ESE 531: Homework 6

Noah Schwab

March 14, 2022

Problem solutions with figures are shown below. Work and code is shown in attachments at end of document.

5.24

$H_{\mathrm{min}}(z) = \frac{4\alpha}{3}\frac{(1-\frac{1}{4}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$, where α is a scale factor.

$H_{\mathrm{ap}}(z) = \frac{(z^{-1}-3)(z^{-1}-\frac{1}{4})}{(1-3z^{-1})(1-\frac{1}{4}z^{-1})}z^{-1}$

See attached work for pole-zero plots. This decomposition is unique up to a scale factor, α. $H_{\mathrm{min}}(z)$ must include the indicated poles and zeros such that they are contained within the unit circle, and $H_{\mathrm{ap}}(z)$ must include the indicated conjugates in order to recover $H(z)$.

5.36

$H_{\mathrm{min}}(z) = \frac{4(1-\frac{1}{2}jz^{-1})(1+\frac{1}{2}jz^{-1})}{(1-\frac{3}{4}z^{-1})(1+\frac{1}{2}z^{-1})}$, ROC: $|z| > \frac{3}{4}$

$H_{\mathrm{ap}}(z) = \frac{(z^{-1}+\frac{1}{2}j)(z^{-1}-\frac{1}{2}j)}{(1-\frac{1}{2}jz^{-1})(1+\frac{1}{2}jz^{-1})}$, ROC: $|z| > \frac{1}{2}$

See attached work for pole-zero plots.

5.42

a) $H(z) = \frac{z^{-2}(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}$, ROC: $\frac{1}{2} < |z| < 3$

b) $n_0 = 2$

$g[0] = -\frac{3}{7}$

c) $F(z) = \frac{4z^{-1}}{3(1+3z^{-1})(1+\frac{1}{3}z^{-1})}$, ROC: $\frac{1}{3} < |z| < 3$

d) $e[n]$ CANNOT be causal.

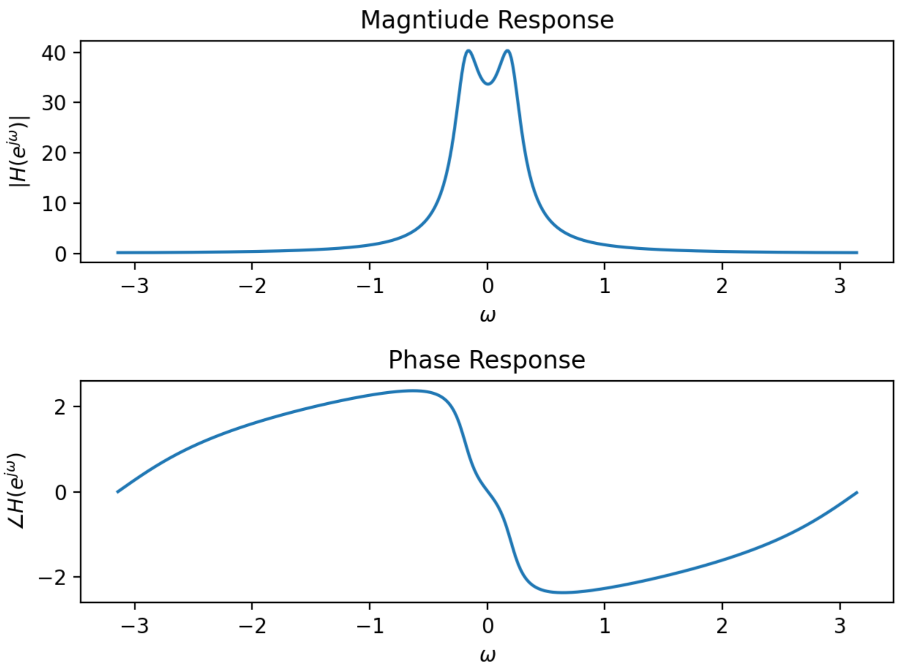
Matlab problem 1: Frequency Response for Difference Equations

1. $y[n] - 1.8\cos\frac{\pi}{16}y[n-1] + 0.81y[n-2] = x[n] + 0.5x[n-1]$

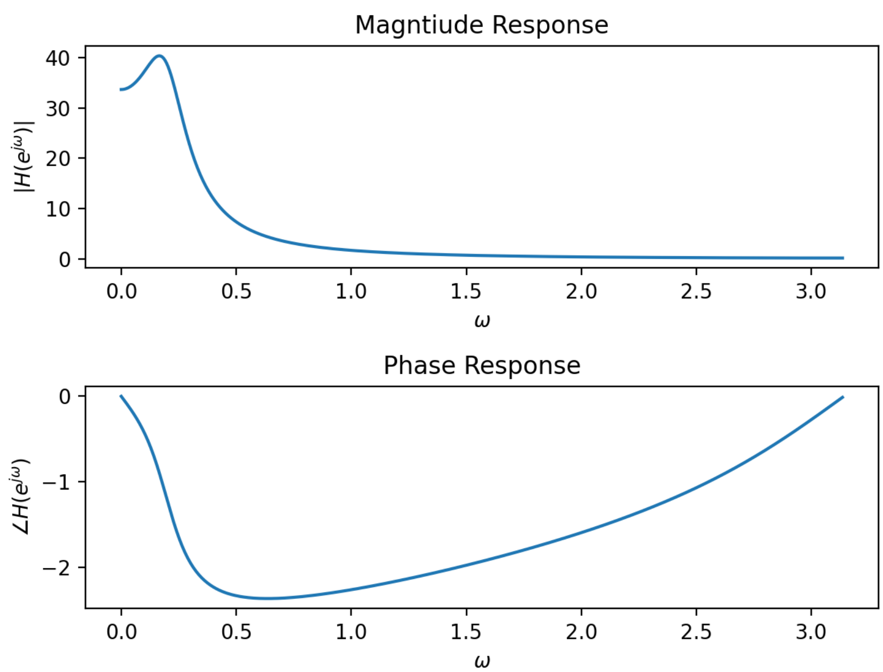
2. $y[n] + 0.13y[n-1] + 0.52y[n-2] + 0.3y[n-3] = 0.16x[n] - 0.48x[n-1] + 0.48x[n-2] - 0.16x[n-3]$

3. $y[n] - 0.268y[n-2] = 0.634x[n] - 0.634x[n-2]$

a) Magnitude and Phase Response for System 1 over the Unit Circle



b) Magnitude and Phase Response for System 1 over Half of the Unit Circle

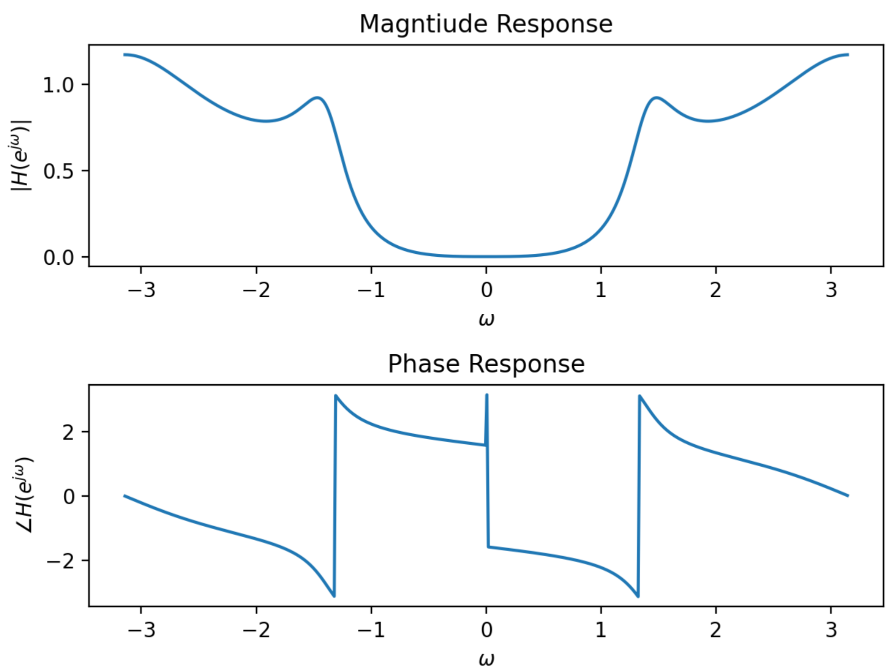


Since the system has real-valued coefficients, the conjugate frequencies evaluate to the same response values. As a result, the frequency response of the system is symmetric about $\omega = 0$, and the responses over the domains $[-\pi, 0]$ and $[0, \pi]$ are mirror images. Therefore, evaluating the response over the upper half of the unit circle is sufficient because it gives us information about the frequency response over the lower half, as well.

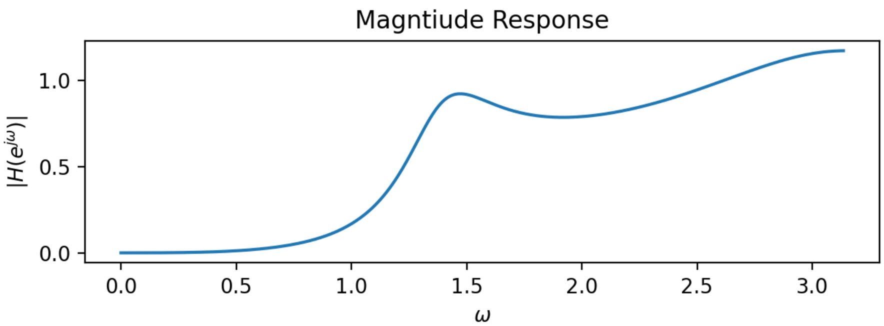
c) As we can see from the magnitude response above, this filter is **Low-Pass**.

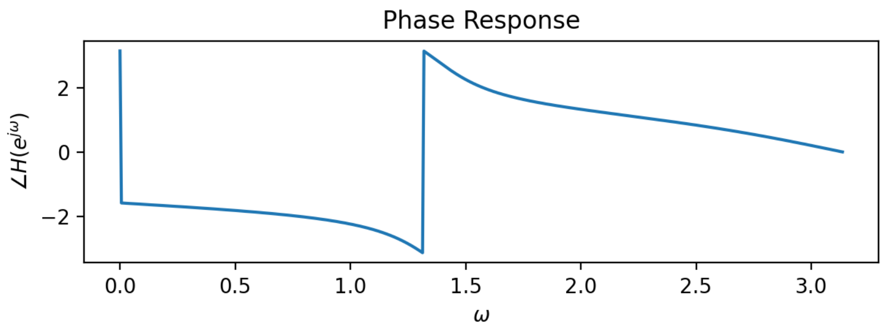
d) Repeat parts (a) - (c) for System 2.

Magnitude and Phase Response for System 2 over the Unit Circle



Magnitude and Phase Response for System 2 over Half of the Unit Circle



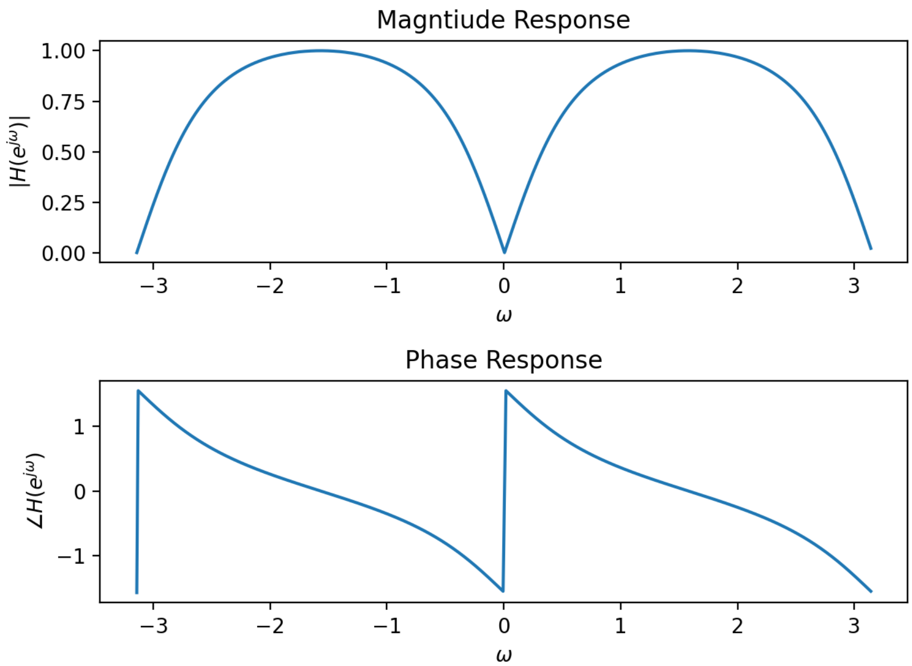


Again, our system has real-valued coefficients, so the frequency response is symmetric about $\omega = 0$.

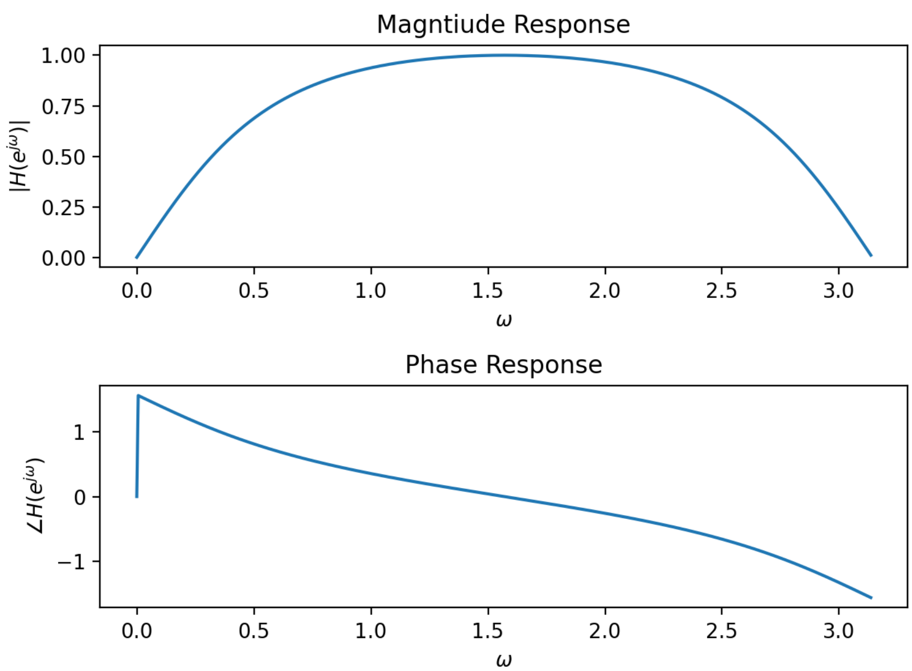
As we can see from the plots above, this filter is **High-Pass**.

e) Repeat parts (a) - (c) for System 3.

Magnitude and Phase Response for System 3 over the Unit Circle



Magnitude and Phase Response for System 3 over Half of the Unit Circle



Again, our system has real-valued coefficients, so the frequency response is symmetric about $\omega = 0$.

As we can see from the plots above, this filter is **Band-Pass**, since it passes all mid-frequencies between 0 and π.

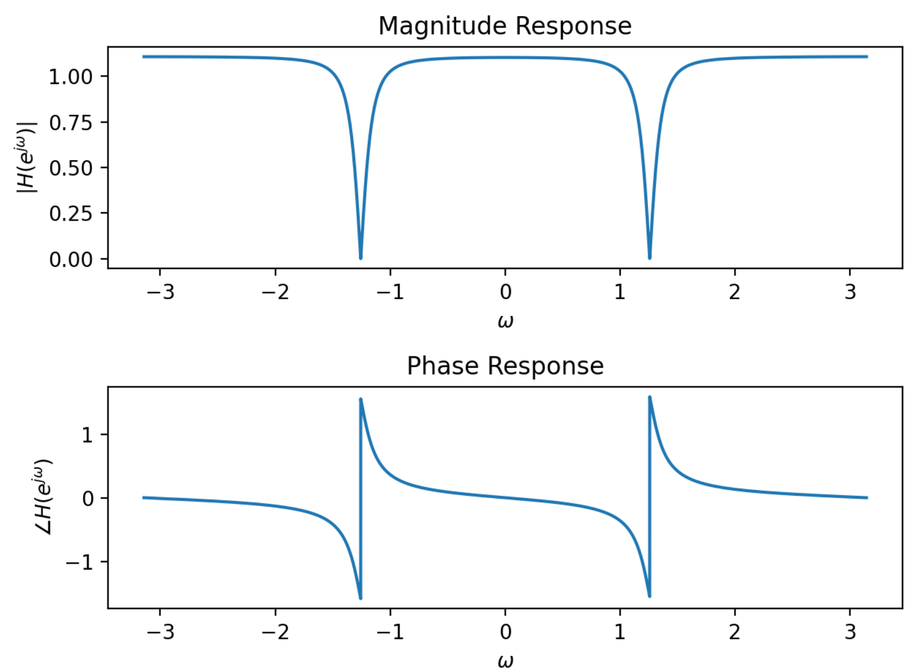
Matlab Problem 2: Frequency Response of a Notch Filter

a) To avoid aliasing, we must satisfy the Nyquist limit: $\Omega_s = \frac{2\pi}{T_s} \geq 2\Omega_N$.

Solving for Ω_N: $\Omega_N = \frac{\pi}{T_s}$

We are given $T_s = 1\,\text{ms}$. Therefore, the Nyquist frequency is: $\Omega_N = 1000\pi$ Hz

b) Frequency Response with $\omega_0 = 2\pi/5$

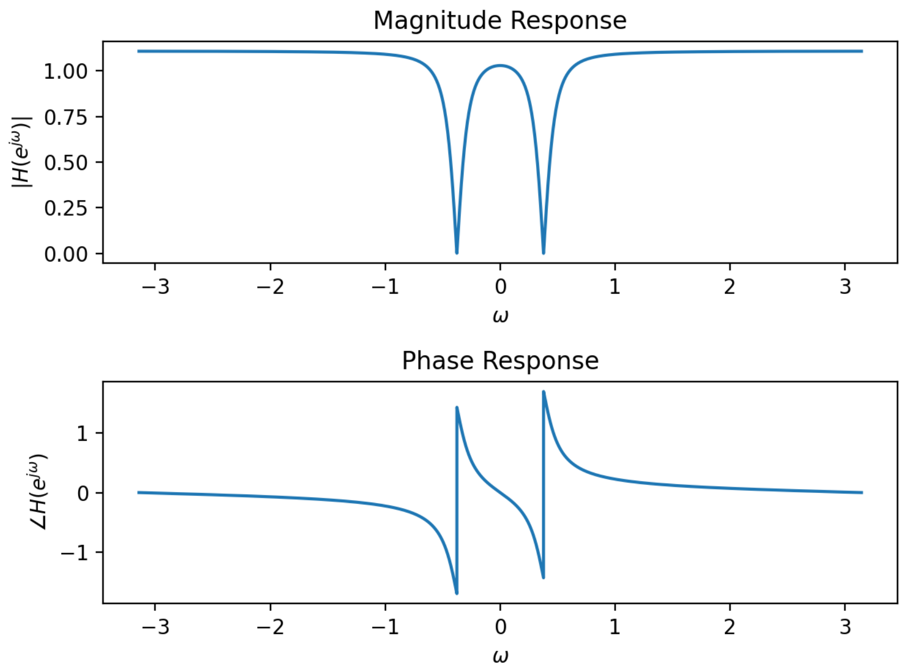


c) In order to eliminate a 60 Hz component, we must choose the discrete frequency ω_0 that corresponds to the component to be elimated.

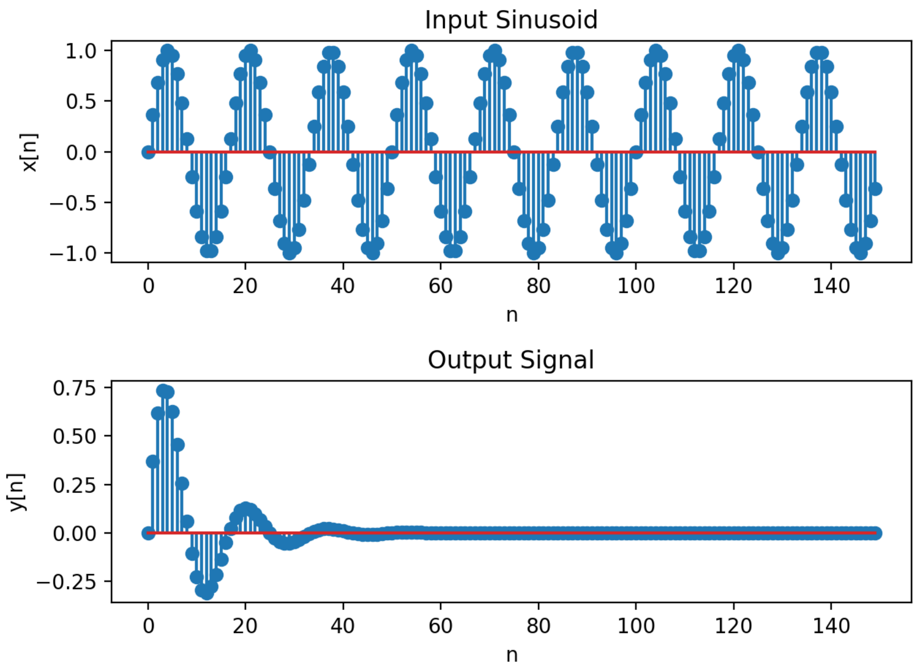
$\omega_0 = 2\pi\Omega_0 T_s = 2\pi(60\text{Hz})(1 \times 10^{-3}\text{s}) = \frac{3\pi}{25}$

Since the notch frequency is close to 0, we can predict that there will be some unintentional attenuation at lower frequencies, and the gain will be a constant of 1 at higher frequencies.

d) Frequency Response with $\omega_0 = 3\pi/25$



e) 60 Hz Sinusoid Input to Notch Filter



f) In order to determine the transcient period, I found the discrete time unit n when the point-wise ratio of the output amplitude to the input amplitude dips below 0.01, and then converted the discrete time unit to a continuous time measurement using $T_s = 1$ ms.

