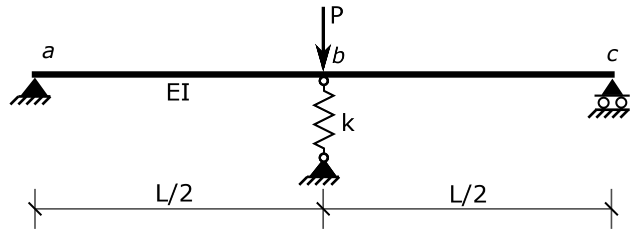
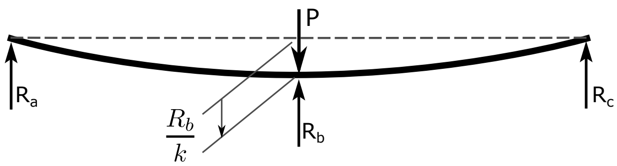
Problem 8.4 - revised Dec 5, 2012: signs of moment diagrams reversed.



Real Structure



We must draw the displacement consistent with the force. If R_b acts **upwards** on b, then the spring must be in compression, therefore it must shorten, therefore the displacement is **downward**.

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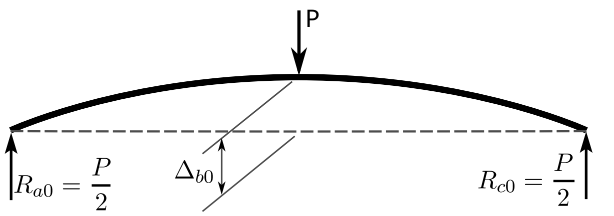
Primary Structure



The displacement is shown upwards because we are going to use the unit redundant to calculate it, and that will be shown upwards to be consistent with the assumed direction of R_b. Thus, a +ive displacement will be **upwards**. This all makes the compatibility equations easier to get right ...

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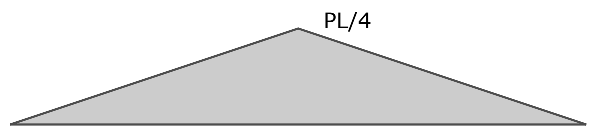

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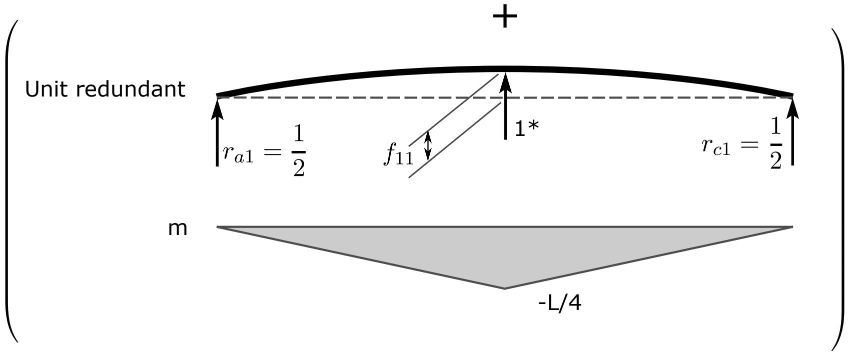
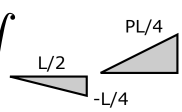


$\times R_b$

The unit load is shown **upwards** because that is the direction assumed for R_b.

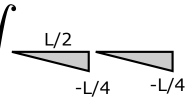
$$\Delta_{b0} = 2 \times \frac{1}{EI} \int$$



$$= 2 \times \frac{1}{EI} \times \frac{1}{3} \times \frac{L}{2} \times -\frac{-L}{4} \times \frac{PL}{4}$$

$$= -\frac{PL^3}{48EI} \qquad (\therefore\downarrow)$$

$$f_{11} = 2 \times \frac{1}{EI} \int$$



$$= 2 \times \frac{1}{EI} \times \frac{1}{3} \times \frac{L}{2} \times \frac{-L}{4} \times \frac{-L}{4}$$

$$= \frac{L^3}{48EI} \qquad (\therefore\uparrow)$$

Compatibility:

$$-\frac{R_b}{k} = \Delta_{b0} + R_b f_{11}$$

$$-\frac{R_b}{k} = -\frac{PL^3}{48EI} + R_b \frac{L^3}{48EI}$$

$$\therefore \; R_b = PL^3 \frac{k}{kL^3 + 48EI} \qquad (\therefore\uparrow)$$

$-R_b/k$ is negative because it has to be downward for an upward value of R_b. Δ_{b0} and f_{11} are positive upward because we have assumed R_b upward, and the unit force is chosen consistent with that.

As k gets very large, the spring approaches being a roller; $k/(kL^3 + 48EI) \to 1/L^3$ and $R_b \to P$. This is correct; all the load gets transferred to the roller and vertical displacement is 0 – therefore no bending in the beams, therefore no end reactions.

When $k = 0$, $k/(kL^3 + 48EI) = 0$ and therefore $R_b = 0$. This is also correct as this is equivalent to no spring at all, therefore no reaction.