Notes on Real Time Collision Detection

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March 1, 2016

1 Introduction

The topic of collision detection is of particular importance in several branches within computer graphics. Some examples include:

- It is used in video games to create realistic animations and interactions.
- It is used to render a frame in a movie.
- It is used in general animations.
- It is used in physical simulations.

As a result, a lot of study has been devoted to this area. In these notes we will discuss two main components: collision detection and space partitioning algorithms.

Before we begin, it is important to determine what we would want in a collision detection system, as well as which are the challenges that we will face when designing it.

1.1 Design Factors

The first thing to consider is how objects are being represented. For the most part, they are represented as triangular meshes (since this is what graphics hardware is designed to work on). That being said, there are other ways of representing objects, such as modelling with implicit functions. While the choice of representation does affect the way the collisions (and other algorithms) will work, let us agree to only consider triangular meshes for the remainder of this discussion.

The second point of discussion is what we are going to be colliding. To be more concise, do we use the rendering mesh or something else? Let's look at an example: suppose that we have a mesh that is composed by 2,000,000 triangles. We wish to intersect this mesh against another model that is also composed of 2,000,000 triangles. The näive way of performing this task would be to check every triangle in the first mesh against every triangle in the second. This, however, results in 4,000,000,000,000 intersection tests! Even with the most powerful computers available, this would take a while. Even worse, it this were part of a game, the amount of time the player would have to wait *per* frame is unacceptable.

Clearly in this case we would want to use something *other* than the render mesh, preferably something that is more compact. This is where bounding volumes kick in. These volumes are called proxy geometry. These are usually optimized for collision detection systems. Unfortunately, they also represent a problem: suppose that all of the proxies are computed in a pre-processing step. What happens when the original mesh is altered? How do we maintain these new geometries?

2 Bounding Volumes

In this section we will discuss bounding volumes. In particular, we will be focusing on the following three:

1. Axis-aligned bounding boxes (AABB),
2. spheres,
3. and oriented bounding boxes (OBB).

We begin our discussion of bounding volumes by asking the following question: what kind of properties do we want in a bounding volume? Ideally, we would want the following:

- Inexpensive intersection tests.
- Tight fitting.
- Inexpensive to compute.
- Easy to transform (rotations are of particular importance).
- Use little memory.

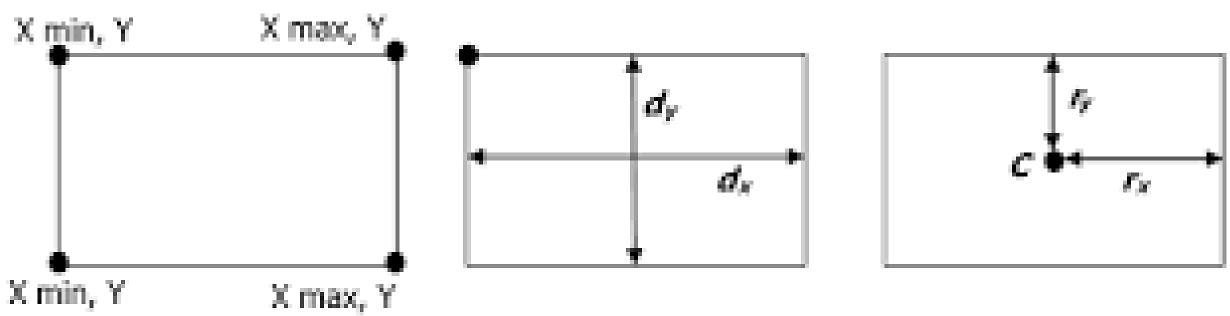
Unfortunately as we will soon see, there have to be trade-offs whenever we select a bounding volume. For example: a sphere is trivial to test and occupies little memory (it only requires 4 floats). Conversely, a convex hull gives us a very tight bound on the object, but requires a lot more memory and is more expensive to test for intersections. So the choice of bounding volume ultimately boils down to the specific requirements of our application.

2.1 Asis-aligned Bounding Boxes

This is one of the most common bounding volumes used today. It is characterized by having each one of its faces aligned with the given coordinate systems. In other words, the face normals will always be parallel with the axes of the coordinate system.

There are three common ways to represent an AABB as shown below:

Figure 1: The three common AABB representations: (from left to right) min-max, min-widths, and centre-radius.



In terms of storage, the centre-radius representation is the most efficient, since we can store the array of halfwidths with less bits than the centre position values. The worst is the min-max representation, since it requires all 6 values to be stored (usually as floats, but they could be doubles). In terms of updating the volume, the last two are the easiest to maintain, while the min-max requires us to translate both points, as opposed to just one.

The intersection test for AABBs is very straight-forward and most of the optimizations here boil down to either re-ordering of the if-statements, or optimizing (if it isn't already) the absolute value function.

2.2 Computing and Updating AABBs

We generally define AABBs in terms of the model space for the particular object. This poses an interesting problem: when we want to perform an intersection test between two different objects, we must transform both AABBs to a common space. The question is: which one?

The reason why this question is interesting is because it is completely possible to construct a case where objects intersect (or not) depending on the choice of space. On top of this, whenever we transform an object from one space to another we are essentially translating them. This translation introduces error into the calculations, which at times may be avoided entirely by selecting a space that leaves the objects close (or on) the origin.

Some bounding volumes (such as convex hulls) are free from orientation, while AABBs are not. This means that whenever an object is rotated, the AABB must be updated in order to preserve its alignment to the axes. The most common strategies for this are:

- Use a loose-fitting AABB that always encloses the object.
- Compute a tight dynamic reconstruction from the original set.
- Computing a tight dynamic reconstruction using hill climbing.
- Computing an approximate dynamic reconstruction from the rotated AABB.

Let us examine each one in a bit more detail.

The first option is to essentially construct an AABB that contains the bounding sphere for that particular mesh. This means that any rotation can be ignored, since spheres are invariant under rotation. This approach has the big drawback of not being particularly tight, and it would be easier (in some cases) to just use the bounding sphere directly instead of wrapping it in an AABB.

Reconstructing an AABB from the point set involves iterating over all the vertices and finding the one that has the smallest and largest points along each of the axes. As you might expect, this is an $O(n)$ operation. It is possible to optimize this by removing all the points that do not lie on the convex hull of the object (since the bounding box must contain points that lie on the convex hull). This requires some pre-processing that needs to be done on the object prior to constructing the AABB. This may not be worth the time (or the memory) it takes to perform these operations, so it may be worth investigating a better bounding volume.

Another way of optimizing the reconstruction of an AABB requires a way to easily obtain the neighbourhood of a specific vertex. Given this, we can construct an AABB by maintaining 6 pointers to the most extreme points in each of the axes. Whenever we rotate the object, we climb along the neighbours of the old vertex until we find a new extremal point. This has an obvious drawback: the objects must be convex in order for this to work properly (otherwise we would get stuck in local minima and would never find the new extremal point). We can further optimize this by reducing the number of comparisons we have to perform. For example, if we are looking for the maximal point in $+x$, then we only need to look at the x coordinate of the vectors. The other obvious problem is that hill-climbing requires a robust implementation for finding neighbours for any given vertices.

The last of the four realignment methods is to simply wrap the rotated AABB in a new AABB. This gives us a new (approximate) AABB that we can use to compute our collision detections. It is important to keep in mind that the new AABB *must* be computed with respect to the original AABB, otherwise the box will just grow indefinitely. The idea is as follows: consider a box A that was transformed by a matrix $\mathbf{M}$ resulting in a box A'. The three columns (or rows depending on the convention) contain the world-coordinate axes of A' in its local frame.
So what does this all mean? Well suppose that A is given using min-max representation and $\mathbf{M}$ is a column matrix. This means that we can find the extents of B by simply projecting each of the rotated vertices onto the world-coordinate axes. Since minimal and maximal extents will be composed by a linear combination of the transformed values of A, then it is only a matter of finding the new min and max. This can be easily done by adding the smaller terms and the largest terms (respectively).

2.3 Spheres

Spheres are another very common bounding volume. Like AABBs, the intersection tests for spheres is very simple to perform. In addition, spheres have the property that they are invariant under rotations, which means that the only transformation that affects them is translation, which is very simple to do. They also have a very compact representation: we only need a point and a radius to fully determine a sphere. That said, the interesting part of spheres isn't in their intersection tests, but rather on how they are computed.