I found the transcient duration to be **45.0 ms**.

```python
# ESE 531: HW6 Problem 2

# Frequency Response for Difference Equations

# libraries
from scipy.signal import freqz
import numpy as np
import matplotlib.pyplot as plt

# part a

# define numerator and denominator polynomial coefficient arrays
b = [1, 0.5]
a = [1, -1.8 * np.cos(np.pi / 16), 0.81]
w, H = freqz(b, a, worN=512, whole=True)

# plot the magnitude and phase of the frequency response
mag = np.roll(np.abs(H), int(len(w)/2))
phase = np.roll(np.angle(H), int(len(w)/2))
w = np.linspace(-np.pi, np.pi, len(w))

fig, axs = plt.subplots(2)

axs[0].plot(w, mag)
axs[0].set_title('Magntiude Response')
axs[0].set_xlabel(r'$\omega$')
axs[0].set_ylabel(r'$|H(e^{j\omega})|$')

axs[1].plot(w, phase)
axs[1].set_title('Phase Response')
axs[1].set_xlabel(r'$\omega$')
axs[1].set_ylabel(r'$\angle H(e^{j\omega})$')

fig.tight_layout()
plt.show()

# part b

# redo frequency response only using upper half of unit circle
w2, H2 = freqz(b, a, worN=512, whole=False)

# plot the magnitude and phase of the frequency response
mag2 = np.abs(H2)
phase2 = np.angle(H2)

fig, axs = plt.subplots(2)

axs[0].plot(w2, mag2)
axs[0].set_title('Magntiude Response')
axs[0].set_xlabel(r'$\omega$')
axs[0].set_ylabel(r'$|H(e^{j\omega})|$')

axs[1].plot(w2, phase2)
axs[1].set_title('Phase Response')
axs[1].set_xlabel(r'$\omega$')
axs[1].set_ylabel(r'$\angle H(e^{j\omega})$')

fig.tight_layout()
plt.show()

# part d

# repeat previous parts with new difference equation
b = [0.16, -0.48, 0.48, -0.16]
a = [1, 0.13, 0.52, 0.3]

w, H = freqz(b, a, worN=512, whole=True)
```

```python
# plot the magnitude and phase of the frequency response
mag = np.roll(np.abs(H), int(len(w)/2))
phase = np.roll(np.angle(H), int(len(w)/2))
w = np.linspace(-np.pi, np.pi, len(w))

fig, axs = plt.subplots(2)

axs[0].plot(w, mag)
axs[0].set_title('Magntiude Response')
axs[0].set_xlabel(r'$\omega$')
axs[0].set_ylabel(r'$|H(e^{j\omega})|$')

axs[1].plot(w, phase)
axs[1].set_title('Phase Response')
axs[1].set_xlabel(r'$\omega$')
axs[1].set_ylabel(r'$\angle H(e^{j\omega})$')

fig.tight_layout()
plt.show()

# redo frequency response only using upper half of unit circle
w2, H2 = freqz(b, a, worN=512, whole=False)

# plot the magnitude and phase of the frequency response
mag2 = np.abs(H2)
phase2 = np.angle(H2)

fig, axs = plt.subplots(2)

axs[0].plot(w2, mag2)
axs[0].set_title('Magntiude Response')
axs[0].set_xlabel(r'$\omega$')
axs[0].set_ylabel(r'$|H(e^{j\omega})|$')

axs[1].plot(w2, phase2)
axs[1].set_title('Phase Response')
axs[1].set_xlabel(r'$\omega$')
axs[1].set_ylabel(r'$\angle H(e^{j\omega})$')

fig.tight_layout()
plt.show()

# part e

# repeat with new difference equation
b = [0.634, 0, -0.634]
a = [1, 0, -0.268]

w, H = freqz(b, a, worN=512, whole=True)

# plot the magnitude and phase of the frequency response
mag = np.roll(np.abs(H), int(len(w)/2))
phase = np.roll(np.angle(H), int(len(w)/2))
w = np.linspace(-np.pi, np.pi, len(w))

fig, axs = plt.subplots(2)

axs[0].plot(w, mag)
axs[0].set_title('Magntiude Response')
axs[0].set_xlabel(r'$\omega$')
axs[0].set_ylabel(r'$|H(e^{j\omega})|$')

axs[1].plot(w, phase)
axs[1].set_title('Phase Response')
axs[1].set_xlabel(r'$\omega$')
axs[1].set_ylabel(r'$\angle H(e^{j\omega})$')
```

```
fig.tight_layout()
plt.show()

# redo frequency response only using upper half of unit circle
w2, H2 = freqz(b, a, worN=512, whole=False)

# plot the magnitude and phase of the frequency response
mag2 = np.abs(H2)
phase2 = np.angle(H2)

fig, axs = plt.subplots(2)

axs[0].plot(w2, mag2)
axs[0].set_title('Magntiude Response')
axs[0].set_xlabel(r'$\omega$')
axs[0].set_ylabel(r'$|H(e^{j\omega})|$')

axs[1].plot(w2, phase2)
axs[1].set_title('Phase Response')
axs[1].set_xlabel(r'$\omega$')
axs[1].set_ylabel(r'$\angle H(e^{j\omega})$')

fig.tight_layout()
plt.show()
```

```python
# ESE 531: HW6 Problem 3

# Frequency Response of a Notch Filter

# libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import lfilter

# part b

# define frequency response function
def H(w, w0):
    return ((1 - np.exp(-1j * (w - w0))) * (1 - np.exp(-1j * (w + w0))) /
            ((1 - 0.9 * np.exp(-1j * (w - w0))) * (1 - 0.9 * np.exp(-1j * (w + w0)))))

# compute frequency response for w0 = 2pi/5
w = np.arange(-np.pi, np.pi, 1 / (1000 * np.pi))
freq_response = H(w, 2 * np.pi / 5)

# plot the magnitude and phase response
fig, axs = plt.subplots(2)

axs[0].plot(w, np.abs(freq_response))
axs[0].set_title("Magnitude Response")
axs[0].set_xlabel(r"$\omega$")
axs[0].set_ylabel(r"$|H(e^{j\omega})|$")

axs[1].plot(w, np.angle(freq_response))
axs[1].set_title("Phase Response")
axs[1].set_xlabel(r"$\omega$")
axs[1].set_ylabel(r"$\angle H(e^{j\omega})$")

fig.tight_layout()
plt.show()

# part d

# frequency response of notch filter with w0 = 3pi/50
w = np.arange(-np.pi, np.pi, 1 / (1000 * np.pi))
freq_response = H(w, 3 * np.pi / 25)

# plot the magnitude and phase response
fig, axs = plt.subplots(2)

axs[0].plot(w, np.abs(freq_response))
axs[0].set_title("Magnitude Response")
axs[0].set_xlabel(r"$\omega$")
axs[0].set_ylabel(r"$|H(e^{j\omega})|$")

axs[1].plot(w, np.angle(freq_response))
axs[1].set_title("Phase Response")
axs[1].set_xlabel(r"$\omega$")
axs[1].set_ylabel(r"$\angle H(e^{j\omega})$")

fig.tight_layout()
plt.show()

# part e

# generate 60-Hz sinusoid and input to filter from part d
Ts = 1e-3 # sec
n = np.arange(0, 150, 1)
f0 = 60 # Hz
fs = 1000 # Hz
sine_wave = np.sin(2 * np.pi * f0 / fs * n)
```

```python
w0 = 3 * np.pi / 25
y = lfilter([1, -2 * np.cos(w0), 1], [1, -1.8 * np.cos(w0), 0.81], sine_wave)

# plot the input and output signals
fig, axs = plt.subplots(2)

axs[0].plot(n, sine_wave)
axs[0].set_title("Input Sinusoid")
axs[0].set_xlabel("n")
axs[0].set_ylabel("x[n]")

axs[1].plot(n, y)
axs[1].set_title("Output Signal")
axs[1].set_xlabel("n")
axs[1].set_ylabel("y[n]")

fig.tight_layout()
plt.show()

# part f

# to measure the transcient response duration, we can find the index n at which the
# ouput response is less than 1% of the input amplitude, and then convert that discrete index
# to a time measurement

ratio_array = np.abs(y / sine_wave)

# binarize ratio_array at a threshold of 0.01
threshold = 0.01
binary_array = np.where(ratio_array > 0.01, 0, 1)

# ignore the first value since 0/0 yields nan
# find the first instance of 1, which corresponds to where in the array the threshold is passed
transcient_index = np.where(binary_array == 1)[0][1]

# sampling period is 1 ms
transcient_time = float(transcient_index) # ms
print(f"The transcient duration is {transcient_time} ms")
```

Noah Schwab
ESE 531

03/06/22

HW 6: All Pass and Min Phase Systems

5.24

Stable systems must contain the unit circle. Since ROC cannot contain any poles, the ROC for this system is

$$\frac{1}{2} < |z| < 3$$

We note that there are poles at $z = \frac{1}{2}$ and $z = 3$, and a zero at $z = 4$ and $z = \infty$

We can re-express the zeros as $z = \frac{1}{c^*}$, $c_1^* = \frac{1}{4}, c_1 = \frac{1}{4}$
$c_2^* = 0, c_2 = 0$

$$H(z) = H_1(z)(1 - cz^{-1}) \frac{z^{-1} - c^*}{1 - cz^{-1}} = H_{min}(z) H_{ap}(z)$$

First construct the all-pass w/ the zeros outside the unit circle

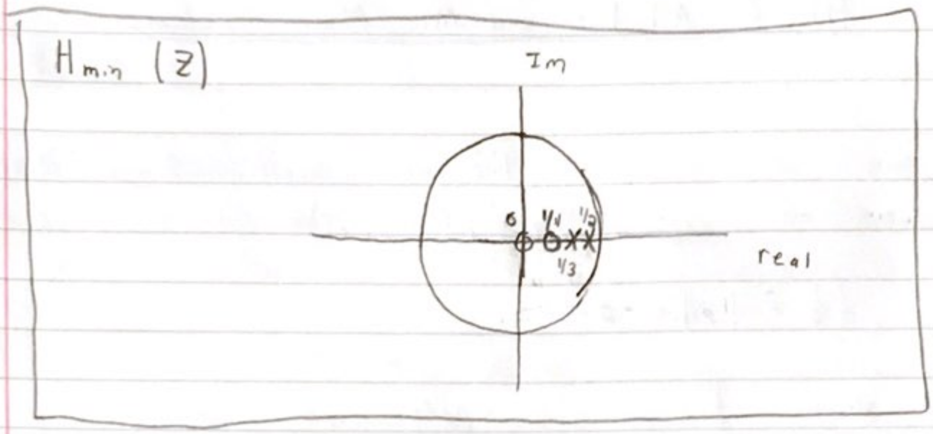
$$\boxed{H_{ap}(z) = \frac{(z^{-1} - \frac{1}{4})(z^{-1} - \frac{1}{\infty})(z^{-1} - 3)}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{\infty}z^{-1})(1 - 3z^{-1})} = \frac{(z^{-1} - \frac{1}{4})(z^{-1} - 3)\, z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - 3z^{-1})}}$$

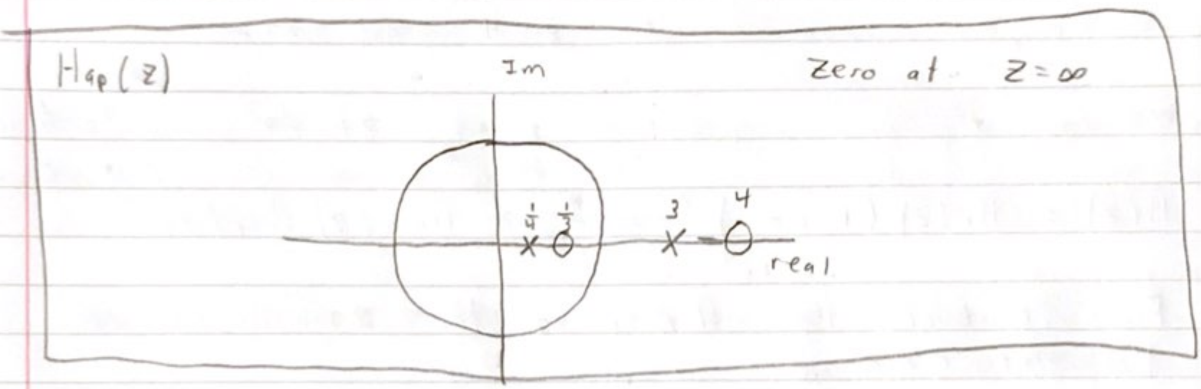
$$H_{min}(z) = \frac{H(z)}{H_{ap}(z)} = \frac{\alpha(1 - 4z^{-1})}{(1 - \frac{1}{2}z^{-1})\cancel{(1 - 3z^{-1})}} \cdot \frac{(1 - \frac{1}{4}z^{-1})\cancel{(1 - 3z^{-1})}}{(z^{-1} - \frac{1}{4})(z^{-1} - 3)\, z^{-1}}$$

$$= \frac{\alpha}{-3} \frac{(1 - 4z^{-1})}{(z^{-1} - \frac{1}{4})} \frac{(1 - \frac{1}{4}z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})\, z^{-1}}$$

$$\boxed{H_{min}(z) = \frac{4\alpha(1 - \frac{1}{4}z^{-1})}{3(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})\, z^{-1}}}$$

$$H(z) = \frac{\alpha(1 - 4z^{-1})\, z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - 3z^{-1})}$$





This decomposition is unique up to a scale factor $H_{min}(z)$ must included the indicated poles and zeros s.t they are contained within the unit circle, and $H_{ap}(z)$ must contain the indicated conjugates in order to recover $H(z)$.

5.36

$$H(z) = \frac{1 + 4z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}} = \frac{1 + 4z^{-2}}{(1 - \frac{3}{4}z^{-1})(1 + \frac{1}{2}z^{-1})}$$

poles at $z = \frac{3}{4}$ and $z = -\frac{1}{2}$

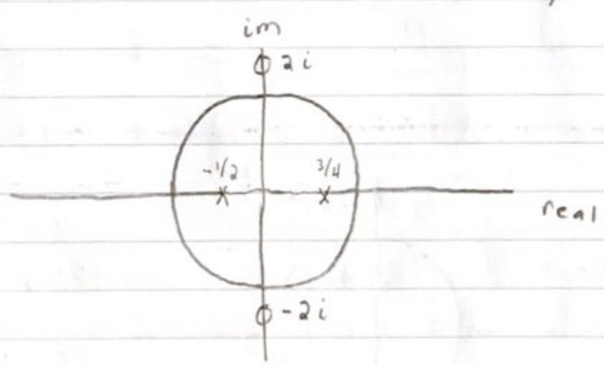
Solve for zeros:

$a = 1,\ b = 0,\ c = 4$

$$\frac{\pm\sqrt{-16}}{2} = \frac{\pm 4i}{2} = \pm 2i$$

$$H(z) = \frac{(1 + 2i z^{-1})(1 - 2i z^{-1})}{(1 - \frac{3}{4}z^{-1})(1 + \frac{1}{2}z^{-1})}$$

pole-zero diagram of $H(z)$



$H_{min}(z)$ contains all poles and zeros inside the unit circle conjugate zeros of zeros outside the unit circle.

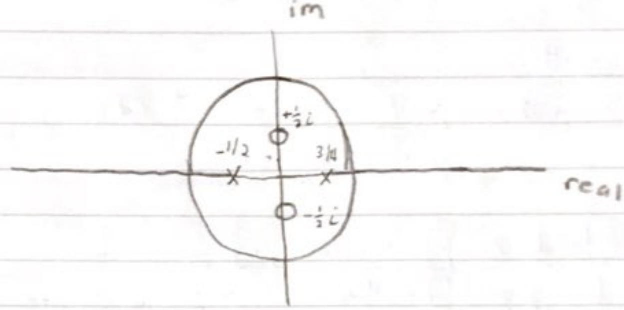
$$\boxed{H_{min}(z) = 4\frac{(1 - \frac{1}{2}i z^{-1})(1 + \frac{1}{2}i z^{-1})}{(1 - \frac{3}{4}z^{-1})(1 + \frac{1}{2}z^{-1})}}$$

factor of 4 from $H(z) = H_{min}(z)\,H_{ap}(z)$

$H_{ap}(z)$ is comprised of all zeros that lie outside the unit circle and all poles s.t. $H(z) = H_{min}(z) H_{ap}(z)$

$$\boxed{H_{ap}(z) = \frac{(z^{-1} + \frac{1}{2} i)(z^{-1} - \frac{1}{2} i)}{(1 - \frac{1}{2} i z^{-1})(1 + \frac{1}{2} i z^{-1})}}$$

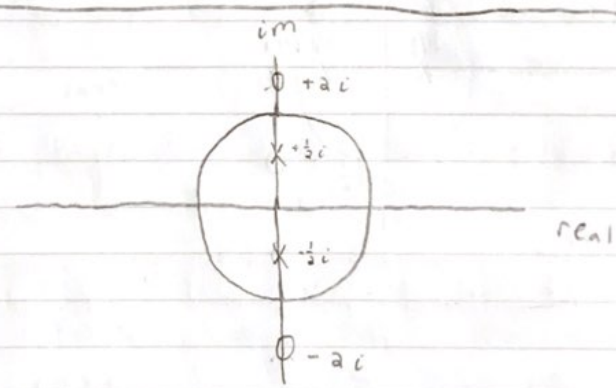
$H_{min}(z)$



$H_{min}(z)$ must be causal and stable.

ROC: $\boxed{|z| > 3/4}$

$H_{ap}(z)$



$H_{ap}(z)$ must be stable.

$\boxed{\text{ROC: } |z| > 1/2}$

missing factor of 4

$$H_{ap}(z) H_{min}(z) = \frac{(z^{-1} + \frac{1}{2} i)(z^{-1} - \frac{1}{2} i)}{\cancel{(1 - \frac{1}{2} i z^{-1})}\cancel{(1 + \frac{1}{2} i z^{-1})}} \frac{\cancel{(1 - \frac{1}{2} i z^{-1})}\cancel{(1 + \frac{1}{2} i z^{-1})}}{(1 - \frac{3}{8} z^{-1})(1 + \frac{1}{8} z^{-1})} = \frac{\frac{1}{4} (-2i z^{-1} + 1)(2i z^{-1} + 1)}{(1 - \frac{3}{8} z^{-1})(1 + \frac{1}{8} z^{-1})}$$

5.42

a) Since $H(z)$ is stable, we know that its ROC must contain the unit circle. Therefore,

$$\boxed{\text{ROC}: \quad \tfrac{1}{2} < |z| < 3}$$

There are poles at $z=0$, $z=\frac{1}{2}$, $z=-3$ and zero at $z=2$

$$H(z) = \alpha \frac{(1-2z^{-1})\, z^{-2}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}$$

We can solve for the scale factor by computing the inverse z-transform and imposing $\sum_{n=-\infty}^{\infty} (-1)^n h[n] = -1$

From the definition of the Z-transform:

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

Evaluated at $z=-1$:

$$H(-1) = \sum_{n=-\infty}^{\infty} h[n]\,(-1)^n = -1$$

We are given the condition that $H(-1) = -1$.

$$H(-1) = \frac{\alpha\,(1-2(-1)^{-1})\,(-1)^{-2}}{(1-\frac{1}{2}(-1)^{-1})(1+3(-1)^{-1})} = -1$$

$$\Rightarrow \frac{\alpha\,(3)(1)}{-\cancel{(\frac{3}{2})}\cancel{(-2)}} = \alpha = 1$$

$$\Rightarrow \boxed{H(z) = \frac{z^{-2}(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}}$$

b) A shift in time domain corresponds to a modulation in frequency.

$$g[n] = h[n+n_0] \iff G(z) = z^{n_0} H(z)$$

$$G(z) = \frac{z^{-2}(1-2z^{-1})\, z^{n_0}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}$$

Given $G(z)\big|_{z=0} = 0$ and $\lim_{z\to\infty} G(z) < \infty$, we know that the limit as $z \to \infty$ must be stable and the limit as $z \to 0$ must be 0

$$G(z) = \frac{z^{n_0-2} + 2z^{n_0-2}}{1 + \frac{5}{2}z^{-1} - \frac{3}{2}z^{-2}}$$

In order for the conditions to be met, the largest power in the numerator must be less than or equal to that of the den st as $z \to \infty$, the limit takes on a convergent value according to the leading coefficients. Therefore, $n_0 \leq 2$. Additionally, $G(z)|_{z=0} = 0$, which means the coefficient of the smallest power must be zero in the numerator. This is achieved when $n_0 > 1$. Therefore, $\boxed{n_0 = 2}$

$$g[n] = h[n+2].$$

To solve for $g[0]$, we can analyze the power series expansion.

$$G(z) = \cdots + g[-2]z^2 + g[-1]z + g[0] + g[1]z^{-1} + g[2]z^{-2} + \cdots$$

$$G(z) = \frac{1 - 2z^{-1}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})} = \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1+3z^{-1}}$$

$$A_1 = \left.\frac{1-2z^{-1}}{1+3z^{-1}}\right|_{z=\frac{1}{2}} = \frac{1-4}{1+6} = \frac{-3}{7}$$

$$A_2 = \left.\frac{1-2z^{-1}}{1-\frac{1}{2}z^{-1}}\right|_{z=-3} = \frac{\frac{5}{3}}{\frac{7}{6}} = \frac{10}{7}$$

$$G(z) = \frac{-3/7}{1-\frac{1}{2}z^{-1}} + \frac{10/7}{1+3z^{-1}}$$

The ROC of $G(z)$ is the same as $H(z)$, $\frac{1}{2} < |z| < 3$.

$$g[n] = -\frac{3}{7}\left(\frac{1}{2}\right)^n u[n] - \frac{10}{7}(-3)^n u[-n-1]$$

$$\boxed{g[0] = -\frac{3}{7}}$$

c) We can use two properties to solve for $F(z)$: Time Reversal and Convolution of a Seauence.

$x[-n] \leftrightarrow X(\frac{1}{z})$ $\qquad$ $x_1[n] * x_2[n] \leftrightarrow X_1(z)\,X_2(z)$

Therefore,

$$F(z) = H(z)\,H(\tfrac{1}{z})$$

$$F(z) = \left[\frac{z^{-2}(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}\right]\left[\frac{z^{2}(1-2z)}{(1-\frac{1}{2}z)(1+3z)}\right]$$

$$= \left[\frac{1-2z^{-1}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}\right]\left[\frac{z(z^{-1}-2)}{z^{2}(z^{-1}-\frac{1}{2})(z^{-1}+3)}\right]$$

$$= \left[\frac{\cancel{(1-2z^{-1})}\,4}{\cancel{(1-\frac{1}{2}z^{-1})}(1+3z^{-1})}\right]\left[\frac{4\cancel{(-2)}\cancel{(1-\frac{1}{2}z^{-1})}}{z\cancel{(-2)}\cancel{(1-2z^{-1})}(3)(1+\frac{1}{3}z^{-1})}\right]$$

$$\boxed{F(z) = \frac{4z^{-1}}{3(1+3z^{-1})(1+\frac{1}{3}z^{-1})}}$$

Its ROC is the the intersection of the ROCs of $H(z)$ and $H(\frac{1}{z})$. The ROC of $H(z)$ is $\frac{1}{2} < |z| < 3$ and of $H(\frac{1}{z})$ is $\frac{1}{3} < |z| < 2$

Therefore, the ROC of $F(z)$ is $\boxed{\frac{1}{3} < |z| < 3}$

d) We are looking for a signal $e[n]$ w/ a z-transform $E(z)$ s.t.

$$E(z)H(z) = \frac{1}{1-z^{-1}},$$

where the right-side of the equation corresponds to the z-transform of the unit step, which has an ROC of $|z| > 1$

We know that $H(z)$ has an ROC of $\frac{1}{2} < |z| < 3$. Multiplication of z-transforms result in an ROC that contains the intersection of the two factor transforms.

Therefore, $|z| > 1$ must contain the intersection of

$\frac{1}{2} < |z| < 3$ and the ROC of $E(z)$. Therefore, the ROC of $E(z)$ must exclude $\frac{1}{2} < |z| < 1$

As a result, the ROC of $E(z)$ must be $1 < |z| < \beta$

where β is some value larger than 1.

In order for there to exist some $e[n]$ that is right-sided, the ROC must extend to ∞, meaning there is a pole at some finite z.

Solving for $E(z)$:

$$E(z) = \frac{1}{1-z^{-1}} \frac{1}{H(z)} = \frac{1}{1-z^{-1}} \frac{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}{z^{-2}(1-2z^{-1})}$$

$$E(z) = \frac{z^2(z-\frac{1}{2})(z+3)}{(z-2)(z-1)} = \frac{z^4 + \frac{5}{2}z^3 - \frac{3}{2}z^2}{z^2-3z+2}$$

$$= \frac{(z^2 + \frac{11}{2}z + 13)(z^2 - 3z + 2) - 28z - 26}{z^2-3z+2}$$

$$E(z) = z^2 + \frac{11}{2}z + 13 - \frac{28z - 26}{z^2-3z+2}$$

The leading term z^2 corresponds to a time term $\delta[n+2]$ $\left(z^2 = 1 \cdot z^2 = \delta[n+2] \right)$

Therefore $e[n]$ depends on future time steps, and cannot be causal.