ESE 531: Homework 2

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Problem solutions with figures are shown below. Work and code is shown in attachments at end of document.

3.32

a) $x[n] = \frac{n+1}{35}(\frac{-1}{2})^{n+1}u[n+1] + \frac{58}{1225}(\frac{-1}{2})^{n}u[n] + \frac{32}{25}(2)^{n}u[-n-1] - \frac{108}{49}(3)^{n}u[-n-1]$

b) $x[n] = \frac{1}{n!}u[n]$

c) $x[n] = \delta[n+2] + 2\delta[n+1] - 2(2)^{n}u[-n-1]$

3.45

a) $H(z) = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$

ROC: $|z| > \frac{3}{4}$

See attached plot.

b) $h[n] = (\frac{3}{4})^{n}u[n] - 2(\frac{3}{4})^{n-1}u[n-1]$

c) $y[n] - \frac{3}{4}y[n-1] = x[n] - 2x[n-1]$

d) The system is stable and causal.

3.48

a) ROC, $Y(z)$: $\frac{1}{2} < |z| < 2$

b) $y[n]$ is a two-sided sequence.

c) ROC, $X(z)$: $|z| > \frac{3}{4}$

d) $x[n]$ is causal.

e) $x[0] = 0$

f) See attached work for plot.

ROC, $H(z)$: $|z| < 2$

g) $h[n]$ is anticausal.

4.21

a) $T_1 \leq \frac{\pi}{\Omega_0}$

b) $T_2 \leq \frac{\pi}{\Omega_0}$

$\frac{1.5\pi}{\Omega_0} \leq T_2 \leq \frac{2\pi}{\Omega_0}$

$T_2 = \frac{3\pi}{\Omega_0}$

c) $H_s(j\Omega) = \begin{cases} T_2 & : \frac{2}{3}\Omega_0 \leq |\Omega| \leq \Omega_0 \\ 0 & : \text{elsewhere} \end{cases}$

See attached plot.

4.23

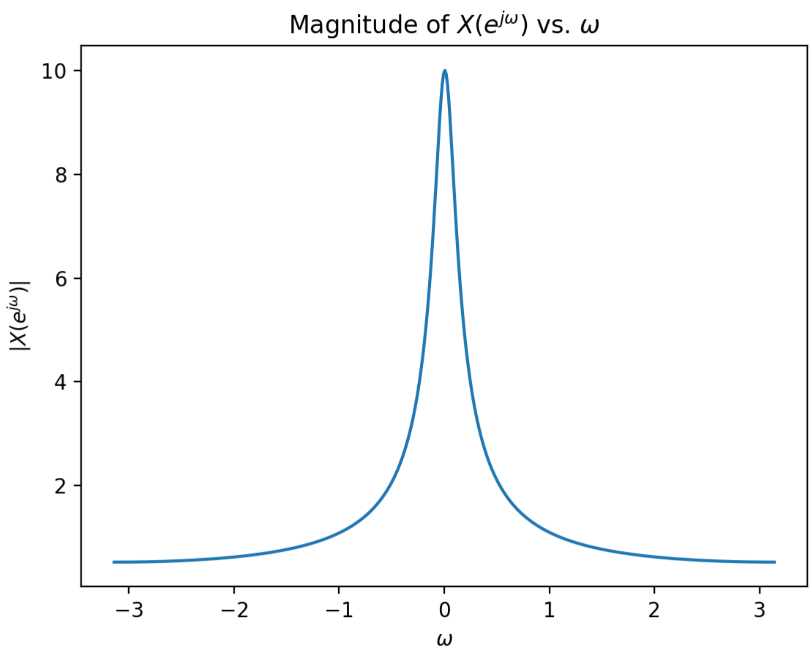
a) See attached plots.

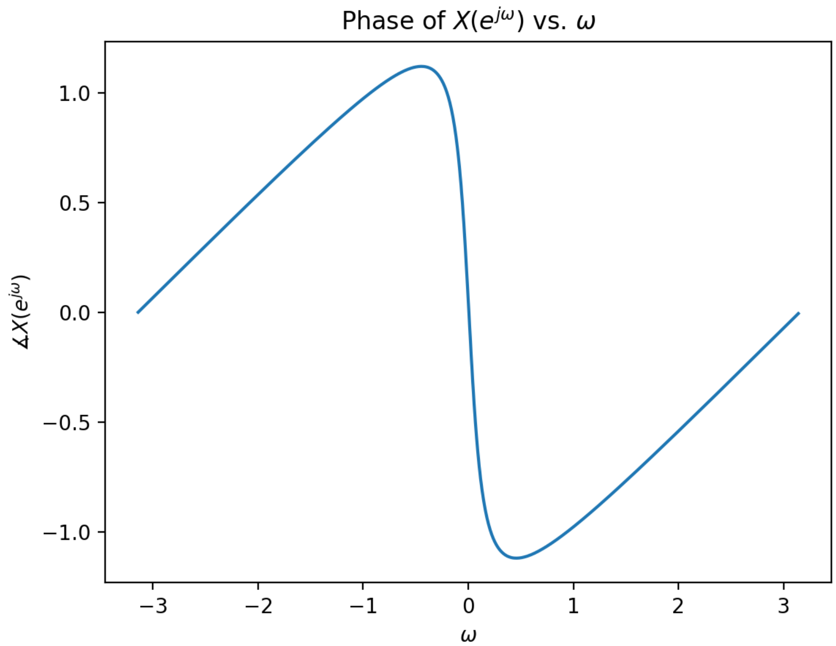
b) $|\omega_c| < \frac{\pi}{3}$

$$H_c(j\Omega) = \begin{cases} 1 & : |\Omega| < \omega_c \times 6 \times 10^3 \\ 0 & : \text{elsewhere} \end{cases}$$

Matlab Problem 1

a) As we can see below, the magnitude of the DTFT is even and the phase is odd, as a function ω. Since our original signal $x[n]$ is a real function, Table 2.1 (Symmetry Properties of the Fourier Transform) tells us that its magnitude will be even and its phase will be odd.





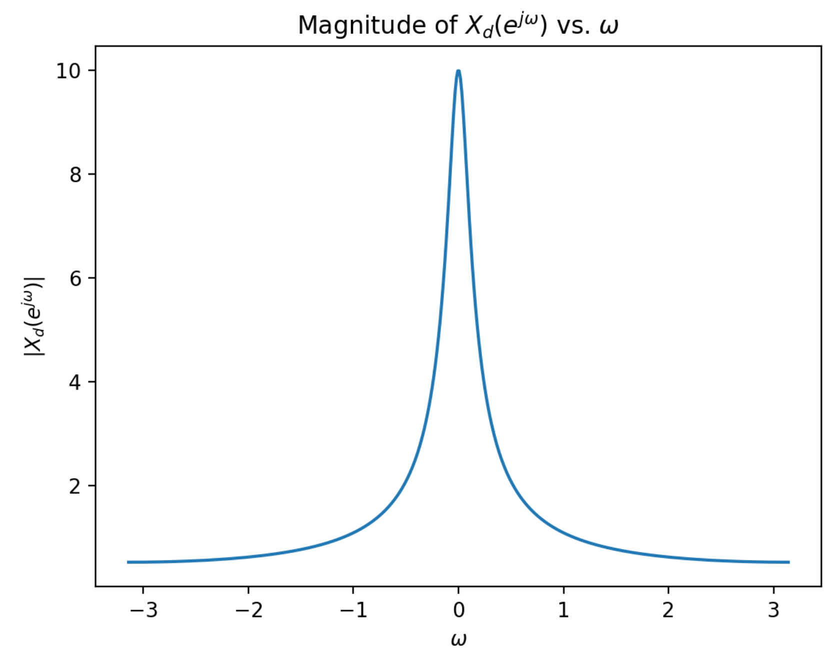
b) Using the table of Fourier Transform pairs, we know that the expression for the DTFT of the our signal is $X(e^{j\omega}) = \frac{1}{1-(0.9)e^{-j\omega}}$.

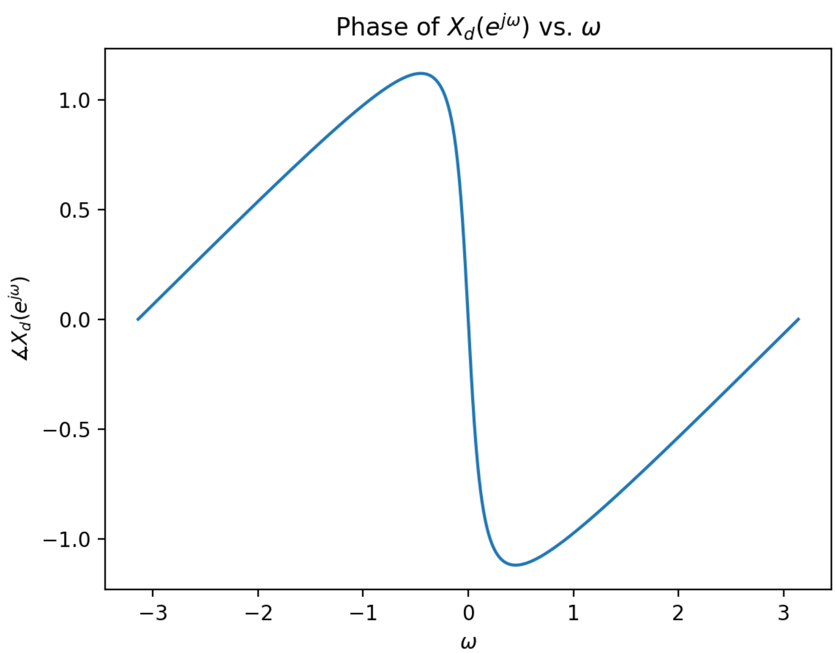
Evaluating Euler's Identity, and computing the magnitude and phase, we get the following expressions:

$|X(e^{j\omega})| = \frac{1}{\sqrt{1.81-1.8\cos\omega}}$

$\angle X(e^{j\omega}) = -\arctan\frac{0.9\sin\omega}{1-0.9\cos\omega}$

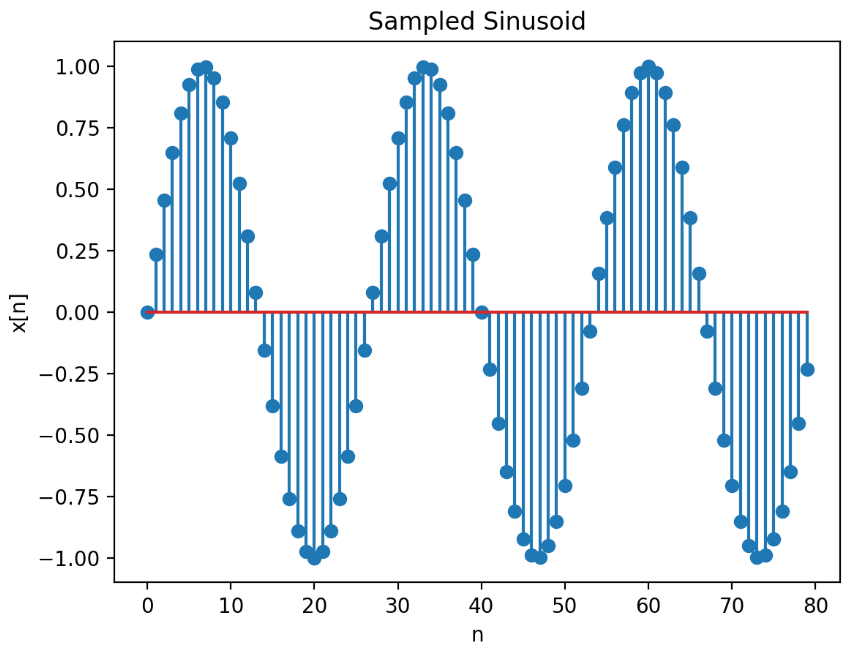
c) As we can see below, our derived formulas for the magnitude and phase of the DTFT yield the same plots.



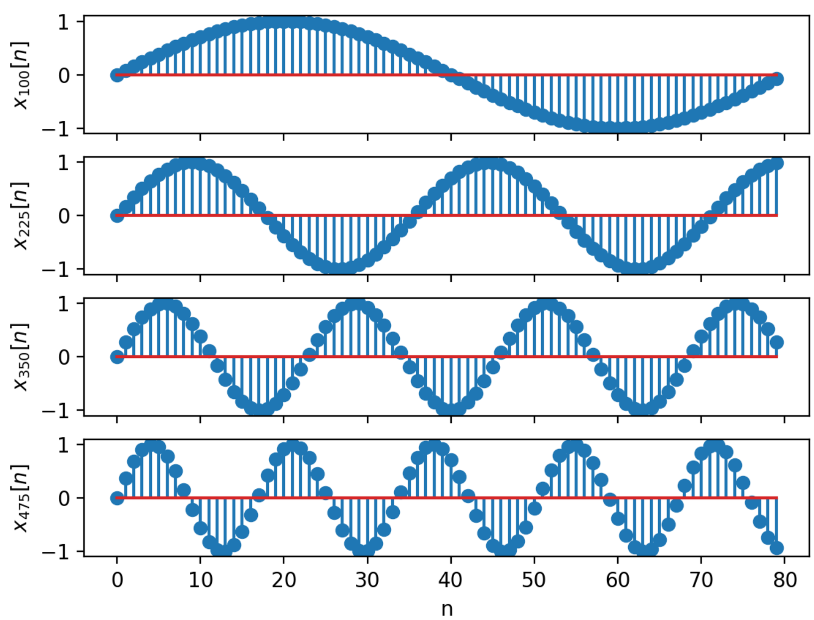


Matlab Problem 2

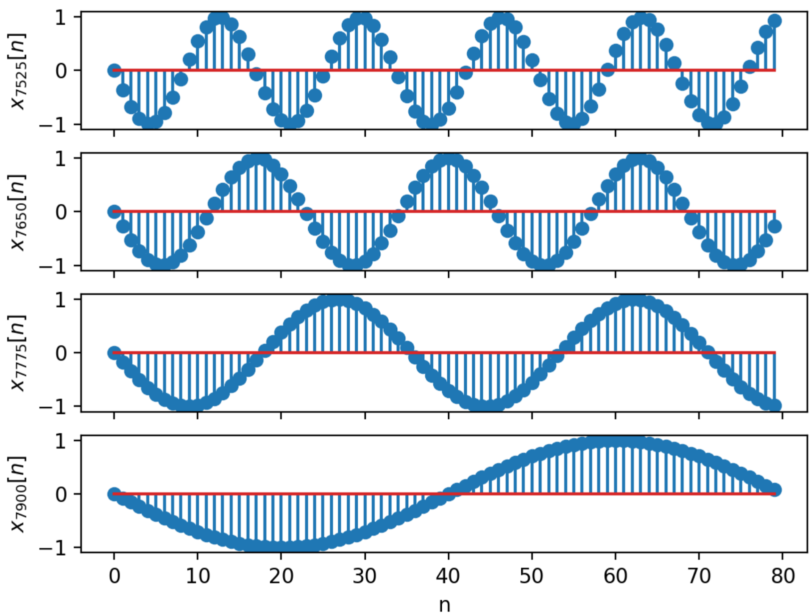
a) Sampled Sinusoid



b) Varied Frequencies



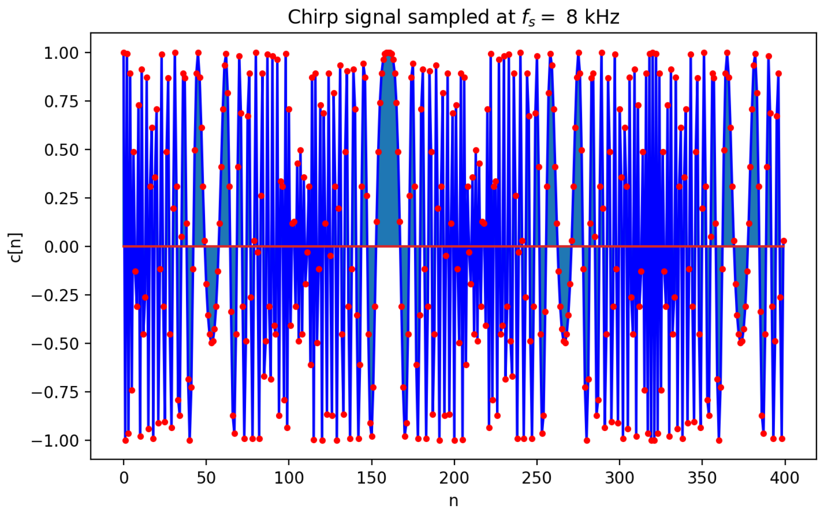
c) Even though the frequencies are increasing, it appears as though they are decreasing. This phenomenon can be attributed to aliasing. Although the sinsuoid frequencies are increasing, the sampling frequency remains constant. As a result, information is lost between samples, and the sampled sinusoid appears to be of a frequency less than what it is actually oscillating at.



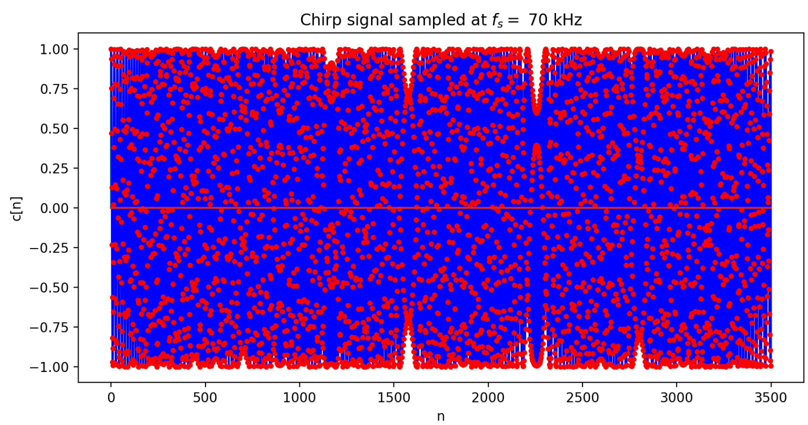
Matlab Problem 3

a) We are given the instantaneous frequency as a function of time as $f_i t = \mu t + f_1$. If the total duration is 50 ms, and $f_1 = 4$ kHz and $\mu = 600$ kHz/s, then the range of frequencies covered by the chirp is 4 kHz to 34 kHz.

b) Chirp signal sampled at 8 kHz



c) Chirp signal sampled at 70 kHz



d) As we can see from the plots above, the chirp signal sampled at 70 kHz includes much higher frequencies, which we can see from the numerous samples and sharp turns between adjacent samples. What this means is that it contains a wider range of frequencies as compared with the chirp signal sampled at 8 kHz. When we listen to the chirps, the one sampled at 8 kHz is a low, dull sound that doesn't vary, whereas the chirp signal sampled at 70 kHz actually sounds like a chirp and increases in pitch as time passes.

When we sample at a higher frequency, we can retain more samples in time, which allows our sampled signal to express more of the chirp frequencies included in the original signal.

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ESE 531

02/06/22

HW 2: DTFT, z-transform, Sampling

3.32

a) $$X(z) = \frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})}$$

$$= \frac{A_1}{(1+\frac{1}{2}z^{-1})^2} + \frac{A_2}{(1+\frac{1}{2}z^{-1})} + \frac{A_3}{(1-2z^{-1})} + \frac{A_4}{(1-3z^{-1})}$$

Solving for coefficents:

$$A_1 = (1+\tfrac{1}{2}z^{-1})^2 X(z)\Big|_{z=-\frac{1}{2}} = \frac{(1+\frac{1}{2}z^{-1})^2}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})}\Bigg|_{z=-\frac{1}{2}}$$

$$\boxed{A_1 = \frac{1}{35}}$$

$$A_3 = (1-2z^{-1})X(z)\Big|_{z=2} = \frac{1}{(1+\frac{1}{2}z^{-1})^2(1-3z^{-1})}\Bigg|_{z=2}$$

$$\boxed{A_3 = \frac{1}{(\frac{5}{4})^2(-\frac{1}{2})} = -\frac{32}{25}}$$

$$A_4 = (1-3z^{-1})X(z)\Big|_{z=3} = \frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})}\Bigg|_{z=3}$$

$$\boxed{A_4 = \frac{1}{(\frac{7}{6})^2(\frac{1}{3})} = \frac{108}{49}}$$

$$1 = A_1(1-2z^{-1})(1-3z^{-1}) + A_2(1+\tfrac{1}{2}z^{-1})(1-2z^{-1})(1-3z^{-1})$$

$$+ A_3(1+\tfrac{1}{2}z^{-1})^2(1-3z^{-1}) + A_4(1+\tfrac{1}{2}z^{-1})^2(1-2z^{-1})$$

$$1 = A_1 + A_2 + A_3 + A_4$$

$$A_2 = 1 - A_1 - A_3 - A_4 = 1 - \frac{1}{35} + \frac{32}{25} - \frac{108}{49}$$

$$= \frac{1225}{1225} - \frac{35}{1225} + \frac{1568}{1225} - \frac{2700}{1225}$$

$$\boxed{A_2 = \frac{58}{1225}}$$

$\Rightarrow$

$$X(z) = \frac{1/35}{(1+\frac{1}{2}z^{-1})^2} + \frac{58/1225}{(1+\frac{1}{2}z^{-1})} - \frac{32/25}{(1-2z^{-1})} + \frac{108/49}{(1-3z^{-1})}$$

Since we know $x[n]$ is stable, its ROC must contain the unit circle. With poles at $z = -\frac{1}{2}, 2, 3,$ the ROC must be $\frac{1}{2} < |z| < 2$. Therefore,

$$\boxed{x[n] = \frac{n+1}{35}\left(-\frac{1}{2}\right)^{n+1} u[n+1] + \frac{58}{1225}\left(-\frac{1}{2}\right)^n u[n] + \frac{32}{25}(2)^n u[-n-1] - \frac{108}{49}(3)^n u[-n-1]}$$

b) $X(z) = e^{z^{-1}}$

We can express e^z as a power series:

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$$

Therefore, $e^{z^{-1}} = \sum_{n=0}^{\infty} \frac{(z^{-1})^n}{n!} = 1 + z^{-1} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + \dots$

Since the Z-transform is given by $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$, by inspection we can find $x[n]$:

$$x[n] = \begin{cases} 0 & n < 0 \\ \frac{1}{n!} & n \geq 0 \end{cases}$$

$\Rightarrow$ $\boxed{x[n] = \frac{1}{n!} u[n]}$

c) $X(z) = \frac{z^3 - 2z}{z-2} = \frac{z^2 - 2}{1-2z^{-1}} = \frac{z^2}{1-2z^{-1}} - \frac{2}{1-2z^{-1}}$ $\quad |z| < 2$

In this case, we can treat the first term as a polynomial, and solve via long division

$$\begin{array}{r|l} & z^2 + 2z \\ \hline 1-2z^{-1} & z^2 \\ & z^2 - 2z \\ \hline & 2z \\ & 2z - 4 \\ \hline & 4 \end{array}$$

$X(z) = z^2 + 2z + \frac{4}{1-2z^{-1}} - \frac{2}{1-2z^{-1}} = z^2 + 2z + \frac{2}{1-2z^{-1}}, \ |z| < 2$

We can find the inverse using known properties of the Z-transform. Linearity and modulation give us:

$$X(z) = z^2 \implies x[n] = \delta[n+2]$$
$$X(z) = 2z \implies x[n] = 2\delta[n+1]$$
$$X(z) = \frac{2}{1-2z^{-1}} \implies x[n] = -2(2)^n u[-n-1]$$

See Table 3.1 and 3.2

Therefore,

$$\boxed{x[n] = \delta[n+2] + 2\delta[n+1] - 2(2)^n u[-n-1]}$$

3.45

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] + 2^n u[-n-1]$$

$$y[n] = 6\left(\tfrac{1}{2}\right)^n u[n] - 6\left(\tfrac{3}{4}\right)^n u[n]$$

a) First, we compute the respective Z-transforms. We can use known properties and transform pairs.

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$|z| > \tfrac{1}{2} \cap |z| < 2$$

$$\boxed{X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}, \quad \tfrac{1}{2} < |z| < |2|}$$

$$\boxed{Y(z) = \frac{6}{1-\frac{1}{2}z^{-1}} - \frac{6}{1-\frac{3}{4}z^{-1}}}$$

$$|z| > \tfrac{1}{2} \cap |z| > \tfrac{3}{4} \Rightarrow \boxed{|z| > \tfrac{3}{4}}$$

Now we can compute $H(z)$:

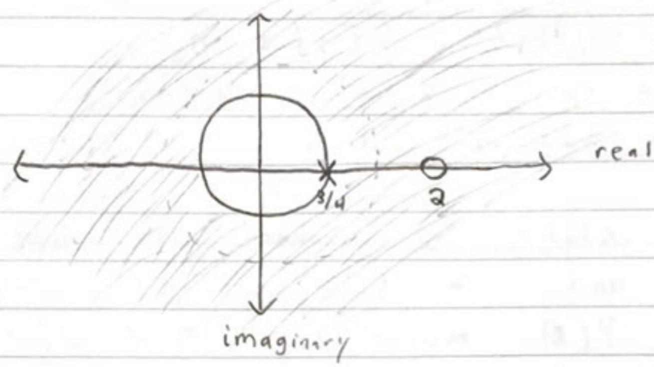
First, simplify $X(z)$ and $Y(z)$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}} = \frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$Y(z) = \frac{6}{1-\frac{1}{2}z^{-1}} - \frac{6}{1-\frac{3}{4}z^{-1}} = 6\left(\frac{-\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})}\right) = \frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\dfrac{\cancel{-\frac{3}{2}z^{-1}}}{\cancel{(1-\frac{1}{2}z^{-1})}(1-\frac{3}{4}z^{-1})}}{\dfrac{\cancel{-\frac{3}{2}z^{-1}}}{\cancel{(1-\frac{1}{2}z^{-1})}(1-2z^{-1})}} = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$$

$$\boxed{H(z) = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}} \qquad |z| > \frac{3}{4}}$$



b) To find $h[n]$, we must compute the inverse Z-transform of $H(z)$. Taking advantage of properties and transform pairs:

$$H(z) = (1-2z^{-1})\left(\frac{1}{1-\frac{3}{4}z^{-1}}\right) \qquad |z| > \frac{3}{4}$$

$$H(z) = \frac{1}{1-\frac{3}{4}z^{-1}} - 2z^{-1}\left(\frac{1}{1-\frac{3}{4}z^{-1}}\right) \qquad |z| > \frac{3}{4}$$

$$\boxed{h[n] = \left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{3}{4}\right)^{n-1} u[n-1]}$$

c) $$H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$$

$$\Rightarrow Y(z) - \frac{3}{4}z^{-1}Y(z) = X(z) - 2z^{-1}X(z)$$

Inverse Z-transform:

$$\boxed{y[n] - \frac{3}{4}y[n-1] = x[n] - 2x[n-1]}$$

d) As we can see from the pole-zero plot in part (a), the unit circle is included in the ROC. Therefore the system is stable. In part (b), we can see that $h[n] = 0$ for $n < 0$, therefore the system is also causal.

3.48

a) $y[n]$ is stable, which means its ROC must include the unit circle. Since the ROC cannot include a pole, ROC, $Y(z)$ must be $\boxed{\frac{1}{2} < |z| < 2}$

b) From part (a), ROC of $Y(z)$ is a ring in the z-plane, which means $y[n]$ is a $\boxed{\text{two-sided sequence}}$

c) Since $x[n]$ is stable, it must include the unit circle. Therefore, ROC, $X(z)$ is $\boxed{|z| > \frac{3}{4}}$

d) Since the ROC extends outward, $x[n]$ is right-sided, which means that it is $\boxed{\text{causal}}$.

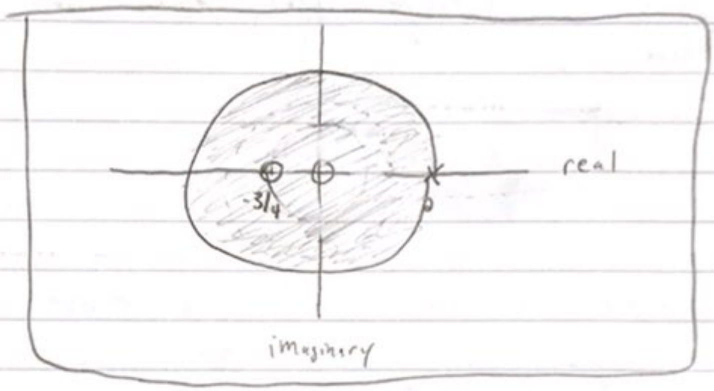
e) $$\lim_{z\to\infty} X(z) = x[0] \qquad [3.57]$$

$$\lim_{z\to\infty} \frac{(1-\frac{1}{4}z^{-1})}{(1+\frac{3}{4}z^{-1})(1-\frac{1}{2}z^{-1})} = \lim_{z\to\infty} \frac{-z^{-1}}{-z^{-2}} = 0$$

$$\boxed{x[0] = 0}$$

f) $$H(z) = \frac{Y(z)}{X(z)} = \frac{\dfrac{A\cancel{\left(1-\frac{1}{2}z^{-1}\right)}z^{-1}}{\cancel{\left(1-\frac{1}{2}z^{-1}\right)}\left(1-2z^{-1}\right)}}{\dfrac{B\cancel{\left(1-\frac{1}{2}z^{-1}\right)}}{\left(1+\frac{3}{4}z^{-1}\right)\cancel{\left(1-\frac{1}{2}z^{-1}\right)}}} = \frac{Az^{-1}\left(1+\frac{3}{4}z^{-1}\right)}{B\left(1-2z^{-1}\right)}$$

$H(z)$ has zeros at $z = 0, -3/4$ and poles at $z = 2$ and $z = \infty$



The ROC for $H(z)$ is $\boxed{|z| < 2}$, since its ROC must contain $\frac{3}{4} < |z| < 2$

g) The ROC of $h[n]$ extends toward zero inward, which means it is a left-side signal. Therefore, $\boxed{h[n] \text{ is anticausal}}$

4.21

a) $x_r(t) = x_c(t)$ is satisfied such that T_1 has a value that avoids aliasing. In other words, it satisfies Nyquist:

$$\frac{2\pi}{T_1} \geq 2\Omega_N$$

From the Fourier Transform of $x_c(t)$, we see that $\Omega_N = \Omega_0$. Therefore,

$$\boxed{T_1 \leq \pi/\Omega_0}$$

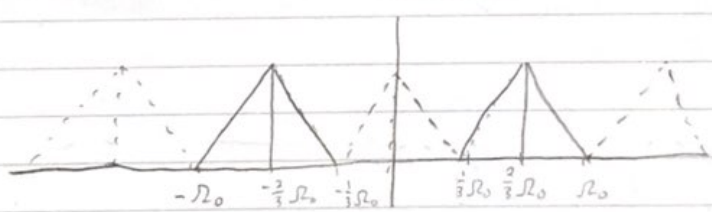
b) As demonstrated in part (a), T_2 must satisfy Nyquist:

$$T_2 \leq \pi/\Omega_0$$

However, the discontinuity in $X_c(j\Omega)$ allows for a larger value of T_2, as long as we reconstruct correctly. There will be an overlap between the triangles for the following conditions

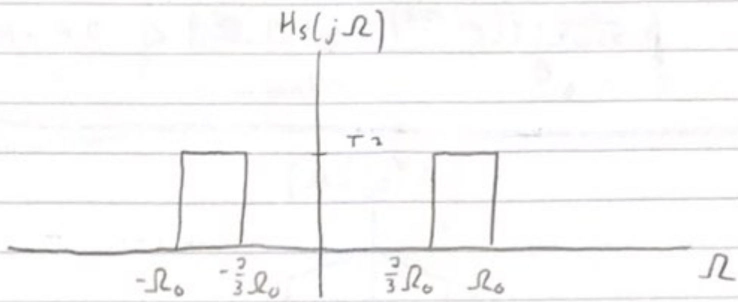
$$T_2 \leq \pi/\Omega_0, \quad \frac{1.5\pi}{\Omega_0} \leq T_2 \leq \frac{2\pi}{\Omega_0}, \quad T_2 = \frac{3\pi}{\Omega_0}$$

$$\boxed{T_2 = \frac{3\pi}{\Omega_0}}$$



To recover the original signal, we can pass the sampled spectrum to a band-pass filter

$$H_s(j\Omega) = \begin{cases} T_2, & \frac{2}{3}\Omega_0 < |\Omega| < \Omega_0 \\ 0, & \text{elsewhere} \end{cases}$$



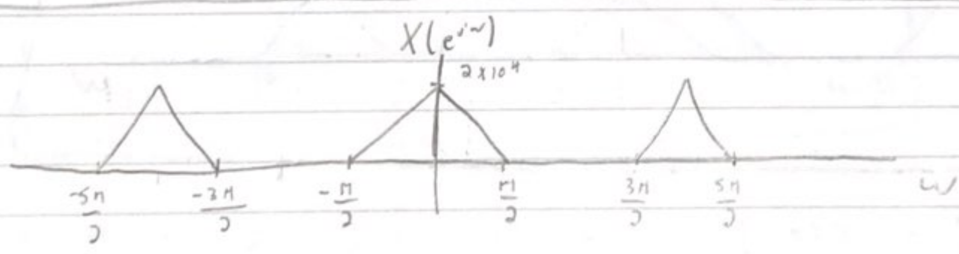
4.23

a)

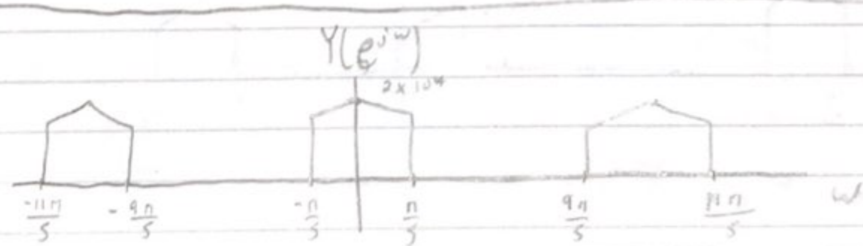
i) $\frac{1}{T_1} = \frac{1}{T_2} = 2 \times 10^4$

In this case, $\Omega_s = \frac{2\pi}{T_1} = 2\pi \times 2 \times 10^4$

$$X(e^{j\omega}) = \frac{1}{T_1} \sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T_1} - \frac{2\pi k}{T_1}\right)\right]$$

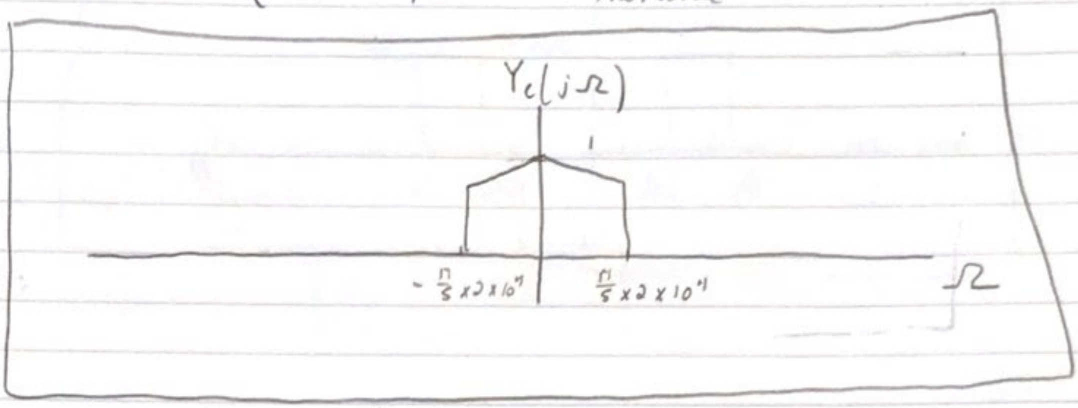


$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$



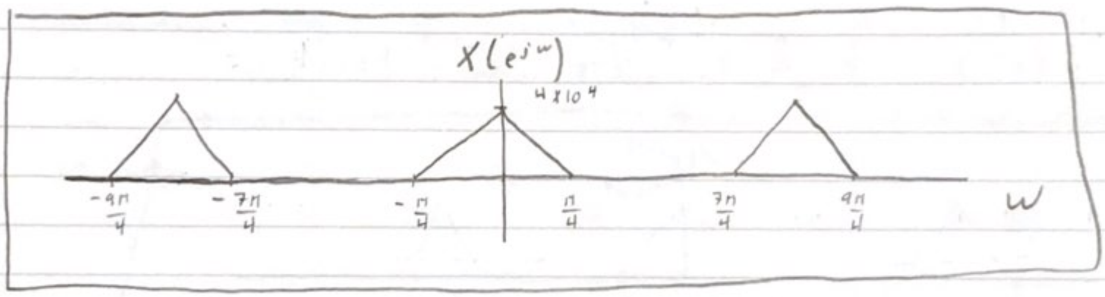
$$Y_c(j\Omega) = \begin{cases} T_2 Y(e^{j\Omega t}), & |\Omega| < \pi/T_2 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{2\times10^4} Y(e^{j\Omega t}), & |\Omega| < 2\pi\times10^4 \\ 0, & \text{otherwise} \end{cases}$$

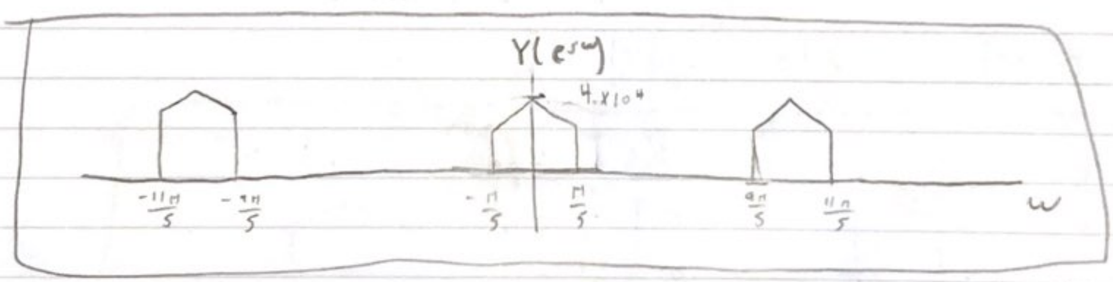


ii) $\frac{1}{T_1} = 4\times10^4$, $\frac{1}{T_2} = 10^4$

$$X(e^{j\omega}) = \frac{1}{T_1}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T_1} - \frac{2\pi k}{T_1}\right)\right]$$

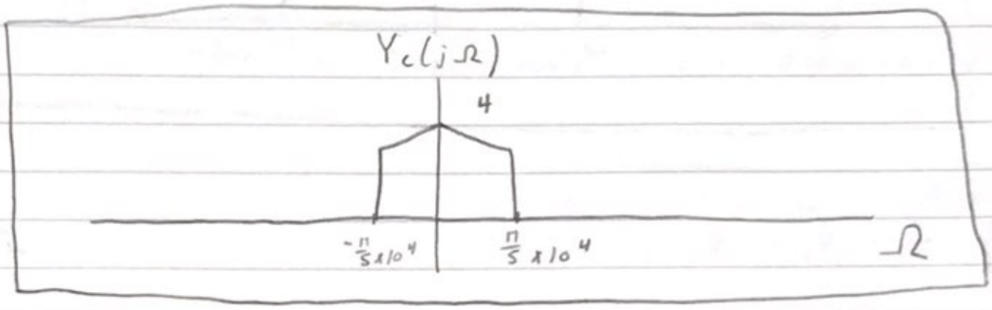


$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$



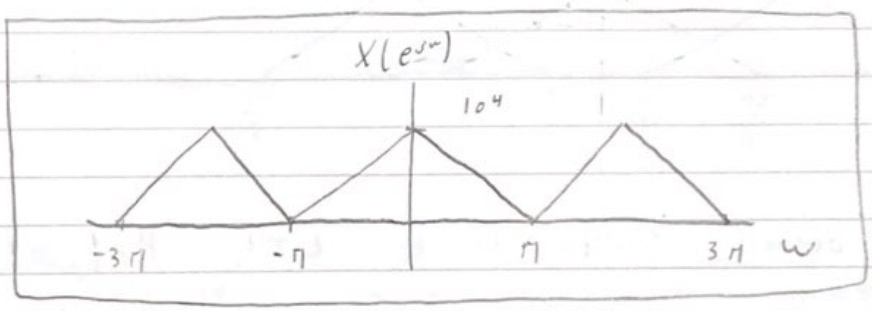
$$Y_c(j\Omega) = \begin{cases} T_2\, Y(e^{j\Omega t}), & |\Omega| < \pi/T_2 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{10^4}\, Y(e^{j\Omega t}), & |\Omega| < \pi \times 10^4 \\ 0, & \text{otherwise} \end{cases}$$

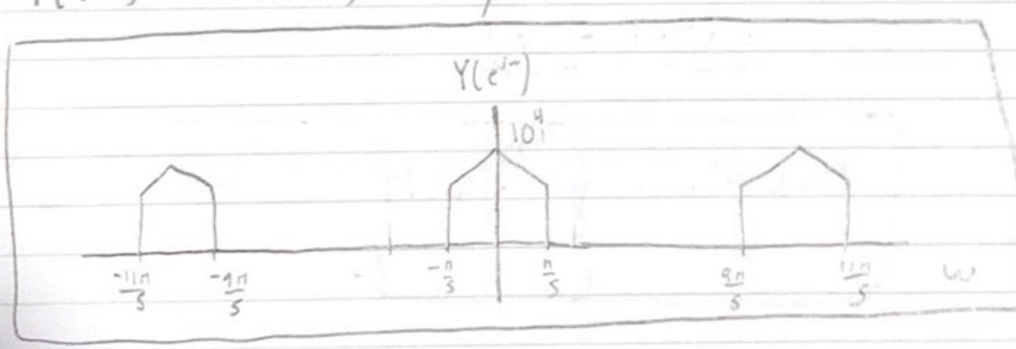


iii) $\frac{1}{T_1} = 10^4, \quad \frac{1}{T_2} = 3\times10^4$

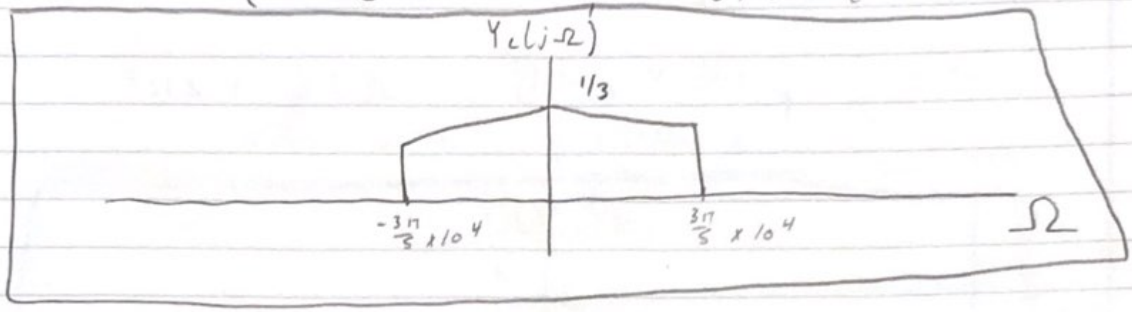
$$X(e^{j\omega}) = \frac{1}{T_1}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T_1} - \frac{2\pi k}{T_1}\right)\right]$$



$$Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})$$

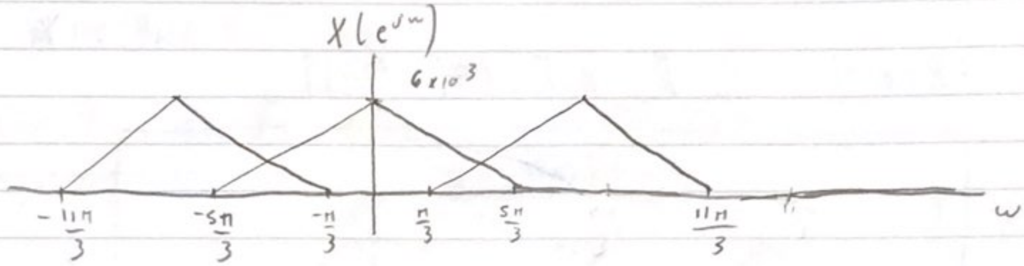


$$Y_c(j\Omega) = \begin{cases} \frac{1}{3\times10^4} Y(e^{j\Omega t}), & |\Omega| < 3\pi\times10^4 \\ 0 & \text{otherwise} \end{cases}$$



b) $\frac{1}{T_1} = \frac{1}{T_2} = 6\times10^3$

$$\Omega_s = \frac{2\pi}{T_1} = 2\pi\times6\times10^3$$



For the overall system to be LTI, $H_c(j\Omega)$ must only retain unaliased signal. Therefore, the max value of w_c is $\frac{\pi}{3}$.

$$\boxed{|w_c| < \frac{\pi}{3}}$$

$$\boxed{H_c(j\Omega) = \begin{cases} 1, & |\Omega| < w_c\times6\times10^3 \\ 0, & \text{else} \end{cases}}$$

```python
# ESE 531, HW2 Problem 2

# libraries
import numpy as np
from scipy import signal
import matplotlib.pyplot as plt

# domain of signal
N = 100

# compute the DTFT of x[n] = (0.9)^n * u[n]
x = np.array([(0.9 ** n) for n in range(0, N + 1)])
w, X = signal.freqz(x, whole=True)

# part a

# shift vectors from [0, 2pi] to [-pi, pi)
X = np.roll(X, int(len(X)/2))
w = np.linspace(-np.pi, np.pi, len(X))

# plot the magnitude
plt.plot(w, np.abs(X))
plt.title("Magnitude of $X(e^{j\omega}$) vs. $\omega$")
plt.xlabel("$\omega$")
plt.ylabel("$|X(e^{j\omega})|$")
plt.show()

# plot the phase
plt.plot(w, np.angle(X))
plt.title("Phase of $X(e^{j\omega}$) vs. $\omega$")
plt.xlabel("$\omega$")
plt.ylabel("$\measuredangle X(e^{j\omega})$")
plt.show()

# part b and c
# formula for magnitude and phase of DTFT of x using tranforms pairs
X_d_mag = [1 / np.sqrt(1.81 - 1.8*np.cos(o)) for o in w]
X_d_phase = [-np.arctan(0.9 * np.sin(o) / (1 - 0.9*np.cos(o))) for o in w]

# plot the magnitude
plt.plot(w, X_d_mag)
plt.title("Magnitude of $X_d(e^{j\omega}$) vs. $\omega$")
plt.xlabel("$\omega$")
plt.ylabel("$|X_d(e^{j\omega})|$")
plt.show()

# plot the phase
plt.plot(w, X_d_phase)
plt.title("Phase of $X_d(e^{j\omega}$) vs. $\omega$")
plt.xlabel("$\omega$")
plt.ylabel("$\measuredangle X_d(e^{j\omega})$")
plt.show()
```

```python
# ESE 531, HW2 Problem 3

# libraries
import numpy as np
import matplotlib.pyplot as plt

# define constant sampling frequency
fs = 8000 # Hz

# part a
fo = 300 # Hz
T = 0.01 # sec
N =  int(fs * T)
x = np.sin([2 * np.pi * fo/fs * n for n in range(N)])

# plot x[n]
plt.stem(x)
plt.title("Sampled Sinusoid")
plt.xlabel("n")
plt.ylabel("x[n]")
plt.show()

# part b

fig, axs = plt.subplots(4, 1, sharex=True)

# list of frequencies
fo_list = np.arange(100, 600, 125) # Hz
for i, f in enumerate(fo_list):
    x_f = np.sin([2 * np.pi * f/fs * n for n in range(N)])
    axs[i].stem(x_f)
    axs[i].set_ylabel(f'$x_{({f})}[n]$')

axs[3].set_xlabel('n')
plt.show()

# part c

fig2, axs2 = plt.subplots(4, 1, sharex=True)

# list of frequencies
fo_list2 = np.arange(7525, 8025, 125) # Hz
for i, f in enumerate(fo_list2):
    x_f = np.sin([2 * np.pi * f/fs * n for n in range(N)])
    axs2[i].stem(x_f)
    axs2[i].set_ylabel(f'$x_{({f})}[n]$')

axs2[3].set_xlabel('n')
plt.show()

# part d

fig3, axs3 = plt.subplots(4, 1, sharex=True)

# list of frequencies
fo_list3 = np.arange(32100, 32600, 125) # Hz
for i, f in enumerate(fo_list3):
    x_f = np.sin([2 * np.pi * f/fs * n for n in range(N)])
    axs3[i].stem(x_f)
    axs3[i].set_ylabel(f'$x_{({f})}[n]$')

axs3[3].set_xlabel('n')
plt.show()
```

```python
# ESE 531, HW2 Problem 4

# libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy.io.wavfile import write

# part a

# parameters
f1 = 4000 # Hz
mu = 600000 # Hz/sec
T = 0.05 # sec

# chirp signal
c_t = np.cos([np.pi * mu * t**2 + 2 * np.pi * f1 * t for t in np.arange(0, 0.05, 0.001)])

# part b

# sampling frequency
fs = 8000 # Hz
N = int(fs * T)

# sampled chirp signal
c_s = np.cos([np.pi * mu * (n/fs)**2 + 2 * np.pi * f1 * n/fs for n in range(N)])

# plot the continuous and sampled signal together
plt.plot(c_s, 'b-')
plt.stem(c_s, markerfmt='r.', linefmt=None)
plt.title("Chirp signal sampled at $f_s =$ 8 kHz")
plt.xlabel("n")
plt.ylabel("c[n]")
plt.show()

# write the chirp signal to a .wav file
write('chirp_8kHz.wav', 44100, c_s)

# part c
fs2 = 70000 # Hz
N2 = int(fs2 * T)

# sampled chirp signal
c_s2 = np.cos([np.pi * mu * (n/fs2)**2 + 2 * np.pi * f1 * n/fs2 for n in range(N2)])

# plot the continuous and sampled signal together
plt.plot(c_s2, 'b-')
plt.stem(c_s2, markerfmt='r.', linefmt=None)
plt.title("Chirp signal sampled at $f_s =$ 70 kHz")
plt.xlabel("n")
plt.ylabel("c[n]")
plt.show()

# write the chirp signal to a .wav file
write('chirp_70kHz.wav', 44100, c_s2)
```