If we linearize the equation around the Schwarzschild-de Sitter metric, we find the first-order equation

$$\delta R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \delta R + \left(\Lambda - \frac{1}{2} R \right) h_{\mu\nu} = 0 \tag{1}$$

where $h_{\mu\nu}$ is the metric perturbation. Writing the curvature perturbations in terms of the metric perturbation, we obtain the equation:

$$\begin{aligned} &\frac{1}{2} \left(\nabla_\lambda \nabla_\mu h_\nu{}^\lambda + \nabla_\lambda \nabla_\nu h_\mu{}^\lambda - \nabla^2 h_{\mu\nu} - \nabla_\mu \nabla_\nu h \right) \\ &\quad - \frac{1}{2} g_{\mu\nu} \left(-h^{\alpha\beta} R_{\alpha\beta} + \nabla_\alpha \nabla_\beta h^{\alpha\beta} - \nabla^2 h \right) \\ &\quad + \left(\Lambda - \frac{1}{2} R \right) h_{\mu\nu} = 0 \end{aligned} \tag{2}$$

The unperturbed metric is the Schwarzschild-de Sitter metric in the usual spherically symmeric coordinates:

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \tag{3}$$

with $f(r) = 1 - \frac{2M}{r} - H^2 r^2$. The nonvanishing Christoffel symbols are:

$$\Gamma^t_{tr} = \frac{f'}{2f} \tag{4}$$

$$\Gamma^r_{tt} = \frac{1}{2} f f' \tag{5}$$

$$\Gamma^r_{rr} = -\frac{f'}{2f} \tag{6}$$

$$\Gamma^r_{\theta\theta} = -rf \tag{7}$$

$$\Gamma^r_{\phi\phi} = -rf \sin^2 \theta \tag{8}$$

$$\Gamma^\theta_{r\theta} = \frac{1}{r} \tag{9}$$

$$\Gamma^\theta_{\phi\phi} = -\cos\theta \sin\theta \tag{10}$$

$$\Gamma^\phi_{r\phi} = \frac{1}{r} \tag{11}$$

$$\Gamma^\phi_{\theta\phi} = \cot\theta \tag{12}$$

and the Ricci tensor and Ricci scalar are given by

$$R_{tt} = \frac{1}{2} f (2 \frac{f'}{r} + f'') \tag{13}$$

$$R_{rr} = -\frac{2f' + rf''}{2rf} \tag{14}$$

$$R_{\theta\theta} = 1 - f - rf' \tag{15}$$

$$R_{\phi\phi} = -\sin^2\theta(-1 + f + rf') \tag{16}$$

$$R = \frac{2 - 2f - 4rf' - r^2 f''}{r^2} \tag{17}$$