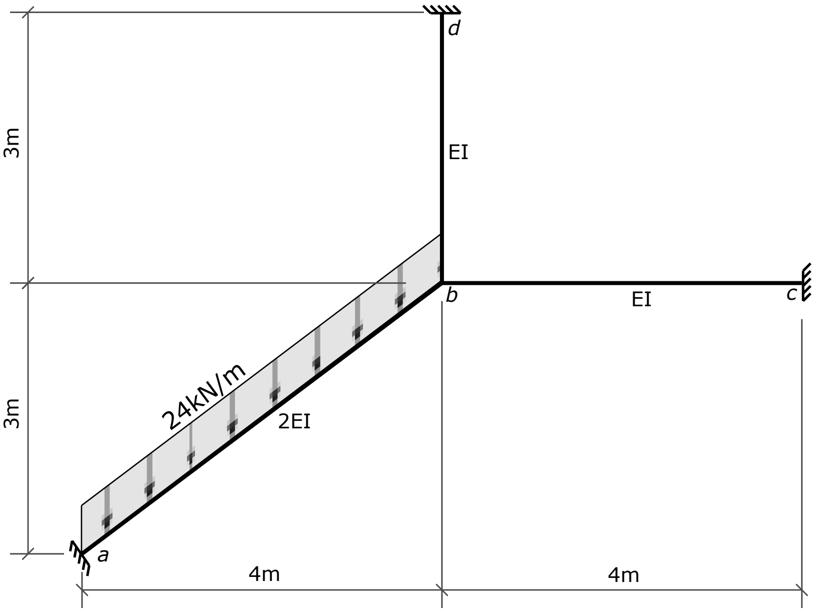
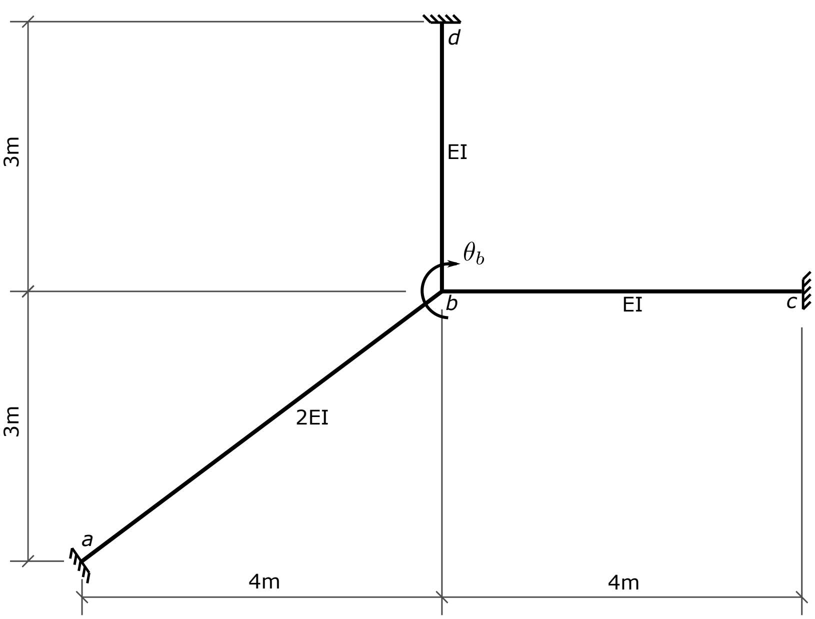
Problem 11 - Solution



1. Identify DOFs

There is 1 - θ_b, the rotation of joint b.



2. Fixed-end moments

The distributed load on member ab must be resolved into its perpendicular component on ab. In this case, the perpendicular component is $0.8 \times 24 = 19.2$ kN/m.

There are no transverse loads on the two other members, so all 4 member fixed end moments for those are zero.

In [1]:
```
Mfab = -0.8*24*5*5/12
Mfba = 0.8*24*5*5/12
Mfbc = Mfcb = Mfbd = Mfdb = 0
```

3. Slope deflection equations

Express member end moments as a function of the unknown joint rotation, θ_b.

In [2]:
```
from sympy import symbols, solve, init_printing
init_printing()
```

In [3]:
```
theta_b, EI = symbols('theta_b EI')
theta_a = theta_c = theta_d = 0      # rotations at the outsaide end
 of each member
```

In [4]:
```
Mab = (2*EI/5)*(4*theta_a + 2*theta_b) + Mfab
Mba = (2*EI/5)*(2*theta_a + 4*theta_b) + Mfba
display(Mab,Mba)
```

$$\frac{4EI\theta_b}{5} - 40.0$$

$$\frac{8EI\theta_b}{5} + 40.0$$

In [5]:
```
Mbc = (EI/4)*(4*theta_b + 2*theta_c) + Mfbc
Mcb = (EI/4)*(2*theta_b + 4*theta_c) + Mfcb
display(Mbc,Mcb)
```

$$EI\theta_b$$

$$\frac{EI\theta_b}{2}$$

In [6]:
```
Mbd = (EI/3)*(4*theta_b + 2*theta_d) + Mfbd
Mdb = (EI/3)*(2*theta_b + 4*theta_d) + Mfdb
display(Mbd,Mdb)
```

$$\frac{4EI\theta_b}{3}$$

$$\frac{2EI\theta_b}{3}$$

4. Equilibrium Equation

The sum of the moments acting on joint b must be zero.

Note that the negatives of the member end forces act on the joint.

In [7]:
```
ee = (Mba + Mbc + Mbd)  # = 0, +ive ccw on joint
ee
```

Out[7]:

$$\frac{59EI\theta_b}{15} + 40.0$$

5. Solve for displacement

In [8]:
```
ans = solve([ee],theta_b)
ans
```

Out[8]:

$$\left\{\theta_b : -\frac{10.1694915254237}{EI}\right\}$$

6. Back-substitute to get member end momentsa

In [9]:
```
mab = Mab.subs(ans).n()
mba = Mba.subs(ans).n()
display(mba, mab)
```

$$23.728813559322$$

$$-48.135593220339$$

In [10]:
```
mbc = Mbc.subs(ans).n()
mcb = Mcb.subs(ans).n()
display(mbc,mcb)
```

$$-10.1694915254237$$

$$-5.08474576271186$$

In [11]:
```
mbd = Mbd.subs(ans).n()
mdb = Mdb.subs(ans).n()
display(mbd,mdb)
```

-13.5593220338983

-6.77966101694915

7. Check joint equilibrium

In [12]:
```
# sum of moments acting on joint, +ive ccw
mba+mbc+mbd
```

Out[12]: $8.88178419700125 \cdot 10^{-15}$

8. Member end shears

As there are no transverse loads and bc and bd, the shears on those are constant (non-changing) over the whole length of each member.

In [13]:
```
vab = -(mab + mba - 0.8*24*5*5/2)/5
vba = 0.8*24*5 - vab
vbc = -(mbc + mcb)/4
vbd = -(mbd + mdb)/3
display(vab,vba, vbc,vbd)
```

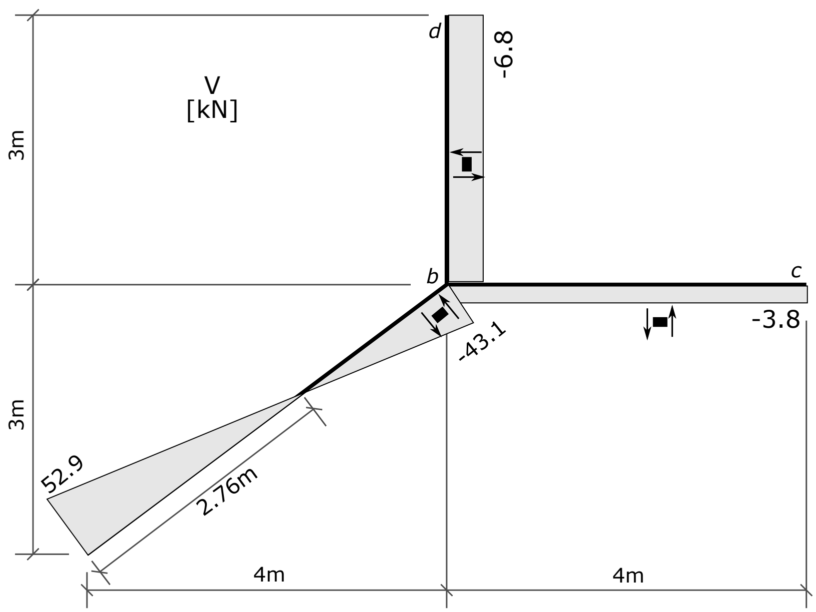
52.8813559322034

43.1186440677966

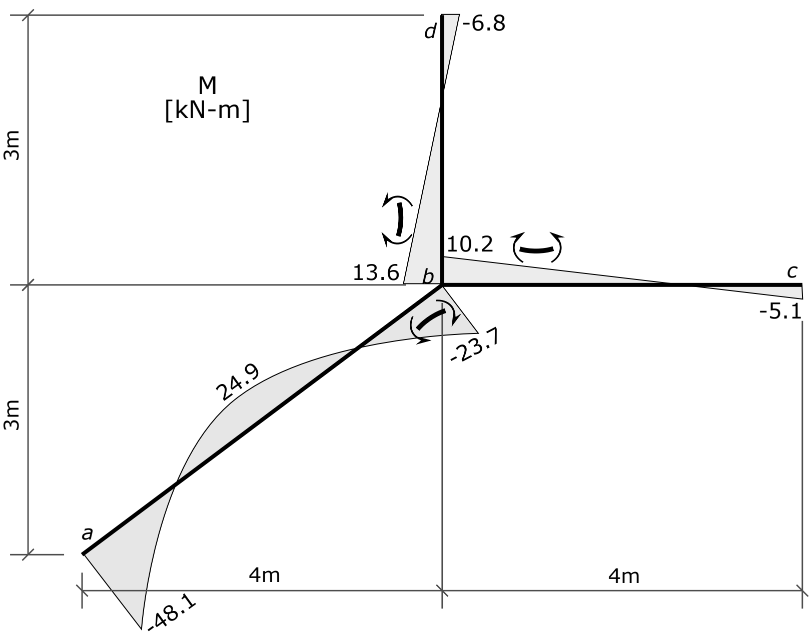
3.8135593220339

6.77966101694915

9. Shear force diagram



10. Bending moment diagram



In [ ]: