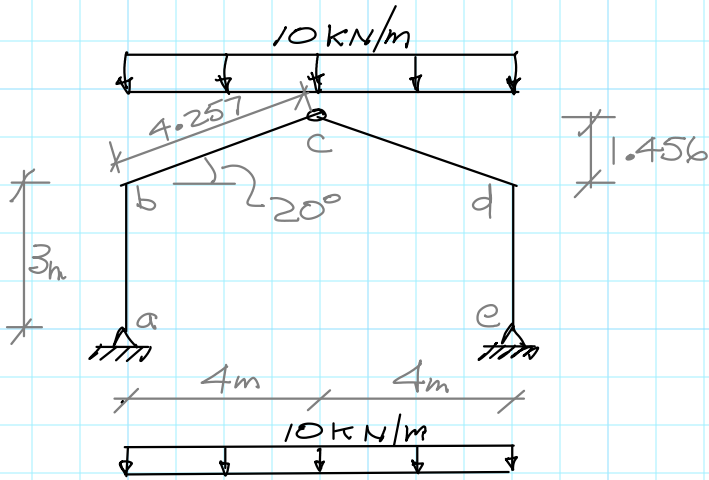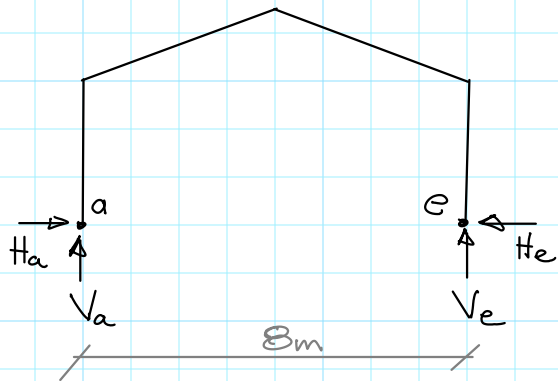# Example-F1   Frames: Member End Forces

Draw FBD's for all members, showing member end forces. Check joint equilibrium & draw NVM diagrams.

4 unknowns, but can solve for 2 on this FBD.

$$\sum M_a = 0 \quad (+\curvearrowleft$$
$$-10 \times 8 \times 4 + V_e \times 8 = 0$$
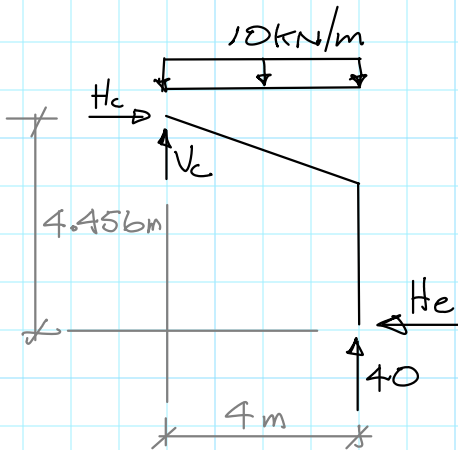$$\underline{V_e = 40 \text{ kN}} \qquad (\therefore \uparrow)$$
$$\sum F_y = 0 \quad +\uparrow$$
$$-10 \times 8 + V_a + V_e = 0$$
$$-80 + V_a + 40 = 0$$
$$\underline{V_a = 40 \text{ kN}} \qquad (\therefore \uparrow)$$

$$\sum M_c = 0 \quad (+\curvearrowright$$
$$-10 \times 4 \times 2 + 40 \times 4 - H_e \times 4.456 = 0$$
$$\underline{H_e = 17.95 \text{ kN}} \qquad (\therefore \leftarrow)$$
$$\sum F_x = 0 \quad \xrightarrow{+}$$
$$\underline{H_c = 17.95 \text{ kN}} \qquad (\therefore \rightarrow)$$
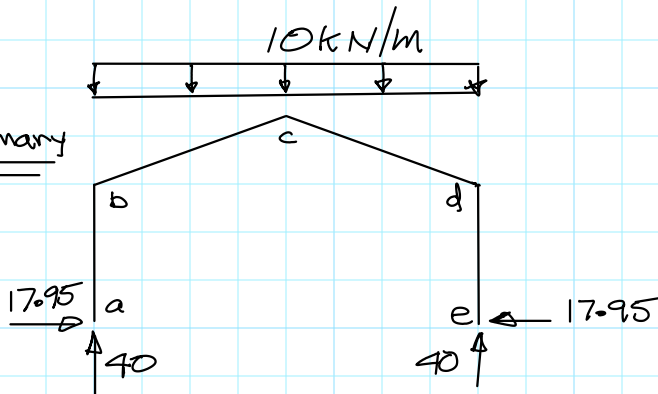$$\sum F_y = 0 \quad +\uparrow$$
$$-10 \times 4 + 40 + V_c = 0$$
$$\underline{V_c = 0}$$
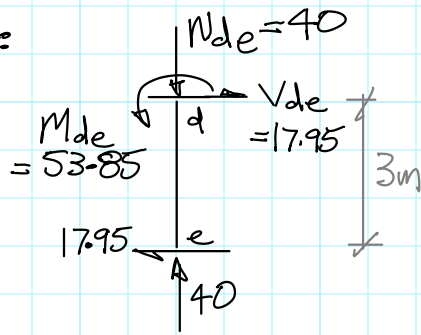
(and must be, for symmetry)

from 1st FBD
$$\sum F_x = 0 \quad \xrightarrow{+}$$
$$H_a - H_e = 0$$
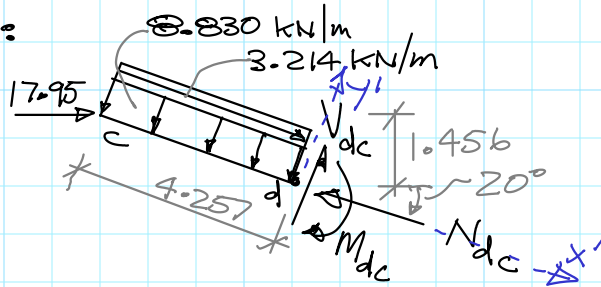$$\underline{H_a = 17.95 \text{ kN}} \qquad (\therefore \rightarrow)$$

Summary

Member End Forces

Member de:

$\sum F_y = 0 \quad +\uparrow$
$$\underline{N_{de} = 40 \, kN}$$
$\sum F_x = 0 \quad \xrightarrow{+}$
$$\underline{V_{de} = 17.95 \, kN}$$
$\sum M_d = 0 \quad (+\circlearrowleft$
$$+M_{de} - 17.95 \times 3 = 0$$
$$\underline{M_{de} = 53.85 \, kN\text{-}m}$$

Member ab: similar (symmetry)

Member cd:

$\perp$ component
$$= \frac{10 \times 4 \times \cos 20°}{4.257} = 8.830 \quad \frac{kN/m}$$

$\parallel$ component
$$= \frac{10 \times 4 \times \sin 20°}{4.257} = 3.214 \quad \frac{kN/m}$$

$\sum F_{x'} = 0 \quad \xrightarrow{+}$

$$17.95 \cos 20 + 3.214 \times 4.257 - N_{dc} = 0$$
$$\underline{N_{dc} = 30.55 \, kN} \qquad (\therefore \nwarrow)$$

$\sum F_{y'} = 0 \quad +\nearrow$

$$17.95 \sin 20 - 8.830 \times 4.257 + V_{dc} = 0$$
$$\underline{V_{dc} = 31.45 \, kN} \qquad (\therefore \nearrow)$$

$\sum M_d = 0 \quad (+\circlearrowleft$
$$-17.95(\sin 20)4.257 + 8.830 \times 4.257 \times \frac{4.257}{2} - M_{dc} = 0$$
$$\underline{M_{dc} = 53.87 \, kN\text{-}m} \qquad (\therefore \circlearrowright)$$

$$17.95 \cos 20° = 16.87 \, kN$$
$$17.95 \sin 20° = 6.14 \, kN$$

Summary cd:

16.87 kN
6.14 kN
31.45 kN
30.55 kN
53.87 kN-m
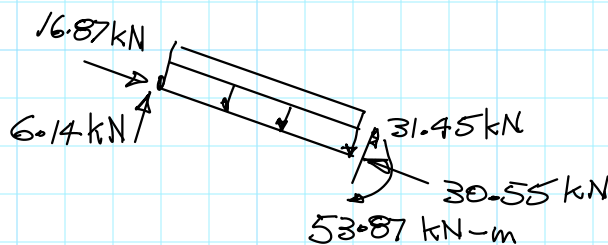
bc: similar

Check joint equilibrium:

Joint d:

30.55   31.45
53.87
53.85   d
17.95   40

(from end forces FBDs)
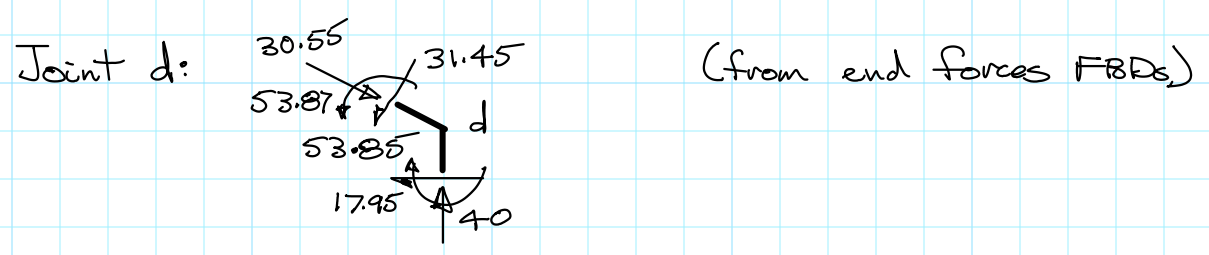
$\Sigma M_d = 53.87 - 53.85 = 0.02$ O.k

$\Sigma F_x = -17.95 + 30.55 \cos 20° - 31.45 \sin 20° = 0.001$ Ok

$\Sigma F_y = 40 - 30.55 \sin 20° - 31.45 \cos 20° = -0.002$ OK

$\therefore$ Equilib of joint d Ok

Joint c:

c
17.95   17.95   Ok.
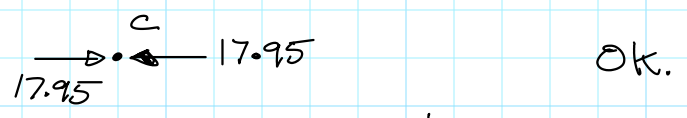
from symmetry

Member forces in bc similar to cd.

Joint b:

Similar to d

NVM Diagrams

Member cd

6.1

16.9

3.24 kN/m
8.830 kN/m

31.5
d
30.6
53.9

N [kN]   16.9   0

30.6
0

V [kN]   6.1

0.70 m

M [kN-m]   2.1

−31.5

−53.9

Member de

40
53.9   17.95
d

40   0   18.0   0   53.9

e

17.95   40   40   0   18.0   0

N [kN]   V [kN]   M [kN-m]
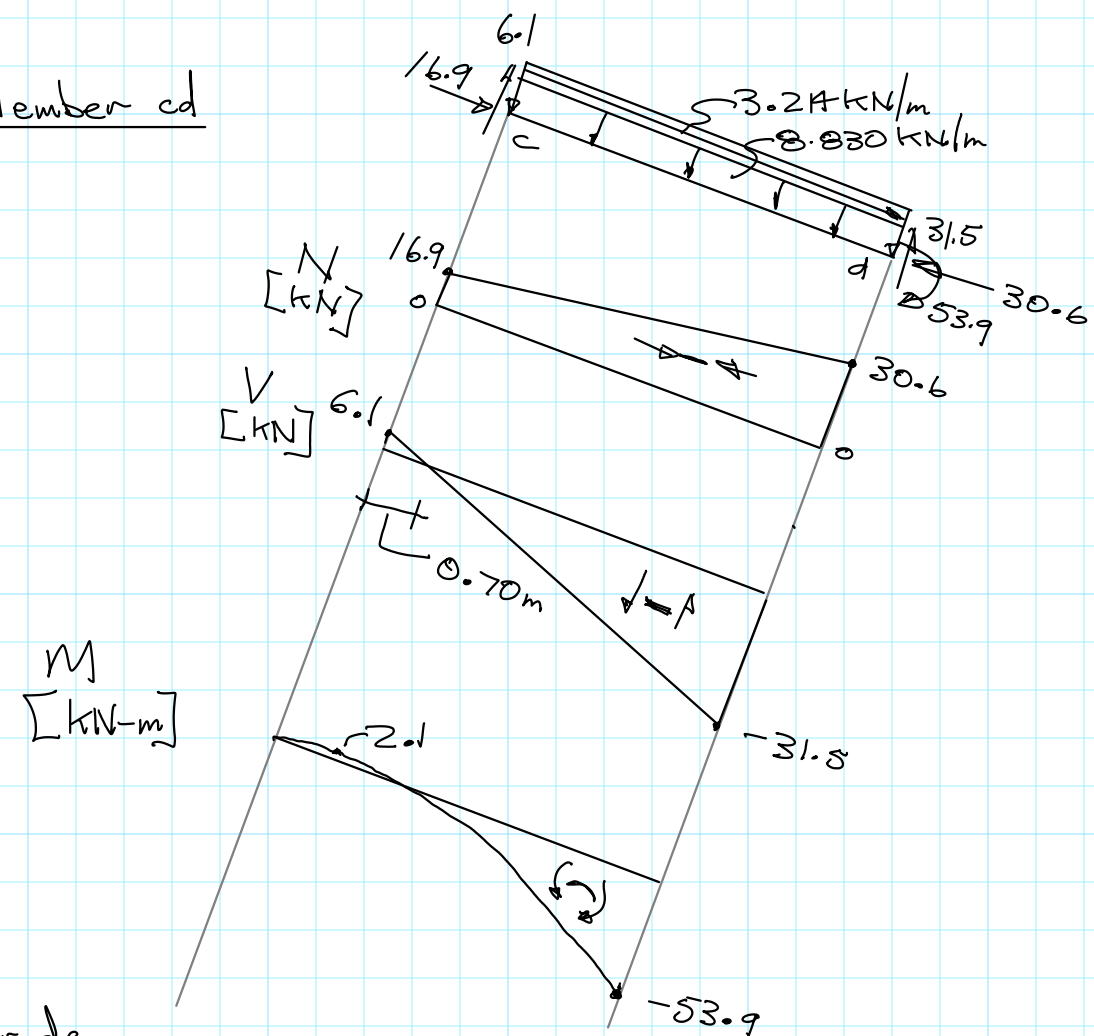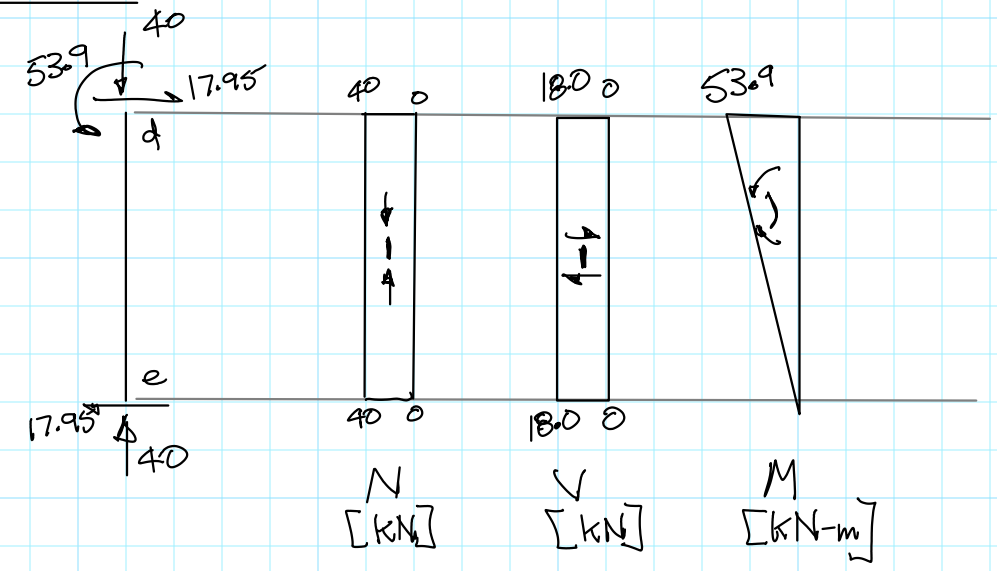
NVM Diagrams (preferable presentation)

$\mathfrak{C}$
sym.

16.9

(c)

30.6

40

(c)

N
[kN]

$\mathfrak{C}$
sym.

0

6.1

0.7m

1-1

18.0

−31.5

1-1

0

V
[kN]

$\mathfrak{C}$
sym.

2.1

−539

M
[kN-m]