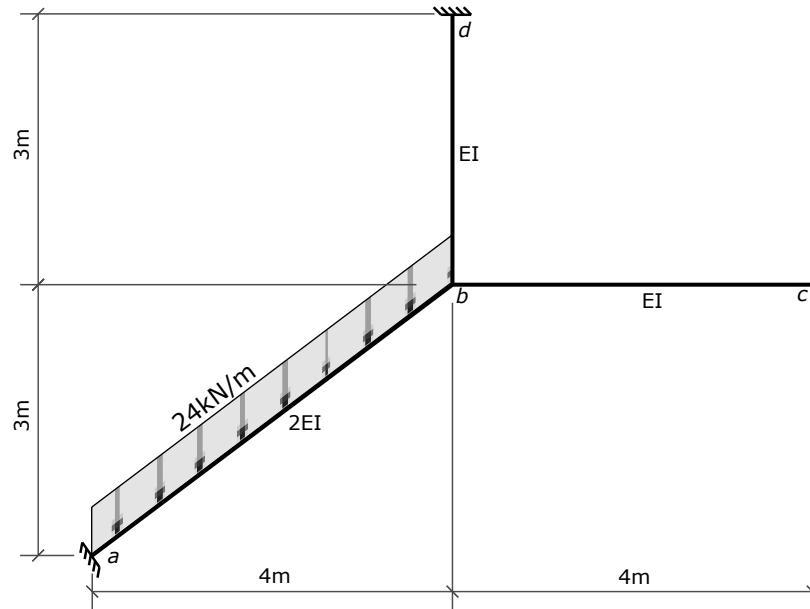
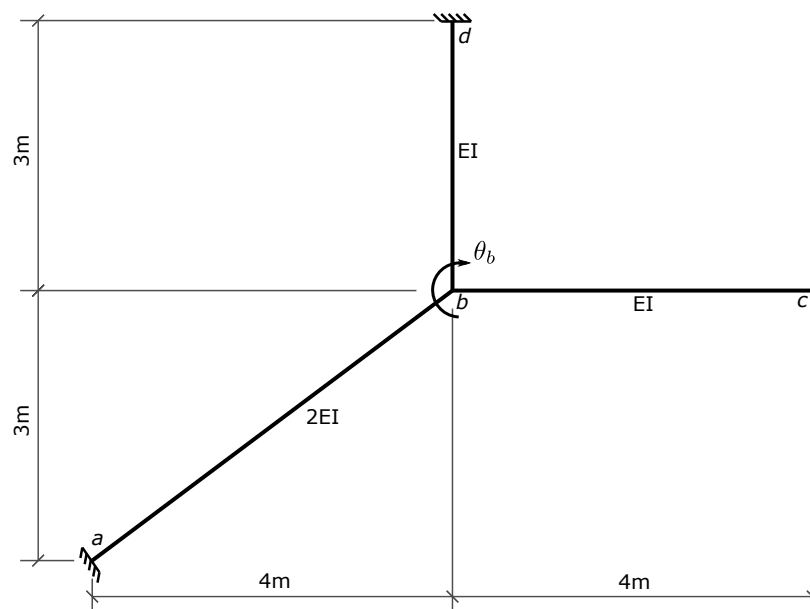


Problem 11 - Solution



1. Identify DOFs

There is 1 - θ_b , the rotation of joint b .



2. Fixed-end moments

The distributed load on member \$ab\$ must be resolved into its perpendicular component on \$ab\$. In this case, the perpendicular component is $0.8 \times 24 = 19.2$ kN/m.

There are no transverse loads on the two other members, so all 4 member fixed end moments for those are zero.

```
In [1]: Mfab = -0.8*24*5*5/12
        Mfba = 0.8*24*5*5/12
        Mfbc = Mfcb = Mfbd = Mfdb = 0
```

3. Slope deflection equations

Express member end moments as a function of the unknown joint rotation, θ_b .

```
In [2]: from sympy import symbols, solve, init_printing
        init_printing()
```

```
In [3]: theta_b, EI = symbols('theta_b EI')
        theta_a = theta_c = theta_d = 0      # rotations at the outside end of e
        ach member
```

```
In [4]: Mab = (2*EI/5)*(4*theta_a + 2*theta_b) + Mfab
        Mba = (2*EI/5)*(2*theta_a + 4*theta_b) + Mfba
        display(Mab, Mba)
```

$$\frac{4}{5} EI \theta_b - 40.0$$

$$\frac{8}{5} EI \theta_b + 40.0$$

```
In [5]: Mbc = (EI/4)*(4*theta_b + 2*theta_c) + Mfbc
        Mcb = (EI/4)*(2*theta_b + 4*theta_c) + Mfcb
        display(Mbc, Mcb)
```

$$EI \theta_b$$

$$\frac{EI}{2} \theta_b^2$$

```
In [6]: Mbd = (EI/3)*(4*theta_b + 2*theta_d) + Mfbd
        Mdb = (EI/3)*(2*theta_b + 4*theta_d) + Mfdb
        display(Mbd, Mdb)
```

$$\frac{4}{3} EI \theta_b^3$$

$$\frac{2}{3} EI \theta_b^3$$

4. Equilibrium Equation

The sum of the moments acting on joint b must be zero.

Note that the negatives of the member end forces act on the joint.

```
In [7]: ee = (Mba + Mbc + Mbd) # = 0, +ive ccw on joint
        ee
```

```
Out[7]:  $\frac{59 EI \theta_b}{15} + 40.0$ 
```

5. Solve for displacement

```
In [8]: ans = solve([ee], theta_b)
        ans
```

```
Out[8]:  $\left\{ \theta_b : -\frac{10.1694915254237}{EI} \right\}$ 
```

6. Back-substitute to get member end moments

```
In [9]: mab = Mab.subs(ans).n()
        mba = Mba.subs(ans).n()
        display(mba, mab)
```

```
 $23.728813559322$ 
```

```
 $-48.135593220339$ 
```

```
In [10]: mbc = Mbc.subs(ans).n()
        mcb = Mcb.subs(ans).n()
        display(mbc, mcb)
```

```
 $-10.1694915254237$ 
```

```
 $-5.08474576271186$ 
```

```
In [11]: mbd = Mbd.subs(ans).n()
        mdb = Mdb.subs(ans).n()
        display(mbd, mdb)
```

```
 $-13.5593220338983$ 
```

```
 $-6.77966101694915$ 
```

7. Check joint equilibrium

```
In [12]: # sum of moments acting on joint, +ive ccw
mba+mbc+mbd
```

```
Out[12]: 8.88178419700125 \cdot 10^{-15}
```

8. Member end shears

As there are no transverse loads and V_{bc} and V_{bd} , the shears on those are constant (non-changing) over the whole length of each member.

```
In [13]: vab = -(mab + mba - 0.8*24*5*5/2)/5
vba = 0.8*24*5 - vab
vbc = -(mbc + mcb)/4
vbd = -(mbd + mdb)/3
display(vab, vba, vbc, vbd)
```

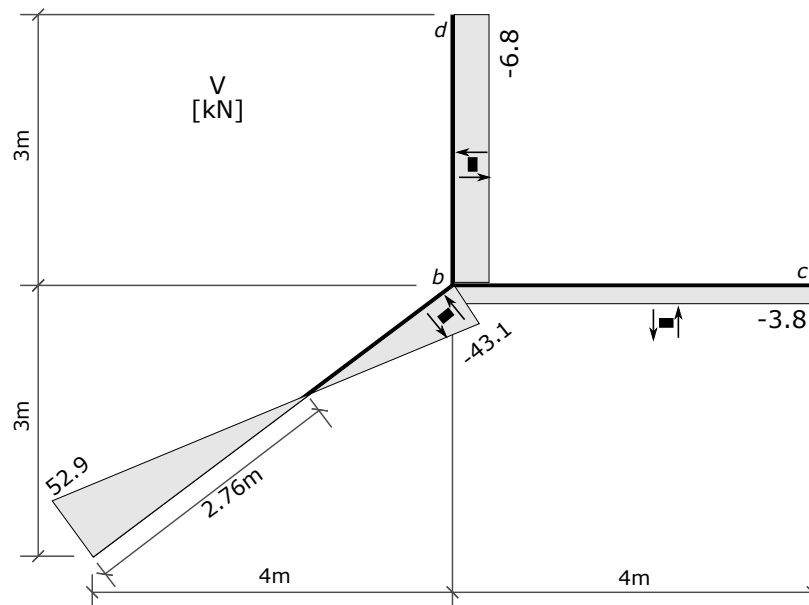
```
52.8813559322034
```

```
43.1186440677966
```

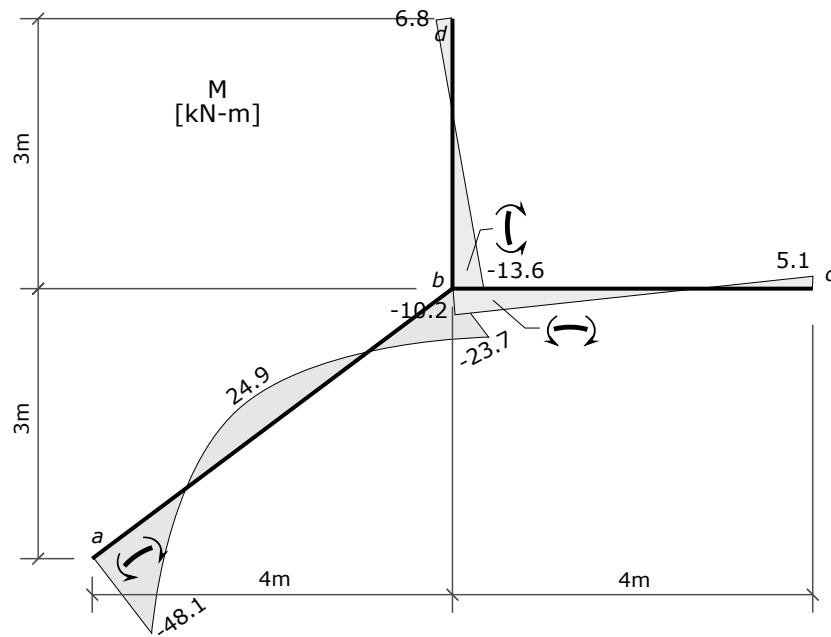
```
3.8135593220339
```

```
6.77966101694915
```

9. Shear force diagram



10. Bending moment diagram



In []: