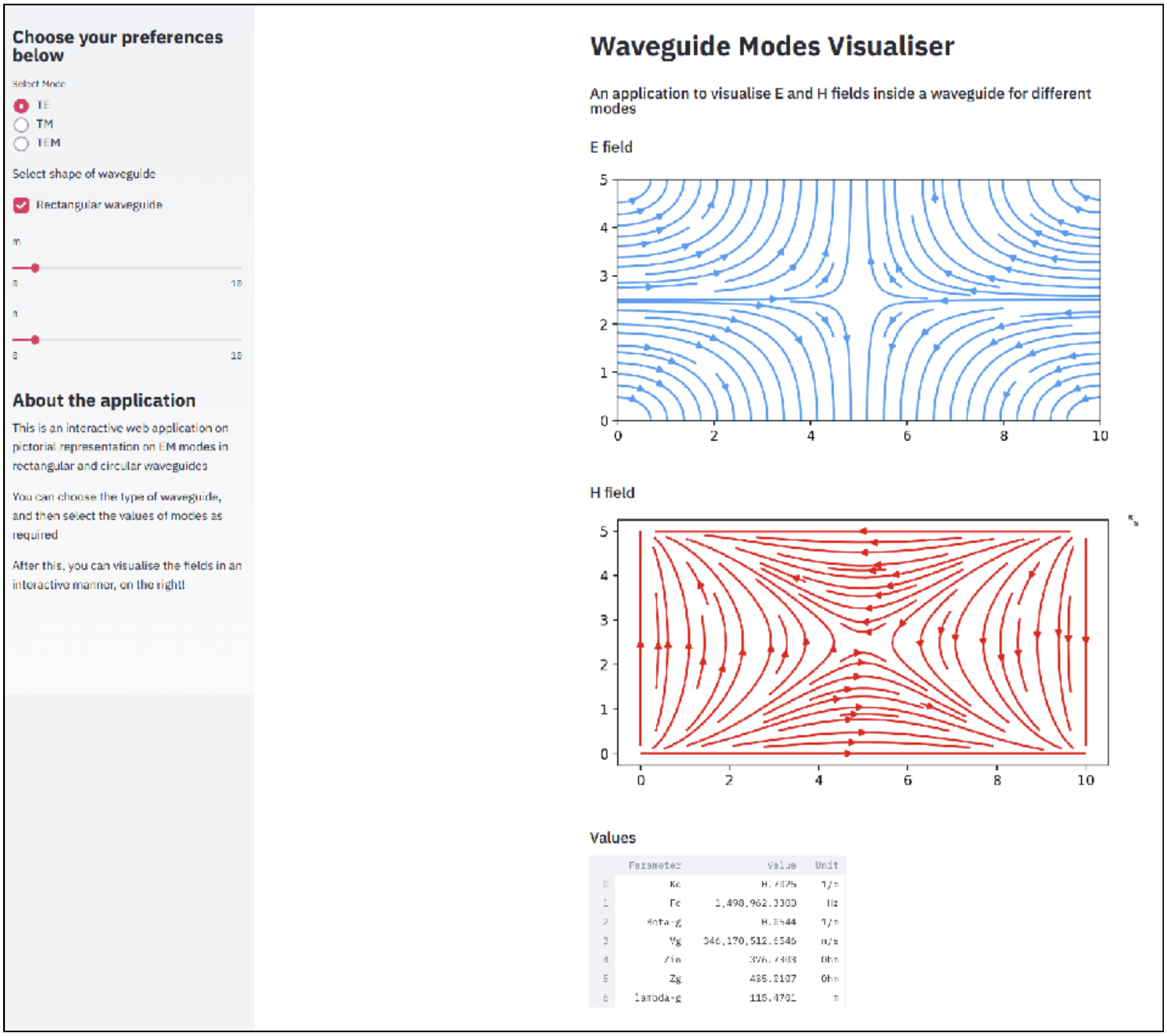
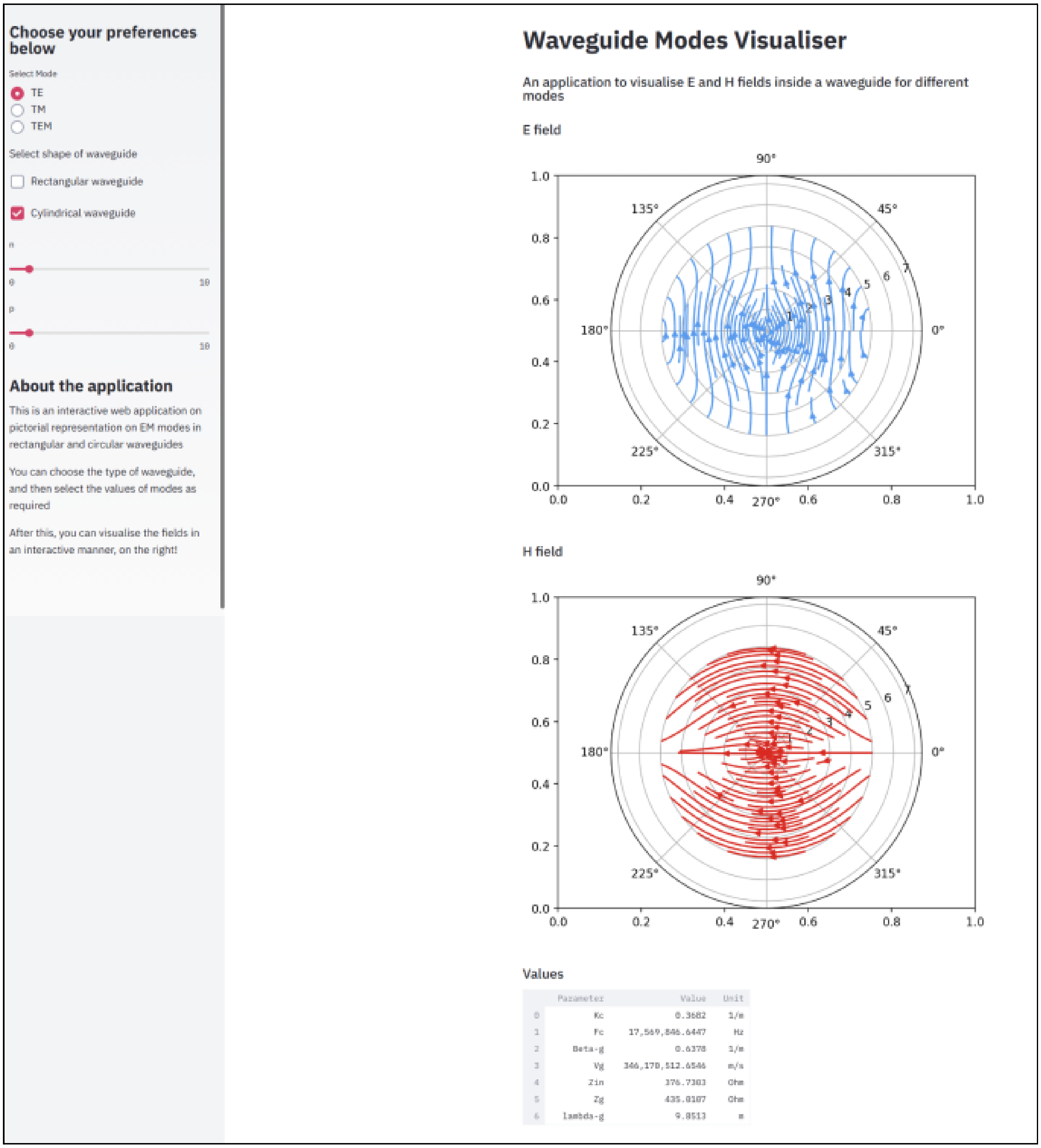


Illustration 2 - TE11 Mode - Circular Waveguide



	Parameter	Value	Unit
0	Kc	0.3682	1/m
1	Fc	17,569,846.6447	Hz
2	Beta-g	0.6378	1/m
3	Vg	346,170,512.6546	m/s
4	Zin	376.7303	Ohm
5	Zg	435.0107	Ohm
6	lambda-g	9.8513	m

Link to Project:

The application that we developed, has been deployed using **Heroku.** It can now be accessed by the public, using the link -

https://wg-visualiser.herokuapp.com/

Conclusion:

A web application for visualising field lines of EM waves was made with Python programming language and other supporting libraries. The images of the resulting website are attached to this report.

References:

- Microwave Devices and Circuits by **Samuel Y Liao**
- https://www.eit.lth.se/fileadmin/eit/courses/eten15/lectureWG1.pdf
- https://www.hep.princeton.edu//~mcdonald//examples/coax_potential.pdf