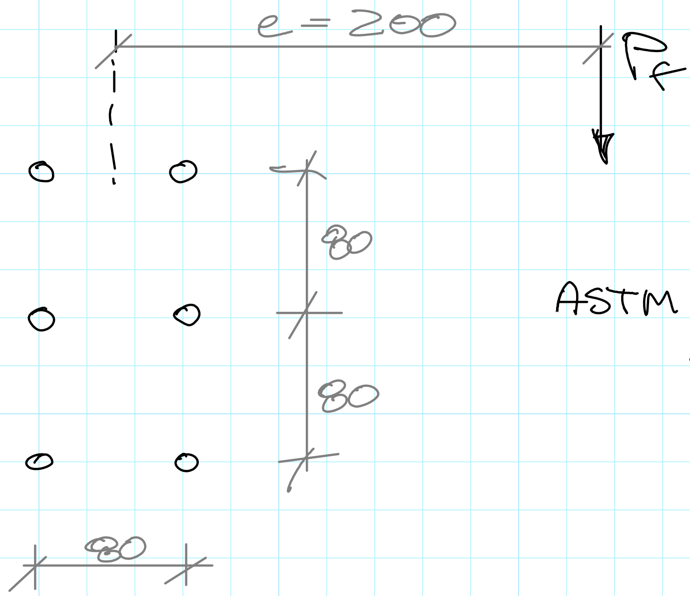
# CIVE 3205

## Example Bolt-4
## Eccentrically Loaded

March 11, 2013

Compute the capacity, $P_r$, of the eccentrically loaded bolt group.




ASTM A325M bolts
20M

Force in a single bolt

$$R = R_u \left(1 - e^{-\mu \Delta}\right)^{\lambda}$$

$R_u$ = ult. shearing force

= 329 kN (unfactored double shear threads excluded)

$\mu$ = 0.394

$\lambda$ = 0.55

$\Delta$ = shearing deformation in bolt, mm.

$\Delta$ is proportional to distance from instant centre of rotation

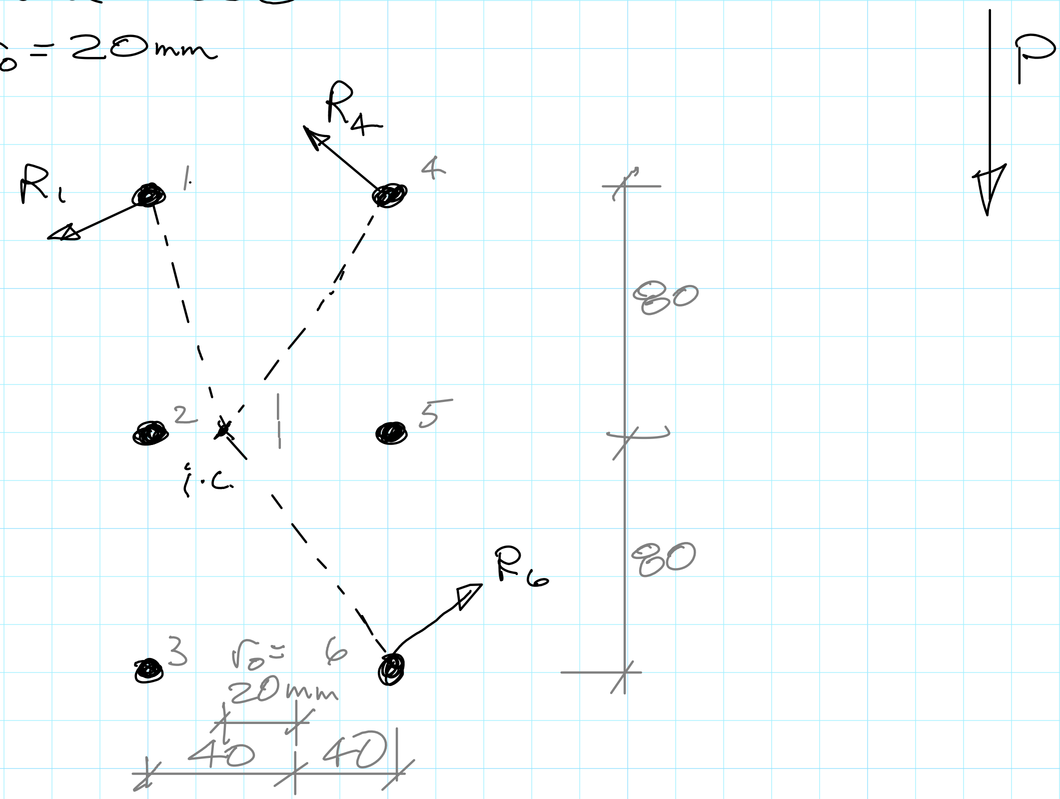
Bolt furthest from i.c. is assummed to be at its ultimate deformation.

$$\Delta_u = 8.64 \text{ mm}$$

Use $r_o$ = dist leftward of i.c from centroid

Choose a position of i.c. on the horizontal axis

try $r_o = 20$mm




Assume that one plate rotates rigidly about i.c. Then shear deformation in each bolt is proportional to distance from i.c.

Bolts 4 & 6 are furthest from i.c.

$$r_{max} = r_4 = r_6 = \sqrt{60^2 + 80^2} = 100 \text{ mm}$$

The deformation of 4 & 6 is

$\Delta_4 = \Delta_6 = \Delta_v = 8.64\,mm$ ✓

The force in 4 & 6 is

$$R_4 = R_6 = 329\left(1 - e^{-0.394 \times 8.64}\right)^{.55}$$

$$= 322.9\ kN$$

Bolts 1 & 3 – distance from i.c.

$$r_1 = r_3 = \sqrt{20^2 + 80^2} = 82.46\ mm$$

shear deformations are:

$$\Delta_i = \frac{r_i}{r_{max}} \Delta_{max}$$

$$r_1 = r_3 = \frac{82.46}{100} \times 8.64$$

$$= 7.125\ mm$$

the forces in bolts 1 & 3

$$R_1 = R_3 = 329\left(1 - e^{-.394 \times 7.125}\right)^{.55}$$

$$= 317.9\ kN$$

Bolt 2:

$$r_2 = 20\ mm$$

$$\Delta_2 = \frac{20}{100} \times 8.64\,mm = 1.728\,mm$$

$$R_2 = 329\left(1 - e^{-.394 \times 1.728}\right)^{.55}$$

$$= 223.2\ kN$$

Bolt 5:

$$r_5 = 60\,mm$$

$$\Delta_5 = \frac{60}{100} \times 8.64\,mm = 5.184\,mm.$$

$$R_5 = 329\left(1 - e^{-.394 \times 5.184}\right)^{.55}$$

$$= 304.8\ kN.$$

Now, each of these forces has a vertical component, $V_i$

$$V_i = \frac{x_i}{r_i} R_i$$

& produces a moment about the i.c., $M_i$

$$M_i = r_i R_i$$

Bolts 4&6

$$V_4 = V_6 = \frac{60}{100} \times 322.9 = 193.7 \text{ kN}$$

$$M_4 = M_6 = 100 \times 322.9 = 32290 \text{ kN-mm}$$

Bolts 1&3

$$V_1 = V_3 = \frac{-20}{82.46} \times 317.9 = -77.10 \text{ kN}$$

$$M_1 = M_3 = 82.46 \times 317.9 = 26214 \text{ kN-mm}$$

Bolt 2

$$V_2 = \frac{-20}{20} \times 223.2 = -223.2 \text{ kN}$$

$$M_2 = 20 \times 223.2 = 4464 \text{ kN-mm}$$

Bolt 5

$$V_2 = \frac{60}{60} \times 304.8 = 304.8 \text{ kN}$$

$$M_2 = 60 \times 304.8 = 18288 \text{ kN-mm}$$

If the i.c. is at the correct place, these forces will be in equilibrium.

First, we can calculate the value of P causing this failure from

$$P_u (e + r_o) = \Sigma M_i$$

$$P_u (200+20) = 32290 \times 2 + 26214 \times 2 + 4464 + 18288$$

$$P_u = 635.3 \text{ kN}$$

this is the ultimate load that is in equilibrium with the moment caused by the bolt forces.

Now check $\Sigma F_y \; +\uparrow$

$$\Sigma F_y = P_u + \Sigma V_i$$

$$= -635.3 + 193.7\times 2 + -77.10\times 2 + -223.2 + 304.8$$

$$= -320.5 \neq 0 \quad \text{N.G.}$$

The ult load, $P_u$, should also be in equilibrium with the vertical components of the bolt forces.

In this case it isn't.

∴ The position chosen for $r_o = 20\text{mm}$ is not correct.

This procedure must be repeated with different values of $r_o$ until

$$P_u = \frac{\Sigma M_i}{e + r_o} = \Sigma V_i$$

For this problem, that happens (closely enough) at:

$r_o = 35.68\text{mm}$

$\Sigma M_i = 145.3\times 10^3$ kN-mm

$\Sigma V_i = 616$ kN

$\underline{\underline{P_u = 617\text{ kN}}} \leftarrow$

In tabular form, the calculations are:

$r_0 = 35.7$ mm

| bolt # | $x_i$ (mm) | $y_i$ (mm) | $r_i$ (mm) | $\Delta_i = \frac{r_i}{r_{max}} \Delta_{max}$ | $R_i$ (kN) | $V_i$ (kN) | $M_i$ (kN-mm) |
|---|---|---|---|---|---|---|---|
| 1 | −4.3 | 80 | 80.12 | 6.286 | 313 | −17 | 25114 |
| 2 | −4.3 | 0 | 4.32 | 0.339 | 105 | −105 | 453 |
| 3 | −4.3 | −80 | 80.12 | 6.286 | 313 | −17 | 25114 |
| 4 | 75.7 | 80 | 110.12 | 8.640 | 323 | 222 | 35561 |
| 5 | 75.7 | 0 | 75.68 | 5.937 | 311 | 311 | 23545 |
| 6 | 75.7 | 80 | 110.12 | 8.640 | 323 | 222 | 35561 |
| | | | | | $\Sigma$ | 616 | $145.3\times10^3$ |

$$P_u = \frac{145.3\times10^3}{35.7+200} = 616 \text{ kN}$$

Equilibrium satisfied

$P_u = \Sigma V_i$

$\therefore r_0$ is correct

$\therefore P_u = 616$ kN

The force $R_u$ for a single bolt is unfactored, double shear A325M M20 bolts, threads excluded.

$P_f \leq \phi_b P_u \leq 0.8 \times 616 \times \frac{1}{2}$

$$\underline{\underline{P_f \leq 246 \text{ kN}}} \quad \text{single shear}$$

The above is not practical for hand calculation.

Tables 3-14 thru 3-20 are design aids for this.

From Table 3-15, 2 rows of bolts

pitch = 80 mm    3 bolts/row

eccentricity = 80mm

$C = 1.91$

$V_r = 125\ kN$ (factored, single shear, threads excluded)

$P_f \le C V_r$

$\le 1.91 \times 125\ kN$    (threads excluded)

$\underline{\underline{P_f \le 239\ kN}}$ ←

(cf 246 kN, above)
(within 3%)

Must ensure threads excluded, otherwise reduce strength to 0.70

Must ensure that bearing resistance is greater than shear resistance of 125 kN/bolt.