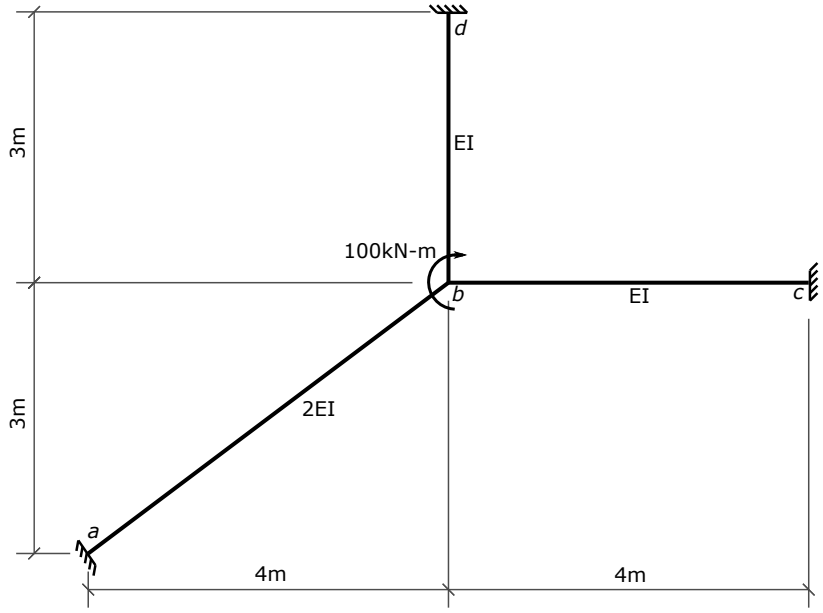
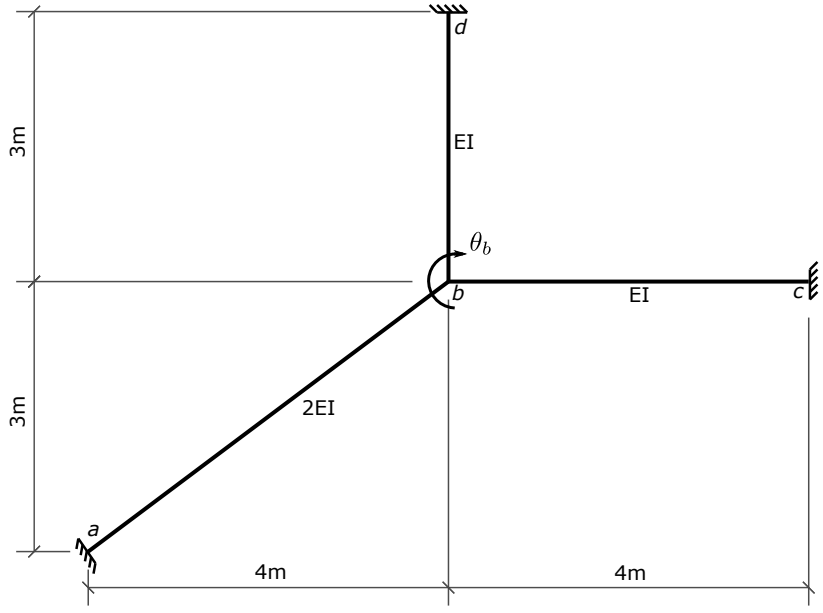
Problem 10 - Solution



1. Identify DOFs

There is 1 - θ_b, the rotation of joint b.



2. Fixed-end moments

There are no transverse loads on any of the 3 members, so all 6 member fixed end moments are zero.

In [1]:
```
Mfab = Mfba = Mfbc = Mfcb = Mfbd = Mfdb = 0
```

3. Slope deflection equations

Express member end moments as a function of the unknown joint rotation, θ_b.

In [2]:
```
from sympy import symbols, solve, init_printing
init_printing()
```

In [3]:
```
theta_b, EI = symbols('theta_b EI')
theta_a = theta_c = theta_d = 0      # rotations at the outsaide end of e
ach member
```

In [4]:
```
Mab = (2*EI/5)*(4*theta_a + 2*theta_b) + Mfab
Mba = (2*EI/5)*(2*theta_a + 4*theta_b) + Mfba
display(Mab,Mba)
```

$$\frac{4 EI \theta_{b}}{5}$$

$$\frac{8 EI \theta_{b}}{5}$$

In [5]:
```
Mbc = (EI/4)*(4*theta_b + 2*theta_c) + Mfbc
Mcb = (EI/4)*(2*theta_b + 4*theta_c) + Mfcb
display(Mbc,Mcb)
```

$$EI \theta_{b}$$

$$\frac{EI \theta_{b}}{2}$$

In [6]:
```
Mbd = (EI/3)*(4*theta_b + 2*theta_d) + Mfbd
Mdb = (EI/3)*(2*theta_b + 4*theta_d) + Mfdb
display(Mbd,Mdb)
```

$$\frac{4 EI \theta_{b}}{3}$$

$$\frac{2 EI \theta_{b}}{3}$$

4. Equilibrium Equation

The sum of the moments acting on joint b must be zero.

Note that the negatives of the member end forces act on the joint.

In [7]:
```
ee = (Mba + Mbc + Mbd) - 100  # = 0, +ive ccw on joint
ee
```

Out[7]: $$\frac{59 EI \theta_{b}}{15} - 100$$

5. Solve for displacement

In [8]:
```
ans = solve([ee],theta_b)
ans
```

Out[8]: $$\left \{ \theta_{b} : \frac{1500}{59 EI}\right \}$$

6. Back-substitute to get member end momentsa

In [9]:
```
mab = Mab.subs(ans).n()
mba = Mba.subs(ans).n()
display(mba, mab)
```

$$40.6779661016949$$

$$20.3389830508475$$

In [10]:
```
mbc = Mbc.subs(ans).n()
mcb = Mcb.subs(ans).n()
display(mbc,mcb)
```

$$25.4237288135593$$

$$12.7118644067797$$

In [11]:
```
mbd = Mbd.subs(ans).n()
mdb = Mdb.subs(ans).n()
display(mbd,mdb)
```

$$33.8983050847458$$

$$16.9491525423729$$

7. Check joint equilibrium

In [12]:
```
# sum of moments acting on joint, +ive ccw
mba+mbc+mbd-100
```

Out[12]: $$0$$

8. Member end shears

As there are no transverse loads, the shears are constant (non-changing) over the whole length of each member.

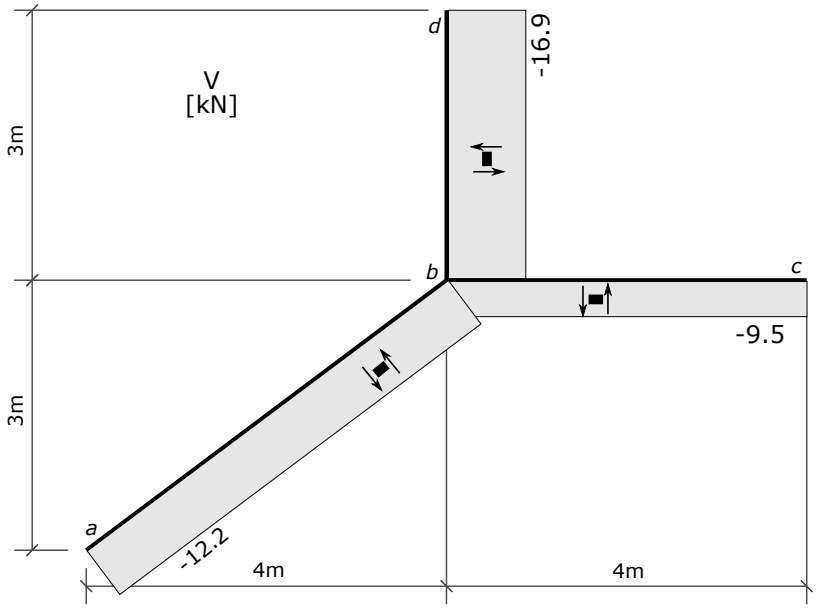
In [13]:
```
vab = -(mab + mba)/5
vbc = -(mbc + mcb)/4
vbd = -(mbd + mdb)/3
display(vab,vbc,vbd)
```

$$-12.2033898305085$$

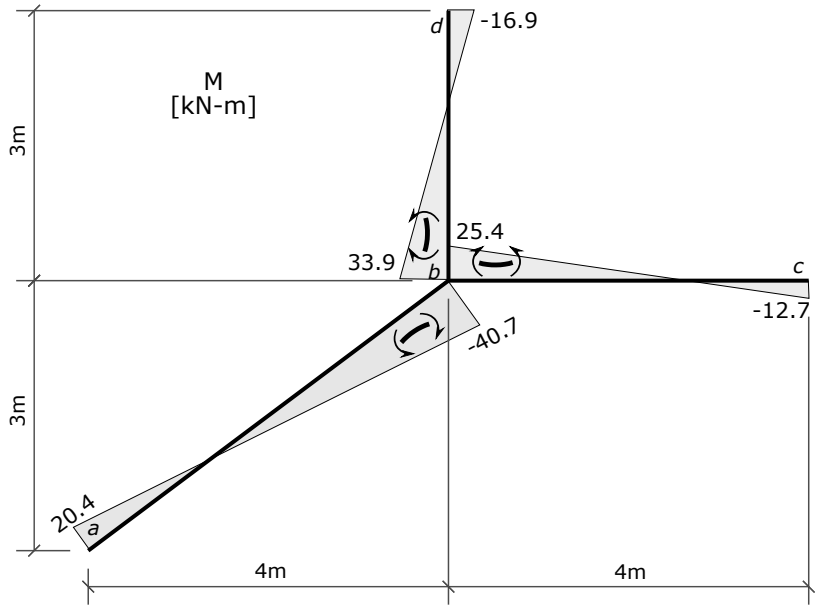
$$-9.53389830508475$$

$$-16.9491525423729$$

9. Shear force diagram



10. Bending moment diagram



In [ ]: