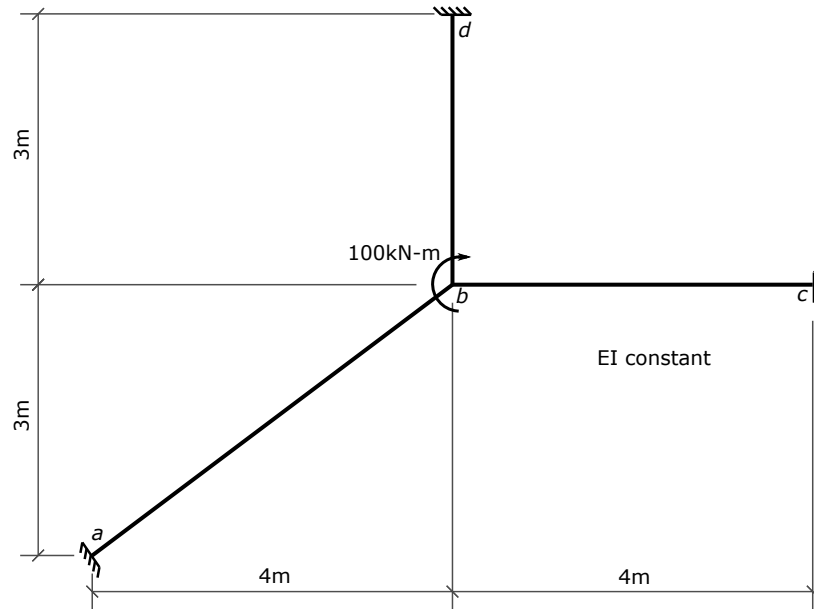
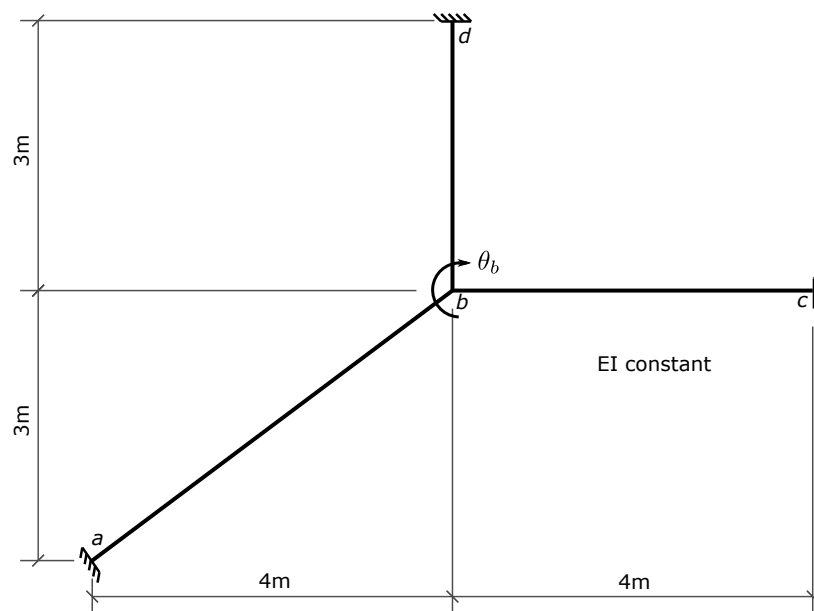


## Problem 09 - Solution



### 1. Identify DOFs

There is 1 -  $\theta_b$ , the rotation of joint  $b$ .



## 2. Fixed-end moments

There are no transverse loads on any of the 3 members, so all 6 member fixed end moments are zero.

```
In [1]: Mfab = Mfba = Mfbc = Mfcb = Mfbd = Mfdb = 0
```

## 3. Slope deflection equations

Express member end moments as a function of the unknown joint rotation,  $\theta_b$ .

```
In [2]: from sympy import symbols, solve, init_printing
init_printing()
```

```
In [3]: theta_b, EI = symbols('theta_b EI')
theta_a = theta_c = theta_d = 0      # rotations at the outside end of e
ach member
```

```
In [4]: Mab = (EI/5)*(4*theta_a + 2*theta_b) + Mfab
Mba = (EI/5)*(2*theta_a + 4*theta_b) + Mfba
display(Mab, Mba)
```

$$\frac{2EI\theta_b}{5}$$

$$\frac{4EI\theta_b}{5}$$

```
In [5]: Mbc = (EI/4)*(4*theta_b + 2*theta_c) + Mfbc
Mcb = (EI/4)*(2*theta_b + 4*theta_c) + Mfcb
display(Mbc, Mcb)
```

$$EI\theta_b$$

$$\frac{EI\theta_b}{2}$$

```
In [6]: Mbd = (EI/3)*(4*theta_b + 2*theta_d) + Mfbd
Mdb = (EI/3)*(2*theta_b + 4*theta_d) + Mfdb
display(Mbd, Mdb)
```

$$\frac{4EI\theta_b}{3}$$

$$\frac{2EI\theta_b}{3}$$

## 4. Equilibrium Equation

The sum of the moments acting on joint  $b$  must be zero.

Note that the negatives of the member end forces act on the joint.

```
In [7]: ee = (Mba + Mbc + Mbd) - 100 # = 0, +ive ccw on joint
        ee
```

```
Out[7]: 
$$\frac{47 EI \theta_b}{15} - 100$$

```

## 5. Solve for displacement

```
In [8]: ans = solve([ee], theta_b)
        ans
```

```
Out[8]: 
$$\left\{ \theta_b : \frac{1500}{47 EI} \right\}$$

```

## 6. Back-substitute to get member end momentsa

```
In [9]: mab = Mab.subs(ans).n()
        mba = Mba.subs(ans).n()
        display(mba, mab)
```

```

$$25.531914893617$$

```

```

$$12.7659574468085$$

```

```
In [10]: mbc = Mbc.subs(ans).n()
         mcb = Mcb.subs(ans).n()
         display(mbc, mcb)
```

```

$$31.9148936170213$$

```

```

$$15.9574468085106$$

```

```
In [11]: mbd = Mbd.subs(ans).n()
         mdb = Mdb.subs(ans).n()
         display(mbd, mdb)
```

```

$$42.5531914893617$$

```

```

$$21.2765957446809$$

```

## 7. Check joint equilibrium

```
In [12]: # sum of moments acting on joint, +ive ccw
        mba+mbc+mbd-100
```

```
Out[12]: 
$$0$$

```

## 8. Member end shears

As there are no transverse loads, the shears are constant (non-changing) over the whole length of each member.

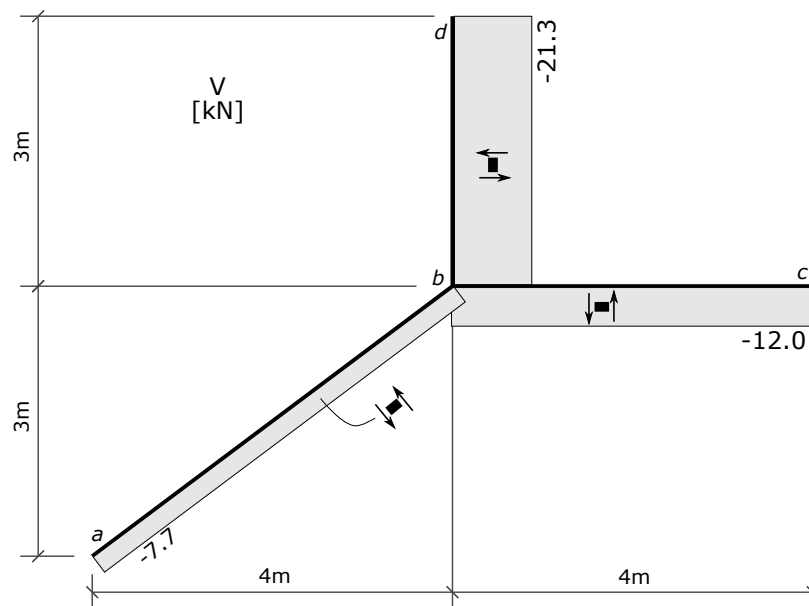
```
In [13]: vab = -(mab + mba) / 5
vbc = -(mbc + mcb) / 4
vbd = -(mbd + mdb) / 3
display(vab, vbc, vbd)
```

\$\$-7.65957446808511\$\$

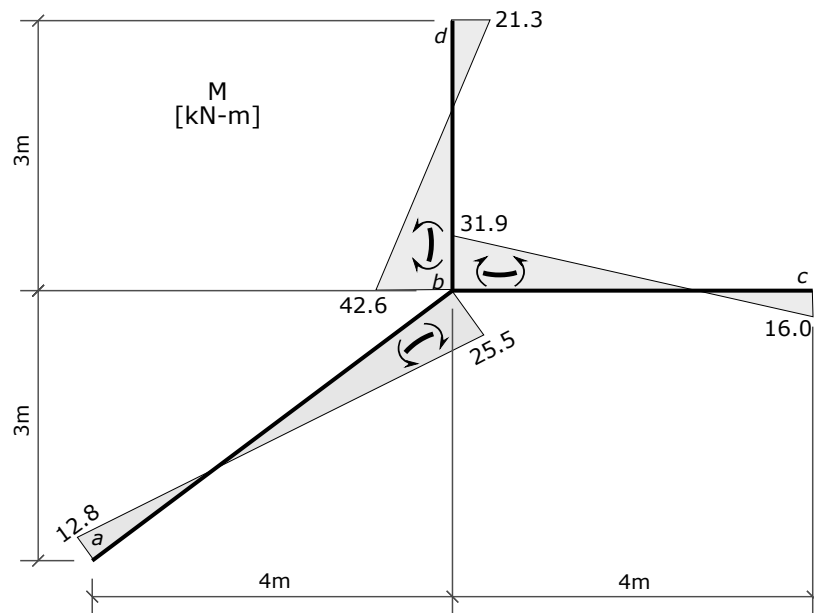
\$\$-11.968085106383\$\$

\$\$-21.2765957446809\$\$

## 9. Shear force diagram



## 10. Bending moment diagram



In [ ]: